Linear imperfections and correction

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References

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Reminder of transverse beam dynamics

Equation reminder

Lorentz equation $$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

E : total energy

T : kinetic energy $$E = \sqrt{p^2c^2 + m_0^2c^4} = T + m_0c^2 = T + E_0$$

p : momentum

β : reduced velocity $$\beta = \frac{v}{c}$$

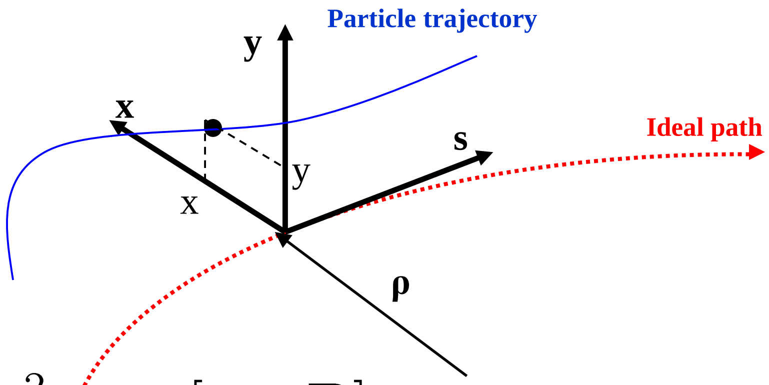
γ : reduced energy $$\gamma = \frac{E}{m_0c^2}$$

$\beta\gamma$: reduced momentum $$\beta\gamma = \frac{pc}{m_0c^2}$$

Reference trajectory

- Cartesian coordinates not useful to describe motion in a circular accelerator (not true for linacs)

- A system following an ideal path along the accelerator is used (**Frenet** reference system) $(\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_z}) \rightarrow (\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_s})$



  - The curvature vector is $\boldsymbol{\kappa} = -\frac{d^2\mathbf{s}}{ds^2}$

  - From Lorentz equation

$$\frac{d\mathbf{p}}{dt} = m_0\gamma\frac{d^2\mathbf{s}}{dt^2} = m_0\gamma v_s^2\frac{d^2\mathbf{s}}{ds^2} = -m_0\gamma v_s^2\boldsymbol{\kappa} = q[\mathbf{v}\times\mathbf{B}]$$

where we used the curvature vector definition and $\frac{d^2}{dt^2} = v_s^2\frac{d^2}{ds^2}$

- Using $m_0\gamma v_s = p_s = (p^2 - p_x^2 - p_y^2)^{1/2} \approx p$, the ideal path of the reference trajectory is defined by

$$\boldsymbol{\kappa}_0 = -\frac{q}{p}\left[\frac{\mathbf{v}}{v_s}\times\mathbf{B_0}\right]$$

Beam guidance

- Consider uniform magnetic field $\mathbf{B} = \{0, B_y, 0\}$ in a direction perpendicular to particle motion. From the reference trajectory equation, after developing the cross product and considering that the transverse velocities $v_x \,, v_y \ll v_s$, the radius of curvature is

$$\frac{1}{\rho} = |k| = |\frac{q}{p}B|$$

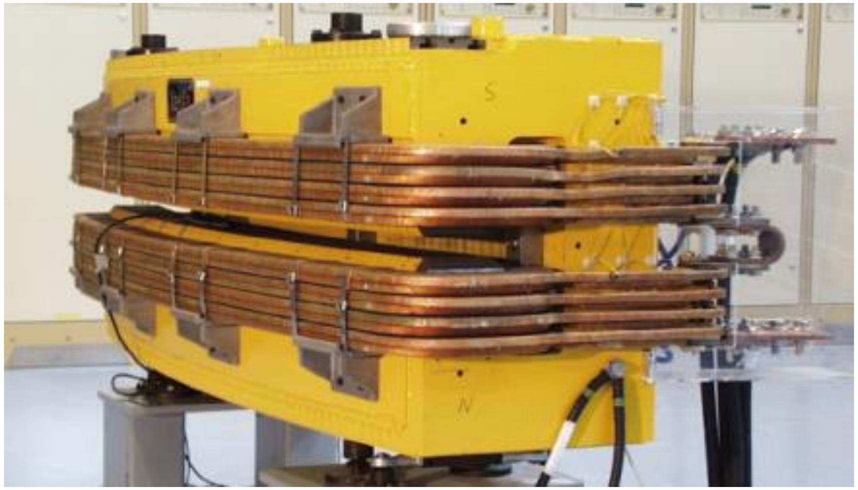
- We define the **magnetic rigidity** $|B\rho| = \frac{p}{q}$

- In more practical units $\boxed{\beta E[GeV] = 0.2998|B\rho|[Tm]}$

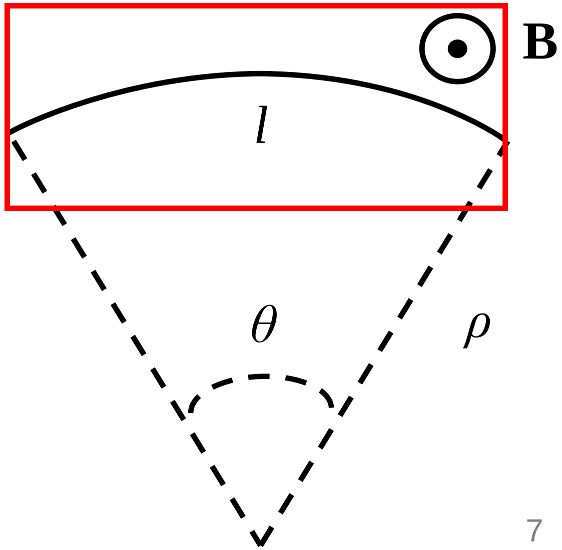
- For ions with charge multiplicity *n* and atomic number *A*, the energy per nucleon is

$$\beta\bar{E}[GeV/u] = 0.2998\frac{n}{A}|B\rho|[Tm]$$

Dipoles



- Consider ring for particles with energy *E* with *N* dipoles of length *L* (or effective length *l*, i.e. measured on beam path)
  - **Bending angle** $\theta = \frac{2\pi}{N}$
  - **Bending radius** $\rho = \frac{l}{\theta}$
  - **Integrated dipole strength** $Bl = \frac{2\pi}{N} \frac{p}{q}$
- Note:
  - By choosing a dipole field, the dipole length is imposed and vice versa
  - The higher the field, the shorter or smaller number of dipoles can be used
  - The ring circumference (cost) is influenced by the field choice

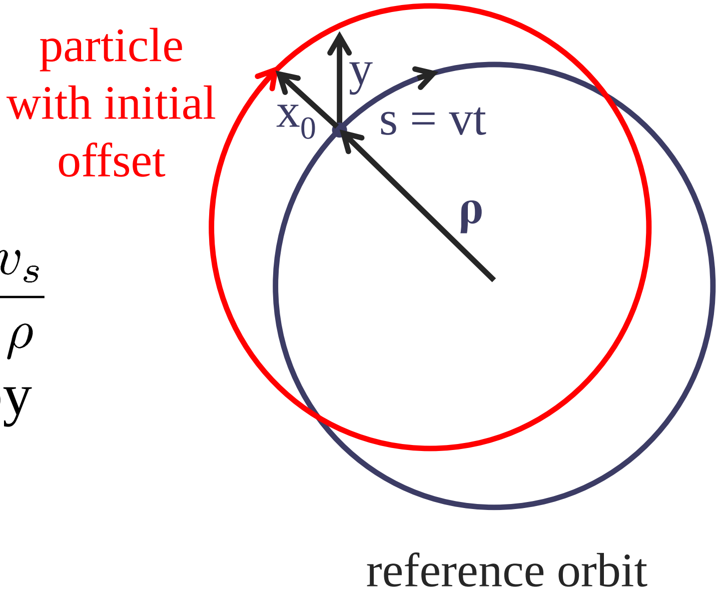


Beam focusing

- Consider a particle in a dipole field
- In the **horizontal plane**
  - it performs harmonic oscillations $x = x_0 \cos(\omega t + \phi)$ with frequency $\omega = \frac{v_s}{\rho}$
  - the horizontal acceleration is described by

$$\frac{d^2x}{ds^2} = \frac{1}{v_s^2}\frac{d^2x}{dt^2} = -\frac{1}{\rho^2}x$$

  - there is a **week focusing** effect in the horizontal plane



- In the **vertical plane**, the only force present is gravitation
  - Particles are displaced vertically following the usual law $\Delta y = \frac{1}{2}a_g \Delta t^2$
  - With $a_g \approx 10$ m/s^2, the particle is displaced by **18 mm** (LHC dipole aperture) in **60 ms** (few hundred turns in LHC) ➔ **need focusing!**

Quadrupoles

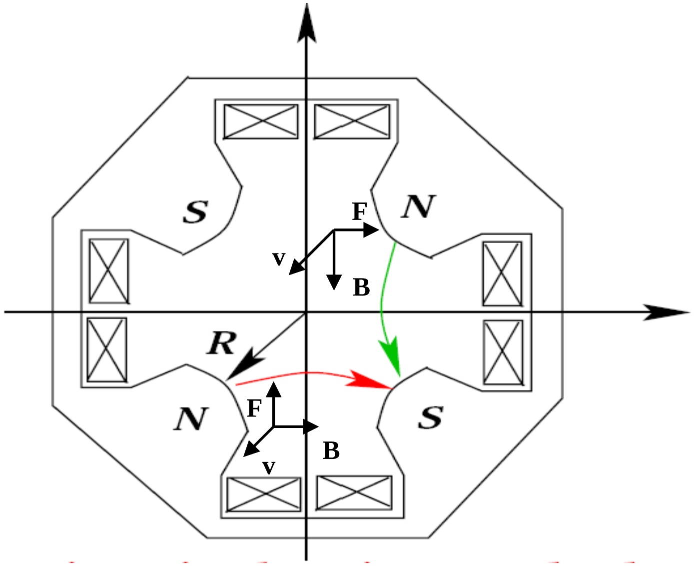
- Quadrupoles are focusing in one plane and defocusing in the other
- The field is $(B_x, B_y) = G(y, x)$
- The resulting force $(F_x, F_y) = k(y, -x)$ with the normalised gradient defined as

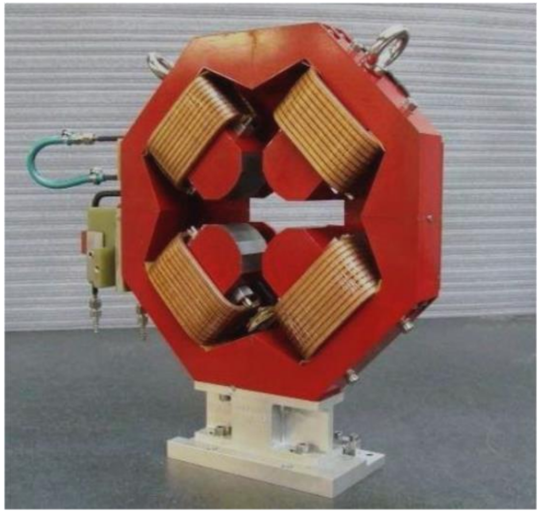
$$k = \frac{qG}{\beta E}$$

- In more practical units:

$$k[m^{-2}] = 0.2998 \frac{G[T/m]}{\beta E[GeV]}$$

- Need to alternate focusing and defocusing to control the beam, i.e. **alternating gradient focusing**





Equations of motion – Linear fields

- Consider s-dependent fields from dipoles and normal quadrupoles

$$B_y = B_0(s) - G(s)x \ , \quad B_x = -G(s)y$$

- The total momentum can be written $p = p_0(1 + \frac{\Delta p}{p})$
- With magnetic rigidity $B_0\rho = \frac{p_0}{q}$ and normalized gradient $k(s) = \frac{G(s)}{B_0\rho}$ the equations of motion are

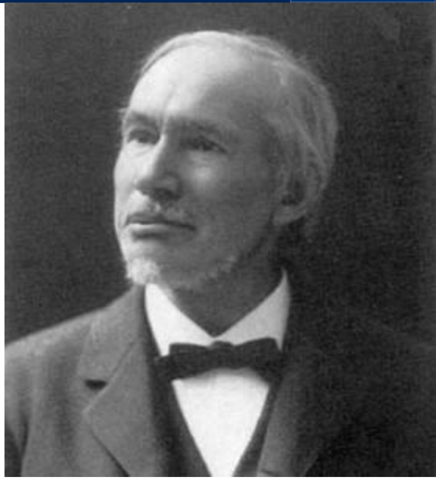
$$\boxed{\begin{aligned} x'' - \left(k(s) - \frac{1}{\rho(s)^2}\right) x &= \frac{1}{\rho(s)} \frac{\Delta p}{p} \\ y'' + k(s)\ y &= 0 \end{aligned}}$$

  - Inhomogeneous equations with s-dependent coefficients
  - The term $\frac{1}{\rho^2}$ corresponds to the dipole **week focusing** and $\frac{1}{\rho}\frac{\Delta p}{p}$ represents **off-momentum** particles

Hill’s equations

- Solutions are combination of the homogeneous and inhomogeneous equations’ solutions
- Consider particles with the design momentum. Equations of motion become

$$\boxed{\begin{aligned} x'' + K_x(s)\ x &= 0 \\ y'' + K_y(s)\ y &= 0 \end{aligned}}$$



George Hill

with $K_x(s) = -\left(k(s) - \frac{1}{\rho(s)^2}\right)\ ,\quad K_y(s) = k(s)$

- **Hill’s equations of linear transverse particle motion**
  - Linear equations with s-dependent coefficients (harmonic oscillator with time dependent frequency)
  - In a ring (or in transport line with symmetries), the **coefficients are periodic** $K_x(s) = K_x(s+C)\ ,\quad K_y(s) = K_y(s+C)$
  - Not straightforward to derive analytical solutions for whole accelerator

Betatron motion

- The on-momentum linear betatron motion of a particle in both planes, is described by

$$\boxed{u(s) = \sqrt{\epsilon\beta(s)}\cos(\psi(s)+\psi_0)} \qquad u \mapsto \{x, y\}$$

with $\alpha,\ \beta,\ \gamma$ the twiss functions $\alpha(s) = -\frac{\beta(s)'}{2},\quad \gamma = \frac{1+\alpha(s)^2}{\beta(s)}$

ψ the **betatron phase** $\psi(s) = \int \frac{ds}{\beta(s)}$

and the **beta function** β is defined by the **envelope equation**

$$\boxed{2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4}$$

- By differentiation, we have that the **angle** is

$$\boxed{u'(s) = \sqrt{\frac{\epsilon}{\beta(s)}}\left(\sin(\psi(s)+\psi_0) + \alpha(s)\cos(\psi(s)+\psi_0)\right)}$$

General transfer matrix

- From the position and angle equations it follows that

$$\cos(\psi(s)+\psi_0) = \frac{u}{\sqrt{\epsilon\beta(s)}}\ , \quad \sin(\psi(s)+\psi_0) = \sqrt{\frac{\beta(s)}{\epsilon}}u' + \frac{\alpha(s)}{\sqrt{\epsilon\beta(s)}}u$$

- Expand the trigonometric formulas and set $\psi(0)=0$ to get the transfer matrix from location 0 to s

$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \mathcal{M}_{0\to s} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

with

$$\mathcal{M}_{0\to s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\psi + \alpha_0 \sin\Delta\psi) & \sqrt{\beta(s)\beta_0}\sin\Delta\psi \\ \frac{(\alpha_0-\alpha(s))\cos\Delta\psi-(1+\alpha_0\alpha(s))\sin\Delta\psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}(\cos\Delta\psi - \alpha(s)\sin\Delta\psi) \end{pmatrix}$$

and $\mu(s) = \Delta\psi = \int_0^s \frac{ds}{\beta(s)}$ the **phase advance**

Periodic transfer matrix

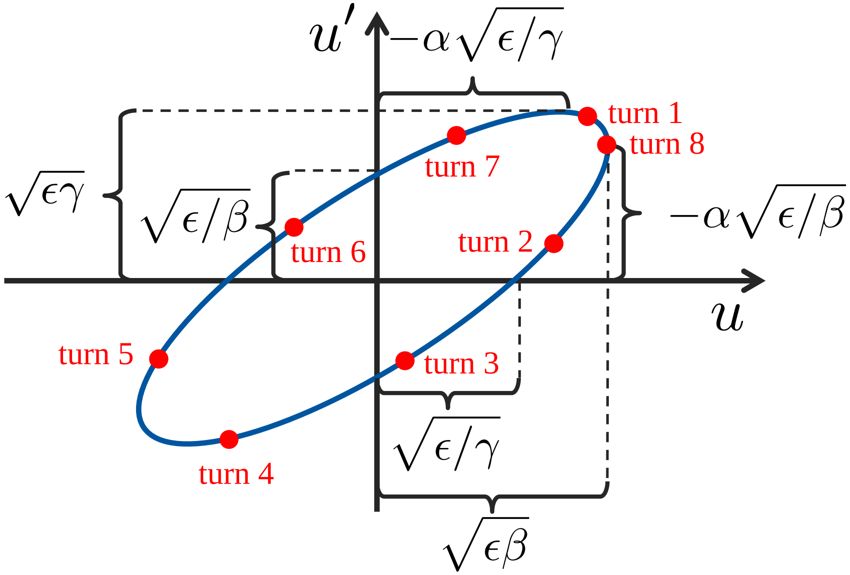
- Consider a periodic cell of length *C*
- The optics functions are $\beta_0 = \beta(C) = \beta\,,\quad \alpha_0 = \alpha(C) = \alpha$

  and the phase advance $\mu = \int_0^C \frac{ds}{\beta(s)}$
- The transfer matrix is $\mathcal{M}_C = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$
- The cell matrix can be also written as

$$\mathcal{M}_C = \mathcal{I}\cos\mu + \mathcal{J}\sin\mu$$

with $\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and the **Twiss matrix** $\boxed{\mathcal{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}}$

Phase space ellipse



- The **phase space coordinates (u, u')** of a single particle at a given location s in the machine **lie on the phase space ellipse** when plotted for several turns.
- The **values of the Twiss parameters** and therefore the orientation of the phase space ellipse **depend on the s location in the machine**.
- The **Twiss parameters are periodic with the machine circumference**. Their values are derived from the transfer matrix and they are uniquely defined at any point in the machine.

Tune and working point

- In a ring, the **tune** is defined from the 1-turn phase advance

$$Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}(s)} = \frac{\nu_{x,y}}{2\pi}$$

i.e. number of betatron oscillations per turn

- Taking the average of the betatron tune around the ring we have in **smooth approximation**

$$\nu = 2\pi Q = \frac{C}{\langle\beta\rangle} \rightarrow Q = \frac{R}{\langle\beta\rangle}$$

  - Extremely useful formula for deriving scaling laws!

- The position of the tunes in a diagram of horizontal versus vertical tune is called a **working point**
- The tunes are imposed by the choice of the quadrupole strengths
- One should try to avoid **resonance conditions**

Transverse linear imperfections and correction

Outline

- Introduction
- Closed orbit distortion (steering error)
  - Beam orbit stability
  - Imperfections leading to closed orbit distortion
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
  - Dispersion and chromatic orbit
- Optics function distortion (gradient error)
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- Coupling error
  - Coupling errors and their effect
  - Coupling correction
- Chromaticity

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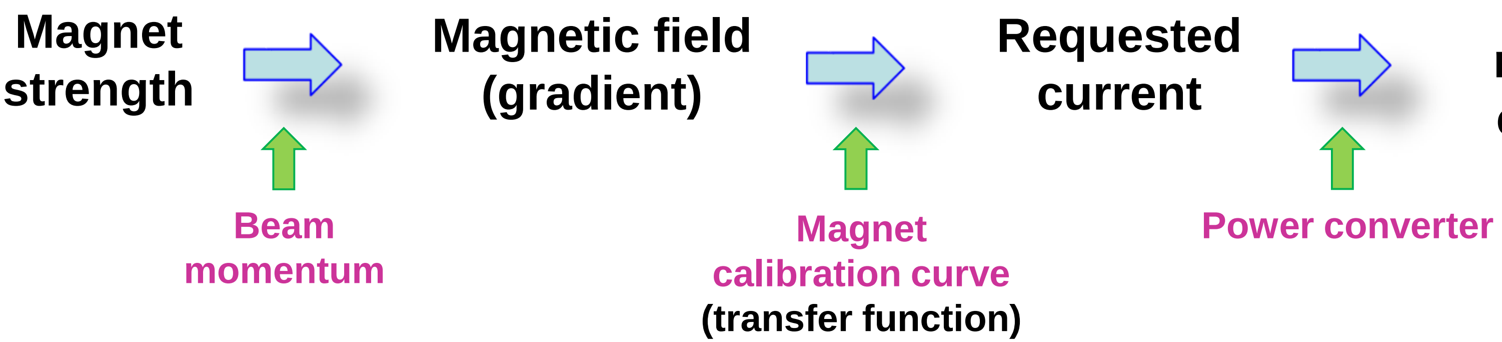
From model to reality - fields

- The physical units of the machine model defined by the accelerator physicist must be converted into magnetic fields and eventually into currents for the power converters that feed the magnet circuits.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by uncertainties or errors on: Beam momentum, magnet calibrations and power converter regulation.



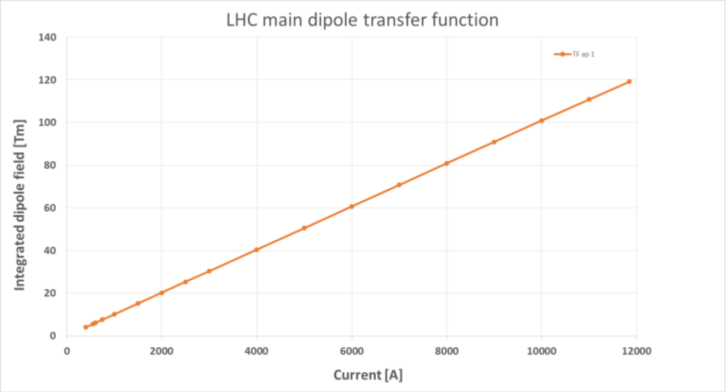
Magnet strength → **Magnetic field (gradient)** → **Requested current** → **Actual magnet current**

Beam momentum

Magnet calibration curve (transfer function)

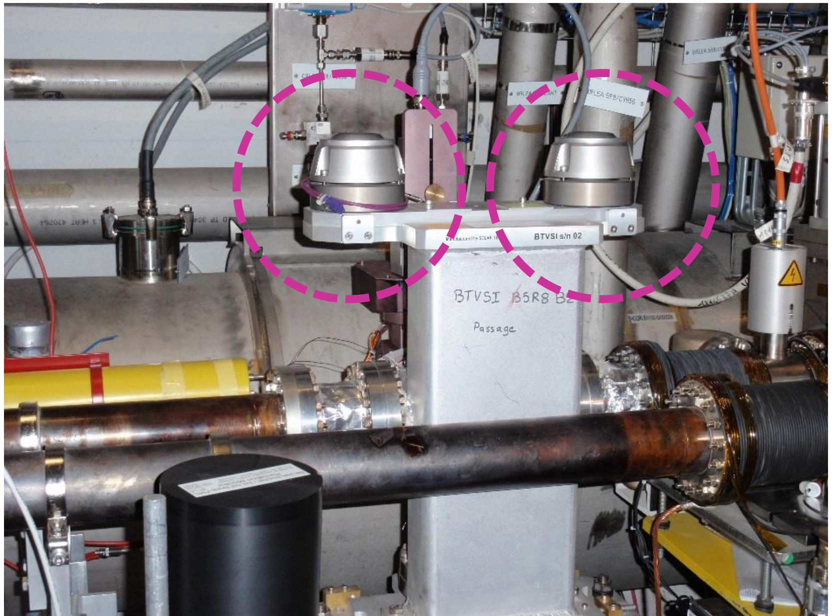
Power converter

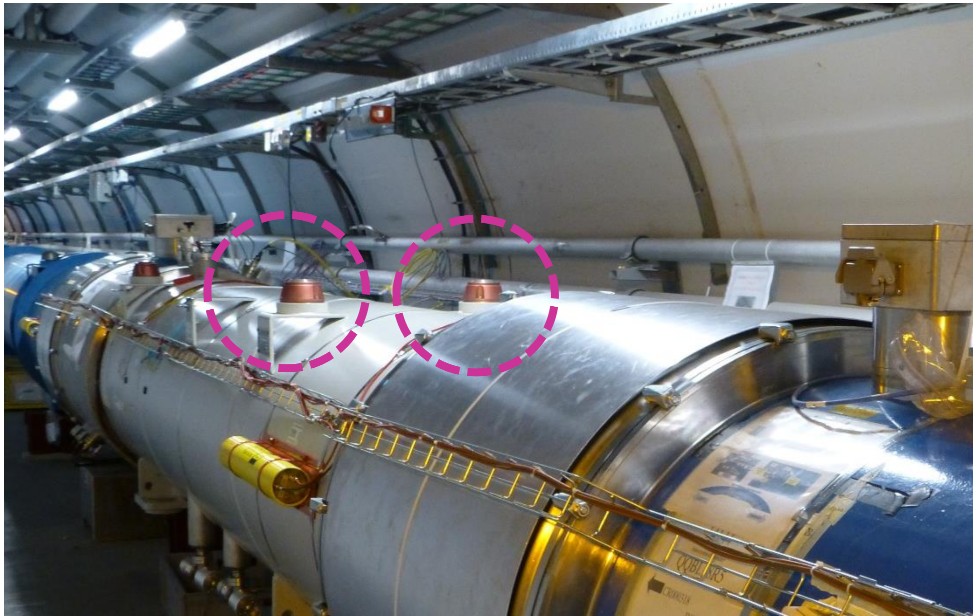
Example of the LHC main dipole calibration curve



From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of micrometres for CLIC !
  - For CERN hadron machines we aim for accuracies of around 0.1 mm.
- The alignment process implies:
  - Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
  - Precise in-situ alignment (position and angle) of the element in the tunnel.
- **Alignment errors are a common source of imperfections.**





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Beam orbit stability

- Beam orbit stability is very critical
  - Injection and extraction efficiency of synchrotrons
  - Stability of collision point in colliders
  - Stability of the synchrotron light spot in the beam lines of light sources
- Consequences of orbit distortion
  - Miss-steering of beams, modification of dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling, modulation of lattice functions, poor injection/extraction efficiency
- Sources for closed orbit drifts
  - **Long term (years - months)**: ground settling, season changes
  - **Medium term (days - hours)**: sun and moon, day-night variations (thermal), rivers, rain, wind, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns
  - **Short term (minutes - seconds)**: ground vibrations, power supplies, experimental magnets, air conditioning, refrigerators/compressors

Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)
   a) Particle injected on the reference orbit … remains on the reference orbit turn after turn

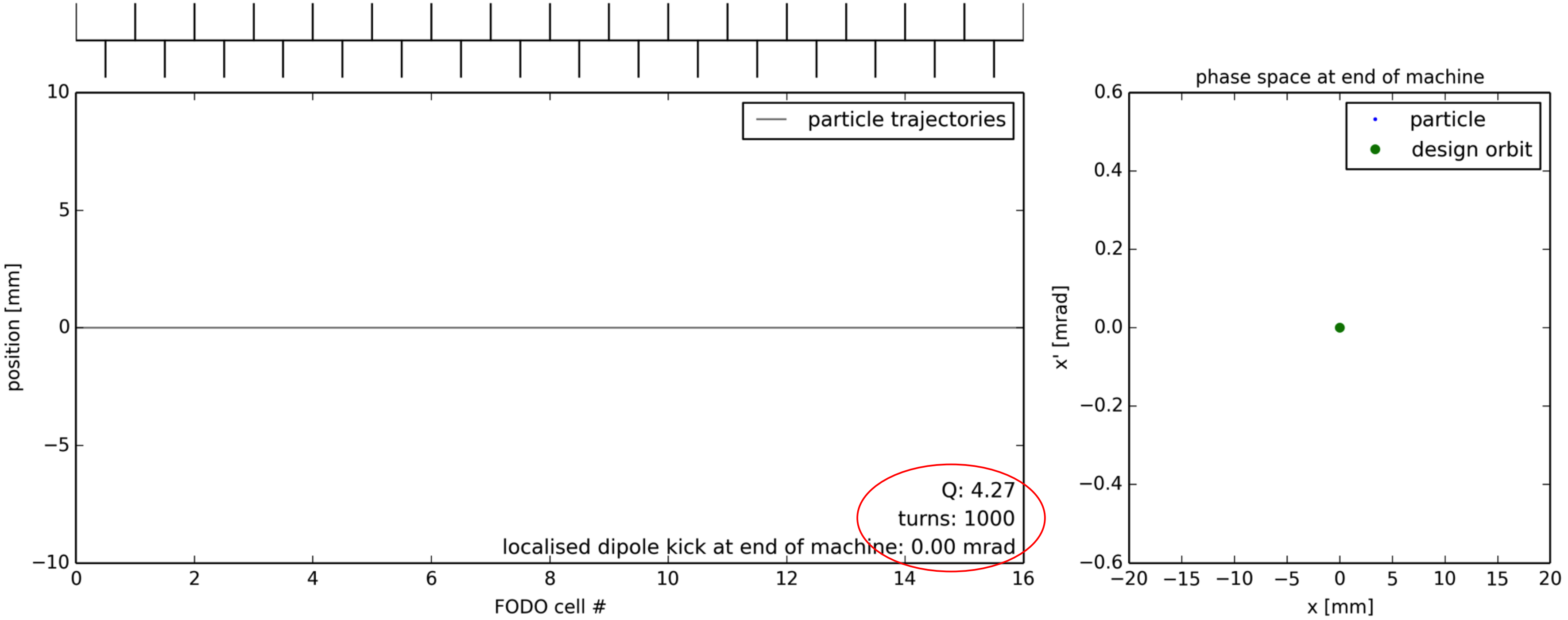

Illustration of closed orbit distortion

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   a) Particle injected on the reference orbit … remains on the reference orbit turn after turn
   b) Particle injected with offset

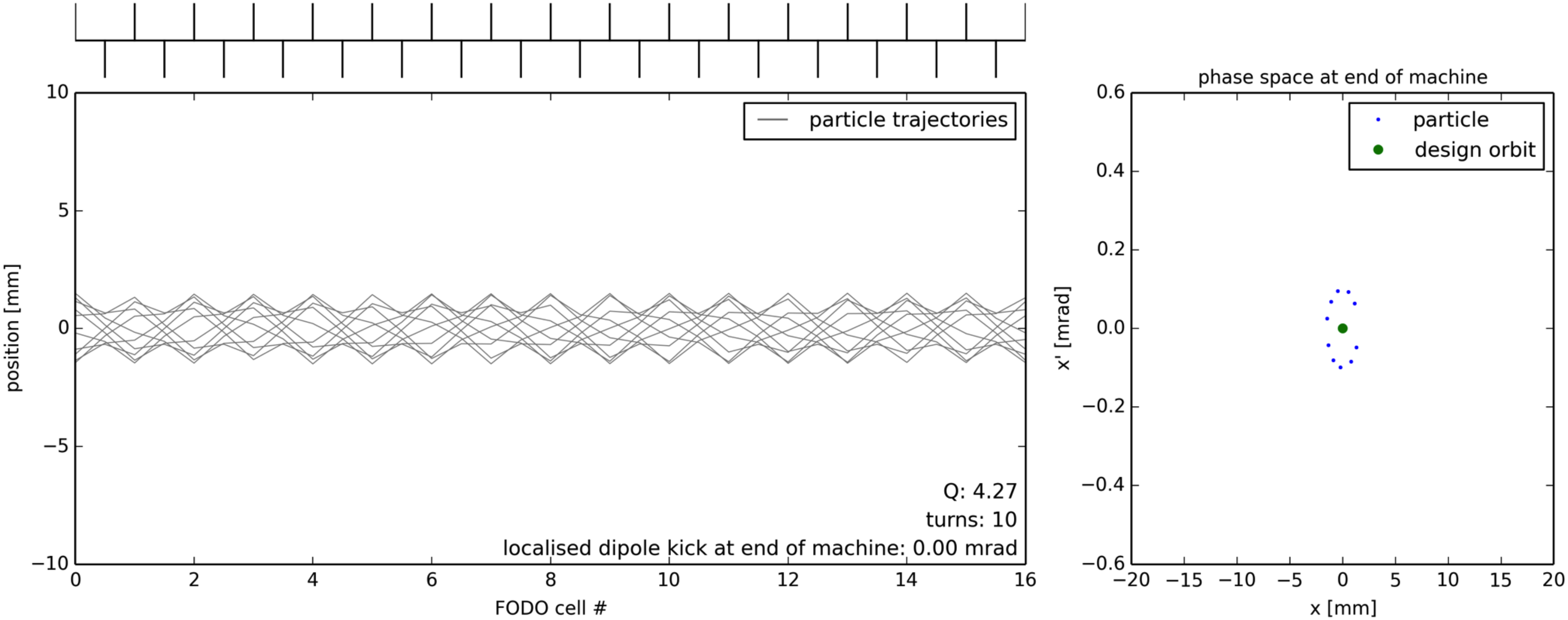


Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)
   a) Particle injected on the reference orbit … remains on the reference orbit turn after turn
   b) Particle injected with offset … performs betatron oscillations around the **closed orbit** which is the same as reference orbit as long as there are no imperfections

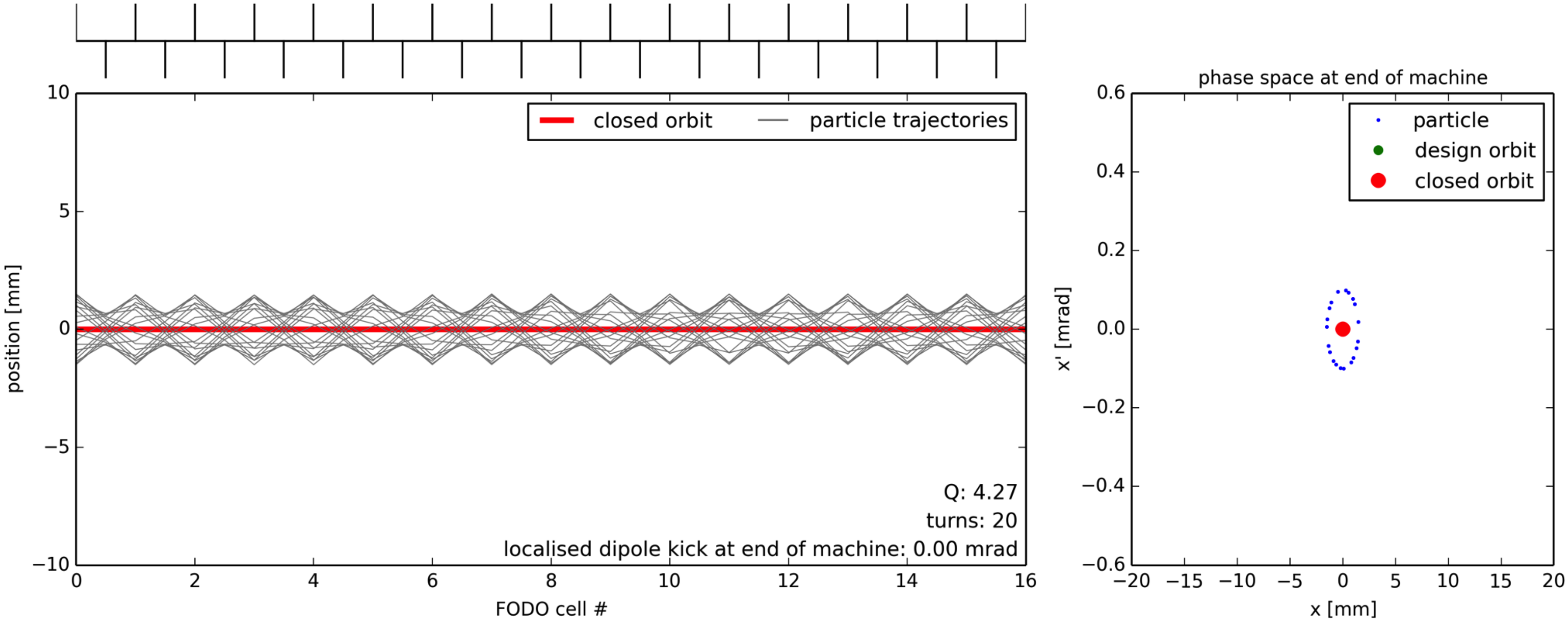


Illustration of closed orbit distortion

2. Ideal machine toy model with dipole error (unintended deflection) at the end of circumference
    a) <u>Particle injected on the reference orbit … receives dipole kick every turn</u>

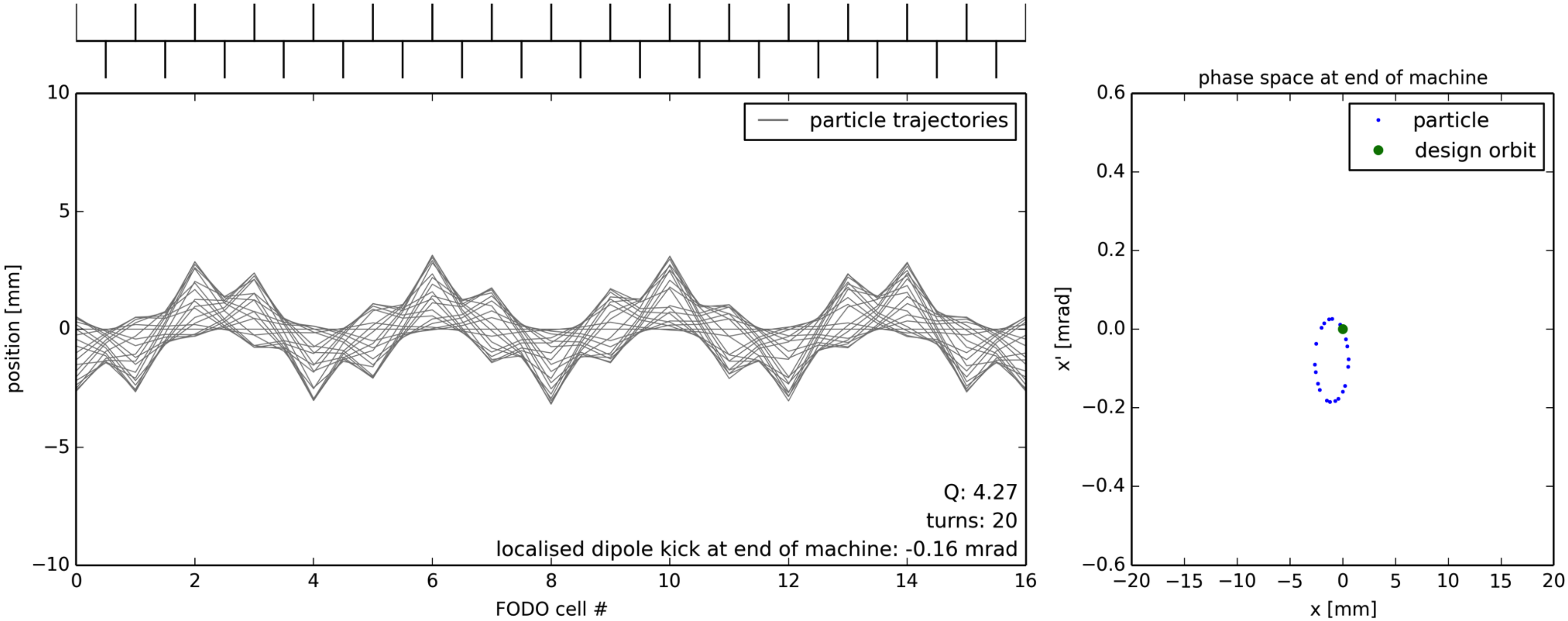
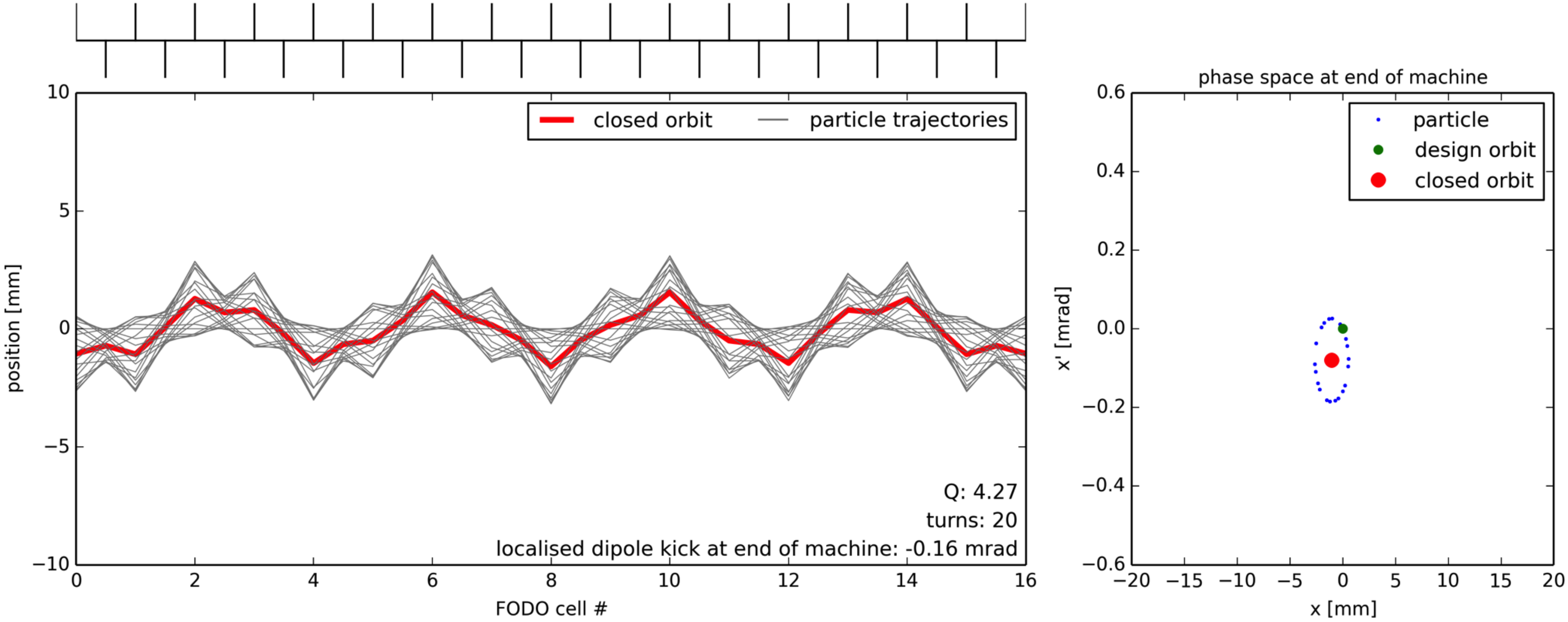


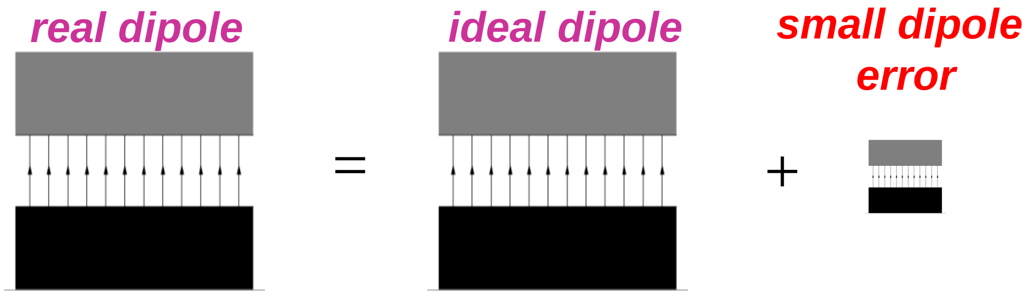
Illustration of closed orbit distortion

2. Ideal machine toy model with dipole error (unintended deflection) at the end of circumference
   a) Particle injected on the reference orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**



Sources of unintended deflections

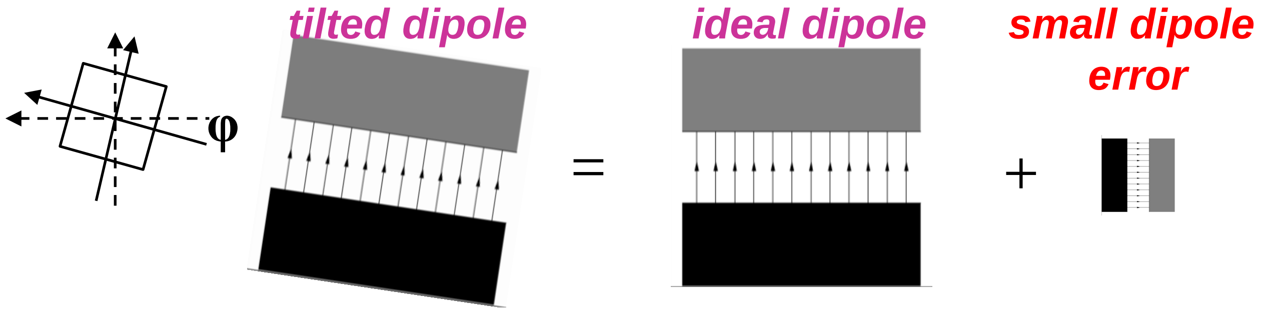
- Field error (deflection error) of a dipole magnet
  - This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc.)
  - The imperfect dipole can be expressed as a perfect one + a small error



→ horizontal kick

$$\theta = \frac{\delta(Bl)}{B\rho}$$

- A small rotation (misalignment) of a dipole magnet has the same effect, but in the other plane

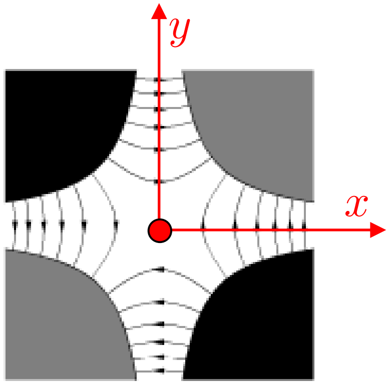


→ vertical kick

$$\theta = \frac{Bl\sin\phi}{B\rho}$$

Misalignments causing feed-down

- Misalignment of a quadupole magnet
  - Equivalent to perfectly aligned quadrupole plus small deflection



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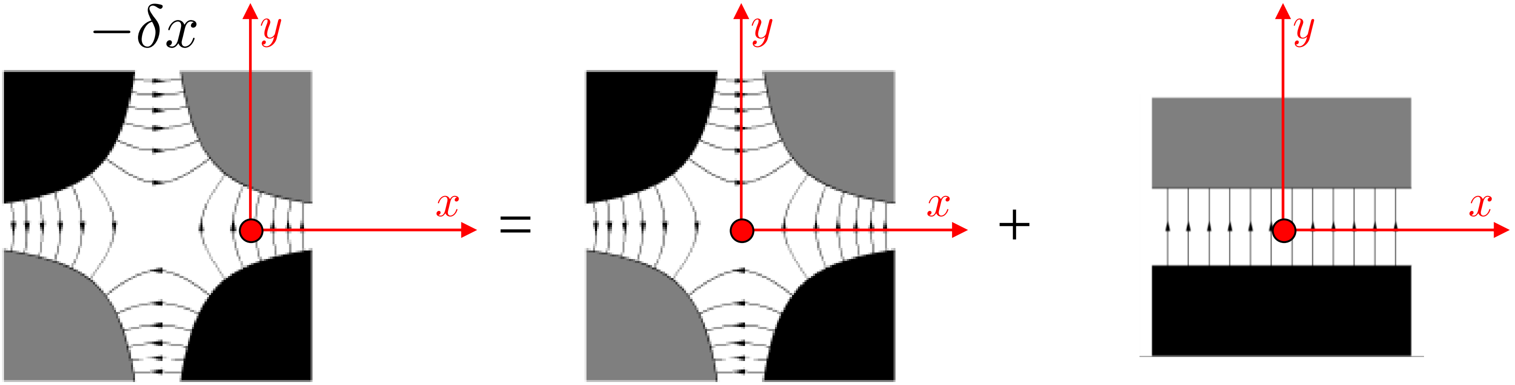


$$B_y = G\bar{x} = G(x+\delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

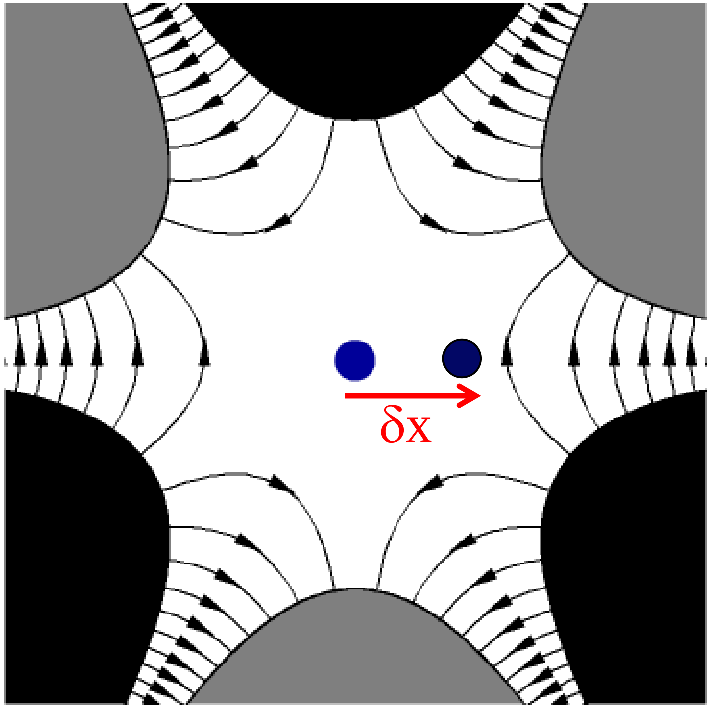
… this is called feed-down

- Effect of **pure horizontal** displacement of any **2(n+1)-pole** magnet

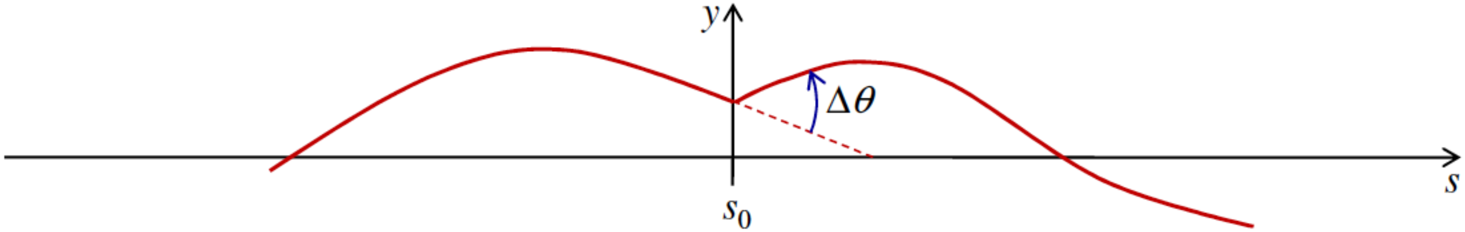
$$\boxed{B_y(y=0) = \frac{B_n}{n!}\bar{x}^n = \frac{B_n}{n!}(x+\delta x)^n = \frac{B_n}{n!}(\underbrace{x^n}_{\textbf{2(n+1)-pole}} + \underbrace{n\delta x x^{n-1}}_{\textbf{2n-pole}} + \underbrace{\frac{n(n-1)}{2}(\delta x)^2 x^{n-2}}_{\textbf{2(n-1)-pole}} + \cdots + \underbrace{(\delta x)^n}_{\textbf{dipole}})}$$

Problem 1

Derive an expression for the resulting magnetic field when a normal sextupole with field $\mathbf{B} = \mathbf{B_2/2\ x^2}$ is displaced by $\boldsymbol{\delta x}$ from its center position. At what type of fields correspond the resulting components? Do the same for an octupole with field $\mathbf{B} = \mathbf{B_3/6\ x^3}$. What is the leading order multi-pole field error when displacing a general **2(n+1)**-pole magnet?



Effect of single dipole kick



- Consider a single dipole kick $\theta = \delta u'_0 = \delta u'(s_0) = \frac{\delta(Bl)}{B\rho}$ at $s=s_0$
- The coordinates before and after the kick are

$$\begin{pmatrix} u_0 \\ u'_0 - \theta \end{pmatrix} = \mathcal{M} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

with the 1-turn transfer matrix

$$\mathcal{M} = \begin{pmatrix} \cos 2\pi Q + \alpha_0 \sin 2\pi Q & \beta_0 \sin 2\pi Q \\ -\gamma_0 \sin 2\pi Q & \cos 2\pi Q - \alpha_0 \sin 2\pi Q \end{pmatrix}$$

- The final coordinates are

$$u_0 = \theta \frac{\beta_0}{2\tan \pi Q} \qquad \text{and} \qquad u'_0 = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan \pi Q}\right)$$

Closed orbit from single dipole kick

- Taking the solutions of Hill’s equations at the location of the kick, the orbit will close to itself only if

$$\sqrt{\epsilon\beta_0}\cos(\phi_0) = \sqrt{\epsilon\beta_0}\cos(\phi_0 + 2\pi Q)$$

$$-\sqrt{\frac{\epsilon}{\beta_0}}(\sin(\phi_0) + \alpha_0\cos(\phi_0)) = -\sqrt{\frac{\epsilon}{\beta_0}}(\sin(\phi_0 + 2\pi Q) + \alpha_0\cos(\phi_0 + 2\pi Q)) - \theta$$

  - This yields the following relations for the invariant and phase (this can be also derived by the equations in the previous slide)

$$\epsilon = \frac{\beta_0\theta^2}{4\sin^2(\pi Q)}, \quad \phi_0 = -\pi Q$$

- For any location around the ring, the orbit distortion is written as

$$\boxed{u(s) = \theta\underbrace{\frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)}}\cos(\pi Q - |\psi(s) - \psi_0|)}$$

Maximum distortion amplitude

Transport of closed orbit distortion

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$u_2 = m_{11}u_1 + m_{12}u_1'$$
$$u_2' = m_{21}u_1 + m_{22}u_1'$$

- Consider a single dipole kick at position 1: $\theta_1 = \frac{\delta(Bl)}{B\rho}$
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u_1' + \theta_1) \to \delta u_2 = m_{12}\theta_1$$

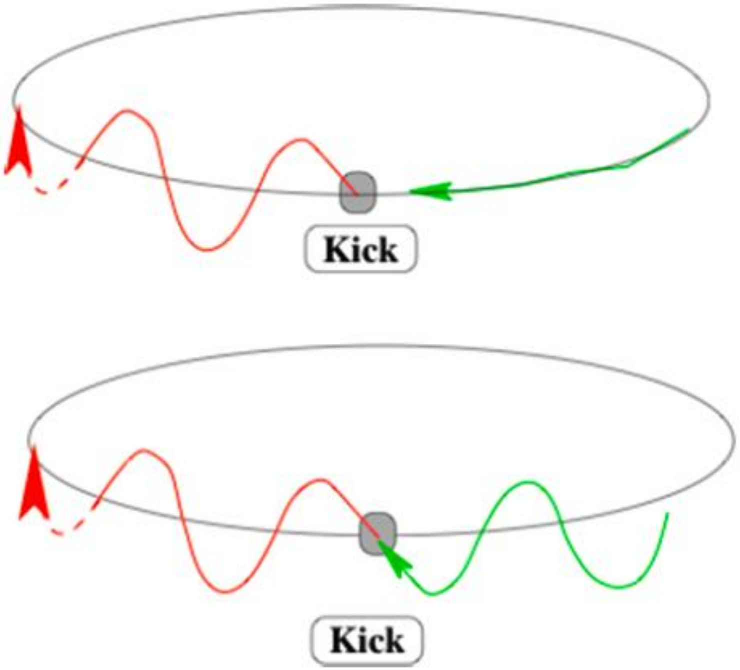
- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\theta_1$$
$$\delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}}[\cos(\psi_{12}) - \alpha_2\sin(\psi_{12})]\theta_1$$

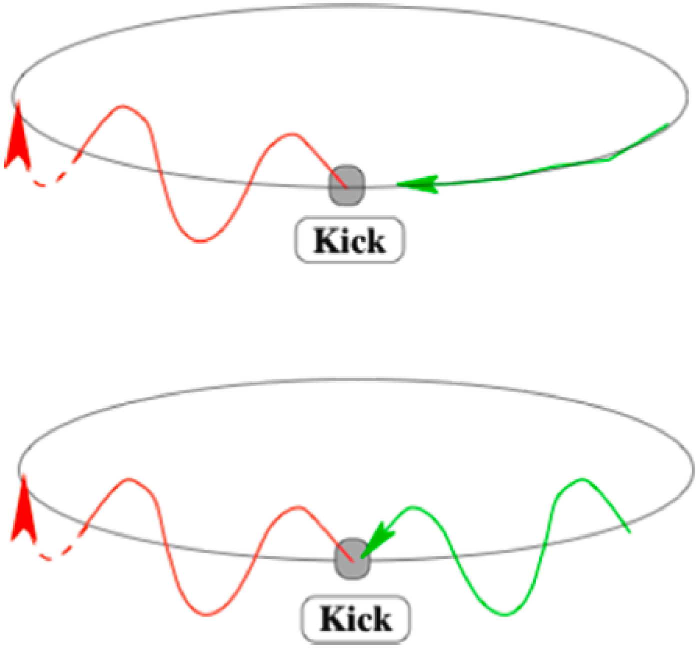
Integer and half integer resonance

$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$

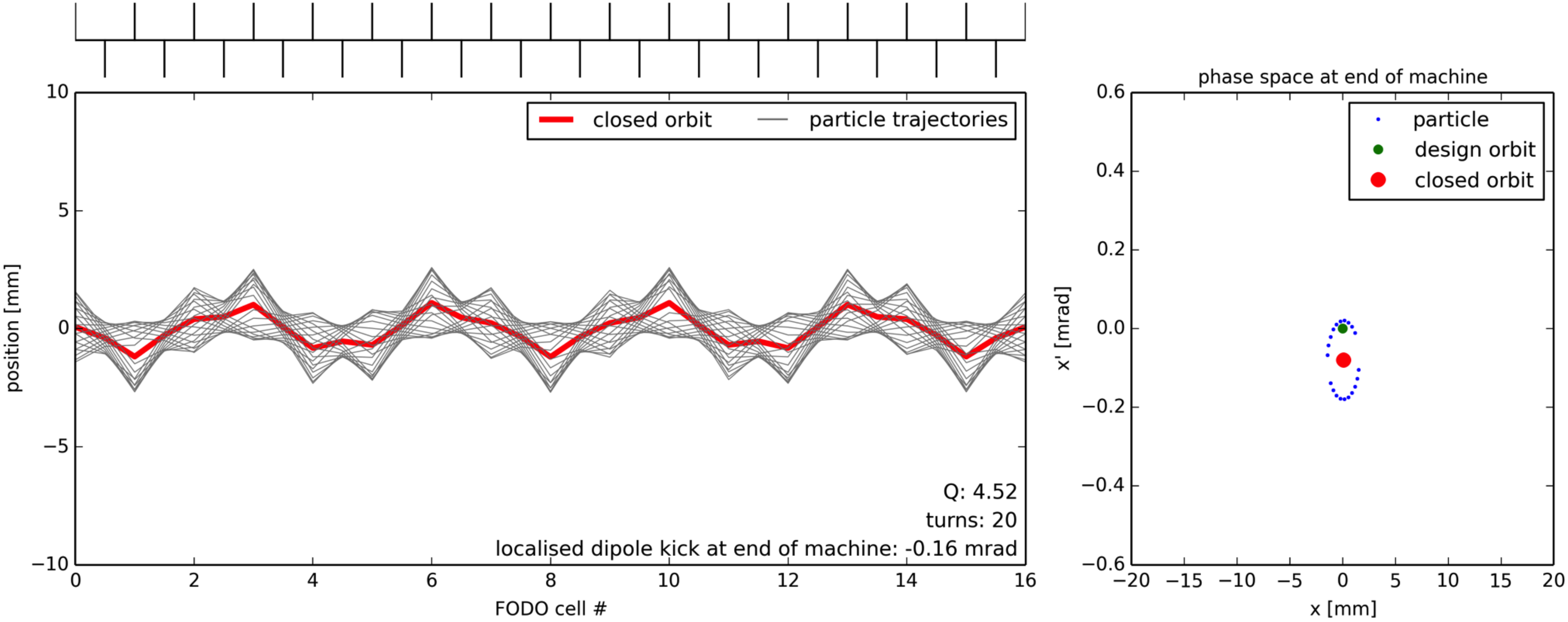
- Dipole perturbations add-up in consecutive turns for $Q = n$
- Integer tune excites orbit oscillations (resonance)



- Dipole kicks get cancelled in consecutive turns for $Q = n/2$
- Half-integer tune cancels orbit oscillations



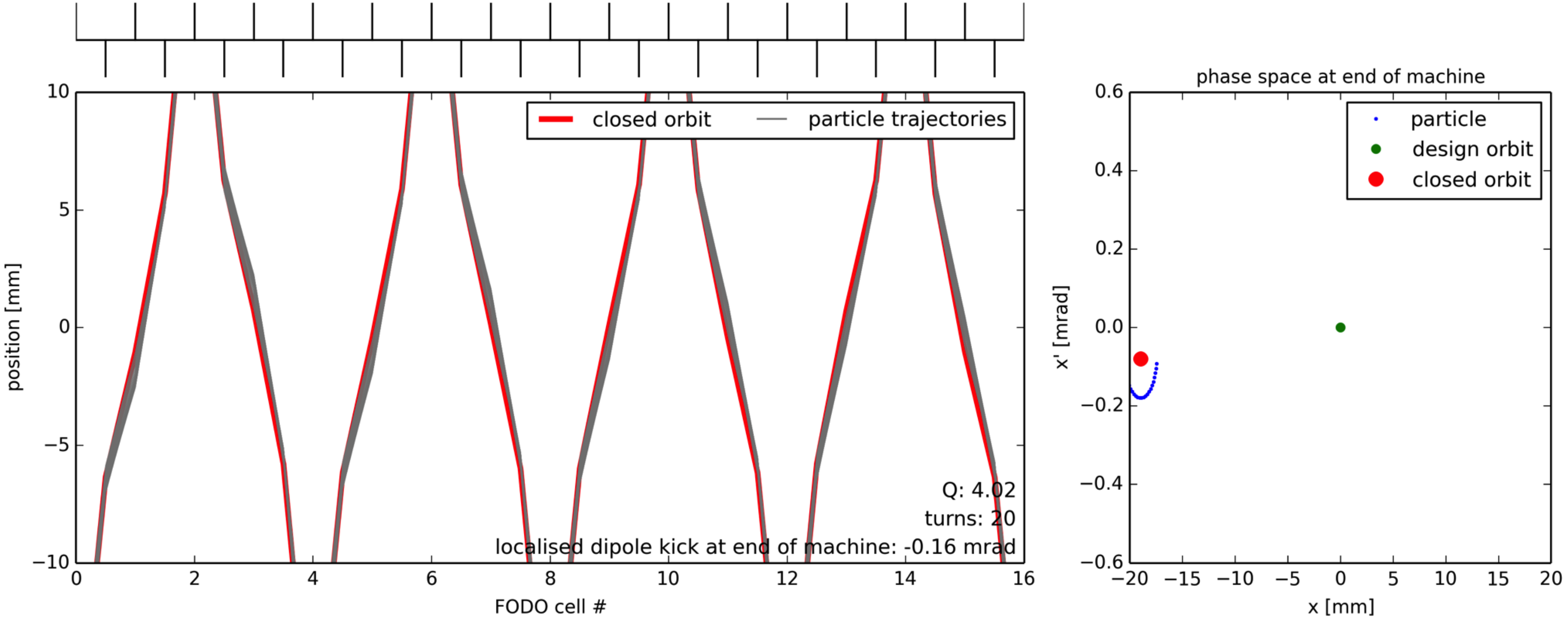
Single dipole kick vs. tune



Single dipole kick vs. tune

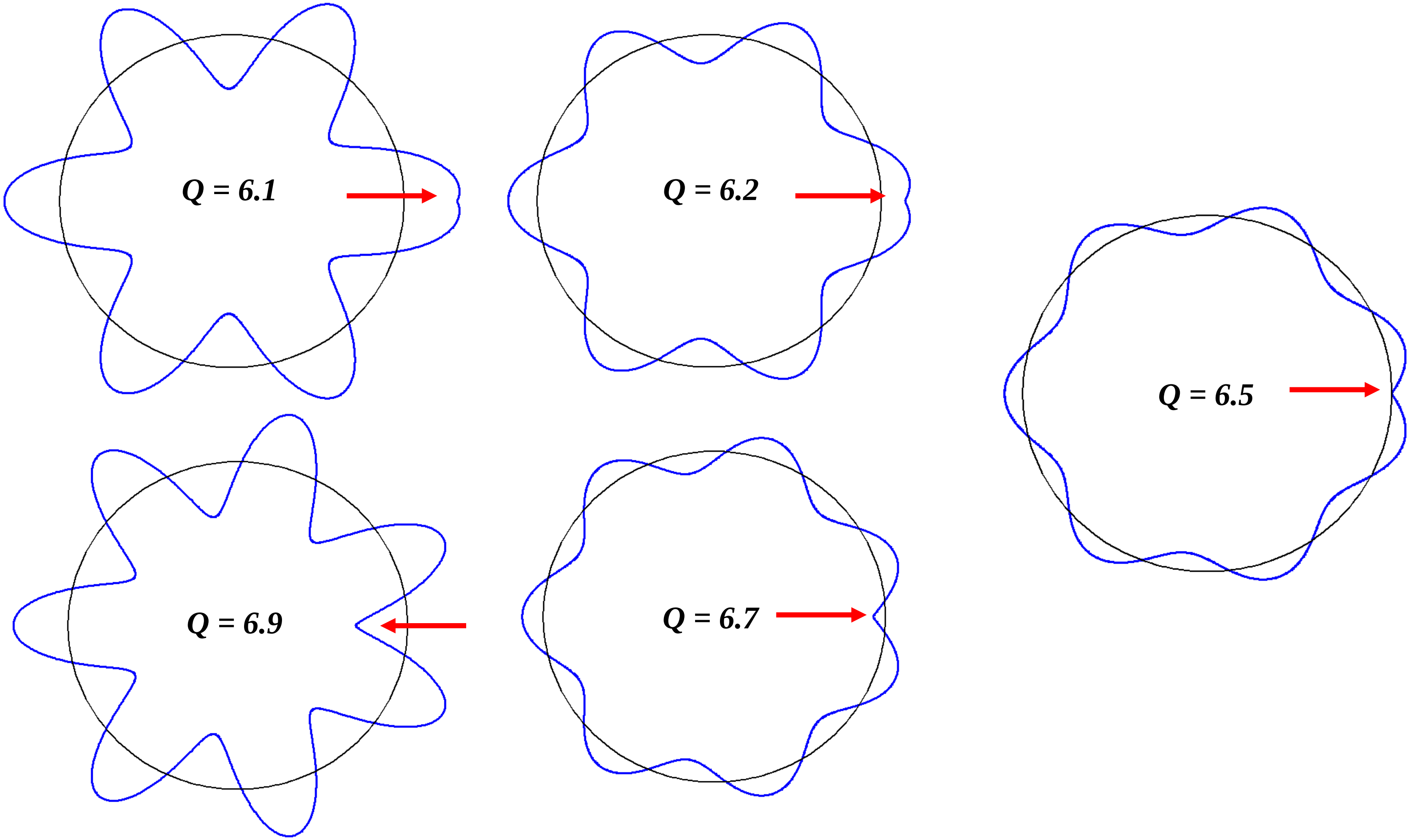
- Closed orbit distortion is most critical for **tunes close to integer → closed orbit becomes unstable**
- The closed orbit distortion propagates with the betatron phase advance (e.g. single kick induces 4 oscillations for a tune of Q=4.x)

$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$



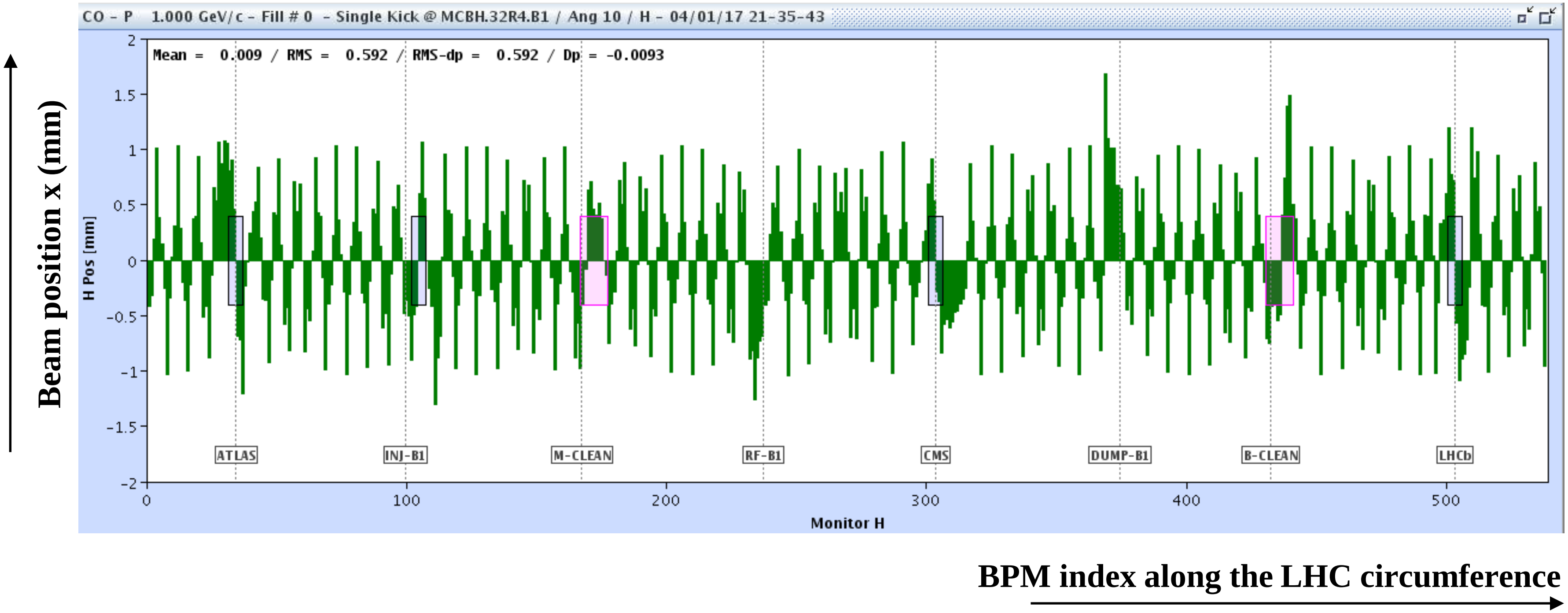
Closed orbit examples

- Example of horizontal closed orbit for a machine with tune Q = 6.x
- The **kink at the location of the deflection** (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.



A deflection at the LHC

- In the example below for the 26.7km long LHC, there is one undesired deflection, leading to a perturbed closed orbit.

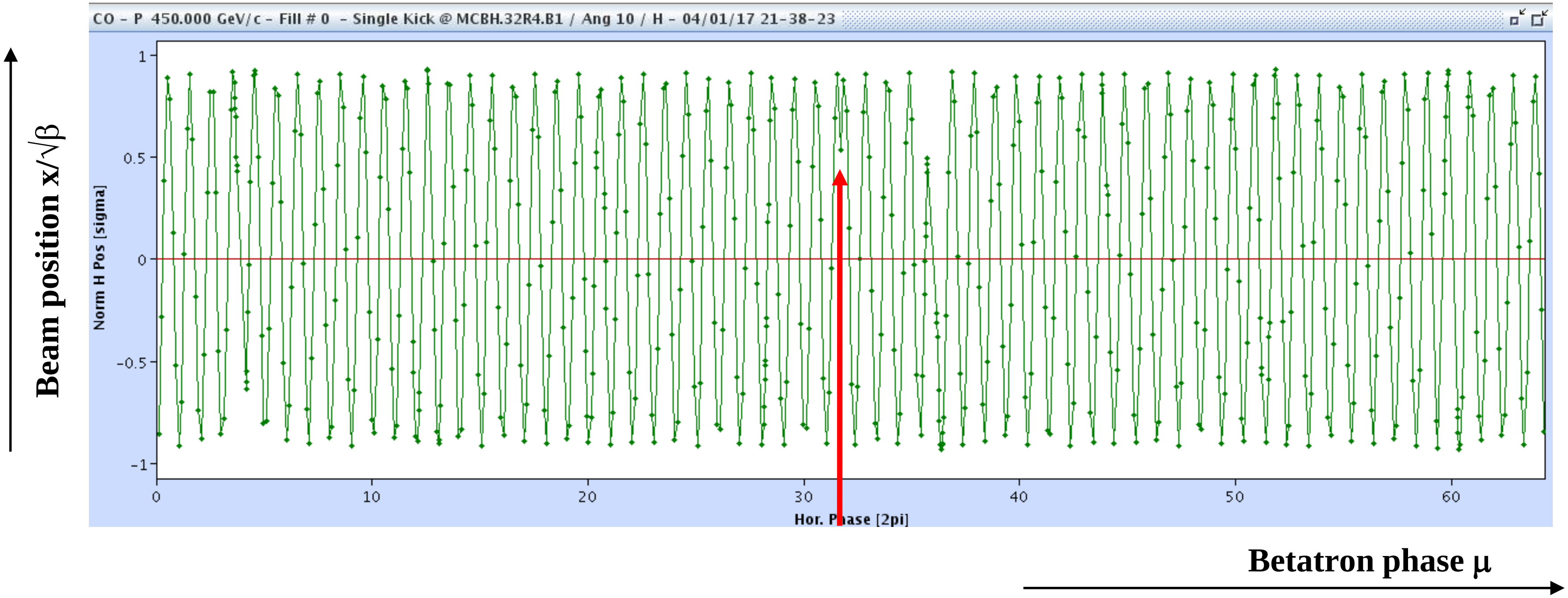


Where is the location of the deflection?

A deflection at the LHC

- To make our life easier we divide the position by √β(s) and replace the BPM index by its phase μ(s) → transform into pure sinusoidal oscillation

$$\frac{u(s)}{\sqrt{\beta(s)}} = \theta \frac{\sqrt{\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$



Can you localize the deflection now?

Global orbit distortion

Courant and Snyder, 1957

- Orbit distortion due to many errors

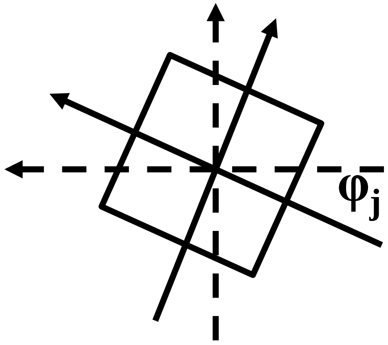
$$u(s)=\frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\int_s^{s+C}\theta(\tau)\sqrt{\beta(\tau)}\cos(\pi Q-|\psi(s)-\psi(\tau)|)d\tau$$

- By approximating the errors as delta functions in *n* locations, the distortion at *i* observation points (Beam Position Monitors) is

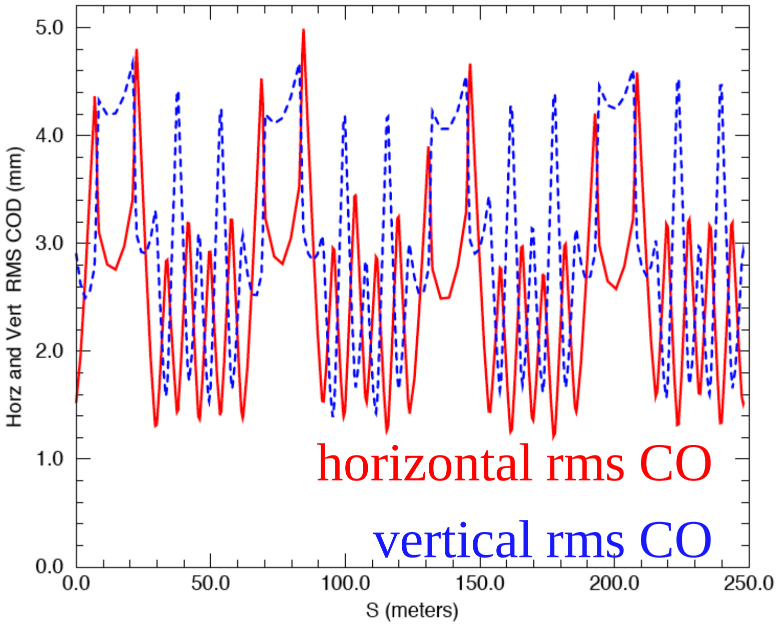
$$u_i=\frac{\sqrt{\beta_i}}{2\sin(\pi Q)}\sum_{j=i+1}^{i+n}\theta_j\sqrt{\beta_j}\cos(\pi Q-|\psi_i-\psi_j|)$$

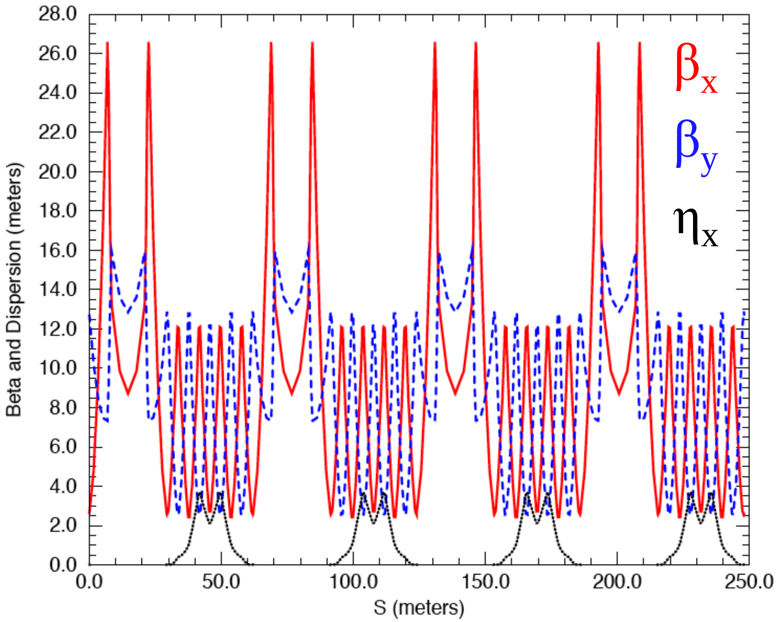
with the kick produced by the *j*th error

- Integrated dipole field error $\theta_j=\frac{\delta(B_j l_j)}{B\rho}$
- Dipole roll $\theta_j=\frac{B_j l_j\sin\phi_j}{B\rho}$
- Quadrupole displacement $\theta_j=\frac{G_j l_j\delta u_j}{B\rho}$



Example: Orbit distortion in SNS





- In the SNS accumulator ring, the beta function is **6m** in the dipoles and **30m** in the quadrupoles, the tune is **6.2**
- Consider dipole error of **1mrad**
- The maximum orbit distortion in dipoles is $u_0 = \frac{\sqrt{6 \cdot 6}}{2\sin(6.2\pi)} \cdot 10^{-3} \approx 5\text{mm}$
- For quadrupole displacement giving the same **1mrad** kick (and betas of 30m) the maximum orbit distortion is 25mm, to be compared to magnet radius of 105mm

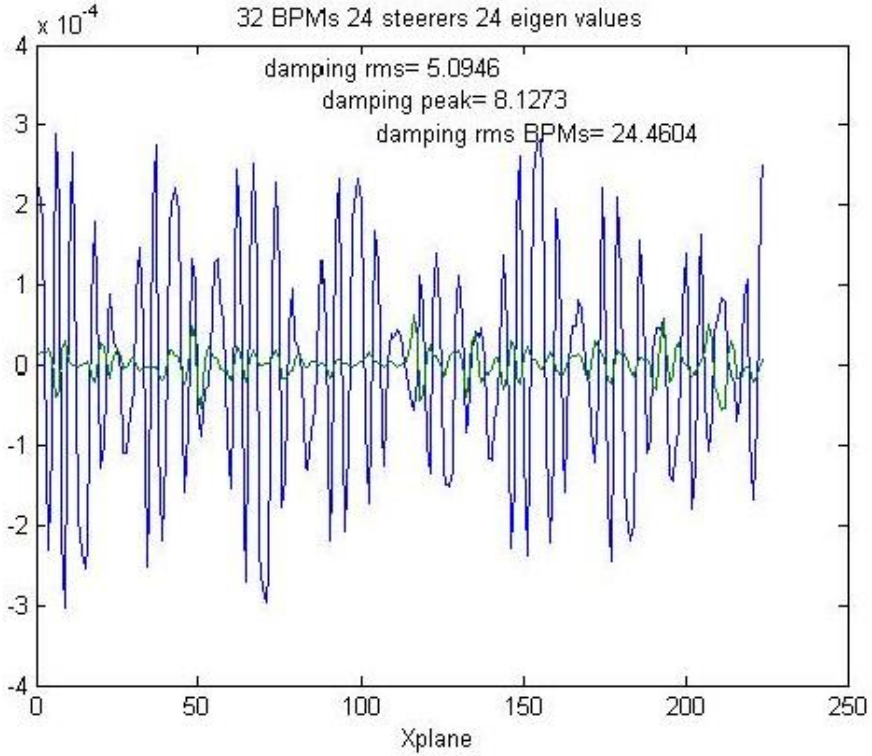
Example: Orbit distortion in ESRF

- In the ESRF storage ring, the beta function is **1.5m** in the dipoles and **30m** in the quadrupoles, the horizontal tune is **36.44**
- Consider dipole error of **1mrad**
- Maximum orbit distortion in dipoles

$$u_0 = \frac{\sqrt{1.5 \cdot 1.5}}{2\sin(36.44\pi)} \cdot 10^{-3} \approx 1\text{mm}$$

- For quadrupole displacement with **1mm**, the distortion is $u_0 \approx 8\text{mm}$!!!
- Magnet alignment is critical

Vertical orbit correction



Statistical estimation of orbit errors

- Consider random distribution of errors in N magnets
  - By squaring the orbit distortion expression and averaging over the angles (considering uncorrelated errors), the expectation (rms) value is given by

$$u_{\rm rms}(s) = \frac{\sqrt{\beta(s)}}{2\sqrt{2}|\sin(\pi Q)|}\Big(\sum_i \sqrt{\beta_i}\theta_i\Big)_{\rm rms} = \frac{\sqrt{N\beta(s)\beta_{\rm rms}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\rm rms}$$

- Example:
  - In the SNS ring, there are **32** dipoles and **54** quadrupoles
  - The rms value of the orbit distortion in the dipoles

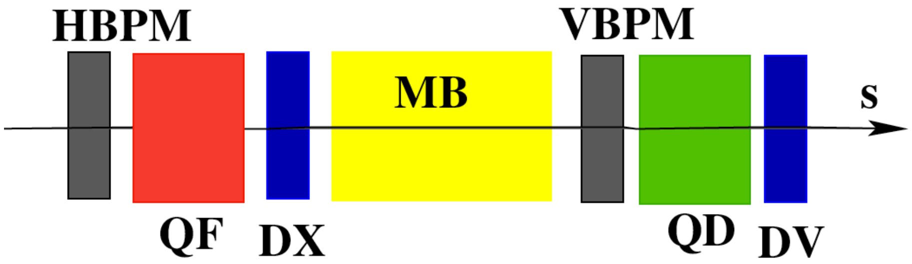
$$u_{\rm rms}^{\rm dip} = \frac{\sqrt{6\cdot 6\sqrt{32}}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 2{\rm cm}$$

  - In the quadrupoles, for equivalent kick

$$u_{\rm rms}^{\rm quad} = \frac{\sqrt{30\cdot 30\sqrt{54}}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 13{\rm cm}$$

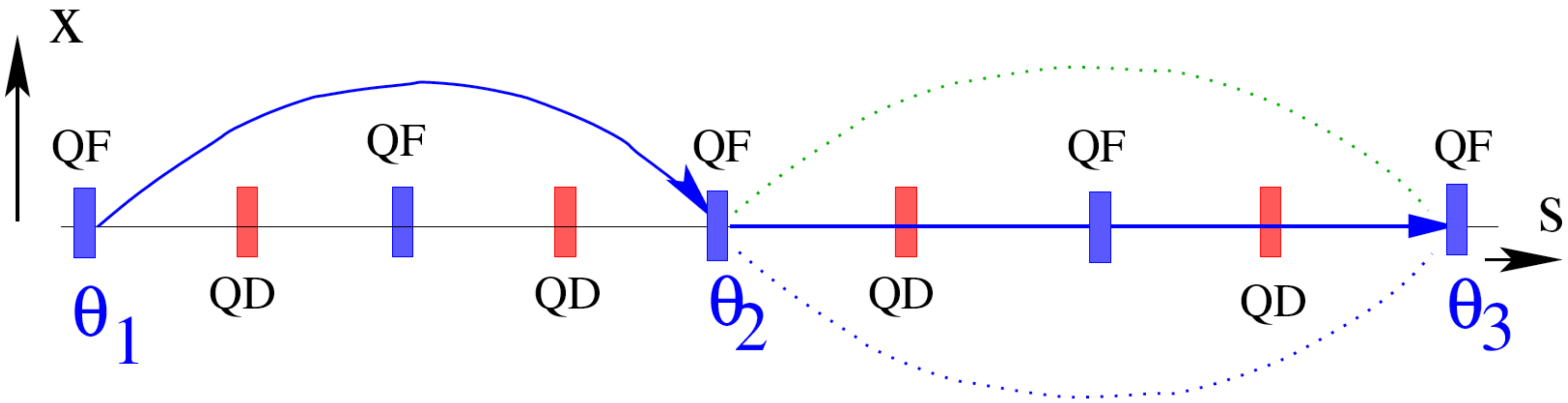
Correcting the orbit distortion

- Place horizontal and vertical dipole correctors and **b**eam **p**osition **m**onitors close to focusing and defocusing quads, respectively



- Measure orbit in BPMs and minimize orbit distortion
  - Globally
    - Harmonic, minimizing components of the orbit frequency response after a Fourier analysis
    - Most efficient corrector (MICADO), finding the most efficient corrector for minimizing the rms orbit
    - Least square minimization using the orbit response matrix of the correctors
  - Locally
    - Sliding Bumps
    - Singular Value Decomposition (SVD)

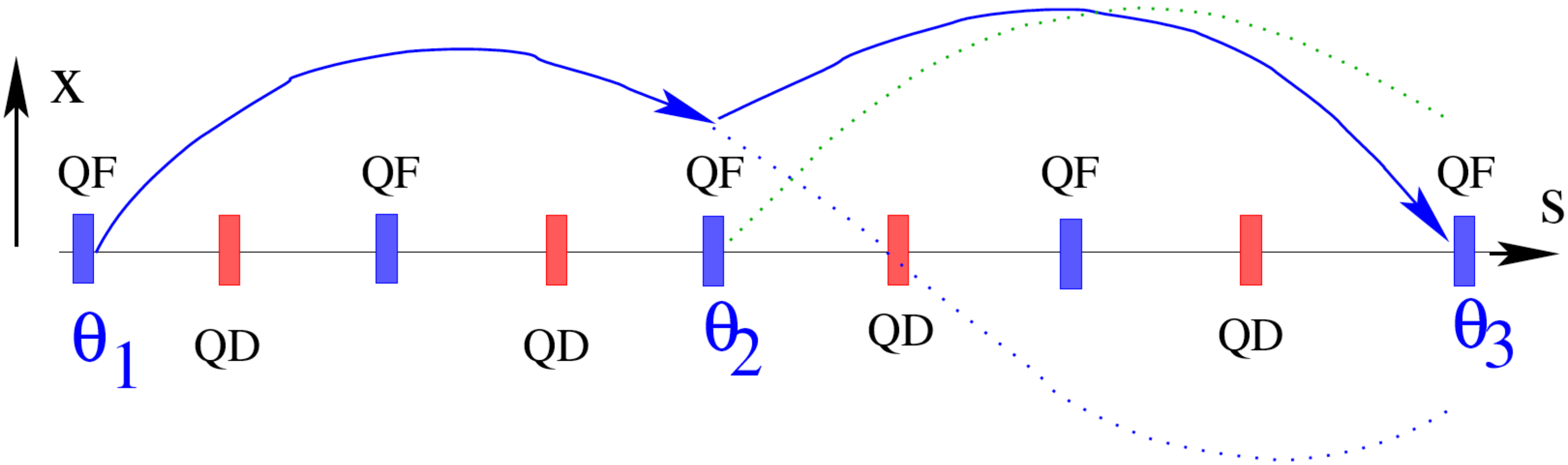
Orbit bumps: 2-bump



- Consider a cell, where correctors are placed close to the focusing quads
- The orbit shift at the 2nd corrector is $\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\,\theta_1$
- This orbit error can be eliminated by choosing a phase advance equal to ***π*** between correctors (this is called a "***π***-bump")
- The angle should satisfy the following equation

$$\boxed{\theta_2 = \delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}}\left[\cos(\psi_{12})\,\theta_1 - \alpha_2\sin(\psi_{12})\right] = \sqrt{\frac{\beta_1}{\beta_2}}\theta_1}$$

Orbit bumps: 3-bump

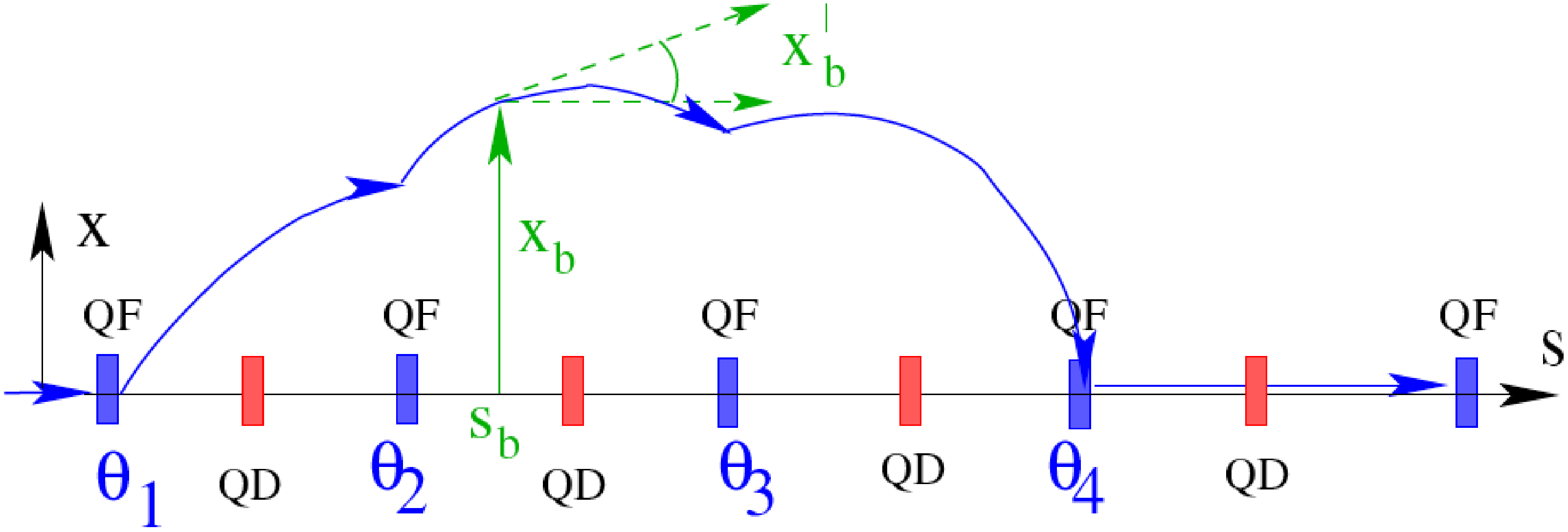


- **3-bump**: works for any phase advance if the three correctors satisfy

$$\boxed{\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3}$$

- Need large number of correctors
- No control of the angles at the entrance and exit of the bump

Orbit bumps: 4-bump



$$\theta_1 = \frac{1}{\sqrt{\beta_1\beta_s}}\frac{\cos\psi_{2s}-\alpha_s\sin\psi_{2s}}{\sin\psi_{12}}x_s - \sqrt{\frac{\beta_s}{\beta_1}}\frac{\sin\psi_{2s}}{\sin\psi_{12}}x'_s$$

$$\theta_2 = \frac{1}{\sqrt{\beta_2\beta_s}}\frac{\cos\psi_{1s}-\alpha_s\sin\psi_{1s}}{\sin\psi_{12}}x_s + \sqrt{\frac{\beta_s}{\beta_2}}\frac{\sin\psi_{1s}}{\sin\psi_{12}}x'_s$$

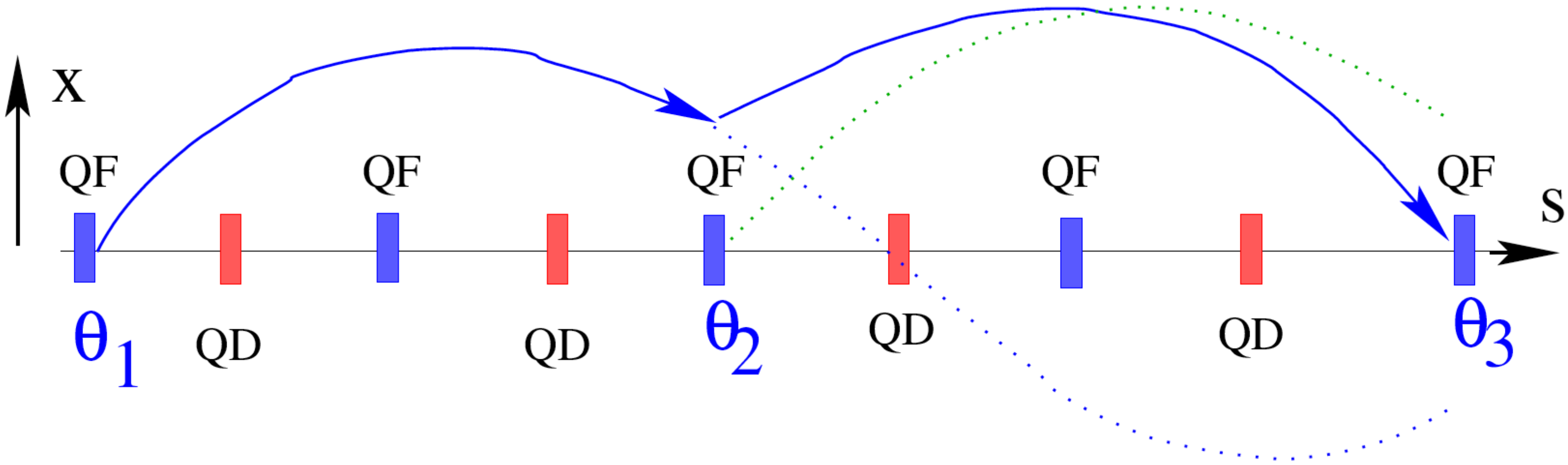
$$\theta_3 = \frac{1}{\sqrt{\beta_3\beta_s}}\frac{\cos\psi_{s4}-\alpha_s\sin\psi_{s4}}{\sin\psi_{34}}x_s - \sqrt{\frac{\beta_s}{\beta_3}}\frac{\sin\psi_{s4}}{\sin\psi_{34}}x'_s$$

$$\theta_4 = \frac{1}{\sqrt{\beta_4\beta_s}}\frac{\cos\psi_{s3}-\alpha_s\sin\psi_{s3}}{\sin\psi_{34}}x_s + \sqrt{\frac{\beta_s}{\beta_4}}\frac{\sin\psi_{s3}}{\sin\psi_{34}}x'_s$$

- **4-bump**: works for any phase advance
- Cancels position and angle outside of the bump
- Can be used for aperture scanning, extraction bumps, …

Problem 2

Three correctors are placed at locations with phase advance of **π/4** between them and beta functions of **12**, **2** and **12m**. How are the corrector kicks related to each other in order to achieve a closed 3-bump.

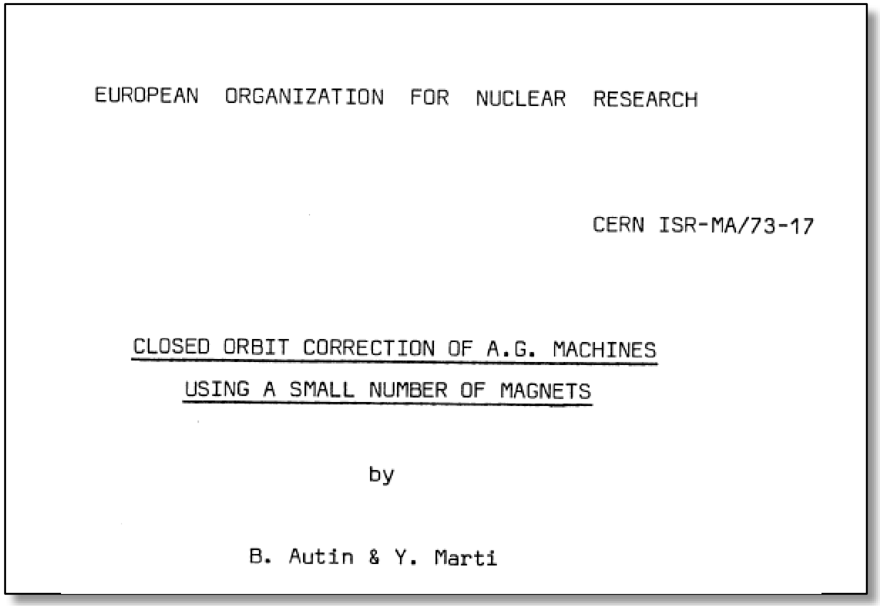


Closed orbit correction: MICADO

- The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.
- B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines: **MICADO***

* MInimisation des CArrés des Distortions d'Orbite.

(Minimization of the quadratic orbit distortions)



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

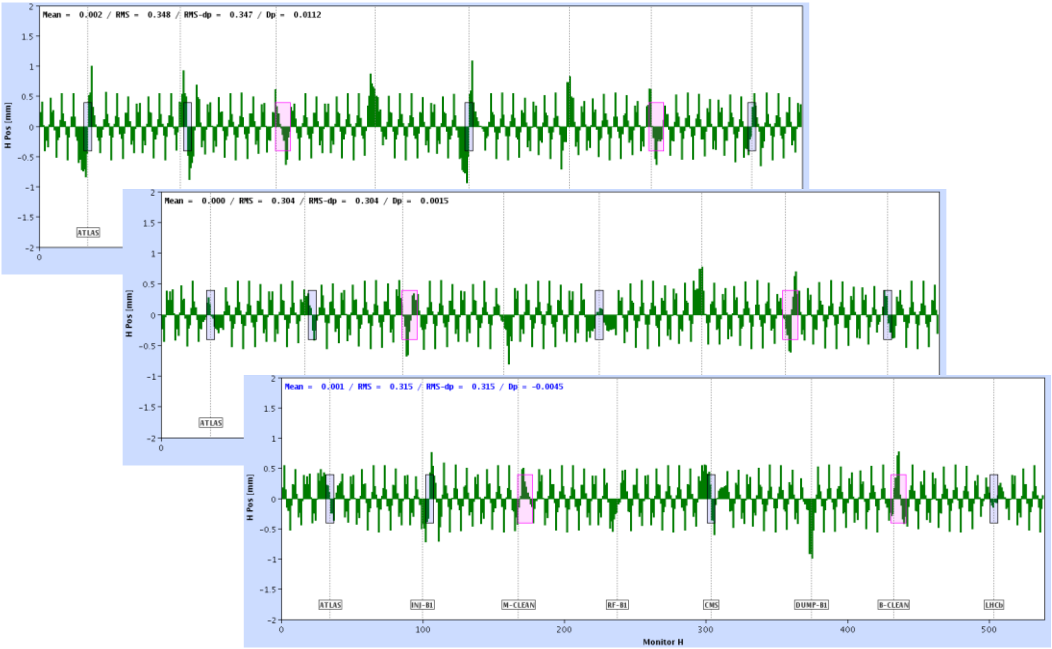
CLOSED ORBIT CORRECTION OF A.G. MACHINES
USING A SMALL NUMBER OF MAGNETS

by

B. Autin & Y. Marti

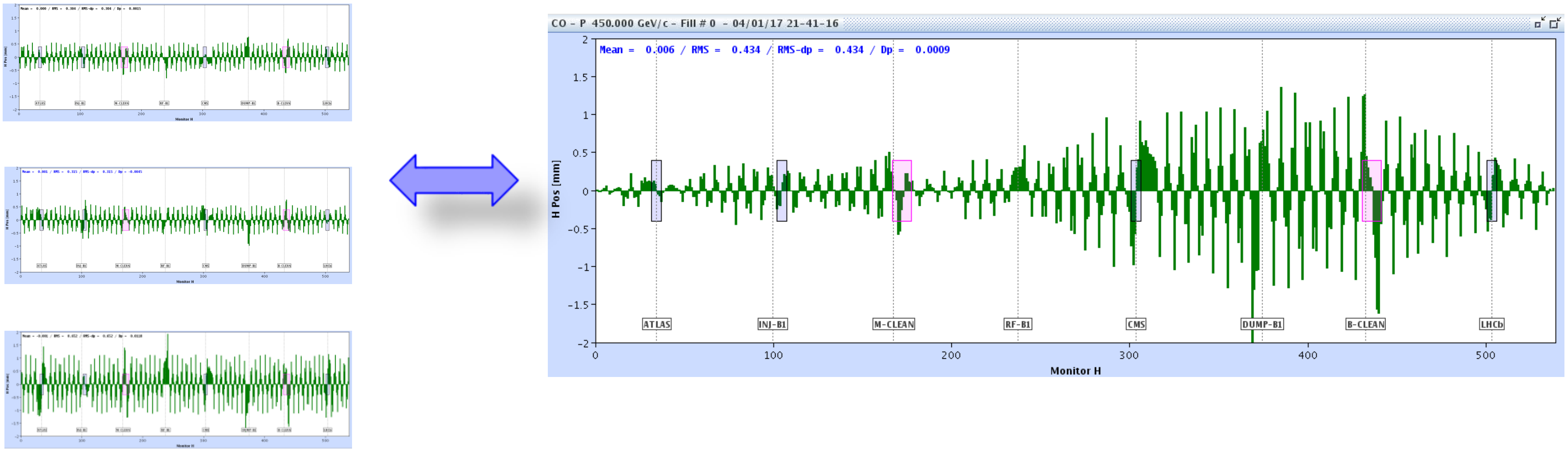
MICADO – how does it work?

- The intuitive principle of MICADO is rather simple.
- Preparation:
  - Need a model of the machine
  - Compute for each orbit corrector what the effect (response) is expected to be on the orbit



MICADO – how does it work?

- MICADO compares the response of every corrector with raw orbit



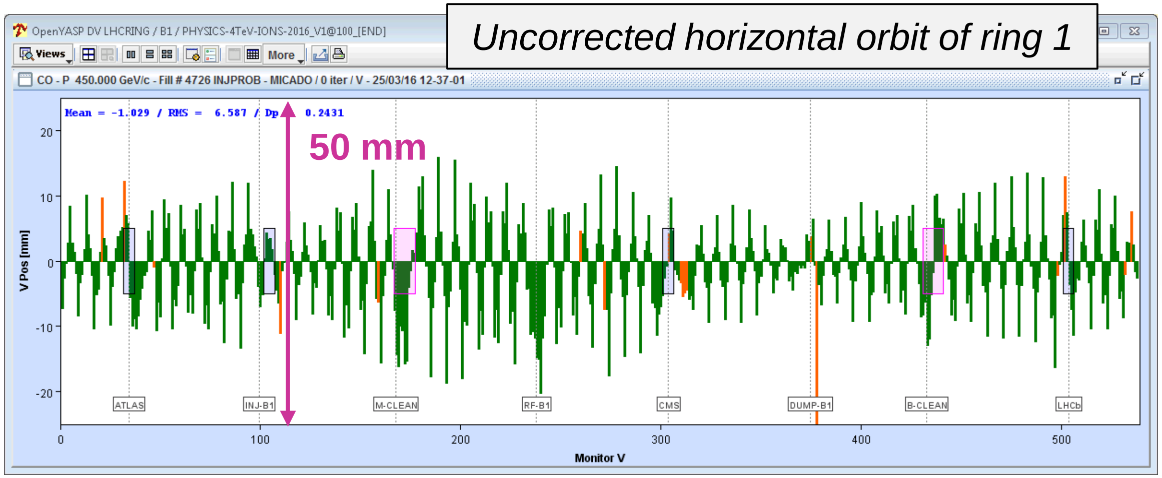
…

- MICADO picks out the corrector that has the **best match** with the orbit, and that will give the largest improvement to the orbit deviation rms

- The procedure can be iterated until the orbit is good enough (or as good as it can be)

MICADO – LHC Orbit example

- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by more than a factor 20.



Uncorrected horizontal orbit of ring 1

OpenYASP DV LHCRING / B1 / PHYSICS-4TeV-IONS-2016_V1@100_[END]

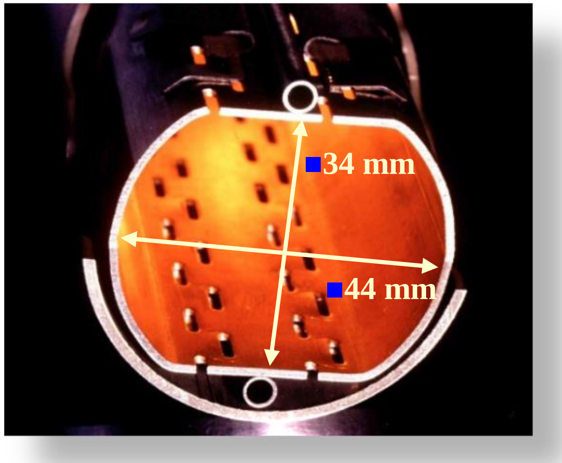
CO - P 450.000 GeV/c - Fill # 4726 INJPROB - MICADO / 0 iter / V - 25/03/16 12-37-01

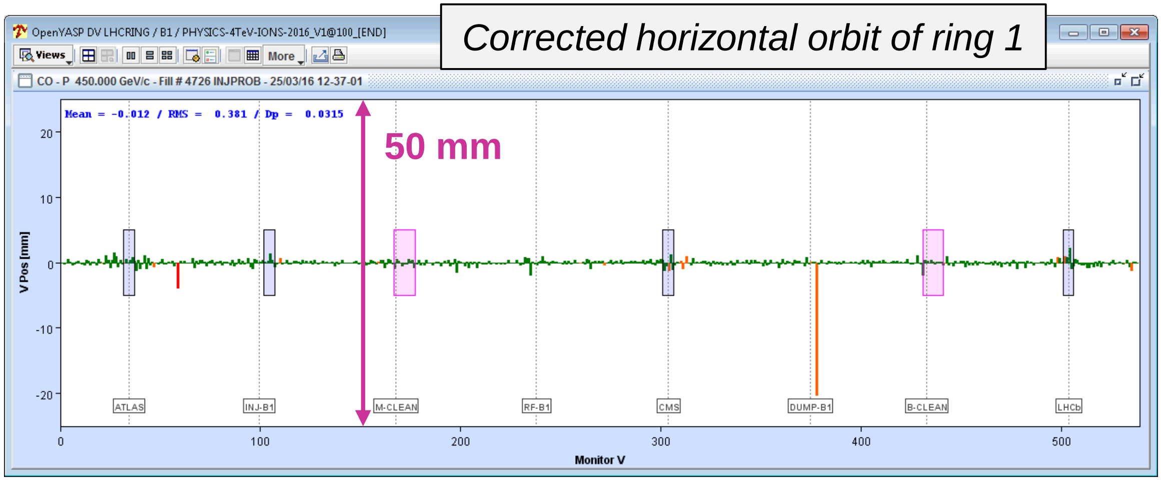
Mean = -1.029 / RMS = 6.587 / Dp = 0.2431

50 mm

MICADO & Co

LHC vacuum chamber





Corrected horizontal orbit of ring 1

OpenYASP DV LHCRING / B1 / PHYSICS-4TeV-IONS-2016_V1@100_[END]

CO - P 450.000 GeV/c - Fill # 4726 INJPROB - 25/03/16 12-37-01

Mean = -0.012 / RMS = 0.381 / Dp = 0.0315

50 mm

At the LHC a good orbit correction is vital !

Response matrix approach

- This approach works for orbit correction when using the measured orbit distortion (but also for beta-beating when using $\Delta\beta/\beta$, etc.)
  - Available set of correctors: $\vec{c}$
  - Available observables (here the Beam Position Monitors): $\vec{m}$
  - Assume the linear approximation is good (small corrections): $\mathbf{A}\vec{c} = \vec{m}$
  - Use optics model to compute response matrix $\mathbf{A}$ (i.e. the orbit change in the i^{th} monitor due to a unit kick from the j^{th} corrector):

$$A_{i,j} = \frac{\sqrt{\beta_i\beta_j}\cos(\pi Q - |\mu_i - \mu_j|)}{2\sin(\pi Q)}$$

… or use, e.g., MADX

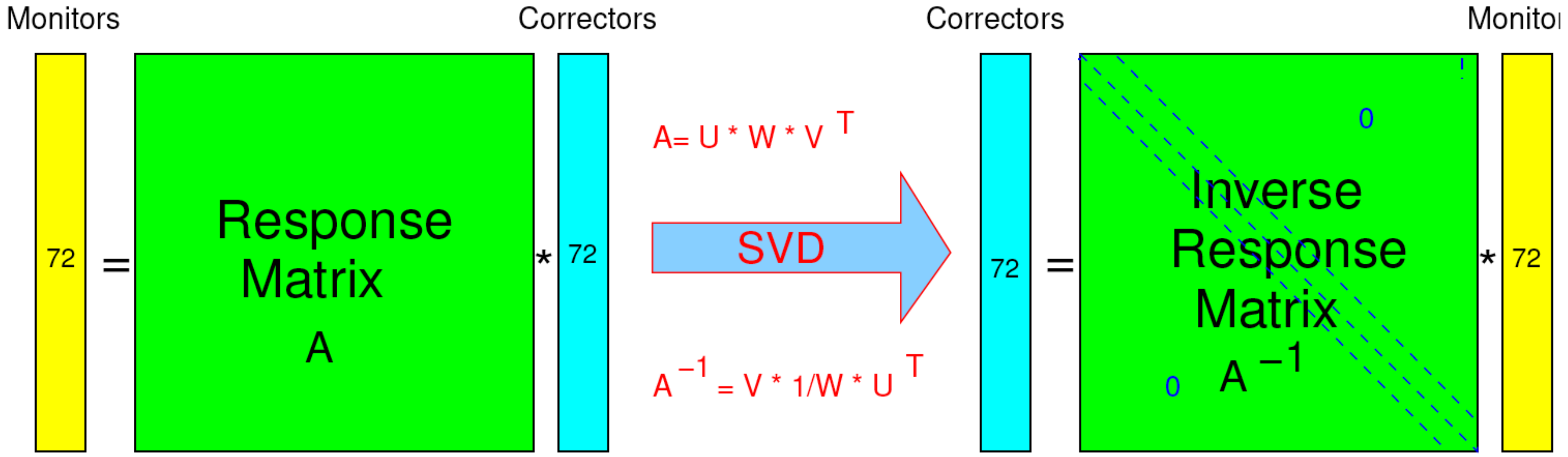
  - Invert or pseudo-invert the response matrix A to compute an effective global correction based on the measured Δm :

$$\Delta\vec{c} = \mathbf{A}^{-1}\Delta m$$

  - In case the number of correctors is not the same as the number of Beam Position Monitors one has to perform a pseudo matrix inversion, for example using the "Singular Value Decomposition (SVD)" algorithm

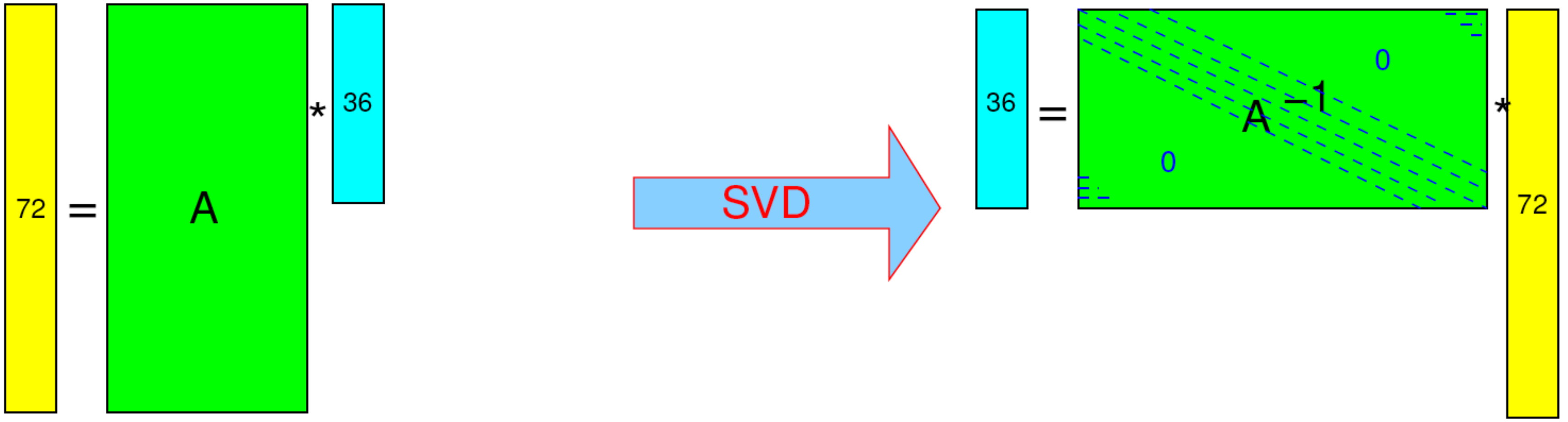
Singular Value Decomposition

N monitors / N correctors



=> Minimization of the RMS orbit (=0 in case of "Matrix Inversion" using all Eigenvalues)

N monitors / M correctors

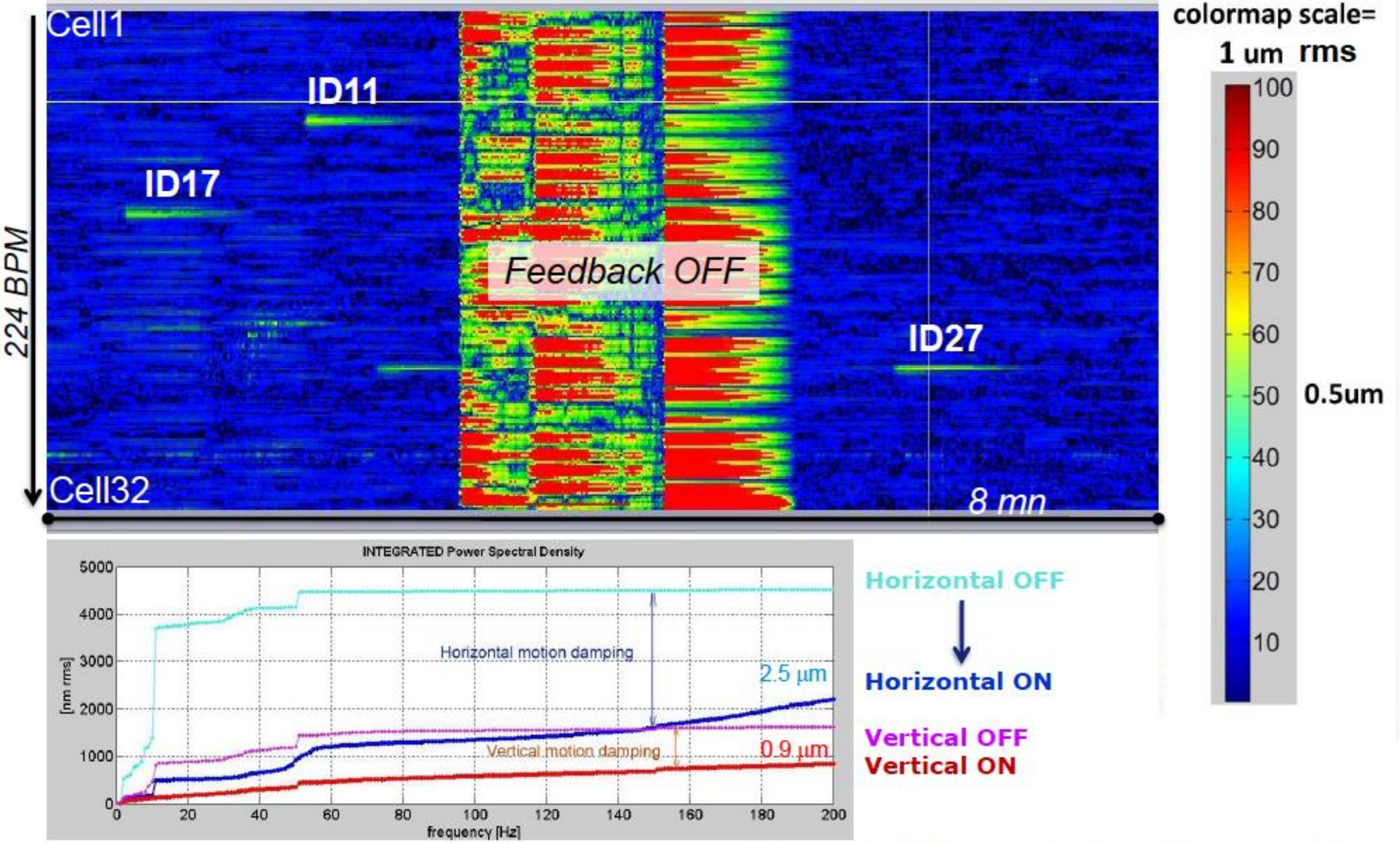


=> Minimization of the RMS orbit (monitor averaging)

Orbit feedback

- Closed orbit stabilization performed using slow and fast orbit feedback system.
- Slow feedback operates every few seconds and uses complete set of BPMs for both planes
- Efficient in correcting distortion due to current decay in magnets or other slow processes
- Fast orbit correction system operates in a wide frequency range
  - correcting distortions induced by quadrupole and girder vibrations (up to 10kHz for the ESRF)
- Local feedback systems used to damp oscillations in areas where beam stabilization is critical (interaction points, insertion devices)

Example: Orbit Feedback at ESRF



~0.1µm stability routinely achieved in V

~1.0µm stability routinely achieved in H

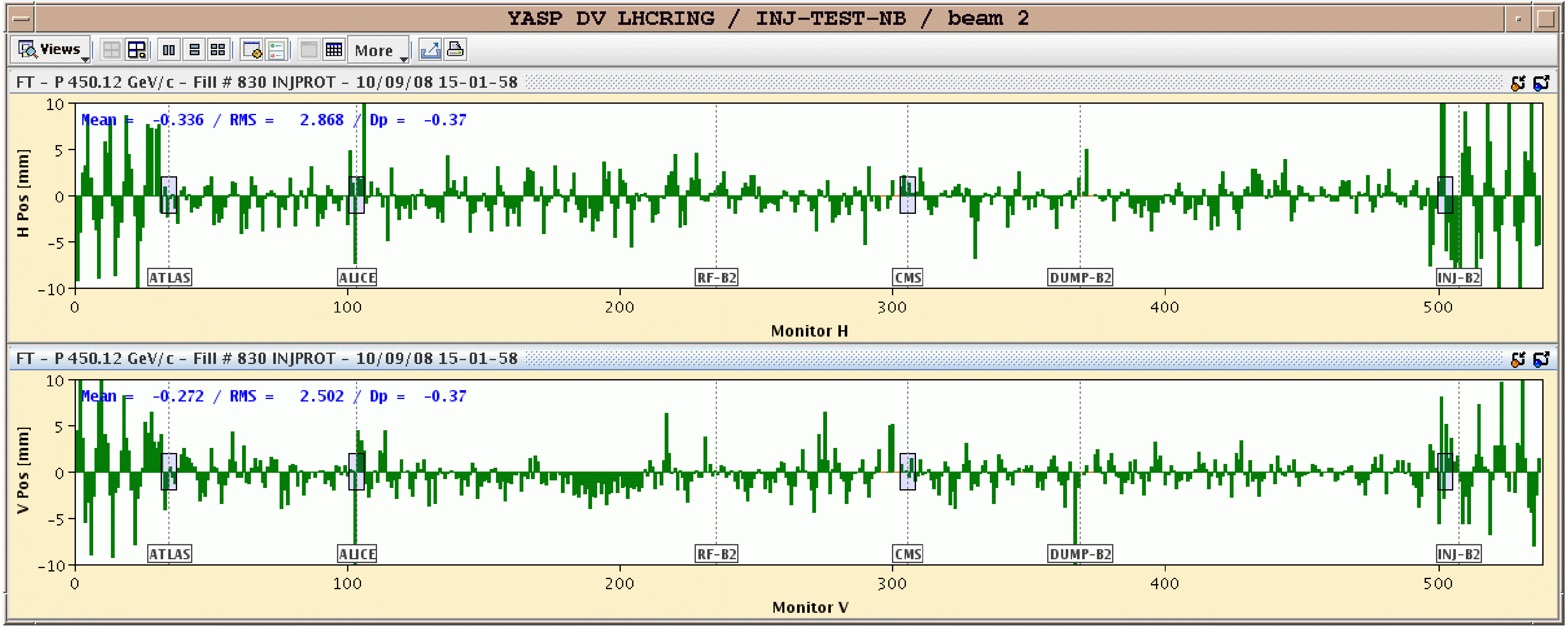
P. Raimondi 2014

Beam threading

- Threading the beam round the LHC ring (very first commissioning)
  - One beam at a time, one hour per beam.
  - Collimators were used to intercept the beam (1 bunch, 2 × 109 protons)
  - Beam through 1 sector (1/8 ring) → correct trajectory, open collimator and move on

Beam 2 threading

BPM availability ~ 99%



YASP DV LHCRING / INJ-TEST-NB / beam 2

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

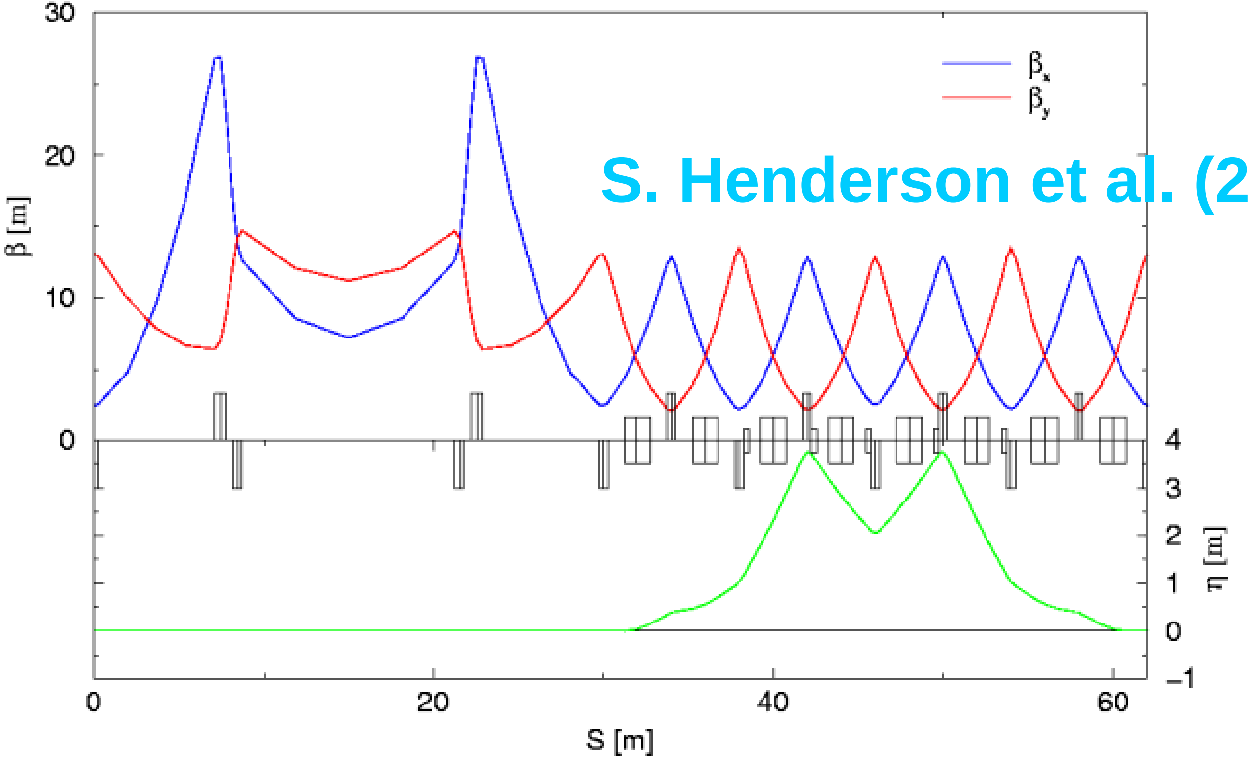
Mean = -0.336 / RMS = 2.868 / Dp = -0.37

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.272 / RMS = 2.502 / Dp = -0.37

Problem 3

SNS: A **proton** ring with kinetic energy of **1GeV** and a **circumference** of **248m** has **18, 1m-long** focusing quads with **gradient** of **5T/m**. In one of the quads, the horizontal and vertical **beta** function is of **12m** and **2m** respectively. The **rms beta** function in both planes on the focusing quads is **8m**. With a **horizontal tune** of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on the single focusing quad given by **horizontal** and by **vertical** misalignments of **1mm** in all the quads. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?



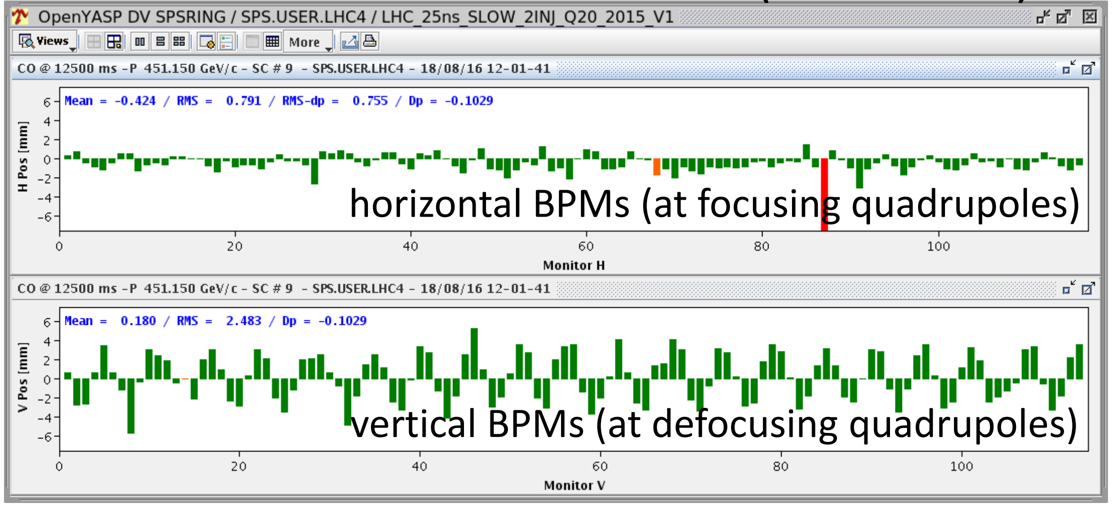
S. Henderson et al. (2005)

Problem 4

<u>SPS:</u> Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads which are **30m** and **108m** for the defocusing ones. The tunes of the machine are **Qx=20.13** and **Qy=20.18**. Due to a mechanical problem, one **<u>focusing quadrupole</u>** was **slowly sinking down** in 2016, resulting in an increasing closed orbit distortion wrt a reference taken in the beginning of the year.

- By how much the quadrupole had shifted down when the maximum vertical closed orbit distortion amplitude in <u>defocusing</u> quadrupoles reached 4 mm?
- Why was there no change of the horizontal orbit measured?
- How big would have been the maximum closed orbit distortion amplitude if it would have been a defocusing quadrupole?

Difference orbit wrt reference (18.08.2016)



OpenYASP DV SPSRING / SPS.USER.LHC4 / LHC_25ns_SLOW_2INJ_Q20_2015_V1

CO @ 12500 ms -P 451.150 GeV/c - SC # 9 - SPS.USER.LHC4 - 18/08/16 12-01-41

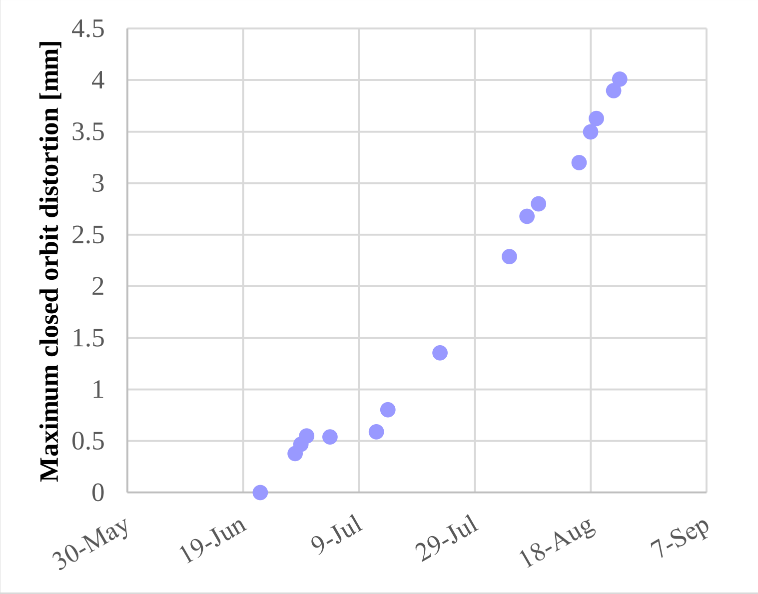
Mean = -0.424 / RMS = 0.791 / RMS-dp = 0.755 / Dp = -0.1029

horizontal BPMs (at focusing quadrupoles)

CO @ 12500 ms -P 451.150 GeV/c - SC # 9 - SPS.USER.LHC4 - 18/08/16 12-01-41

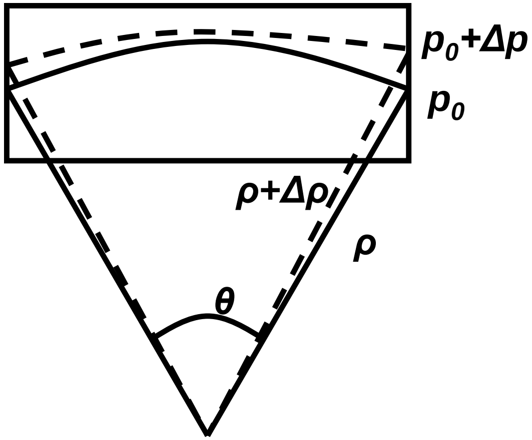
Mean = 0.180 / RMS = 2.483 / Dp = -0.1029

vertical BPMs (at defocusing quadrupoles)



Off-momentum particles in a dipole

- Up to now all particles had the same momentum p_0
- What happens for off-momentum particles, i.e. particles with momentum $p_0+\Delta p$?
  - Consider a dipole with field B and bending radius ρ
  - Recall that the magnetic rigidity is $B\rho = \frac{p_0}{q}$ and for off-momentum particles



$$B(\rho+\Delta\rho) = \frac{p_0+\Delta p}{q} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p_0}$$

  - Considering the effective length of the dipole unchanged

$$\theta\rho = l = \text{const.} \Rightarrow \rho\Delta\theta + \theta\Delta\rho = 0 \Rightarrow \frac{\Delta\theta}{\theta} = -\frac{\Delta\rho}{\rho} = -\frac{\Delta p}{p_0}$$

- Off-momentum particles get different deflection (different orbit)

$$\boxed{\Delta\theta = -\theta\frac{\Delta p}{p_0}}$$

Dispersion equation

- Consider the equations of motion for off-momentum particles

$$x'' + K_x(s)x = \frac{1}{\rho(s)}\frac{\Delta p}{p}$$

  - The solution is a sum of the **homogeneous** (on-momentum) and the **inhomogeneous** (off-momentum) equation solutions

$$x(s) = x_H(s) + x_I(s)$$

  - In that way, the equations of motion are split in two parts

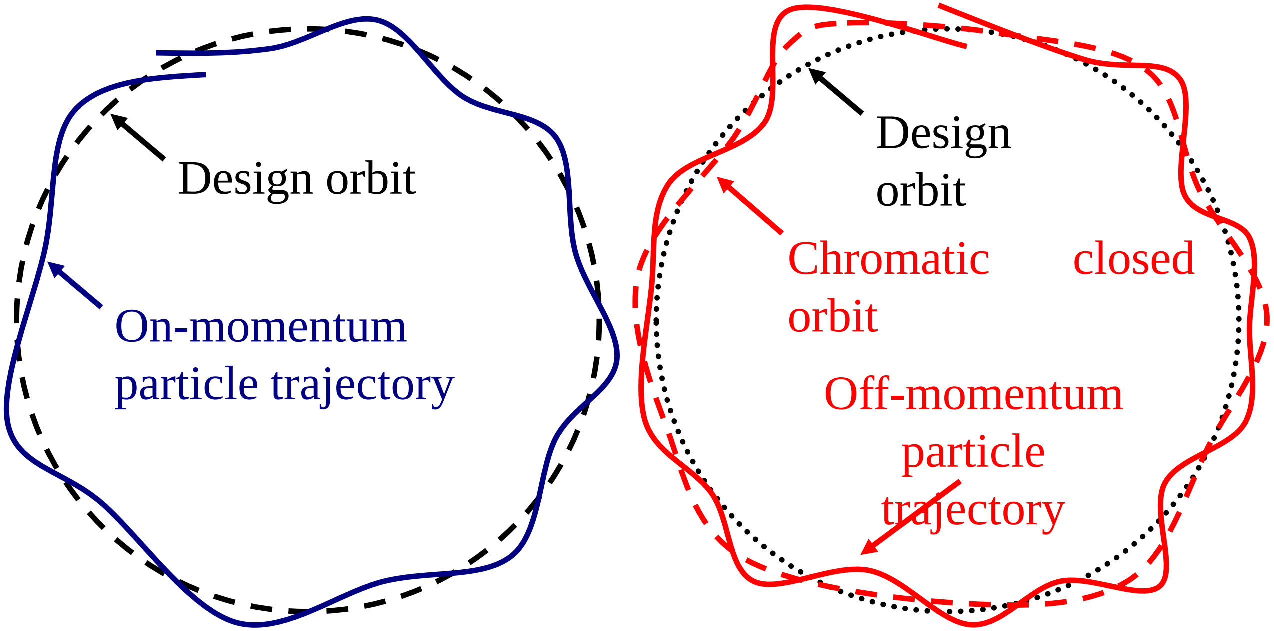
$$\begin{aligned} x''_H + K_x(s)x_H &= 0 \\ x''_I + K_x(s)x_I &= \frac{1}{\rho(s)}\frac{\Delta p}{p} \end{aligned}$$

- The **dispersion function** can be defined as $D(s) = \dfrac{x_I(s)}{\Delta p/p}$
- The dispersion equation is

$$\boxed{D''(s) + K_x(s)\ D(s) = \frac{1}{\rho(s)}}$$

Closed orbit including dispersion

- ❑ Design orbit defined by main dipole field
- ❑ On-momentum particles oscillate around design orbit (w/o errors)
- ❑ Off-momentum particles are not oscillating around design orbit, but around "chromatic" closed orbit, defined by the dispersion function times the momentum offset



$$x = x_{CO}(s) + x_{\beta}(s) + D(s)\frac{\Delta p}{p}$$

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  - ❑ Effect of single and multiple dipole kicks
  - ❑ Closed orbit correction methods
  - ❑ Dispersion and chromatic orbit
- ❑ Optics function distortion (gradient error)
  - ❑ Imperfections leading to optics distortion
  - ❑ Tune-shift and beta distortion due to gradient errors
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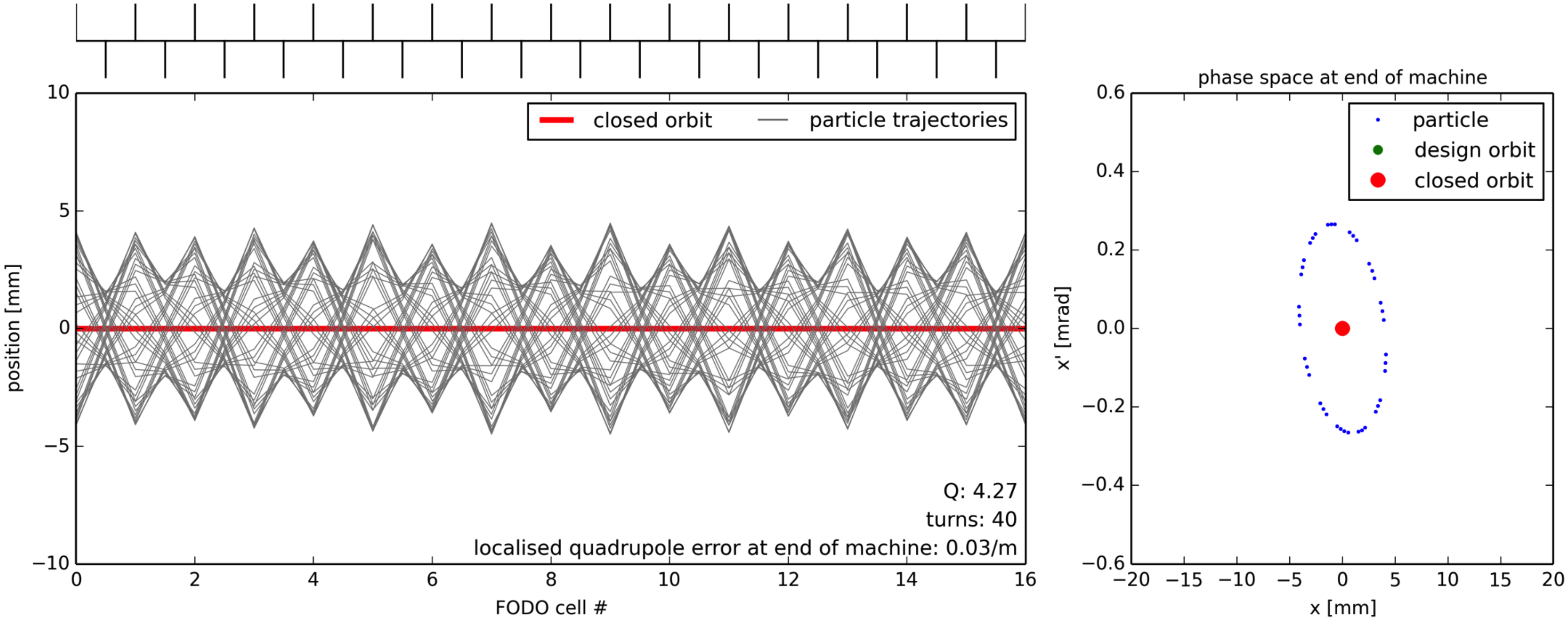
- ❑ Coupling error
  - ❑ Coupling errors and their effect
  - ❑ Coupling correction
- ❑ Chromaticity

Gradient error and optics distortion

- Optics functions perturbation can induce aperture restrictions
- Tune perturbation can lead to dynamic aperture loss
- Broken super-periodicity → excitation of all resonances
  - In a ring made out of *N* identical cells, only resonances that are integer multiples of *N* can be excited
- Sources
  - Errors in quadrupole strengths (random and systematic)
  - Injection elements
  - Higher-order multi-pole magnets and errors
- Observables
  - Tune-shift
  - Beta-beating
  - Excitation of integer and half integer resonances

Illustration of optics distortion

- Ideal machine toy model with regular FODO lattice and quadrupole error at the end of circumference
  - Particle injected with offset performs betatron oscillations but gets additional focusing from quadrupole error
  - Beam envelope is distorted around the machine … "beta-beating"



Gradient error

- Consider the transfer matrix for 1-turn

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a gradient error in a quad. In thin element approximation the quad matrix with and without error are

$$m_0 = \begin{pmatrix} 1 & 0 \\ -K_0(s)ds & 1 \end{pmatrix} \text{ and } m = \begin{pmatrix} 1 & 0 \\ -(K_0(s) + \delta K)ds & 1 \end{pmatrix}$$

- The new 1-turn matrix is $\mathcal{M} = m m_0^{-1} \mathcal{M}_0 = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$

  which yields

$$\mathcal{M} = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ \delta K ds(\cos(2\pi Q) - \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Consider a new matrix after 1 turn with a new tune $\chi = 2\pi(Q+\delta Q)$

$$\mathcal{M}^{\star} = \begin{pmatrix} \cos(\chi)+\alpha_0\sin(\chi) & \beta_0\sin(\chi) \\ -\gamma_0\sin(\chi) & \cos(\chi)-\alpha_0\sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal

$$\mathrm{Tra}(\mathcal{M}^{\star}) = \mathrm{Tra}(\mathcal{M})$$

which gives $2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q+\delta Q))$

- Developing the right hand side

$$\cos(2\pi(Q+\delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{2\pi\delta Q}$$

and finally $4\pi\delta Q = \delta K ds \beta_0$

- For a quadrupole of finite length, we have $\boxed{\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds}$

Gradient error and beta distortion

- Consider the unperturbed transfer matrix for one turn

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \quad \text{with} \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

$$m_{12}^\star = (\beta_0 + \delta\beta)\sin(2\pi(Q + \delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

where we used $\sin(2\pi\delta Q) \approx 2\pi\delta Q$ and $\cos(2\pi\delta Q) \approx 1$

Gradient error and beta distortion

- On the other hand

$$a_{12} = \sqrt{\beta_0 \beta(s_1)}\, \sin\psi, \;\; b_{12} = \sqrt{\beta_0 \beta(s_1)}\, \sin(2\pi Q - \psi)$$

and $m^{\star}_{12} = \underbrace{b_{11} a_{12} + b_{12} a_{22}}_{m_{12}} - a_{12} b_{12} \delta K ds = m_{12} - a_{12} b_{12} \delta K ds$

- Equating the two terms

$$\delta\beta \sin(2\pi Q) + 2\pi \delta Q \beta_0 \cos(2\pi Q) = -\, a_{12} b_{12} \delta K ds$$

- Integrating through the quad

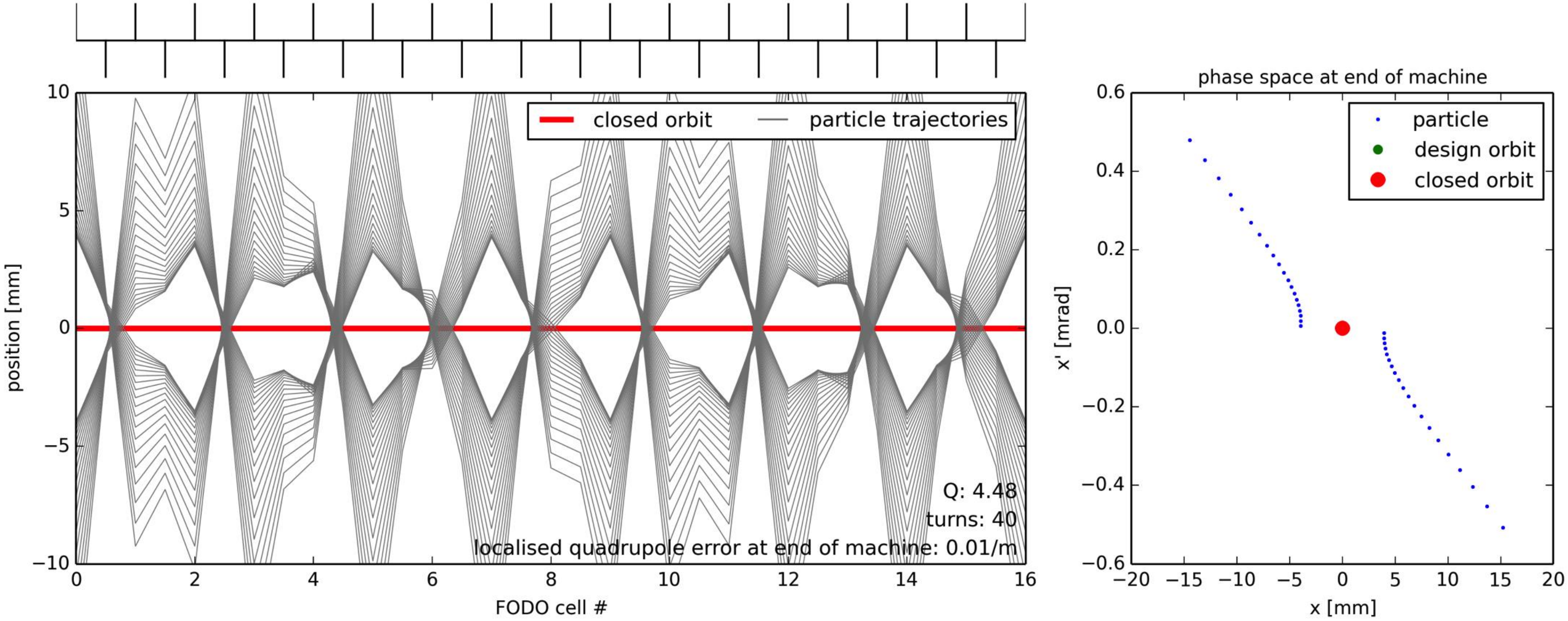
$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)} \int_{s_1}^{s_1+l} \beta(s) \delta K(s) \cos(2\psi - 2\pi Q) ds$$

- There is also an equivalent effect on dispersion

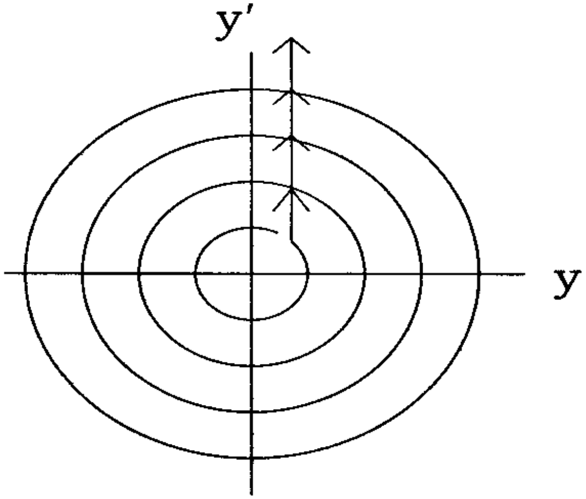
Optics distortion vs. tune

- Quadrupole errors have biggest impact **close to integer and half integer tunes → envelope (or beam size) becomes unstable**
- Optics distortion propagates with twice the tune (count the peaks)

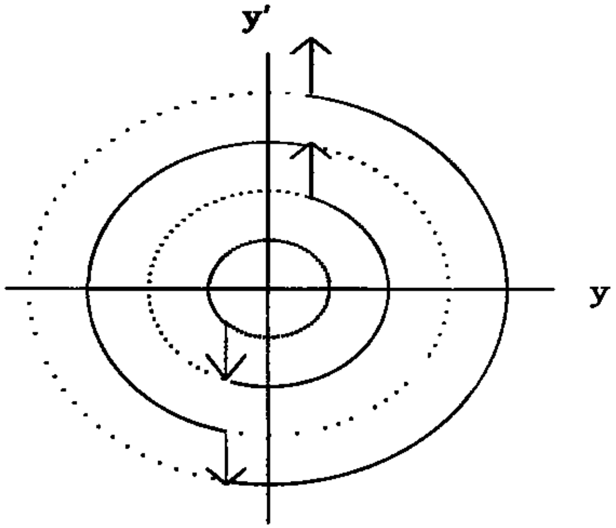
$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+l}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$



Quadrupole error in phase space



Q = N (integer)
→ kicks from quadrupoles add up (same as for kicks from dipoles)



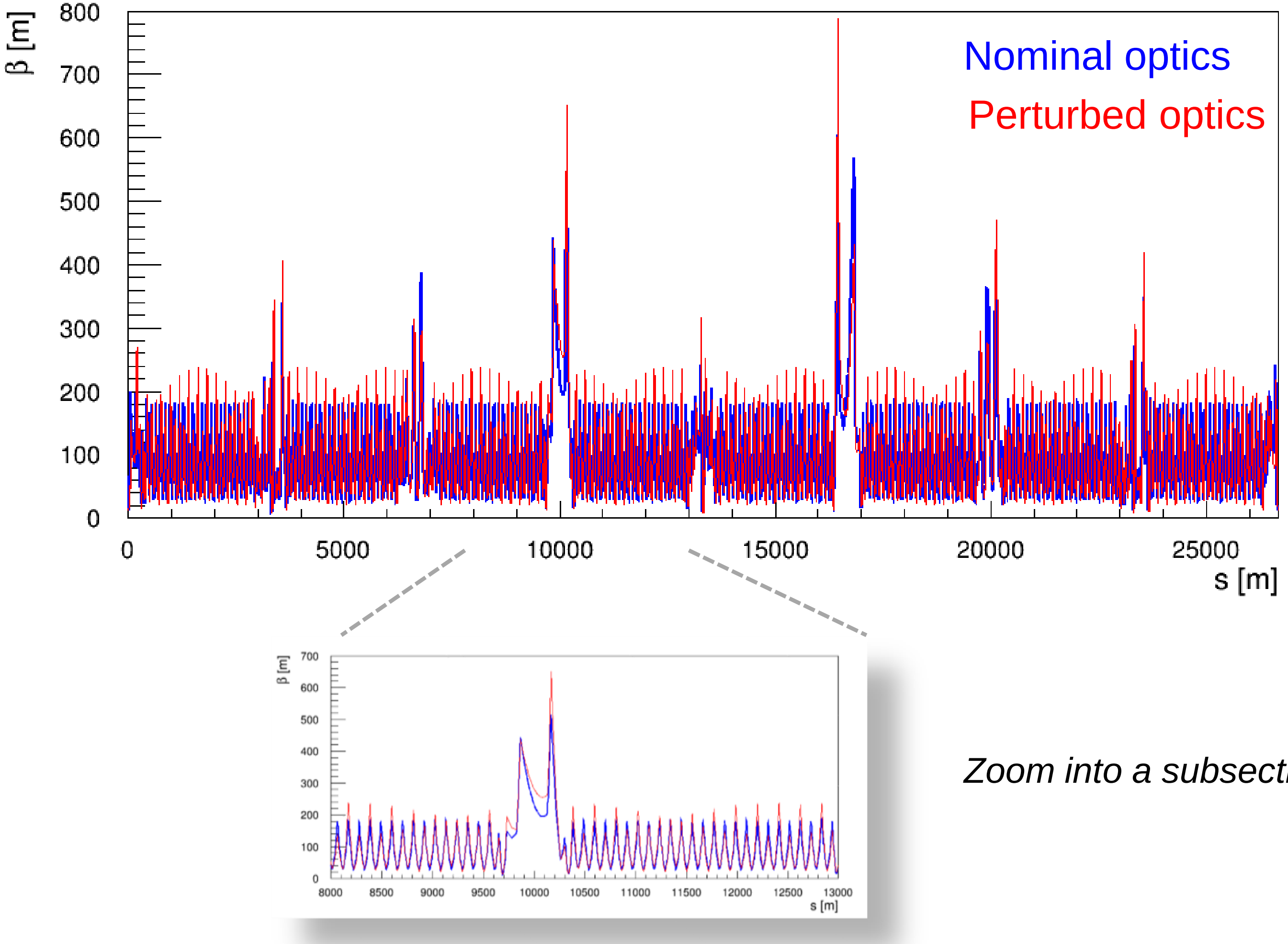
Q = N/2 (half integer)
→ kicks from quadrupoles add up
(while kicks from dipoles cancel)

- Therefore **integer tunes and half integer tunes need to be avoided for machine operation** to avoid beam envelope becoming unstable due to quadrupole errors
- Recall: for integer tunes dipole errors drive the closed orbit unstable

Optics distortion characteristics

- Let’s take a look at the LHC …

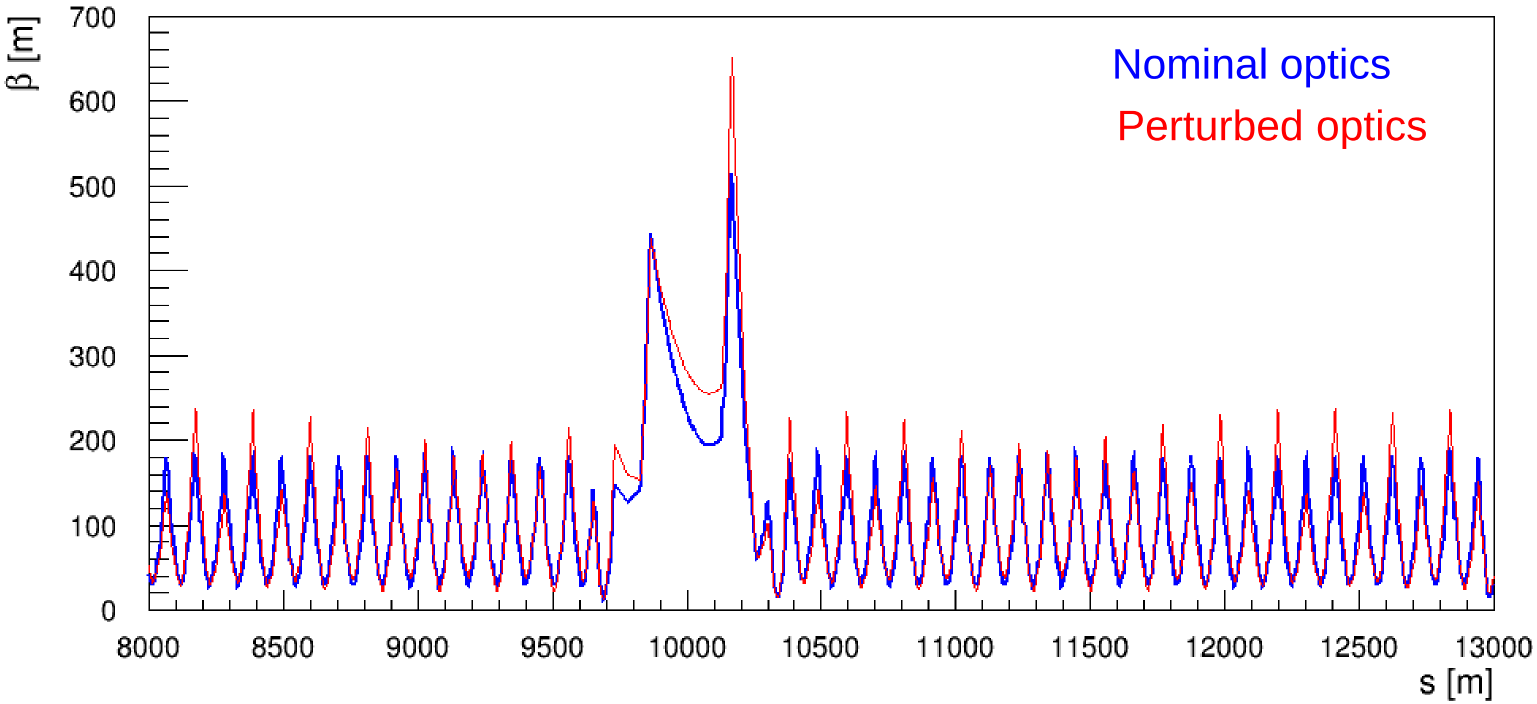
Example: one quadrupole gradient is incorrect



Zoom into a subsection

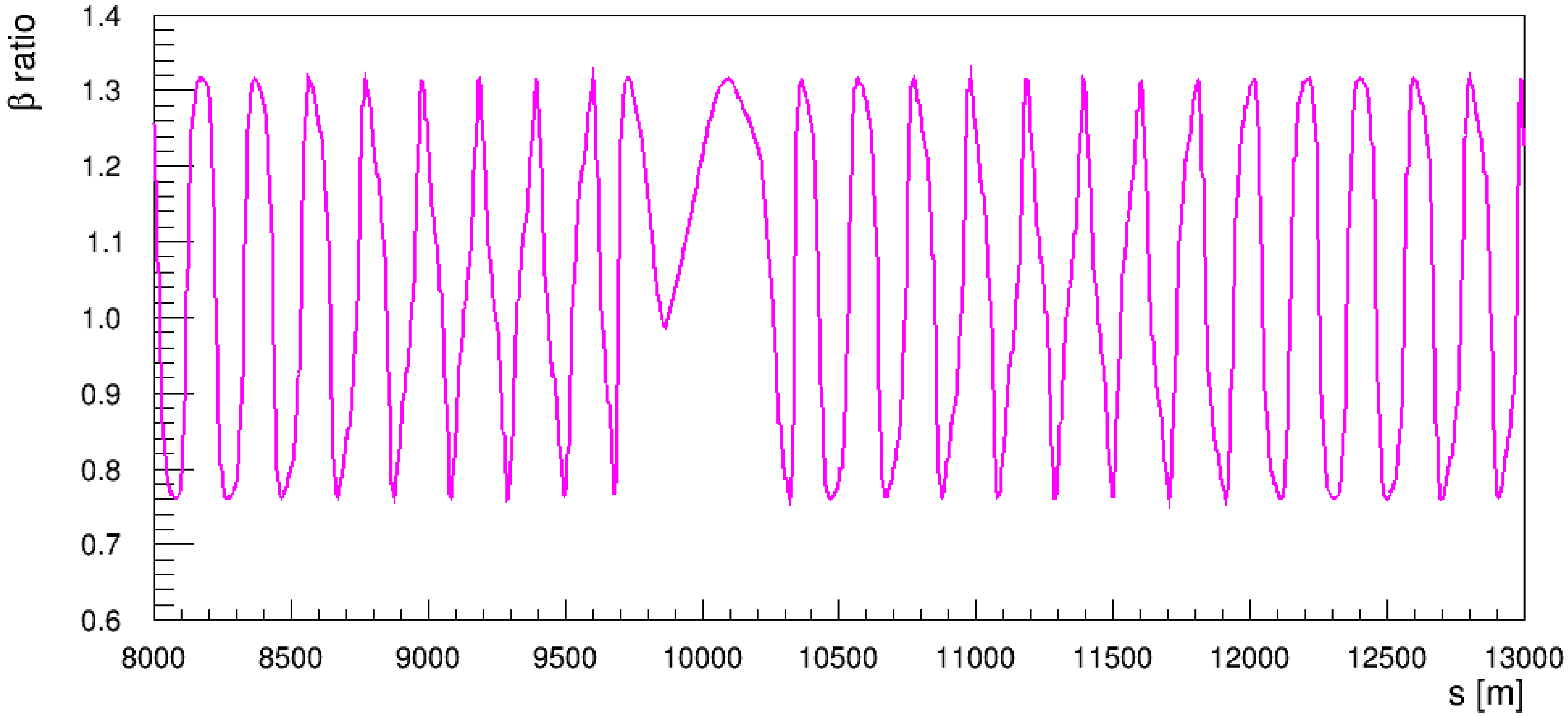
Optics distortion characteristics

- The local beam optics perturbation
- … note the oscillating pattern



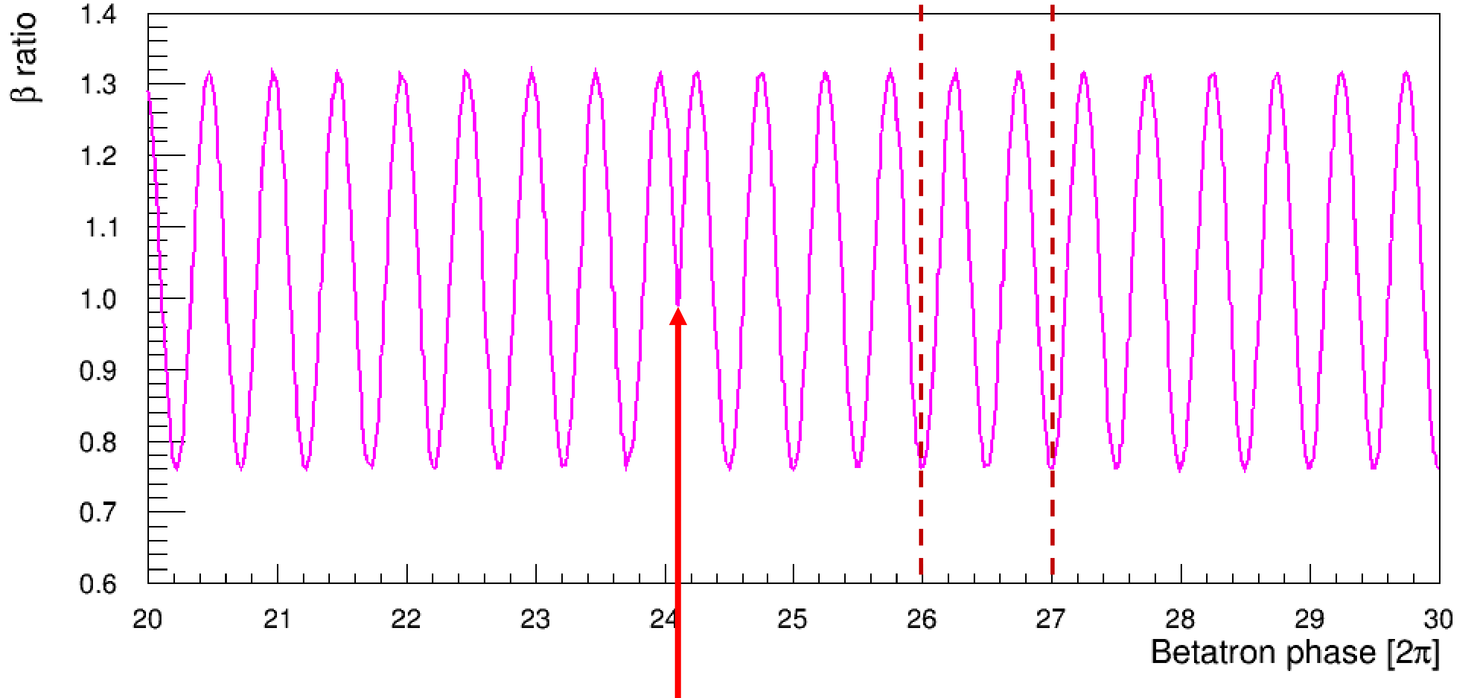
Optics distortion characteristics

- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** (‘beta-beating’). The amplitude of the perturbation is the same all over the ring!

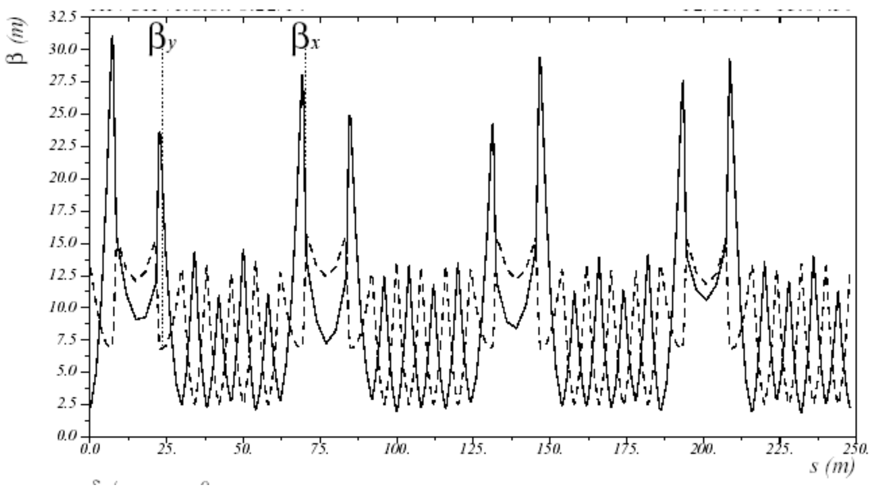


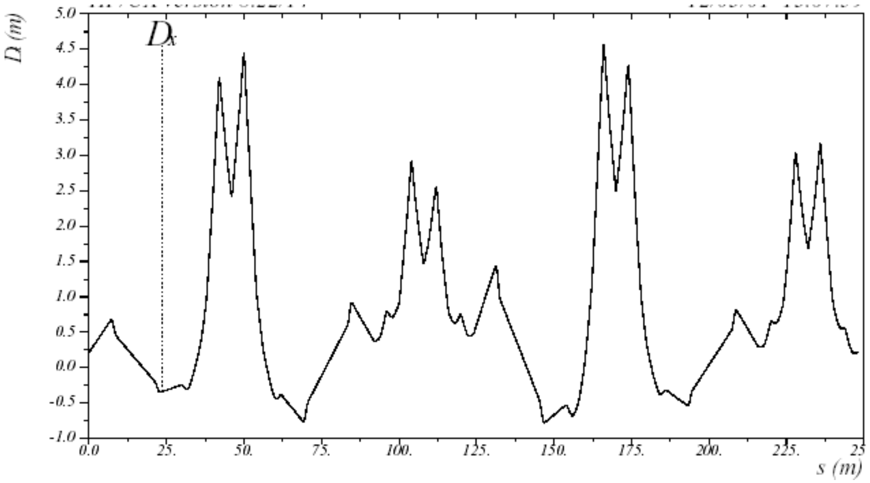
Optics distortion characteristics

- The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance
- The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink!
- If you watch closely you will observe that there are two oscillation periods per 2π (360 deg) phase. The beta-beating frequency is twice the frequency of the orbit!



Example: Gradient error in SNS





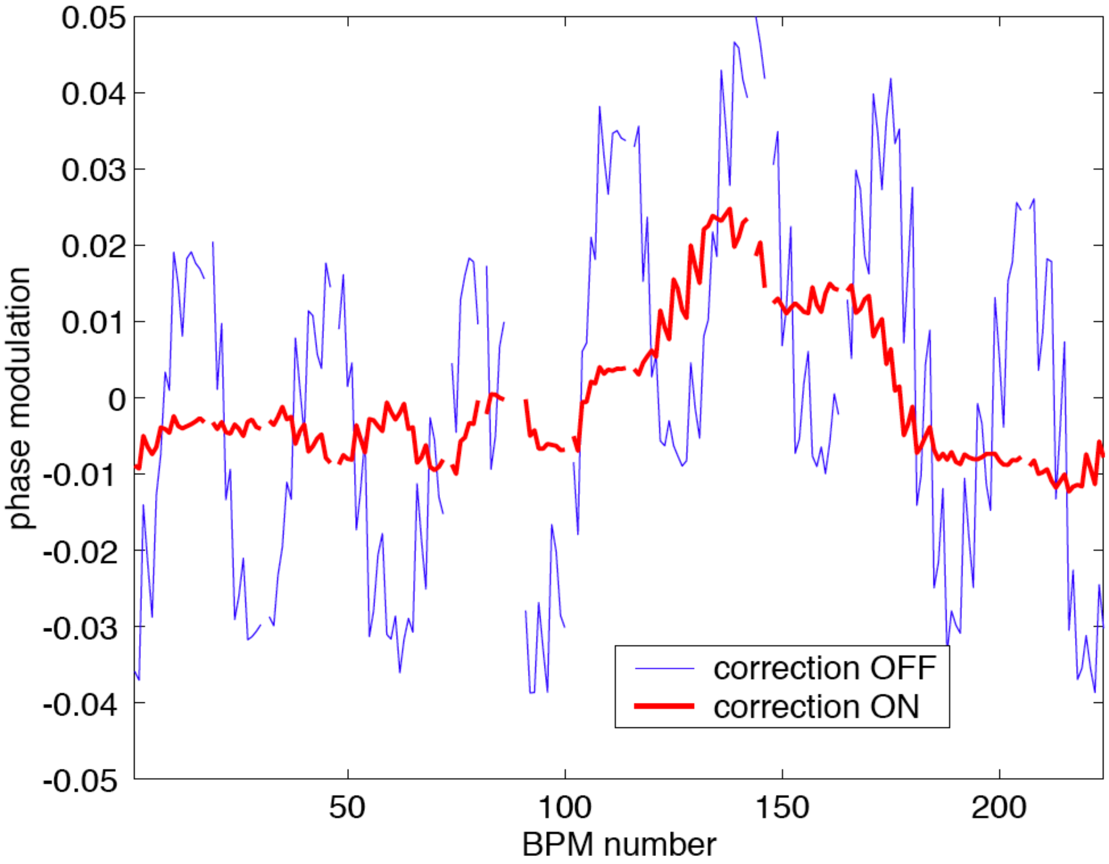
- Consider **18** focusing quads in the SNS ring with **0.01T/m** gradient error. In this location β=**12m**. The length of the quads is **0.5m** and the magnetic rigidity is **5.6567Tm**
  - The tune-shift is $\delta Q = \frac{1}{4\pi} 18 \cdot 12 \frac{0.01}{5.6567} 0.5 = 0.015$
- For a random distribution of errors the beta beating is

$$\frac{\delta\beta}{\beta_0}_{\text{rms}} = -\frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2 \beta_i^2\right)^{1/2}$$

  - Optics functions beating > **20%** by random errors (1% of gradient) in high dispersion quads of the SNS ring … justifies correctors strengths

Example: Gradient error in ESRF

- Consider **128** focusing arc quads in the ESRF storage ring with **0.001T/m** gradient error. In this location β**=30m**. The length of the quads is around **1m.** The magnetic rigidity of the ESRF is **20Tm.**



- The tune-shift is

$$\delta Q = \frac{1}{4\pi} 128 \cdot 30 \frac{0.001}{20} 1 = 0.014$$

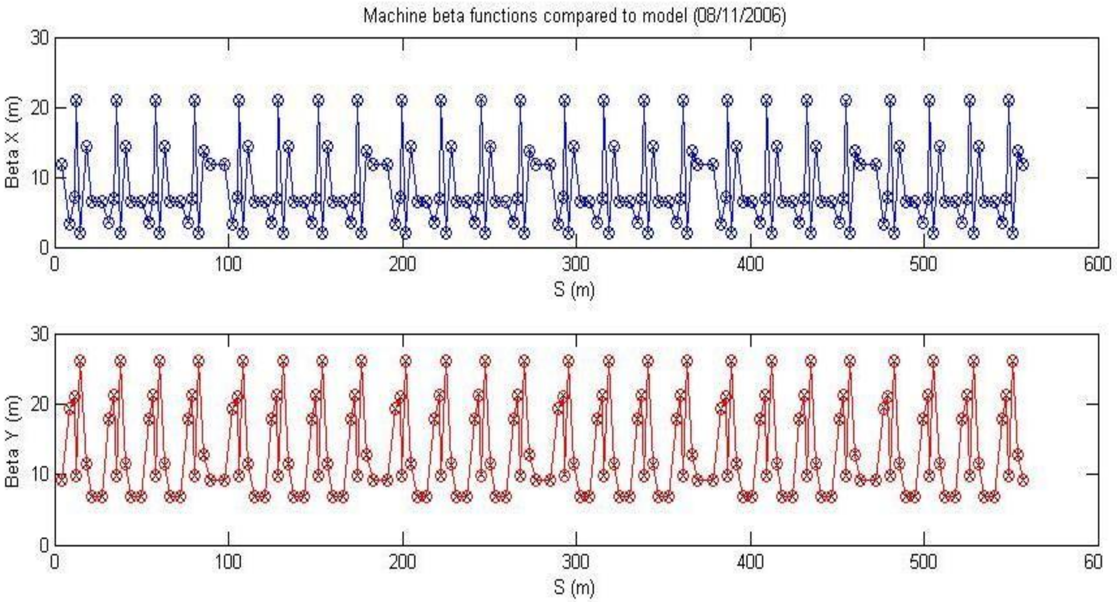
Gradient error correction

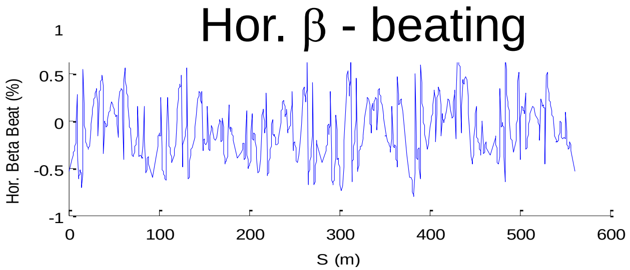
- Quadrupole correctors
  - Individual correction magnets
  - Windings on the core of the quadrupoles (trim windings)
- Methods & approaches
  - Compute tune-shift and optics function beta distortion
  - Move working point close to integer and half integer resonance to increase sensitivity
  - Minimize beta wave or quadrupole resonance width with trim windings
  - Individual powering of trim windings can provide flexibility and beam based alignment of BPM
- Modern methods of response matrix analysis (LOCO) can fit optics model to real machine and correct optics distortion

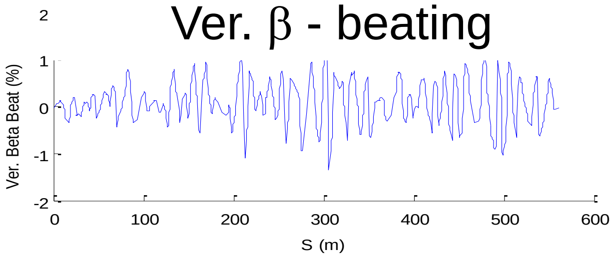
Linear Optics from Closed Orbit

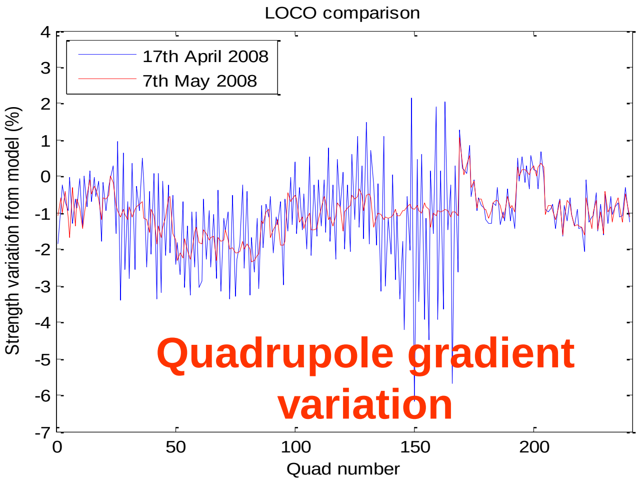
R. Bartolini, LER2010

J. Safranek et al.









Quadrupole gradient variation

Modified version of LOCO with constraints on gradient variations **(see ICFA Newsl, Dec' 07)**

β - beating reduced to 0.4% rms

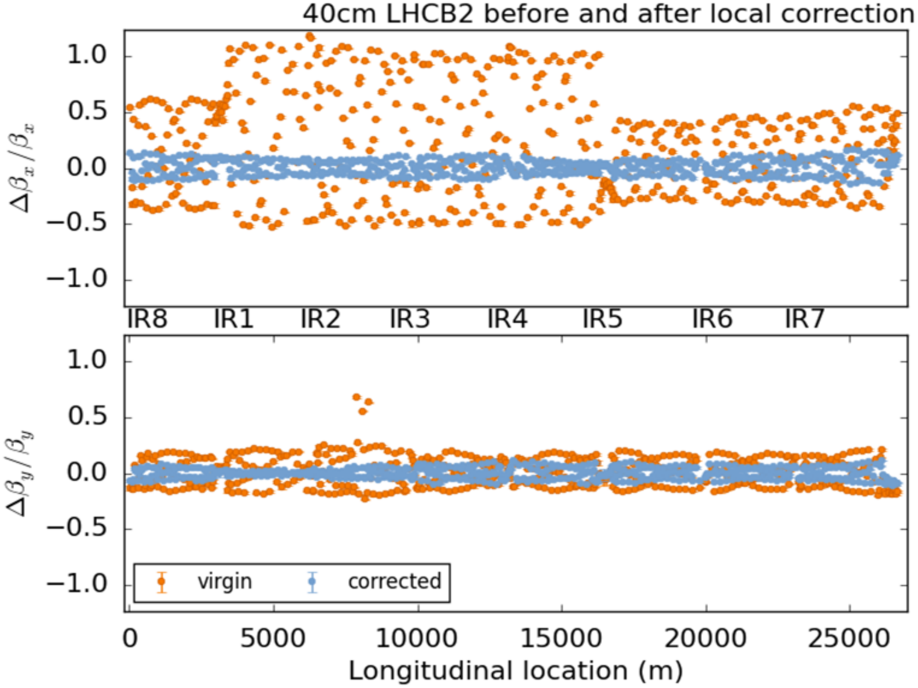
Quadrupole variation reduced to 2%

Results compatible with mag. meas. and calibration

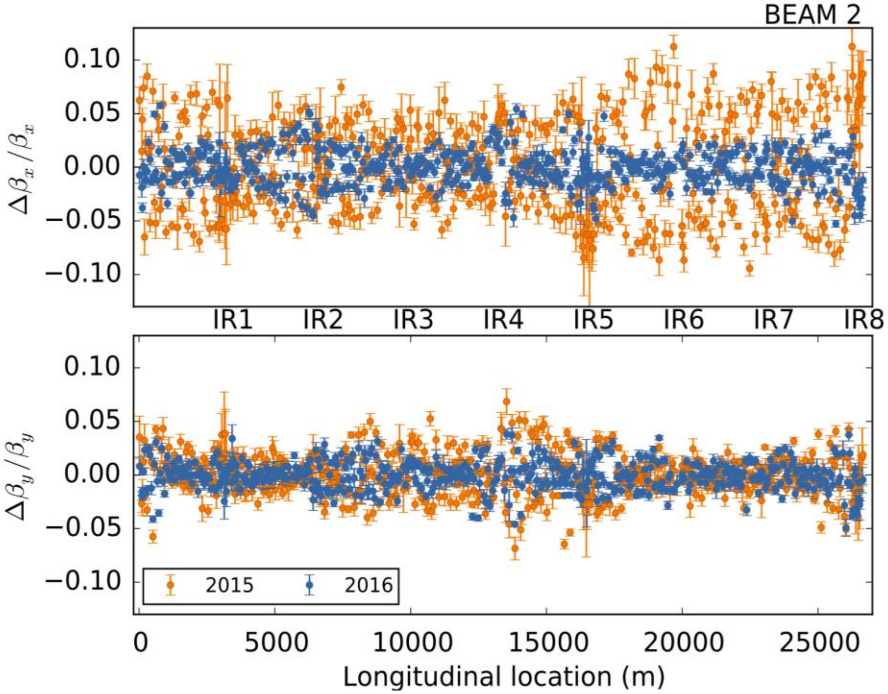
LOCO allowed remarkable progress with the correct implementation of the linear optics

Example: LHC optics corrections

- At β*=40cm, the bare machine has a beta-beat of more than 100%
- After global and local corrections, β-beating was reduced to few %



final corrections



R. Tomas et al. 2016

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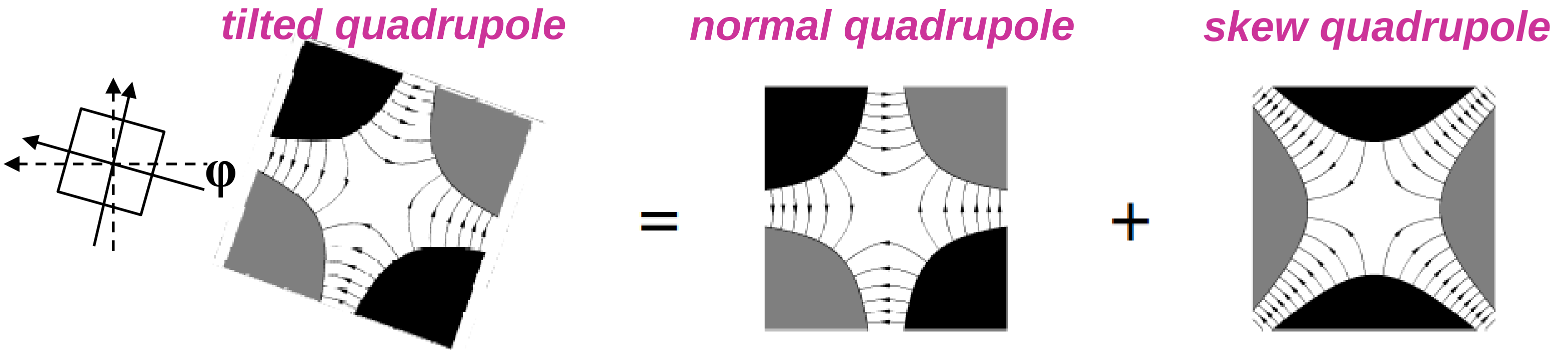
Coupling error

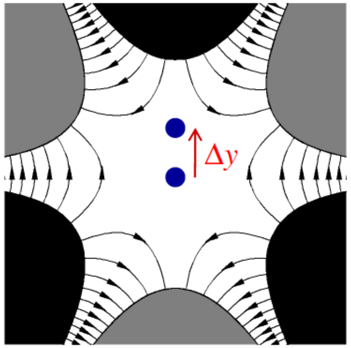
- Coupling errors lead to transfer of horizontal betatron motion and dispersion into the vertical plane
- Coupling may result from rotation of a quadrupole, so that the field contains a skew component



- A vertically displaced sextupole has the same effect as a skew quadrupole. The sextupole field for the displacement of ***δy*** becomes

$$B_x = B_2 x\bar{y} = B_2 xy + \underbrace{B_2 x\delta y}_{\text{skew quadrupole}}$$

$$B_y = \frac{B_2}{2}(x^2 - \bar{y}^2) = \overbrace{-B_2 y\delta y} + \frac{B_2}{2}(x^2 - y^2) - \frac{B_2}{2}\delta y^2$$



4x4 Matrices

- Combine the matrices for each plane

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

to get a total 4x4 matrix

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

Uncoupled motion

with skew quadrupoles these matrix elements are non-zero

Effect of coupling

- Betatron motion is coupled in the presence of skew quadrupoles
- The field is $(B_x, B_y) = k_s(x, y)$ and Hill's equations are coupled
- Motion still linear with two **new eigen-mode tunes**, which are **always split**. In the case of a thin skew quad:

$$\delta Q \propto |k_s|\sqrt{\beta_x\beta_y}$$

- Coupling coefficients represent the degree of coupling

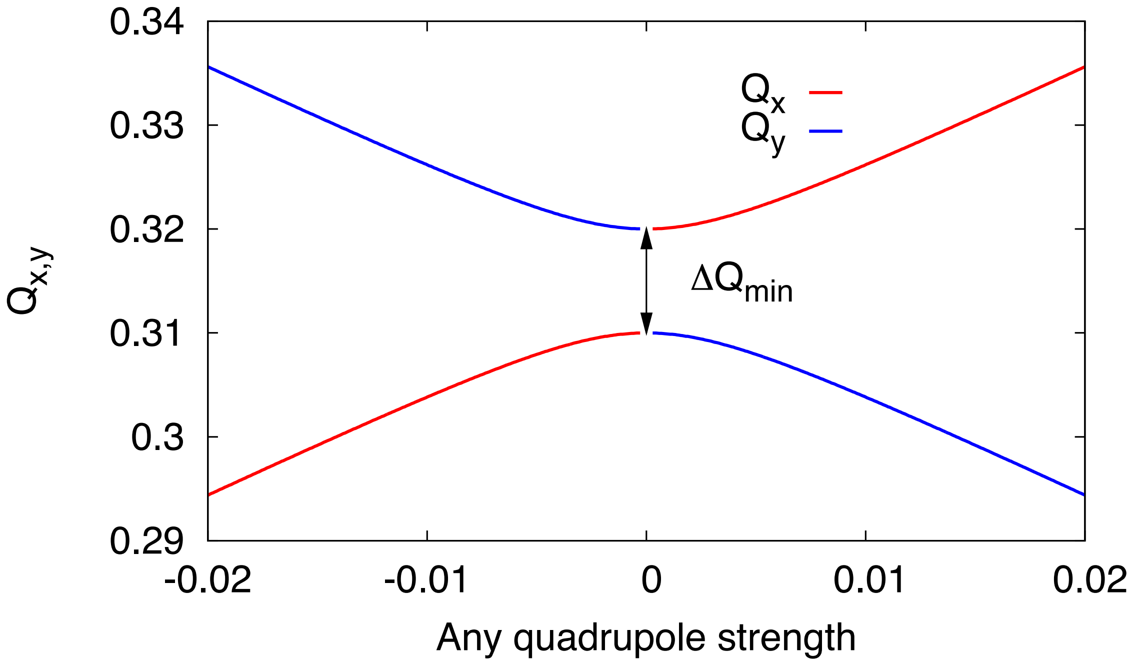
$$|C_\pm| = \left|\frac{1}{2\pi}\oint ds k_s(s)\sqrt{\beta_x(s)\beta_y(s)}e^{i(\psi_x\pm\psi_y-(Q_x\pm Q_y-q_\pm)2\pi s/C)}\right|$$

… complex number characterizing the difference resonance

$$Q_x - Q_y = N$$

- As motion is coupled, vertical dispersion and optics function distortion appears

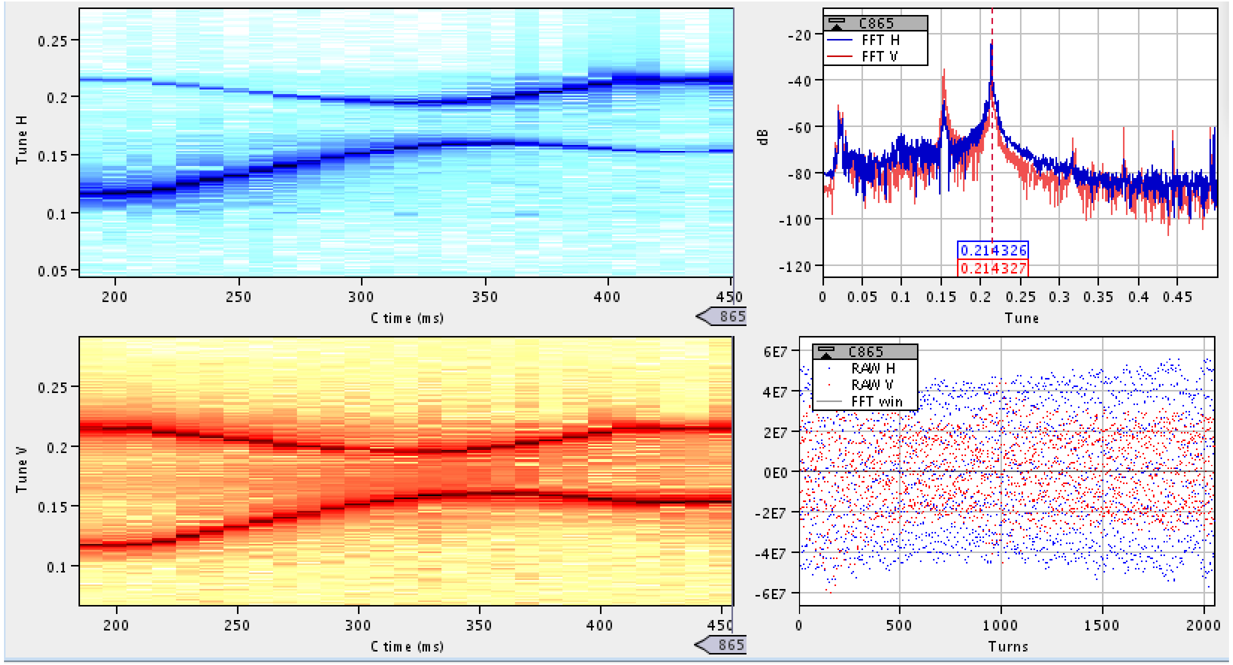
Closest tune approach



- Coupling makes it impossible to approach tunes below $\Delta Q_{min} = |C^-|$, where C^- is again the coupling coefficient

Closest tune approach

Measurement in the CERN PS



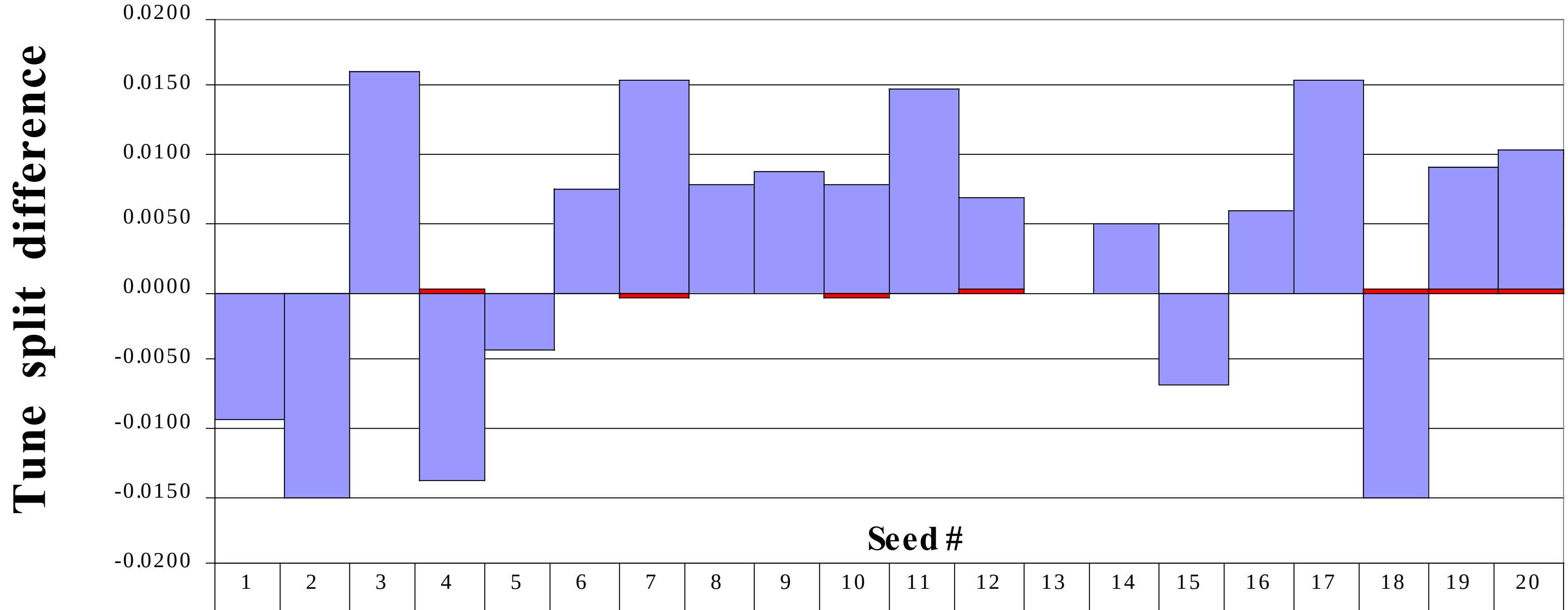
- Coupling makes it impossible to approach tunes below $\Delta Q_{min} = |C^-|$, where C^- is again the coupling coefficient
- The coupling coefficient C^- can be measured very easily by trying to approach the tunes and measure the minimum distance

Linear coupling correction

- Coupling correctors
  - Introduce skew quadrupoles into the lattice
  - If skew quadrupoles are not available, one can make vertical closed orbit bumps in sextuple magnets (used in JPARC main ring until installation of skew quadrupole correctors)
- Methods & approaches
  - Correct globally/locally coupling coefficient (or resonance driving term)
  - Correct optics distortion (especially vertical dispersion)
  - Move working point close to coupling resonances and repeat
- Remarks
  - Correction especially important for beams with unequal emittances "flat beams" (coupling leads to emittance exchange)
  - The (vertical) orbit correction may be critical for reducing coupling (e.g. due to feed-down sextupoles)

Example: SNS coupling correction

- Local decoupling by super period using 16 skew quadrupole correctors
- Results of Q_x=6.23 Q_y=6.20 after a **2mrad** quad roll
- Additional 8 correctors used to compensate vertical dispersion



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
▨ Before Correction	-0.009	-0.014	0.016	-0.013	-0.004	0.007	0.015	0.008	0.008	0.007	0.014	0.006	0.000	0.005	-0.006	0.006	0.015	-0.015	0.009	0.010
🟥 After correction	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000

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Chromaticity

- Linear equations of motion depend on the energy (term proportional to dispersion)
- Chromaticity is defined as: $\xi_{x,y} = \frac{\delta Q_{x,y}}{\delta p/p}$
- Recall that the gradient is $k = \frac{G}{B\rho} = \frac{eG}{p} \rightarrow \frac{\delta k}{k} = \mp \frac{\delta p}{p}$
- This leads to dependence of tunes and optics function on the particle's momentum
- For a linear lattice the tune shift is:

$$\delta Q_{x,y} = \frac{1}{4\pi} \oint \beta_{x,y} \delta k(s) ds = -\frac{1}{4\pi} \frac{\delta p}{p} \oint \beta_{x,y} k(s) ds$$

- So the **natural** chromaticity is:

$$\boxed{\xi_{x,y} = -\frac{1}{4\pi} \oint \beta_{x,y} k(s) ds}$$

- Sometimes the **normalized chromaticity** is quoted $\boxed{\overline{\xi_{x,y}} = \frac{\xi_{x,y}}{Q_{x,y}}}$

Example: Chromaticity in SNS

- In the SNS ring, the **natural chromaticity is –7**
- Consider that momentum spread ***δp/p*** **= ±1%**
- The tune-shift for off-momentum particles is

$$\delta Q_{x,y} = \xi_{x,y}\delta p/p = \pm 0.07$$

- In order to correct chromaticity introduce particles which can focus off-momentum particle

↓

Sextupoles

Chromaticity from sextupoles

- The sextupole field component in the *x*-plane is: $B_y = \frac{S}{2} x^2$
- In an area with non-zero dispersion $x = x_0 + D\frac{\delta P}{P}$
- Then the field is $B_y = \frac{S}{2} x_0^2 + \underbrace{SD\frac{\delta P}{P} x_0}_{\text{quadrupole}} + \underbrace{\frac{S}{2} D^2 \frac{\delta P}{P}^2}_{\text{dipole}}$
- With $k_2 = \frac{S}{B\rho}$

  sextupoles introduce an equivalent focusing correction $\delta k = k_2 D \frac{\delta P}{P}$
- The sextupole induced chromaticity is

$$\boxed{\xi^S_{x,y} = -\frac{1}{4\pi} \oint \mp \beta_{x,y}(s) k_2(s) D_x(s)\, ds}$$

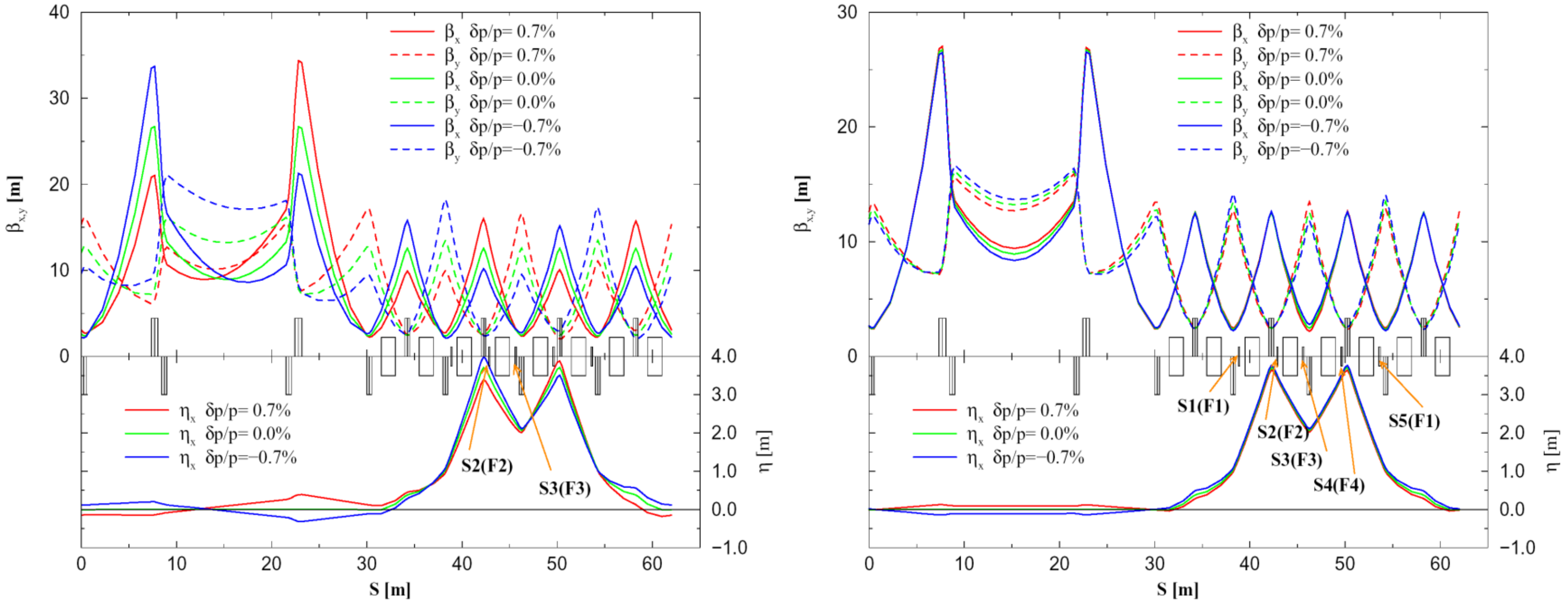
- The total chromaticity is the sum of the natural and sextupole induced chromaticity

$$\boxed{\xi^{tot}_{x,y} = -\frac{1}{4\pi} \oint \beta_{x,y}(s) (k(s) \mp k_2(s) D_x(s))\, ds}$$

Chromaticity correction

- Introduce sextupoles in high-dispersion areas
- Tune them to achieve desired chromaticity
- Two families are able to control horizontal and vertical chromaticity
- Sextupoles introduce non-linear fields (chaotic motion)
- Sextupoles introduce tune-shift with amplitude
- Example:
  - The SNS ring has natural chromaticity of **–7**
  - Placing two sextupoles of length **0.3m** in locations where *β*=**12m**, and the dispersion *D*=**4m**
  - For getting **0** chromaticity, their strength should be $k2 = \frac{7 \cdot 4\pi}{12 \cdot 4 \cdot 2 \cdot 0.3} \approx 3\mathrm{m}^{-3}$ or a gradient of **S=17.3 T/m²**

Two vs. four sextupole families

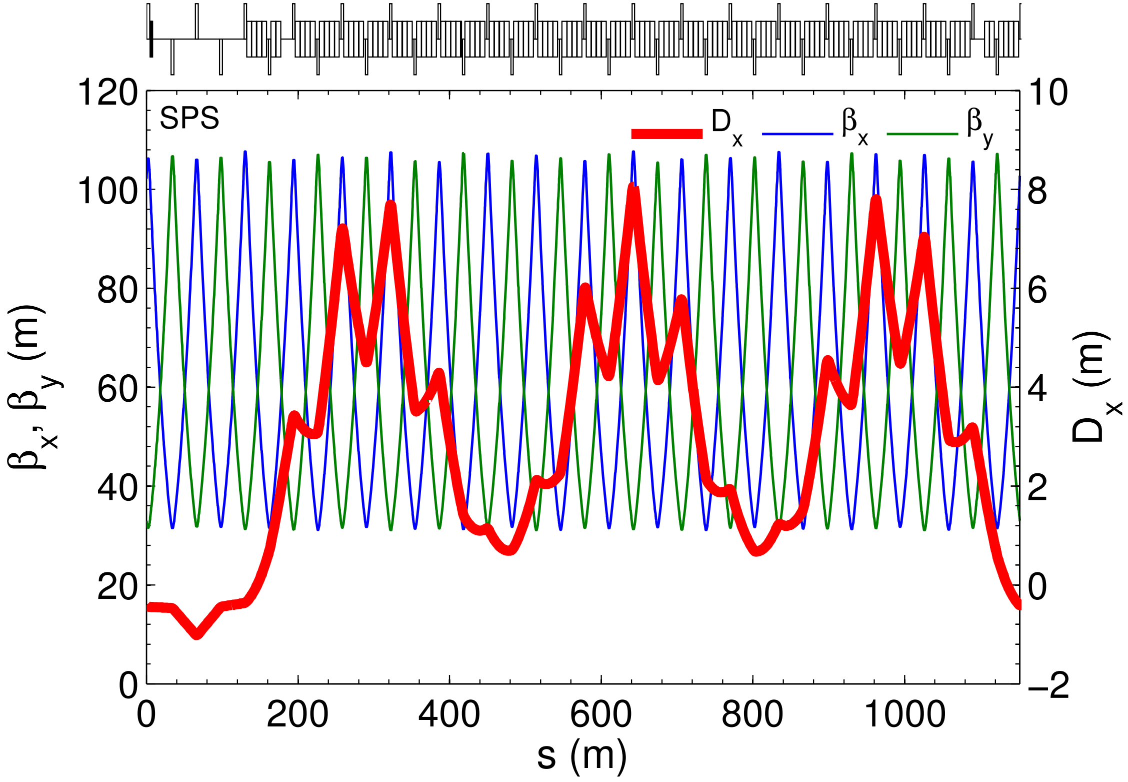


- Two families of sextupoles not enough for correcting off-momentum optics functions' distortion and second order chromaticity
- Possible solutions:
  - Place sextupoles accordingly to eliminate second order effects (difficult)
  - Use more families (4 in the case of the SNS ring)
- Large optics function distortion for momentum spreads of ±0.7%,when using only two families of sextupoles; Correction of off-momentum optics beating with four families

Problem 5

SPS: Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads, and horizontal and vertical **beta** of **30m** and **108m** for the defocusing ones.

- Find the tune change for systematic gradient errors of **1%** in the focusing and **0.5%** in the defocusing quads.
- What is the chromaticity of the machine?



Problem 6

CLIC pre-damping rings: Consider a 2.86 GeV electron storage ring with a racetrack shape of 389 m circumference. Each arc is composed of **17 regular “TME” cells**, each consisting of 2 half dipoles, 2 focusing and 2 defocusing quadrupoles. The beta functions are around **β_x=4m (2m)** and **β_y=4.2m (9m)** in the **focusing (defocusing) quadrupoles** and the normalized quadrupole gradients are **2.49/m^2 (2.07/m^2)**. The **quadrupoles** have a **length** of **0.28m**. The **natural chromaticity** of the machine is about **-19** and **-23** in the horizontal and vertical plane, respectively.

- How big is the chromaticity contribution from the arcs?
- Where would you install sextupole magnets for correcting chromaticity?