Introduction to Magnets

Attilio Milanese

Attilio.Milanese@cern.ch

JUAS
26 Feb. 2018

Thanks to the many colleagues from which I borrowed much material, in particular Davide Tommasini

JUAS - TIMETABLE 2018 - WEEK 8

Schedule 2018	Monday Feb 26th	Tuesday Feb 27th	Wednesday Feb 28th	Thursday March 1st	Friday March 2nd
09:00	**Introduction to Magnets I** lecture *A. Milanese*	**Superconducting magnets** lecture *P. Ferracin*	**Mini-workshop** **Normal conducting Magnets** *J. Bauche & T. Zickler*	***Bus leaves at 8:00 from JUAS*** ***(Lunch at CERN, offered by ESI)*** **PRACTICAL DAYS AT CERN** RF coordinator: **F. Caspers** **M. Wendt** VACUUM coordinator: **V. Baglin** MAGNETS coordinators: **J. Bauche** **L. Fiscarelli** SUPERCONDUCTIVITY coordinator: **J. Fleiter** CLEAR coordinators: **R. Corsini** **W. Farabolini** ***Bus leaves at 17:30 from CERN***	***Bus leaves at 8:00 from JUAS*** ***(Lunch at CERN, offered by ESI)*** **PRACTICAL DAYS AT CERN** RF coordinator: **F. Caspers** VACUUM coordinator: **V. Baglin** MAGNETS coordinators: **J. Bauche** **L. Fiscarelli** SUPERCONDUCTIVITY coordinator: **J. Fleiter** CLEAR coordinators: **R. Corsini** **W. Farabolini** ***Bus leaves at 17:30 from CERN***
10:00	**Introduction to Magnets II** lecture *A. Milanese*				
10:15		**Coffee Break**	**Coffee Break**		
10:30	**Coffee Break** **10:45** **Normal Conducting magnets** lecture *T. Zickler*	**Superconducting magnets** lecture *P. Ferracin*	**Mini-workshop** **Normal conducting Magnets** *J. Bauche & T. Zickler*		
11:15		**Superconducting magnets: cryogenics** lecture *Ph. Lebrun*	**Mini-workshop** **Normal conducting Magnets** *J. Bauche & T. Zickler*		
12:15	**WORKING LUNCH**	**BREAK**	**BREAK**		
14:00	**Normal Conducting magnets** lecture - *T. Zickler*	**Superconducting magnets** lecture *P. Ferracin*	**Mini-workshop** **Superconducting Magnets** *P. Ferracin & P. Lebrun*		
15:00	**Normal Conducting magnets** lecture - *T. Zickler*	**Normal Conducting magnets** lecture - *T. Zickler*	**Mini-workshop** **Superconducting Magnets** *P. Ferracin & P. Lebrun*		
16:00	**Coffee Break**	**Coffee Break**	**Coffee Break**		
16:15	**Superconducting magnets** lecture *P. Ferracin*	**Normal Conducting magnets** lecture - *T. Zickler*	**Mini-workshop** **Superconducting Magnets** *P. Ferracin & P. Lebrun*		
17:15	**Superconducting magnets** lecture *P. Ferracin*	**Normal Conducting magnets** lecture - *T. Zickler*	**Building Large Accelerators with Industry** Seminar *Ph. Lebrun*		
18:15			**AFTER WORK AT ESI**		

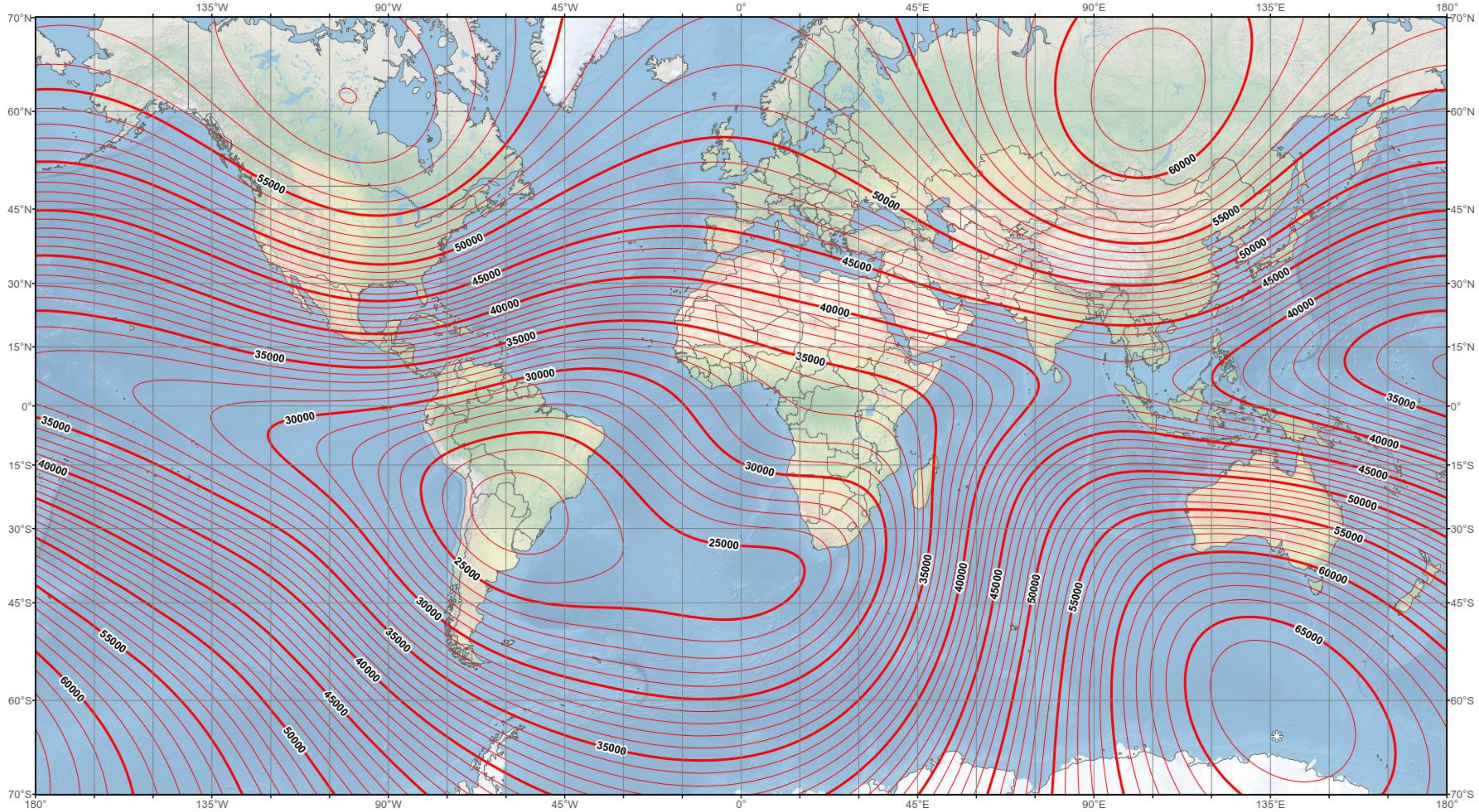
1. Introduction

2. Fundamentals 1: Maxwell and friends

3. Fundamentals 2: harmonics

In Archamps, on 26/02/2018, the (estimated) magnetic field is $|B|$ = 47435 nT = 0.047435 mT = $4.7435 \cdot 10^{-5}$ T ≈ 0.5 Gauss

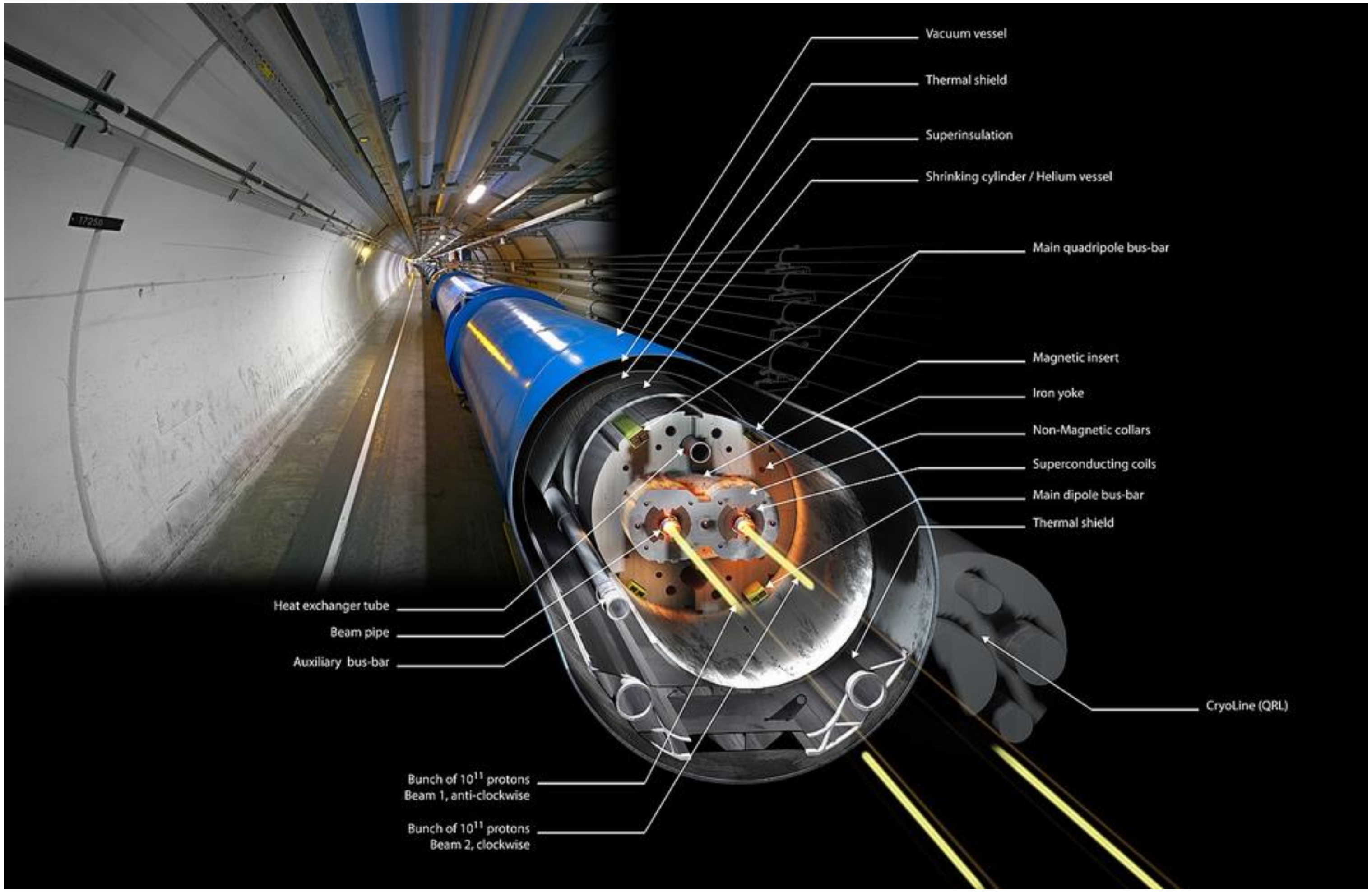
US/UK World Magnetic Model - Epoch 2015.0
Main Field Total Intensity (F)



Main Field Total Intensity (F)
Contour interval: 1000 nT.
Mercator Projection.
✺: Position of dip poles

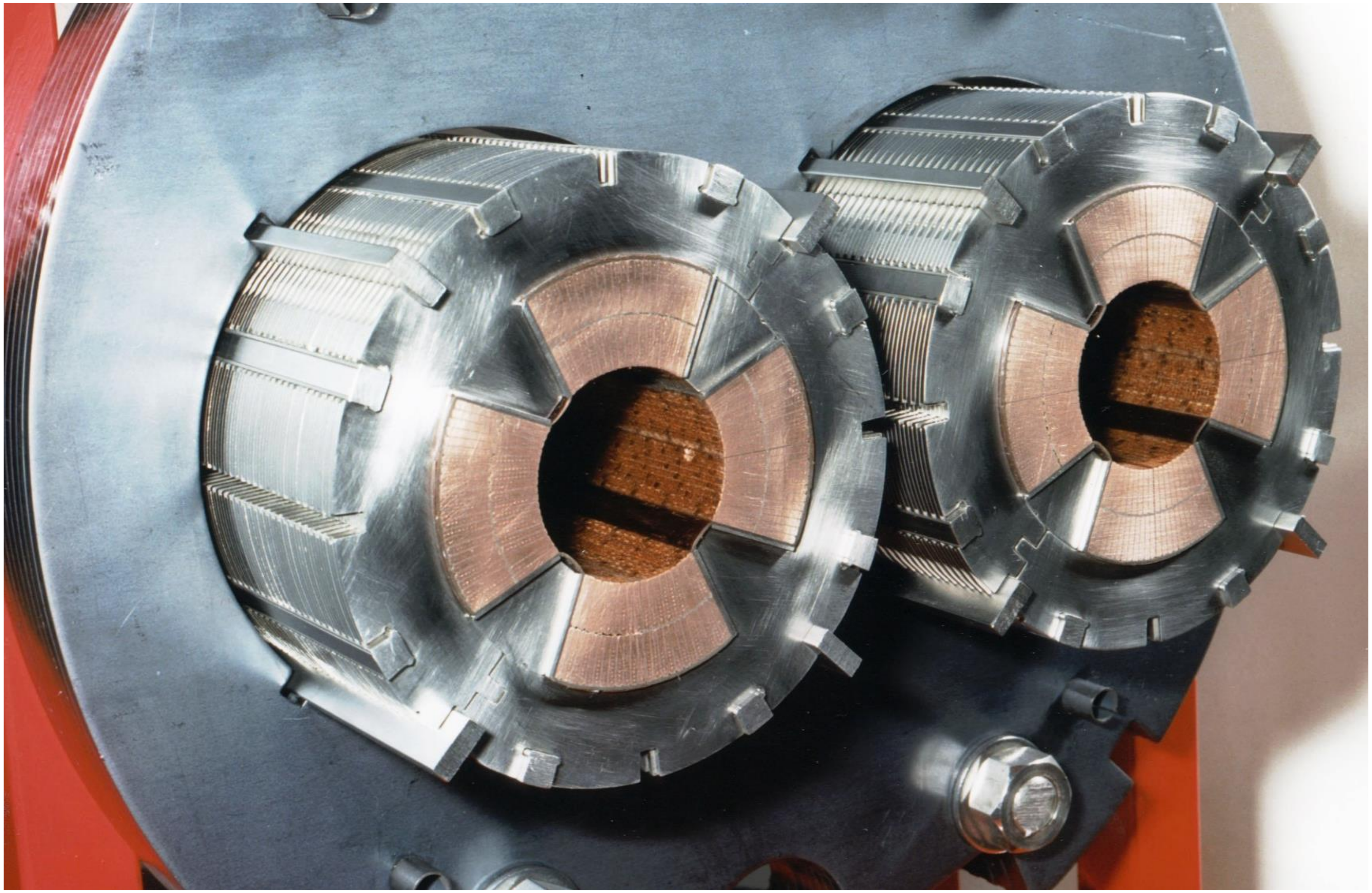
Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM
Map reviewed by NGA and BGS
Published December 2014

This is a main dipole of the LHC at CERN: 8.3 T × 14.3 m



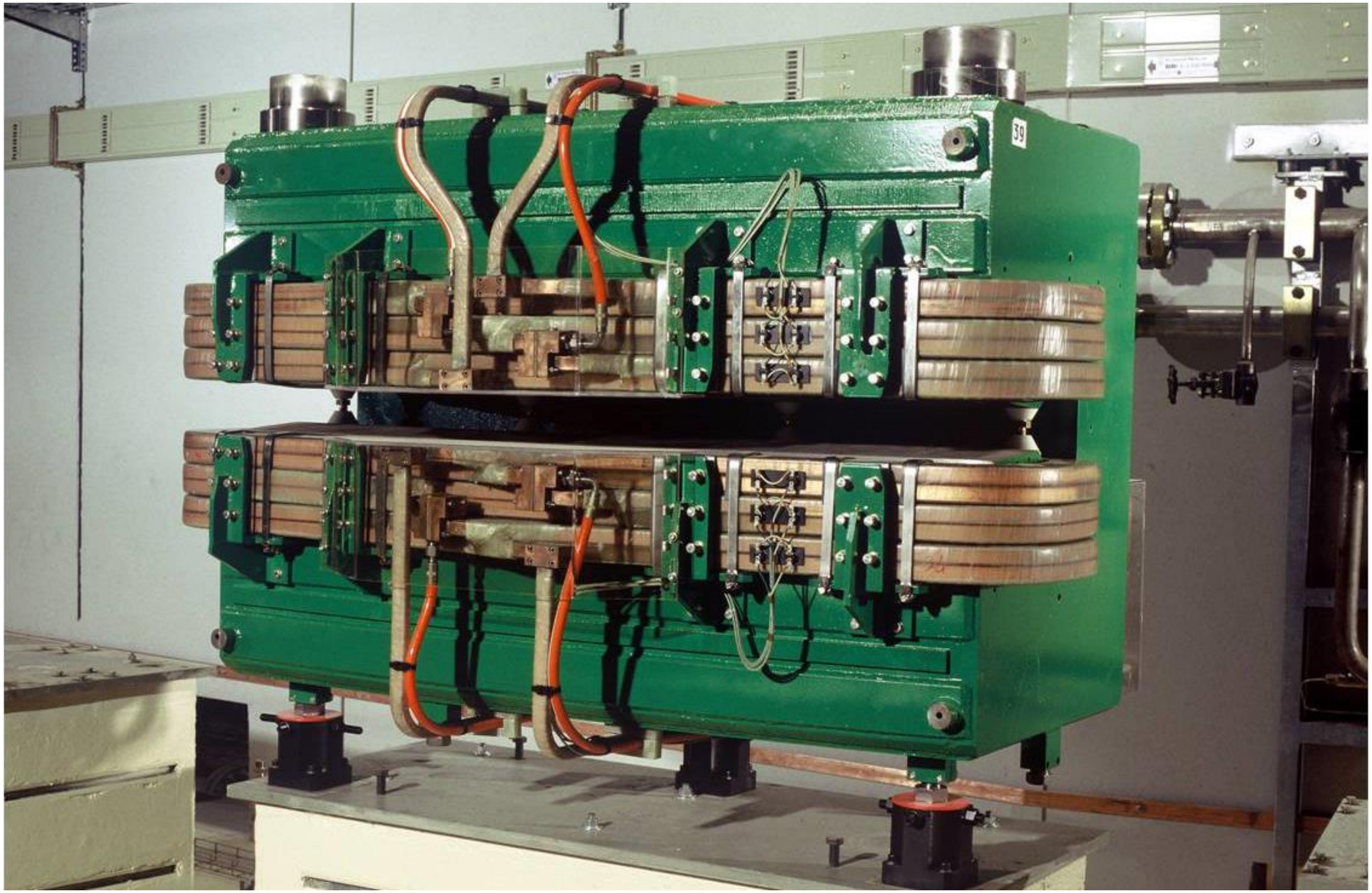
These are main dipoles of the SPS at CERN: 2.0 T × 6.3 m

This is a cross section of a main quadrupole of the LHC at CERN: 223 T/m × 3.2 m

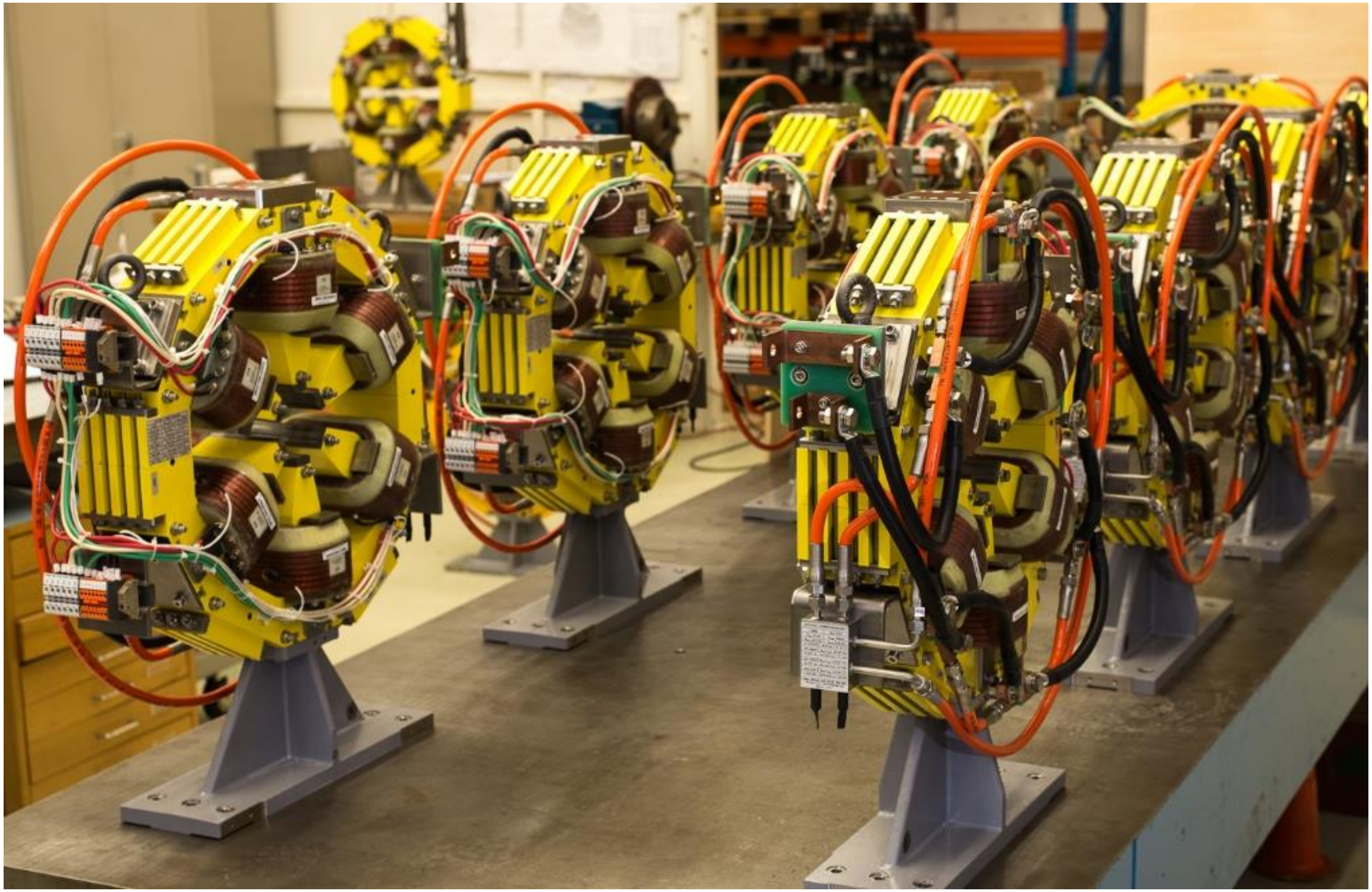


These are main quadrupoles of the SPS at CERN: 22 T/m × 3.2 m

This is a combined function bending magnet of the ELETTRA light source



These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



We can classify magnets based on their geometry (that is, what they do to the beam)

dipole

quadrupole

sextupole

octupole

kicker / septum

solenoid

combined function bending

corrector

skew magnet

undulator / wiggler

We can also classify magnets based on their technology

electromagnet

permanent magnet

iron dominated

coil dominated

normal conducting (resistive)

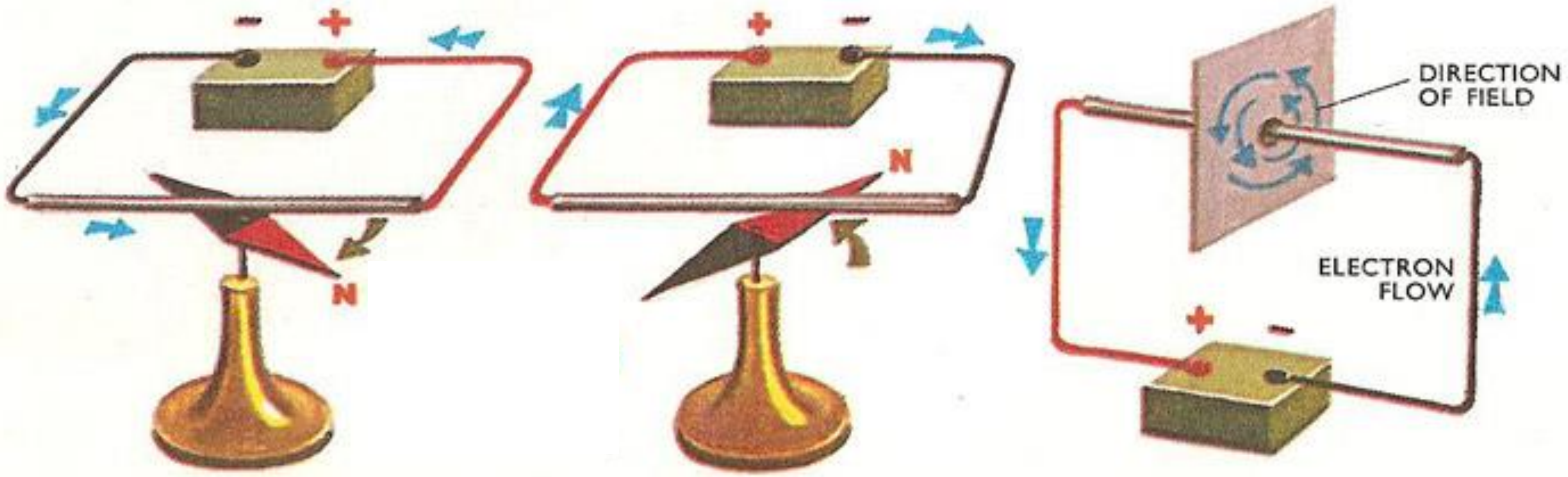
superconducting

static

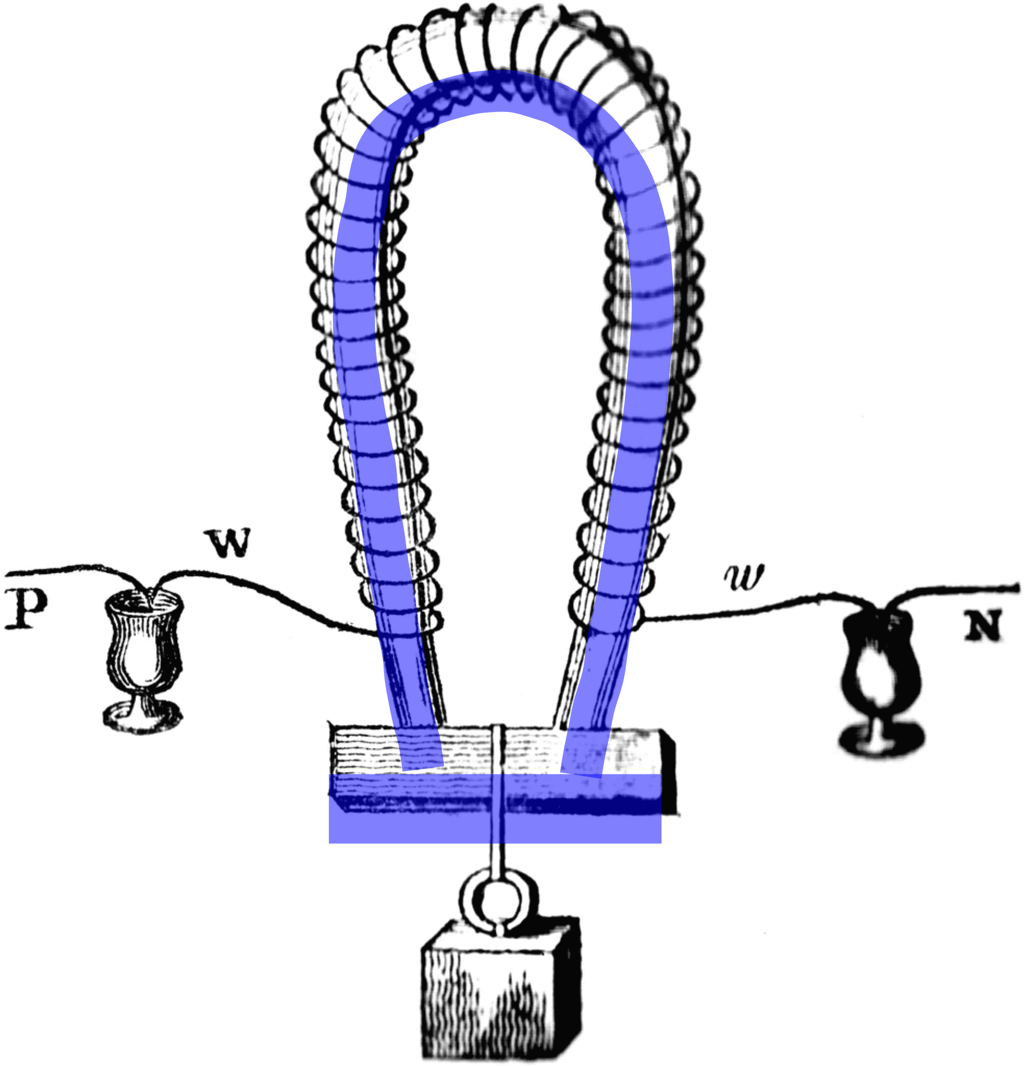
cycled / ramped slow pulsed

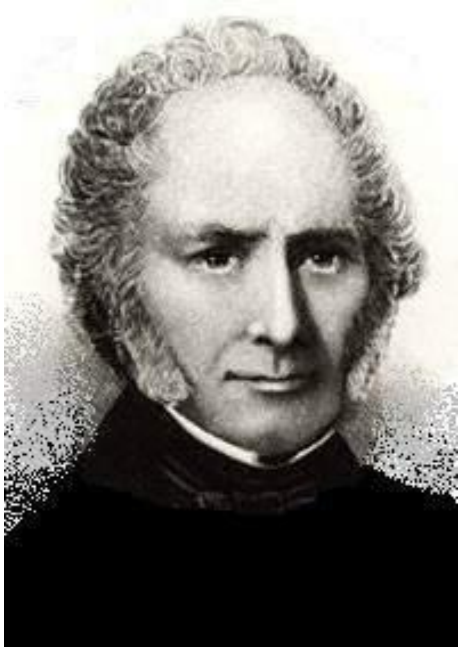
fast pulsed

Ørsted showed in 1820 that electricity and magnetism were somehow related

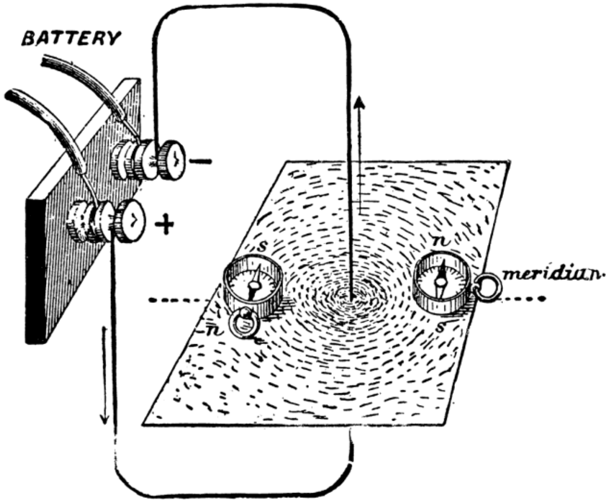


The first electromagnet was built in 1824 by Sturgeon

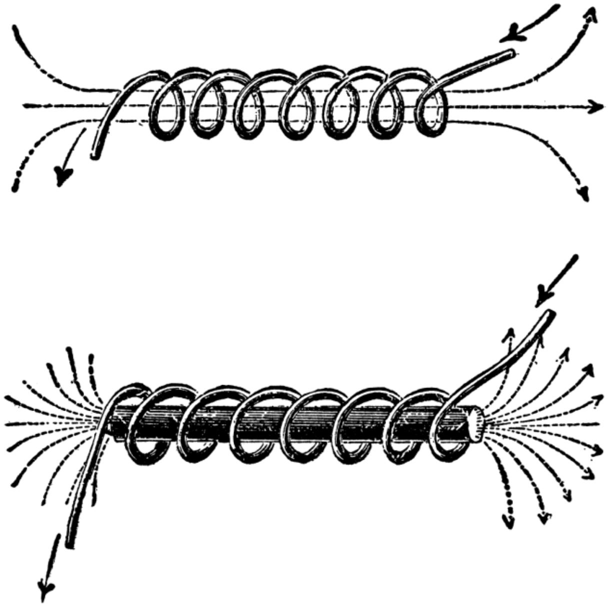




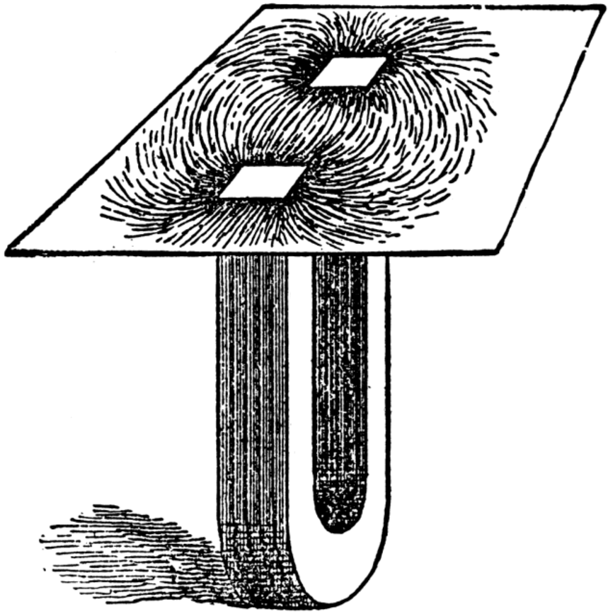
Our magnets work on a few basic principles (steady state only)



an electrical current induces a magnetic effect

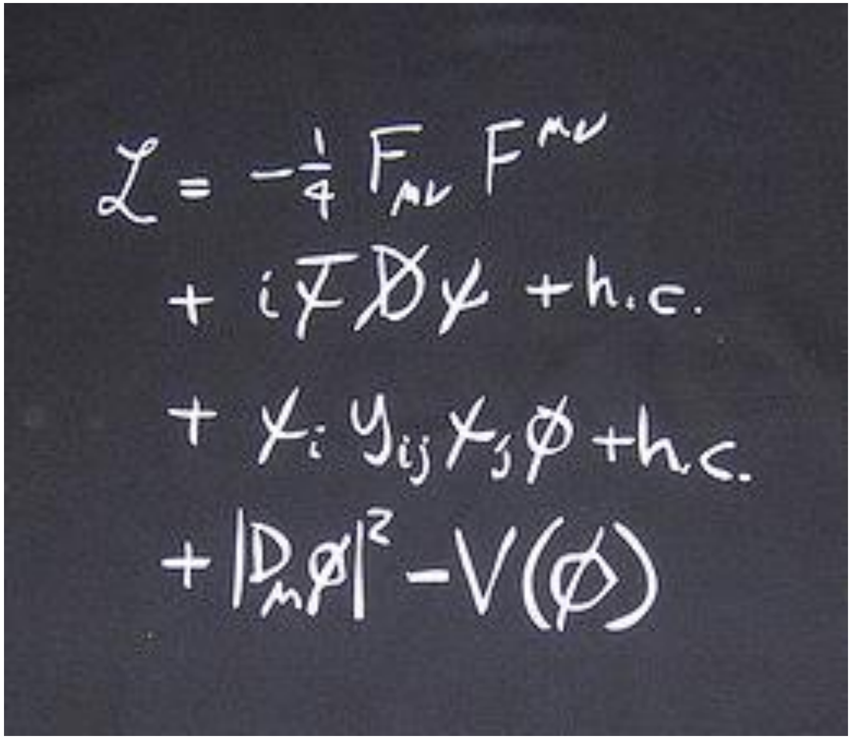


some materials (e.g. iron) greatly enhance these effects



some other materials produce these effects even without electrical currents

So, how do we properly describe all this?



$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &+ i \bar{\psi} \not{D} \psi + h.c. \\ &+ \psi_i y_{ij} \psi_j \phi + h.c. \\ &+ |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

1. Introduction

2. Fundamentals 1: Maxwell and friends

3. Fundamentals 2: harmonics

We need to agree on some nomenclature first

B	magnetic field B field magnetic flux density magnetic induction	T (Tesla)
H	H field magnetic field strength magnetic field	A/m (Ampere/m)
μ_0	permeability of vacuum	$4\pi\cdot10^{-7}$ H/m (Henry/m)
μ_r	relative permeability	dimensionless
μ	permeability, $\mu = \mu_0\mu_r$	H/m

What is B? For us, it is defined by its effect on moving charged particles (or electrical currents), through Lorentz force

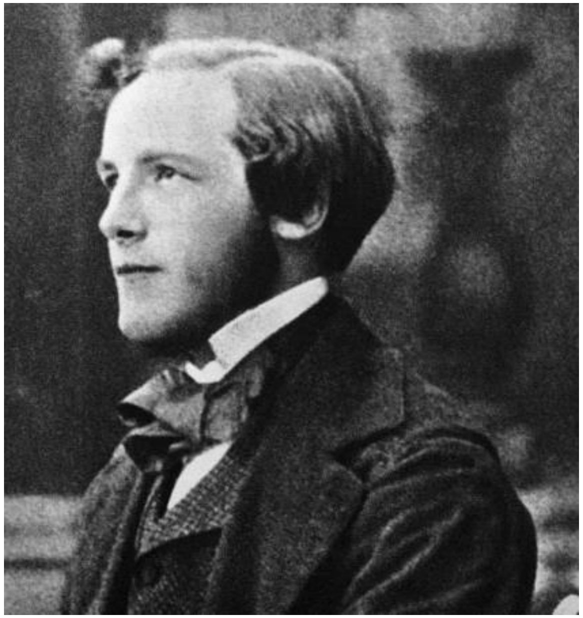
$$\overrightarrow{F_m} = q\left(\vec{v} \times \vec{B}\right)$$

for charged beams

$$\overrightarrow{F_m} = I\vec{\ell} \times \vec{B}$$

for conductors

Maxwell describes it all using vector calculus



$\text{div}\,\vec{D} = \rho$ Gauss law (electricity)

$\text{div}\,\vec{B} = 0$ Gauss law (magnetism)

$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday-Lenz law

$\text{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ Ampère law (with correction)

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$\vec{B} = \mu_0 \mu_r \vec{H}$

constitutive laws
for (simple) materials

Let’s have a closer look at the 3 equations that describe magnetostatics

(1) $\operatorname{div} \vec{B} = 0$ always holds

(2) $\operatorname{rot} \vec{H} = \vec{J}$ holds for magnetostatics

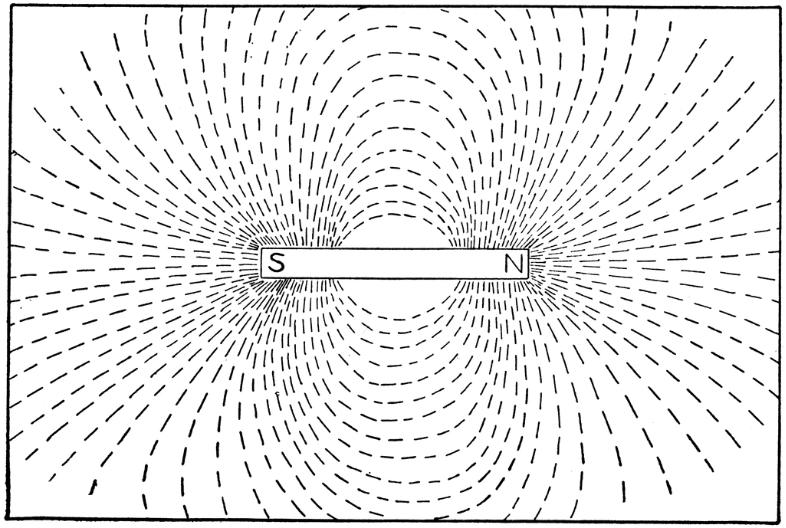
(3) $\vec{B} = \mu_0 \mu_r \vec{H}$ holds for linear materials

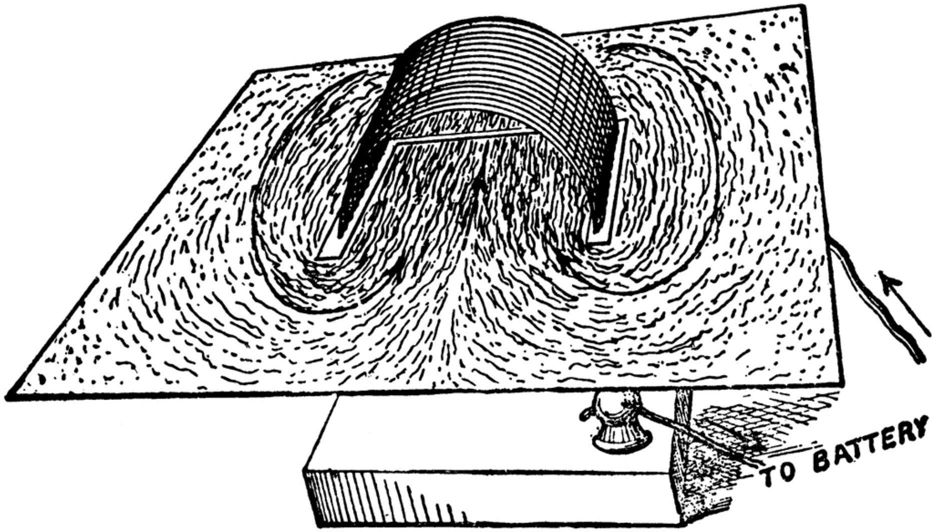
Eq. 1: the magnetic flux tubes wrap around, with neither sources nor sinks

$$\mathrm{div}\,\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\oiint \vec{B} \cdot \overrightarrow{dS} \underset{\uparrow}{=} \iiint \mathrm{div}\,\vec{B}\, dV = 0$$

divergence / Gauss theorem



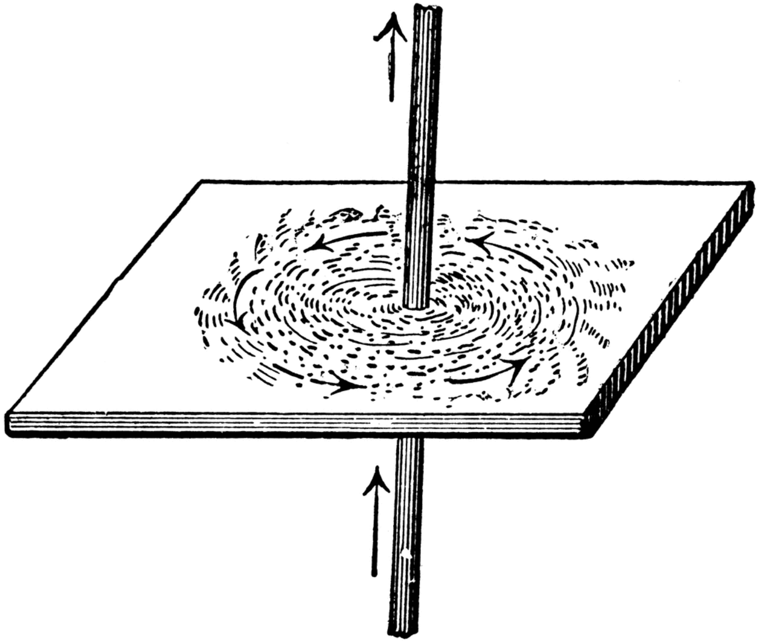


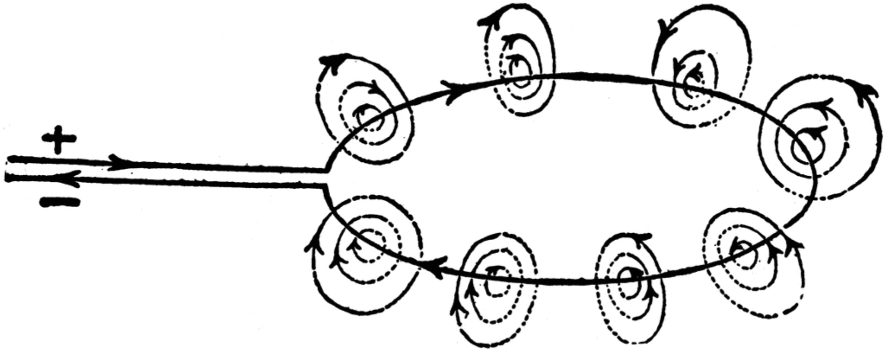
Eq. 2: electrical currents generate (“stir up”) a magnetic field

$$\mathrm{rot}\,\vec{H} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\vec{i_x} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\vec{i_y} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\vec{i_z} = \vec{J}$$

$$\oint \vec{H} \cdot \vec{dl} \underset{\uparrow}{=} \iint \mathrm{rot}\,\vec{H}\,dS = \iint \vec{J}\,dS = NI$$

Kelvin–Stokes theorem



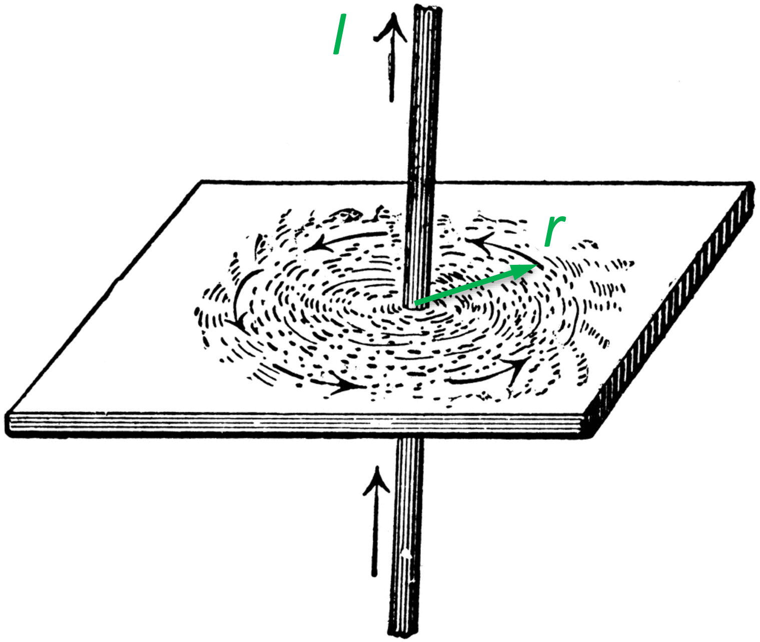


From Eqs. 2 and 3 we can derive Biot-Savart law

$$\oint \vec{H} \cdot \vec{dl} = I$$

$$H\,(2\pi r) = I$$

$$H = \frac{I}{2\pi r}$$



$$\vec{B} = \mu_0 \vec{H}$$

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r}$$

Eq. 3 relates the effect (B) to the cause (H)

In a linear material

$\vec{H}$

produces

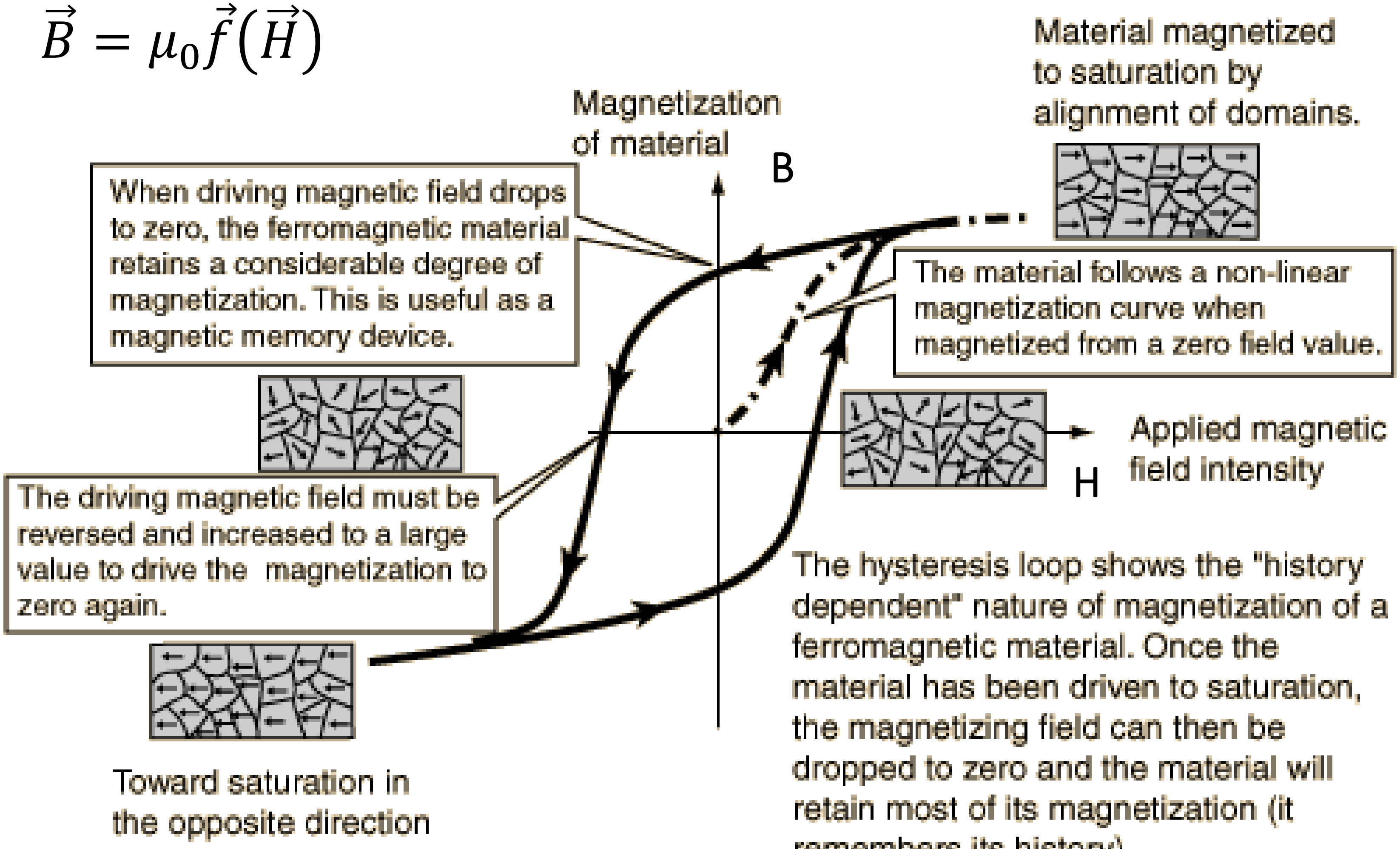
$\vec{B}$

according to

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

In a nonlinear material (with for ex. saturation and hysteresis), the constitutive law becomes more complex

$$\vec{B} = \mu_0 \vec{f}(\vec{H})$$



Material magnetized to saturation by alignment of domains.

When driving magnetic field drops to zero, the ferromagnetic material retains a considerable degree of magnetization. This is useful as a magnetic memory device.

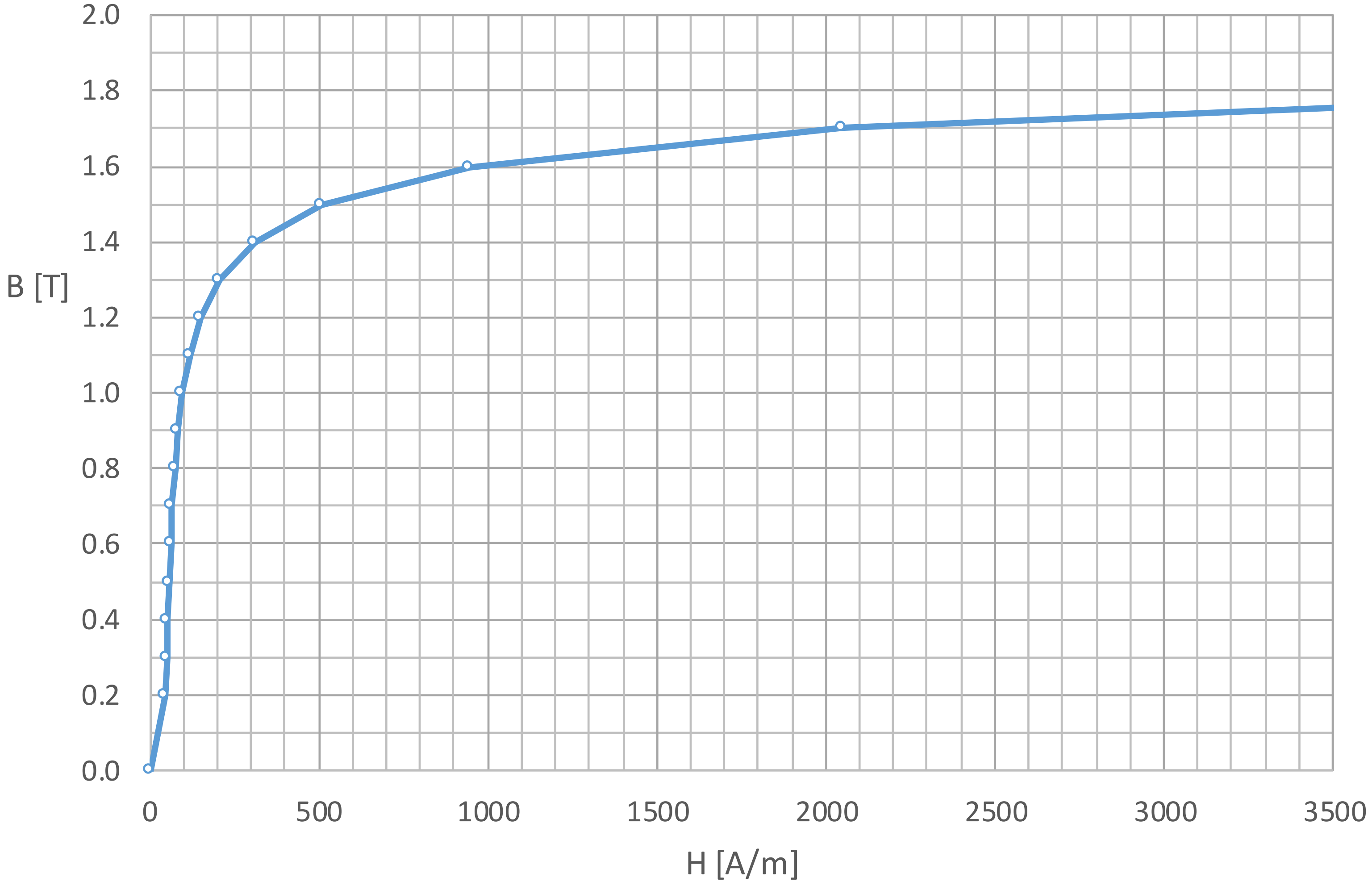
The material follows a non-linear magnetization curve when magnetized from a zero field value.

The driving magnetic field must be reversed and increased to a large value to drive the magnetization to zero again.

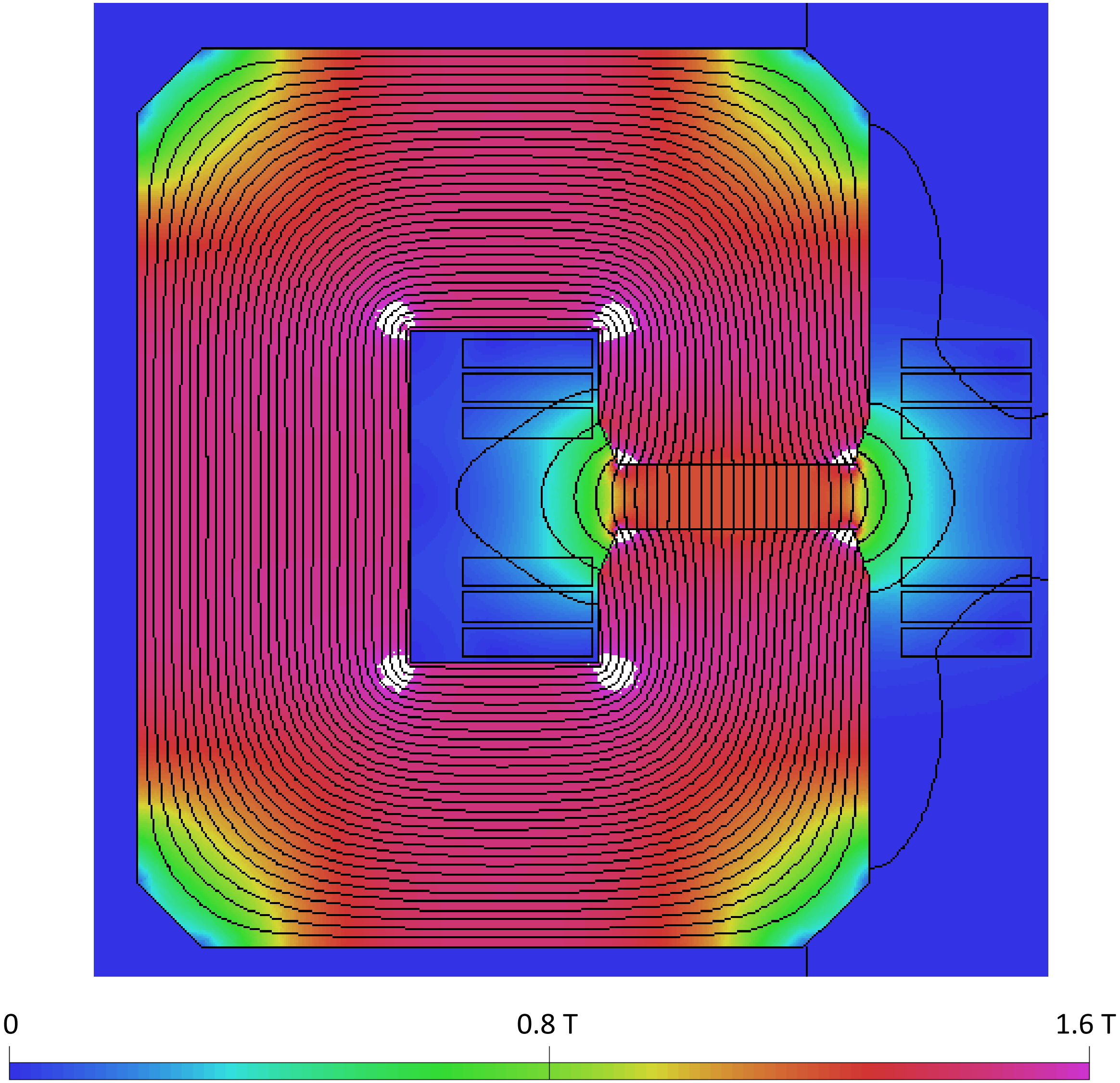
Toward saturation in the opposite direction

The hysteresis loop shows the "history dependent" nature of magnetization of a ferromagnetic material. Once the material has been driven to saturation, the magnetizing field can then be dropped to zero and the material will retain most of its magnetization (it remembers its history).

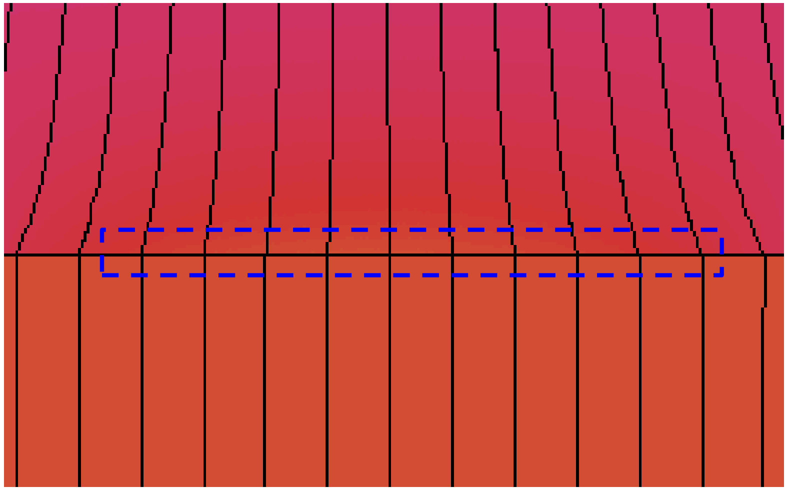
In most of our simulations we use a simple BH model for the material: this is a typical curve for an electrical steel



Now, why do the flux lines come out perpendicular to the iron?



Because they obey to Maxwell!



iron $\mu_r \gg 1$

air $\mu_r = 1$

$$H_{\parallel,\,\mathrm{air}} = H_{\parallel,\,\mathrm{iron}}$$

$$B_{\parallel,\,\mathrm{air}} = \frac{B_{\parallel,\,\mathrm{iron}}}{\mu_{r,\mathrm{iron}}} \approx 0$$

$$B_{\perp,\,\mathrm{air}} = B_{\perp,\,\mathrm{iron}}$$

This is an “advanced introduction”, so let’s introduce the vector potential (3D)

$$\vec{B} = \operatorname{rot} \vec{A}$$

always holds

In magnetostatics, we can combine Eqs. 1 to 3 in a more compact form (3D)

$$\nabla^2\vec{A}=\vec{0}$$

$$\left.\begin{array}{l}\operatorname{div}\vec{B}=0\\[2em] \operatorname{rot}\vec{H}=0\\[2em] \vec{B}=\mu_0\vec{H}\end{array}\right\}\quad\nabla^2\vec{A}=\vec{0}$$

holds for
magnetostatics
and in air

In 2D this becomes a scalar Laplace equation

$$\nabla^2 A_z = 0$$

$$\frac{\partial^2 A_{\boldsymbol{z}}}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0$$

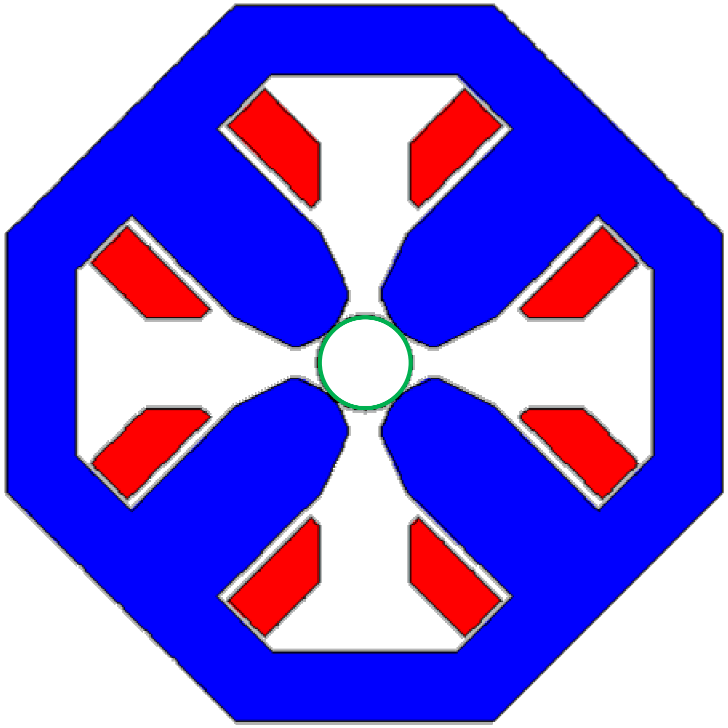
holds for
magnetostatics
and in air

1. Introduction

2. Fundamentals 1: Maxwell and friends

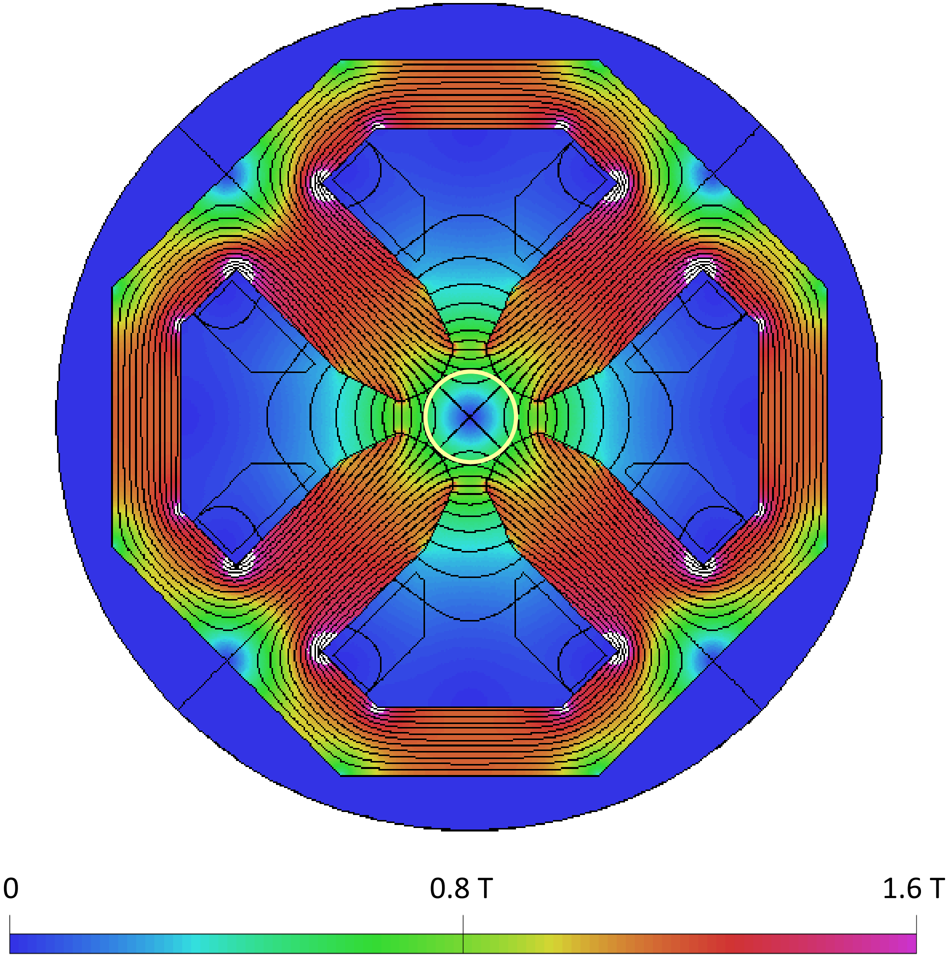
3. Fundamentals 2: harmonics

We look at the 2D first: how can we conveniently describe the field in the aperture, for ex. in a quadrupole?

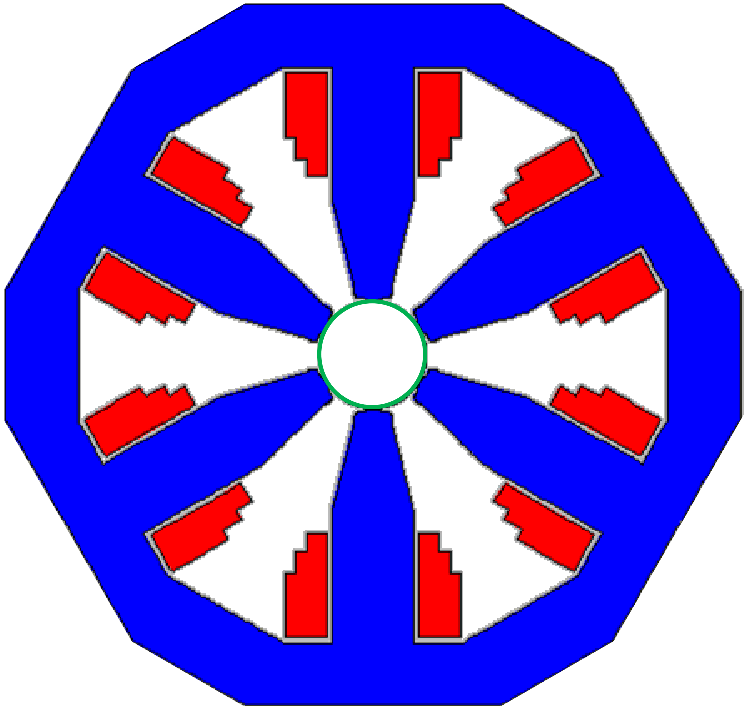


SESAME quadrupole

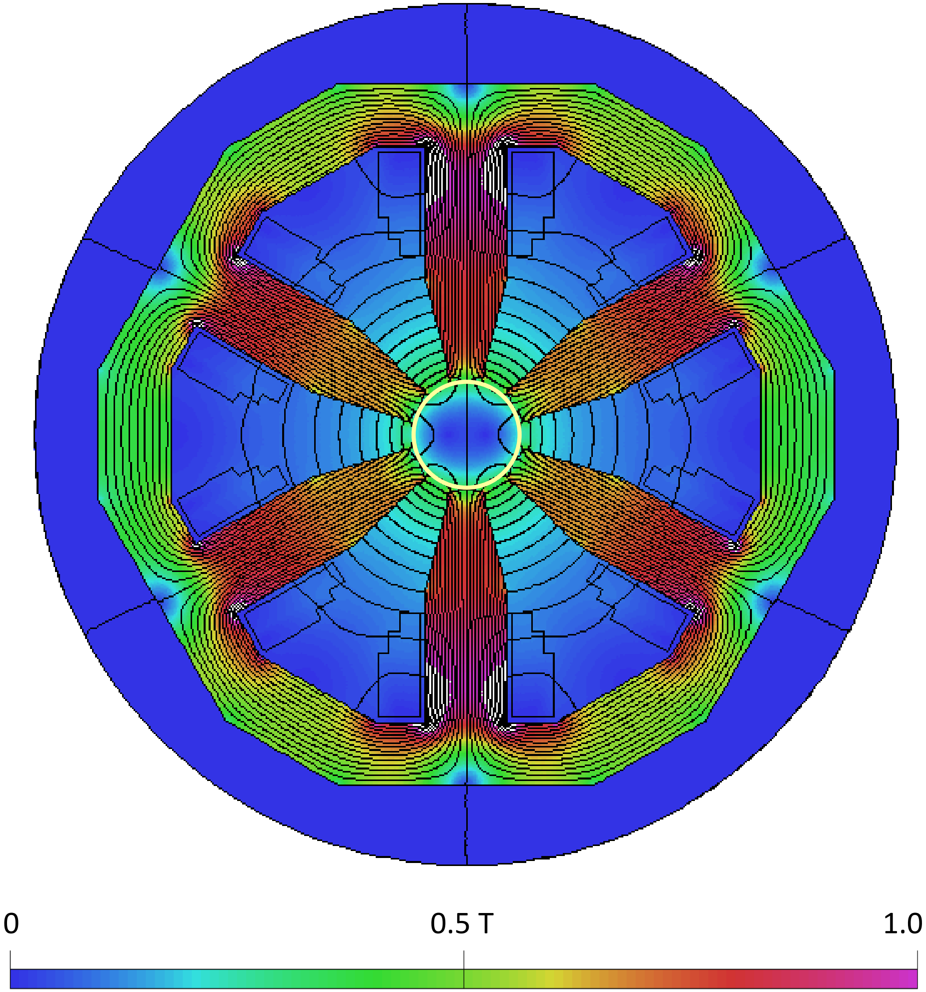
$B_{pole} = 0.6$ T



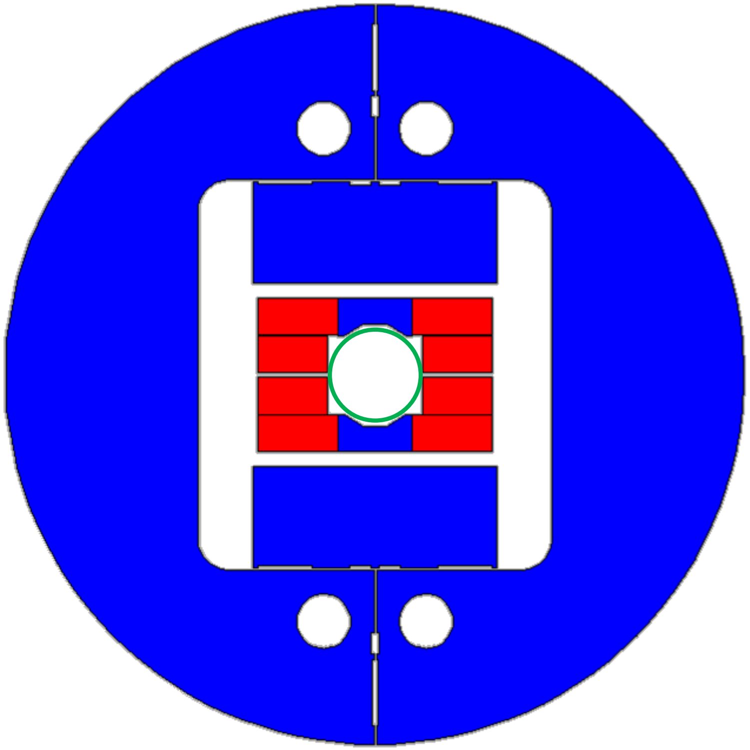
And in another resistive magnet, with a different configuration?



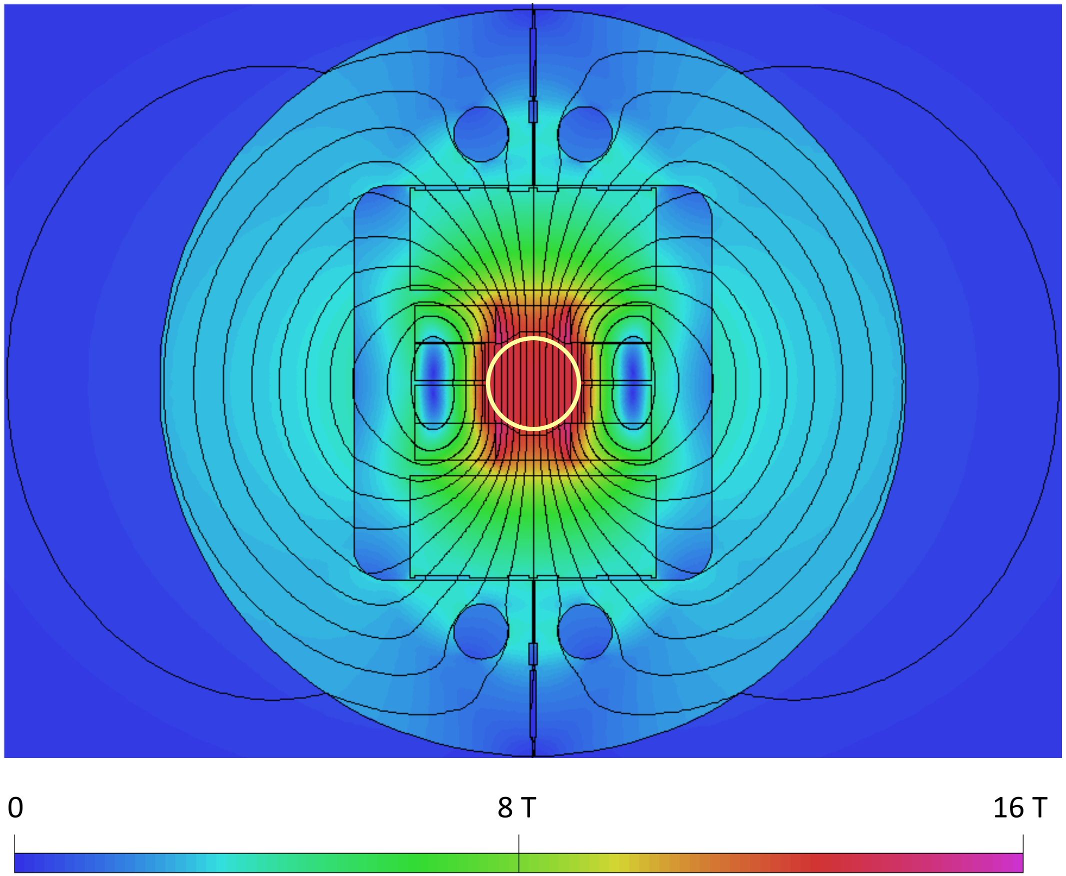
SESAME sextupole
+ vertical dipole corrector



Can the same formalism also describe the field in the aperture of a superconducting dipole?

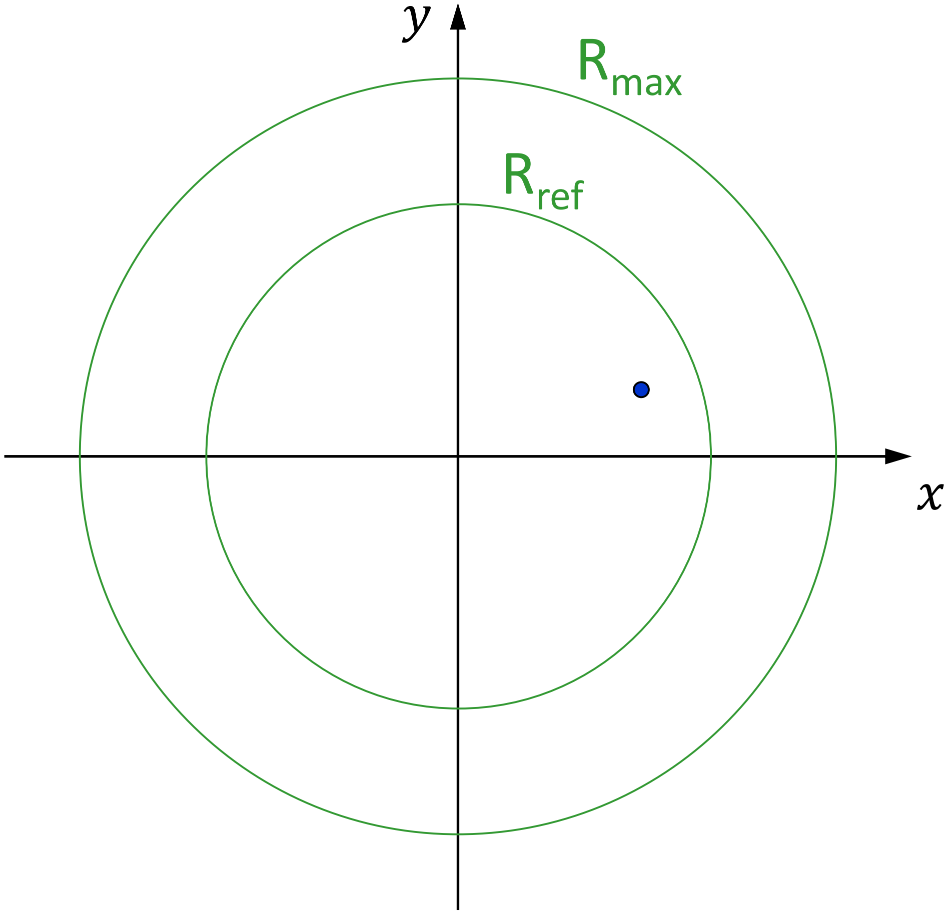


FRESCA2 dipole
13 T



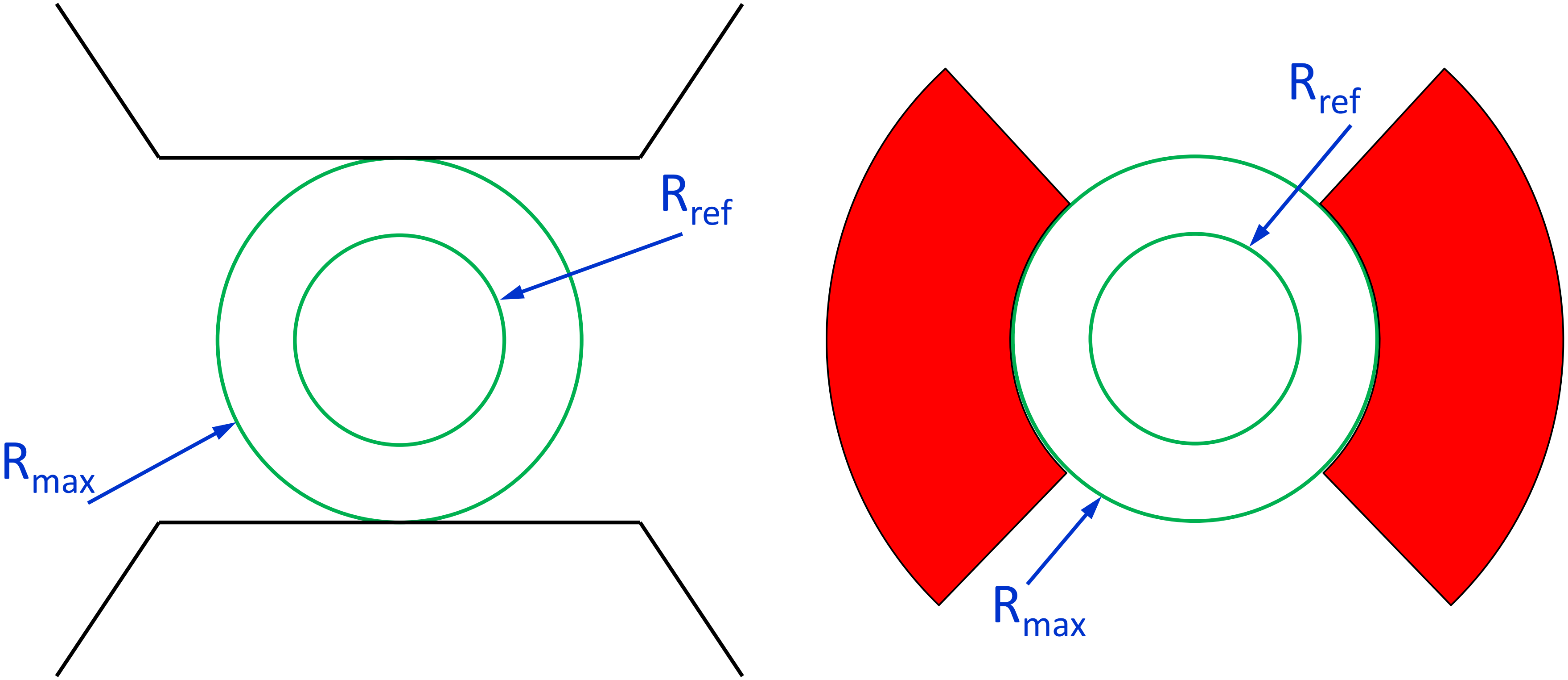
The solution is a harmonic (or multipole) expansion, describing the field (within a circle of validity) with scalar coefficients

$$(4) \qquad B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}} \right)^{n-1}$$



$$z = x + iy = re^{i\theta}$$

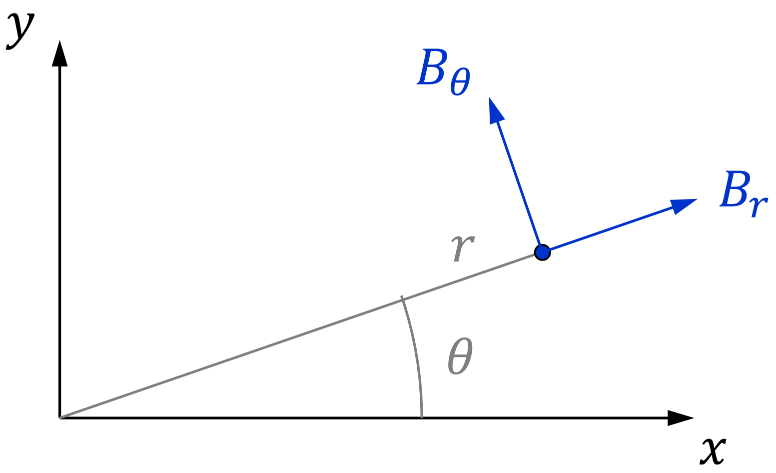
This decomposition has two characteristic radii: R_{ref} and R_{max}



Expanding Eq. 4 in terms of radial and tangential components, we find sin and cos terms

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$



In most cases, there is a main fundamental component, to which the other terms are normalized

$$\boxed{(4) \qquad B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}} \right)^{n-1}}$$

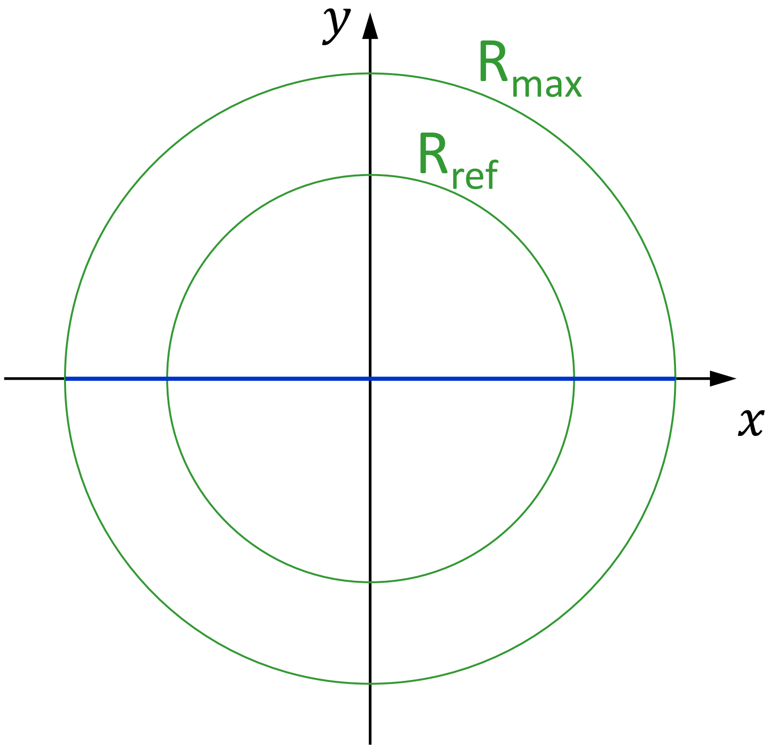
$$b_n = 10000 \frac{B_n}{B_N} \qquad a_n = 10000 \frac{A_n}{B_N}$$

$$B_y(z) + iB_x(z) = \underset{\substack{\uparrow \\ \text{field strength}}}{B_N} \underbrace{\sum_{n=1}^{\infty} \frac{b_n + ia_n}{10000} \left(\frac{z}{R_{ref}} \right)^{n-1}}_{\substack{\uparrow \\ \text{field shape}}}$$

Another useful expansion derived from Eq. 4 is that of B_y on the midplane, *i.e.* at y = 0

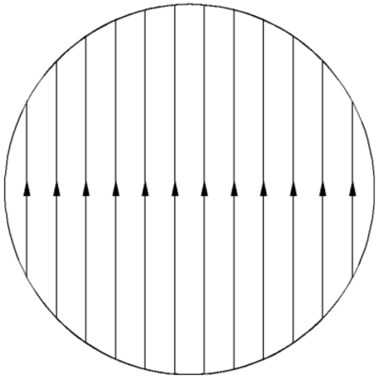
$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R_{ref}}\right)^{n-1} = B_1 + B_2 \frac{x}{R_{ref}} + B_3 \left(\frac{x}{R_{ref}}\right)^2 + \cdots$$

$$B_x(x) = \sum_{n=1}^{\infty} A_n \left(\frac{x}{R_{ref}}\right)^{n-1} = A_1 + A_2 \frac{x}{R_{ref}} + A_3 \left(\frac{x}{R_{ref}}\right)^2 + \cdots$$

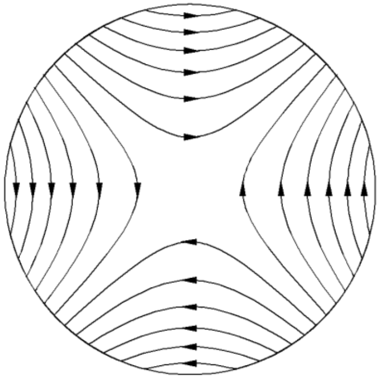


Each multipole corresponds to a field distribution: adding them up, we can describe everything (compatibly with Maxwell)

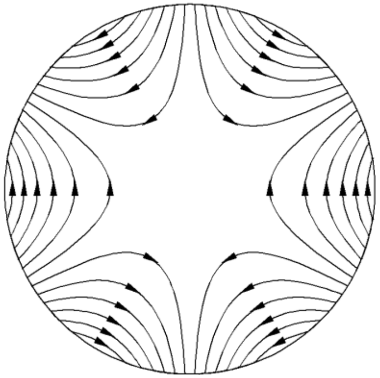
B_1: normal dipole



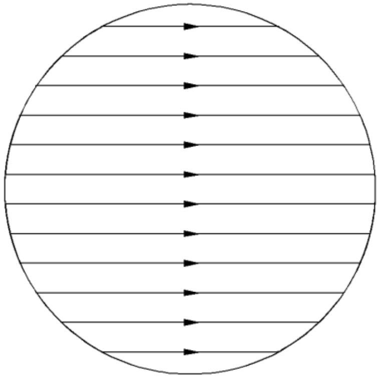
B_2: normal quadrupole



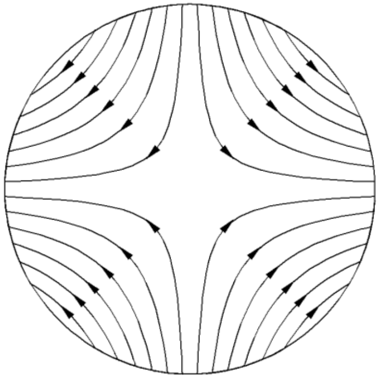
B_3: normal sextupole



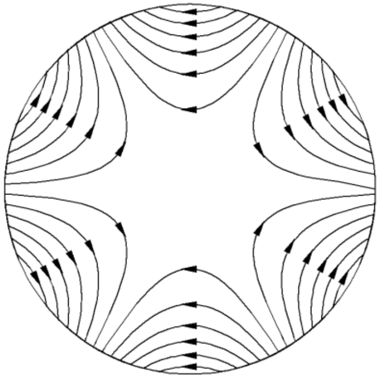
A_1: skew dipole



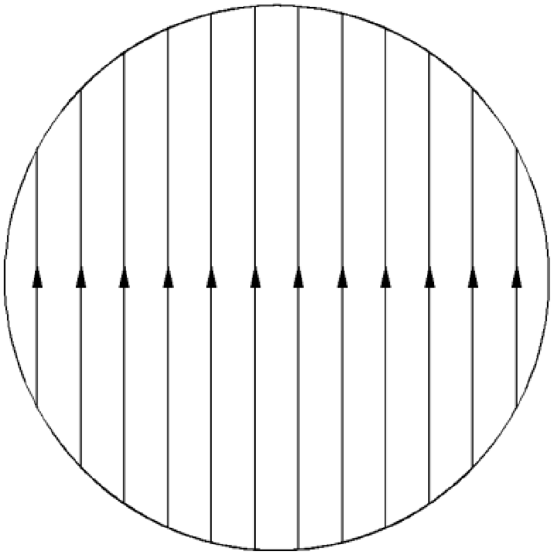
A_2: skew quadrupole

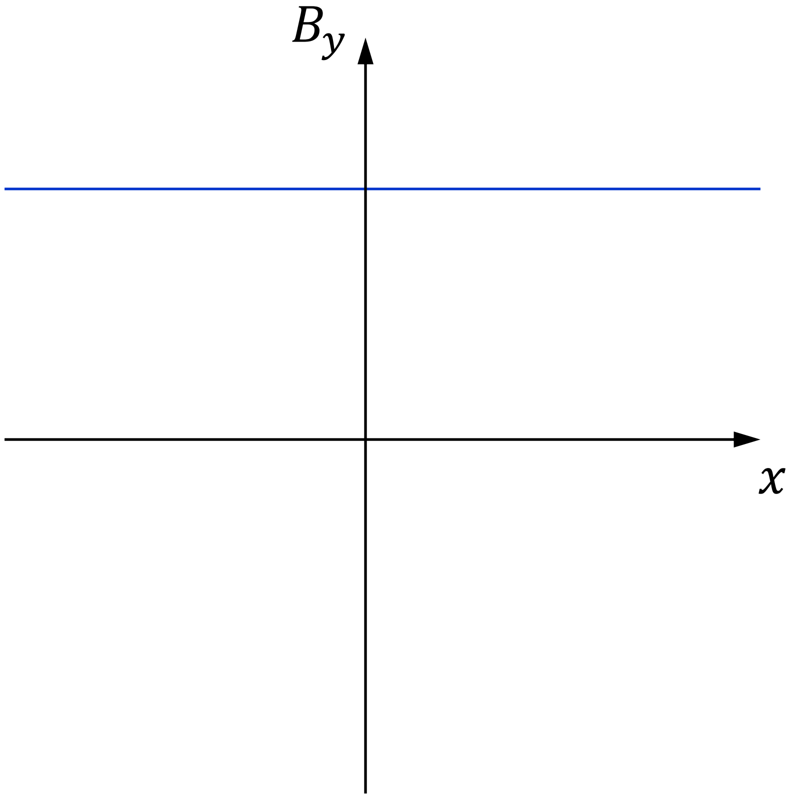


A_3: skew sextupole

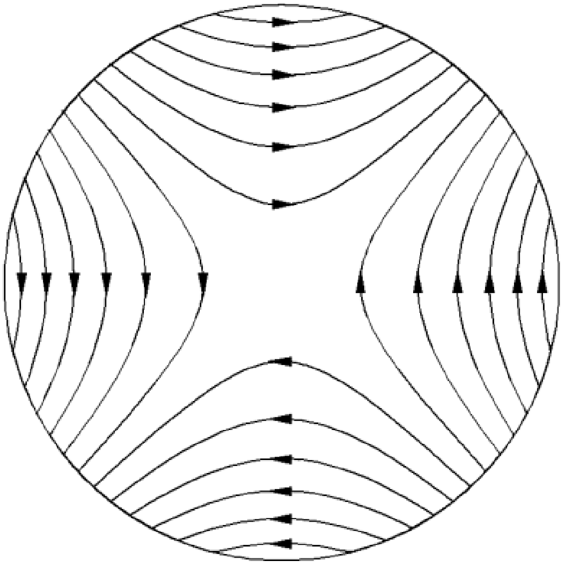


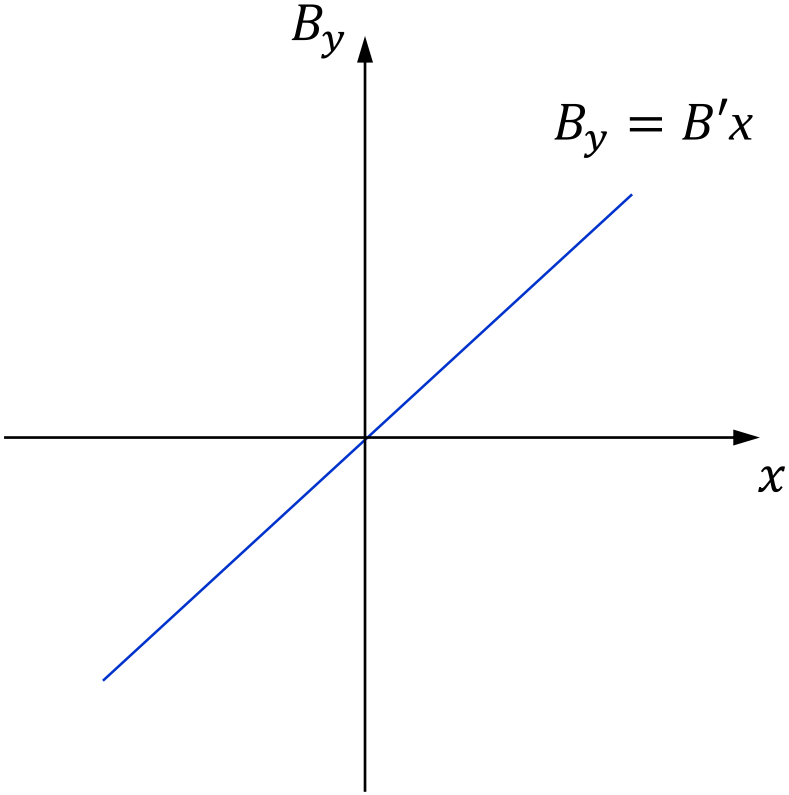
B_1 is the normal dipole





B_2 is the normal quadrupole

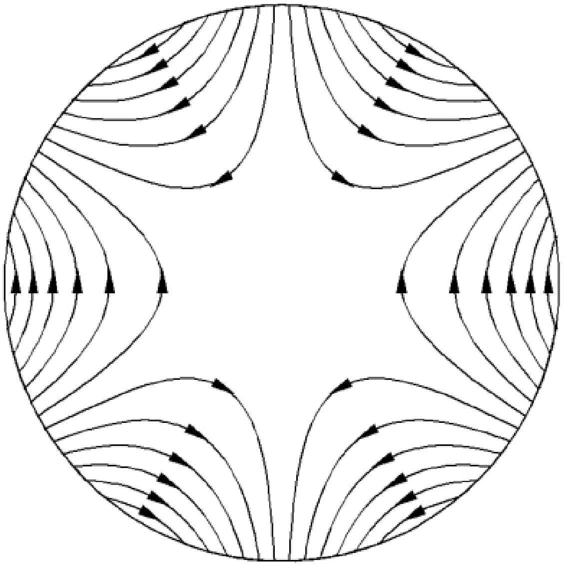




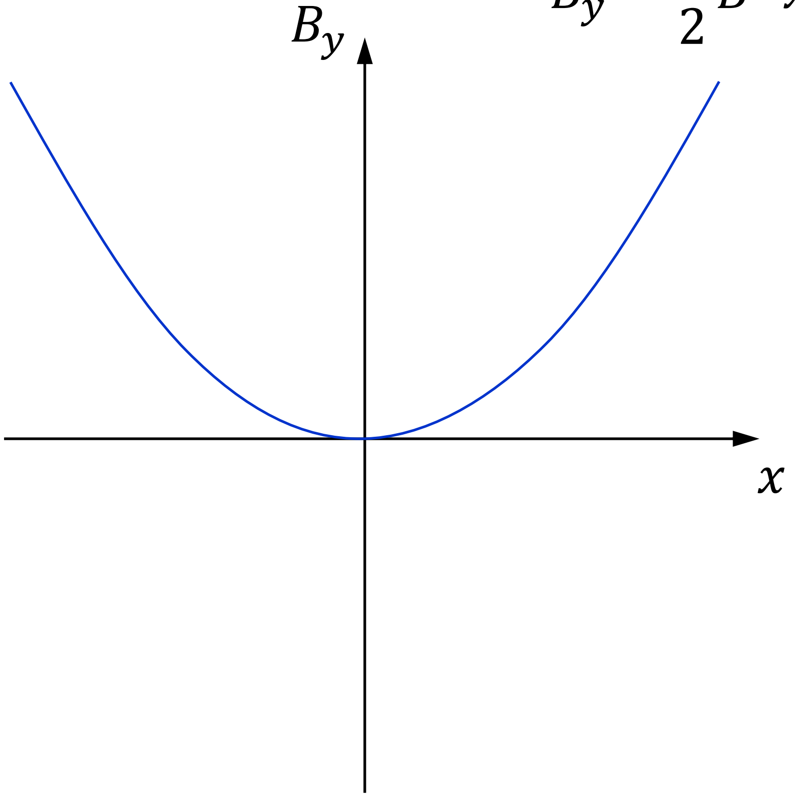
$$G = \frac{B_2}{R} = \frac{\partial B_y}{\partial x} = B'$$

$$B_{pole} = B'R_{pole}$$

B_3 is the normal sextupole



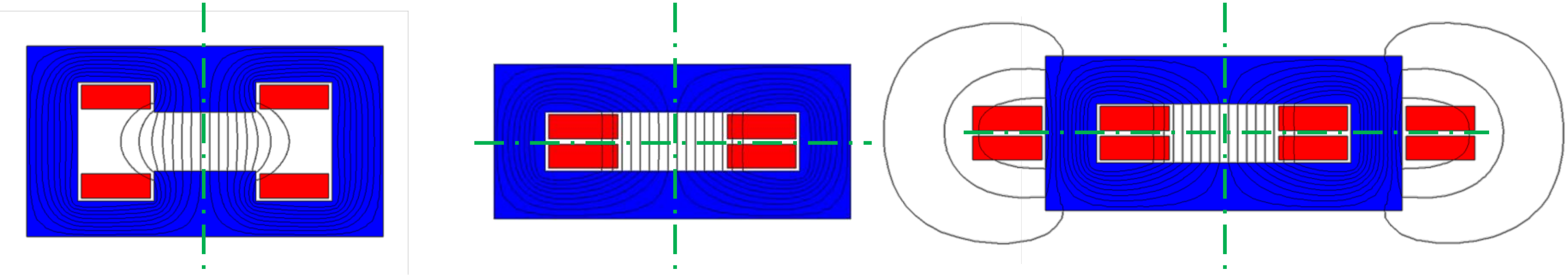
$$B_y = \frac{1}{2}B''x$$



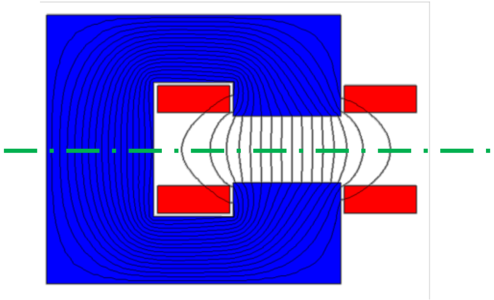
$$B'' = \frac{\partial^2 B_y}{\partial x^2} = \frac{2B_3}{R^2}$$

$$B_{pole} = \frac{1}{2}B''R_{pole}$$

The allowed / not-allowed harmonics refer to the terms that shall / shall not cancel out thanks to design symmetries

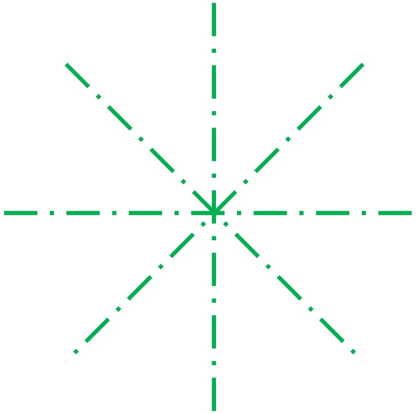


<u>fully symmetric dipoles</u>: only B_1, b_3, b_5, b_7, b_9, etc.

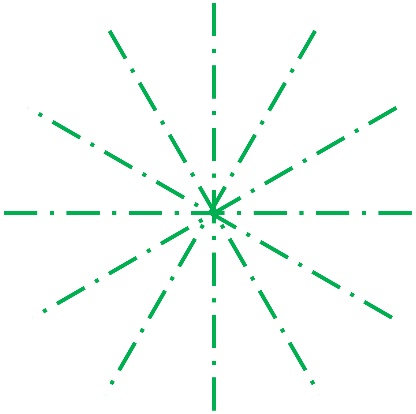


<u>half symmetric dipoles</u>: B_1, b_2, b_3, b_4, b_5, etc.

These are the allowed harmonics for fully symmetric quadrupoles and sextupoles

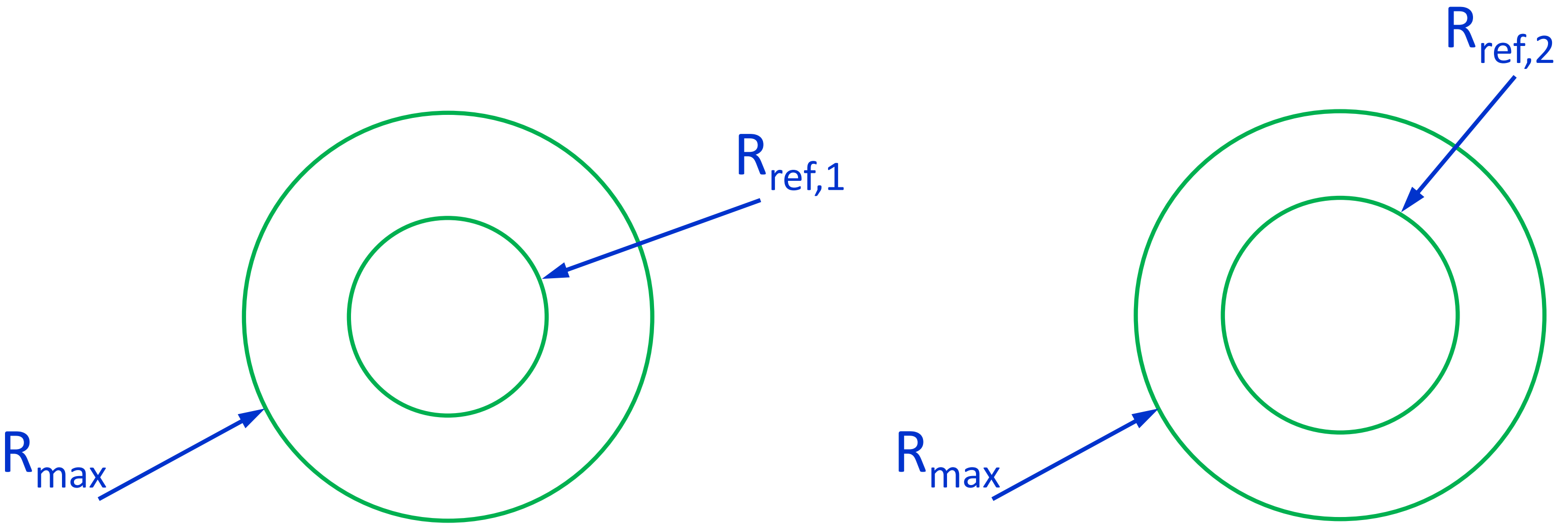


<u>fully symmetric quadrupoles</u>: B_2, b_6, b_{10}, b_{14}, b_{18}, etc.



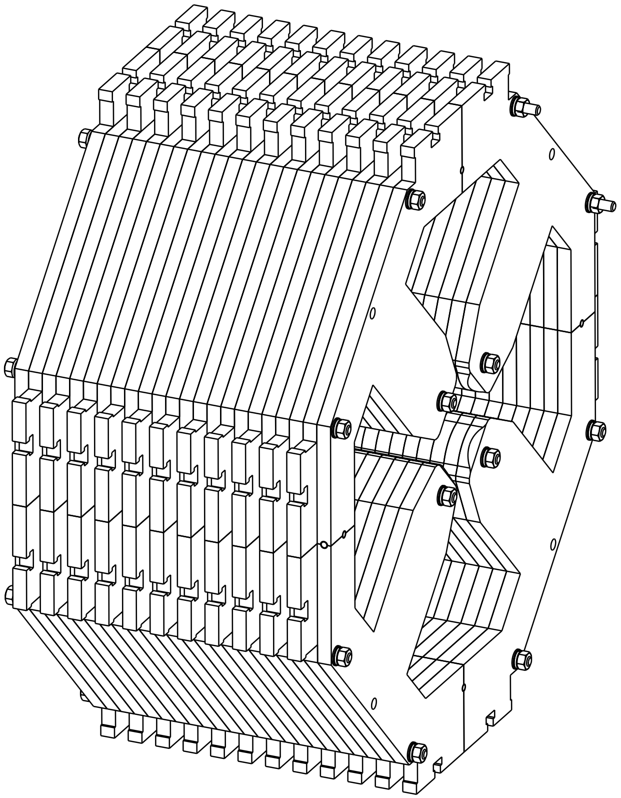
<u>fully symmetric sextupoles</u>: B_3, b_9, b_{15}, b_{21}, etc.

We can change R_{ref} and scale up (or down) the harmonics



$$B_{n,2} = B_{n,1}\left(\frac{R_{ref,2}}{R_{ref,1}}\right)^{n-1} \qquad b_{n,2} = b_{n,1}\left(\frac{R_{ref,2}}{R_{ref,1}}\right)^{n-N}$$

Let’s have a look at a real case: the measurements of 33 quadrupoles built for SESAME

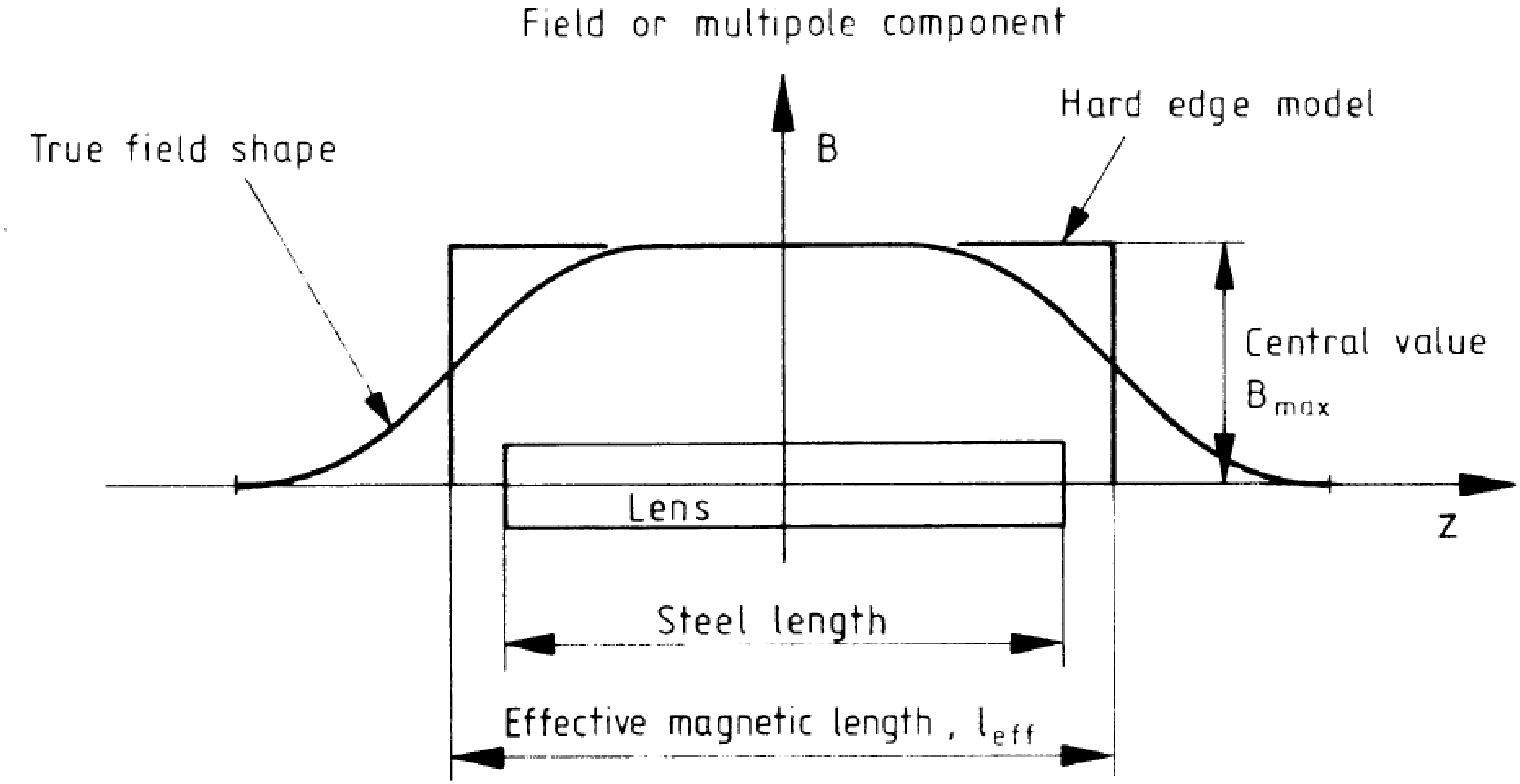


SESAME QF

mean ± rms	QF @ 250 A
b_3	-0.2 ± 0.8
a_3	-0.1 ± 0.9
b_4	0.3 ± 0.4
a_4	-0.3 ± 0.1
b_5	0.0 ± 0.1
a_5	0.0 ± 0.1
b_6	-0.1 ± 0.1
b_{10}	-0.3 ± 0.0
b_{14}	0.3 ± 0.0

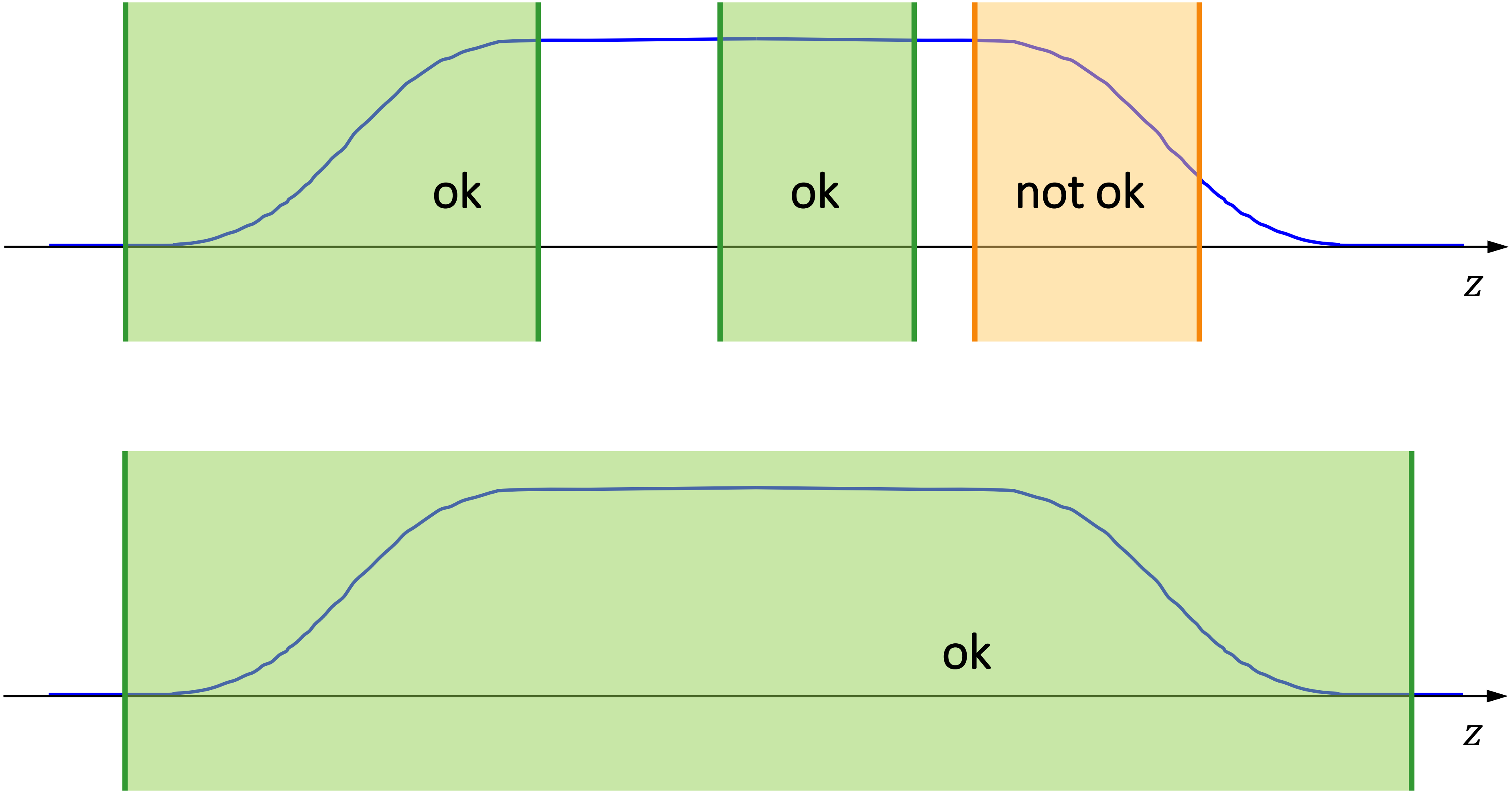
harmonics in 10^{-4} at 24 mm radius

Now, are our magnets 2D or 3D? In most cases what matters is the integrated strength = central strength × magnetic length



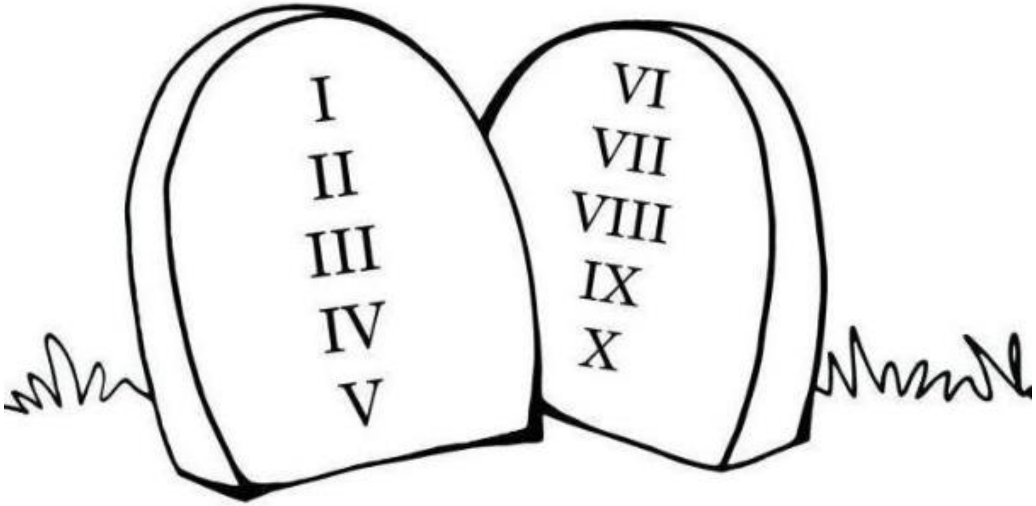
$$l_m B_0 = \int_{-\infty}^{\infty} B(z)dz$$

This 2D decomposition holds also for the integrated 3D field, as long as at the start / end B is constant along z



Thank you

If you want to know more…



1. Lectures about magnets in CERN Accelerator Schools
2. Special CAS edition on magnets, Bruges, Jun. 2009
3. N. Marks, Magnets for Accelerators, J.A.I., Jan. 2015
4. D. Tommasini, Practical Definitions & Formulae for Normal Conducting Magnets
5. Superconducting magnets for particle accelerators in USPAS
6. J. Tanabe, Iron Dominated Electromagnets
7. P. Campbell, Permanent Magnet Materials and their Application
8. K.-H. Mess, P. Schmüser, S. Wolff, Superconducting Accelerator Magnets
9. M. N. Wilson, Superconducting Magnets
10. A. Devred, Practical Low-Temperature Superconductors for Electromagnets