Joint Universities Accelerator School

February 7th, 2018

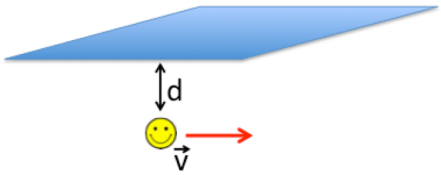
SPACE CHARGE EXAMINATION

(M. Migliorati)

1) Consider two separate charges q which are moving in parallel along the same direction with the same velocity v. Determine the ratio between the force due to the magnetic field with respect to the one due to the electric field. **(value = 3 points)**

2) Obtain the coherent and incoherent betatron tune shift of a uniform proton beam of radius $a = 5$ mm, length $l_0 = 1$ m, inside a circular perfectly conducting pipe of radius $b = 35$ mm, with constant kinetic energy $E_0 = 2$ GeV (other parameters: total number of protons $N = 1\times10^{11}$, machine bending radius $\rho_x = 100$ m, betatron tune $Q_0 = 4.15$, classical radius of proton $r_p = 1.53\times10^{-18}$ m, proton rest mass = 0.938 GeV). **(value = 4 points)**

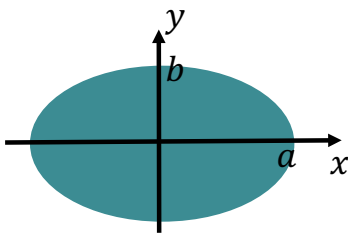
3) Evaluate the electromagnetic force acting on a charge q at a distance d from an infinite, perfectly conducting plane, moving with a relativistic velocity v parallel to the plane (see figure).
(value = 3 points)



4) If we have a uniform coasting beam with elliptic cross section, as in the figure, the direct space charge force can be written as

$$\vec{F}^{self} = \frac{eI}{\pi\varepsilon_0\beta c\gamma^2(a+b)}\left(\frac{x}{a}\hat{\imath}+\frac{y}{b}\hat{\jmath}\right)$$

with a and b the horizontal and vertical semi axes, which vary along the machine due to the betatron function. Determine the horizontal and vertical incoherent tune shifts inside a perfectly conducting circular pipe if the emittances of the beam ε_x and ε_y are known. **(value = 5 points)**



5) Obtain the vertical indirect space charge force of a uniform d.c. current (uniform beam distribution in longitudinal and transverse plane) inside a dipole magnet, which can be considered as two parallel plates of ferromagnetic material knowing that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ under the approximation that $g \gg a$ with $2g$ the dipole gap and a the beam radius. **(value = 5 points)**