Introduction to RF: selected exercises

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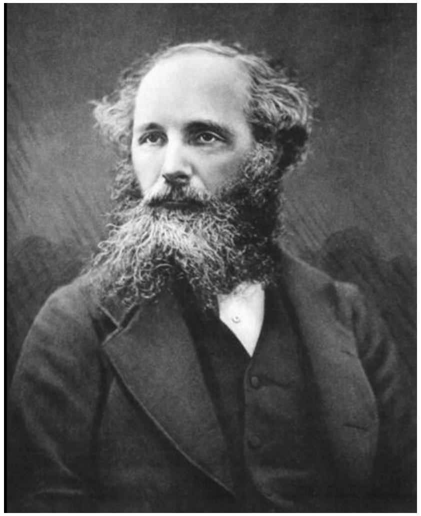

Transverse Electric Magnetic mode

Exercise

Look for a Transverse Electric Magnetic mode $E_z = H_z = 0$

$$\vec{E},\ \vec{H},\ v_p?$$

Hint 1 **Start from a TM mode (vector potential A)** $H_z = 0$

$$\nabla = \nabla_t + \hat{z}\frac{\partial}{\partial z} \qquad \nabla \cdot \vec{A} = \cdots$$

Hint 2

$$\vec{E}_A = \cdots$$

Solution

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Hint 1 **Start from a TM mode (vector potential A)** $H_z = 0$

$$\nabla = \nabla_t + \hat{z}\frac{\partial}{\partial z} \qquad \nabla \cdot \vec{A} = \cdots = -j\beta A_z e^{-j\beta z}$$

Hint 2

$$\vec{E}_A = \cdots = -j\omega\hat{z}A_z e^{-j\beta z} - \frac{j}{\omega\mu\epsilon}\left[\nabla_t + \hat{z}\frac{\partial}{\partial z}\right](-j\beta)A_z e^{-j\beta z} =$$

$$= -\frac{j}{\omega\mu\epsilon}\left[\omega^2\mu\epsilon - \beta\right] A_z e^{-j\beta z}\hat{z} - \frac{\beta}{\omega\mu\epsilon}\nabla_t A_z\ e^{-j\beta z}$$

$$\text{if } \beta^2 = \omega^2\mu\epsilon = k^2 \quad \Longrightarrow \quad e_z = 0$$

Solution **For a given** A_z

$$\vec{H} = \frac{1}{\mu}\nabla_t \times (\hat{z}A_z)\, e^{-j\omega\sqrt{\mu\epsilon}z} \qquad \vec{E} = -\frac{1}{\sqrt{\mu\epsilon}}\ \nabla_t A_z\ e^{-j\omega\sqrt{\mu\epsilon}z}$$

1. $\nabla_t^2 A_z = -\left(k^2 - \beta^2\right) A_z = 0$ **The transverse E field is "electrostatic"**

2. **As plane waves:** $\ldots e^{-j\omega\sqrt{\mu\epsilon}z} \Longrightarrow v_p = 1/\sqrt{\mu\epsilon}$

$$\vec{h}_t = \sqrt{\frac{\epsilon}{\mu}}\ \hat{z} \times \vec{e}_t = \frac{1}{Z_{TEM}}\hat{z} \times \vec{e}_t$$

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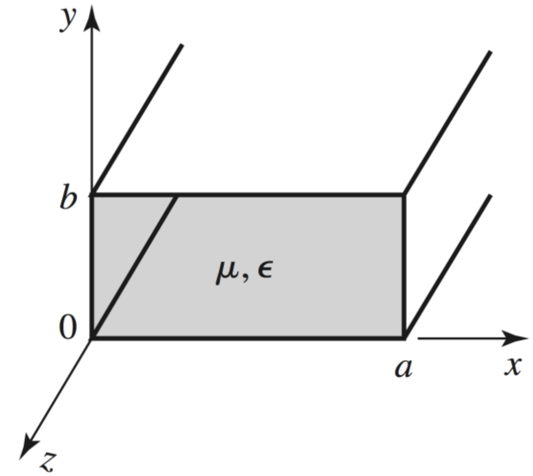
Single mode operation of a rectangular waveguide

Exercise



1. **Find the smallest ratio a/b allowing the largest bandwidth of single mode operation**

2. **State the largest bandwidth of single mode operation**

3. **Defining the single mode bandwidth as**

$$1.25\ (f_c)_1 < f < 0.95\ (f_c)_2$$

Find the single mode BW for WR-90 waveguide (a=22.86mm and b=10.16 mm)

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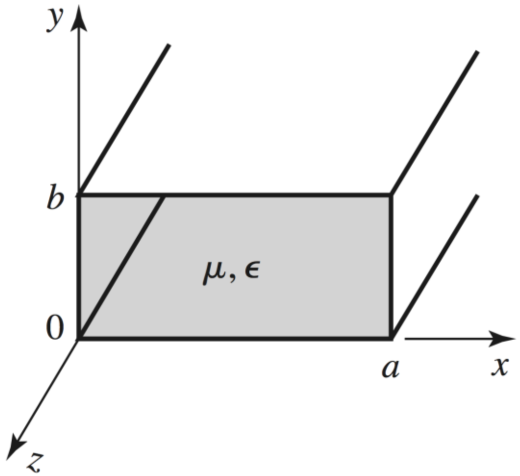

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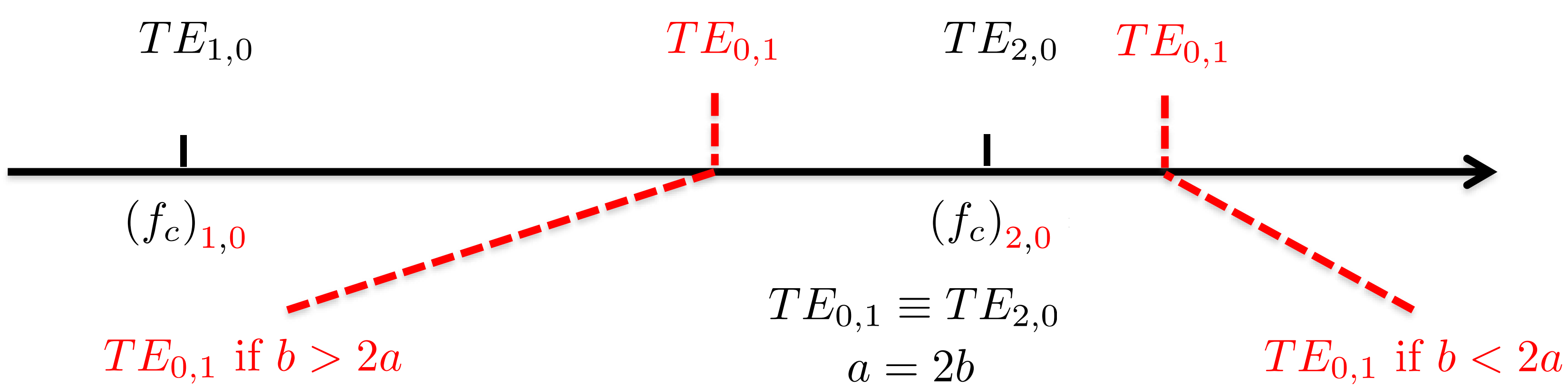
Find the single mode BW for WR-90 waveguide (a=22.86mm and b=10.16 mm)

$$(f_c)_{1,0} = \frac{1}{2\sqrt{\mu\epsilon}a} \qquad (f_c)_{2,0} = \frac{1}{\sqrt{\mu\epsilon}a} = 2\,(f_c)_{2,0} \qquad (f_c)_{0,1} = \frac{1}{\sqrt{\mu\epsilon}b}$$



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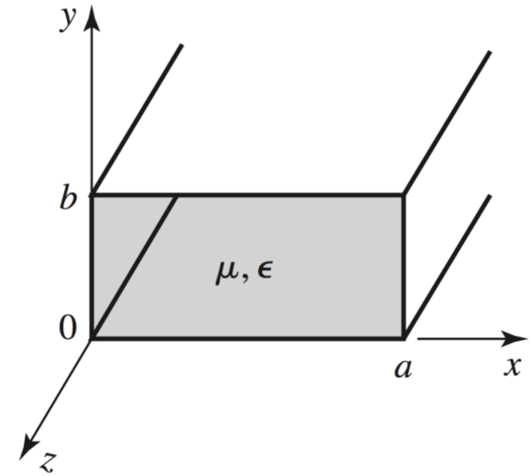
Single mode operation of a rectangular waveguide

Exercise



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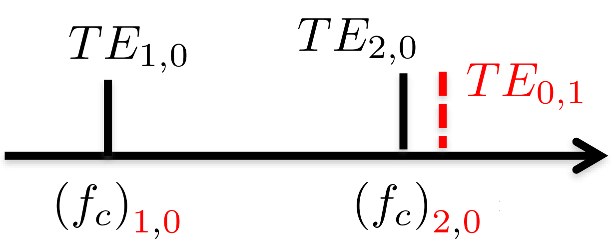
Find the single mode BW for WR-90 waveguide (a=22.86mm and b=10.16 mm)

a=0.9 inches b=0.4 inches



$$(f_c)_{1,0} = c/2a = 3\ 10^8/(2\ 22.86\ 10^{-3}) = 6.56\ \text{GHz}$$

$$(f_c)_{2,0} = c/a = 3\ 10^8/(22.86\ 10^{-3}) = 13.12\ \text{GHz}$$

Single mode BW $6.56\ 1.25 = 8.2\ \text{GHz} < f < 12.4\ \text{GHz} = 13.12\ 0.95$

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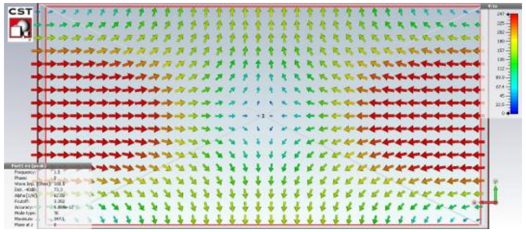
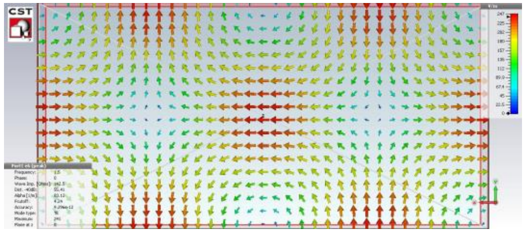
Field pattern at the cross section

$TE^{+z}_{m,n}$ **m (n) is the number of half periods (or maxima/minima) along the x (y) axis in the cross-section.**

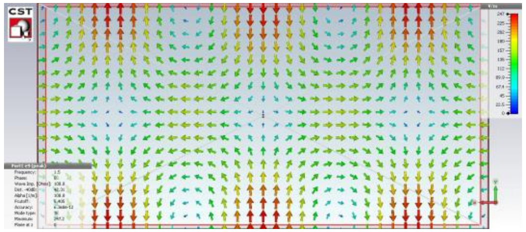
TE11



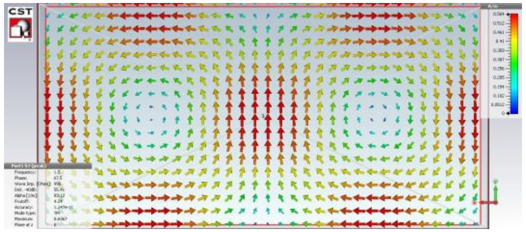
TE21

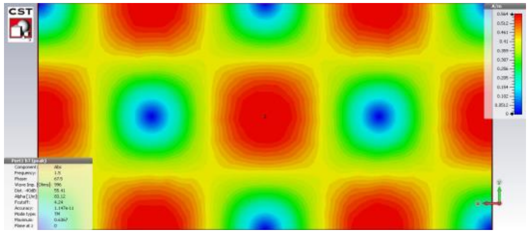


TE31

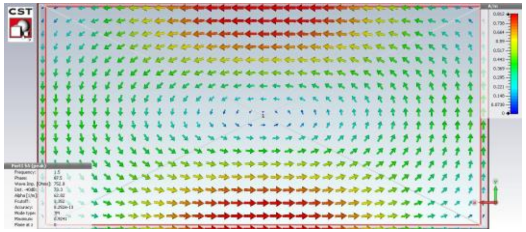


TM21





TM11



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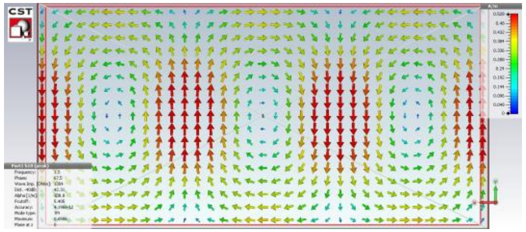

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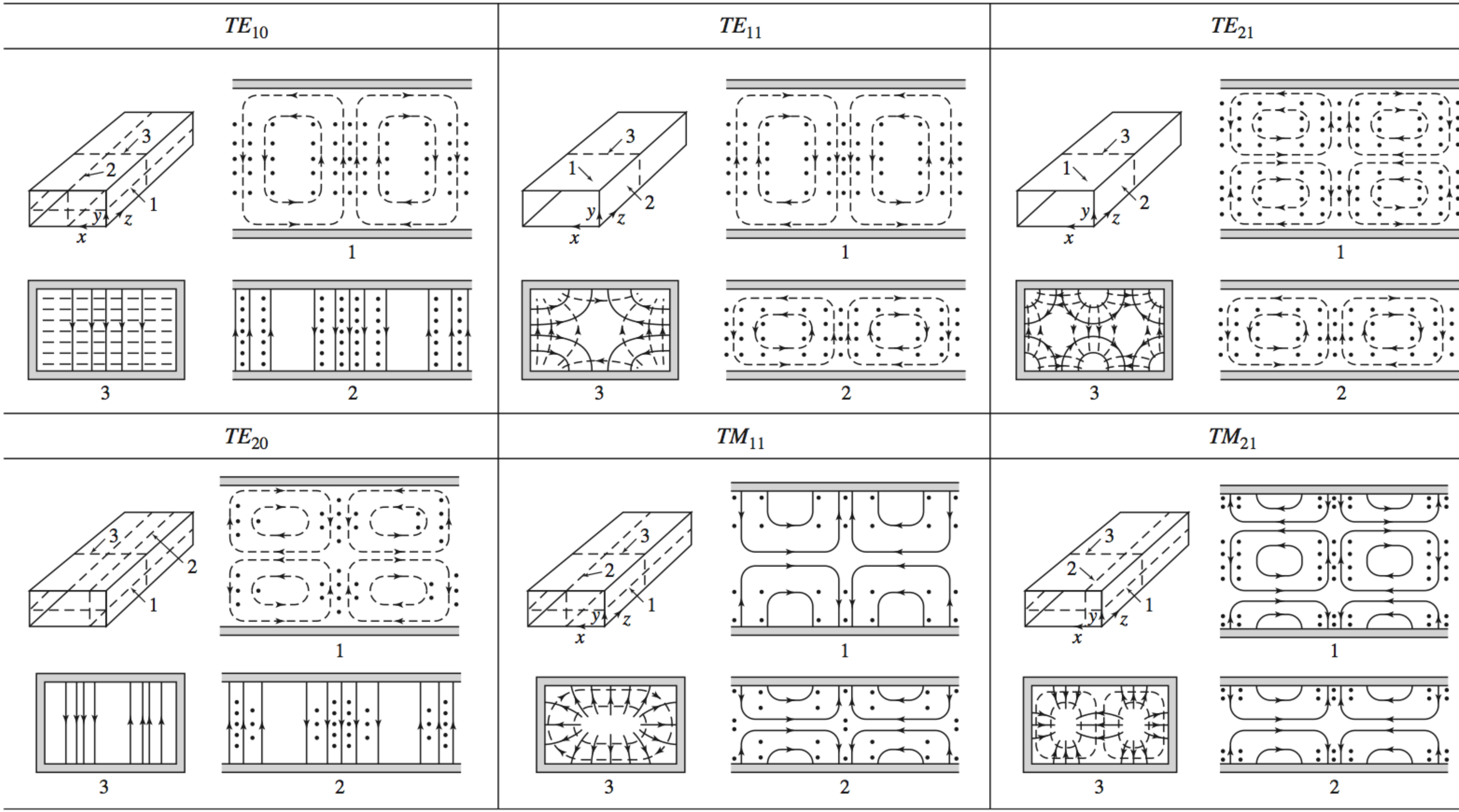
Field pattern (TE mode, rect. WG)

$TE_{m,n}^{+z}$ **m (n) is the number of half periods (or maxima/minima) along the x (y) axis in the cross-section.**



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