The emittance characterizes the whole beam quality, assuming linear behavior as described by second order differential equation.

It is defined within the phase space as: $\varepsilon_x = \frac{1}{\pi} \int_A dx dx'$

The measurement is based on determination of:

either profile width $\boldsymbol{\sigma_x}$ and angular width $\boldsymbol{\sigma_x'}$ at one location
or $\boldsymbol{\sigma_x}$ at different locations and linear transformations.

Different devices are used at transfer lines:

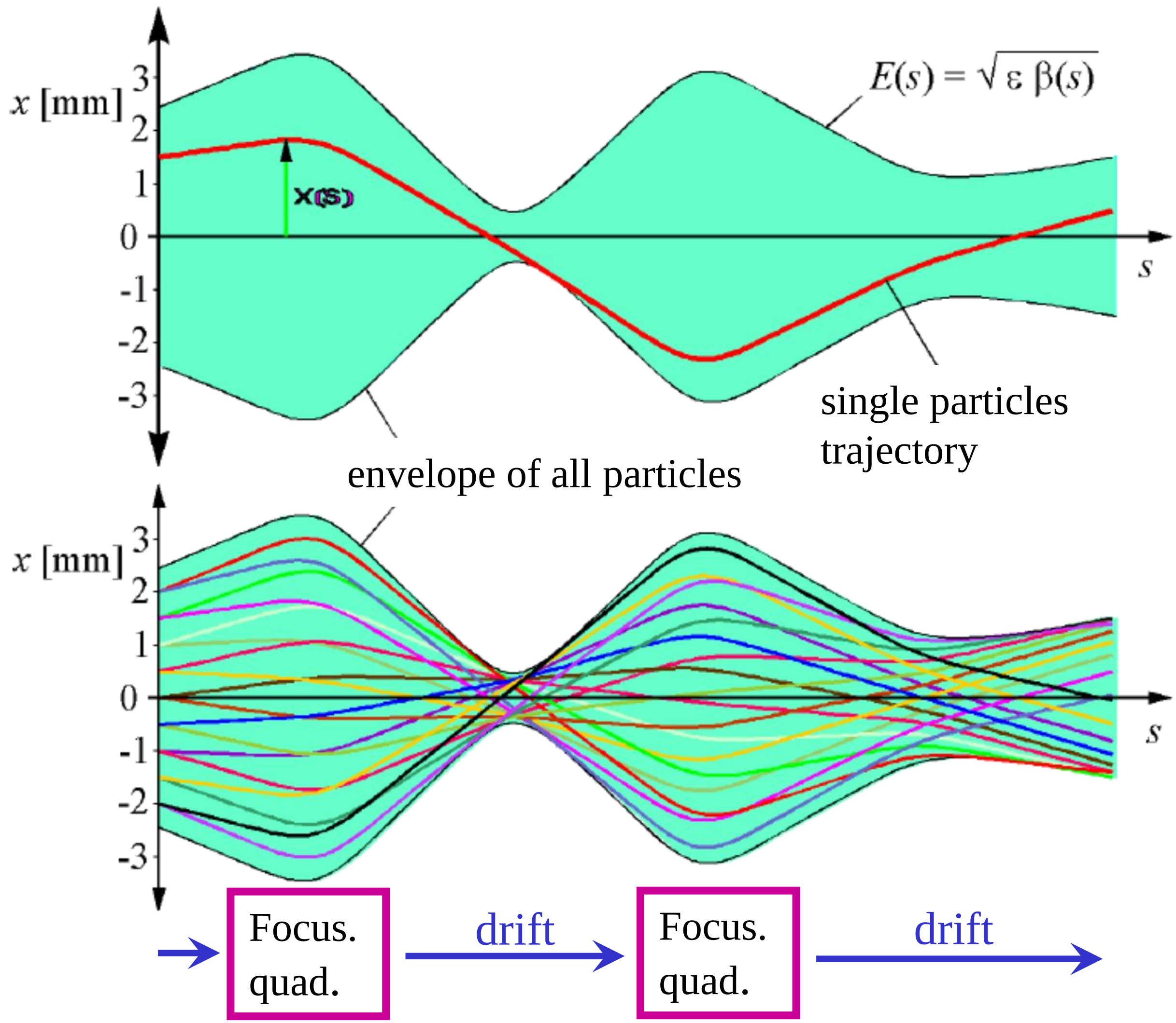
- Lower energies $\boldsymbol{E_{kin}}$ < 100 MeV/u: slit-grid device, pepper-pot (suited in case of non-linear forces).
- All beams: Quadrupole variation & 'three grid' method using linear transformations (**not** well suited in the presence of non-linear forces)

Synchrotron: lattice functions results in stability criterion

$\Rightarrow$ beam width delivers emittance: $\varepsilon_x = \frac{1}{\beta_x(s)}\left[\sigma_x^2 - \left(D(s)\frac{\Delta p}{p}\right)\right]$ and $\varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}$

Outline:

- Definition and some properties of transverse emittance
- Slit-Grid device: scanning method
- Quadrupole strength variation and position measurement
- Summary



- Single particle trajectories are forming a beam
- They have a distribution of start positions and angles

$\Rightarrow$ Characteristic quantity is the **beam envelope**

- **Goal:**

Transformation of envelope

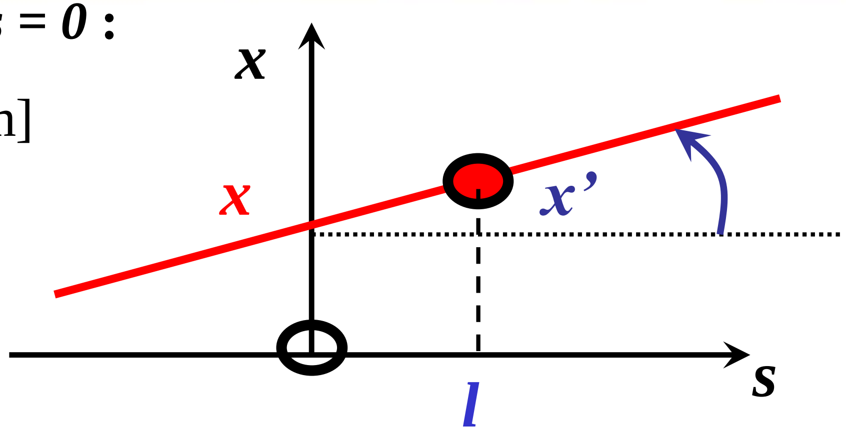
Behavior of whole ensemble

Plot: Wille

Horizontal and vertical coordinates at $s = 0$:

- x **:** Offset from reference orbit in [mm]
- x'**:** Angle of trajectory in unit [mrad]

  $x' = dx / ds$



Assumption: par-axial beams:

- x is small compared to ρ_0
- Small angle with $p_x / p_s << 1$

Longitudinal coordinate:

- Longitudinal orbit difference: $l = -v_0 \cdot (t - t_0)$ in unit [mm]
- Momentum deviation: $\delta = (p - p_0) / p_0$ sometimes in unit [mrad] = [‰]

For **continuous** beam: l has no meaning $\Rightarrow$ set $l \equiv 0$!

Reference particle: no horizontal and vertical offset $x \equiv y \equiv 0$ and $l \equiv 0$ for all s

The basic vector is 6 dimensional:

$$\vec{x}(s) = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{longitudinal deviation} \\ \text{momentum deviation} \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [‰] \end{pmatrix}$$

The transformation from a location s_0 to s_1 is given by the Transfer Matrix R

$$\vec{x}(s_1) = \mathrm{R}(s) \cdot \vec{x}(s_0) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ l_0 \\ \delta_0 \end{pmatrix}$$

Remark: At ring accelerator a comparable (i.e. a bit different) matrix is called **M**

The 2-dim sub-space (***x,x'***) can be used in case there is coupling like dispersion $R_{16} = (x \mid \delta) = 0$

Important examples are:

- Drift with length ***L***: $\mathbf{R}_{\text{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
- Horizontal **focusing** with quadrupole constant ***k*** end effective length ***l***:

$$\mathbf{R}_{\text{focus}} = \begin{pmatrix} \cos\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sin\sqrt{k}\,l \\ -\sqrt{k}\cdot\sin\sqrt{k}\,l & \cos\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{focus}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

- Horizontal **<u>de</u>-focusing** with quadrupole constant ***k*** end effective length ***l***:

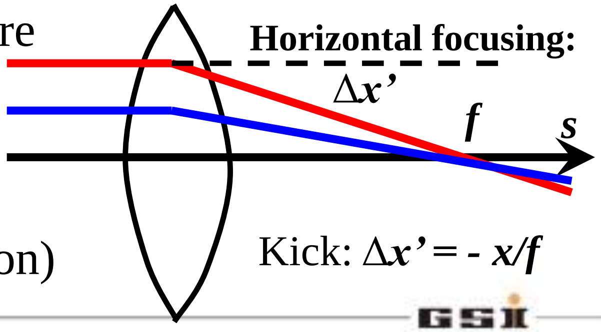
$$\mathbf{R}_{\text{de-focus}} = \begin{pmatrix} \cosh\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sinh\sqrt{k}\,l \\ \sqrt{k}\cdot\sinh\sqrt{k}\,l & \cosh\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{de-focus}} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Ideal quad.: field gradient $g = B_{pole}/a$, B_{pole} field at poles, ***a*** aperture

$\rightarrow$ quadrupole constant $k = |g| / (B\rho)_0$

Thin lens approximation: $l \rightarrow 0 \Rightarrow kl \rightarrow \text{const} \Rightarrow kl \equiv 1/f$

$\Rightarrow$ simple transfer matrix (math. proof by 1st order Taylor expansion)



Liouville's Theorem:

The phase space density can not changes with conservative e.g. linear forces.

The beam distribution at one location s_0 is described by the beam matrix $\sigma(s_0)$

This beam matrix is transported from location s_0 to s_1 via the transfer matrix

$$\sigma(s_1) = R \cdot \sigma(s_0) \cdot R^T$$

6-dim beam matrix with *decoupled* horizontal and vertical plane:

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & 0 & 0 & \sigma_{25} & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ \sigma_{15} & \sigma_{25} & 0 & 0 & \sigma_{55} & \sigma_{56} \\ \sigma_{16} & \sigma_{26} & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$

Horizontal beam matrix:

$$\sigma_{11} = \langle x^2 \rangle$$

$$\sigma_{12} = \langle x \cdot x' \rangle$$

$$\sigma_{22} = \langle x'^2 \rangle$$

Beam width for the three coordinates:

$$x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}}$$

$$y_{rms} = \sqrt{\langle y^2 \rangle} = \sqrt{\sigma_{33}}$$

$$l_{rms} = \sqrt{\langle l^2 \rangle} = \sqrt{\sigma_{55}}$$

Definition of transverse Emittance

The emittance characterizes the whole beam quality: $\varepsilon_x = \frac{1}{\pi}\int_A dx\,dx'$

Ansatz:

Beam matrix at one location: $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$ with $\vec{x} = \begin{pmatrix} x \\ x' \end{pmatrix}$

It describes a 2-dim probability distr.

The value of emittance is:

$$\varepsilon_x = \sqrt{\det \boldsymbol{\sigma}} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

For the profile and angular measurement:

$$x_\sigma = \sqrt{\sigma_{11}} = \sqrt{\varepsilon\beta} \quad \text{and}$$

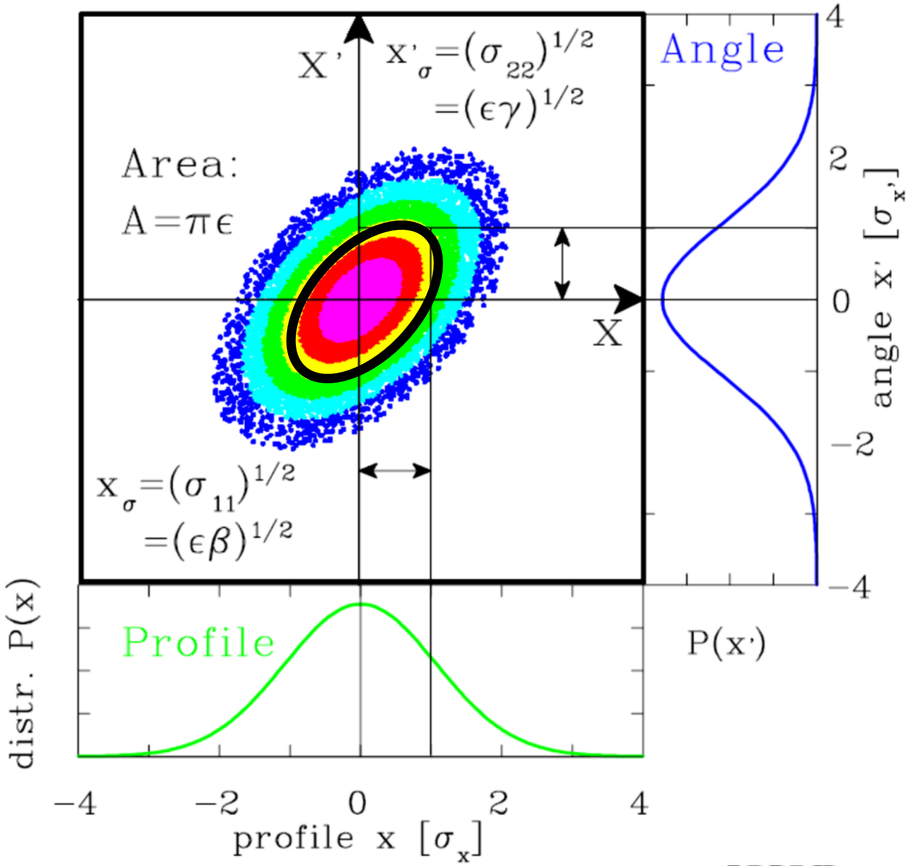
$$x'_\sigma = \sqrt{\sigma_{22}} = \sqrt{\varepsilon\gamma}$$

Geometrical interpretation:

All points $\boldsymbol{x}$ fulfilling $\boldsymbol{x^t \cdot \sigma^{-1} \cdot x = 1}$

are located on a **ellipse**

$$\boldsymbol{\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2} = \det \boldsymbol{\sigma} = \boldsymbol{\varepsilon_x^2}$$



The density function for a 2-dim Gaussian distribution is:

$$\rho(x,x') = \frac{1}{2\pi\epsilon} \exp\left[-\frac{1}{2}\ \vec{x}^T\ \sigma^{-1}\ \vec{x}\ \right]$$

$$= \frac{1}{2\pi\epsilon} \exp\left[\frac{-1}{2\det\sigma}\left(\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2\right)\right]$$

It describes an ellipse with the characteristics

profile and angle Gaussian distribution of width

$$x_\sigma \equiv \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}} \quad \text{and}$$

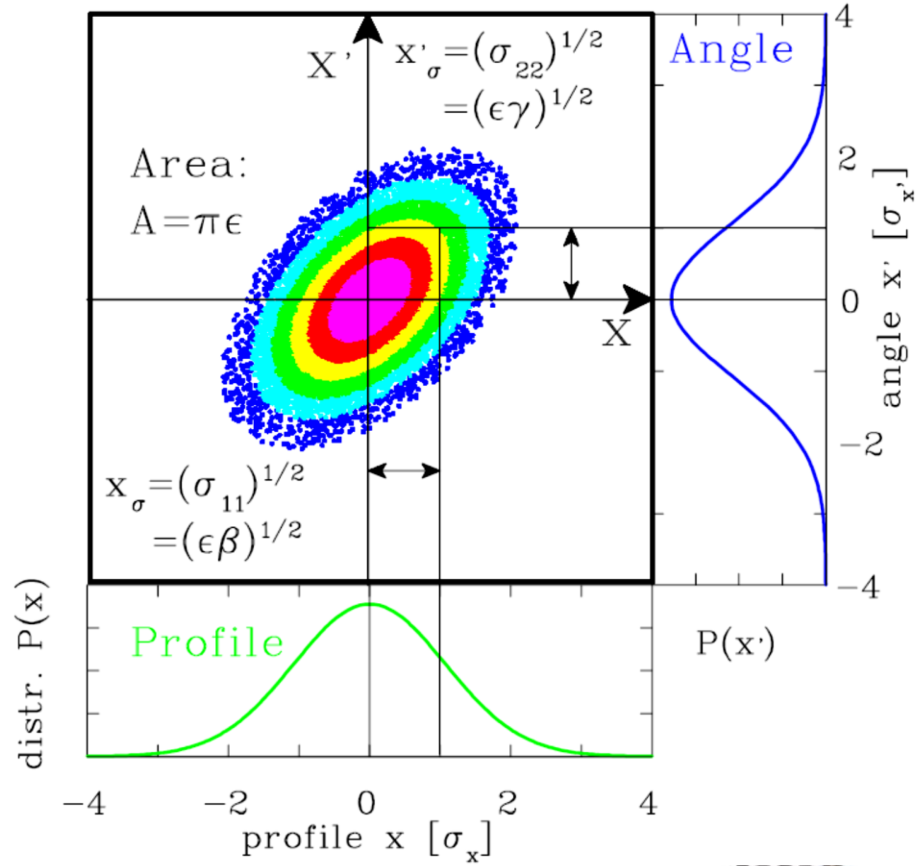
$$x'_\sigma \equiv \sqrt{\langle x'^2 \rangle} = \sqrt{\sigma_{22}}$$

and the correlation or covariance

$$\text{cov} \equiv \sqrt{\langle xx' \rangle} = \sqrt{\sigma_{12}}$$

For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ it is $\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

assuming $\det(\mathbf{A}) = ad\text{-}bc \neq 0 \Leftrightarrow$ matrix invertible



The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance

using terms of 2-dim distribution:

Variances

Covariance
i.e. correlation

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

It describes the value for 1 standard derivation

$$\langle x \rangle \equiv \mu = \frac{\iint x \cdot \rho(x,x')\, dx dx'}{\iint \rho(x,x')\, dx dx'} \qquad \langle x' \rangle \equiv \mu' = \frac{\iint x' \cdot \rho(x,x')\, dx dx'}{\iint \rho(x,x')\, dx dx'}$$

$$\langle x^n \rangle = \frac{\iint (x-\mu)^n \cdot \rho(x,x')\, dx dx'}{\iint \rho(x,x')\, dx dx'} \qquad \langle x'^n \rangle = \frac{\iint (x'-\mu')^n \cdot \rho(x,x')\, dx dx'}{\iint \rho(x,x')\, dx dx'}$$

$$\text{covariance}: \langle xx' \rangle = \frac{\iint (x-\mu)(x'-\mu') \cdot \rho(x,x')\, dx dx'}{\iint \rho(x,x')\, dx dx'}$$

For discrete distribution:

$$\langle x \rangle = \frac{\sum_{i,j} \rho(i,j) \cdot x_i x'_j}{\sum_{i,j} \rho(i,j)}$$

and correspondingly for all other moments

The Emittance for Gaussian and non-Gaussian Beams

The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance

using terms of 2-dim distribution:

It describes the value for 1 stand. derivation

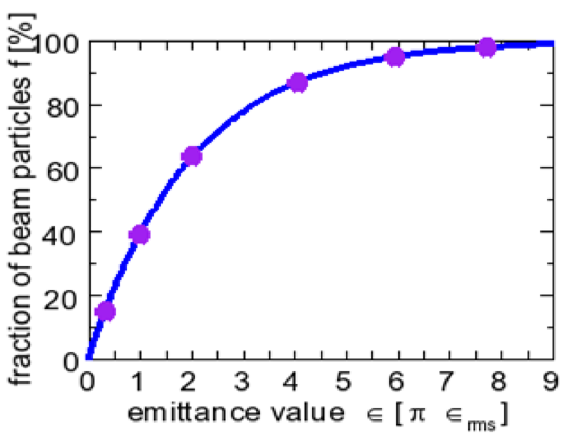
Variances

Covariance i.e. correlation

$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

For <u>Gaussian</u> beams only: ε_{rms} $\leftrightarrow$ interpreted as area containing a fraction ***f*** of ions:

$$\varepsilon(f) = -2\pi\varepsilon_{rms} \cdot \ln(1-f)$$



Emittance *ε(f)*	Fraction *f*
$1 \cdot \varepsilon_{rms}$	15 %
$\pi \cdot \varepsilon_{rms}$	39 %
$2\pi \cdot \varepsilon_{rms}$	63 %
$4\pi \cdot \varepsilon_{rms}$	86 %
$8\pi \cdot \varepsilon_{rms}$	98 %

Care:
No common definition of emittance concerning the fraction ***f***

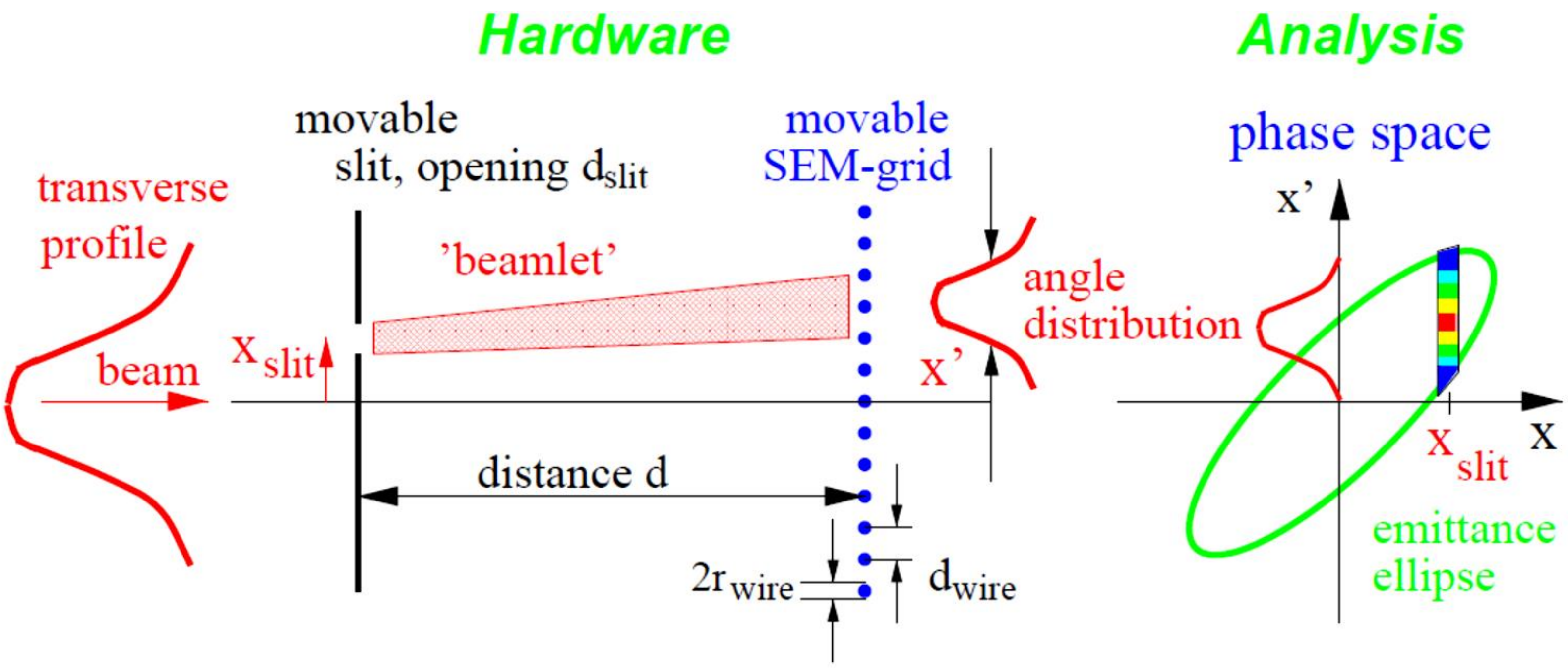
Outline:

- Definition and some properties of transverse **emittance**
- Slit-Grid device: scanning method

  scanning slit → beam position & grid → angular distribution
- Quadrupole strength variation and position measurement
- Summary

Slit-Grid: Direct determination of position and angle distribution.

Used for protons/heavy ions with E_{kin} < 100 MeV/u ⇒ range R < 1 cm.



Slit: position ***P(x)*** with typical width: 0.1 to 0.5 mm

Distance: 10 cm to 1 m (depending on beam velocity)

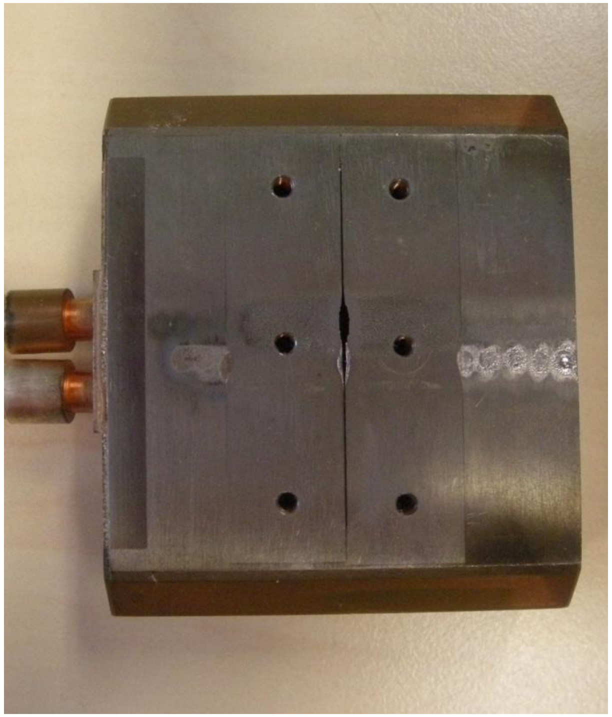
SEM-Grid: angle distribution ***P(x')***

Slit with e.g. 0.1 mm thickness

→ Transmission only from **Δx**.

Example: Slit of width 0.1 mm (defect)

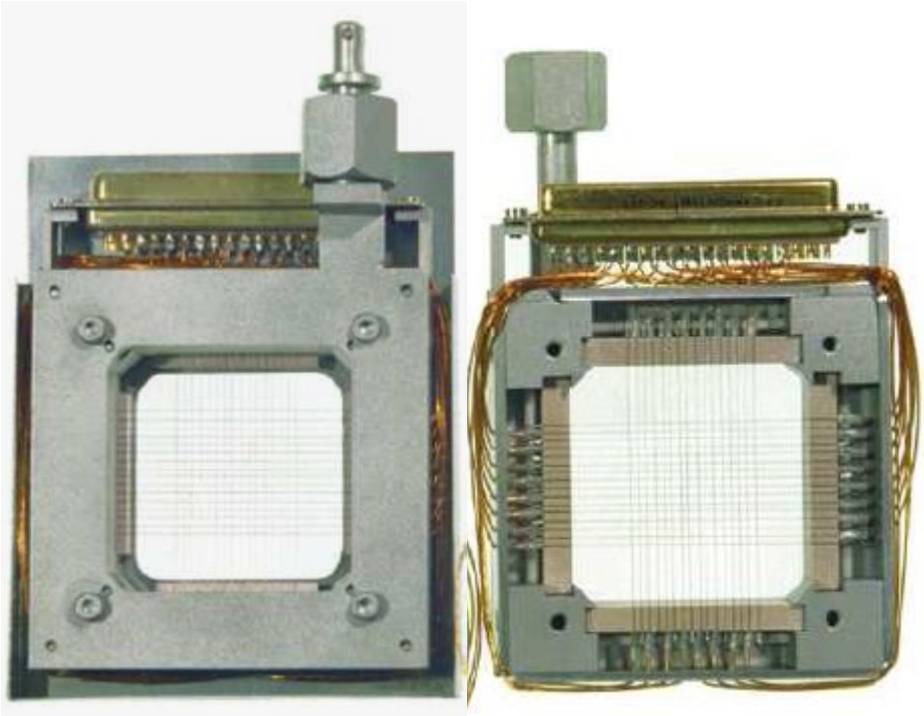
Moved by stepping motor, 0.1 mm resolution



Beam surface interaction: e⁻ emission

→ measurement of current.

Example: 15 wire spaced by 1.5 mm:



Each wire is equipped with one I/U converter

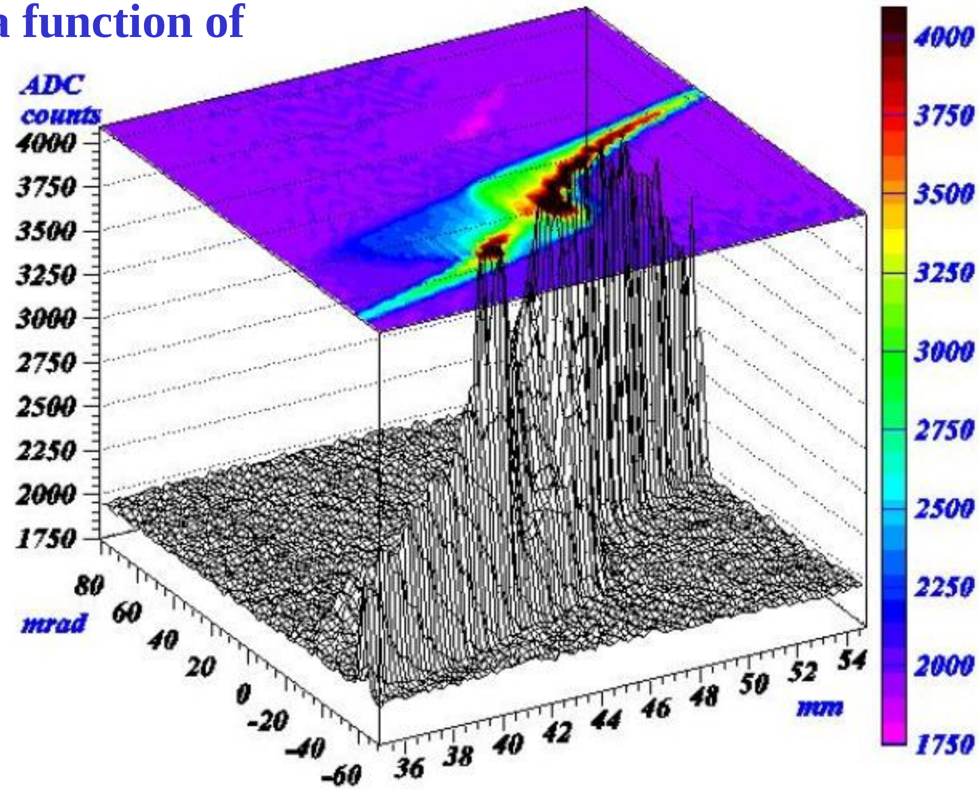
different ranges settings by R_i

→ very large dynamic range up to 10^6.

Display of Measurement Results

The distribution of the ions is depicted as a function of
- Position [mm]
- Angle [mrad]

The distribution can be visualized by
- Mountain plot
- Contour plot



Calc. of 2nd moments $\langle x^2 \rangle$, $\langle x'^2 \rangle$ & $\langle xx' \rangle$

Emittance value ε_{rms} from

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

⇒ **Problems:**

- Finite **binning** results in limited resolution
- **Background** → large influence on $\langle x^2 \rangle$, $\langle x'^2 \rangle$ and $\langle xx' \rangle$

Or fit of distribution i.e. ellipse to data

⇒ **Effective emittance only**

Beam: Ar^{4+}, 60 KeV, 15 µA

at Spiral2 Phoenix ECR source.

Plot from P. Ausset, DIPAC 2009

The Resolution of a Slit-Grid Device

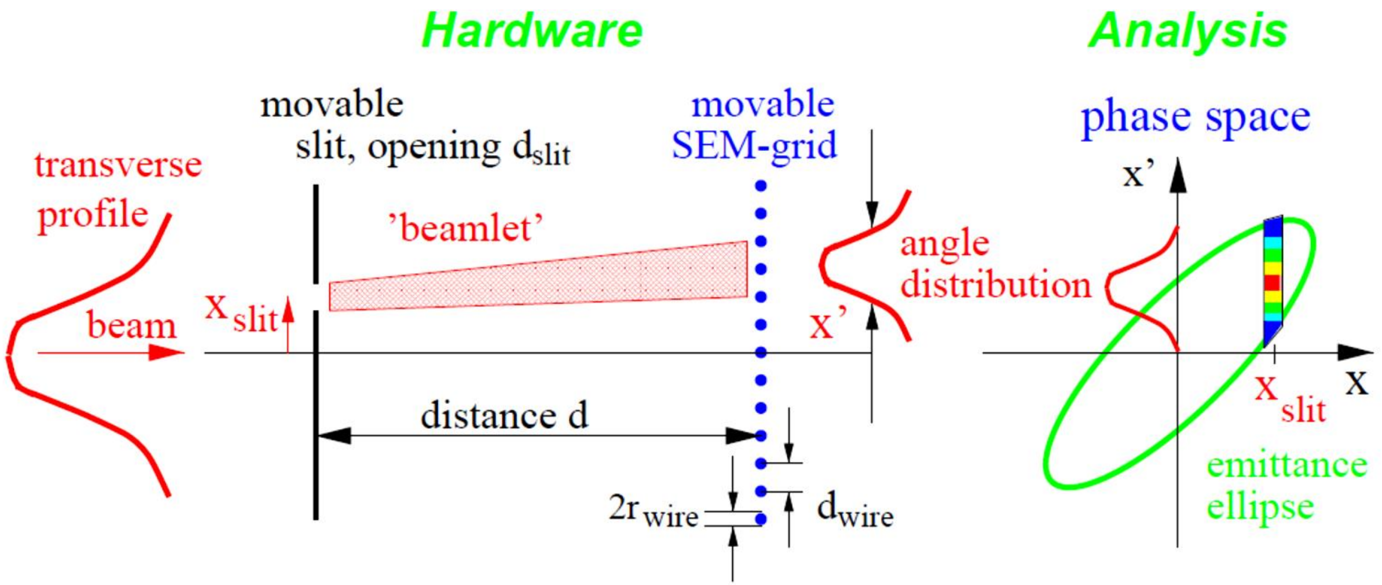
The width of the slit d_{slit} gives the resolution in space $\Delta x = d_{slit}$.

The angle resolution is $\Delta x' = (d_{wire} + 2r_{wire})/d$

$\Rightarrow$ discretization element $\Delta x \cdot \Delta x'$.

By scanning the SEM-grid the angle resolution can be improved.

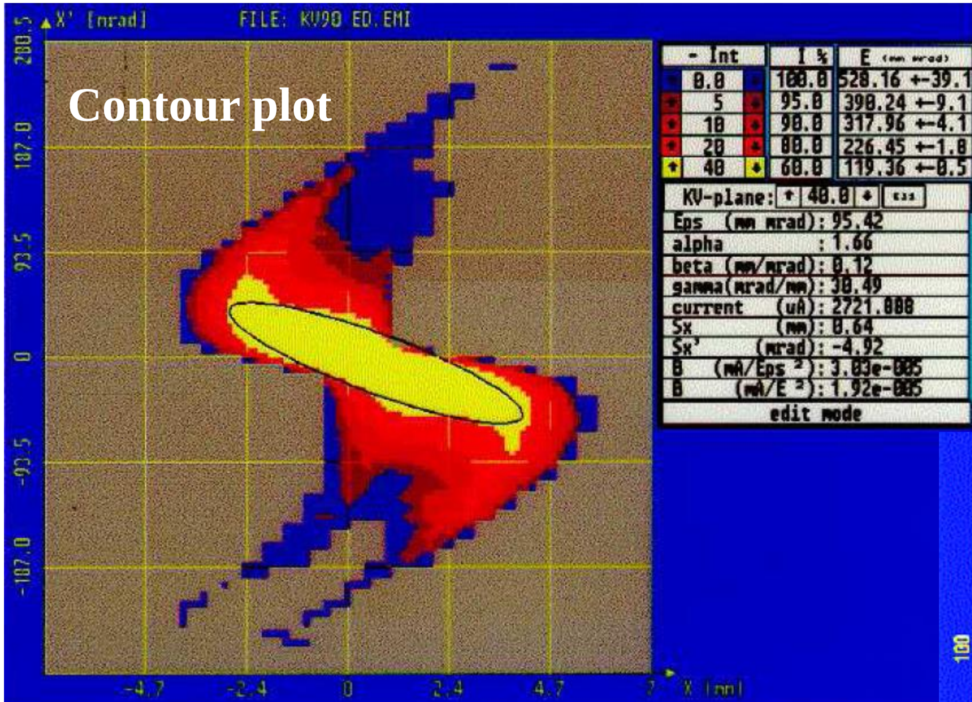
Problems for small beam sizes or parallel beams.

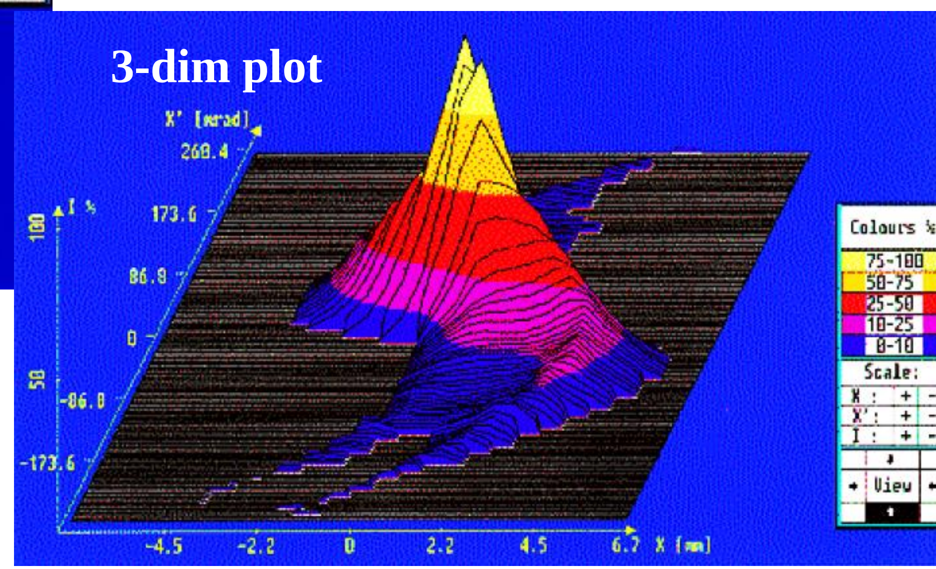


For pulsed LINACs: Only one measurement each pulse → long measuring time required.

Result for a beam behind ion source:

- here aberration in quadrupoles due to large beam size
- different evaluation and plots possible
- can monitor any distribution





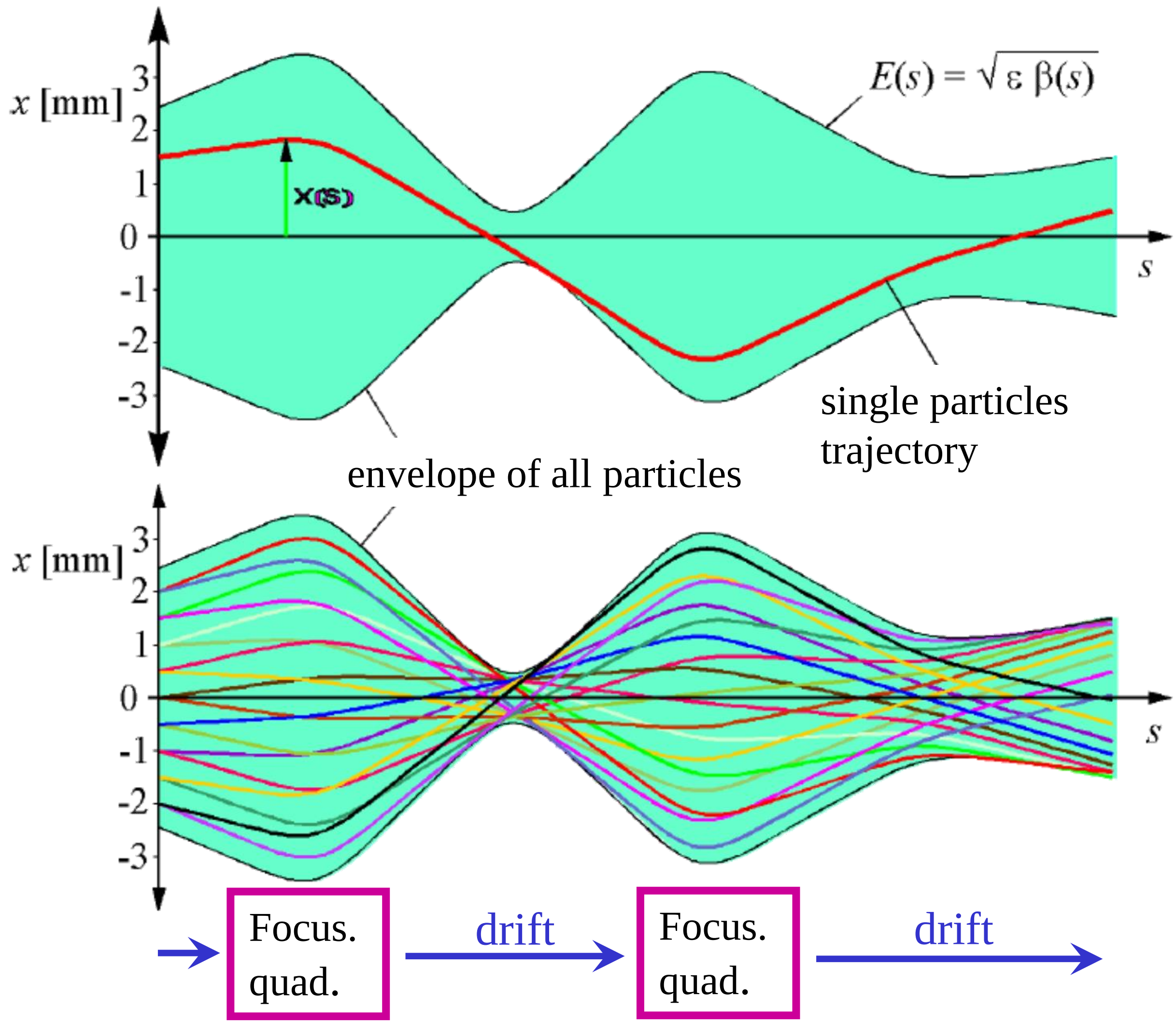
Low energy ion beam:
→ well suited for emittance showing space-charge effects or aberrations.

Outline:

- Definition and some properties of transverse emittance
- Slit-Grid device: scanning method

  scanning slit → beam position & grid → angular distribution
- **Quadrupole strength variation and position measurement**

  emittance from several profile measurement and beam optical calculation
- Summary



- Single particle trajectories are forming a beam
- They have a distribution of start positions and angles

⇒ Characteristic quantity is the **beam envelope**

- **Goal:**

Transformation of envelope

Behavior of whole ensemble

Plot: Wille

Liouville's Theorem:

The phase space density can not changes with conservative e.g. linear forces.

The beam distribution at one location s_0 is described by the beam matrix $\boldsymbol{\sigma}(s_0)$

This beam matrix is transported from location s_0 to s_1 via the transfer matrix

$$\boldsymbol{\sigma}(s_1) = \mathbf{R} \cdot \boldsymbol{\sigma}(s_0) \cdot \mathbf{R}^T$$

6-dim beam matrix with *decoupled* horizontal and vertical plane:

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & 0 & 0 & \sigma_{25} & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ \sigma_{15} & \sigma_{25} & 0 & 0 & \sigma_{55} & \sigma_{56} \\ \sigma_{16} & \sigma_{26} & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$

Horizontal beam matrix:

$$\sigma_{11} = \langle x^2 \rangle$$

$$\sigma_{12} = \langle x \cdot x' \rangle$$

$$\sigma_{22} = \langle x'^2 \rangle$$

Beam width for the three coordinates:

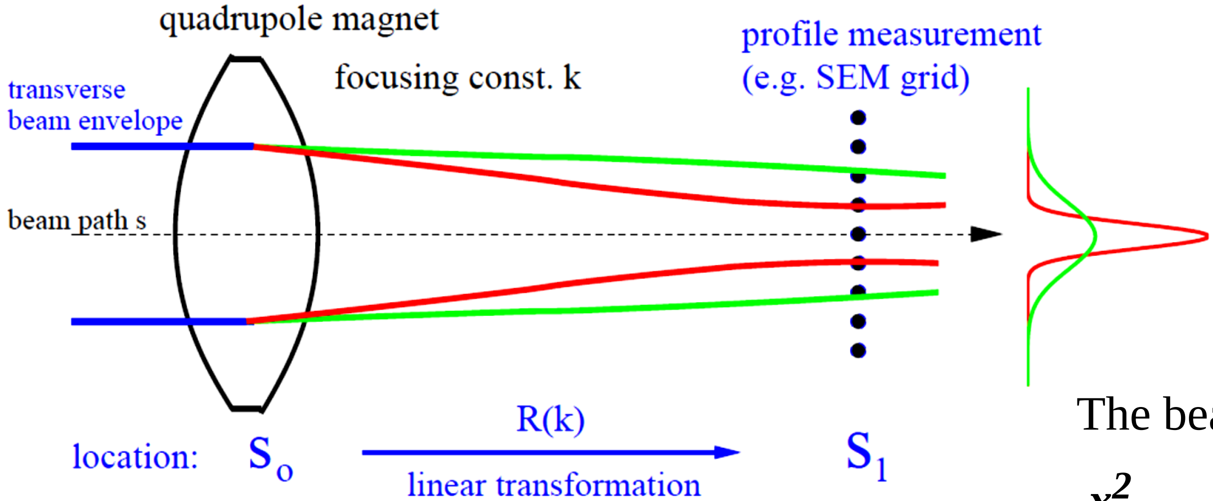
$$x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}}$$

$$y_{rms} = \sqrt{\langle y^2 \rangle} = \sqrt{\sigma_{33}}$$

$$l_{rms} = \sqrt{\langle l^2 \rangle} = \sqrt{\sigma_{55}}$$

Emittance Measurement by Quadrupole Variation

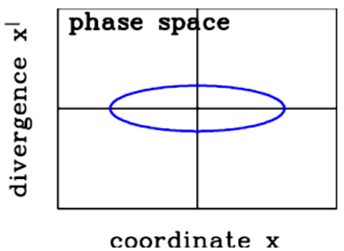
From a profile determination, the emittance can be calculated via linear transformation, if a well known and constant distribution (e.g. Gaussian) is assumed.



The beam width x_{max} and

$x^2_{max} = \sigma_{11}(1, k)$ is measured,

matrix **R(k)** describes the focusing.

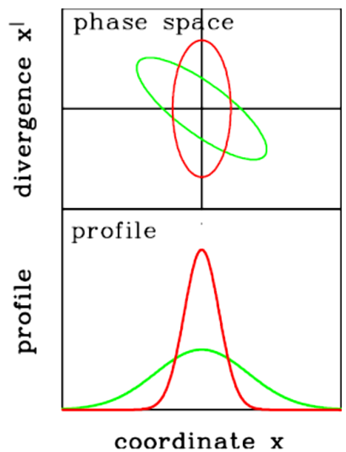


beam matrix:

(Twiss parameters)

$\sigma_{11}(0)$, $\sigma_{12}(0)$, $\sigma_{22}(0)$

to be determined



measurement:

$x^2(k) = \sigma_{11}(1,k)$

Measurement of transverse Emittance

- The beam width x_{max} at s_1 is measured, and therefore $\sigma_{11}(1, k_i) = x^2_{max}(k_i)$.
- Different focusing of the quadrupole $k_1, k_2 ... k_n$ is used: $\Rightarrow \mathbf{R_{focus}}(k_i)$, including the drift, the transfer matrix is changed $\mathbf{R}(k_i) = \mathbf{R_{drift}} \cdot \mathbf{R_{focus}}(k_i)$.
- Task: Calculation of *beam* matrix $\sigma(0)$ at entrance s_0 (size and orientation of ellipse)
- The transformations of the beam matrix are: $\sigma(1, k) = \mathbf{R}(k) \cdot \sigma(0) \cdot \mathbf{R^T}(k)$.

$\Longrightarrow$ Resulting in a redundant system of linear equations for $\sigma_{ij}(0)$:

$$\begin{aligned}
\sigma_{11}(1, k_1) &= R^2_{11}(k_1) \cdot \sigma_{11}(0) + 2R_{11}(k_1)R_{12}(k_1) \cdot \sigma_{12}(0) + R^2_{12}(k_1) \cdot \sigma_{22}(0) \quad \text{focusing } k_1 \\
&\vdots \\
\sigma_{11}(1, k_n) &= R^2_{11}(k_n) \cdot \sigma_{11}(0) + 2R_{11}(k_n)R_{12}(k_n) \cdot \sigma_{12}(0) + R^2_{12}(k_n) \cdot \sigma_{22}(0) \quad \text{focusing } k_n
\end{aligned}$$

- To learn something on possible errors, $n > 3$ settings have to be performed. A setting with a focus close to the SEM-grid should be included to do a good fit.

- *Assumptions:*
  - Only elliptical shaped emittance can be obtained.
  - No broadening of the emittance e.g. due to space-charge forces.
  - If *not* valid: A self-consistent algorithm has to be used.

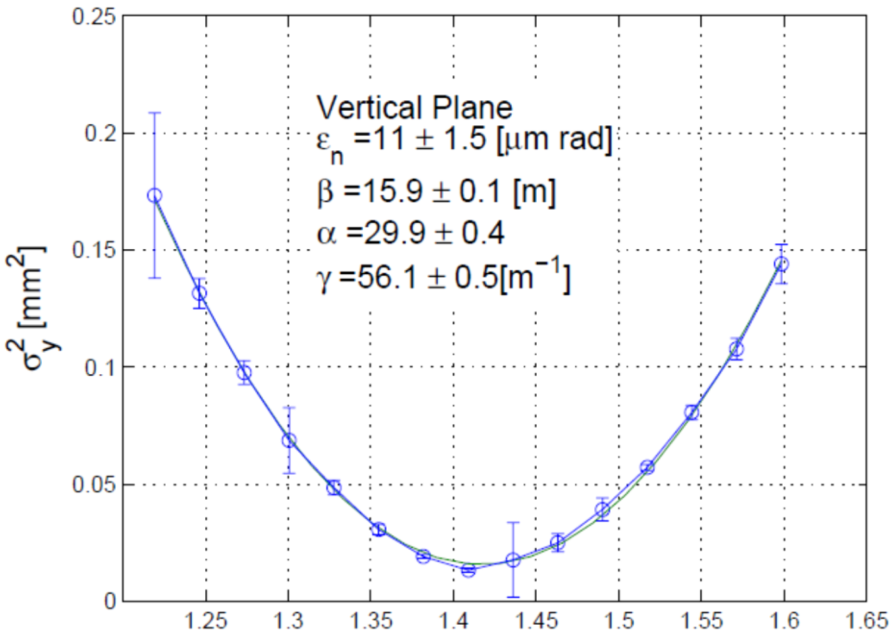
Measurement of transverse Emittance

Using the 'thin lens approximation' i.e. the quadrupole has a focal length of f:

$$\mathrm{R}_{focus}(K) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \Rightarrow \mathrm{R}(L,K) = \mathrm{R}_{drift}(L)\cdot \mathrm{R}_{focus}(K) = \begin{pmatrix} 1+LK & L \\ K & 1 \end{pmatrix}$$

Measurement of the matrix-element $\sigma_{11}(1,K)$ from $\sigma(1,K) = \mathrm{R}(K)\cdot\sigma(0)\cdot\mathrm{R}^T(K)$

Example: Square of the beam width at ELETTRA 100 MeV e^- Linac, YAG:Ce:



Focusing strength K [m^{-1}]

G. Penco (ELETTRA) et al., EPAC'08

For completeness: The relevant formulas

$$\begin{aligned} \sigma_{11}(1,K) = {} & L^2\sigma_{11}(0)\cdot K^2 \\ & + 2\cdot(\ L\sigma_{11}(0) + L^2\sigma_{12}(0)\ )\cdot K \\ & + L^2\sigma_{22}(0) + \sigma_{11}(0) \\ \equiv {} & a\cdot K^2 \ - \ 2ab\cdot K \ + \ ab^2 + c \end{aligned}$$

The three matrix elements at the quadrupole:

$$\sigma_{11}(0) = \frac{a}{L^2}$$

$$\sigma_{12}(0) = -\frac{a}{L^2}\left(\frac{1}{L} + b\right)$$

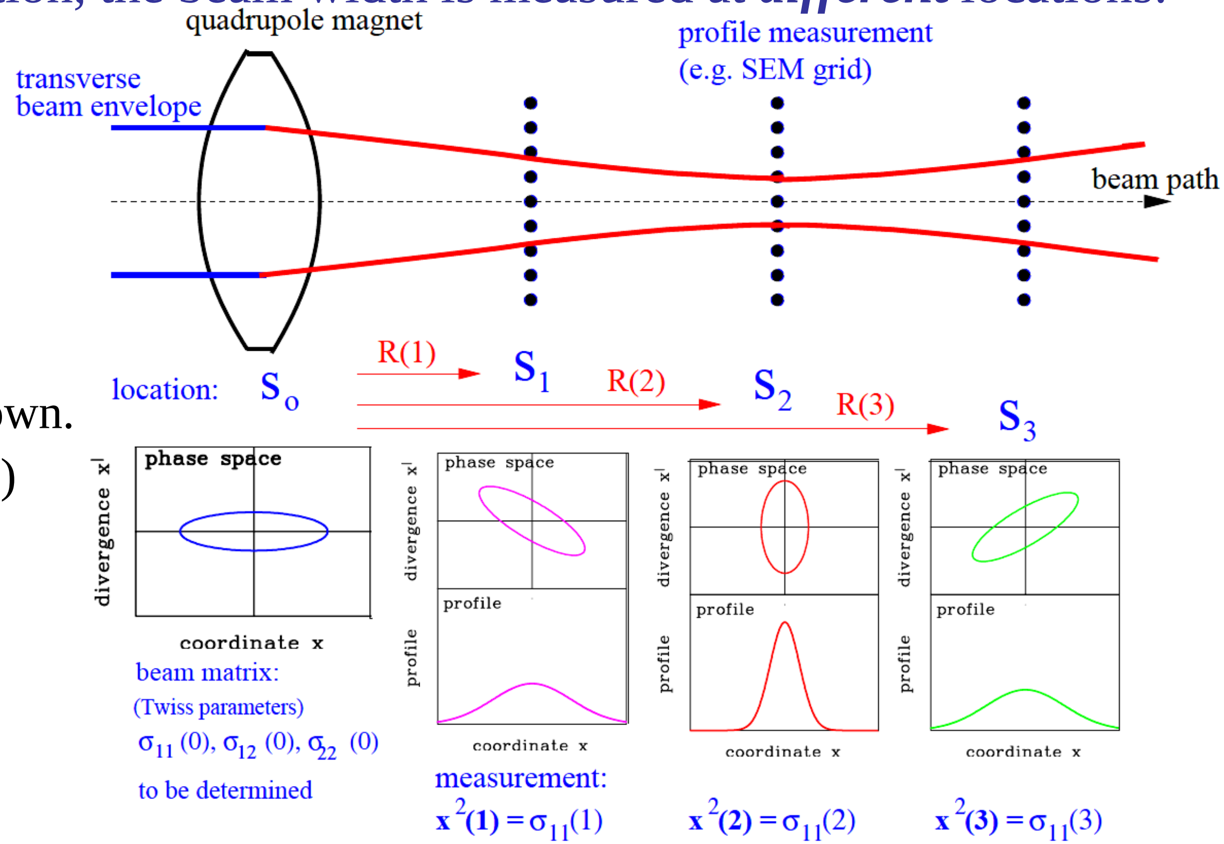
$$\sigma_{22}(0) = \frac{1}{L^2}\left(ab^2 + c + \frac{2ab}{L} + \frac{a}{L^2}\right)$$

$$\varepsilon_{rms} \equiv \sqrt{\det \sigma(0)} = \sqrt{\sigma_{11}(0)\cdot\sigma_{22}(0) - \sigma_{12}^2(0)} = \sqrt{ac}\,/\,L^2$$

Instead of quadrupole variation, the beam width is measured at ***different*** locations:

The procedure is:

- Beam width $x(i)$ measured at the locations s_i
  $\Rightarrow$ beam matrix element $x^2(i) = \sigma_{11}(i)$.
- The transfer matrix **R(*i*)** is known. (without dipole a 3 × 3 matrix.)
- The transformations are:
  $\sigma(i) = R(i)\sigma(0)R^T(i)$
  $\Rightarrow$ redundant equations:

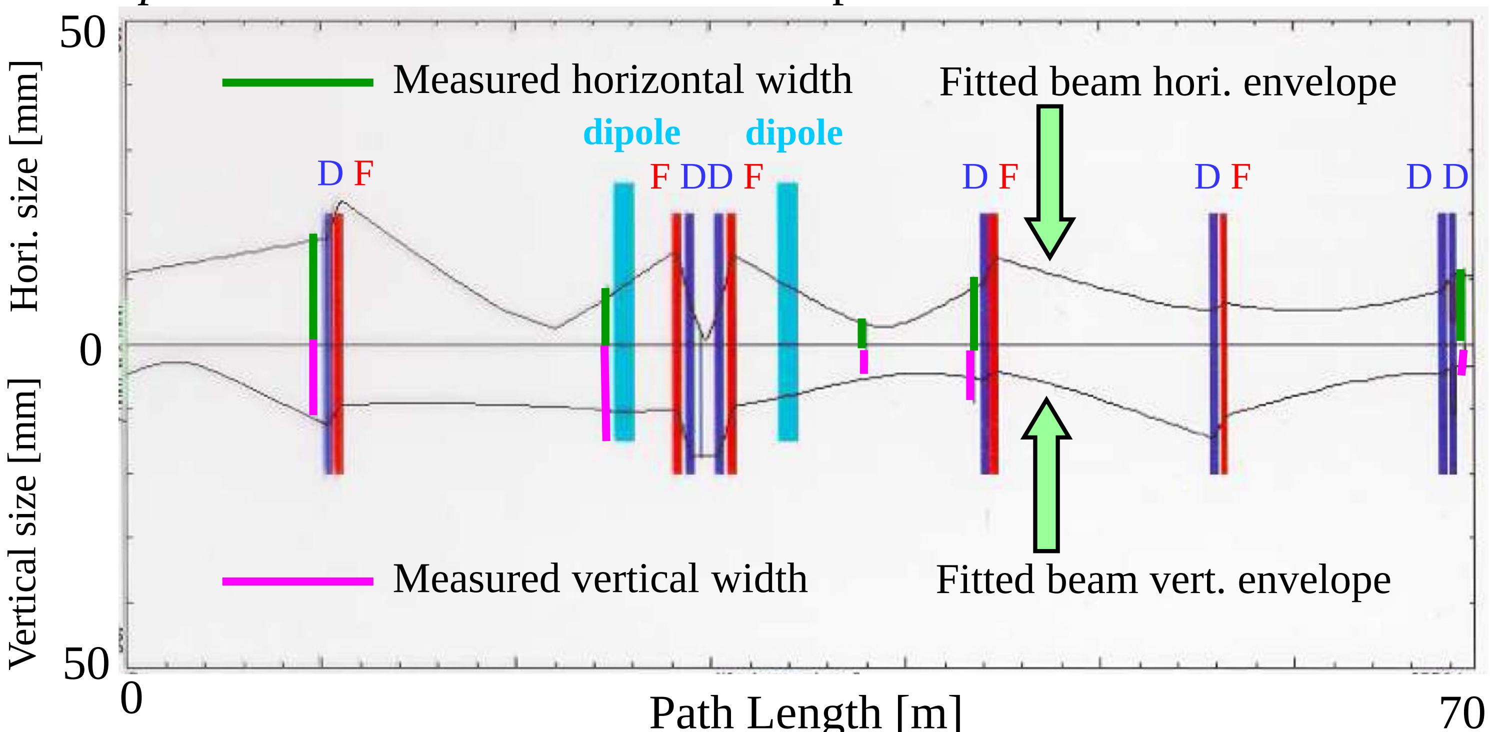


$$\begin{aligned}
\sigma_{11}(1) &= R_{11}^2(1)\cdot\sigma_{11}(0) + 2R_{11}(1)R_{12}(1)\cdot\sigma_{12}(0) + R_{12}^2(1)\cdot\sigma_{22}(0) && \mathbf{R}(1): s_0 \rightarrow s_1 \\
\sigma_{11}(2) &= R_{11}^2(2)\cdot\sigma_{11}(0) + 2R_{11}(2)R_{12}(2)\cdot\sigma_{12}(0) + R_{12}^2(2)\cdot\sigma_{22}(0) && \mathbf{R}(2): s_0 \rightarrow s_2 \\
&\vdots \\
\sigma_{11}(n) &= R_{11}^2(n)\cdot\sigma_{11}(0) + 2R_{11}(n)R_{12}(n)\cdot\sigma_{12}(0) + R_{12}^2(n)\cdot\sigma_{22}(0) && \mathbf{R}(n): s_0 \rightarrow s_n
\end{aligned}$$

Results of a 'Three Grid Method' Measurement

Solution: Solving the linear equations like for quadrupole variation or fitting the profiles with linear optics code (e.g. TRANSPORT, WinAgile, MadX).

Example: The hor. and vert. beam envelope and the beam width at a transfer line:



Assumptions: ➢ constant emittance, in particular no space-charge broadening

➢100 % transmission i.e. no loss due to vacuum pipe scraping

➢ no misalignment, i.e. beam center equals center of the quadrupoles.

Emittance measurements are very important for comparison to theory.

It includes size (value of ε) and orientation in phase space (σ_{ij} or α, β and γ)

(three independent values)

Techniques for transfer lines (synchrotron: width measurement sufficient):

Low energy beams → direct measurement of x- and x'-distribution

- ***Slit-grid:*** movable slit → ***x***-profile, grid → ***x'***-profile
- Variances exists: slit-slit, slit-kick, pepperpot .... method

All beams → profile measurement + linear transformation:

- ***Quadrupole variation:*** one location, different setting of a quadrupole
- ***'Three grid method':*** different locations
- ***Assumptions:*** ➢ well aligned beam, no steering
  ➢ no emittance blow-up due to space charge.

Slit Grid Emittance: Standard device, total 9 device

Pepper-pot Emittance: Special device, total 1 device