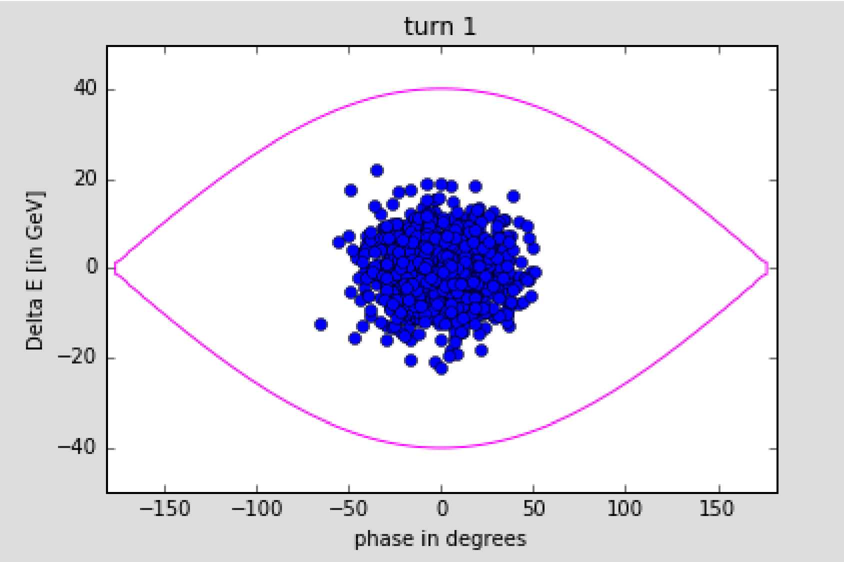
Exercise 1: longitudinal evolution in phase space [90 pts]

a. Case A



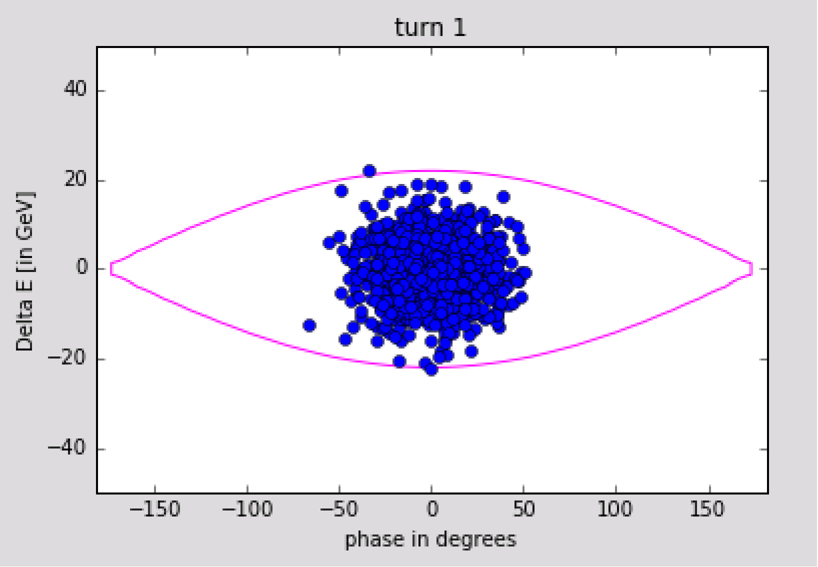
Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

Synchronous phase =0 → we are below transition

Below transition, a particle with energy higher than the synchronous particle means that its revolution frequency increases and it will arrive earlier at the cavity, i.e. with a smaller phase. The particle therefore moves counterclockwise in the [phase, Delta E] phase space.

As said in the text, the bunch is matched, and the bunch will rotate inside the bucket without losses or emittance growth.

b. Case B



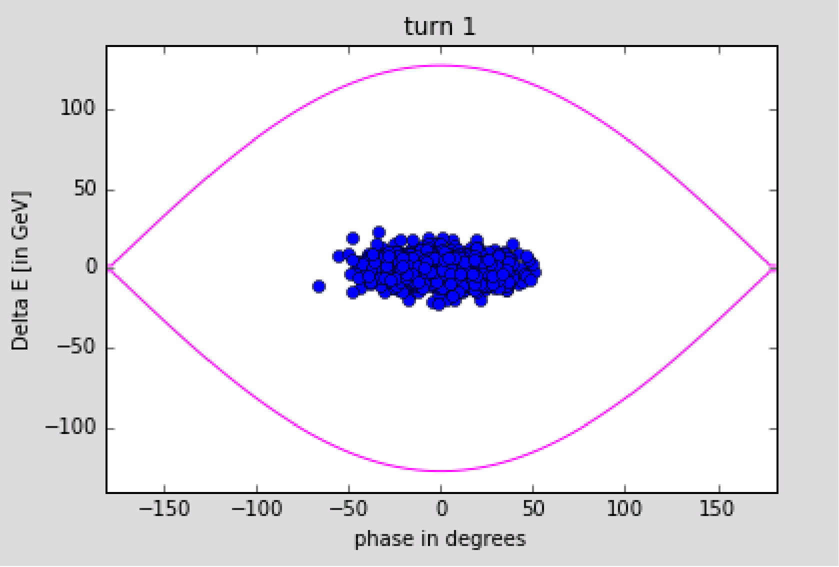
Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

Synchronous phase =0 → we are below transition. The particle therefore moves counterclockwise in the [phase, Delta E] phase space.

Compared to case A, the bunch is not matched: some particles are very close to the separatrix and some particles are even beyond the separatrix. The former particles will fill the bucket, leading to emittance blow up and the latter particles will not be trapped in the bucket and will likely be lost in case of acceleration.

Possible reasons for such a situation is that the RF voltage was abruptly reduced, or the bunch was injected into a machine with lower energy acceptance compared to the upstream machine. This is not a wanted situation.

c. Case C



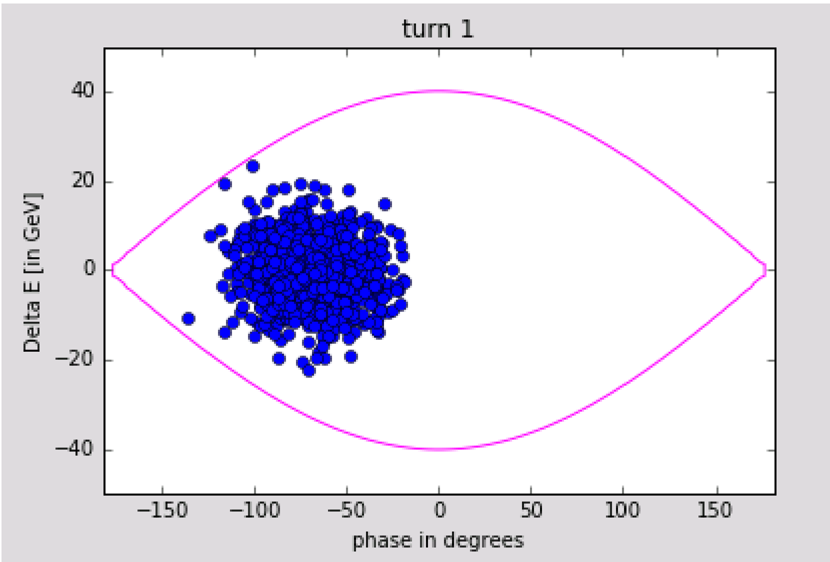
Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

Synchronous phase =0 → we are below transition. The particle therefore moves counterclockwise in the [phase, Delta E] phase space.

Compared to case A, the RF bucket seems to have too high acceptance compared to the bunch distribution. The bunch will rotate counterclockwise as a whole inside the bucket, and will eventually filament.

Possible reasons for such a situation is that the RF voltage was abruptly increased, or the bunch was injected into a machine with higher energy acceptance compared to the upstream machine. This could be a desired effect to reduce bunch length and extract the bunch to a machine with smaller acceptance after a rotation of pi/2. If this is not done, this will lead to emittance blow up but no losses, which is generally not wanted unless there is a good reason.

d. Case D



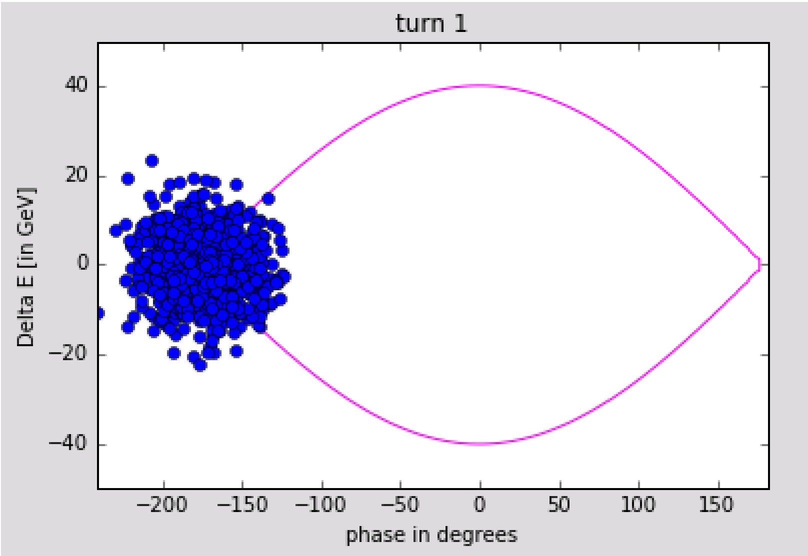
Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

Synchronous phase =0 → we are below transition. The particle therefore moves counterclockwise in the [phase, Delta E] phase space.

Compared to case A, the average phase of the bunch is not at the synchronous phase. The bunch will rotate counterclockwise as a whole inside the bucket, and will eventually filament and fill a much larger proportion of the bucket.

Possible reasons for such a situation is that the RF phase was abruptly changed by ~60 degrees, or the bunch was injected into a machine with a phase error compared to the RF phase. This will lead to emittance blow up but no losses in this case (as no particle is outside of the bucket). This is generally not wanted unless there is a good reason.

e. Case E



Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

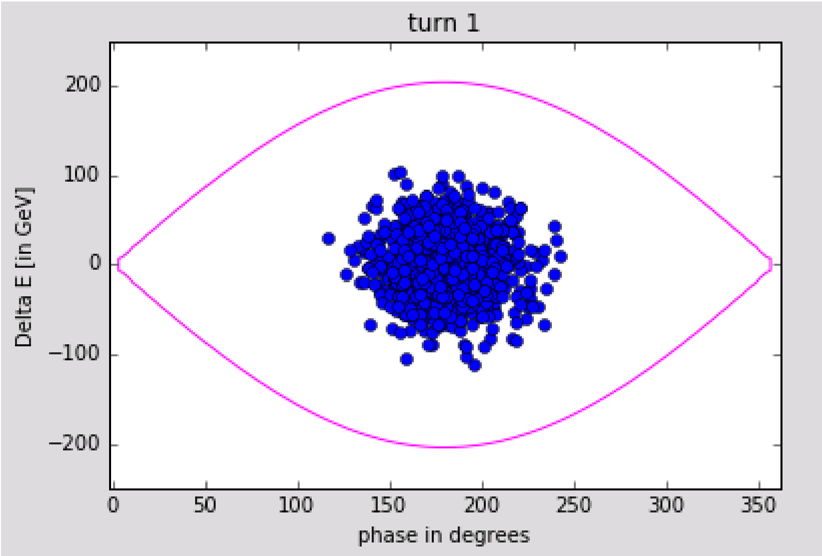
Synchronous phase =0 → we are below transition. The particle therefore moves counterclockwise in the [phase, Delta E] phase space.

This is the same situation as the previous case, with a phase error of 180 degrees.

Particles outside of the bucket will not be trapped in the bucket and will likely be lost in case of acceleration. Particles inside the two neighbouring buckets will be trapped and filament separately inside each bucket.

This will lead to emittance blow up and large losses. This is generally not wanted unless one wants to split two bunches from one machine to another that has a higher RF frequency. To do that in the same machine would be better done adiabatically.

f. Case F

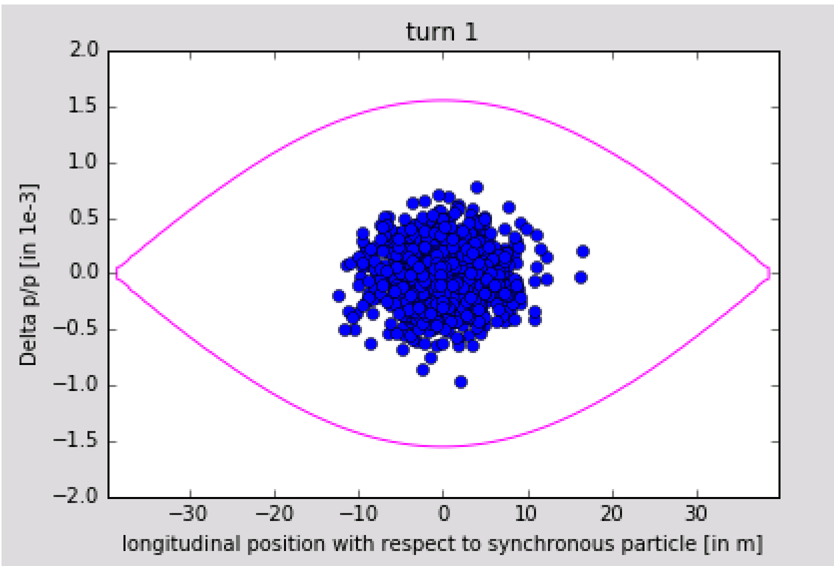


Phase acceptance =2 pi → we are in a stationary bucket (i.e. no acceleration)

Synchronous phase = 180 degrees → we are above transition. The particle therefore moves clockwise in the [phase, Delta E] phase space.

As in case A, the bunch is matched, and the bunch will rotate inside the bucket without losses or emittance growth.

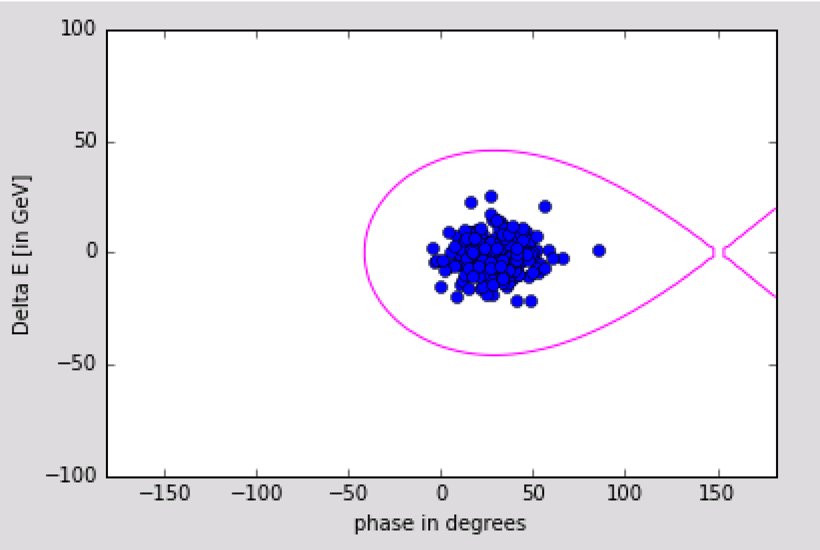
g. Case G



It is the same case as case A with a change of variables. the shape of the bucket is symmetric and therefore not accelerating. the synchronous phase is 0, therefore we are below transition.

The particles move in the clockwise direction due to the minus sign in the phase/position relationship.

h. Case H

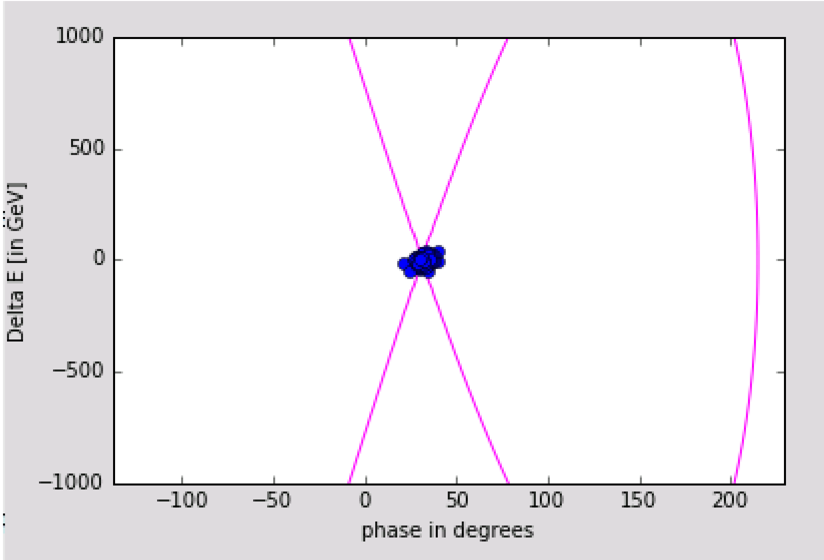


Phase acceptance < 2 pi → we are in non stationary bucket (i.e. acceleration or deceleration)

Synchronous phase is between 0 and 90 degrees → we are below transition and it is an accelerating bucket. The particle therefore moves anticlockwise in the [phase, Delta E] phase space.

It is not obvious whether the bunch is matched or not, but there should be no losses. It seems a normal situation for a bunch in an accelerating bucket

i. Case I



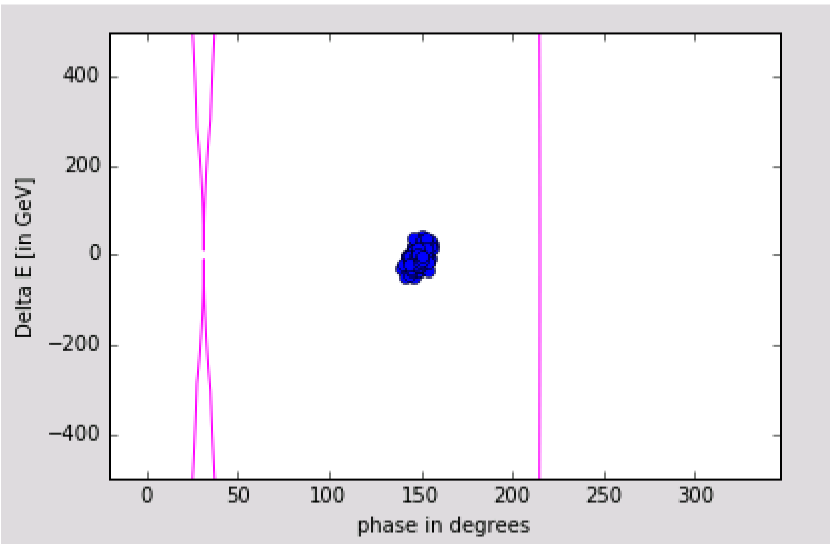
Phase acceptance < 2 pi → we are in non stationary bucket (i.e. acceleration or deceleration)

The unstable phase is between 0 and 90 degrees, which means that the synchronous phase is between 90 and 180 degrees → we are above transition and it is an accelerating bucket. The particle therefore moves clockwise in the [phase, Delta E] phase space.

Most of the particles are outside the bucket and will not be trapped and mostly lost.

The phase seems to be at exactly the unstable phase, as if the required phase change at transition energy was not performed.

j. Case J



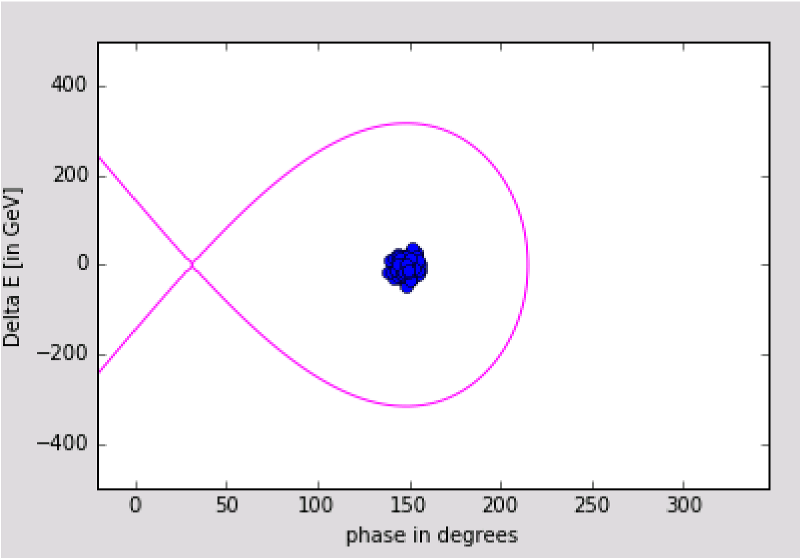
Phase acceptance < 2 pi → we are in non stationary bucket (i.e. acceleration or deceleration)

The unstable phase is between 0 and 90 degrees, which means that the synchronous phase is between 90 and 180 degrees → we are above transition and it is an accelerating bucket. The particle therefore moves clockwise in the [phase, Delta E] phase space.

All the particles are inside the bucket and will remain trapped.

Compared to the case before, the required phase change at transition energy was performed.

k. Case K



Same case as before, but at higher energy.

4) Describe the actions that need to be taken when the beam is about to cross transition energy (from the point of view of single particle longitudinal beam dynamics). [2 pts]

RF phase needs to be changed from phi_s to pi-phi_s to follow the abrupt change of stable bucket area with the change of sign of the slippage factor (since eta*cos(phi_s)>0 is required for stability).

Exercise 2:

Compute the beam energy in collision and the relativistic factor gamma. [6 pts]

```
# Since we are looking at collision of hadrons,
# the energy of the beam is half the collision energy
Ecol=70.6e12
Etot=Ecol/2*e
circumference=54.4e3
print 'Etot[TeV]=', Etot/e/1e12

gamma=Etot/(m_p*c**2)
print 'gamma=', gamma
```

```
Etot[TeV]= 35.3
gamma= 37578.1769794
```

1) Compute the bending radius in collision and the percentage of length that should be covered by dipoles. [6 pts]

```
B=20
beta=np.sqrt(1-1/gamma**2)
p_0=Etot/c*beta
bending_radius=p_0/(e*B)
print 'bending radius [m]=', bending_radius
propDipoles=2*np.pi*bending_radius/circumference
print 'proportion of dipoles=', propDipoles
```

```
bending radius [m]= 5883.33333125
proportion of dipoles= 0.679523410738
```

2) What is the magnetic field at injection? What is the magnetic field swing (the ratio between the maximum and minimum field required from the magnets)? Compare to the magnetic swing required from CERN LHC magnets (8.36 T at top energy and 0.54 T at injection energy), and for the same magnets in the HL-LHC era (9 T at top energy and 0.54 T at injection energy). [6 pts]

```
Etot_i=2.1e12*e
gamma_i=Etot_i/(m_p*c**2)
beta_i=np.sqrt(1-1/gamma_i**2)
p_0_i=Etot_i/c*beta_i
B_i=p_0_i/(e*bending_radius)
print 'B at injection [T]=', B_i
print 'magnetic swing=', B/B_i
print 'magnetic swing for LHC', 8.36/0.54
print 'magnetic swing for HL-LHC', 9/0.54
```

```
B at injection [T]= 1.1898015811
magnetic swing= 16.8095254853
magnetic swing for LHC 15.4814814815
magnetic swing for HL-LHC 16.6666666667
```

3) The RF voltage of the main RF system is 16 MV and the harmonic number is h=73079. The synchrotron tune at top energy is Qs = 0.088 10^{-3}.

a. Compute the revolution frequency at injection and collision energy. Discuss the impact of this on the requirements on the frequency of the main RF system. [5 pts]

```
# a)
f_inj=beta_i*c/circumference
f_top=beta*c/circumference
print 'frev at injection', f_inj
print 'frev at col', f_top
print 'Delta frev', f_top-f_inj
```

```
frev at injection 5514.70533062
frev at col 5514.7058804
Delta frev 0.000549783956558
```

b. Explain in a few words what the synchrotron tune represents. [2 pt]

See the course

c. At top energy, compute the slippage factor, the momentum compaction factor and the transition energy [6 pts]. Explain in a few words what these three parameters represent [6 pts]. Does the SPPC need to cross transition during its magnetic cycle? [3 pts]

```
# b)
Vrf=16e6
Qs=0.088e-3
harmonic=73079

# if we assume no negative momentum compaction factor
eta=-(2*np.pi*beta**2*Etot)*Qs**2/(e*Vrf*harmonic)
alpha_p=1/gamma**2-eta
gamma_tr=1/np.sqrt(alpha_p)
E_tr=gamma_tr*(m_p*c**2)
#eta=1/gamma**2-1/gamma_tr**2
print  'if we assume no negative momentum compaction factor'
print '   eta=', eta
print '   alpha_p=', alpha_p
print '   gamma_tr=', gamma_tr
print '   E_tr[GeV]=', E_tr/e/1e12

# if we assume negative momentum compaction factor
eta=(2*np.pi*beta**2*Etot)*Qs**2/(e*Vrf*harmonic)
alpha_p=1/gamma**2-eta
gamma_tr=1/np.sqrt(-alpha_p)
E_tr=gamma_tr*(m_p*c**2)
print  'if we assume negative momentum compaction factor'
print '   eta=', -eta
print '   alpha_p=', alpha_p
print '   gamma_tr=', gamma_tr, 'j'
print '   E_tr[GeV]=', E_tr/e/1e12 ,'j'
```

```
if we assume no negative momentum compaction factor
   eta= -1.46895110133e-06
   alpha_p= 1.46965925675e-06
   gamma_tr= 824.8817074
   E_tr[GeV]= 0.774873253889
if we assume negative momentum compaction factor
   eta= -1.46895110133e-06
   alpha_p= -1.46824294591e-06
   gamma_tr= 825.279464224 j
   E_tr[GeV]= 0.775246896706 j
```

Note: The comparison between 1/gamma^2 and abs(eta) allows defining the negative sign of eta, provided we assume that the momentum compaction factor remains positive. The SPPC does not need to cross transition since the whole energy range of SPPC is above transition energy.

d. Do a qualitative sketch of the longitudinal phase space, highlighting the separatrix and the synchronous phase at injection. [6 pts]

See plot in the course of separatrix above transition with a phase acceptance of 2 pi and a synchronous phase of pi.

e. How would the separatrix qualitatively change (1) during acceleration and (2) at flat-top? Do sketches of the longitudinal phase space to illustrate what happens in both cases. [8 pts]

See plot in the course of separatrix during acceleration with a reduced phase acceptance and come back to a similar plot at top energy than in question 4 d.

4) Still assuming an RF voltage of 16 MV,
   a. Compute the maximum magnetic field rate $\dot{B}$ that is reachable during acceleration with an RF voltage of 16 MV. [3 pts]

```
# a)
Bdot=Vrf/(2*np.pi*bending_radius*circumference/(2*np.pi))
print 'max Bdot[T/s]=', Bdot
```

```
max Bdot[T/s]= 0.049991668073
```

   b. The magnetic field in the CERN LHC is ramped from 0.54 T to 8.36 T in 1210 s. Assuming the magnetic field rate in SPPC is the same as in LHC and maintained constant for both machines during the whole ramp from injection to collision energy, compute how long the ramp from injection to collision energy would be in SPPC and compute the synchronous phase in SPPC during acceleration. [3 pts]

```
# b)
BdotLHC=(8.36-0.54)/1210
print 'Bdot LHC [T/s]', BdotLHC
timeRamp=(20-B_i)/BdotLHC
print 'time of ramp [s]', timeRamp
phi_s=np.arcsin(2*np.pi*bending_radius*circumference/(2*np.pi)*BdotLHC/Vrf)
print 'phi_s [deg]', (np.pi-phi_s)*180/np.pi
```

```
Bdot LHC [T/s] 0.00646280991736
time of ramp [s] 2910.52942287
phi_s [deg] 172.572142264
```

Note: Since the bucket is accelerating above transition, the synchronous phase is between pi/2 and pi.