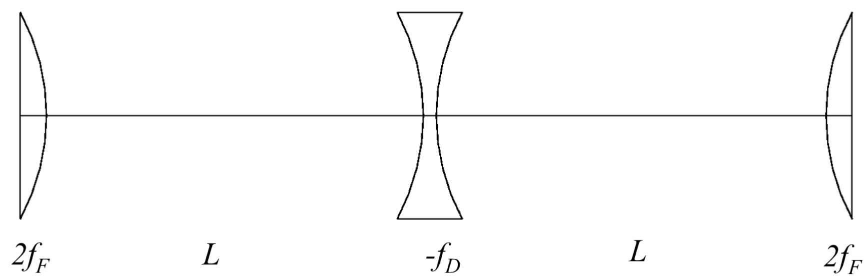
Transverse Beam Dynamics

JUAS 2018 - tutorial 3

1 Exercise: Chromaticity in a FODO cell

Consider a ring made of N_{cell} identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ.

1. Calculate the maximum and the minimum betatron function in the FODO cell. (*Use the thin-lens approximations*)



2. Calculate the natural chromaticities for this ring.

3. Show that for short quadrupoles, if $f_F \simeq f_D$,

$$\xi_N \simeq -\frac{N_{cell}}{\pi} \tan\frac{\mu}{2}.$$

4. Design the FODO cell such that it has: phase advance $\mu = 90$ degrees, a total length of 10 m, and a total bending angle of 5 degrees. What are β_{max}, β_{min}, D_{max}, D_{min}?

5. Add two sextupoles at appropriate locations to correct horizontal and vertical chromaticities. (hints: use 1 sextupole for the horizontal plane and 1 for the vertical plane; do not consider geometric aberrations).

6. If the gradient of all focusing quadrupoles in the ring is wrong by $+10\%$, how much is the tune-shift with and without sextupoles?

2 Exercise: Measurement of Twiss parameters

One of the possible ways to determine experimentally the Twiss parameters at a given point makes use of a so-called quadrupole scan. One can measure the transverse size of the beam in a profile monitor, called Wire Beam Scanner (WBS), located at a distance L downstream a focusing quadrupole, as a function of the normalised gradient in this quadrupole. This allows to compute the emittance of the beam, as well as the β and the α functions at the entrance of the quadrupole.

Let's consider a quadrupole Q with a length of $l = 20$ cm. This quadrupole is installed in an electron transport line where the particle momentum is 300 MeV$/c$. At a distance $L = 10$ m from the quadrupole the transverse beam size is measured with a WBS, for various values of the current I_Q. The maximum value of the quadrupole gradient G is obtained for a current of 100 A, and is $G = 1$ T/m.

Hint: G is proportional to the current. **Advice:** use thin-lens approximation.

1. How does the normalised focusing strength K vary with I_Q?
2. Give the expression Σ_2 as function of α_1, β_1, and γ_1
3. Show that β_2 can be written in the form: $\beta_2 = A_2 (Kl)^2 + A_1 (Kl) + A_0$, and express A_0, A_1, and A_2 as a function of L, α_1, β_1, and γ_1.
4. Express the final beam size, σ_2, as a function of Kl, and find its minimum, which will correspond to $(Kl)_{\text{min}}$.
5. How does σ_2 vary with Kl when $|Kl - (Kl)_{\text{min}}| \gg 1/\beta_1$?
6. Deduce the values of α_1, β_1, and γ_1 from the measurement σ_2, as a function of the quadrupole current I_Q.