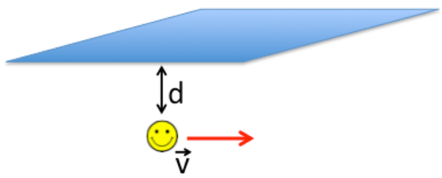
Joint Universities Accelerator School February 7th, 2018

SPACE CHARGE EXAMINATION
(M. Migliorati)

1) Consider two separate charges q which are moving in parallel along the same direction with the same velocity v. Determine the ratio between the force due to the magnetic field with respect to the one due to the electric field. **(value = 3 points)**

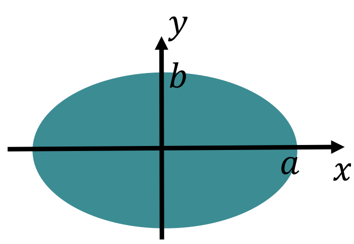
2) Obtain the coherent and incoherent betatron tune shift of a uniform proton beam of radius $a = 5$ mm, length $l_0 = 1$ m, inside a circular perfectly conducting pipe of radius $b = 35$ mm, with constant kinetic energy $E_0 = 2$ GeV (other parameters: total number of protons $N = 1\times10^{11}$, machine bending radius $\rho_x = 100$ m, betatron tune $Q_0 = 4.15$, classical radius of proton $r_p = 1.53\times10^{-18}$ m, proton rest mass = 0.938 GeV). **(value = 4 points)**

3) Evaluate the electromagnetic force acting on a charge q at a distance d from an infinite, perfectly conducting plane, moving with a relativistic velocity v parallel to the plane (see figure).
(value = 3 points)



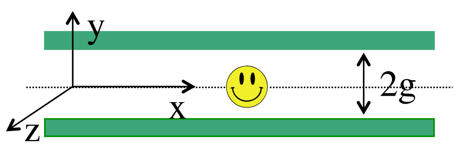
4) If we have a uniform coasting beam with elliptic cross section, as in the figure, the direct space charge force can be written as

$$\vec{F}^{self} = \frac{eI}{\pi\varepsilon_0\beta c\gamma^2(a+b)}\left(\frac{x}{a}\hat{\imath} + \frac{y}{b}\hat{\jmath}\right)$$



with a and b the horizontal and vertical semi axes, which vary along the machine due to the betatron function. Determine the horizontal and vertical incoherent tune shifts inside a perfectly conducting circular pipe if the emittances of the beam ε_x and ε_y are known. **(value = 5 points)**

5) Obtain the vertical indirect space charge force of a uniform d.c. current (uniform beam distribution in longitudinal and transverse plane) inside a dipole magnet, which can be considered as two parallel plates of ferromagnetic material knowing that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$ under the approximation that $g \gg a$ with $2g$ the dipole gap and a the beam radius. **(value = 5 points)**



Solutions

1)

$$\frac{F_B}{F_E} = \frac{qvB}{qE} = \frac{v\beta/cE}{E} = \beta^2$$

2)

$$\Delta Q_c = -\frac{\rho_x^2 N r_p}{b^2\beta^2\gamma Q_0 l_0} = -0.107, \qquad \Delta Q_{inc} = -\frac{\rho_x^2 N r_p}{a^2\beta^2\gamma^3 Q_0 l_0} = -0.536$$

3) It is the same as that of two charges at distance $2d$ moving with the same velocity on parallel trajectories:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\gamma 4d^2}$$

4)

$$\frac{\partial F_x}{\partial x} = \frac{eI}{\pi\varepsilon_0\beta c\gamma^2 a(a+b)} \qquad \frac{\partial F_y}{\partial y} = \frac{eI}{\pi\varepsilon_0\beta c\gamma^2 b(a+b)}$$

$$\Delta Q_x = -\frac{1}{4\pi\beta^2\varepsilon_x E_0}\oint a^2\frac{\partial F_x}{\partial x}ds = -\frac{\rho_x eI}{2\pi\varepsilon_0\beta^3 c\gamma^2\varepsilon_x E_0}\langle\frac{a}{a+b}\rangle$$

$$\Delta Q_x = -\frac{2\rho_x r_0 I}{ec\beta^3\gamma^3\varepsilon_x}\langle\frac{a}{a+b}\rangle \qquad \Delta Q_y = -\frac{2\rho_x r_0 I}{ec\beta^3\gamma^3\varepsilon_y}\langle\frac{b}{a+b}\rangle$$

5) The magnetic field can be obtained by removing the screens and considering image currents flowing in the same direction. If the current is along 'z', then the horizontal magnetic field in a position 'y' inside the beam is

$$B_x^{im}(z,y) = \frac{\mu_0\beta c\bar{\lambda}(z)}{2\pi}\sum_{n=1}^{\infty}\left[\frac{1}{2ng-y} - \frac{1}{2ng+y}\right]$$

By using the approximation *h>>a>y*, the magnetic field due to the image currents is

$$B_x^{im}(z,y) \cong \frac{\mu_0\beta c\bar{\lambda}(z)y}{4\pi g^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\mu_0\beta c\bar{\lambda}(z)\pi^2 y}{24\pi g^2}$$

and the corresponding force is $F_y^{im}(z,y) = \frac{e\beta^2\bar{\lambda}(z)\pi^2}{24\pi\varepsilon_0 g^2}y$