JUAS 2018 exam on synchrotron radiation (90 minutes)

Total number of points: 25 – Marks will not be renormalized to 20

The following constant will be used in the exercises

velocity of light	$c = 2.998\cdot10^{8}$ m/s
reduced Planck constant	$\eta = h/2\pi = 1.05\cdot10^{-34}$ Kg m^2/ s
vacuum dielectric constant	$\varepsilon_0 = 8.854\cdot10^{-12}$ F/m
electron rest energy	0.511 MeV
electron rest mass	$9.1\cdot10^{-31}$ Kg
classical electron radius	$r_e = 1/(4\pi\varepsilon_0)\, e^2/(mc^2) = 2.81\cdot10^{-15}$ m

$$C_q = \frac{55}{32\sqrt{3}}\frac{\eta}{mc} = 3.84\cdot10^{-13}\text{m}$$

Ex. 1:

A synchrotron light source with an electron beam of 2.5 GeV operates in the conditions of minimum emittance for a 10 cells DBA lattice (i.e. 20 equal bendings). The vertical (Y) emittance is defined by the coupling ratio $k = \varepsilon_y/\varepsilon_x$ of 10% (i.e. k = 0.1).

a) Compute the emittances of the ring (assuming $J_x \sim 1$) in both planes.
b) Assuming that the brightness is exclusively dictated by the electron beams and the dispersion is zero at the undulator, compute the increase in brightness if the coupling is reduced to k = 1%
c) At what wavelengths the finite beam size and divergence of the photons will change the results in b)?

Ex. 2:

a) An electron flying through the space with the energy E passes the earth. It is bent by the magnetic field of the earth B_{earth} = 3e-5 T and emits at this time a photon spectrum with a critical energy of λ_c = 600 nm (i.e. red light). What is the energy E of the electron.

b) The electron travels, in vacuum, along a trajectory of total length L = 1000 km. How much energy has the electron lost along this passage due to synchrotron radiation?

Ex. 3:

A linear accelerator provides an electron beam of 200 MeV with very small transverse dimensions. At the end of the LINAC there is a permanent magnet undulator with period λ_u = 30 mm. The pole tip field B_r is 1T. This undulator produces coherent radiation at λ = 300 nm at the fundamental harmonic (n=1). Using the relation

$$B = 2B_r \exp(-\pi\frac{g}{\lambda_u}) \text{ in Tesla}$$

a) Calculate the required gap of the undulator
b) How long is the undulator to obtain a bandwidth of $\Delta\lambda/\lambda$ = 5e-3?

c) Under what conditions the emission spectrum becomes a broadband (dipole-like)?

Ex. 4:

In order to design a modern storage ring for synchrotron radiation a very small beam emittance is required. Describe shortly the most important design criteria to fulfil these requirements. At least 3 criteria should be specified.

Exercise 1 solution:

The DBA-ME (Chasman-Green) condition states that the minimum emittance is

$$\varepsilon = \frac{1}{4\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x}$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\eta}{mc} = 3.84 \cdot 10^{-13} m$$

The text states there are 10 DBA cells i.e. 20 dipoles i.e. the bending angle is

$$\theta = \frac{2\pi}{20} = 314 \text{ mrad}$$

For a 2.5 GeV beam and $J_x = 1$, $\gamma = 2500/0.511 = 4892$, the DBA-ME is

$$\varepsilon_x = \frac{1}{4\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} = \frac{1}{4} \frac{1}{3.87} 3.84 \cdot 10^{-13} (4892)^2 (0.314)^3 = 18 \text{ nm}$$

In the vertical plane with k = 0.1 the emittance is 1.8 nm.

If the brightness is dominated by the electrons, and coupling is reduced to k=0.01 (i.e. vertical emittance 0.18 nm), the increase in brightness is

$$R = \frac{\Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'} (k = 0.1)}{\Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'} (k = 0.01)} = \frac{k \varepsilon_x^2 (k = 0.1)}{k \varepsilon_x^2 (k = 0.01)} = 10$$

The photon beam size and divergence is comparable to the beam size and divergence, for wavelength such that

$\lambda/2\pi > 18$ nm in H and 1.8 nm in V
$\lambda/2\pi > 18$ nm in H and 0.18 nm in V

Therefore for

$\lambda/2\pi > 11$ nm in V for k = 0.1
$\lambda/2\pi > 1.1$ nm in V for k = 0.01

Exercise 2 solution:

a)
We know the energy E, we know the field B and the critical energy

Form the field we compute the radius of curvature.
From the critical frequency we compute the energy

$$\omega_c = \frac{3}{2} \frac{c}{\rho} \gamma^3 = \frac{2\pi c}{\lambda_c} \qquad \rightarrow \qquad \gamma^3 = \frac{4\pi\rho}{3\lambda_c}$$

To work out γ we need ρ. Using

$$\frac{e}{p} = \frac{1}{B\rho} = \frac{ec}{pc} = \frac{ec}{\beta E} \qquad \rightarrow \qquad \frac{1}{\rho} = \frac{ecB}{\beta E}$$

hence

$$\gamma^3 = \frac{4\pi\rho}{3\lambda_c} = \frac{4\pi\beta E}{3\lambda_c ecB} = \frac{4\pi\beta mc\gamma}{3\lambda_c eB} \qquad \rightarrow \qquad \gamma^2 = \frac{4\pi\beta mc}{3\lambda_c eB}$$

putting the numbers

$$\gamma = \sqrt{\frac{4\pi\cdot 9.1\cdot 10^{-31} 3\cdot 10^{8}}{3\cdot 600\cdot 10^{-9} 1.6\cdot 10^{-19} 3\cdot 10^{-5}}} \sim 19800$$

E ~ 10.1 GeV

b)
To compute the energy lost we need to compute the total instantaneous power radiated and the time it takes to travel 1000 km. We know the energy from a) and v ~ c.

The instantaneous power lost is (see slides)

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\vec{\beta}}\right|^2 \gamma^4 = \frac{e^4}{6\pi\varepsilon_0 m^4 c^5} B^2 E^2 = \frac{e^4}{6\pi\varepsilon_0 m^2 c} B^2 \gamma^2$$

The time it takes to travel 1 km is

$$T = \frac{L}{\beta c} \sim 3.3\mu s$$

The energy lost is

$$E = PT = \frac{e^4}{6\pi\varepsilon_0 m^2 c} B^2 \gamma^2 \frac{L}{\beta c}$$

Putting the numbers

$$E = PT = \frac{e^4 \beta^2}{6\pi\varepsilon_0 m^2 c} B^2 \gamma^2 \frac{L}{\beta c} \sim \frac{e^4}{6\pi\varepsilon_0 m^2 c^2} B^2 \gamma^2 L \sim 0.1 meV$$

Exercise 3 solution:

a)
We know the energy of the beam, the period, the field of the undulator and the wavelength the energy of the beam. We can compute K and from the relation K vs gap we compute the gap

Using

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

and

$$K = \frac{e\lambda_u B}{2\pi mc}$$

and

$$B(T) = 2B_r \exp(-\pi \frac{g}{\lambda_u})$$

In order to have 300 nm at the fundamental harmonic, emitted by 200 MeV electrons (γ = 391) in λ_u = 30 mm period undulator

$$K = \sqrt{2\left(\frac{2\gamma^2\lambda}{\lambda_u} - 1\right)} = 2.25$$

Since K ~ 93.3 $B_0\lambda_u$, which means

B = 0.80 T

Can work out the gap from the exponential relation

$$g = -\frac{\lambda_u}{\pi}\ln\left(\frac{B_0}{2B_r}\right) = 8.7 \text{ mm}$$

b)
The linewidth for N period is

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{N}$$

If the linewidth is $5\cdot10^{-3}$ then N = 200 and the undulator is long 6 m.

Exercise 4 solution:

The main criteria for low emittance rings are dictated by the formula

low <H> at dipoles
large J_x
small gamma

and using the limit for small bend angles

reduce the bend angle
use many bending magnets (many cells and many bends per cell: MBA)
large circumference

can use damping wigglers to further decrease the emittance