JUAS RF Course 2018 Tutorial 1

Some physical constants (in SI units, but without the units attached) :

```
c = QuantityMagnitude[UnitConvert[Quantity["SpeedOfLight"]]]
```

299 792 458

```
μ = N[QuantityMagnitude[UnitConvert[Quantity["VacuumPermeability"]]]]
```

1.25664×10^{-6}

```
ϵ = N[QuantityMagnitude[UnitConvert[Quantity["VacuumPermittivity"]]]]
```

8.85419×10^{-12}

Other resources :

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

376.73

1.) Design of a "pillbox" cavity

Problem : Design a simple cavity of the "Pillbox" type with the following parameters :

frequency :

$\lambda = 1.0;$

$$f = \frac{c}{\lambda}$$

2.99792×10^{8}

$\omega = 2 \pi f$

1.88365×10^{9}

```
EngineeringForm[%]
```

1.88365×10^{9}

Wall material: copper

$\sigma_{Cu} = 5.8 \times 10^{7}$

5.8×10^{7}

Skin depth: copper (see also page 59)

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{Cu}\, \mu}}$$

3.81677×10^{-6}

axial length:

```
h = 0.2;
```

Questions

1 : Find from the analytical formula

cavity radius a

The fundamental mode is the TM_{010} mode, with

```
μr = 1;
ϵr = 1;
m = 0;
n = 1;
p = 0;
a = .
```

$$f_{mnp} = \text{FullSimplify}\left[N\left[\frac{c}{2\,\pi\,\sqrt{\mu_r\,\epsilon_r}}\sqrt{\left(\frac{\text{BesselJZero}[m,\ n]}{a}\right)^2 + \left(\frac{p\,\pi}{h}\right)^2}\right],\ a > 0\right];$$

with the zero of the Bessel function:

```
N[BesselJZero[m, n]]
```

2.40483

see also page 41

```
f010 = %%
```

$$\frac{1.14743 \times 10^{8}}{a}$$

$$\lambda_0 = \frac{c}{f_{010}}$$

2.61274 a

see also page 49

```
result = Solve[λ == %, a]
```

{{a → 0.38274}}

```
result[[1]]
{a → 0.38274}

a /. result[[1]]
0.38274

a = % λ
0.38274
```

see also page 47

cavity quality factor Q

$$Q = \frac{a}{\delta}\left(1 + \frac{a}{h}\right)^{-1}$$

```
34 416.2
```

see also page 49

"geometry factor", also known as "characteristic impedance" R/Q

```
BesselJ[1, N[BesselJZero[0, 1]]]
0.519147
```

exact result:

RoverQ =

$$\frac{4\,\eta}{\text{BesselJZero}[0,1]^3\,\pi\,\text{BesselJ}[1,\text{N}[\text{BesselJZero}[0,1]]]^2}\;\frac{\text{Sin}\left[\frac{\text{BesselJZero}[0,1]}{2}\,\frac{h}{a}\right]^2}{\frac{h}{a}}$$

```
84.6096
```

approximation:

$$\text{RoverQapprox} = 185\,\frac{h}{a}$$

```
96.6714
```

see also page 50

Error due to sin(x) ≈ x approximation in %:

```
100 (1 - RoverQ / RoverQapprox)
12.4771
```

Is the cavity completely determined?

With the 3 given parameters the intrinsic cavity is fully determined!

2 : Find the equivalent circuit of the intrinsic cavity.

```
rApprox = RoverQapprox Q
```

3.32706×10^{6}

$$\text{lApprox} = \frac{\text{RoverQapprox}}{\omega}$$

5.13213×10^{-8}

EngineeringForm[%]

51.3213×10^{-9}

$$\text{cApprox} = \frac{1}{\text{RoverQapprox}\,\omega}$$

5.49163×10^{-12}

rExact = RoverQ Q

2.91194×10^{6}

$$\text{lExact} = \frac{\text{RoverQ}}{\omega}$$

4.49178×10^{-8}

EngineeringForm[%]

44.9178×10^{-9}

$$\text{cExact} = \frac{1}{\text{RoverQ}\,\omega}$$

6.27451×10^{-12}

see also page 66-67

3 : Find the 3-dB bandwidth of the intrinsic cavity.

$$\Delta f = \frac{f}{Q}$$

8710.79

see also page 72

4 : Calculate the necessary RF power for a gap voltage of:

In critical coupling source and load impedance are equal, and the amplifier sees them in parallel: R/2

$V = 100 \times 10^3;$

$$P = \frac{V^2}{2\,(\text{rExact}/2)}$$

3434.13

see also page 67

5 : The cavity shall be fed by an amplifier designed for a load impedance of

```
Zl = 50;
```

Determine:

the peak voltage at the cavity input.

$$vPeak = \sqrt{2 Z_l P}$$

586.015

the necessary transformer ratio k of the input coupler.

$$k = \sqrt{\frac{rExact}{Z_l}}$$

241.327

see also page 67

2.) Multiple choice questions:

1. How will the resonant frequency f_{res} of the E010 (TM010) mode of a pill box cavity change if height of the cavity is doubled? (check 1)
The f_{res} will not change.

2. A critically coupled aluminum pill-box cavity is driven by an RF generator with an output power of 100 kW. How much power would be dissipated by the cavity if it were made of silver?
σ_{Al} = 38*106 S/m, σ_{Ag} = 63*106 S/m. Note: the silver cavity would also be critically coupled. (check 1)
The power dissipation will not change

3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals.
$\sigma_{Cu} = 58 \times 10^6$ S/m, $\mu = \mu_0 \mu_r$, $u_0 = 4 \pi 10^{-7}$ Vs/Am (check 1)
46.7 mm

4. A rectangular waveguide has a width (long side!) of a = 10 cm. (check 2)
The mode TE10 or H10 has a cutoff frequency of 1.5 GHz.
The electric field is orthogonal to the side with the larger dimension

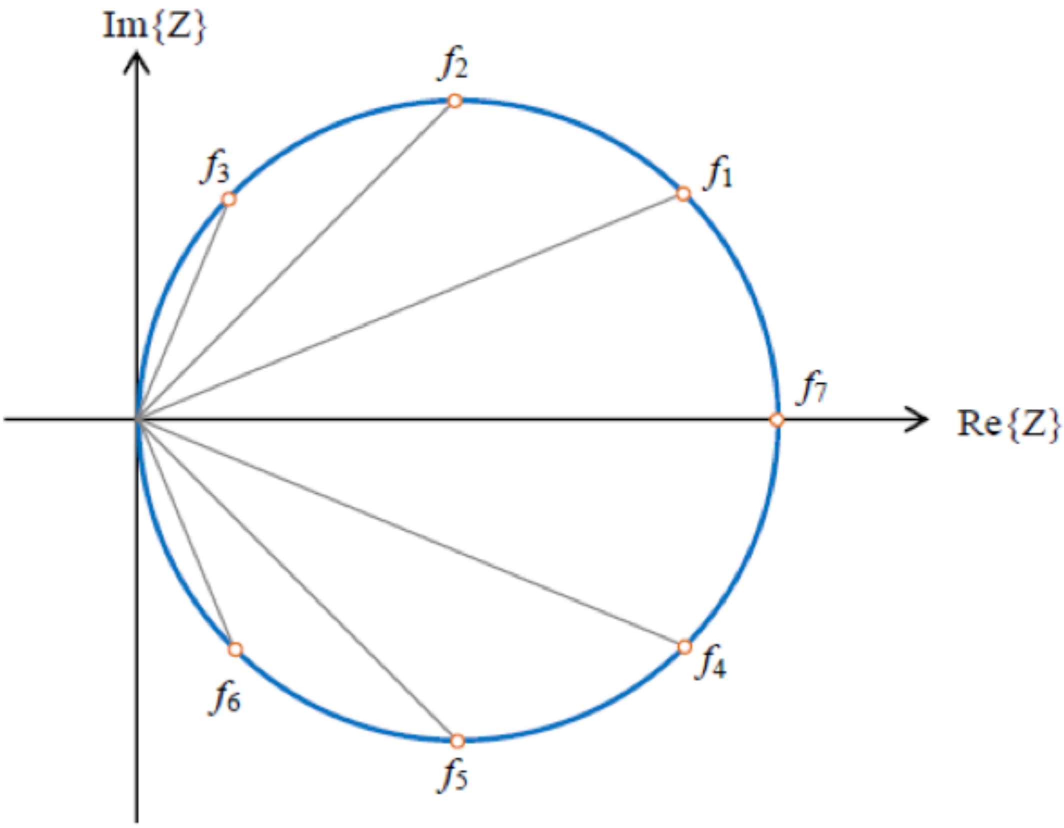
5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section without inner conductor? (check 1)
TE

6. Adding capacitive loading to a cavity (check 1)
lowers the resonant frequency

7. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)
99 %

3.) Impedances in the complex plane

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies, f1…f7. The result is shown in the complex Z-plane:



□	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f / MHz	105.11	105.05	104.94	105.29	105.35	105.46	105.20
Z / kΩ	$200.0\, e^{i30°}$	$162.6\, e^{i45°}$	$115.0\, e^{i36°}$	$200.0\, e^{-i30°}$	$162.6\, e^{-i45°}$	$115.0\, e^{-i60°}$	$230.0\, e^{i0°}$

Questions

1 : Determine the resonant frequency.

$f_7 = 105.2 \times 10^6$;

$f_{res} = f_7$

1.052×10^8

2 : Determine the 3-dB bandwidth (BW) of this resonator.
(Hint: The bandwidth of a resonator is defin ed as the frequency

difference
between the upper and lower 3-dB frequency points.)

$f_5 = 105.35 \times 10^6;$

$f_2 = 105.05 \times 10^6;$

$BW = f_5 - f_2$

300 000.

In order to evaluate the properties of a resonator,
it is common to model it as equivalent circuit with lumped RLC elements.

3 : Sketch the equivalent circuit for the measured resonator.

R-L-C parallel circuit

4 : Determine R.

`r = 230;`

(at $f_{res} = f_7 = 105.2$ MHz)

5 : Draw the locus of admittance of this circuit in the Y-plane, and indicate lower and upper 3-dB points.

Straight line parallel to the Im{Y} axes, crossing Re{Y} at 1/230 mS. The 3-dB points are located on the locus as points crossing with lines from the origin under ±45°.

6 : Determine the Q-value, as well as L and C for this circuit.

$Q = f_{res} / BW$

350.667

$$l = \frac{r}{2\pi f_{res} Q}$$

9.92288×10^{-10}

`EngineeringForm[%]`

992.288×10^{-12}

$$c = \frac{1}{(2\pi f_{res})^2 l}$$

2.30659×10^{-9}

```
f2 =.
f5 =.
fres =.
BW =.
Q =.
r =.
l =.
c =.
```

4.) Transmission-lines

Given is a coaxial transmission-line with an inner diameter of the outer conductor of 100 mm, the dielectric is air (so-called “air-line”).

Questions

1 : What is the outer diameter of the inner conduction to achieve a characteristic impedance of 50Ω ?

```
dOut = 100 × 10^-3;
rOut = dOut / 2;
Z = 50;
result = Solve[Z == 60 Log[rOut / rIn], rIn]
```

$\left\{\left\{rIn \to \frac{1}{20\, e^{5/6}}\right\}\right\}$

```
result[[1]]
```

$\left\{rIn \to \frac{1}{20\, e^{5/6}}\right\}$

```
rIn /. result[[1]] // N
```

0.0217299

```
dIn = 2 × %
```

0.0434598

see also page 10

2 : With which velocity is a wave travelling in this line?

```
v = c
```

299 792 458

see also page 9

3 : Specify the capacitance and inductance per meter length of this transmission-line?

$$\text{Cprime} = \frac{1}{v\,Z} \text{ // N}$$

6.67128×10^{-11}

```
EngineeringForm[%]
```

66.7128×10^{-12}

$$\text{Lprime} = \frac{Z}{v} \text{ // N}$$

1.66782×10^{-7}

```
EngineeringForm[%]
```

166.782×10^{-9}

see also page 9

Instead of an air dielectric this transmission line is now homogeneously filled with Teflon (ϵ_r = 2)

4 : Determine the phase velocity, characteristic impedance, as well as capacitance and inductance per meter length?

```
ϵr = 2;
```

$$v = \frac{c}{\sqrt{\epsilon r}} \text{ // N}$$

2.11985×10^{8}

$$Z = \frac{60}{\sqrt{\epsilon r}} \text{Log}\left[\frac{\text{dOut}}{\text{dIn}}\right]$$

35.3553

$$\text{Cprime} = \frac{1}{v\,Z} \text{ // N}$$

1.33426×10^{-10}

```
EngineeringForm[%]
```

133.426×10^{-12}

```
Lprime = Z/v // N
```

1.66782×10^{-7}

```
EngineeringForm[%]
```

166.782×10^{-9}

see also page 9 and 10

5.) Waves on a transmission line Z = 50 Ω

Problem : Convert the couple (volatge V, current I) into the equivalent couple (forward wave a, reflected wave b) and vise versa using the relations

$a = \frac{V + IZ}{2}$ $\quad V = a + b$

$b = \frac{V - IZ}{2}$ $\quad IZ = a - b$

Questions

1 : In a 50 Ω system, a directional coupler measured the forward and reflected waves a and b at a certain plane as a = 100∠0° and b = 60∠45°

Calculate the corresponding voltage V and current I.

```
a = 100;
```

```
b = N[ExpToTrig[60 Exp[I 45 Degree]]]
```

42.4264 + 42.4264 i

```
v = a + b
```

142.426 + 42.4264 i

```
{Abs[v], Arg[v] / Degree}
```

{148.611, 16.5879}

```
z = 50;
```

```
i = (a - b)/z
```

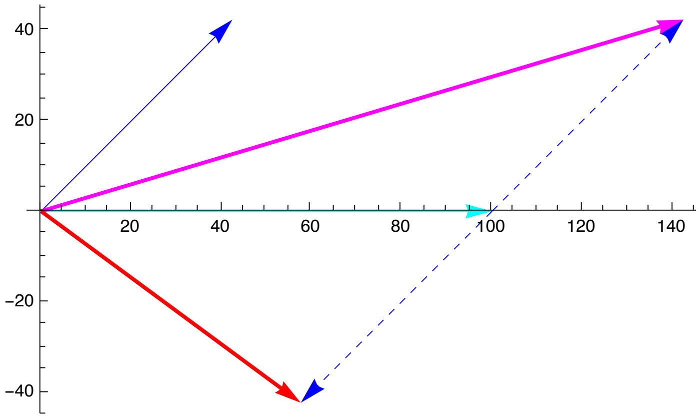
1.15147 − 0.848528 i

```
{Abs[i], Arg[i] / Degree}
{1.43035, -36.3868}

i z
57.5736 - 42.4264 i

{Abs[i z], Arg[i z] / Degree}
{71.5173, -36.3868}

Graphics[
 {{Cyan, Arrow[{{0, 0}, {Re[a], 0}}]}, {Blue, Arrow[{{0, 0}, {Re[b], Im[b]}}]},
  {Blue, Dashed, Arrow[{{Re[a], 0}, {Re[a + b], Im[0 + b]}}]},
  {Magenta, Thick, Arrow[{{0, 0}, {Re[a + b], Im[0 + b]}}]},
  {Blue, Dashed, Arrow[{{Re[a], 0}, {Re[a - b], Im[0 - b]}}]},
  {Red, Thick, Arrow[{{0, 0}, {Re[i z], Im[i z]}}]}},
 Axes → True, AspectRatio → Automatic]
```



2 : At some plane of a 50 Ω system, a voltage of V = 100∠0° and a current of I = 1.0∠-45° are measured.

Calculate the corresponding foreward and backward waves a and b.

```
v = 100;

i = N[ExpToTrig[1.0 Exp[-I 45 Degree]]]
0.707107 - 0.707107 i
```

$$a = \frac{v + i\, z}{2}$$

```
67.6777 - 17.6777 i

{Abs[a], Arg[a] / Degree}
{69.9483, -14.6388}
```

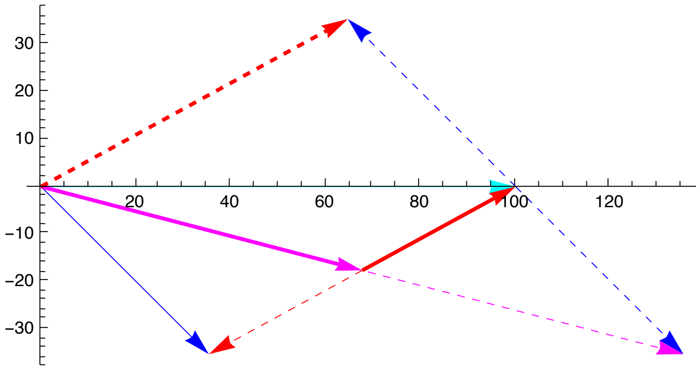
$$b = \frac{v - i\, z}{2}$$

```
32.3223 + 17.6777 i
```

```
{Abs[b], Arg[b] / Degree}
```

```
{36.8406, 28.6751}
```

```
Graphics[{{Cyan, Arrow[{{0, 0}, {Re[v], 0}}]},
  {Blue, Arrow[{{0, 0}, {Re[i z], Im[i z]}}]},
  {Blue, Dashed, Arrow[{{Re[v], 0}, {Re[v] + Re[i z], 0 + Im[i z]}}]},
  {Blue, Dashed, Arrow[{{Re[v], 0}, {Re[v] - Re[i z], 0 - Im[i z]}}]},
  {Magenta, Dashed, Arrow[{{0, 0}, {Re[2 a], Im[2 a]}}]},
  {Magenta, Thick, Arrow[{{0, 0}, {Re[a], Im[a]}}]},
  {Red, Thick, Arrow[{{Re[a], Im[a]}, {Re[a + b], Im[a + b]}}]},
  {Red, Dashed, Arrow[{{Re[a], Im[a]}, {Re[a - b], Im[a - b]}}]},
  {Red, Dashed, Thick, Arrow[{{0, 0}, {Re[2 b], Im[2 b]}}]}},
 Axes → True, AspectRatio → Automatic]
```



6.) “Pillbox” cavity characteristics

The following data has been determined on a cavity:

Inductance:

```
l = 15.915 × 10^-9;
```

Capacitance:

```
c = 1.5915 × 10^-12;
```

3-dB bandwidth:

```
Δf = 50 × 10^3;
```

Questions

Determine

- the resonance frequency

$$f_{res} = \frac{1}{2\pi}\frac{1}{\sqrt{l\,c}}$$

1.00003×10^{9}

see also page 66

- the characteristic impedance R/Q

$$\text{RoverQ} = \sqrt{\frac{l}{c}}$$

100.

see also page 67

- the quality factor Q

$$Q = \frac{f_{res}}{\Delta f}$$

20 000.6

see also page 72

- the time constant τ

$$\tau = \frac{Q}{\pi\, f_{res}}$$

6.3662×10^{-6}

see also page 78

- the peak induced voltage immediately after the passage of a short particle bunch with charge

$$q = 15.916 \times 10^{-9};$$

$$V_{step} = \text{Abs}\left[0 - \frac{q}{c}\right]$$

10 000.6

see also page 80

- the remaining cavity voltage 10 μs after the bunch passage

$$t = 10 \times 10^{-6};$$

$$V_{end} = V_{step}\, e^{-\frac{t}{\tau}}$$

2078.93

see also page 80

7.) Gap-width optimization of a cavity

The following parameters of a 100-MHz cavity have been evaluated by a cavity design program as a function of the gap width g:
R/Q (characteristic impedance),
Q (quality factor)

```
f = 100 × 10^6;
```

The cavity is connected to an amplifier delivering a power of 1 kW

```
p = 1 × 10^3;
```

The cavity beam has a relative velocity of

```
β = 0.15;
```

Questions

Calculate for each gap width:

- shunt impedance R

```
g = {0.1, 0.2, 0.3};
RoverQ = {100, 150, 200};
q = {5000, 7000, 9000};
r = RoverQ q
{500 000, 1 050 000, 1 800 000}
```

- intrinsic cavity voltage V_{cav} for 100 kW power

```
p = 100 × 10^3;
Vcav = N[√(2 r p)]
{316 228., 458 258., 600 000.}
```

see also page 67

- phase angle Θ of the passage through the gap

Velocity v of the beam through the cavity gap g:
$v = \beta c_0 = \frac{g}{t}$
Time t through the gap:
$t = \frac{g}{\beta c_0} = \frac{g}{\beta f \lambda}$
The total phase angle accounts for $\Theta = 2\pi$,
the phase Θ is given by multiplying both sides by 2π/f (remember: f t = 1):
$2\pi f t = 2\pi \frac{g f}{\beta f \lambda} \Longrightarrow 2\pi = \Theta = 2\pi \frac{g}{\beta \lambda} = \frac{2\pi f g}{\beta c_0}$

$$\Theta = \frac{2\,\pi\,f\,g}{\beta\,c}$$

{1.39723, 2.79446, 4.19169}

in degree:

$$\frac{\Theta\,180}{\pi}$$

{80.0554, 160.111, 240.166}

- transit time factor T

The transient time factor T is related to the phase angle Θ:

$$\frac{g\,\omega}{2\,\beta\,c_0} = \frac{2\,\pi\,f\,g}{2\,\beta\,c_0} = \frac{\Theta}{2}$$

$$T = \frac{\mathrm{Sin}[\Theta/2]}{(\Theta/2)}$$

{0.920618, 0.704948, 0.412864}

see also page 94

- beam voltage V_{beam} maximally seen by the beam taking the transit time factor T into account

$V_{beam} = V_{cav}\,T$

{291 125., 323 048., 247 719.}

Which of the three versions gives the highest beam voltage?

Version 2 with a gap of g = 200 mm gives the highest beam voltage V_{beam} = 323 kV

Supposing a field enhancement factor of ~1.5 in the gap region, evaluate the danger of voltage breakdown for each design using the Kilpatrick limit as a yardstick.

$1.5\,V_{cav}$

{474 342., 687 386., 900 000.}

$V_{Kilpatrick}$ (@ 100 MHz) = {~1.15 MV, ~2 MV, ~3.3 MV}

For all gap configurations there is no danger of a voltage breakdown, the cavity voltage is always well below the Kilpatrick limit.

see also page 109

8.) A higher order-mode (HOM) in a cavity

A RF cavity has an unwanted high-order mode (HOM) at

$f_{HOM} = 600 \times 10^6;$

with a shunt impedance of

$r_{HOM} = 6 \times 10^6;$

a 3-dB bandwidth of

$\Delta f_{HOM} = 15 \times 10^3;$

and a transit time factor of

$T = 1;$

The beam consists of very short bunches, following each other at interval of

$t_{bunch} = 20 \times 10^{-6};$

The circulating current is

$i_{beam} = 0.1;$

(Remember: I = charge per time)

Questions

Calculate:

- Q, R/Q, and C at the HOM frequency

$$Q_{HOM} = \frac{f_{HOM}}{\Delta f_{HOM}}$$

40 000

see also page 72

$$\text{RoverQ} = \frac{r_{HOM}}{Q_{HOM}}$$

150

$$C_{HOM} = N\left[\frac{1}{2\,\pi\, f_{HOM}\, \text{RoverQ}}\right]$$

1.76839×10^{-12}

see also page 66

- HOM voltage induced by a single bunch

Assuming a single bunch in the ring :

$q_{bunch} = i_{beam}\, t_{bunch}$

$2. \times 10^{-6}$

$\Delta V = \frac{q_{bunch}}{C_{HOM}}$

1.13097×10^{6}

see also page 78

- The time constant τ of the cavity

$\tau = 2\, r_{HOM}\, C_{HOM}$

0.0000212207

```
EngineeringForm[%]
```

21.2207×10^{-6}

see also page 78

- The HOM voltage at the arrival of the next bunch

$V_{end}\, e^{-t/\tau} = V_{step} \implies V_{next} = \Delta V\, e^{-t/\tau}$

$V_{next} = \Delta V\, e^{-t_{bunch}/\tau}$

440696.

- The total HOM voltage in steady state, after the passage of an infinite number of bunches, supposing that the HOM resonance lies at an exact multiple of the beam revolution frequency

$V_{end} = \frac{q_{bunch}}{C_{HOM}\left(1 - e^{-t_{bunch}/\tau}\right)}$

1.85303×10^{6}