Problem 1 solution

Derive an expression for the resulting magnetic field when a normal sextupole with field **$B_y = B_2/2\ x^2$** is displaced by **δx** from its center position. At what type of fields correspond the resulting components? Do the same for an octupole with field **$B_y = B_3/6\ x^3$**. What is the leading order multi-pole field error when displacing a general **2(n+1)**-pole magnet?

- The vertical field of a sextupole is $B_y = \frac{B_2}{2} x^2$
- Considering a displacement $x \mapsto x + \delta x$ the field is written as

$$B_y = \frac{B_2}{2}(x+\delta x)^2 = \frac{B_2}{2}(\underbrace{x^2}_{\text{sextupole}} + \underbrace{2(\delta x)x}_{\text{quadrupole}} + \underbrace{(\delta x)^2}_{\text{dipole}})$$

- For an octupole

$$B_y = \frac{B_3}{6}(x+\delta x)^3 = \frac{B_3}{6}(\underbrace{x^3}_{\text{octupole}} + \underbrace{3(\delta x)x^2}_{\text{sextupole}} + \underbrace{3(\delta x)^2 x}_{\text{quadrupole}} + \underbrace{(\delta x)^3}_{\text{dipole}})$$

- The vertical field for a 2(n+1)-pole is

$$B_y(y=0) = \frac{B_n}{n!}\bar{x}^n$$

Problem 1 solution

- By displacing it $x \mapsto x + \delta x$, the vertical field is

$$B_y(y=0) = \frac{B_n}{n!}\bar{x}^n = \frac{B_n}{n!}(x+\delta x)^n = \frac{B_n}{n!}\left(x^n + n\delta x x^{n-1} + \frac{n(n-1)}{2}(\delta x)^2 x^{n-2} + \cdots + (\delta x)^n\right)$$

- So the leading order feed-down is a **2n-pole**

Problem 2 solution

Three correctors are placed at locations with phase advance of **π/4** between them and beta functions of **12**, **2** and **12m**. How are the corrector kicks related to each other in order to achieve a closed 3-bump.

- The relations for achieving a 3-bump are

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

- The phase advances are $\psi_{12} = \psi_{23} = \pi/4$ and $\psi_{13} = \psi_{12} + \psi_{23} = \pi/2$ which gives $\psi_{31} = -\pi/2$

- So $\theta_1 = \theta_3$ and $\theta_2 = -\theta_1\sqrt{12}$

Problem 3 solution

<u>SNS:</u> A **proton** ring with kinetic energy of **1GeV** and a **circumference** of **248m** has **18, 1m-long** focusing quads with **gradient** of **5T/m**. In one of the quads, the horizontal and vertical **beta** function is of **12m** and **2m** respectively. The **rms beta** function in both planes on the focusing quads is **8m**. With a **horizontal tune** of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on the single focusing quad given by **horizontal** and by **vertical** misalignments of **1mm** in all the quads. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?

- The rms orbit distortion is given by

$$u_{\rm rms}(s) = \frac{\sqrt{N\beta(s)\beta_{\rm rms}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\rm rms}$$

- We need to determine the rms kick angle, which in the case of a quadrupole displacement is given by

$$\theta_{\rm rms} = \frac{Gl}{B\rho}(\delta u)_{\rm rms}$$

- The magnetic rigidity is

$$B\rho \;\; [\mathrm{T\ m}] = \frac{1}{0.2998}\beta_r E \;\; [\mathrm{GeV}]$$

Problem 3 solution

- We need to compute the total energy which is $E = T + E_0 = 1.938 \text{ GeV}$
- Now we need to compute the relativistic beta. First we compute the relativistic gamma

$\gamma_r = \frac{E}{E_0} = 2.07$ and the relativistic beta is $\beta_r = \sqrt{1 - 1/\gamma_r^2} = 0.875$

- The magnetic rigidity is then $B\rho = 5.657 \text{ Tm}$ and the rms angle in both planes is

$\theta_{\text{rms}} = 8.8 \times 10^{-4} \text{ rad}$

- Now we can calculate the rms orbit distortion on the single focusing quad

$$x_{\text{rms}}(s) = \frac{\sqrt{N\beta_x(s)\beta_{x\text{rms}}}}{2\sqrt{2}|\sin(\pi Q_x)|}\theta_{x\text{rms}} = \frac{\sqrt{18 \times 12 \times 8}}{2\sqrt{2}|\sin(6.23\pi)|}8.8 \times 10^{-4} = 19.6\text{mm}$$

- The vertical is

$$y_{\text{rms}}(s) = \frac{\sqrt{N\beta_y(s)\beta_{y\text{rms}}}}{2\sqrt{2}|\sin(\pi Q_y)|}\theta_{y\text{rms}} = \frac{\sqrt{18 \times 2 \times 8}}{2\sqrt{2}|\sin(6.20\pi)|}8.8 \times 10^{-4} = 9\text{mm}$$

- For $Q_x = 6.1$ the horizontal orbit distortion becomes $x_{\text{rms}}(s) = 41.9\text{mm}$
- For $Q_x = 6.01$ we have $x_{\text{rms}}(s) = 0.41 \text{ m}$
- The vertical remains unchanged…

Problem 4 solution

<u>SPS:</u> Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads, and horizontal and vertical **beta** of **30m** and **108m** for the defocusing ones. The tunes of the machine are **Qx=20.13** and **Qy=20.18**. Due to a mechanical problem, one **<u>focusing</u> quadrupole** was **slowly sinking down** in 2016, resulting in an increasing closed orbit distortion with respect to a reference taken in the beginning of the year.
- By how much the quadrupole had shifted down when the maximum vertical closed orbit distortion amplitude in <u>defocusing</u> quadrupoles reached 4 mm?
- Why was there no change of the horizontal orbit measured?
- How big would have been the maximum closed orbit distortion amplitude if it would have been a defocusing quadrupole?

- The magnetic rigidity is $B\rho \;\; [\mathrm{T\ m}] = \dfrac{1}{0.2998}\beta_r E \;\; [\mathrm{GeV}]$
- For 400GeV, the relativistic beta is almost 1 and then the magnetic rigidity is

$$B\rho = 1334 \;\; \mathrm{T\ m}$$

- The focusing normalized gradient is $K_F = \dfrac{G_F}{B\rho} = \dfrac{15}{1334} = 0.011\,\mathrm{m}^{-2}$
- The defocusing one is just the same with opposite sign $K_D = -0.011\,\mathrm{m}^{-2}$

Problem 4 solution

- The closed orbit distortion from a single dipole error is given by

$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$

- We are interested in the peak orbit distortion

$$\hat{y} = \theta \frac{\sqrt{\hat{\beta}_y \beta_0}}{2\sin(\pi Q)}$$

- From this we can calculate the required kick

$$\theta = \frac{\hat{y}\, 2\sin(\pi Q)}{\hat{\beta}_y \beta_0} = \frac{0.004 \times 2 \times \sin(\pi 20.18)}{\sqrt{108 \times 30}} = 75\,\mu\text{rad}$$

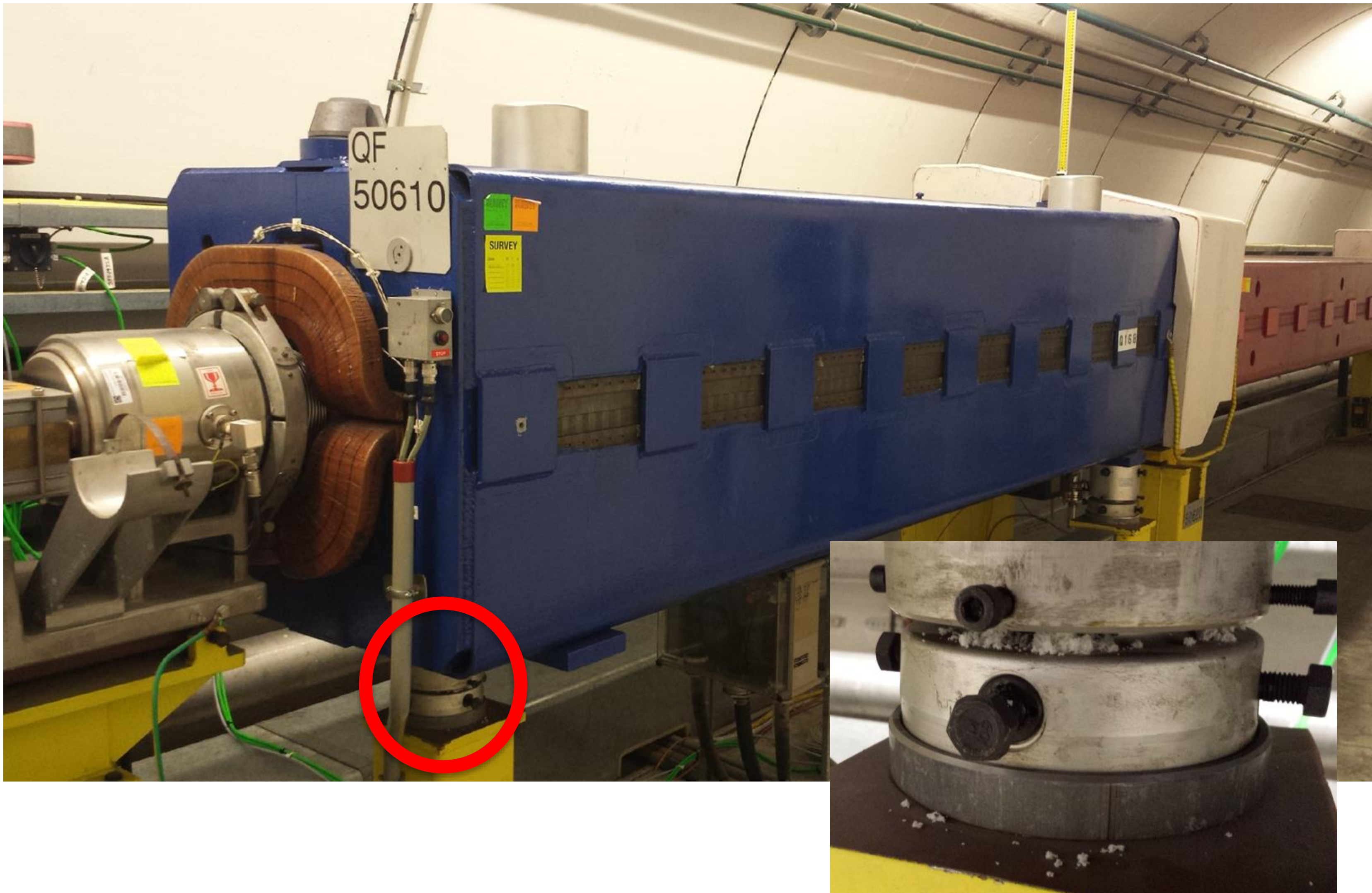
- And finally the required quadrupole displacement to produce this deflection

$$\theta = \frac{Gl\delta y}{B\rho} = K_F l_F \delta y$$

$$\delta y = \frac{\theta}{K_F l_F} = \frac{75 \times 10^{-6}}{0.011 \times 3.22}\text{m} = 2\,\text{mm}$$

Problem 4 solution

- In fact what was found in the machine was that one of the supports of the quadrupole was broken and on that end it was shifted down by 4 mm, resulting in 2 mm average shift



Problem 4 solution

- No horizontal orbit change was observed, because the quadrupole shifted only in the vertical plane resulting in a pure vertical kick.
- If it would have been a defocusing quadrupole, the kick would have been the same but with opposite sign. However, the impact on the closed orbit would have been bigger since the vertical β-function is bigger in the defocusing quadupole, such that the peak orbit distortion would reach

$$\hat{y} = \theta \frac{\sqrt{\hat{\beta}_y \beta_0}}{2\sin(\pi Q)} = \theta \frac{\sqrt{\hat{\beta}_y \hat{\beta}_y}}{2\sin(\pi Q)} = 75 \times 10^{-6} * \frac{\sqrt{108 \times 108}}{2\sin(\pi 20.18)} \,\mathrm{m} = 7.5\,\mathrm{mm}$$

Problem 5 solution

SPS: Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads, and horizontal and vertical **beta** of **30m** and **108m** for the defocusing ones.
- Find the tune change for systematic gradient errors of **1%** in the focusing and **0.5%** in the defocusing quads.
- What is the chromaticity of the machine?

- The magnetic rigidity is $B\rho \ \ [\mathrm{T\ m}] = \frac{1}{0.2998}\beta_r E \ \ [\mathrm{GeV}]$
- For 400GeV, the relativistic beta is almost 1 and then the magnetic rigidity is

$$B\rho = 1334 \ \ \mathrm{T\ m}$$

- The focusing normalized gradient is $K_F = \frac{G_F}{B\rho} = \frac{15}{1334} = 0.011\,\mathrm{m}^{-2}$
- The defocusing one is just the same with opposite sign $K_D = -0.011\,\mathrm{m}^{-2}$
- Now, the tune change is given by

$$\delta Q_u = \frac{1}{4\pi}\sum_i \beta_u K_i \left(\frac{\delta K}{K}\right)_i l_i$$

Problem 5 solution

- By splitting the focusing and defocusing quads, we have

$$\delta Q_u = \frac{1}{4\pi}\left(N_F \beta_u^F K_F \left(\frac{\delta K}{K}\right)_F l_F + N_D \beta_u^D K_D \left(\frac{\delta K}{K}\right)_D l_D\right)$$

- As $N_F = N_D = N,\ \ l_F = l_D = l$ and $K_F = -K_D = K$ the tune shift can be rewritten as

$$\delta Q_{x,y} = \frac{1}{4\pi} N l K \left(\pm \beta_{x,y}^F \left(\frac{\delta K}{K}\right)_F \mp N_D \beta_{x,y}^D \left(\frac{\delta K}{K}\right)_D\right)$$

- This gives a horizontal and vertical tune shift of

$$\delta Q_x = \frac{108 \times 3.22 \times 0.011}{4\pi}(+108 \times 0.01 - 30 \times 0.005) = 0.3$$

$$\delta Q_y = \frac{108 \times 3.22 \times 0.011}{4\pi}(-30 \times 0.01 + 108 \times 0.005) = 0.07$$

- The chromaticity of the machine is

$$\xi_{x,y} = -\frac{1}{4\pi}\sum_i \beta_{x,y}^i K_{x,y}^i$$

Problem 5 solution

- By splitting again the focusing and defocusing quads' contribution, we have

$$\xi_{x,y} = -\frac{1}{4\pi} NlK(\pm\beta^F_{x,y} \mp \beta^D_{x,y})$$

- This gives in both planes

$$\xi_{x,y} = -\frac{108 \times 3.22 \times 0.011}{4\pi}(108 - 30) = -24$$

Problem 6 solution

<u>CLIC pre-damping rings:</u> Consider a 2.86 GeV electron storage ring with a racetrack shape of 389 m circumference. Each arc is composed of **17 regular "TME" cells**, each consisting of 2 half dipoles, 2 focusing and 2 defocusing quadrupoles. The beta functions are around **β_x=4m (2m)** and **β_y=4.2m (9m)** in the **focusing (defocusing) quadrupoles** and the normalized quadrupole gradients are **2.49/m^2 (2.07/m^2)**. The **quadrupoles** have a **length** of **0.28m**. The **natural chromaticity** of the machine is about **-19** and **-23** in the horizontal and vertical plane, respectively.

- How big is the chromaticity contribution from the arcs?
- Where would you install sextupole magnets for correcting chromaticity?
-

- The chromaticity from quadrupoles is given by

$$\xi_{x,y} = -\frac{1}{4\pi}\oint \beta_{x,y} k(s) ds$$

- For one TME cell we get

$$\xi_{x,cell} = -\frac{1}{4\pi}\cdot(2\beta_{x,qf}k_{qf}l_{qf} - 2\beta_{x,qd}k_{qd}l_{qd})$$

$$= -\frac{1}{4\pi}(2\cdot 4\cdot 2.49\cdot 0.28 - 2\cdot 2\cdot 2.07\cdot 0.28) = -0.28$$

$$\xi_{y,cell} = -\frac{1}{4\pi}\cdot(-2\beta_{y,qf}k_{qf}l_{qf} + 2\beta_{y,qd}k_{qd}l_{qd})$$

$$= -\frac{1}{4\pi}(-2\cdot 4.2\cdot 2.49\cdot 0.28 + 2\cdot 9\cdot 2.07\cdot 0.28) = -0.36$$

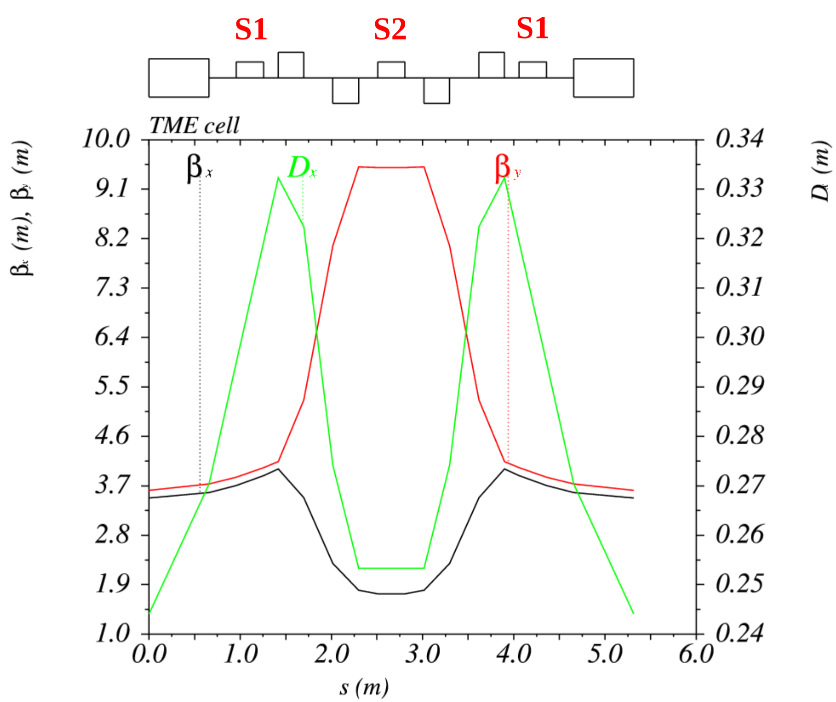
Problem 6 solution

- The chromaticity contribution from the arcs is then

$$\xi_{x,arc} = 2 \cdot N_{cells/arc} \cdot \xi_{x,cell} = 2 \cdot 17 \cdot (-0.28) = -9.42$$

$$\xi_{y,arc} = 2 \cdot N_{cells/arc} \cdot \xi_{y,cell} = 2 \cdot 17 \cdot (-0.36) = -12.4$$

- The optimal location of the sextupoles is at locations with high β and high dispersion to minimize their strength



Problem 6 solution

- The total chromaticity is given by

$$\xi_{x,y}^{\text{tot}} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)\left(k(s) \mp S(s)D_x(s)\right) ds$$

- The sextupole gradients required to correct the total chromaticity can be estimated using the dispersion values from the figure

$$\xi_x^{tot} = \xi_x^{nat} - \frac{Ncells}{4\pi}(-\beta_{x1}D_1S1l_{S1} - \beta_{x2}D_2S2l_{S2} - \beta_{x1}D_1S1l_{S1}) = 0 \longrightarrow$$

$$S1 = \frac{-4\pi\xi_x^{nat} - D_2l_{S2}NcellsS2\beta_{x2}}{2D_1l_{S1}Ncells\beta_{x1}}$$

$$\xi_y^{tot} = \xi_y^{nat} - \frac{Ncells}{4\pi}(\beta_{y1}D_1S1l_{S1} + \beta_{y2}D_2S2l_{S2} + \beta_{y1}D_1S1l_{S1}) = 0 \longrightarrow$$

$$S1 = \frac{4\pi\xi_y^{nat} - D_2l_{S2}NcellsS2\beta_{y2}}{2D_1l_{S1}Ncells\beta_{y1}}$$

Problem 6 solution

- Equalizing the two expressions for S1 we can derive S2:

$$S2 = -\frac{4\pi(\beta_{y1}\xi_x^{nat} + \beta_{x1}\xi_y^{nat})}{D2l_{s2}Ncells(\beta_{x2}\beta_{y1} - \beta_{x1}\beta_{y2})} = -32.8m^{-3}$$

Substituting this into one of the two expressions for S1 we get: $S1 = 14.8m^{-3}$