Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

Total "raw" number of points: 138 (92 for exercise 1, 46 for exercise 2)

The marks will be normalized out of 20

Exercise 1: longitudinal evolution in phase space [92 pts]

Several initial states in phase space are given below assuming the sign and phase conventions of the course (cases A to K, with the separatrix in purple and sample of bunch particles in blue).

As defined in the course:

Phase corresponds to the phase difference with respect to the phase of the synchronous particle. *Delta E* corresponds to the energy variation compared to the energy of the synchronous particle. *Delta p/p* corresponds to the normalized momentum variation with respect to the momentum of the synchronous particle.

For each case, address the following questions:

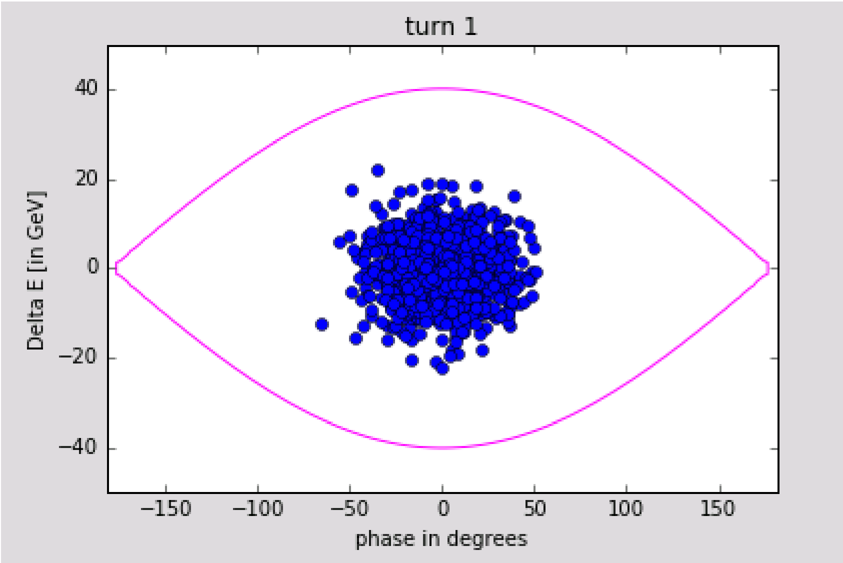
1) Describe the situation **qualitatively** in terms of :
   - beam energy (is energy changing? Is the beam energy below or above transition energy?), [2 pts each]
   - direction of rotation of the particles in phase space [1 pt each]
   - matching of the bunch to the bucket (assuming the beam is well matched to the bucket in the reference case A) [1 pt each]

2) Explain qualitatively what is likely to happen and draw a qualitative phase space sketch of the final state of the bunch if we keep these conditions for some time. [2 pts each]

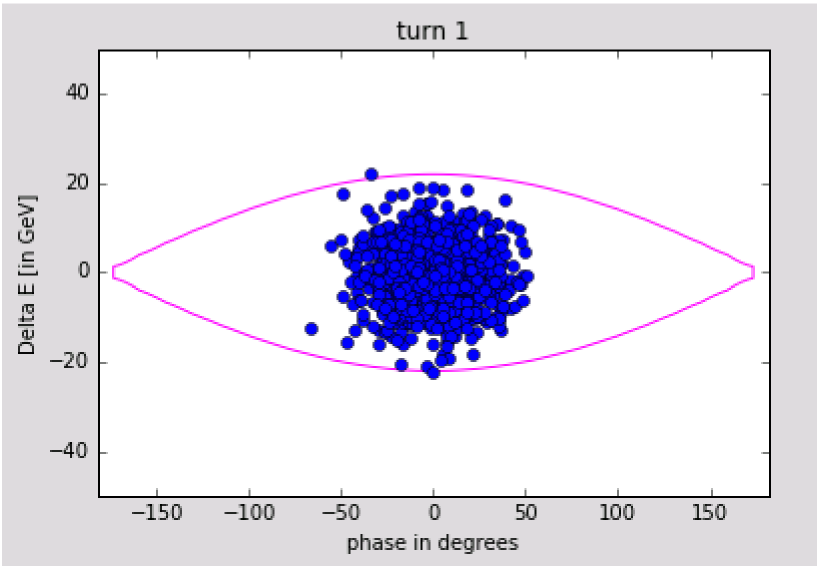
3) In each of the cases above, give scenario(s) that could have led to such initial states, and say if this situation should be desirable for performance or not. In particular, are losses or longitudinal emittance blow up expected? [2 pts each]

Note: it is not expected that variations in energy acceptance of longitudinal position [in m] are discussed in this exercise.

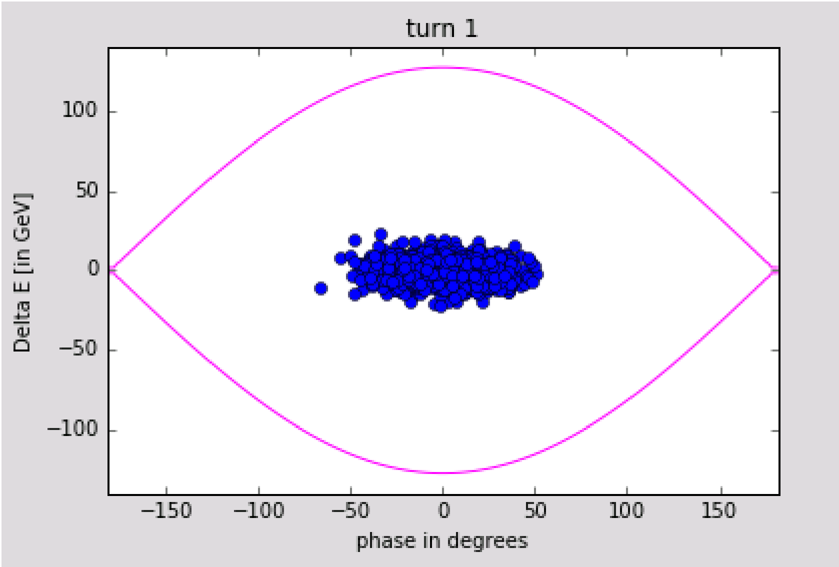
a. Case A



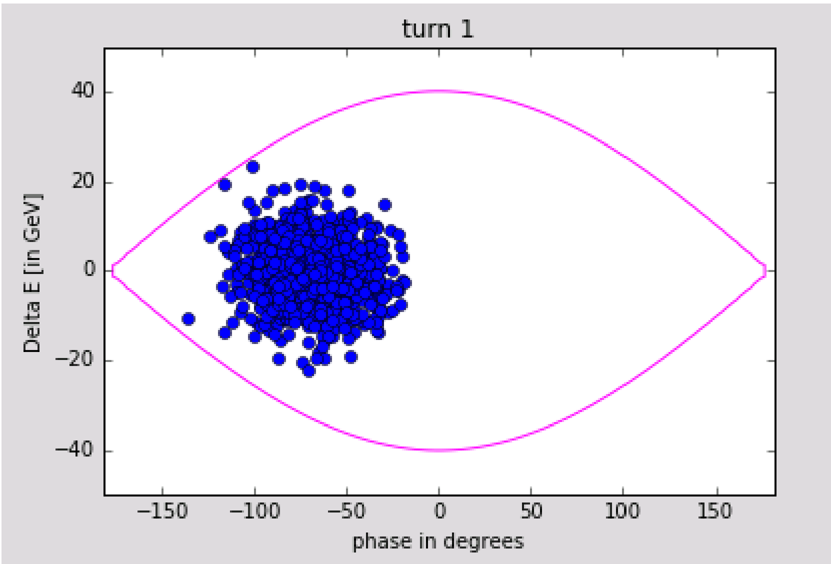
b. Case B



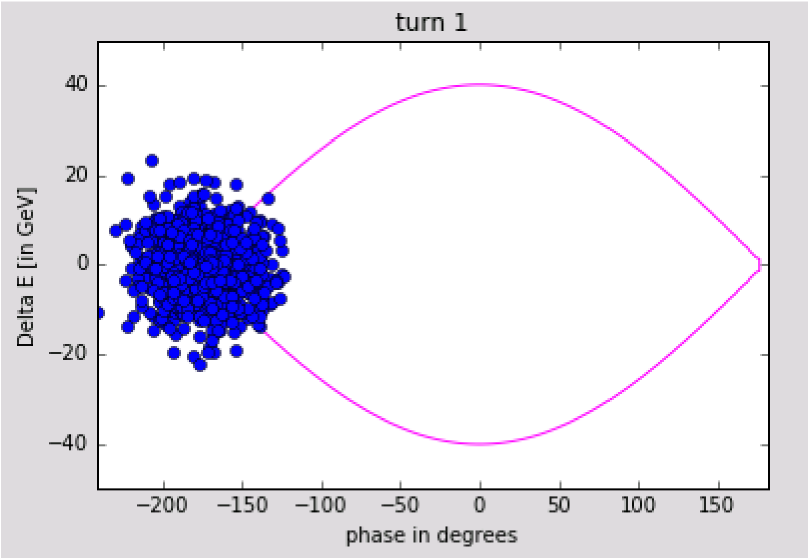
c. Case C



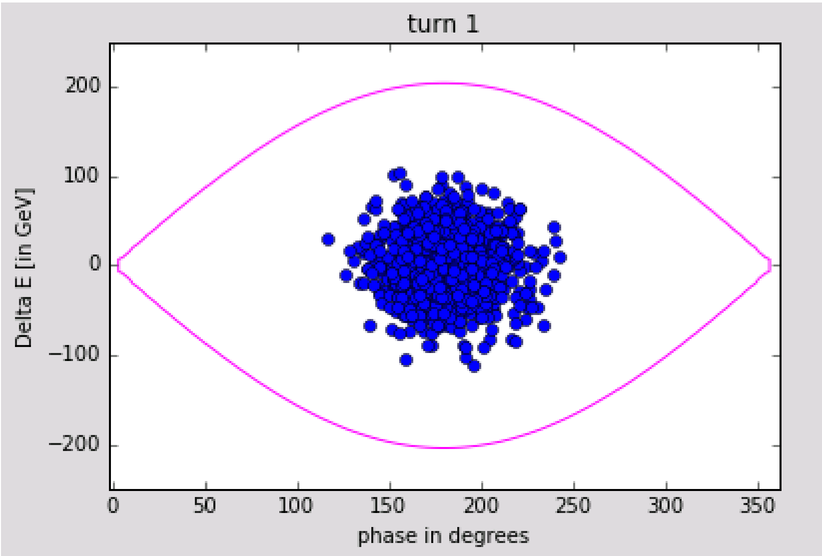
d. Case D



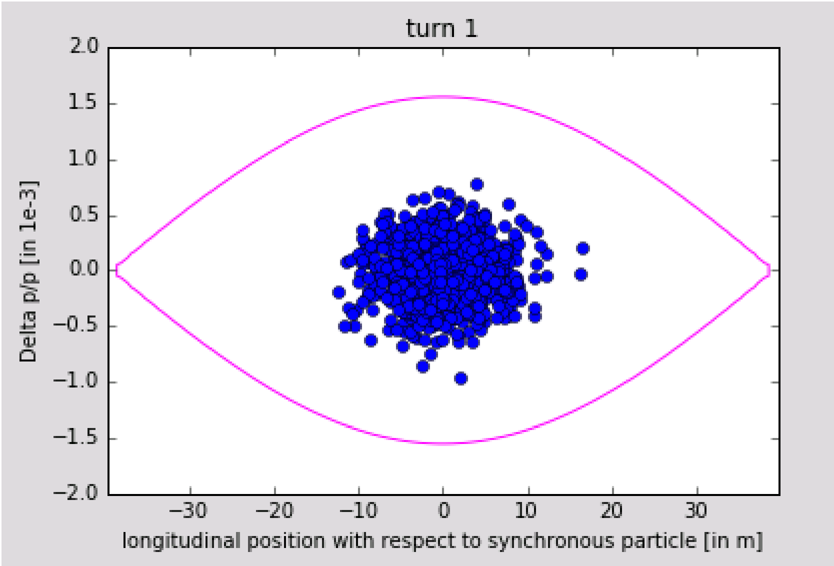
e. Case E



f. Case F



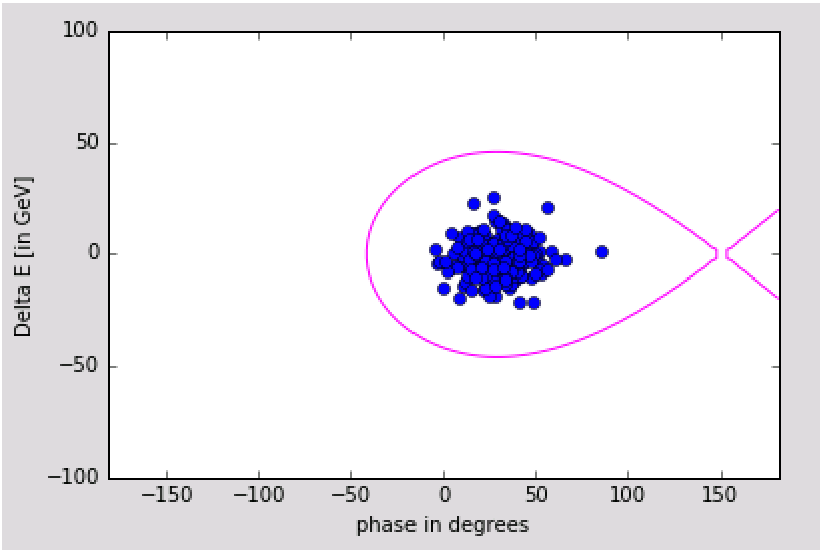
g. Case G



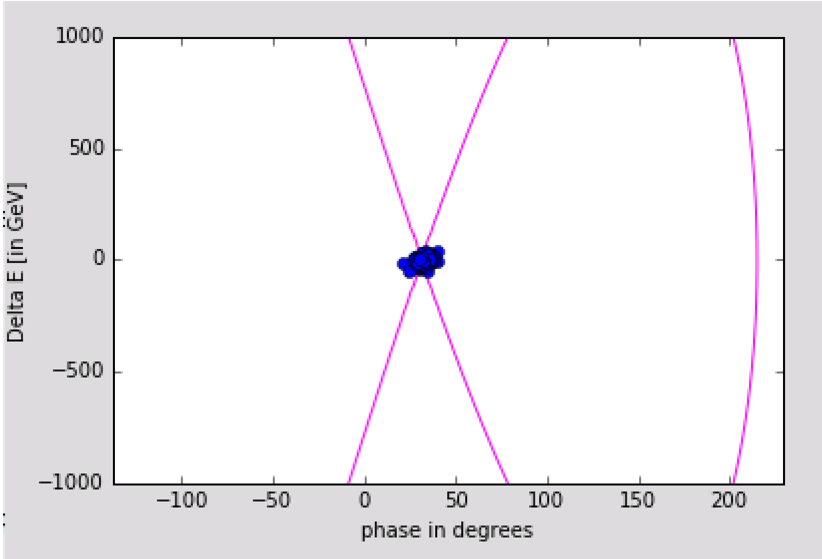
For this case, we assume that the longitudinal position with respect to the synchronous particle is linked to the phase Phi by

$$\Delta\phi[rad] = -\frac{h}{R}\, z[m]$$

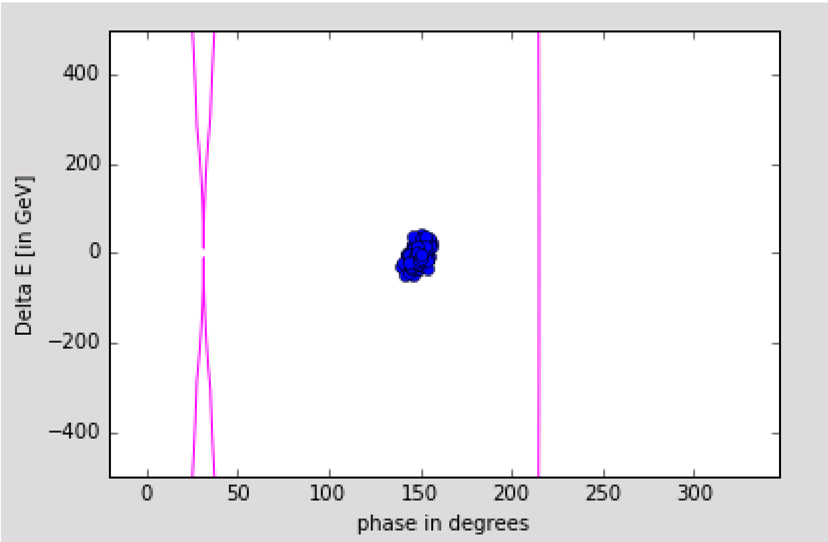
h. Case H



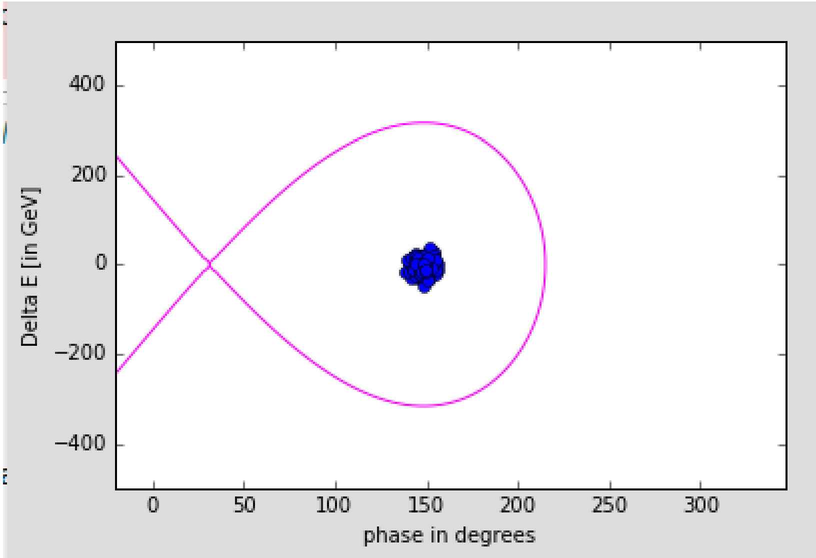
i. Case I



j. Case J



k. Case K (in this case you may compare the Delta E between case J and K)



4) Describe the actions that need to be taken when the beam is about to cross transition energy (from the point of view of single particle longitudinal beam dynamics) and why we need to take these actions [4 pts]

Exercise 2: the Chinese hadron collider project [46 pts]

Here is a table available in the literature of machine parameters of the Super Proton-Proton Collider (SPPC), the project for a hadron collider in China.

Table 1: Key SPPC parameters

Parameter	Value	Unit
Collision energy (C. of M.)*	70.6	TeV
Peak luminosity	1.2×10^{35}	$cm^{-2}s^{-1}$
Number of IPs**	2	
Circumference	54.4	km
Injection energy	2.1	TeV
Overall cycle time	~15	hours
Dipole field	20	T

* *C. of M.* : center of mass

** *IPs*: Interaction points

In this exercise, we consider the acceleration of protons in the SPPC ring.

Show that the beam energy in collision is 35.3 TeV and compute the relativistic factor gamma at collision energy. [6 pts]

1) Compute the bending radius in collision and the percentage of circumference that should be covered by dipoles. [6 pts]
2) What is the magnetic field at injection? What is the magnetic field swing (the ratio between the maximum and minimum field required from the magnets)? Compare to the magnetic swing required from CERN LHC magnets (8.36 T at top energy and 0.54 T at injection energy), and for the same magnets in the HL-LHC era (9 T at top energy and 0.54 T at injection energy). [6 pts]
3) The RF voltage of the main RF system is 16 MV and the harmonic number is h=73079.
   a. Compute the revolution frequency at injection and collision energy. Discuss the impact of this on the requirements on the frequency of the main RF system. [5 pts]
   b. Explain in a few words the physical meaning of the synchrotron tune. [2 pt]
   c. The synchrotron tune at top energy is Qs = $0.088\ 10^{-3}$. Compute the slippage factor, the momentum compaction factor and the transition energy [6 pts]. Explain in a few words the physical meaning of these three parameters [6 pts]. Does the SPPC need to cross transition during its magnetic cycle? [3 pts]
4) Still assuming an RF voltage of 16 MV,
   a. Compute the maximum magnetic field ramping rate $\dot{B}$ that is reachable during acceleration with an RF voltage of 16 MV. [3 pts]
   b. The magnetic field in the CERN LHC is ramped from 0.54 T to 8.36 T in 1210 s. Assuming the magnetic field ramping rate in SPPC is the same as in LHC and maintained constant for both machines during the whole ramp from injection to collision energy, compute how long the ramp from injection to collision energy would be in SPPC and compute the synchronous phase in SPPC during acceleration. [3 pts]

Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: $c = 3.00\ 10^{8}$ m/s