JUAS 2016 exam

The following constant will be used in the exercises

elementary electric charge	$e = 1.6\cdot10^{-19}$ C
velocity of light	$c = 2.998\cdot10^{8}$ m/s
reduced Planck constant	$\eta = h/2\pi = 1.05\cdot10^{-34}$ Kg m^2/ s
vacuum dielectric constant	$\varepsilon_0 = 8.854\cdot10^{-12}$ F/m
electron rest energy	0.511 MeV
proton rest energy	938 MeV
classical electron radius	$r_e = 1/(4\pi\varepsilon_0)\, e^2/(mc^2) = 2.81\cdot10^{-15}$ m

$$C_q = \frac{55}{32\sqrt{3}}\frac{\eta}{mc} = 3.84\cdot10^{-13}\text{m}$$

Solve 3 of the following 4 problems.

Ex. 1:
The formula for the energy loss per turn for an electron with energy E in a storage ring with dipoles having bending radius ρ is given, in handy units, by

$$U_0(\text{keV}) = 88.46\frac{E(\text{GeV})^4}{\rho(\text{m})} \qquad (1)$$

a) Compute the energy loss per turn by an electron beam with energy 5 GeV travelling in a ring with equal dipoles having bending radius ρ = 10m.
b) Assuming constant energy loss per turn (1), how many turns it would take for the electrons to radiate all their energy. How does this value compare to the transverse damping time?
c) If all the energy losses are due to photons emitted at the critical energy ε_c, how many photons per turn would be emitted?
d) write down the analogous of formula (1) for a proton.

Ex. 2:
We want to build an electron storage ring with the design energy of 2 GeV, using N identical bending magnets with identical beam optics in the TME lattice. The target for the horizontal emittance is 2 nm-rad.

a) How many bending magnets are required?
b) How long are the bending (i.e. the length of the trajectory in the magnet), if the field strength is B = 1.4 T?
c) How do things change for a DBA lattice?

Ex. 3:
Assume we have a 3 GeV storage ring with a horizontal emittance of ε_x = 3 nm. We want to increase the brightness by building a new lattice reducing the horizontal emittance by a factor 10, to 300 pm. Assume the optics functions at the centre of the undulators are the same in both lattices and they are $\beta_x = \beta_y$ =2m and $D_x = D_y = D_x' =$

$D_y' = \alpha_x = \alpha_y = 0$. Assume that the vertical emittance is always 1% of the horizontal emittance.

a) Compute the photon beam size and divergence for an undulator with L = 2 m emitting in the fundamental wavelength at λ = 1A and for λ = 10 nm;
b) What is the percentage increase in brightness at λ = 10 nm after reducing the emittance as indicated above;
c) At what wavelengths the increase in brightness becomes negligible;

Ex. 4:

A storage ring operates at 3 GeV and has a horizontal emittance ε_x = 3 nm. We want to reduce the emittance to 300 pm by reducing the energy of the ring

a) What should be the new electron beam energy?
b) Would this be a good strategy to increase the brightness in a light source? Explain why. [Hint: What happens to the spectrum of the bending radiation? And the undulator radiation?]
c) What are the strategies currently proposed to achieve ultra low emittance lattices? Explain why.

Ex 1: solution

a)

For a 5 GeV electron beam in a banding with radius 10 m we have

$$U_0(\text{keV}) = 88.46\frac{5^4}{10} = 5.5\,\text{MeV}$$

b)

To lose all the energy it will take N turns given by

$$N = \frac{E}{U_0} = \frac{3\varepsilon_0}{e^2}\frac{E_0^4}{E^4}\rho E = \frac{3\varepsilon_0}{e^2}\frac{E_0^4}{E^3}\rho$$

i.e.

$$N = \frac{E}{U_0} = \frac{E(\text{keV})}{U_0(\text{keV})} = \frac{E(\text{keV})\rho(\text{m})}{88.46 \cdot E(\text{GeV})^4} = 905\text{ turns}$$

Check this is 500 turns

This number scales with γ^3 and inversely with ρ as the damping time.

c)

If all the energy loss are due to photons emitted at the critical energy

$$\varepsilon_c = \frac{3\eta c\gamma^3}{2\rho}$$

The number of photon emitted would be

$$N_{ph} = \frac{U_0}{\varepsilon_c} = \frac{e^2}{3\varepsilon_0}\frac{\gamma^4}{\rho}\frac{2\rho}{3\eta c\gamma^3} = \frac{2e^2\gamma}{9\eta c\varepsilon_0} = 0.21\cdot\gamma = 0.21 * 9800 = 2050\text{ photons}$$

Check numbers again here

a)

The formula for the energy loss per turn reads

$$U_0 = \frac{e^2}{3\varepsilon_0}\frac{E^4}{E_0^4}\frac{1}{\rho}$$

For electrons it is written

$$U_0(\text{keV}) = 88.46\frac{E(\text{GeV})^4}{\rho(\text{m})}$$

For protons it is written

$$U_0(\mathrm{keV}) = 7.3 \cdot 10^{-12} \frac{E(\mathrm{GeV})^4}{\rho(\mathrm{m})}$$

Ex 2: solution

a)

The TME condition states that the minimum emittance is

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x}$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\eta}{mc} = 3.84 \cdot 10^{-13} \mathrm{m}$$

For a 2 GeV beam and Jx = 1 ad an emittance of 2nm we have

$$\theta^3 = \frac{12\sqrt{15} J_x \varepsilon}{C_q \gamma^2} \quad \rightarrow \qquad \theta \sim 250 \text{ mrad}$$

Therefore we need 25 dipoles at least.

b)

If the field is B = 1.4 T, the bending angle is given by

$$\theta = \frac{BL}{B\rho} = \frac{e}{p} BL = \frac{ec}{\beta E} BL \quad \rightarrow \qquad L = \frac{\theta \beta E}{ecB} \sim 1.7 \text{ m}$$

c)

For a DBA lattice

$$\varepsilon = \frac{1}{4\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} \rightarrow \qquad \theta \sim 175 \text{ mrad}$$

Therefore 36 dipoles, i.e. 18 DBA cells

Ex 3: solution

a)

An undulator of L = 2 m at λ = 1 A will have

Photon beam size	$\sigma_{ph} = \frac{\sqrt{2L\lambda}}{2\pi}$	3.2 um
Photon beam divergence	$\sigma_{ph'} = \sqrt{\frac{\lambda}{2L}}$	5 urad

An undulator of L = 2 m at λ = 10 nm will have

Photon beam size	$\sigma_{ph} = \frac{\sqrt{2L\lambda}}{2\pi}$	32 um
Photon beam divergence	$\sigma_{ph'} = \sqrt{\frac{\lambda}{2L}}$	50 urad

b)
The brightness will be

$$B = \frac{\Phi}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

with

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2}$$

Given the optics function quoted

$$\sigma_x = \sqrt{\varepsilon_x \beta_x} \qquad = 77 \text{ um}$$
$$\sigma_{x'} = \sqrt{\frac{\varepsilon_x}{\beta_x}} \qquad = 39 \text{ urad}$$
$$\sigma_y = \sqrt{\varepsilon_y \beta_y} \qquad = 7.7 \text{ um}$$
$$\sigma_{y'} = \sqrt{\frac{\varepsilon_y}{\beta_y}} \qquad = 3.9 \text{ urad}$$

Therefore at 10 nm

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = \sqrt{77*77 + 32*32} \qquad \sim 83 \text{ um}$$
$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = \sqrt{39*39 + 50*50} \qquad \sim 63 \text{ urad}$$
$$\Sigma_y = \sqrt{\sigma_y^2 + \sigma_{ph}^2} = \sqrt{7.7*7.7 + 32*32} \qquad \sim 33 \text{ um}$$
$$\Sigma_{y'} = \sqrt{\sigma_{y'}^2 + \sigma_{ph'}^2} = \sqrt{3.9*3.9 + 50*50} \qquad \sim 50 \text{ urad}$$

With the new emittance we have

$$\sigma_x = \sqrt{\varepsilon_x \beta_x} \qquad = 25 \text{ um}$$
$$\sigma_{x'} = \sqrt{\frac{\varepsilon_x}{\beta_x}} \qquad = 13 \text{ urad}$$
$$\sigma_y = \sqrt{\varepsilon_y \beta_y} \qquad = 2.5 \text{ um}$$
$$\sigma_{y'} = \sqrt{\frac{\varepsilon_y}{\beta_y}} \qquad = 1.3 \text{ urad}$$

Therefore at 10 nm

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = \sqrt{25*25 + 32*32} \qquad \sim 41 \text{ um}$$
$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = \sqrt{13*13 + 50*50} \qquad \sim 52 \text{ urad}$$
$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = \sqrt{2.5*2.5 + 32*32} \qquad \sim 33 \text{ um}$$

$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = \sqrt{1.3*1.3 + 50*50}$$ ~ 50 urad

At 10 nm, if the emittance is reduced by a factor of 10, we have

83*63*33*50/(41*52*33*50) is reduced by 2.45

c)
For the brightness increase to be negligible the emittance must become smaller than the wavelength, such that

$$\varepsilon < \frac{\lambda}{2\pi}$$

i.e.

$\lambda > 2\pi *$ 300pm = 1.9 nm i.e. soft Xrays

Ex. 4:
a)
To reduce the emittance by a factor 10 the energy must be decreased by a factor sqrt(10), i.e. from 3 GeV to 0.95 GeV

b)
All the spectrum range is significantly reduced
Bending critical energy scales as γ^3
Undulator radiation scales as γ^2

c)
MBA lattices: see slides.