

Cosmology Course
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Lecture 1

Afterglow Light Pattern
375,000 yrs.

Dark Ages

Inflation

Inflation

Quantum Fluctuations

CMB

1st Stars
about 400 million yrs.



Lecture 2

Development of
Galaxies, Planets, etc.

Dark Matter

PBH

Big Bang Expansion

13.77 billion years

GW

Lecture 3

Dark Energy
Accelerated Expansion

Dark Energy Surveys

WMAP

LSS

Outline

Lecture 1

- Shortcomings of the Hot Big Bang
- The Inflationary Paradigm
- Homogeneous Scalar Field Dynamics
- Slow roll approximation
- Quantum Fluctuations in de Sitter
- CMB anisotropies (temp. + polar.)
- Planck 2017

Shortcomings of the Hot Big Bang

- **The space-time structure of the observable Universe**
 - Why is the Universe so close to spatial flatness?
 - Why is matter so homogeneously distributed on large scales?
- **The origin of structures in the Universe**
 - How did primordial spectrum of density perturbations originate?
- **The origin of matter and radiation**
 - Where does all the energy in the Universe come from?
 - How did the matter-antimatter asymmetry arise?
- **The initial singularity**
 - Did the Universe have a beginning?
 - What is the global structure of the Universe beyond our observable patch?

Einstein-Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \quad ij + 00$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad 00$$

Energy density conservation

$$D_{\mu}T^{\mu}_{\nu} = 0 \Rightarrow \dot{\rho}(t) + 3\frac{\dot{a}}{a}(\rho(t) + p(t)) = 0$$

$$p(t) = w\rho(t) \quad \text{barotropic fluid}$$

Time evolution of density params

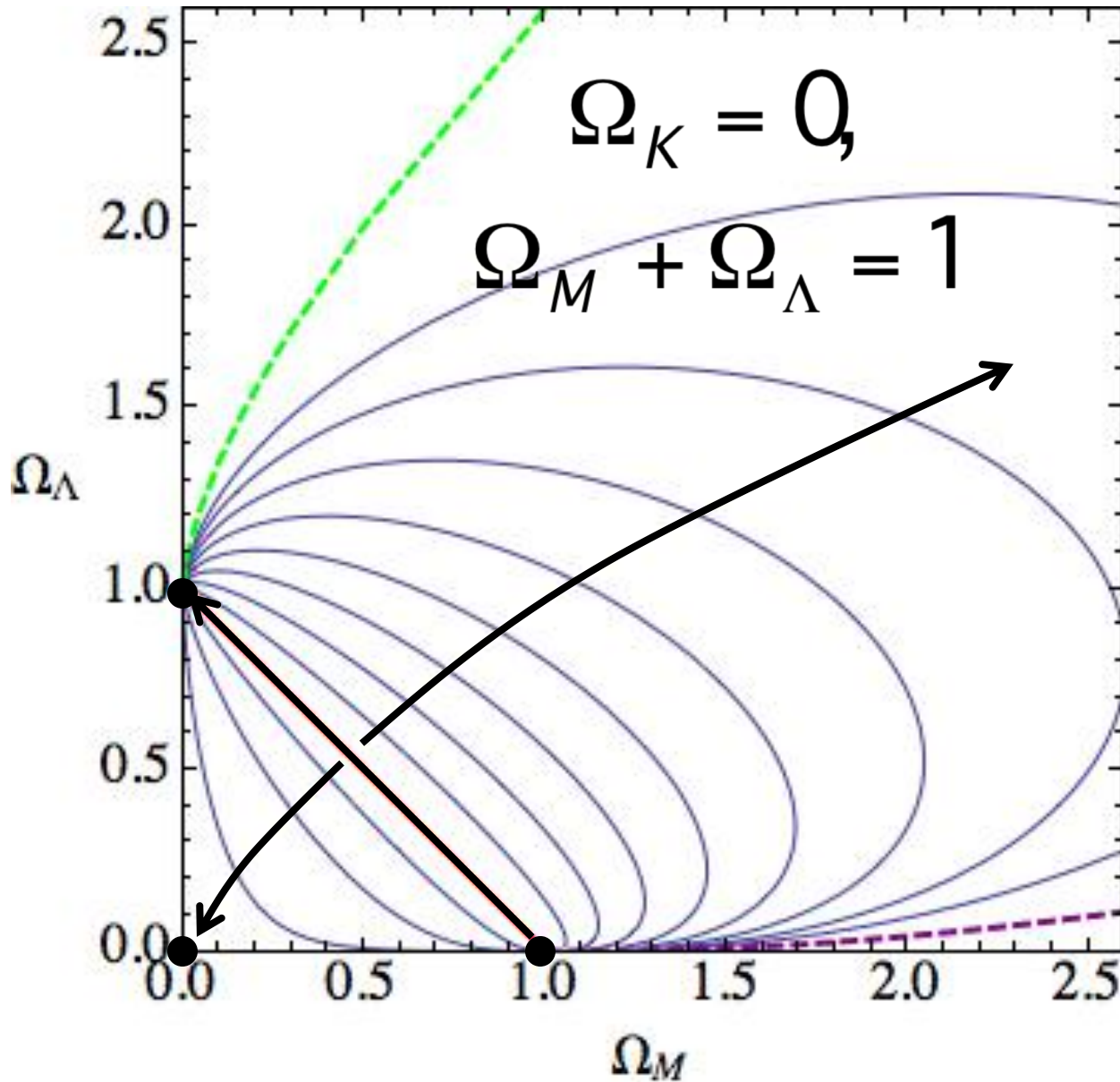
$$x \equiv \Omega_M(a) = \frac{8\pi G \rho_M(a)}{3H^2(a)} = \frac{\Omega_M}{\Omega_M + \Omega_K a + \Omega_\Lambda a^3}$$

$$y \equiv \Omega_\Lambda(a) = \frac{\Lambda}{3H^2(a)} = \frac{\Omega_\Lambda a^3}{\Omega_M + \Omega_K a + \Omega_\Lambda a^3}$$

Homogeneous system of eqs.

$$\begin{aligned} x' &\equiv \frac{dx}{dN} = -x(1-x+2y) \\ y' &\equiv \frac{dy}{dN} = +y(x+2(1-y)) \end{aligned} \quad \begin{array}{l} \text{critical points} \\ N = \ln a \\ (x=0, y=0) \\ (x=1, y=0) \\ (x=0, y=1) \end{array}$$

Flatness is unstable



critical
points

$$(x = 0, y = 0)$$

$$(x = 1, y = 0)$$

$$(x = 0, y = 1)$$

Flatness

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}, \quad \Omega = \frac{8\pi G}{3H^2} \rho, \quad x \equiv \frac{\Omega - 1}{\Omega} = \frac{3K / 8\pi G}{\rho a^2}$$

$$\frac{d \ln \rho}{d \ln a} = \frac{\rho'}{\rho} = -3(1+w) \Rightarrow x' = (1+3w)x$$

Matter & Radiation

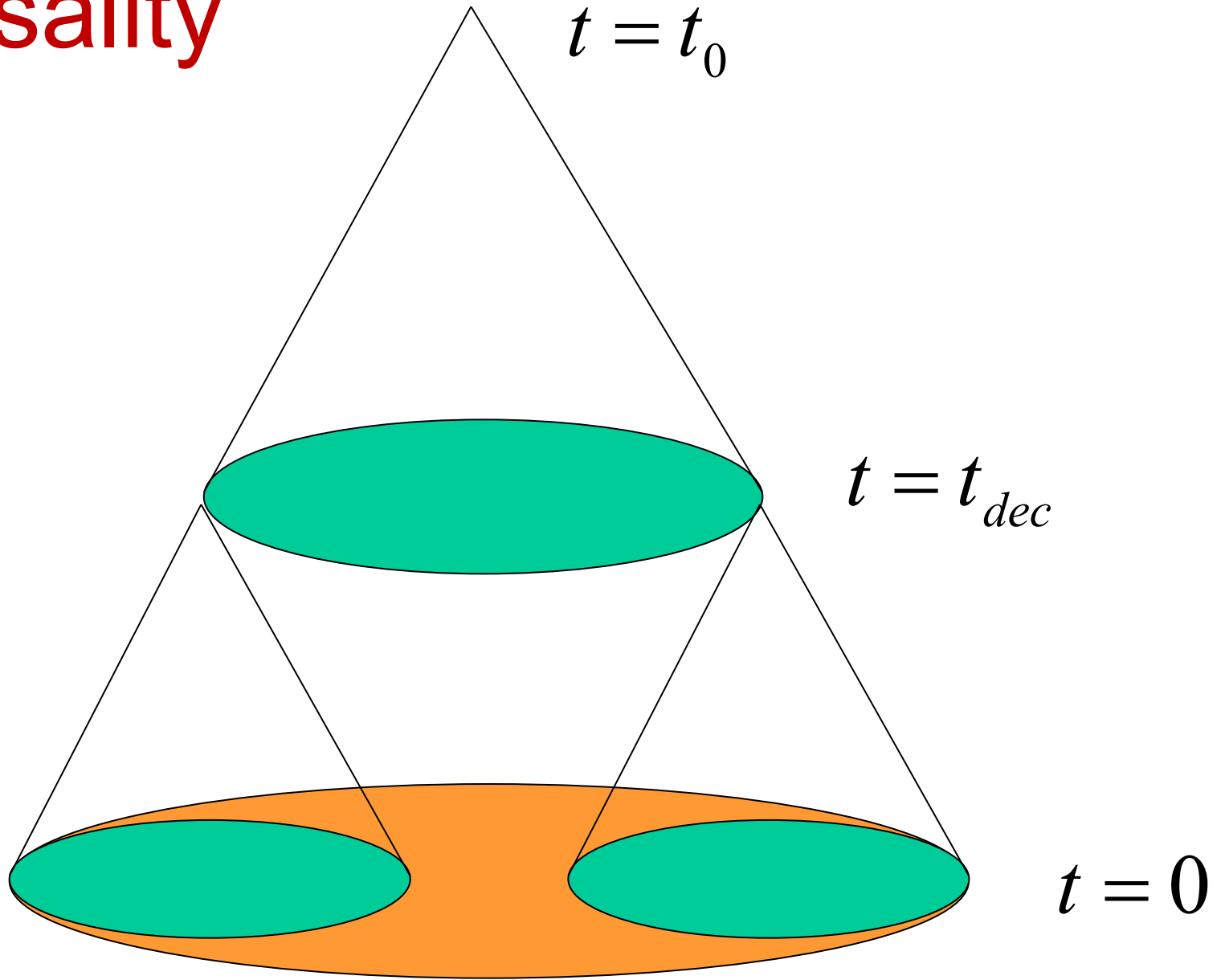
$x=0$ unstable $x \propto a^2, a$

Vacuum energy

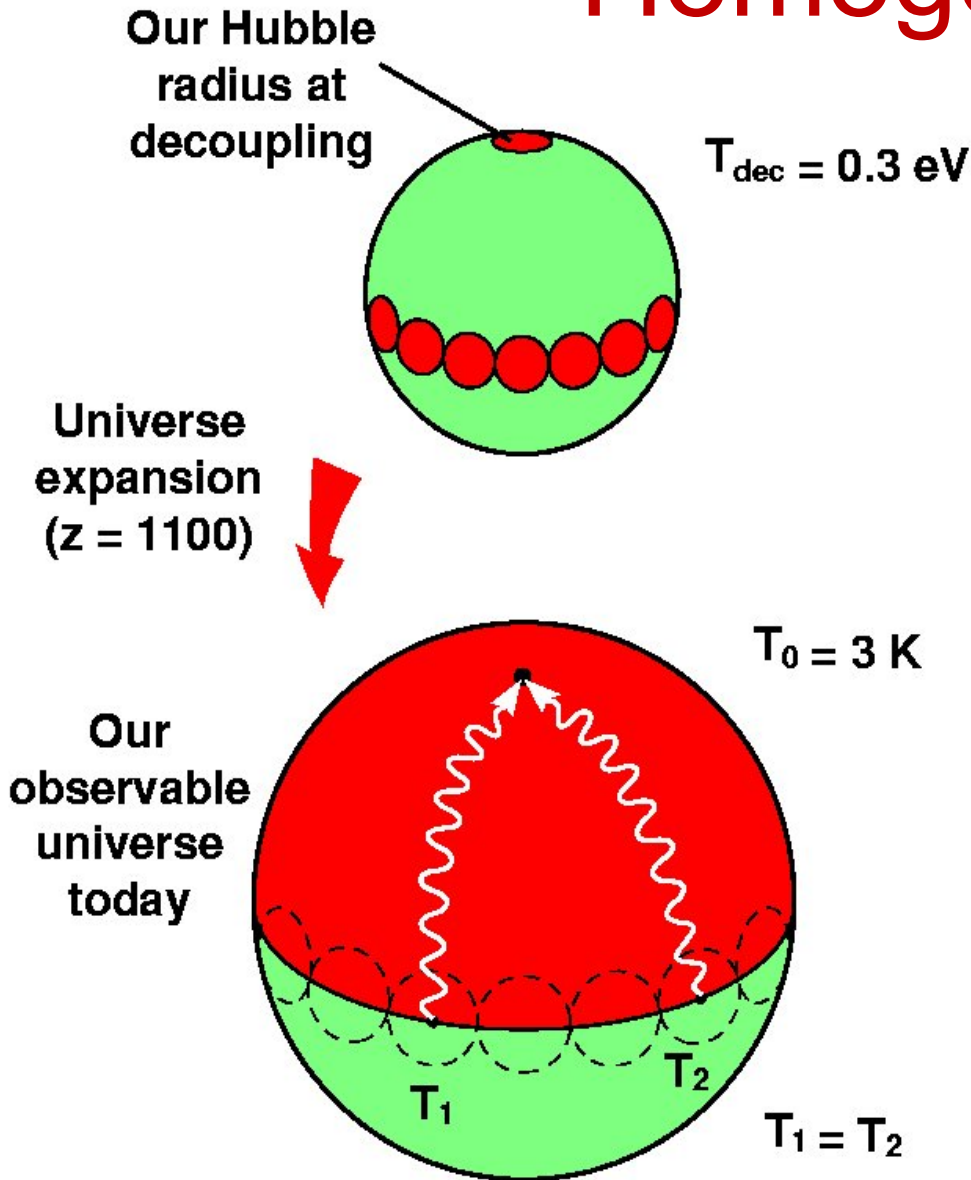
$x=0$ stable $x \propto a^{-2}$

$$x_0 = x_{in} \left(\frac{T_{in}}{T_{eq}} \right)^2 (1+z_{eq}) < 10^3 \quad \text{e.g. } x_{BBN} < 10^{-1\epsilon}$$

Causality



Homogeneity



Scale Factor

$$a(t) \propto t^p \quad p < 1$$

Particle Horizon

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \propto t$$

Causally Disconnected

$$N_{CD}(z) \approx \left(\frac{a}{d_H} \right)^3 \approx (1+z)^{3/2}$$

Homogeneity

Size
Universe

$$a \propto t^{1/2}$$

Horizon

$$d_H \propto t$$

Causally
Disconnected
Regions

$$N_{CD}(z) \approx (1+z)^{3/2}$$

$$a_0 \equiv d_H(t_0)$$

?

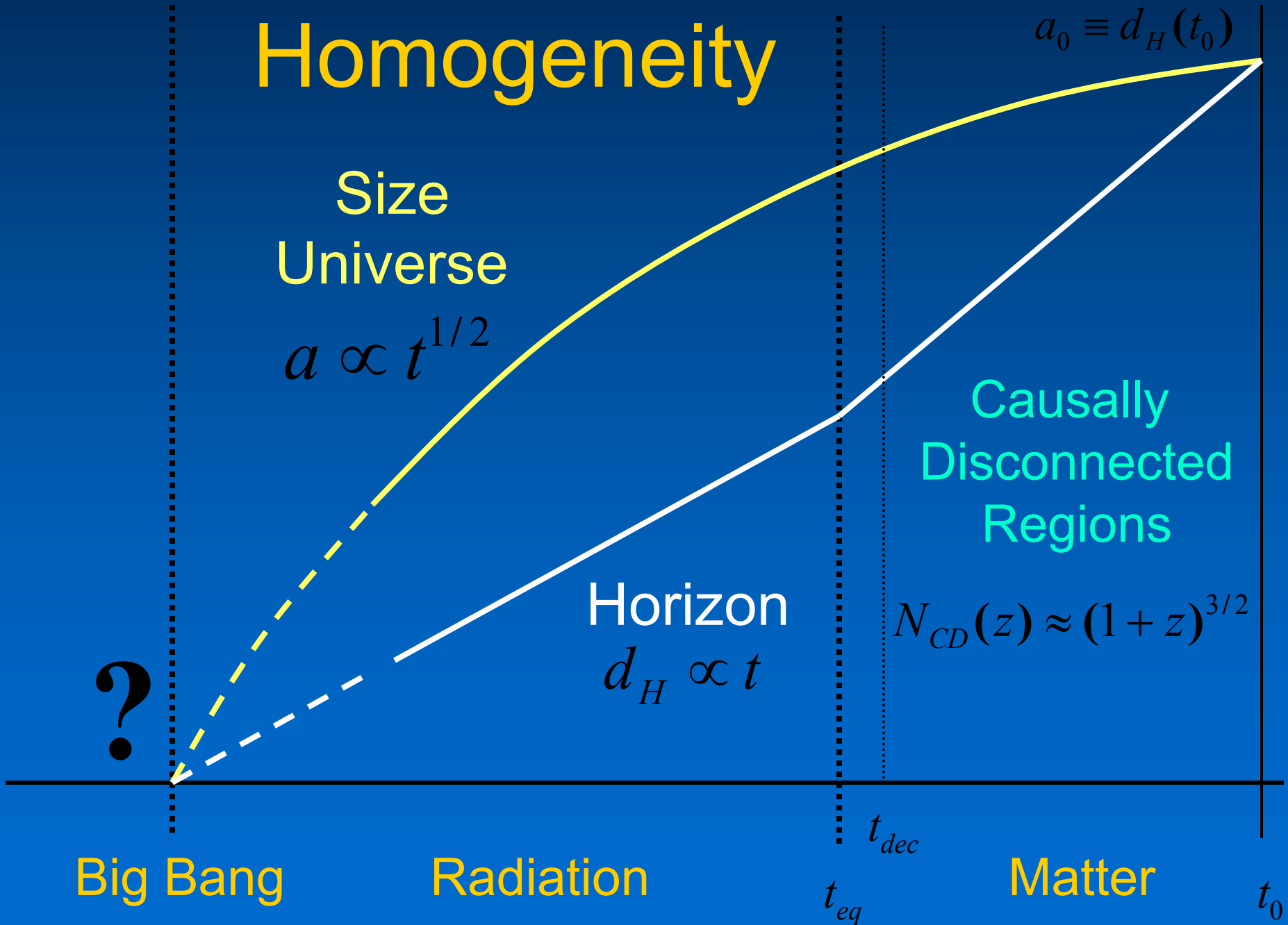
Big Bang

Radiation

Matter

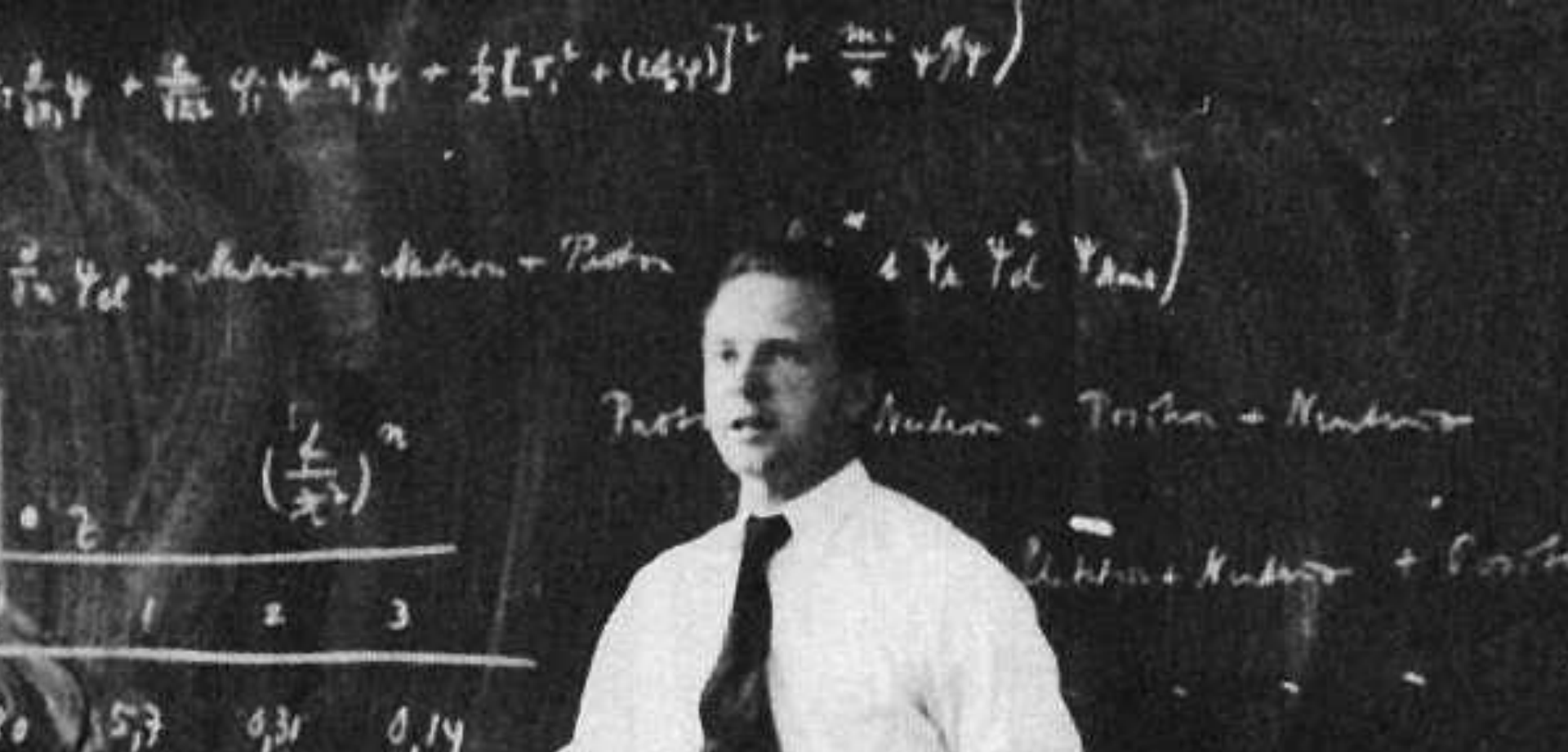
t_{dec}
 t_{eq}

t_0



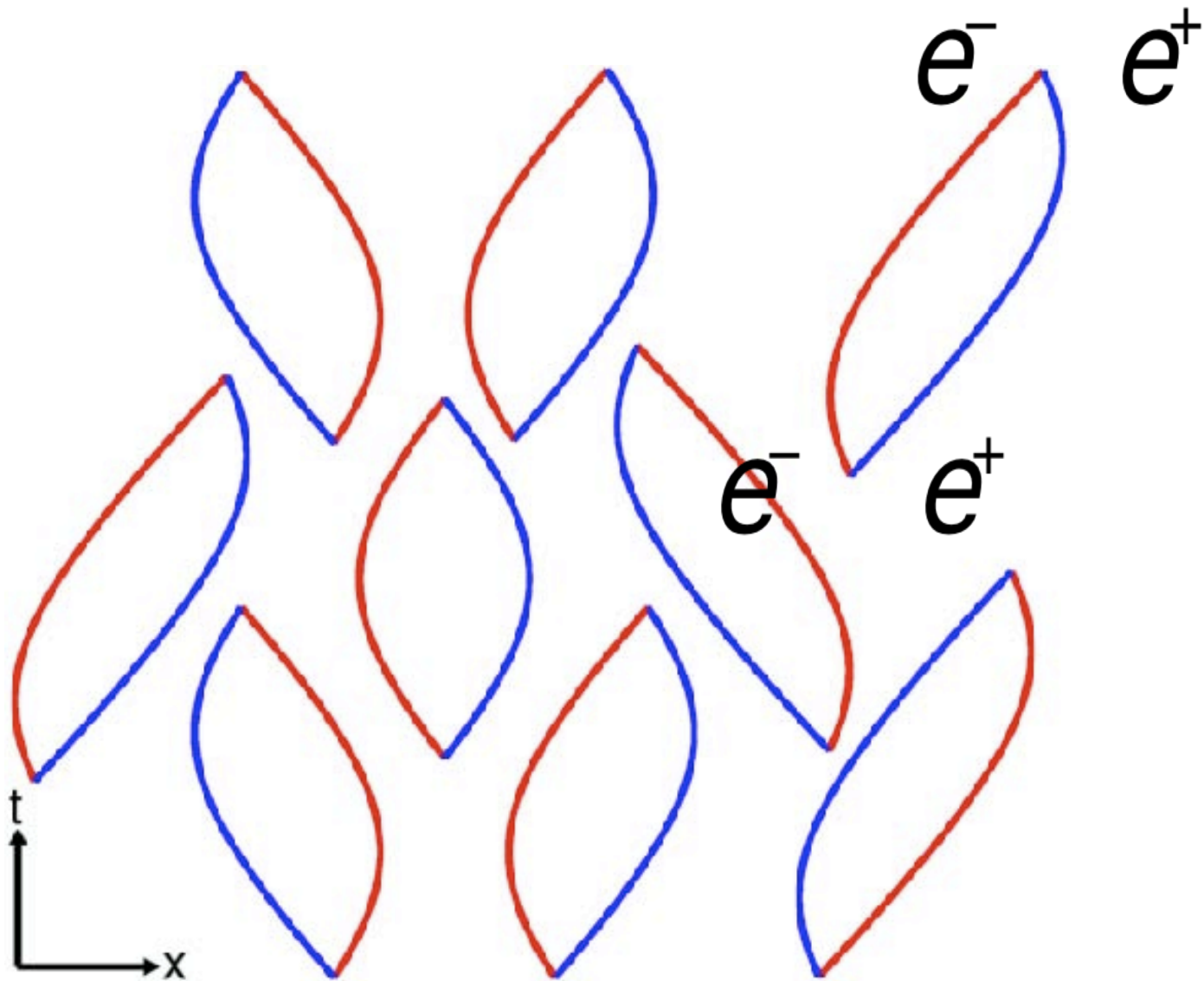
A very elegant solution:

INFLATION



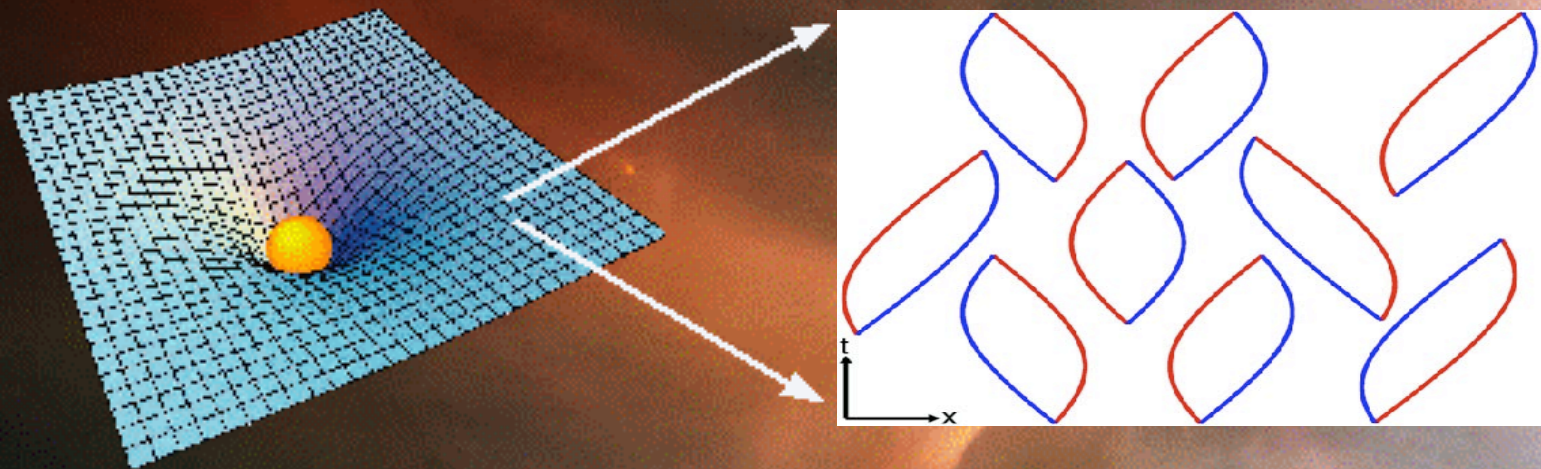
1 fm 200MeV \approx 1

Vacuum fluctuations



The universe itself could be a product of quantum uncertainty.

“empty space” is a sea of virtual particles winking in and out of existence:



Inflation

Our Universe could be the result of a quantum fluctuation



Alan Guth

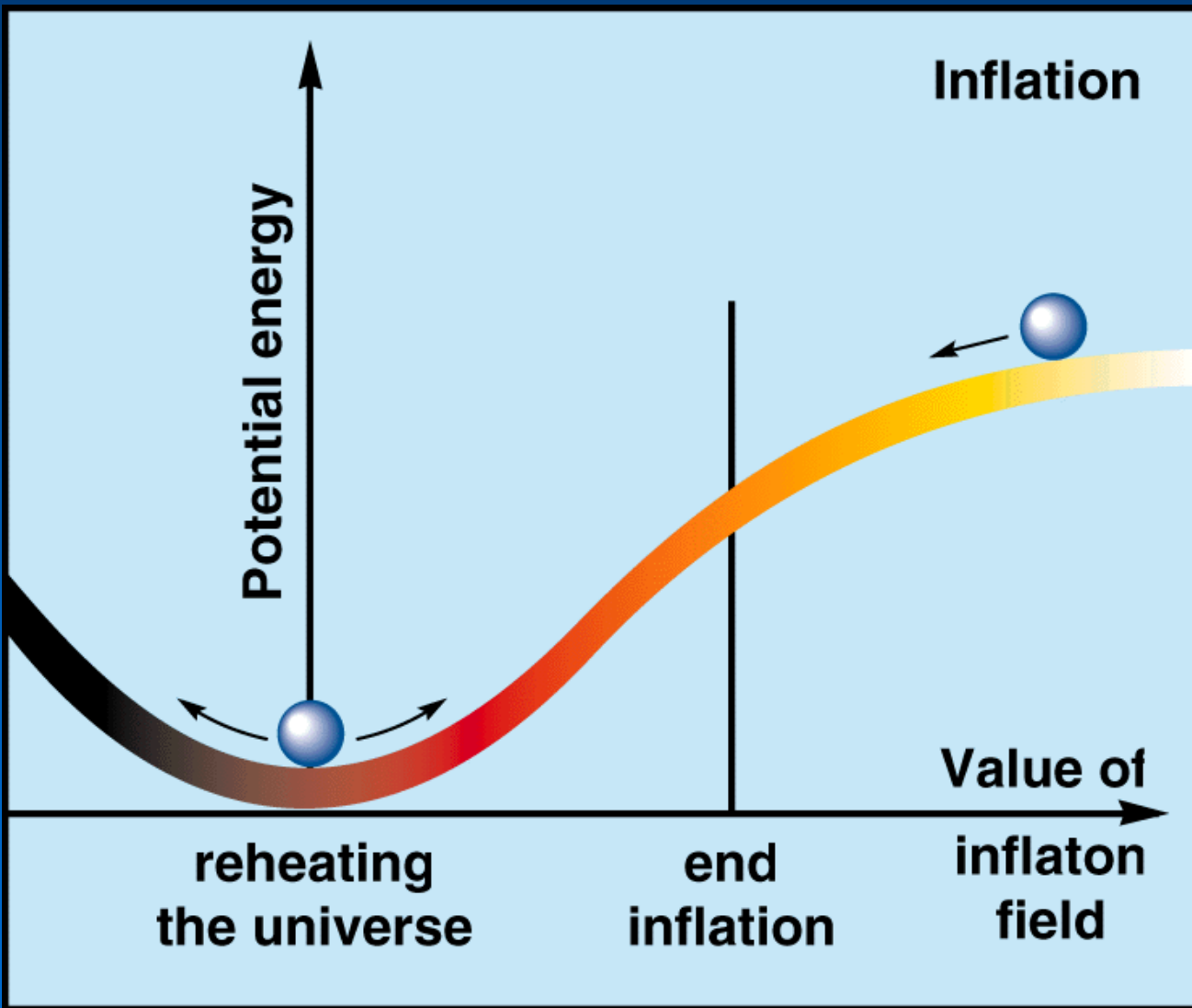


Andrei Linde



A small bubble of quantum vacuum expands very rapidly until it encompasses all our Universe

Effective description (scalar field)

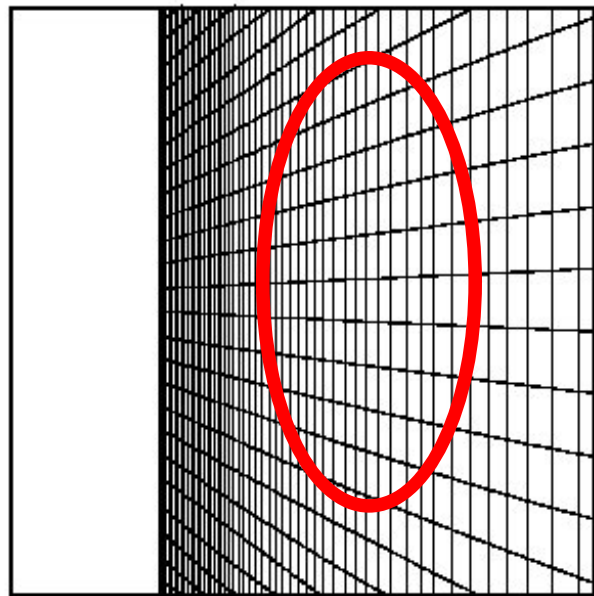
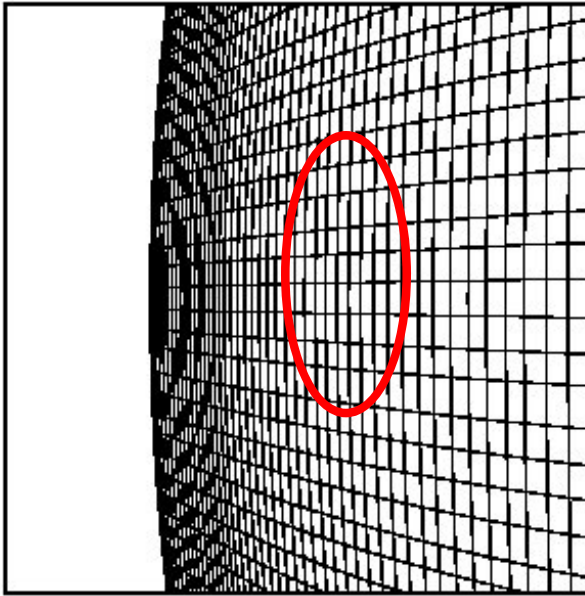
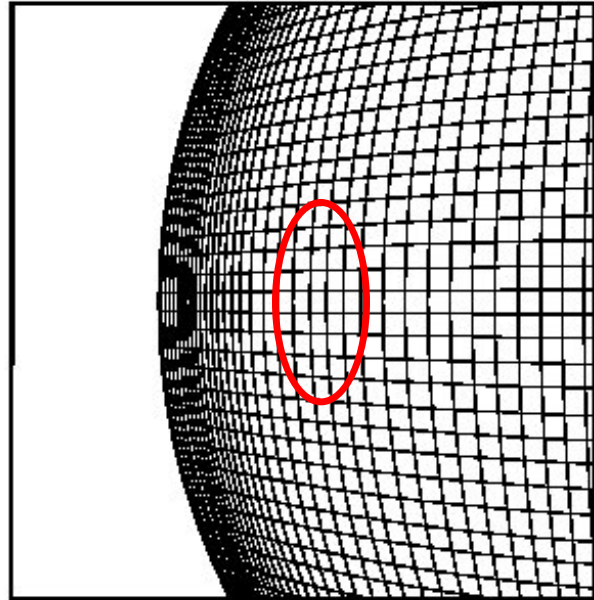
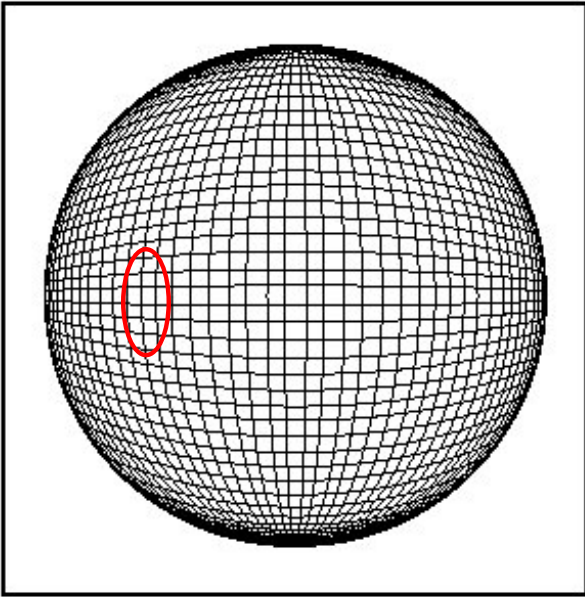


Constant density

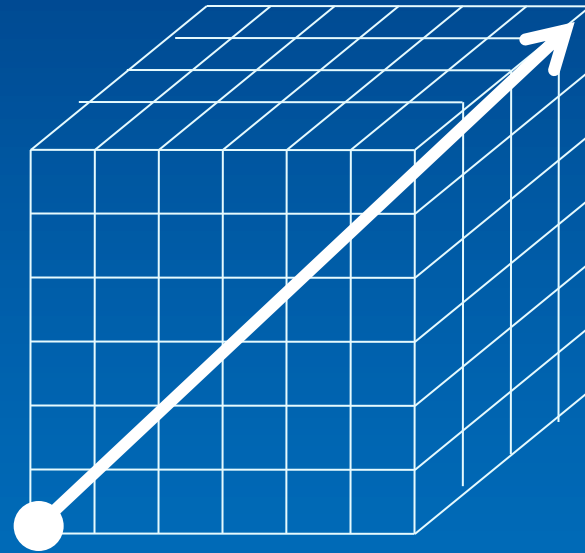
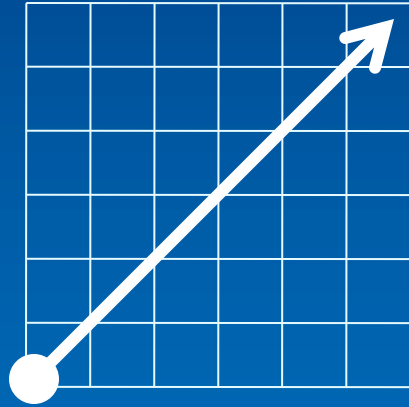
GR

Exponential Growth

Flat spatial homogeneous sections



Flat = Euclidean



Straight trajectory

SCALAR FIELD DYNAMICS

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)$$

Hamiltonian and momentum constraint equations

$$H^2 = \frac{\kappa^2}{3} \left[\frac{1}{2} (\Pi\phi)^2 + V(\phi) \right],$$

$$H_{|i} = -\frac{\kappa^2}{2} \Pi\phi \phi_{|i},$$

Evolution equations

$$\dot{H} = -\frac{\kappa^2}{2} (\Pi\phi)^2$$

$$\dot{\Pi}\phi + 3H\Pi\phi + \frac{\partial V}{\partial\phi} = 0.$$

HAMILTON-JACOBI EQUATION

Constraint equations $\rightarrow \left(\frac{\partial H}{\partial t}\right)_{\phi} = 0, \quad \left(\frac{\partial \Pi^{\phi}}{\partial t}\right)_{\phi} = 0.$

Then $H \equiv H(\phi(t, x^i)),$

$$3H^2(\phi) = \frac{2}{\kappa^2} \left(\frac{\partial H}{\partial \phi}\right)^2 + \kappa^2 V(\phi),$$

$$\dot{\phi} = -\frac{2}{\kappa^2} \left(\frac{\partial H}{\partial \phi}\right) = \Pi^{\phi}, \quad \frac{\dot{a}}{a} = H(\phi),$$

$$\dot{H} = -\frac{2}{\kappa^2} \left(\frac{\partial H}{\partial \phi}\right)^2 = -\frac{\kappa^2}{2} (\Pi^{\phi})^2,$$

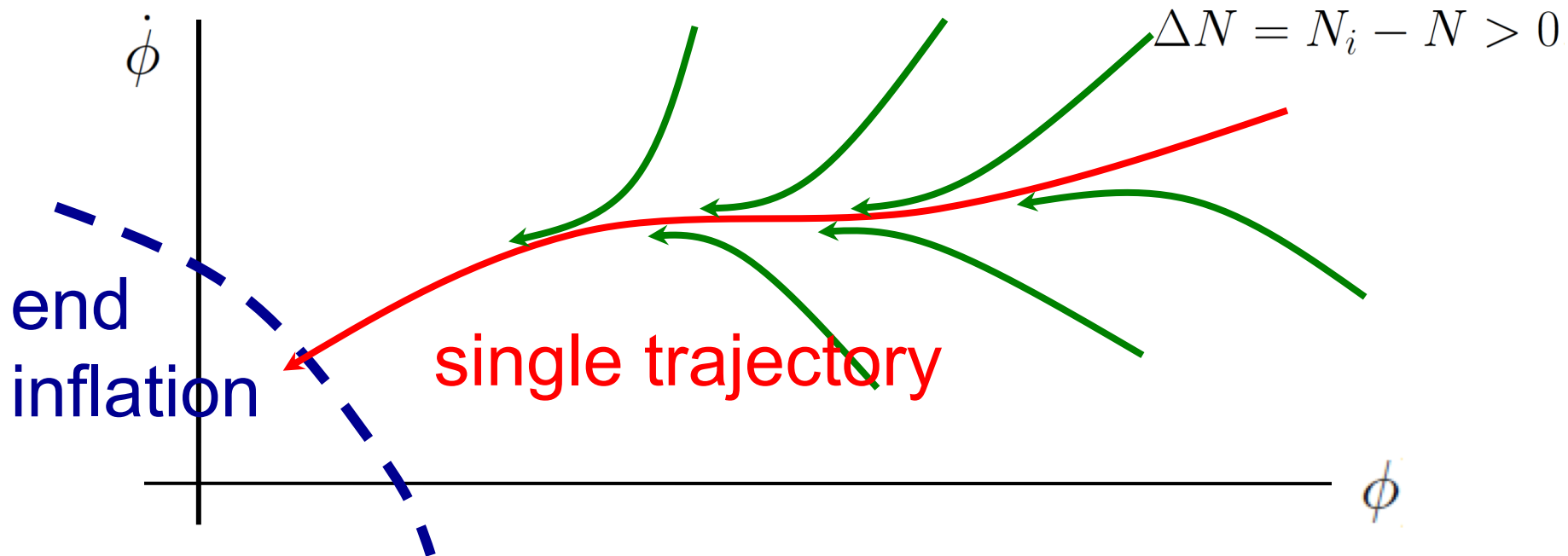
SLOW-ROLL ATTRACTOR

$H_0(\phi)$ exact particular solution

$H(\phi) = H_0(\phi) + \delta H(\phi)$ linear perturbation

Then $H'_0(\phi) \delta H'(\phi) = (3\kappa^2/2) H_0 \delta H$ with solution:

$$\delta H(\phi) = \delta H(\phi_i) \exp\left(\frac{3\kappa^2}{2} \int_{\phi_i}^{\phi} \frac{H_0(\phi) d\phi}{H'_0(\phi)}\right) = \delta H(\phi_i) \exp(-3\Delta N)$$



SLOW-ROLL PARAMETERS

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{2}{\kappa^2} \left(\frac{H'(\phi)}{H(\phi)} \right)^2 = -\frac{\partial \ln H}{\partial \ln a},$$

$$\delta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{2}{\kappa^2} \left(\frac{H''(\phi)}{H(\phi)} \right) = -\frac{\partial \ln H'}{\partial \ln a},$$

The scalar field ϕ acts as a new “time”

$$N_e \equiv \ln \frac{a_{\text{end}}}{a(t)} = \int_t^{t_{\text{end}}} H dt = -\frac{\kappa^2}{2} \int_{\phi}^{\phi_{\text{end}}} \frac{H(\phi) d\phi}{H'(\phi)}$$

The number of “e”-folds N_e to the end inflation

SLOW-ROLL APPROXIMATION

$$H^2 \left(1 - \frac{\epsilon}{3}\right) \simeq H^2 = \frac{\kappa^2}{3} V(\phi),$$
$$3H\dot{\phi} \left(1 - \frac{\delta}{3}\right) \simeq 3H\dot{\phi} = -V'(\phi)$$

just dynamics

$$\epsilon = \frac{2}{\kappa^2} \left(\frac{H'(\phi)}{H(\phi)} \right)^2 \simeq \frac{1}{2\kappa^2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \equiv \epsilon_V \ll 1,$$

$$\delta = \frac{2}{\kappa^2} \frac{H''(\phi)}{H(\phi)} \simeq \frac{1}{\kappa^2} \frac{V''(\phi)}{V(\phi)} - \frac{1}{2\kappa^2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \equiv \eta_V - \epsilon_V \ll 1,$$

$$\xi = \frac{4}{\kappa^4} \frac{H'(\phi)H'''(\phi)}{H^2(\phi)} \simeq \frac{1}{\kappa^4} \frac{V'(\phi)V'''(\phi)}{V^2(\phi)} - \frac{3}{2\kappa^4} \frac{V''(\phi)}{V(\phi)} \left(\frac{V'(\phi)}{V(\phi)} \right)^2$$

$$+ \frac{3}{4\kappa^4} \left(\frac{V'(\phi)}{V(\phi)} \right)^4 \equiv \xi_V - 3\eta_V\epsilon_V + 3\epsilon_V^2 \ll 1.$$

$$N \simeq \int_{\phi_i}^{\phi_e} \frac{\kappa d\phi}{\sqrt{2\epsilon_V(\phi)}} = \kappa^2 \int_{\phi_i}^{\phi_e} \frac{V(\phi) d\phi}{V'(\phi)},$$

INFLATIONARY SOLUTIONS OF HBBP

$$\begin{aligned} \rho &= \frac{1}{2} \dot{\phi}^2 + V(\phi), \\ p &= \frac{1}{2} \dot{\phi}^2 - V(\phi). \end{aligned} \quad \dot{\rho} + 3H(\rho + p) = 0$$

$$V(\phi) \gg \dot{\phi}^2 \Rightarrow p \simeq -\rho \Rightarrow \rho \simeq \text{const.} \Rightarrow H(\phi) \simeq \text{const.}$$

$$a(t) \sim \exp(Ht) \Rightarrow \frac{\ddot{a}}{a} > 0 \quad \text{accelerated expansion}$$

$$x_0 = x_{\text{in}} e^{-2N} \frac{a_{\text{rh}}^2 \rho_{\text{rh}}}{a_{\text{end}}^2 \rho_{\text{end}}} \frac{T_{\text{rh}}^2}{T_{\text{eq}}^2} (1 + z_{\text{eq}}) \simeq e^{-2N} 10^{56} \leq 1 \Rightarrow N \geq 65$$

$$\begin{aligned} {}^{(3)}R &= \frac{6K}{a^2} = {}^{(3)}R_{\text{in}} e^{-2N} \longrightarrow 0, & \rho_{\text{M}} \propto a^{-3} \sim e^{-3N} &\longrightarrow 0, \\ \delta_k &\sim \left(\frac{k}{aH}\right)^2 \Phi_k \propto e^{-2N} \longrightarrow 0, & \rho_{\text{R}} \propto a^{-4} \sim e^{-4N} &\longrightarrow 0, \end{aligned}$$

curvature

matter

LINEAR METRIC PERTURBATIONS

$$ds^2 = a(\eta)^2 \left[-(1 + 2\Phi)d\eta^2 + (1 - 2\Phi)d\mathbf{x}^2 + h_{ij} dx^i dx^j \right]$$

linear pert. eqs.

$$\Phi'' + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa^2}{2}[\phi'\delta\phi' - a^2V'(\phi)\delta\phi],$$

$$-\nabla^2\Phi + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = -\frac{\kappa^2}{2}[\phi'\delta\phi' + a^2V'(\phi)\delta\phi],$$

$$\Phi' + \mathcal{H}\Phi = \frac{\kappa^2}{2}\phi'\delta\phi,$$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2V''(\phi)\delta\phi = 4\phi'\Phi' - 2a^2V'(\phi)\Phi.$$

$$u'' - \nabla^2 u - \frac{z''}{z}u = 0,$$

$$\nabla^2\Phi = \frac{\kappa^2}{2} \frac{\mathcal{H}}{a^2} (zu' - z'u),$$

$$\left(\frac{a^2\Phi}{\mathcal{H}} \right)' = \frac{\kappa^2}{2} zu.$$

Mukhanov variables

$$u \equiv a\delta\phi + z\Phi,$$

$$z \equiv a \frac{\phi'}{\mathcal{H}}.$$

QUANTUM FLUCTUATIONS IN QdS

$$\delta S = \frac{1}{2} \int d^3x d\eta \left[(u')^2 - (\nabla u)^2 + \frac{z''}{z} u^2 \right]$$

$$\hat{u}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[u_k(\eta) \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + u_k^*(\eta) \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right],$$

scalar field's Fock space

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta^3(\mathbf{k} - \mathbf{k}'),$$

$$\hat{a}_{\mathbf{k}} |0\rangle = 0.$$

equal-time commutation relations

$$[\hat{u}(\eta, \mathbf{x}), \hat{\Pi}_u^\dagger(\eta, \mathbf{x}')] = i\hbar \delta^3(\mathbf{x} - \mathbf{x}')$$

QUANTUM FLUCTUATIONS IN QdS

normalization condition on the modes u_k

$$u_k u_k^{*'} - u_k' u_k^* = i$$

the Wronskian of the mode equation

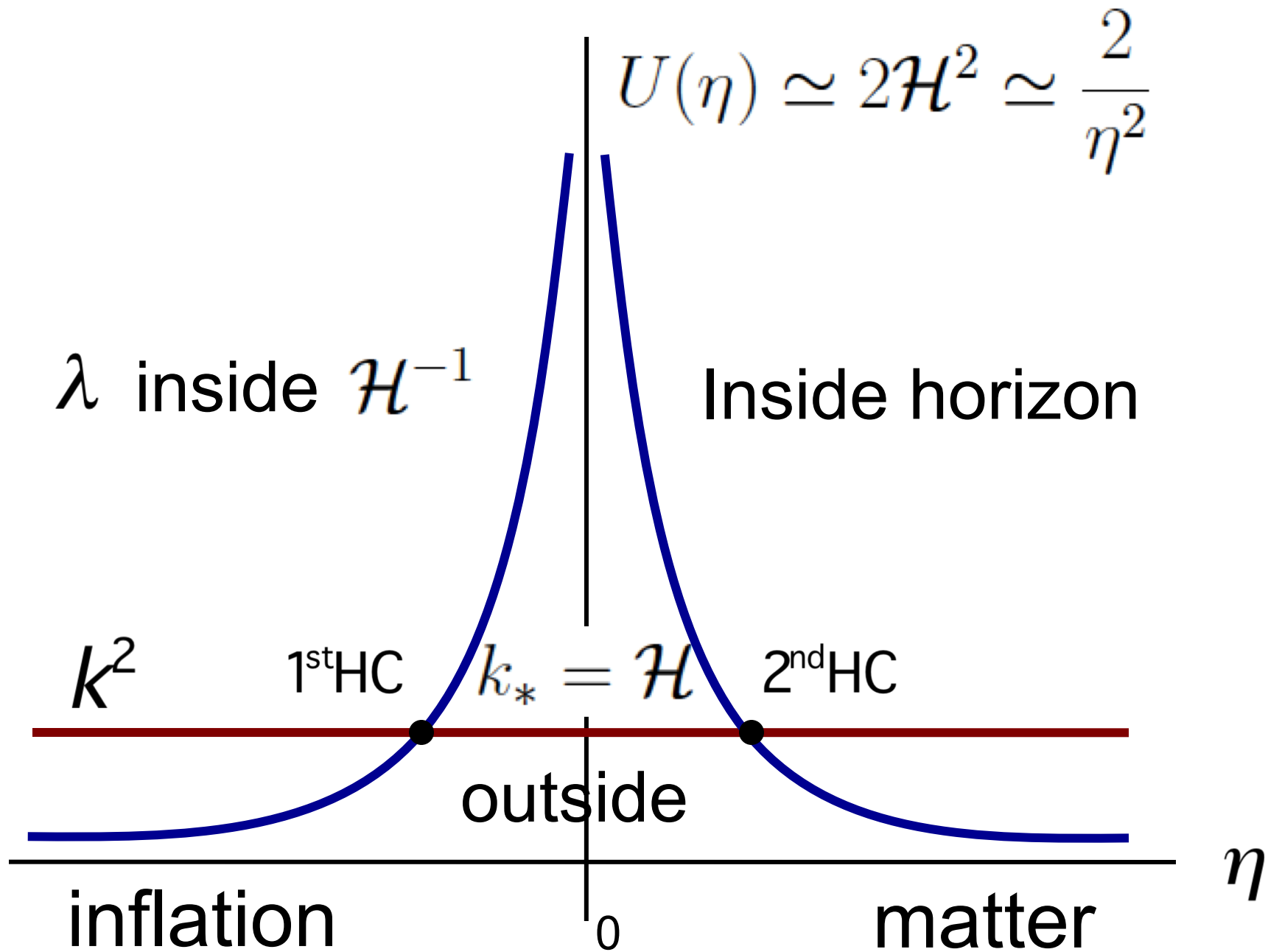
$$u_k'' + \left(k^2 - \frac{z''}{z} \right) u_k = 0$$

Schrödinger like equation

$$-u_k'' + U(\eta) u_k = k^2 u_k$$

time-dependent potential

$$U(\eta) = z''/z$$



SOLUTIONS OF MODE EQUATIONS

slow-roll parameters

$$\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{\kappa^2 z^2}{2 a^2},$$

$$\delta = 1 - \frac{\phi''}{\mathcal{H}\phi'} = 1 + \epsilon - \frac{z'}{\mathcal{H}z},$$

$$\xi = - \left(2 - \epsilon - 3\delta + \delta^2 - \frac{\phi'''}{\mathcal{H}^2\phi'} \right)$$

approx. constant

$$\epsilon' = 2\mathcal{H} \left(\epsilon^2 - \epsilon\delta \right) = \mathcal{O}(\epsilon^2),$$

$$\delta' = \mathcal{H} \left(\epsilon\delta - \xi \right) = \mathcal{O}(\epsilon^2).$$

for constant slow-roll parameters, we can write

$$\eta = \frac{-1}{\mathcal{H}} + \int \frac{\epsilon da}{a\mathcal{H}} \simeq \frac{-1}{\mathcal{H}} \frac{1}{1 - \epsilon},$$

$$\frac{z''}{z} = \mathcal{H}^2 \left[(1 + \epsilon - \delta)(2 - \delta) + \mathcal{H}^{-1}(\epsilon' - \delta') \right] \simeq \frac{1}{\eta^2} \left(\nu^2 - \frac{1}{4} \right),$$

where $\nu = \frac{1 + \epsilon - \delta}{1 - \epsilon} + \frac{1}{2}$

EXACT SOLUTIONS OF MODE EQS.

two asymptotic regimes,

$$u_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} \quad k \gg aH \quad \text{Minkowsky}$$

$$u_k = C_1(k) z \quad k \ll aH \quad \text{superhorizon}$$

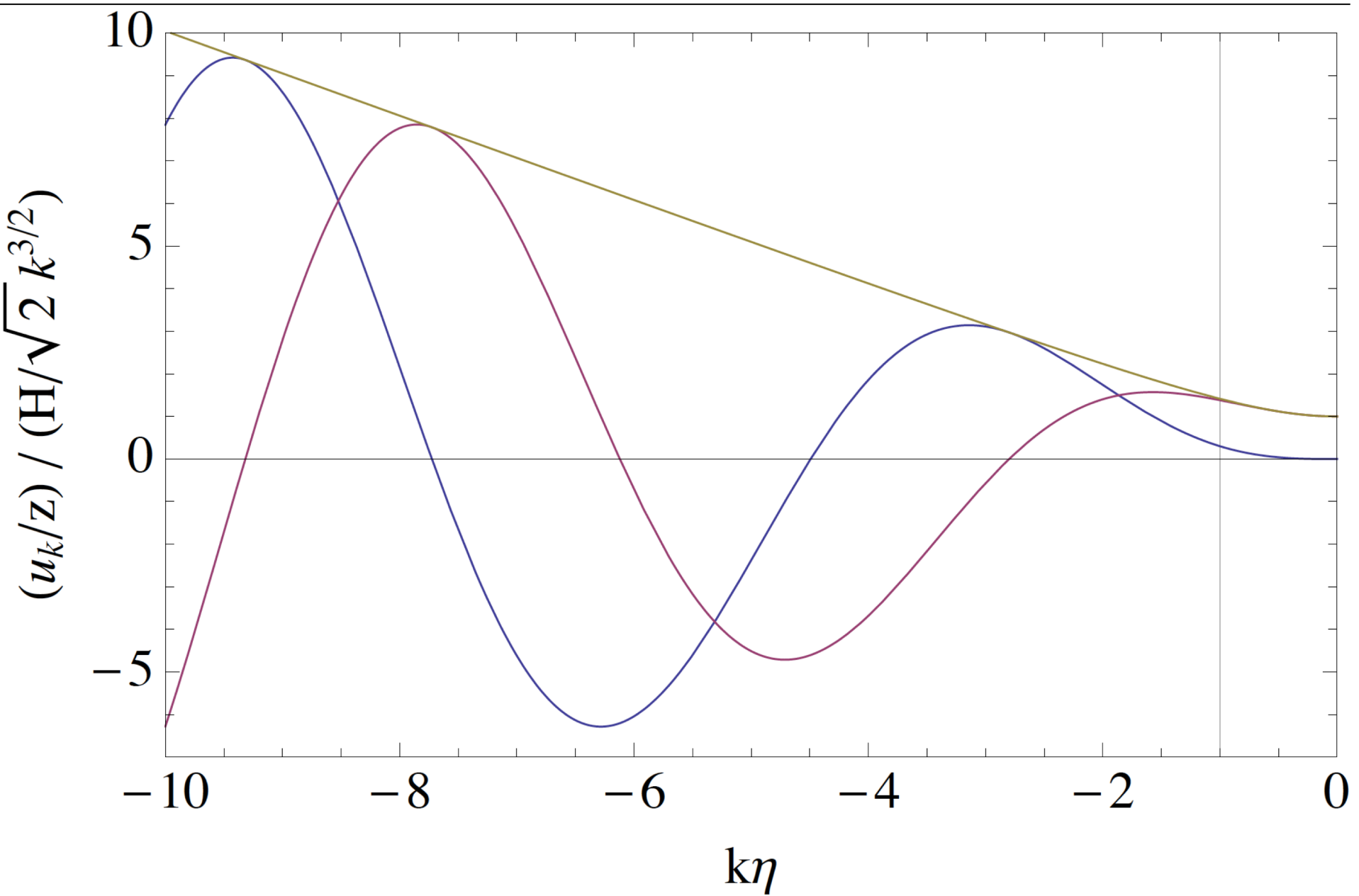
exact solution that connects the two regimes

$$u_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} (-\eta)^{1/2} H_\nu^{(1)}(-k\eta)$$

where $H_\nu^{(1)}(z)$ is the Hankel function of the first kind

$$\text{e.g. } H_{3/2}^{(1)}(x) = -e^{ix} \sqrt{2/\pi x} (1 + i/x),$$

$$\text{and } \nu \text{ is given by } \nu = \frac{1 + \epsilon - \delta}{1 - \epsilon} + \frac{1}{2}$$



EXACT SOLUTIONS OF MODE EQS.

limit $k\eta \rightarrow 0$, the solution becomes

$$|u_k| = \frac{2^{\nu-\frac{3}{2}} \Gamma(\nu)}{\sqrt{2k} \Gamma(\frac{3}{2})} (-k\eta)^{\frac{1}{2}-\nu} \equiv \frac{C(\nu)}{\sqrt{2k}} \left(\frac{k}{aH}\right)^{\frac{1}{2}-\nu},$$

$$C(\nu) = 2^{\nu-\frac{3}{2}} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} (1-\epsilon)^{\nu-\frac{1}{2}} \simeq 1 \quad \text{for } \epsilon, \delta \ll 1$$

compute Φ and $\delta\phi$ from the super-Hubble-scale mode

$$\Phi = C_1 \left(1 - \frac{\mathcal{H}}{a^2} \int a^2 d\eta\right) + C_2 \frac{\mathcal{H}}{a^2},$$

$$\frac{\delta\phi}{\phi'} = \frac{C_1}{a^2} \int a^2 d\eta - \frac{C_2}{a^2}.$$

C_1 growing mode

C_2 decaying mode

SCALAR CURVATURE PERTURBATIONS

gauge invariant quantity constant for superhorizon modes of adiabatic perturbations,

$$\zeta \equiv \Phi + \frac{1}{\epsilon\mathcal{H}} (\Phi' + \mathcal{H}\Phi) = \frac{u}{z},$$

ζ is the gauge invariant curvature perturbation \mathcal{R}_c on constant energy density hypersurfaces,

$$\zeta = \mathcal{R}_c + \frac{1}{\epsilon\mathcal{H}^2} \nabla^2 \Phi$$

$$\zeta' = \frac{1}{\epsilon\mathcal{H}} \nabla^2 \Phi \simeq 0 \quad \text{constant}$$

for (adiabatic) superhorizon modes, $k \ll aH$

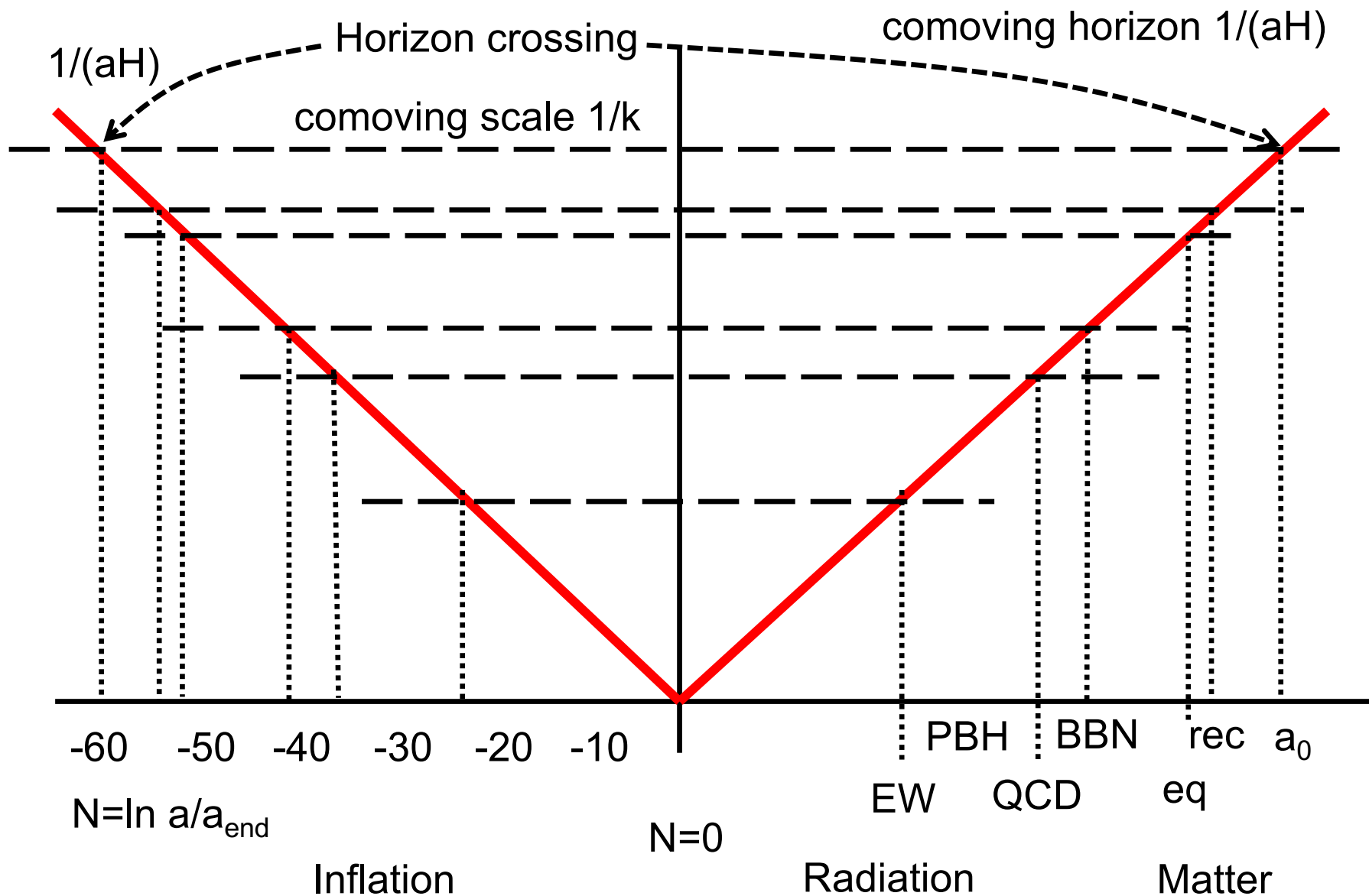
SCALAR CURVATURE PERTURBATIONS

Therefore, we can evaluate the Newtonian potential Φ_k when the perturbation reenters the horizon during radiation/matter eras in terms of the curvature perturbation \mathcal{R}_k when it left the Hubble scale during inflation,

$$\Phi_k = \left(1 - \frac{\mathcal{H}}{a^2} \int a^2 d\eta\right) \mathcal{R}_k = \frac{3 + 3\omega}{5 + 3\omega} \mathcal{R}_k = \begin{cases} \frac{2}{3} \mathcal{R}_k & \text{radiation era,} \\ \frac{3}{5} \mathcal{R}_k & \text{matter era.} \end{cases}$$

These expressions will be of special importance later. (CMB)

Inflation



GRAVITATIONAL WAVE PERTURBATIONS

$$\delta S = \frac{1}{2} \int d^3x d\eta \frac{a^2}{2\kappa^2} \left[(h'_{ij})^2 - (\nabla h_{ij})^2 \right]$$

tensor field h_{ij} considered as a quantum field,

$$\hat{h}_{ij}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=1,2} \left[h_k(\eta) e_{ij}(\mathbf{k}, \lambda) \hat{a}_{\mathbf{k},\lambda} e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \right],$$

$e_{ij}(\mathbf{k}, \lambda)$ are the two polarization tensors,

satisfying symmetric, transverse and traceless conditions

$$e_{ij} = e_{ji}, \quad k^i e_{ij} = 0, \quad e_{ii} = 0,$$

$$e_{ij}(-\mathbf{k}, \lambda) = e_{ij}^*(\mathbf{k}, \lambda), \quad \sum_{\lambda} e_{ij}^*(\mathbf{k}, \lambda) e^{ij}(\mathbf{k}, \lambda) = 4,$$

TENSOR MODE EQUATION

gauge invariant tensor amplitude

$$v_k(\eta) = \frac{a}{\sqrt{2\kappa}} h_k(\eta)$$

decoupled in linear perturbation theory,

$$v_k'' + \left(k^2 - \frac{a''}{a} \right) v_k = 0$$

For constant slow-roll parameters, the potential becomes

$$\frac{a''}{a} = 2\mathcal{H}^2 \left(1 - \frac{\epsilon}{2} \right) = \frac{1}{\eta^2} \left(\mu^2 - \frac{1}{4} \right),$$

$$\mu = \frac{1}{1 - \epsilon} + \frac{1}{2}.$$

EXACT SOLUTIONS

in the two asymptotic regimes,

$$v_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} \quad k \gg aH \quad \text{Minkowsky}$$

$$v_k = C_3(k) a \quad k \ll aH \quad \text{superhorizon}$$

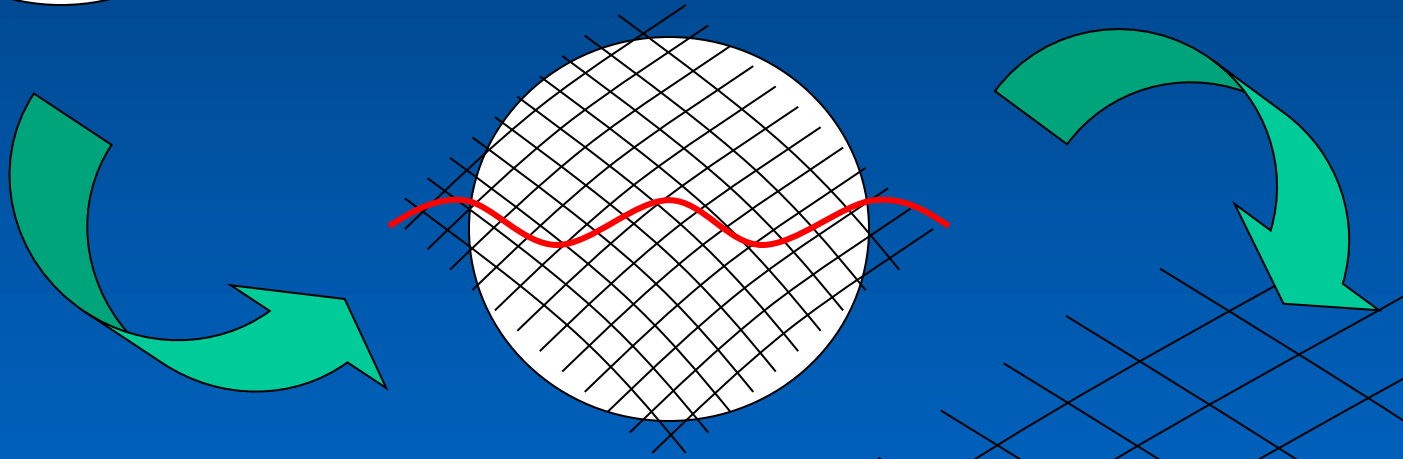
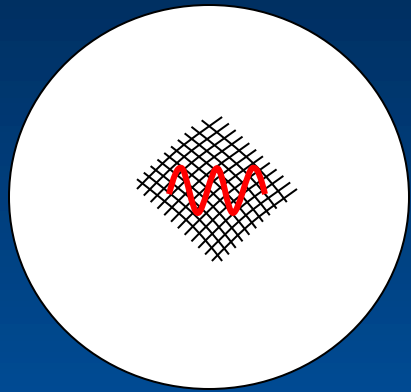
exact solutions

$$v_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\mu+\frac{1}{2})\frac{\pi}{2}} (-\eta)^{1/2} H_\mu^{(1)}(-k\eta)$$

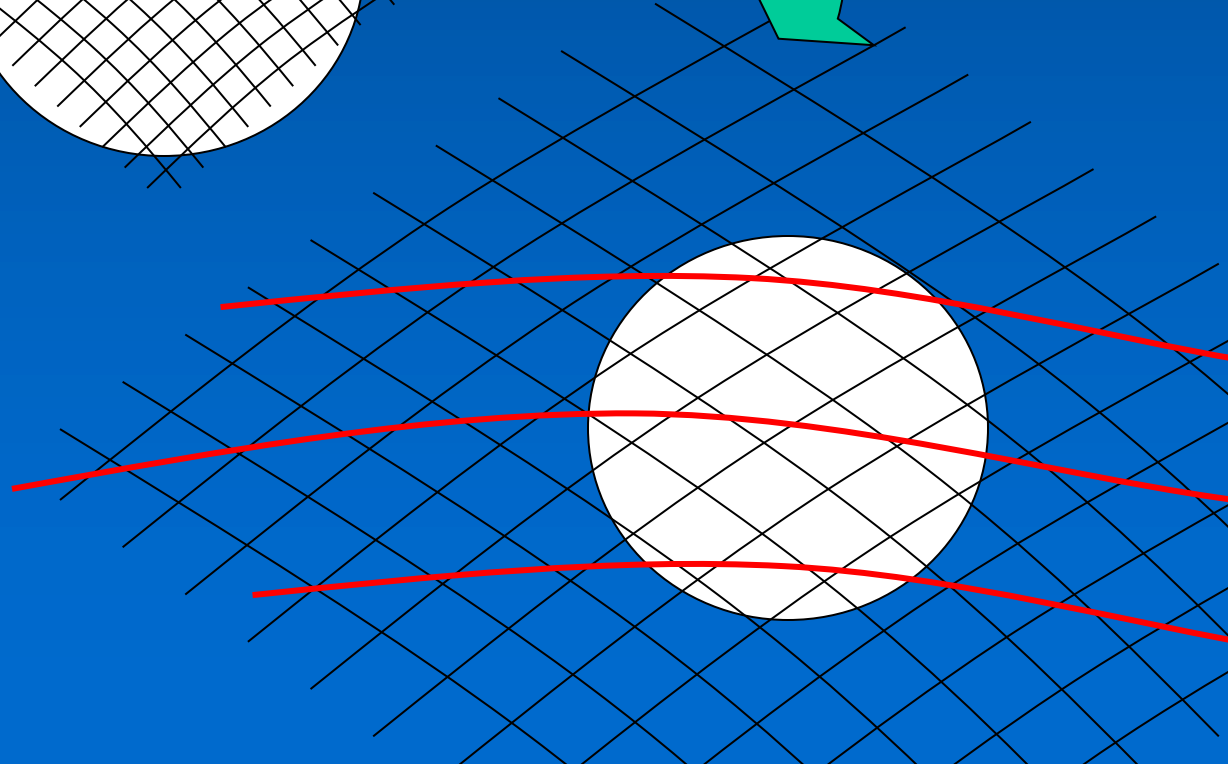
$$\text{In the limit } k\eta \rightarrow 0, \quad |v_k| = \frac{C(\mu)}{\sqrt{2k}} \left(\frac{k}{aH}\right)^{\frac{1}{2}-\mu}$$

Since h_k becomes constant on superhorizon scales, evaluate the tensor metric perturbation when it reentered during the radiation or matter era directly in terms of its value during inflation.

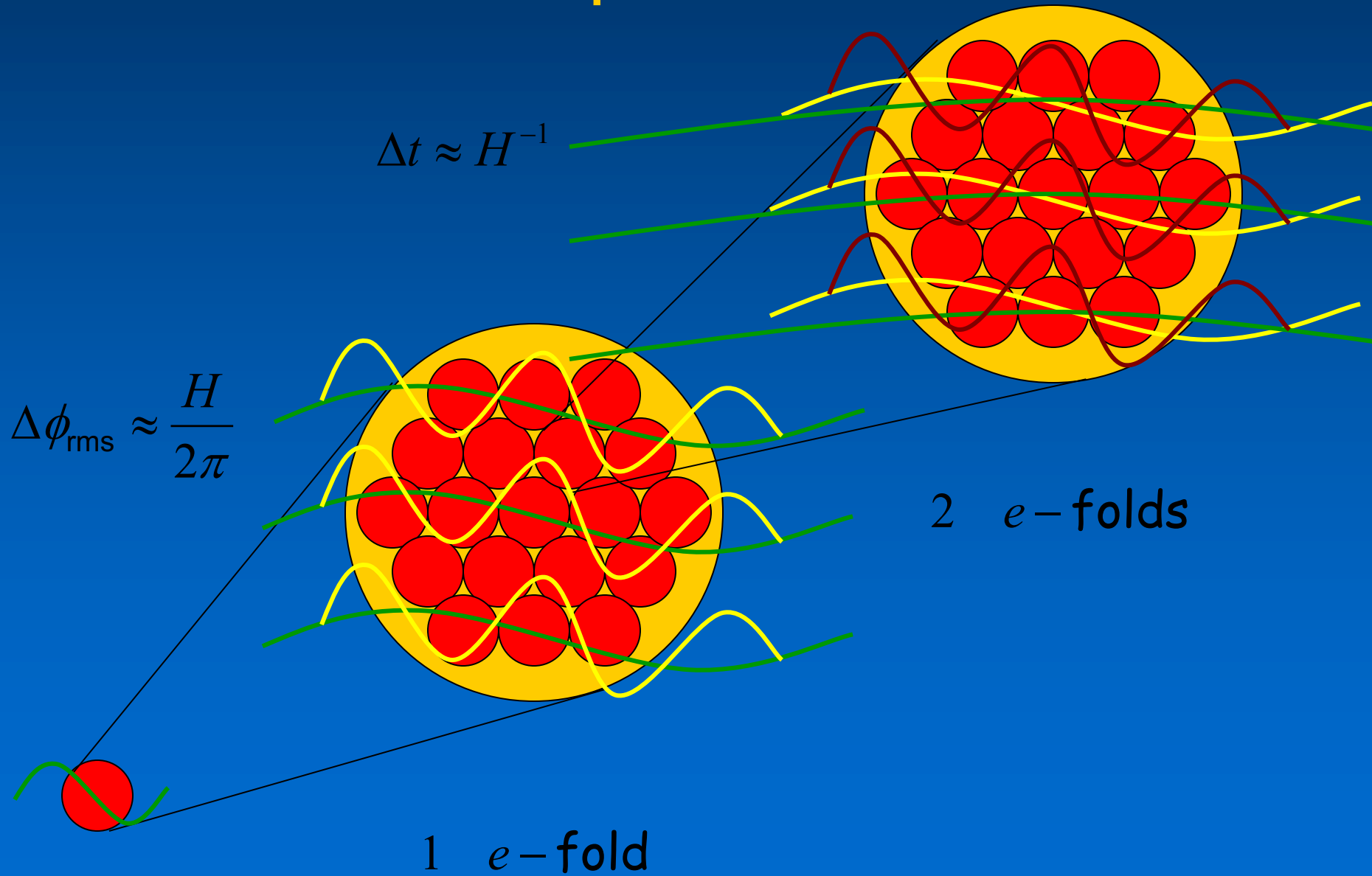
Quantum Fluctuations within the horizon

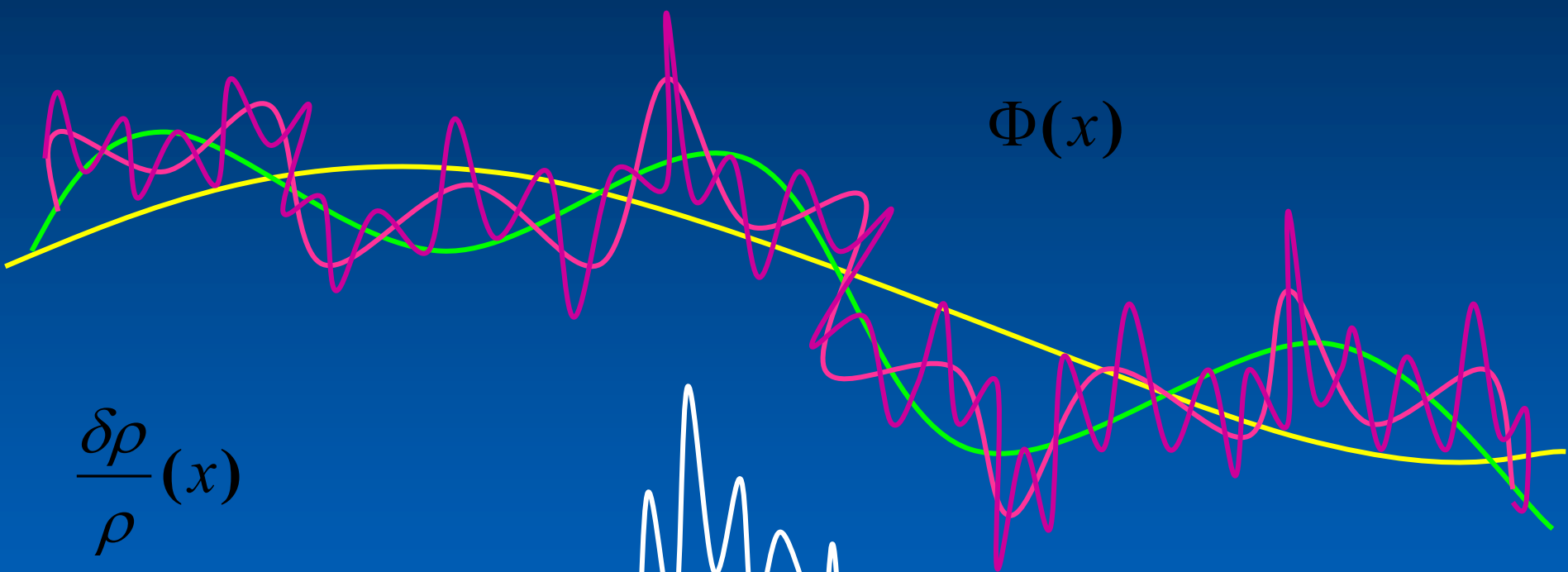


Classical Metric
perturbations

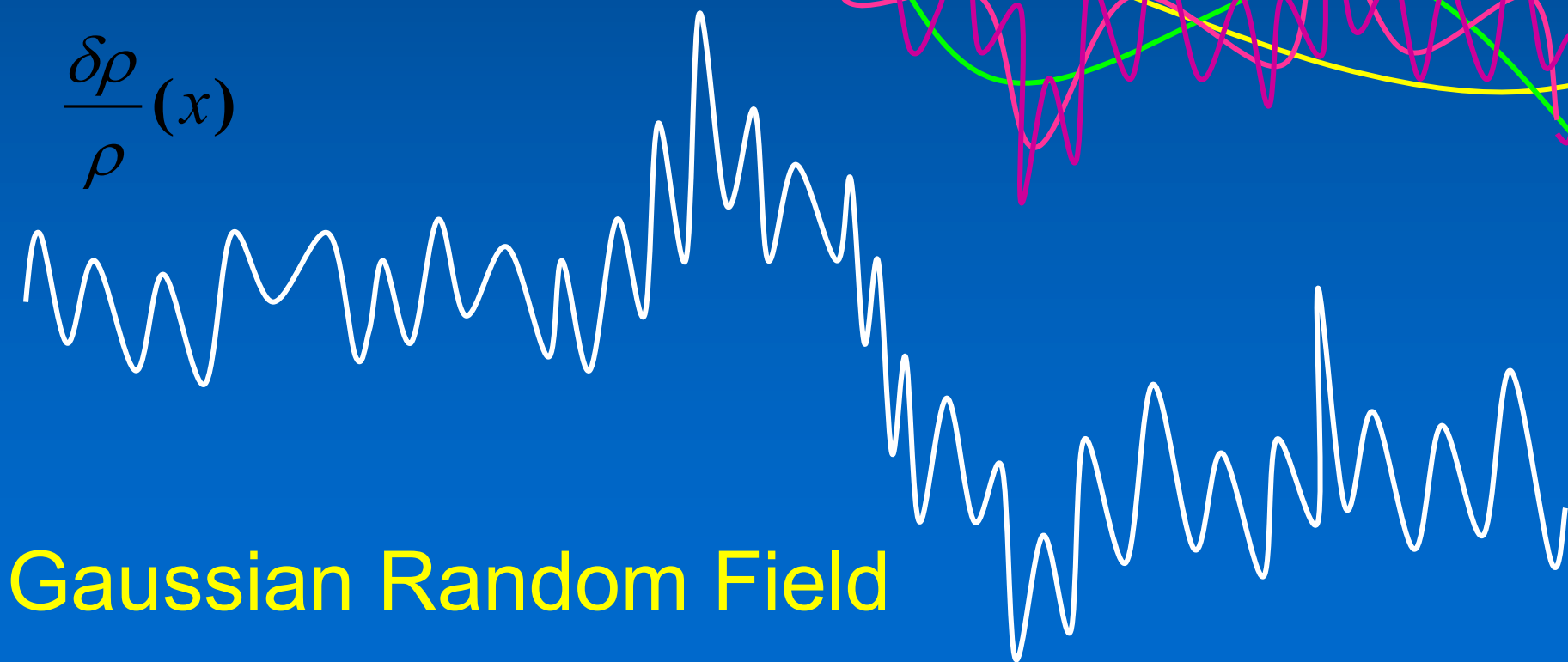


Scale Invariant Spectrum





$$\frac{\delta\rho}{\rho}(x)$$



Horizon Crossing

perturbation

horizon

causal region

Inflation

Radiation

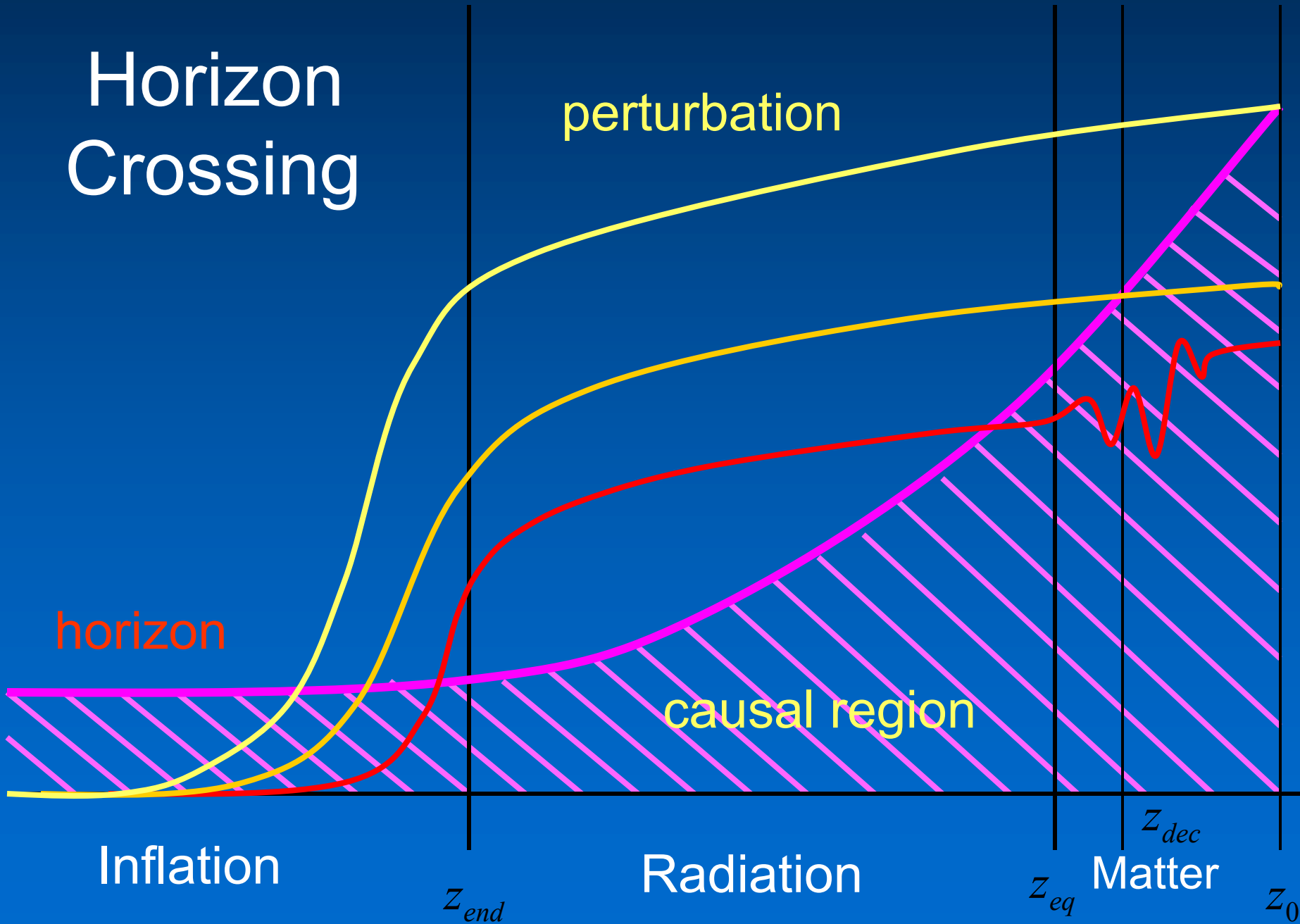
Matter

z_{end}

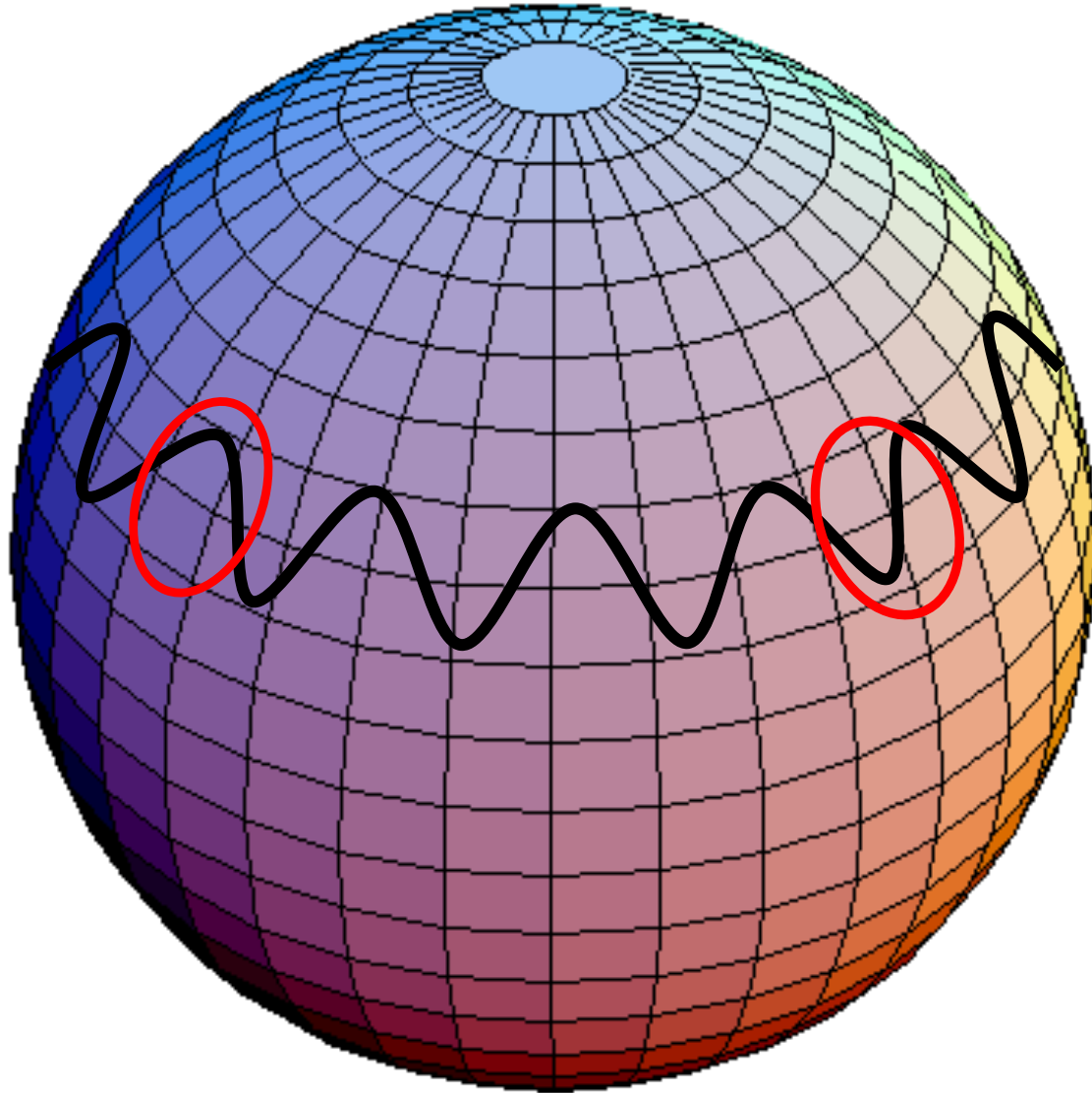
z_{eq}

z_{dec}

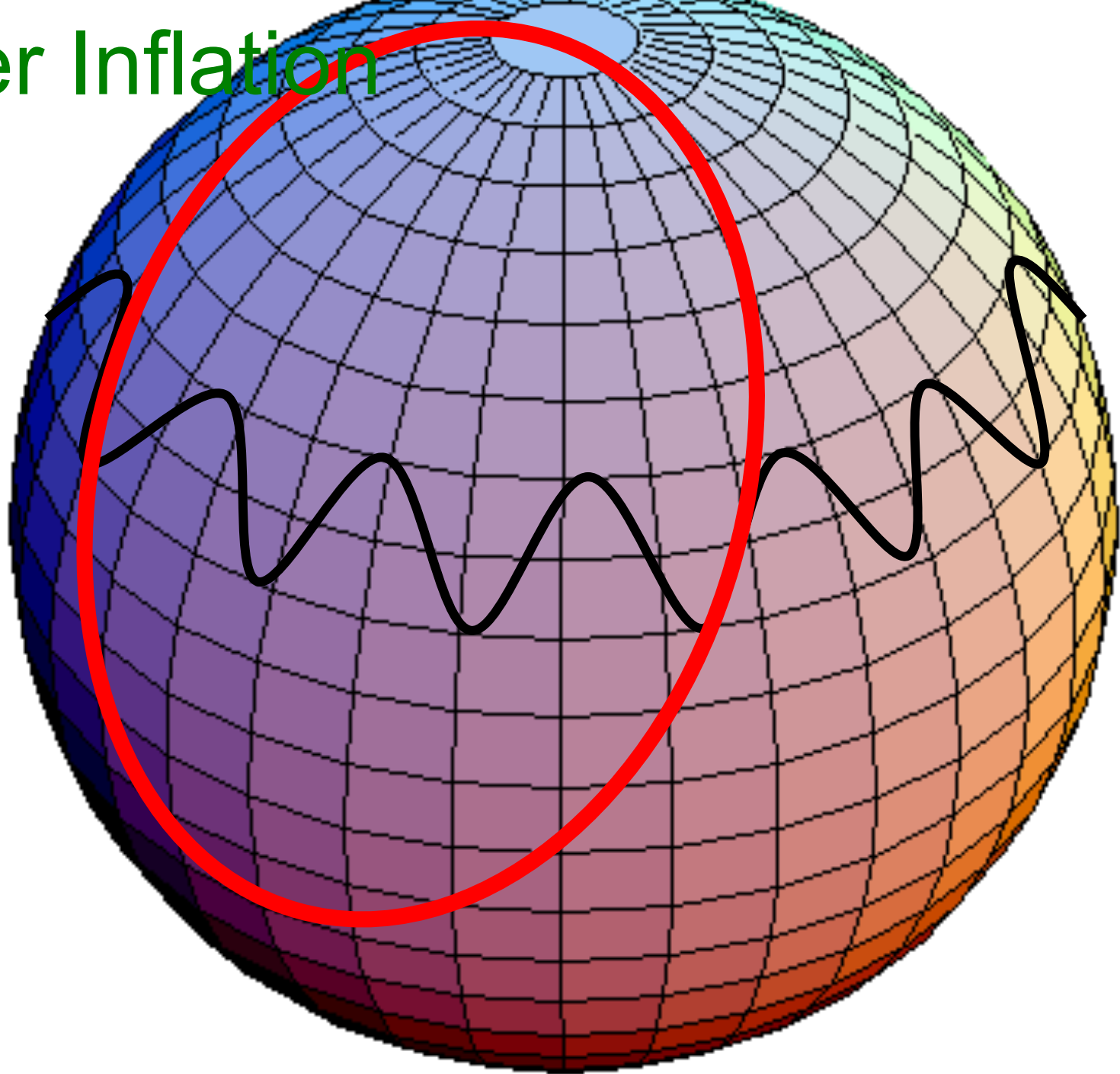
z_0



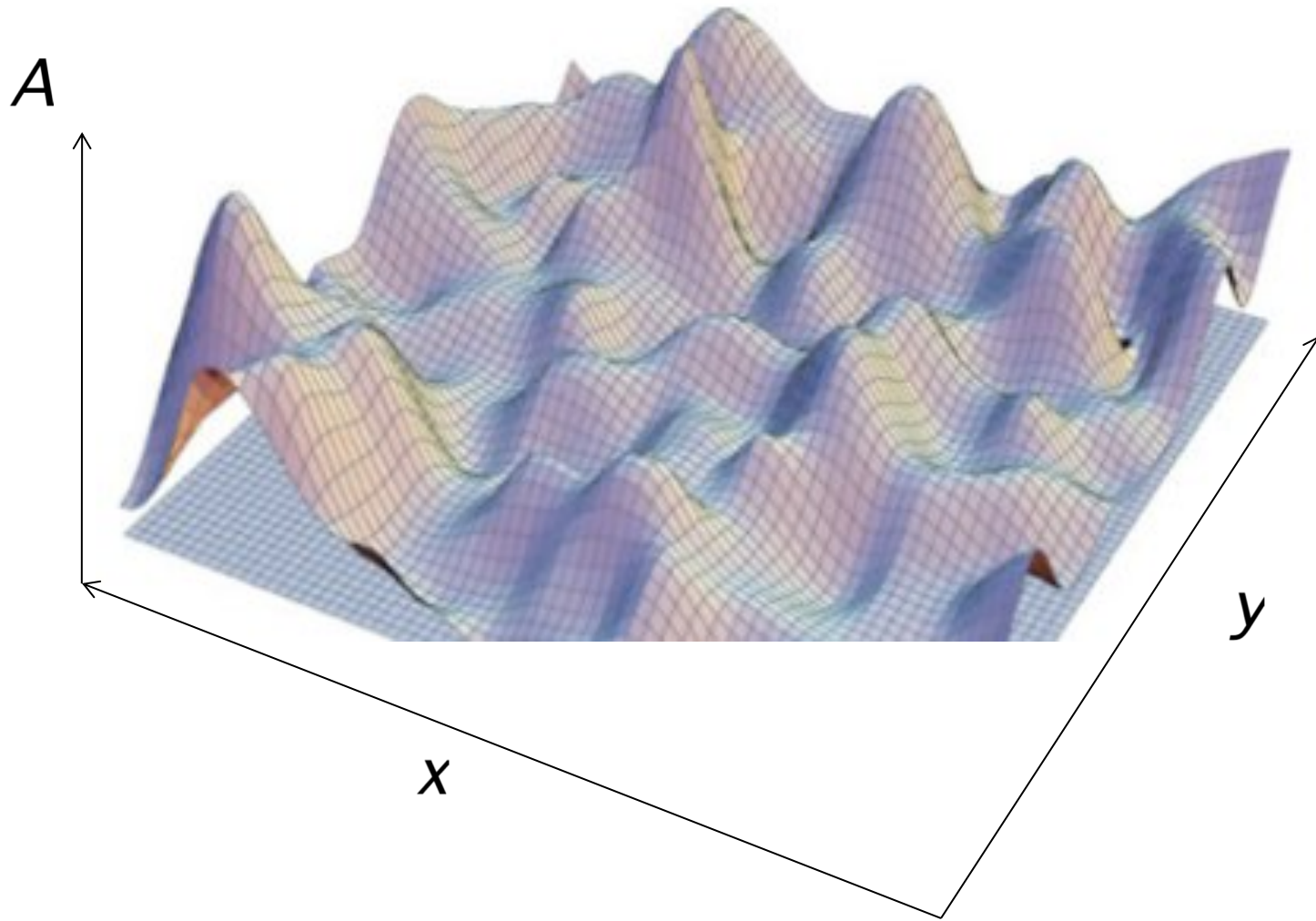
During Inflation



After Inflation



Ripples in Space



Stretched to cosmological distances

Predictions of Inflation

BIG BANG

Inflation

Quantum
fluctuations

Radiation background
anisotropies

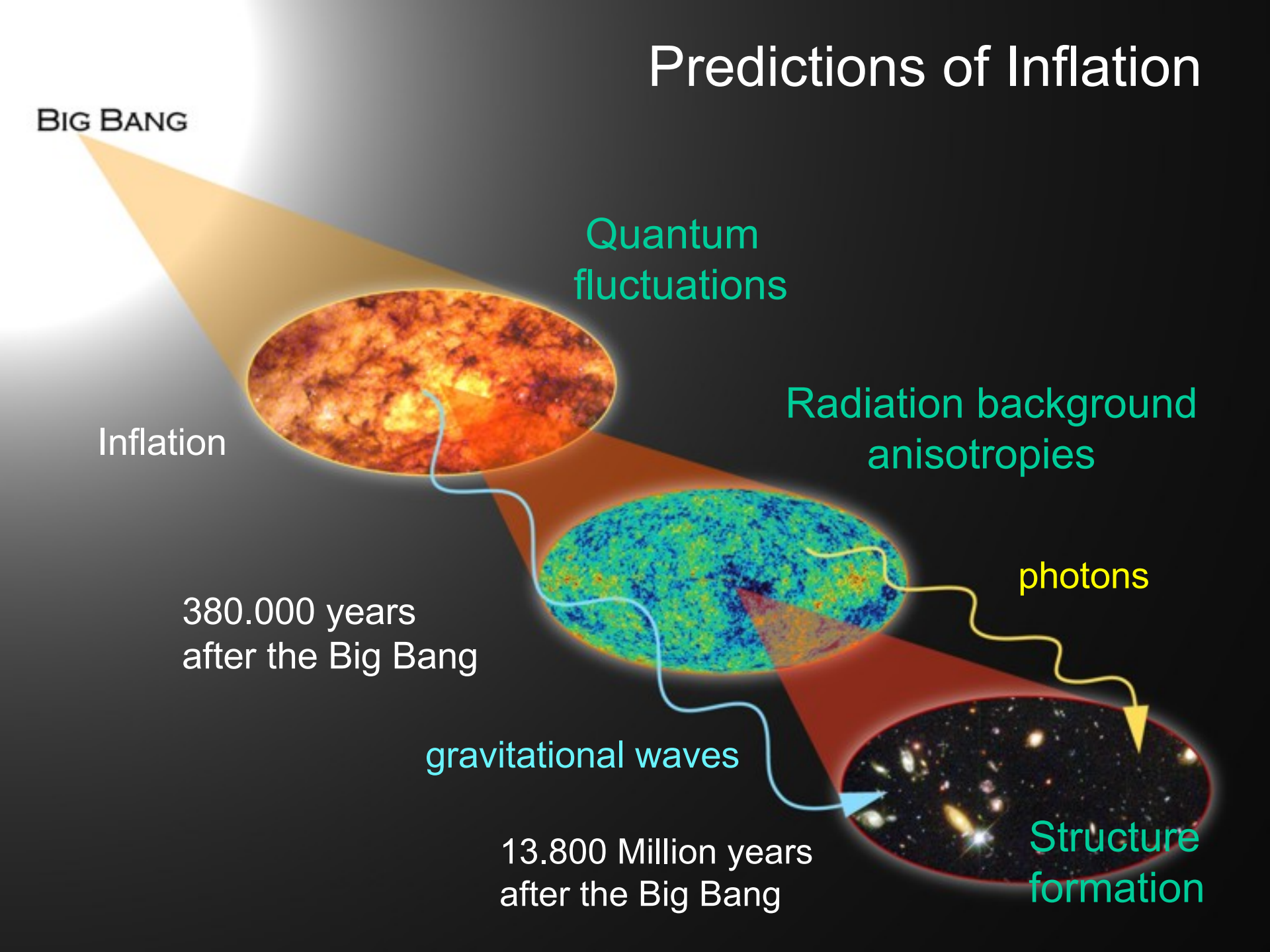
380.000 years
after the Big Bang

photons

gravitational waves

13.800 Million years
after the Big Bang

Structure
formation



Basic Inflationary Predictions

Geometry and matter:

- Homogeneity (acausal origin)
- Flat spatial sections (exp. growth)
- No appreciable topology (exp.growth)
- Origin matter & radiation (reheating)

Metric Perturbations:

- Gaussian spectrum (ground state)
- Aprox. scale invariant (slow roll cond.)
- Adiabatic density fluctuations (single fluid)
- Gravitational waves (tensor metric pert.)
- No vector perturbations (no cosmic defects)

SCALAR POWER SPECTRA

two-point correlation function in Fourier space

$$\langle 0 | \mathcal{R}_k^* \mathcal{R}_{k'} | 0 \rangle = \frac{|u_k|^2}{z^2} \delta^3(\mathbf{k} - \mathbf{k}') \equiv \frac{\mathcal{P}_{\mathcal{R}}(k)}{4\pi k^3} (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'),$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|u_k|^2}{z^2} = \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\nu} \equiv \underline{A_S^2} \left(\frac{k}{aH}\right)^{n_s-1}$$

$$\mathcal{R}_k = \zeta_k = \frac{u_k}{z} \quad \text{enter the horizon at } a = k/H$$

$$A_S^2 = \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi}\right)^2 = \frac{1}{\pi\epsilon} \frac{H^2}{M_p^2}$$

amplitude and tilt,

scale invariant

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k} = 3 - 2\nu = 2 \left(\frac{\delta - 2\epsilon}{1 - \epsilon} \right) \simeq \underline{2\eta_V - 6\epsilon_V} \ll 1$$

running of the tilt

$$\frac{dn_s}{d \ln k} = -\eta \mathcal{H} \left(2\xi + 8\epsilon^2 - 10\epsilon\delta \right) \simeq 2\xi_V + 24\epsilon_V^2 - 16\eta_V\epsilon_V$$

TENSOR POWER SPECTRA

tensor (gravitational wave) metric perturbation

$$\sum_{\lambda} \langle 0 | h_{\mathbf{k},\lambda}^* h_{\mathbf{k}',\lambda} | 0 \rangle = 4 \frac{2\kappa^2}{a^2} |v_{\mathbf{k}}|^2 \delta^3(\mathbf{k} - \mathbf{k}') \equiv \frac{\mathcal{P}_g(k)}{4\pi k^3} (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')$$

$$\mathcal{P}_g(k) = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\mu} \equiv \underline{A_T^2} \left(\frac{k}{aH}\right)^{n_T}$$

$$h_{\mathbf{k}} = \kappa\sqrt{2} \frac{v_{\mathbf{k}}}{a} \quad \text{enter the horizon at } a = k/H$$

$$A_T^2 = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 = \frac{16}{\pi} \frac{H^2}{M_P^2}$$

amplitude and tilt,

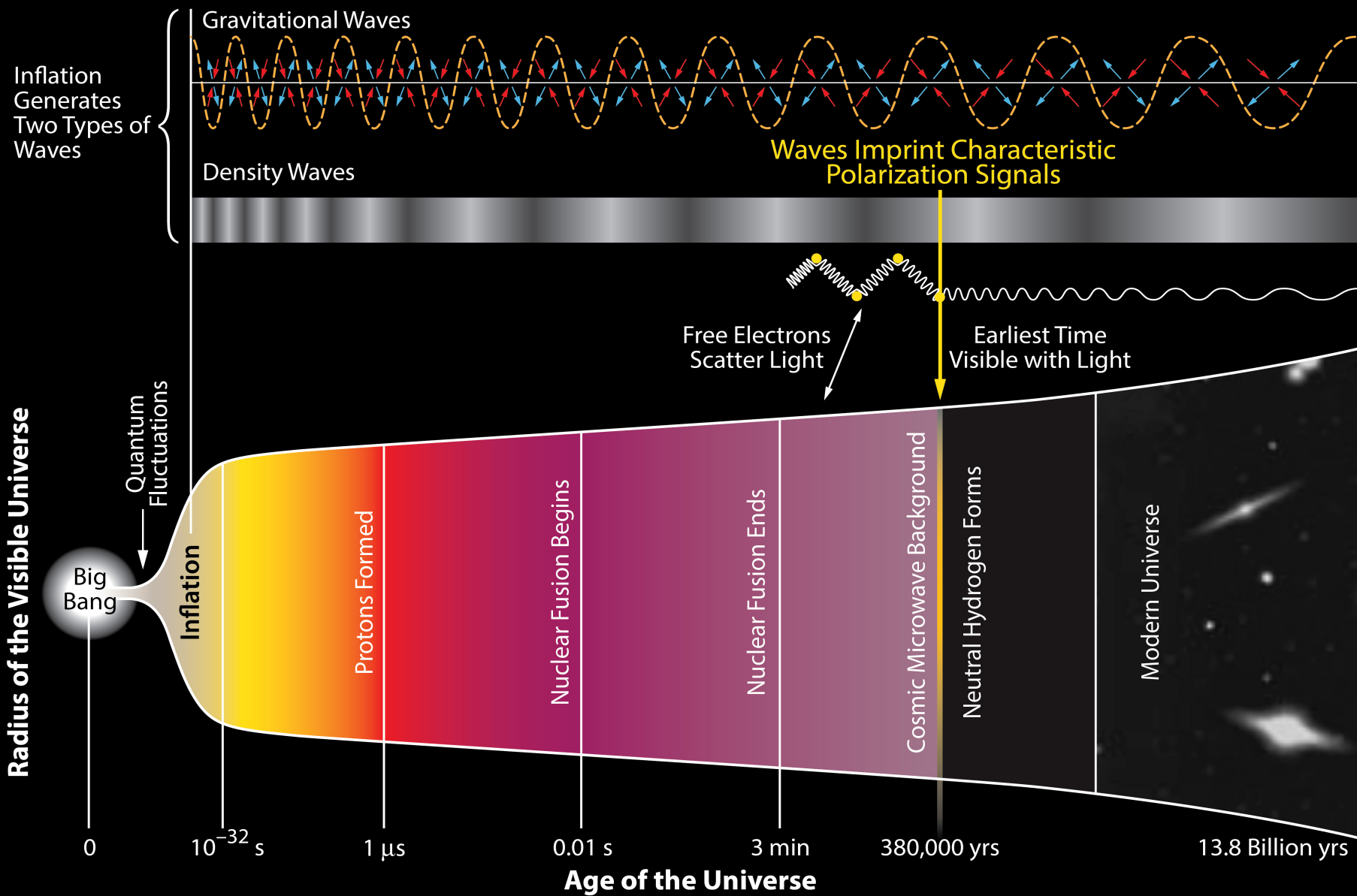
scale invariant

$$n_T \equiv \frac{d \ln \mathcal{P}_g(k)}{d \ln k} = 3 - 2\mu = \frac{-2\epsilon}{1 - \epsilon} \simeq \underline{-2\epsilon_V} < 0 \quad \ll 1$$

running of the tilt

$$\frac{dn_T}{d \ln k} = -\eta \mathcal{H} (4\epsilon^2 - 4\epsilon\delta) \simeq 8\epsilon_V^2 - 4\eta_V \epsilon_V$$

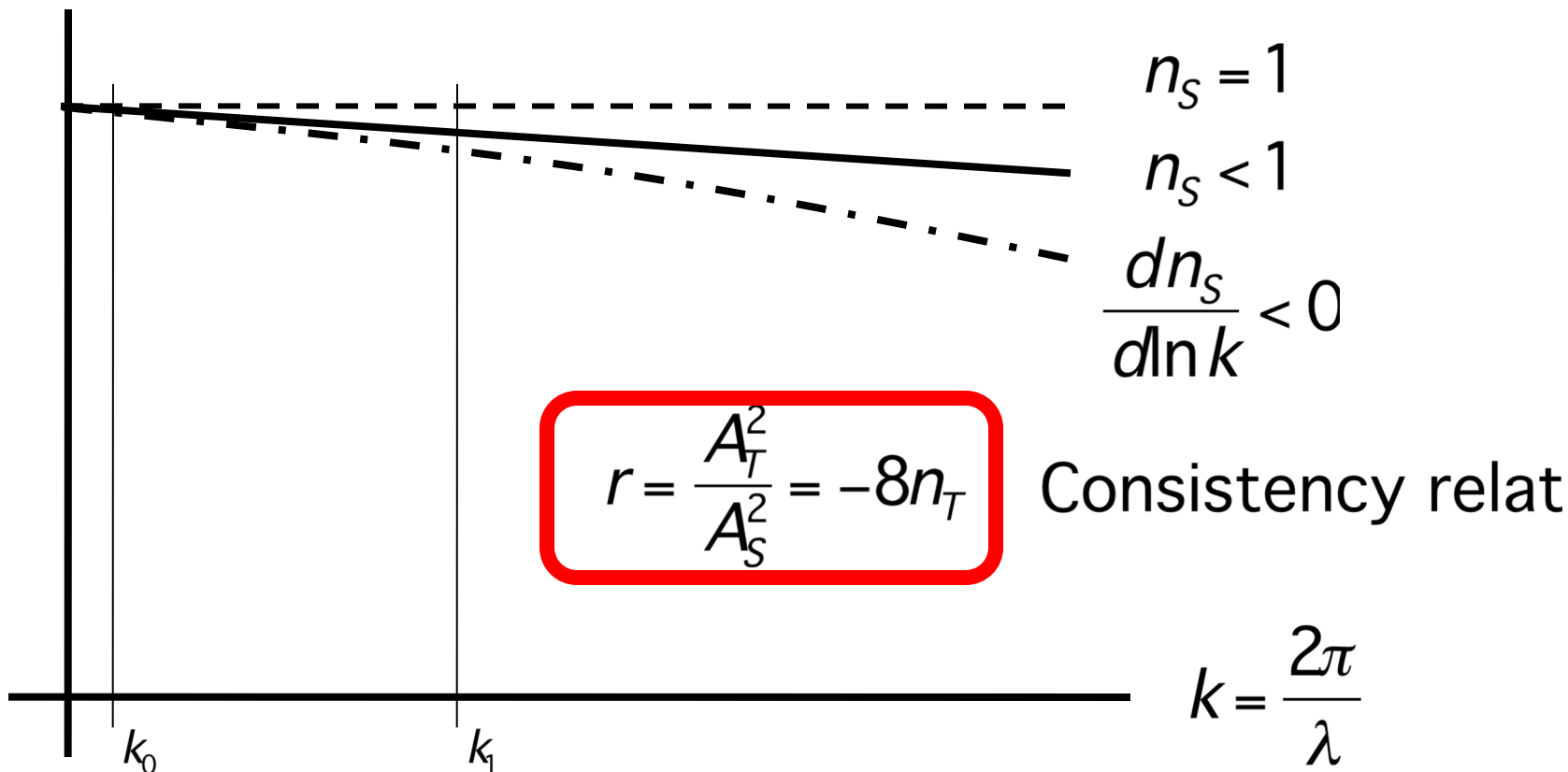
History of the Universe



two types {

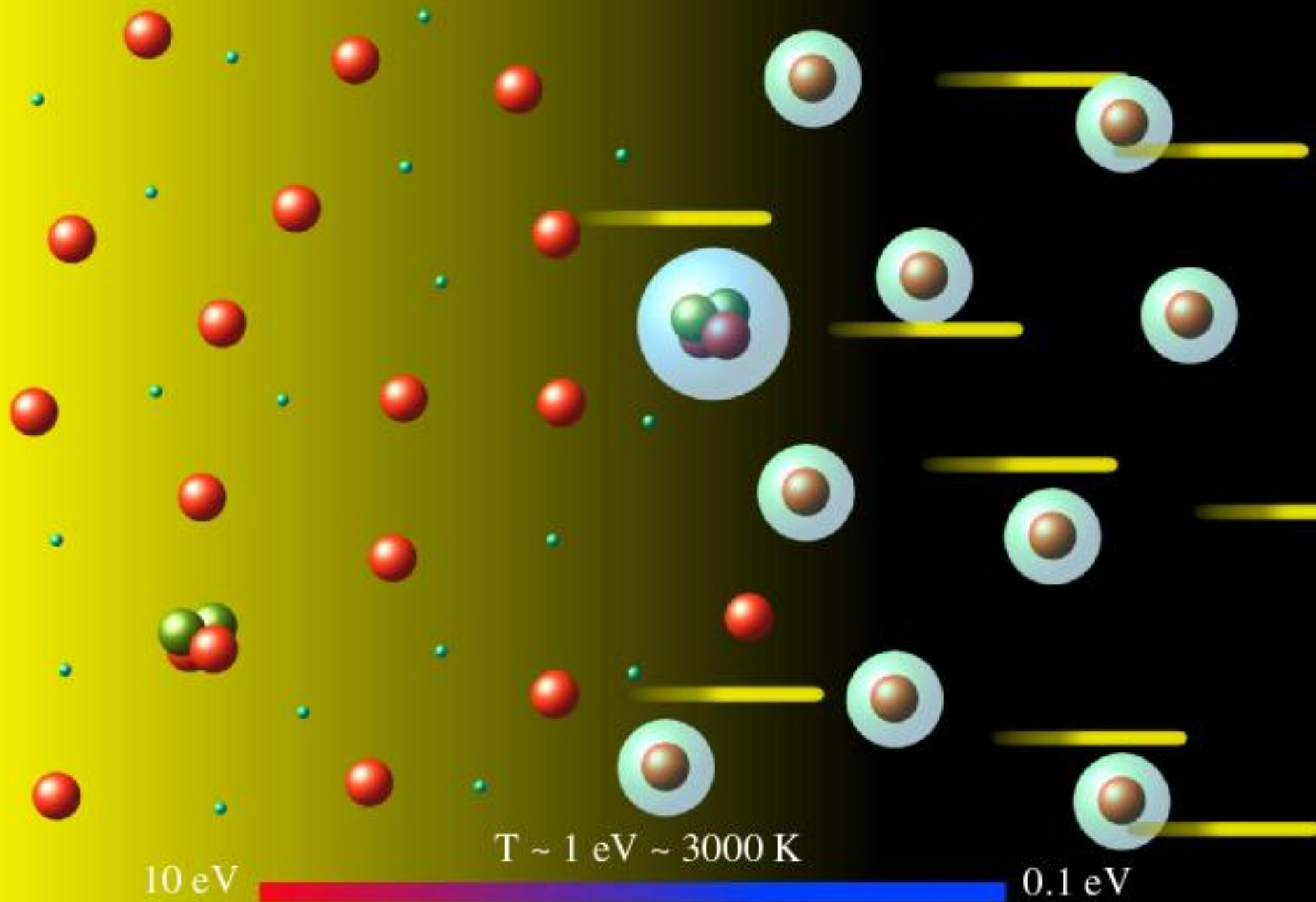
- Scalar (density) spectrum $P_S(k) = A_S^2(k_0) \left(\frac{k}{k_0}\right)^{n_S-1}$
- Tensor (GW) spectrum $P_T(k) = A_T^2(k_0) \left(\frac{k}{k_0}\right)^{n_T}$

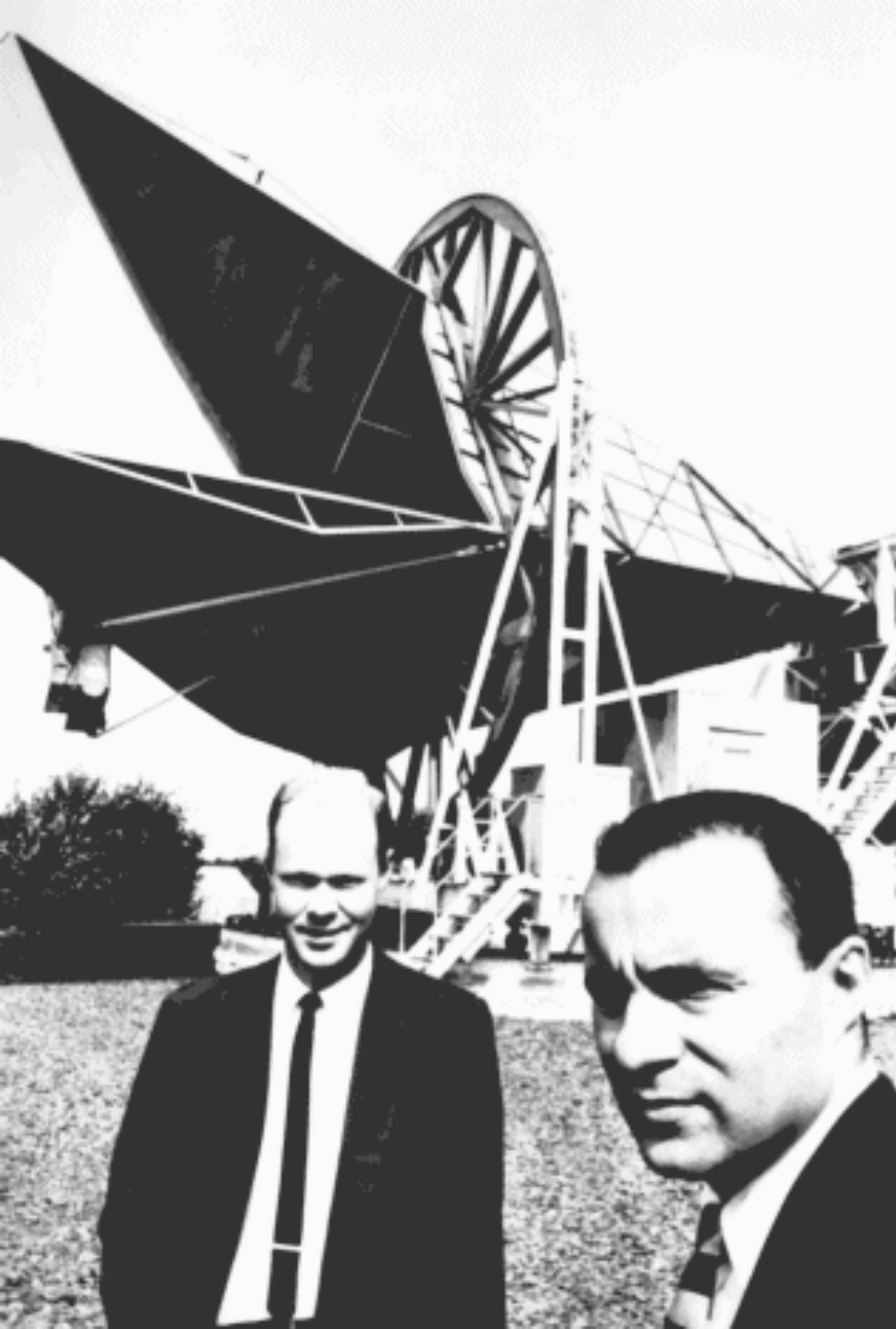
$P(k)$



Cosmic Microwave Background

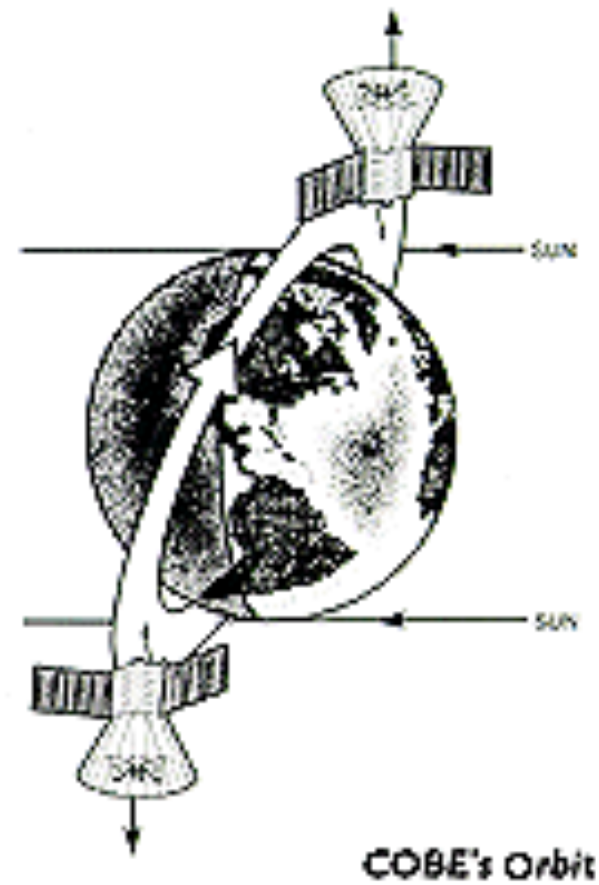
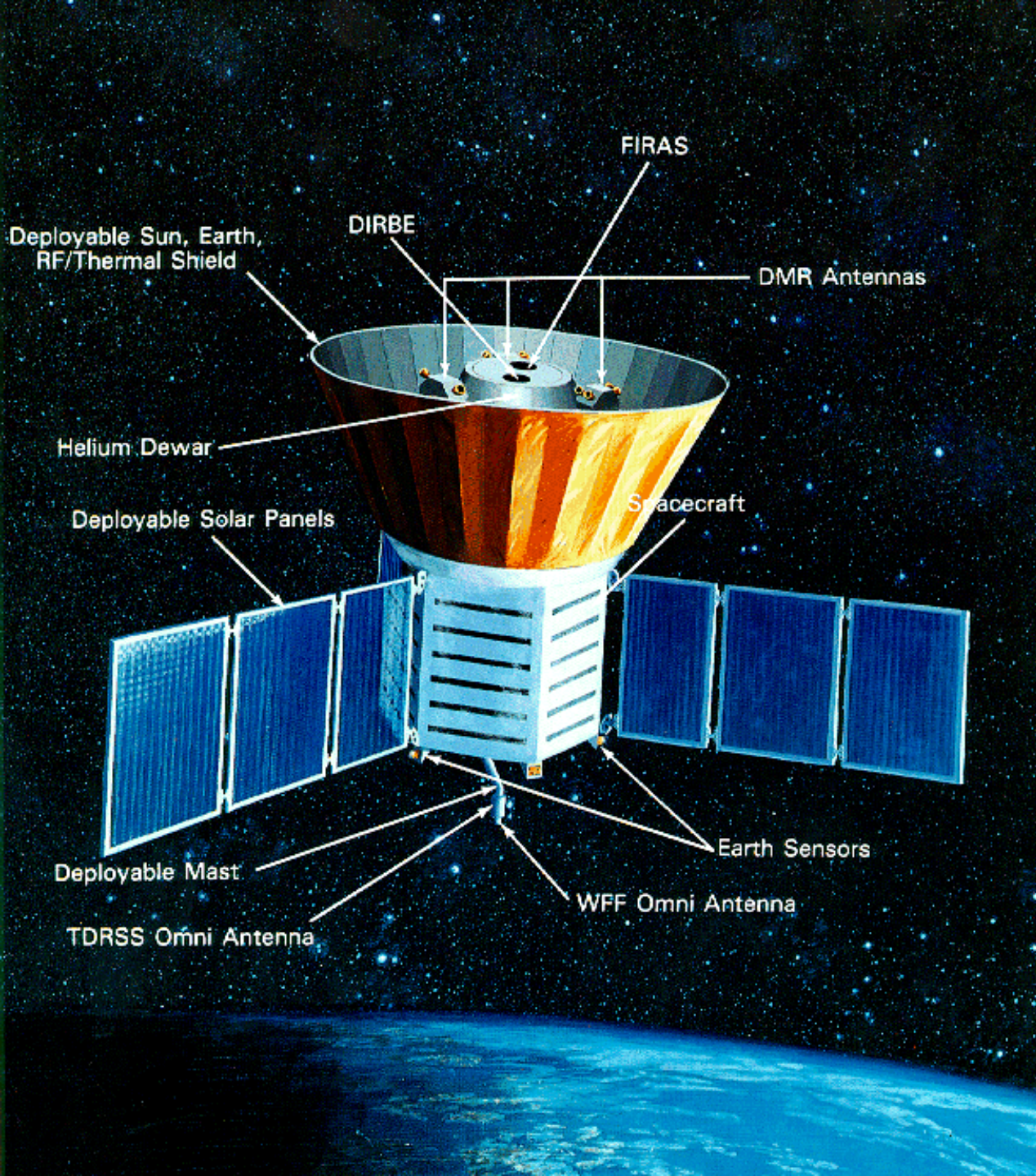
Recombination



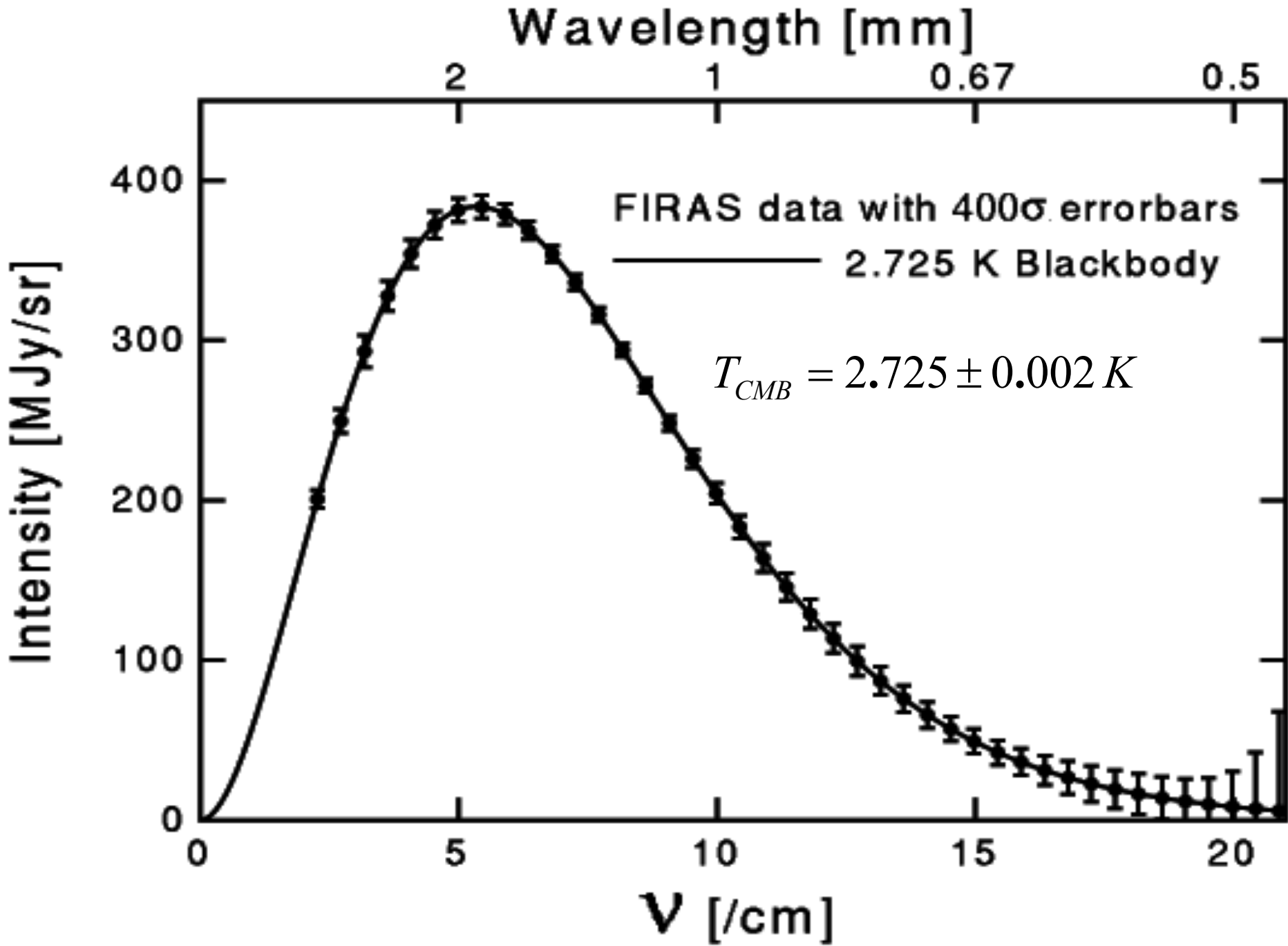


Discovery of CMB

Arno Penzias
Robert Wilson
(1965)



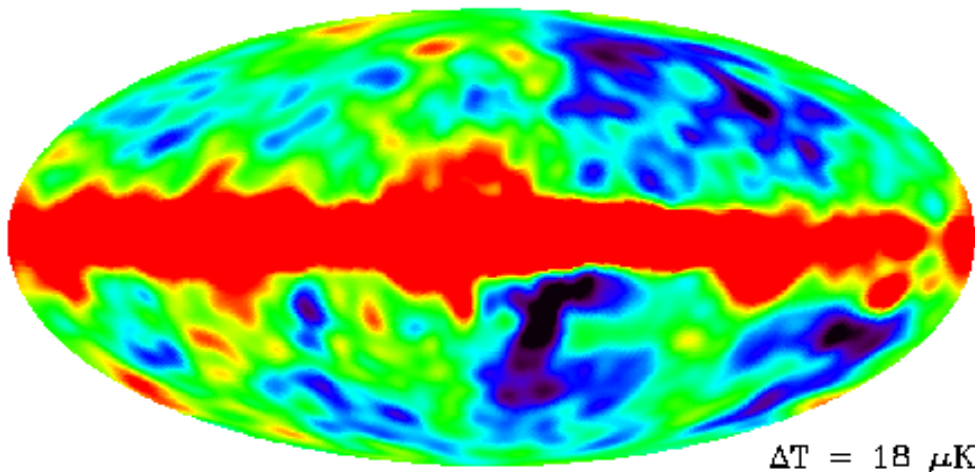
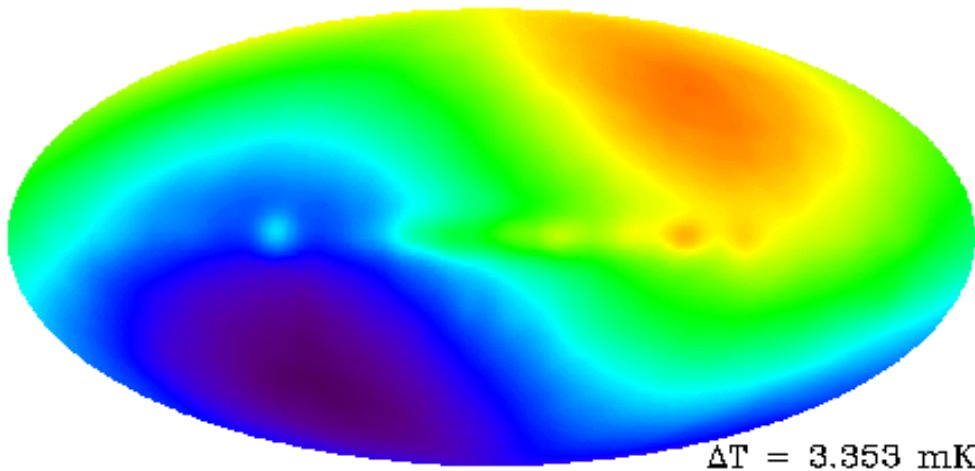
COBE
(1989-1992)



COBE 4-year
Measurements
(1992-1996)

First
Measurements
Temperature
Anisotropies
(1992)

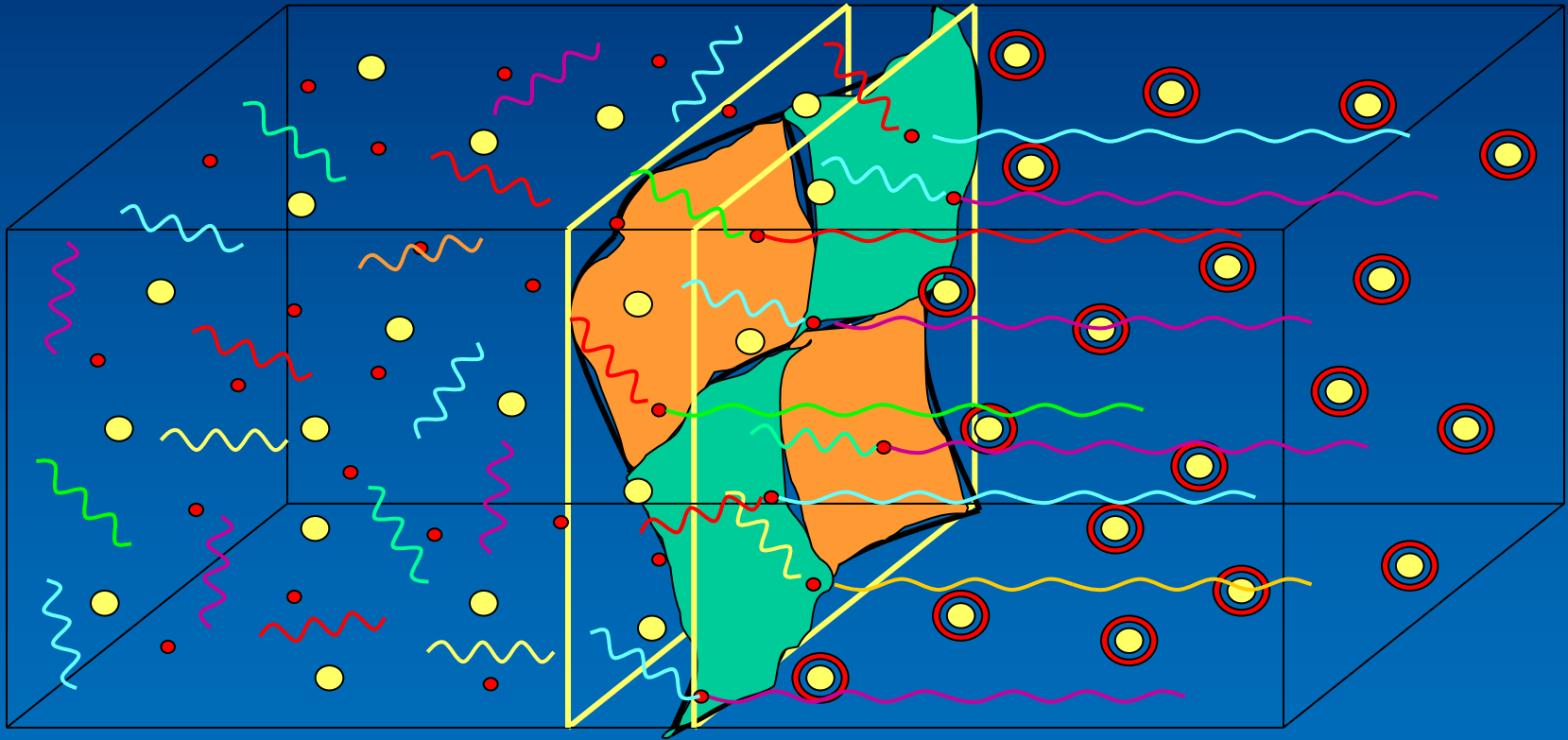
$$\frac{\Delta T}{T_0} \approx 10^{-5}$$



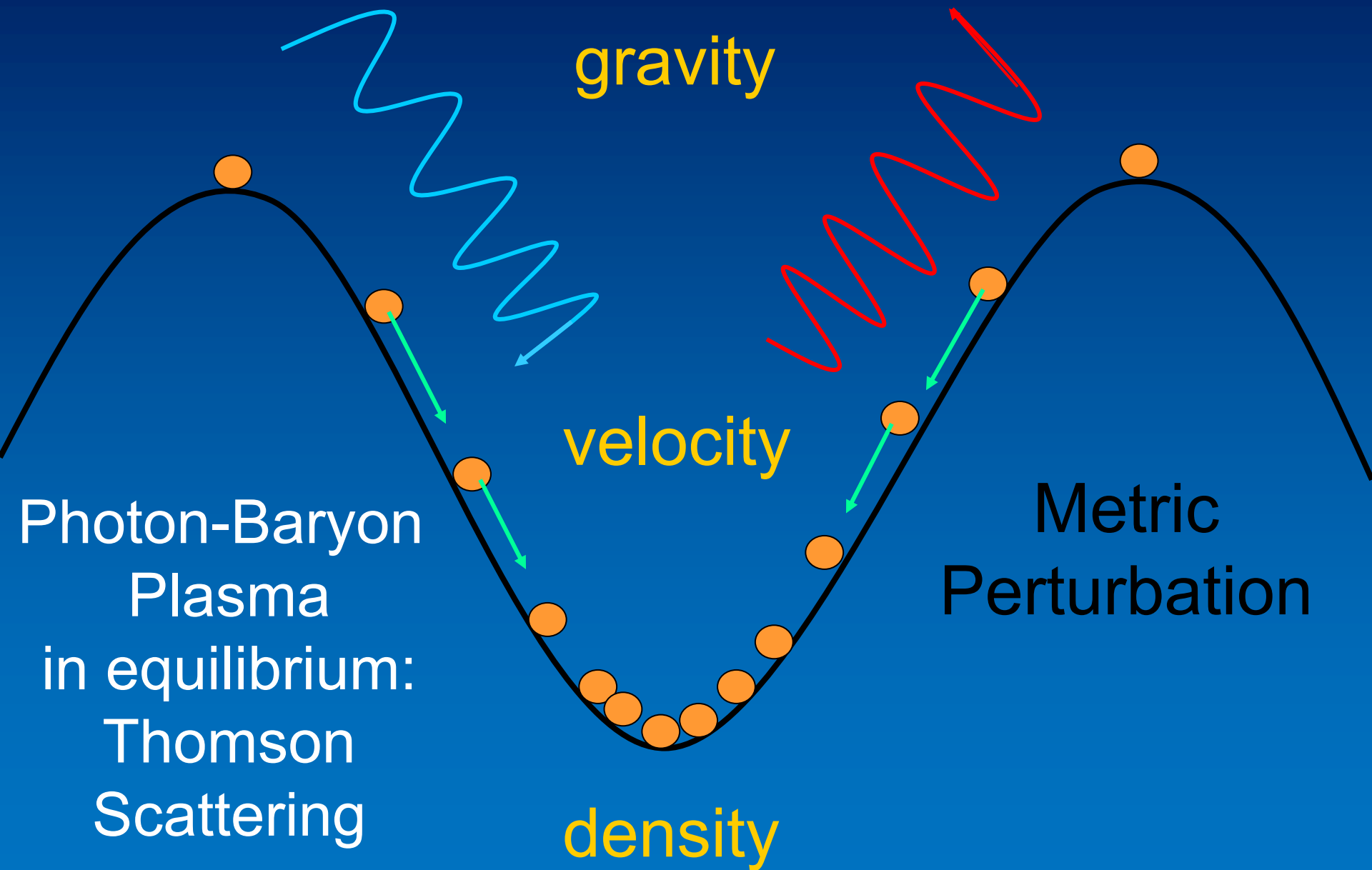
CMB

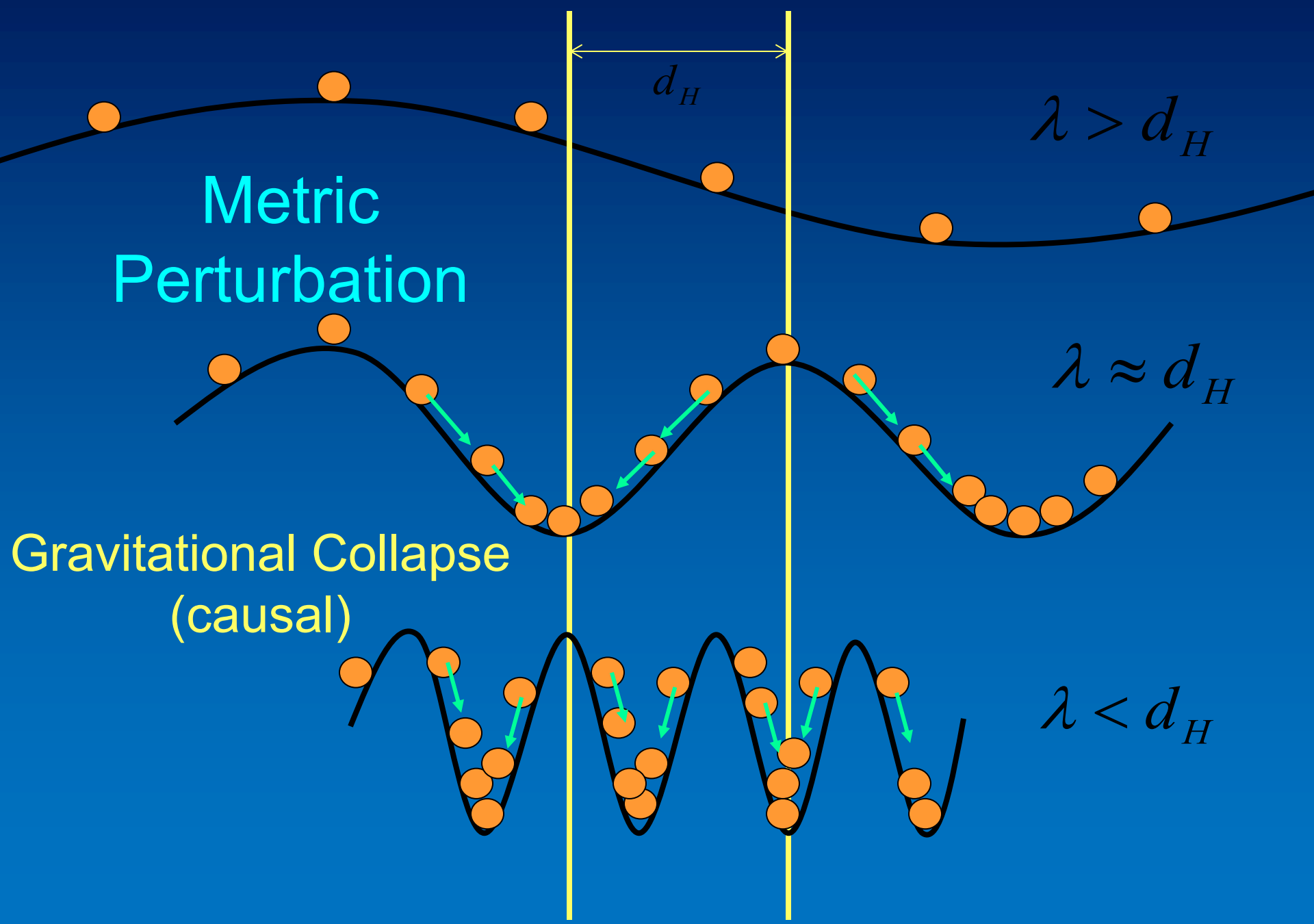
**Temperature
Anisotropies**

The microwave background is a snapshot
of the last scattering surface

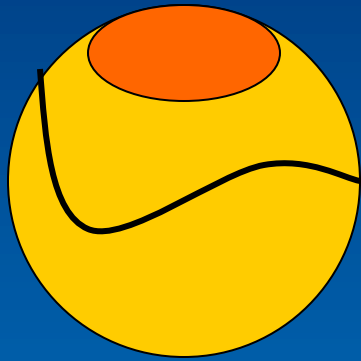


The anisotropies reflect the perturbations
in the surface of last scattering



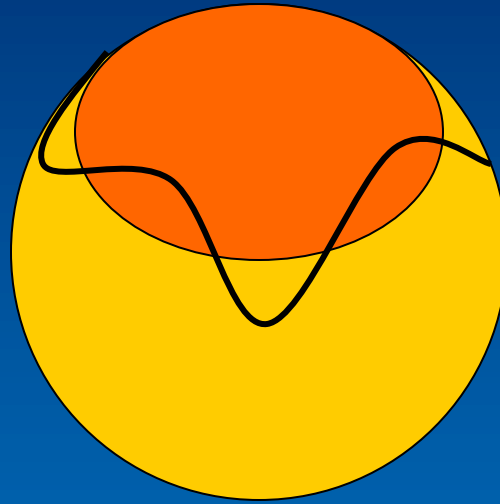


After Inflation



$$\lambda > d_H$$

Outside
Horizon



$$\lambda \approx d_H$$

Enters
Horizon



$$\lambda < d_H$$

Inside
Horizon

Horizon Crossing

perturbation

horizon

causal region

Inflation

Radiation

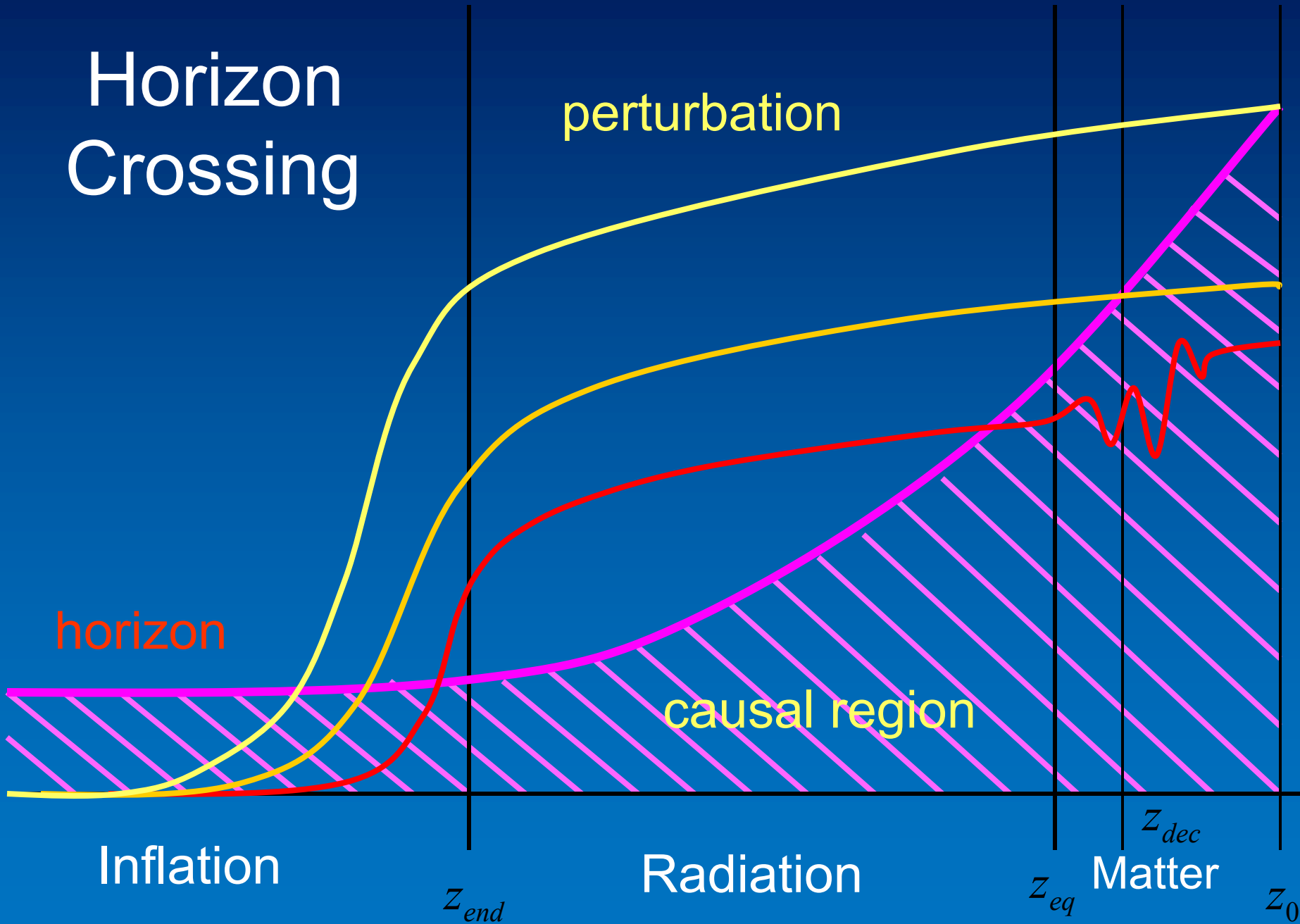
Matter

z_{end}

z_{eq}

z_{dec}

z_0



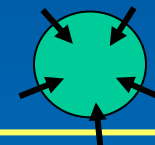
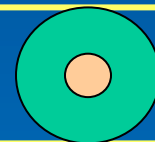
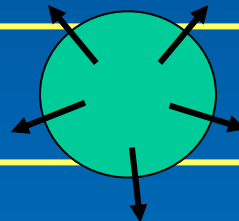
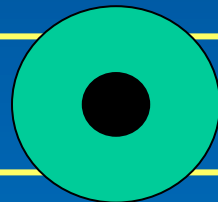
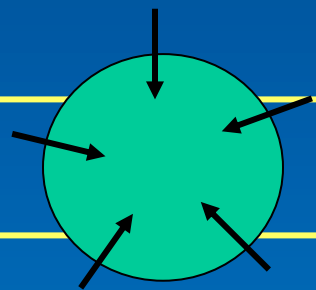
z **Gravitational collapse and radiation pressure**



Acoustic oscillations

Δz

SLS



z_{dec}

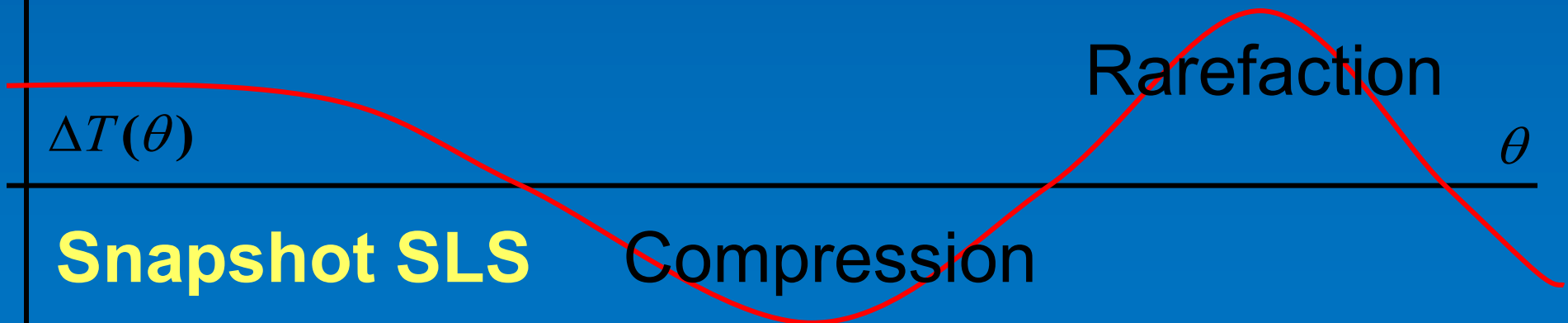
Rarefaction

$\Delta T(\theta)$

θ

Snapshot SLS

Compression



CMB TEMPERATURE ANISOTROPIES

gravity + density + velocity

$$\frac{\delta T}{T}(\mathbf{r}) = \Phi(\mathbf{r}, t_{\text{dec}}) + 2 \int_{t_{\text{dec}}}^{t_0} \dot{\Phi}(\mathbf{r}, t) dt + \frac{1}{3} \frac{\delta \rho}{\rho}(\mathbf{r}, t_{\text{dec}}) - \frac{\mathbf{r} \cdot \mathbf{v}}{c}$$

The Sachs-Wolfe effect on large angular scales

$\delta \rho / \rho = -2\Phi$ (for adiabatic perturbations)

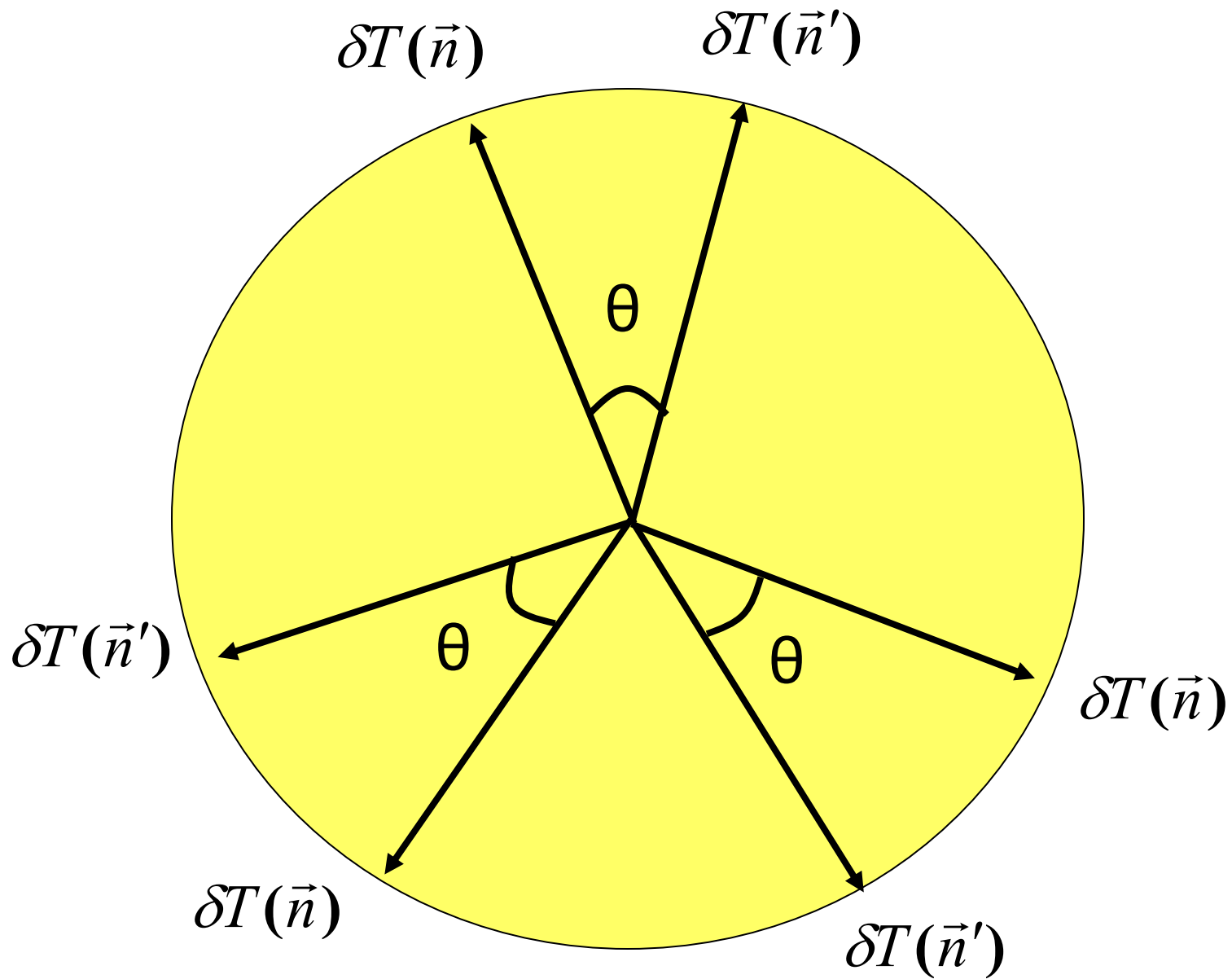
$$\frac{\delta T}{T}(\theta, \phi) = \frac{1}{3} \Phi(\eta_{\text{LS}}) Q(\eta_0, \theta, \phi) + 2 \int_{\eta_{\text{LS}}}^{\eta_0} dr \Phi'(\eta_0 - r) Q(r, \theta, \phi)$$

$Q(\mathbf{x})$ eigenfunctions of the Laplacian $\Phi(\eta, \mathbf{x}) \equiv \Phi(\eta) Q(\mathbf{x})$

$$\nabla^2 Q_{klm}(r, \theta, \phi) = -k^2 Q_{klm}(r, \theta, \phi)$$

$$Q_{klm}(r, \theta, \phi) = \Pi_{kl}(r) Y_{lm}(\theta, \phi) \quad \Pi_{kl}(r) = \sqrt{\frac{2}{\pi}} k j_l(kr)$$

$$\Phi'' + 3\mathcal{H} \Phi' + a^2 \Lambda \Phi - 2K \Phi = 0$$



Angular power spectrum

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$$

$$C(\theta) = \left\langle \delta T^*(\vec{n}) \delta T(\vec{n}') \right\rangle_{\vec{n} \cdot \vec{n}' = \cos \theta} = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos \theta)$$

$$C_l^S = \frac{4\pi}{25} \int_0^{\infty} \frac{dk}{k} P_S(k) j_l^2(kr) \quad P_S(k) = A_S^2 \left(\frac{k}{k_*} \right)^{n-1}$$

Sachs-Wolfe plateau $\frac{l(l+1)C_l^S}{2\pi} = \frac{A_S^2}{25} = \text{const}$

Angular Power Spectrum

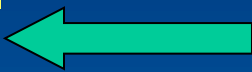
Superhorizon

Subhorizon

Gravitational
Potential

Acoustic
Harmonic
Oscillations

Compression



Rarefaction

Compression

Rarefaction

Compression

$$\delta T \approx [l(l+1) C_l]^{1/2}$$

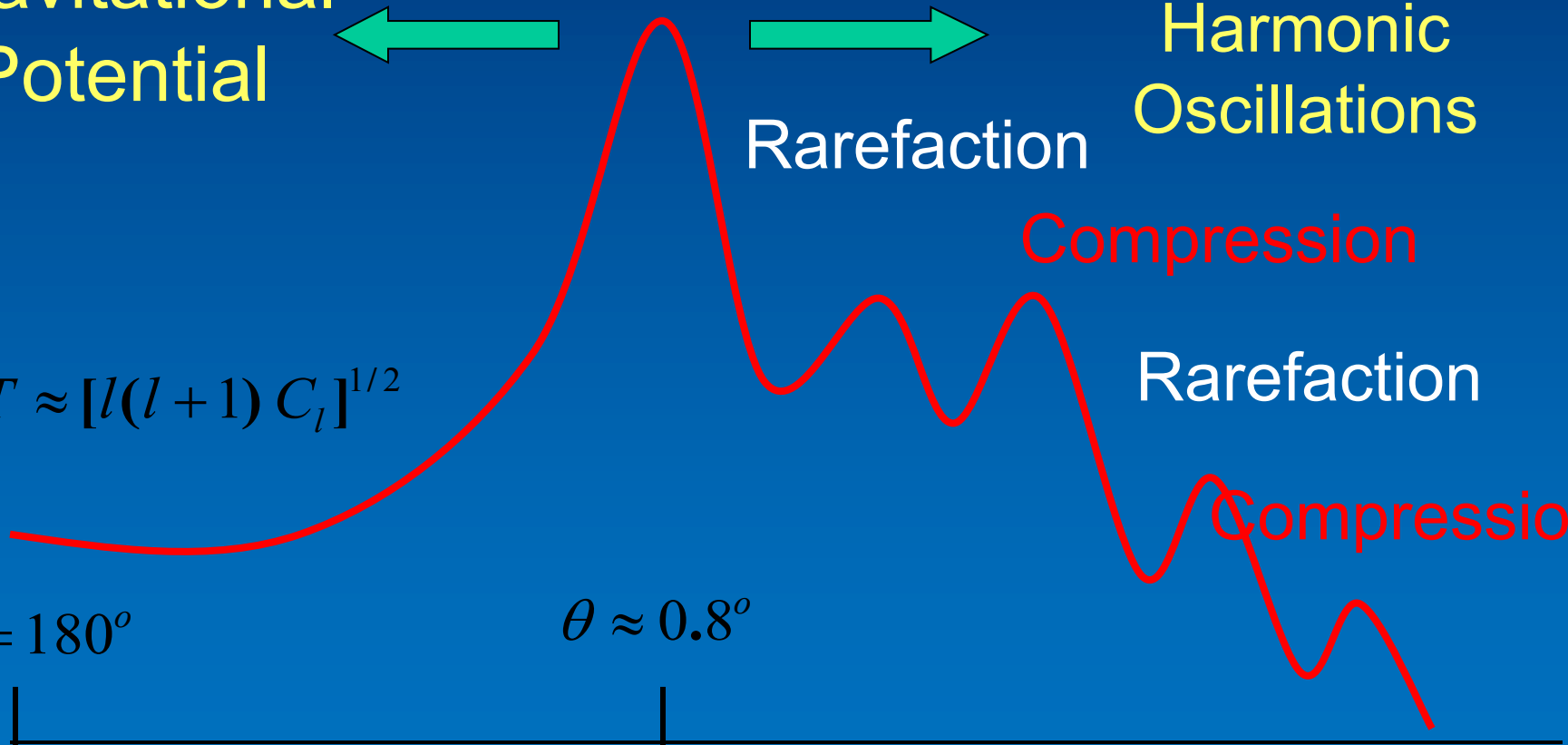
$\theta = 180^\circ$

$\theta \approx 0.8^\circ$

$l \approx \theta^{-1}$

$l = 2$

$l \approx 220$



SACHS-WOLFE PLATEAU

$$\frac{\delta T}{T}(\theta, \phi) = \frac{1}{3} \Phi(\eta_{\text{LS}}) Q = \frac{1}{5} \mathcal{R} Q(\eta_0, \theta, \phi) \equiv \sum_{l=2}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi)$$

two-point correlation function

$$C_l = \langle |a_{lm}|^2 \rangle$$

$$C(\theta) = \left\langle \frac{\delta T^*}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle_{\mathbf{n} \cdot \mathbf{n}' = \cos \theta} = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos \theta)$$

$$C_l^{(S)} = \frac{4\pi}{25} \int_0^{\infty} \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k) j_l^2(k\eta_0) \quad \underline{\mathcal{P}_{\mathcal{R}}(k) = A_S^2 (k\eta_0)^{n-1}}$$

$$C_l^{(S)} = \frac{2\pi}{25} A_S^2 \frac{\Gamma[\frac{3}{2}] \Gamma[1 - \frac{n-1}{2}] \Gamma[l + \frac{n-1}{2}]}{\Gamma[\frac{3}{2} - \frac{n-1}{2}] \Gamma[l + 2 - \frac{n-1}{2}]}$$

$$\frac{l(l+1) C_l^{(S)}}{2\pi} = \frac{A_S^2}{25} = \text{constant, for } n = 1$$

SACHS-WOLFE PLATEAU

gauge-invariant tensor perturbation

$$h_k'' + 2\mathcal{H} h_k' + (k^2 + 2K) h_k = 0 \quad \Rightarrow \quad h_k(\eta) = 3h j_1(k\eta)/k\eta$$

$$\frac{\delta T}{T}(\theta, \phi) = \int_{\eta_{\text{LS}}}^{\eta_0} dr h'(\eta_0 - r) Q_{rr}(r, \theta, \phi)$$

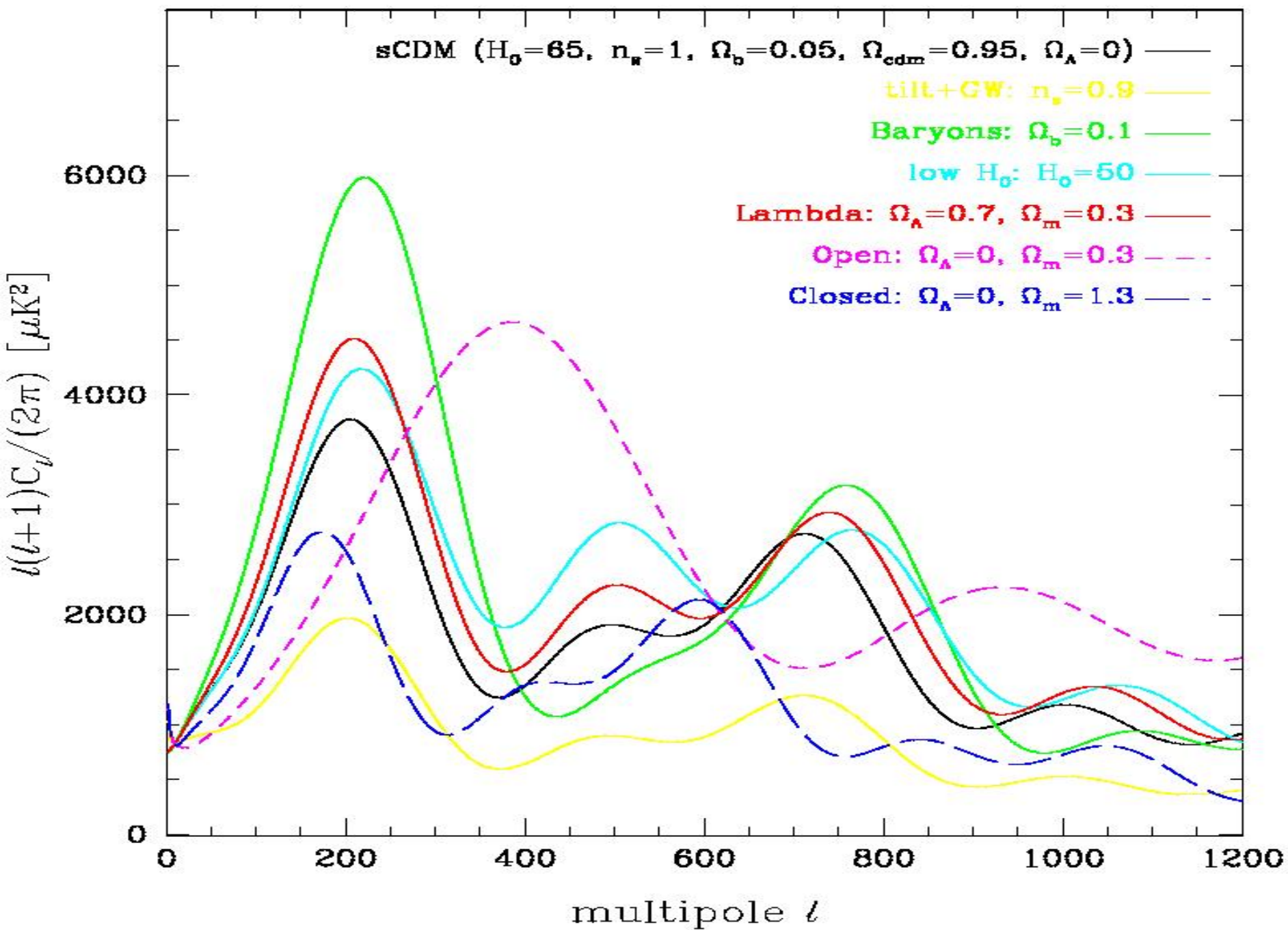
Q_{rr} is rr -component of tensor harmonic along line of sight

$$Q_{kl}^{rr}(r) = \left[\frac{(l-1)l(l+1)(l+2)}{\pi k^2} \right]^{1/2} \frac{j_l(kr)}{r^2} \quad I_{kl} = \int_0^{x_0} dx \frac{j_2(x_0 - x)j_l(x)}{(x_0 - x)x^2}$$

$$C_l^{(T)} = \frac{9\pi}{4} (l-1)l(l+1)(l+2) \int_0^\infty \frac{dk}{k} \mathcal{P}_g(k) I_{kl}^2 \quad \underline{\mathcal{P}_g(k) = A_T^2 (k\eta_0)^{n_T}}$$

$$l(l+1) C_l^{(T)} = \frac{\pi}{36} \left(1 + \frac{48\pi^2}{385} \right) A_T^2 B_l \quad \text{for } n_T = 0$$

$$B_l = (1.1184, 0.8789, \dots, 1.00) \quad \text{for } l = 2, 3, \dots, 30$$



Degeneracies in the determination of parameters

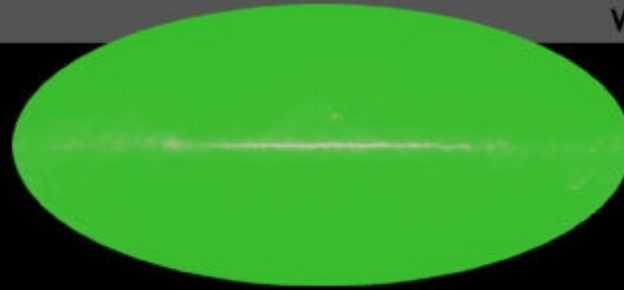


It is necessary to make a multiparameter fit with the largest possible data set

1965



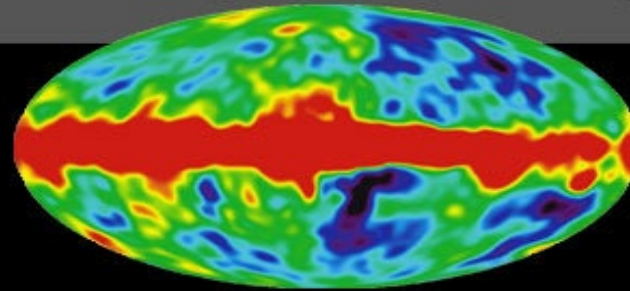
Penzias and
Wilson



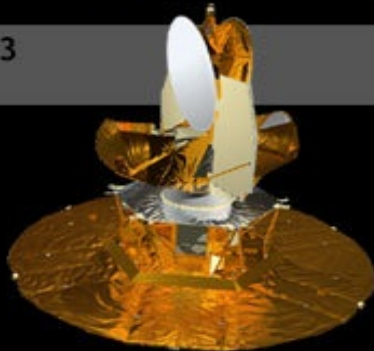
1992



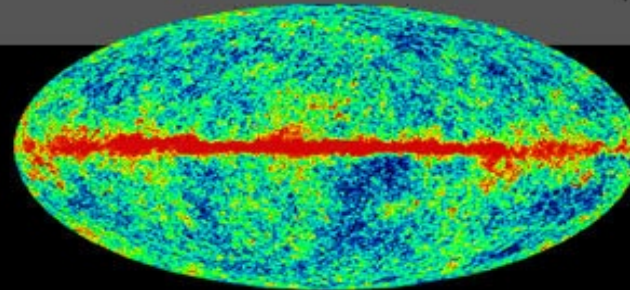
COBE



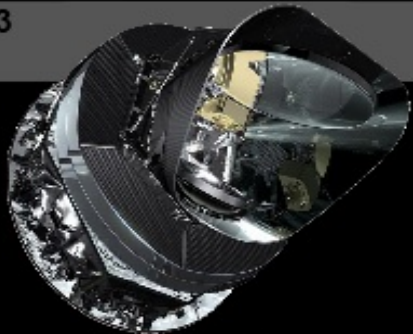
2003



WMAP

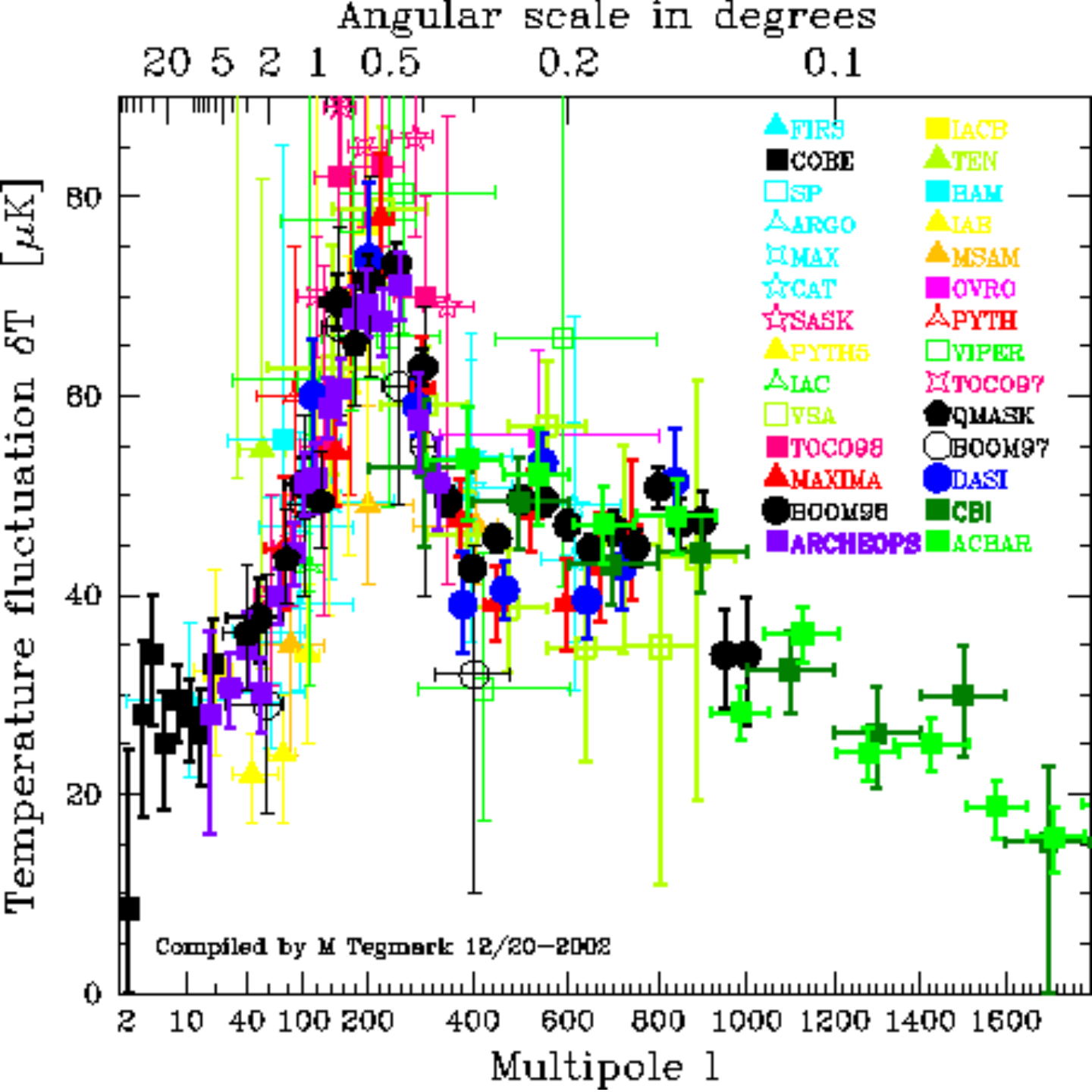


2013



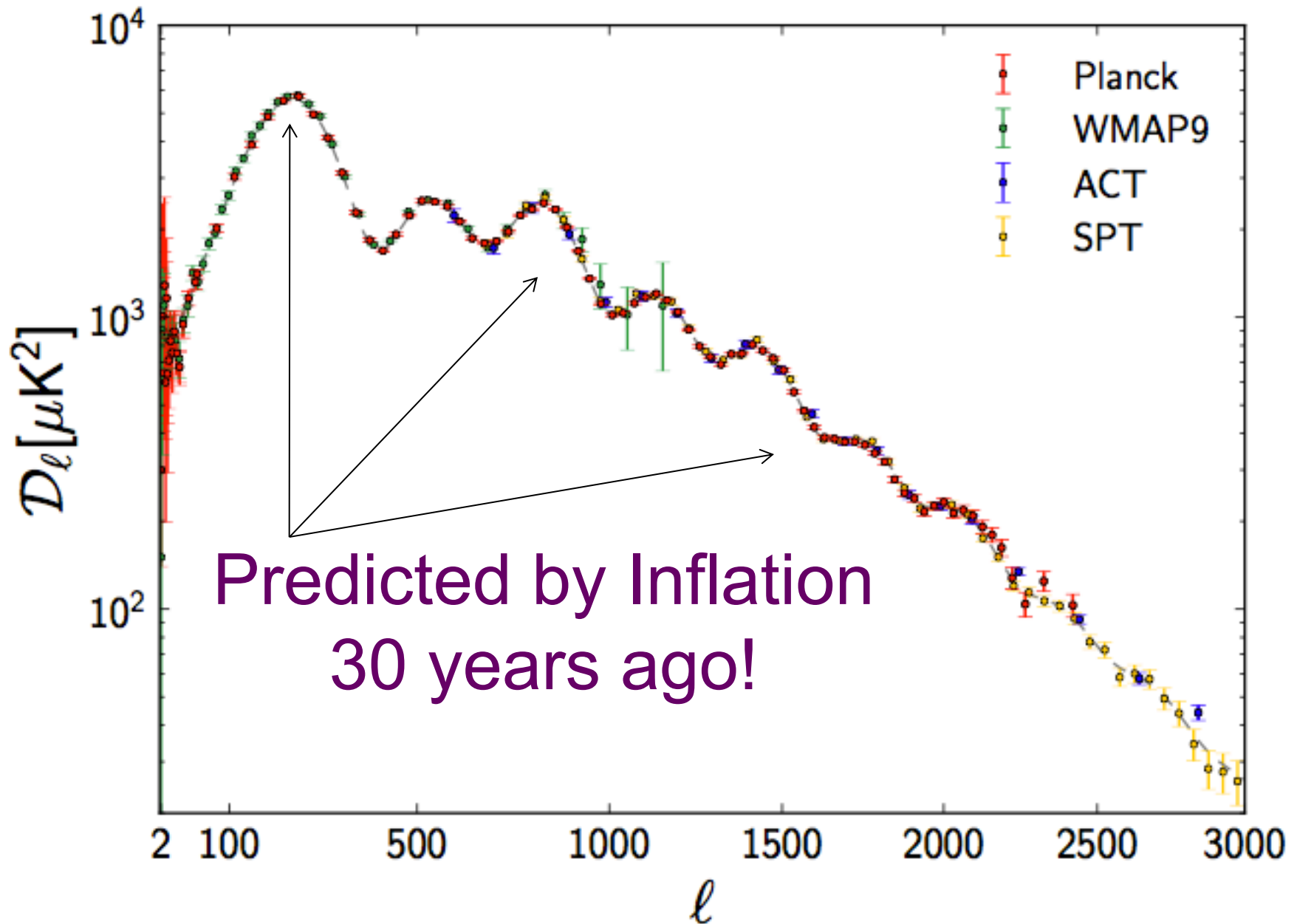
Planck

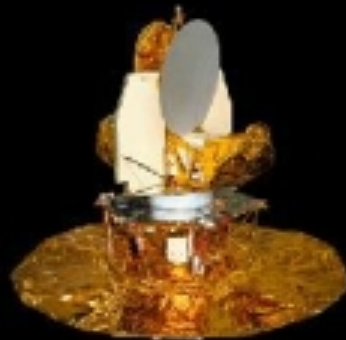




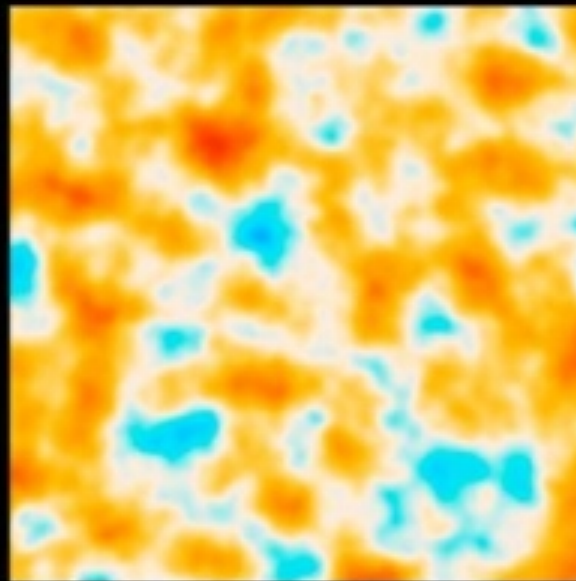
All
CMB
Exp.
(2002)

Planck Power Spectrum (2015)

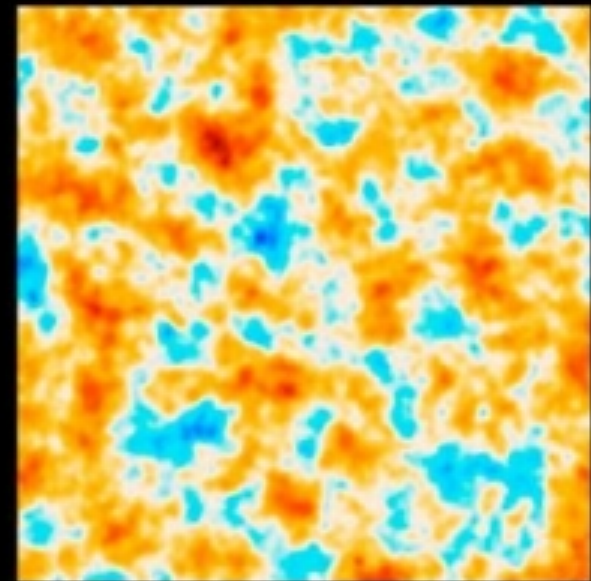




COBE

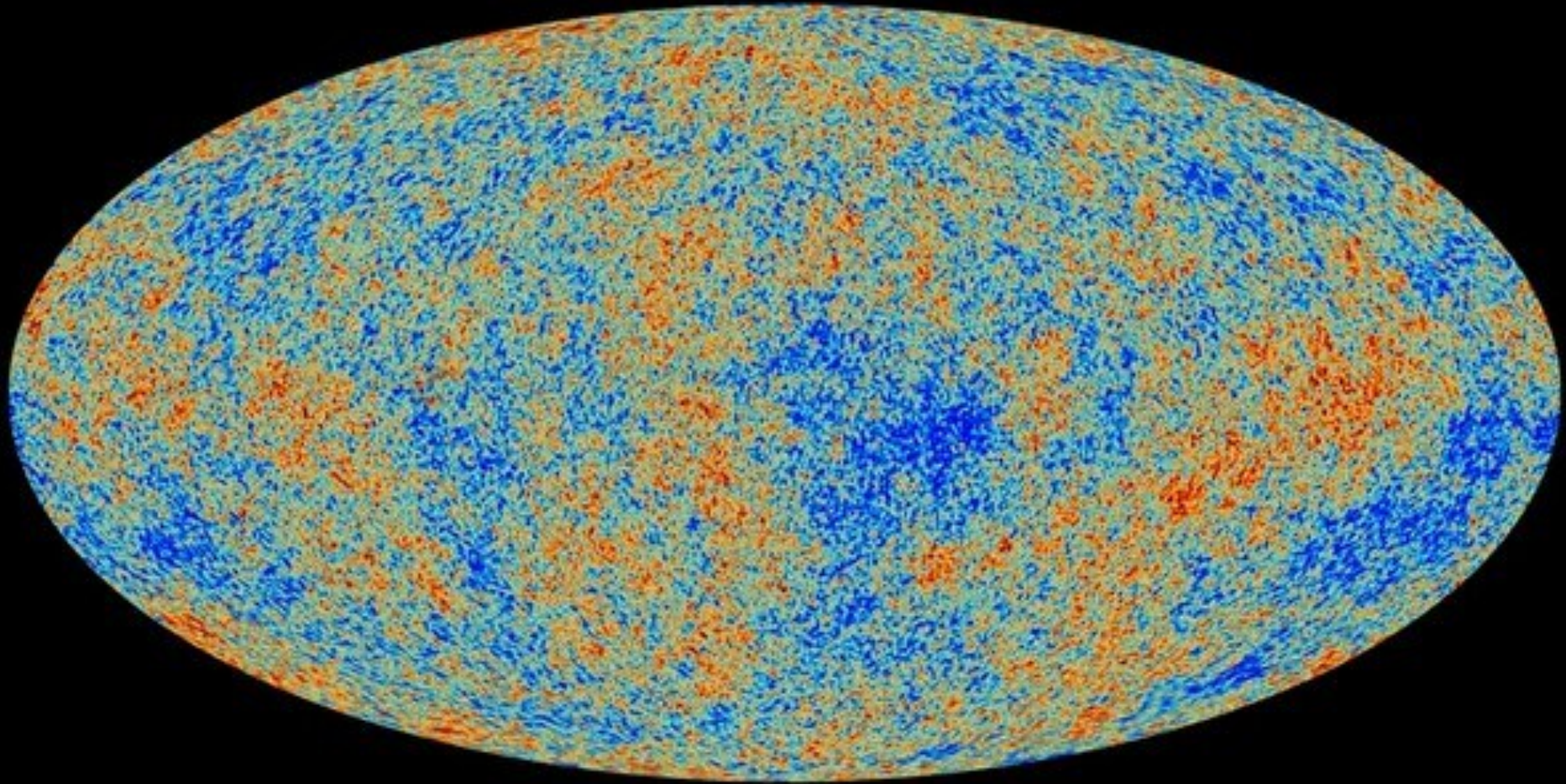


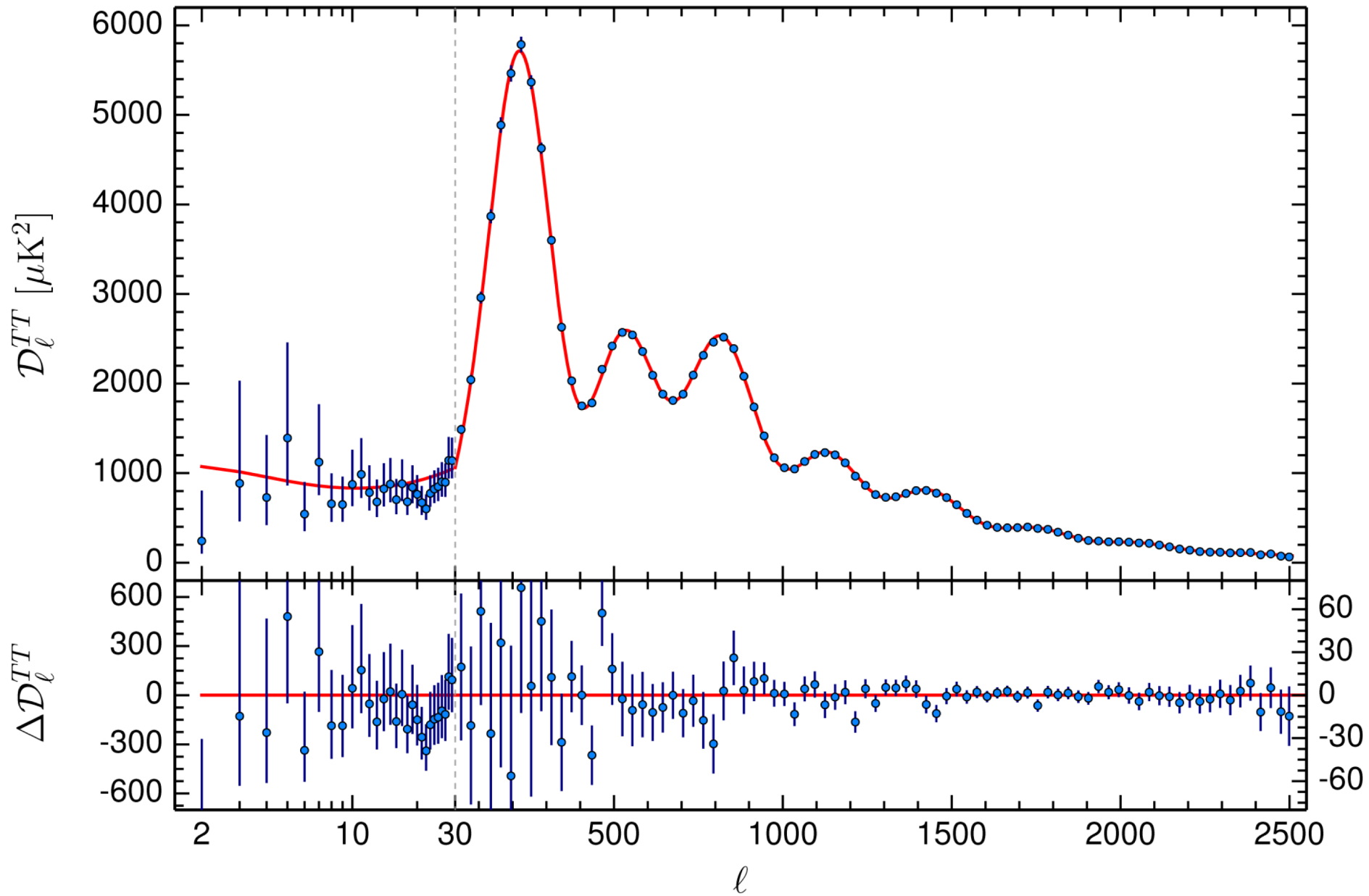
WMAP



Planck

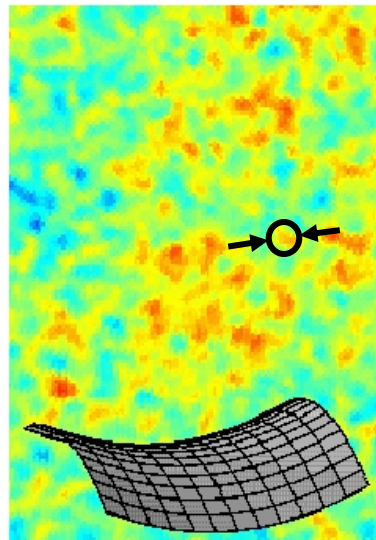
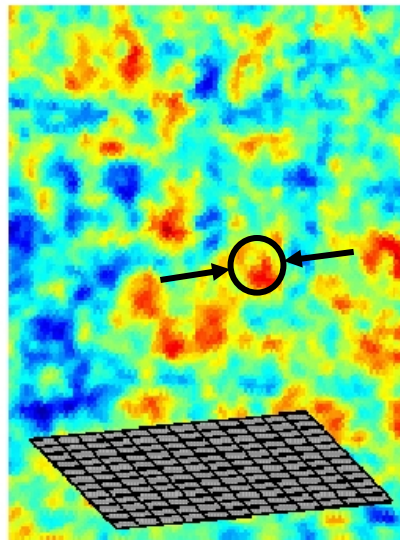
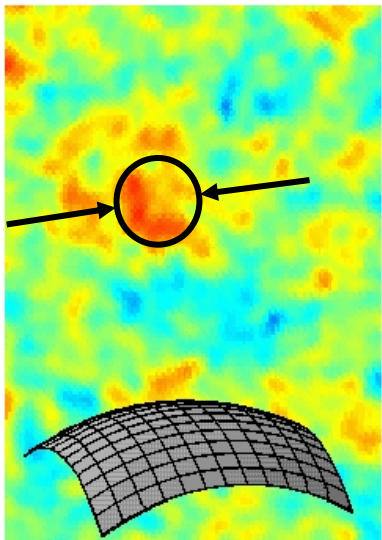
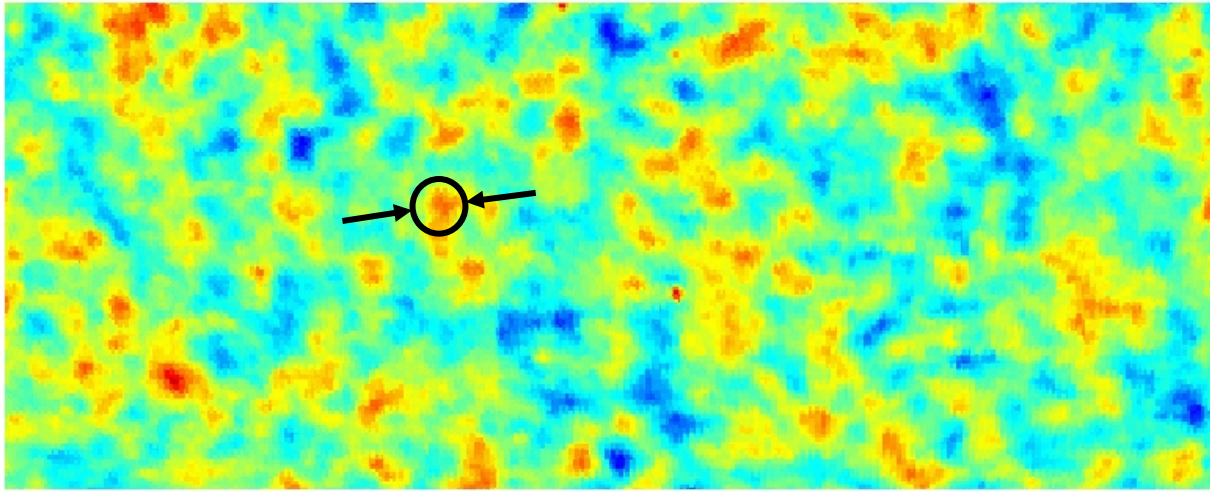
Planck (2015)



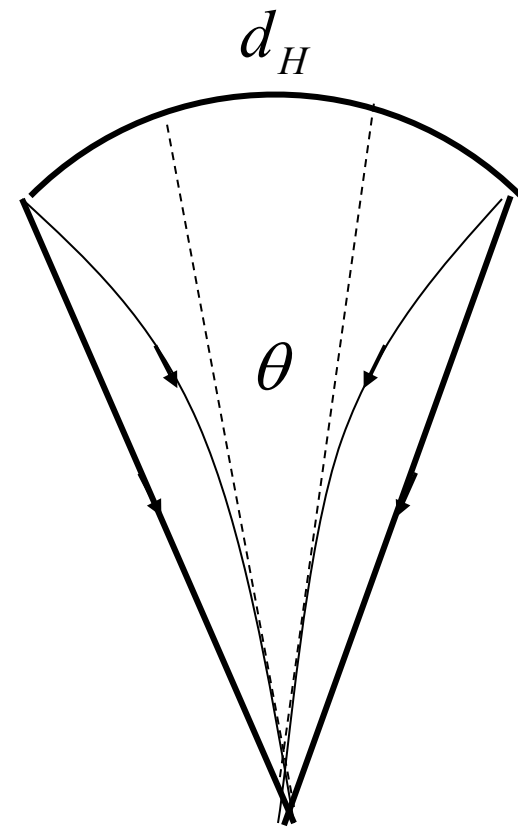


$$\Omega_K = |\Omega_0 - 1| < 0.0005 \pm 0.0001 \text{ Planck (2015)}$$

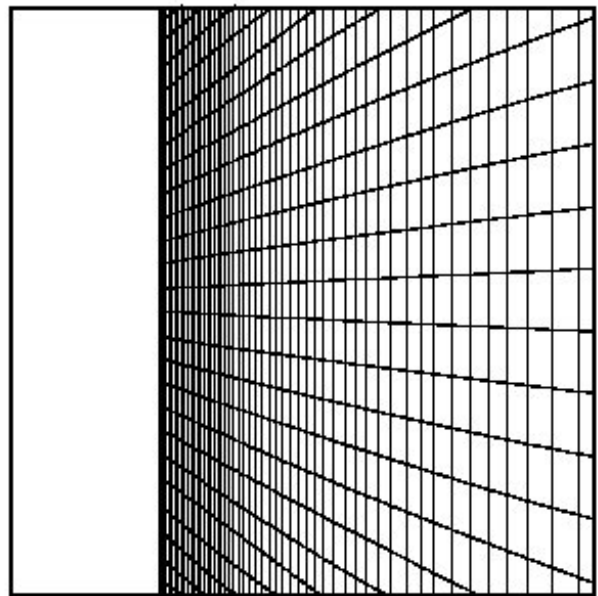
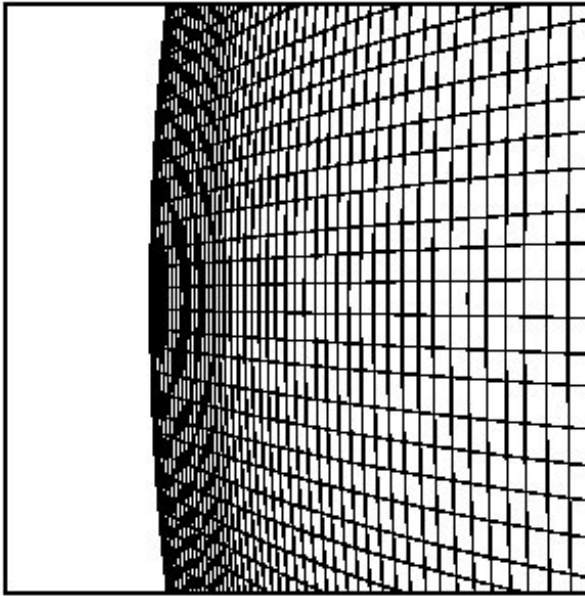
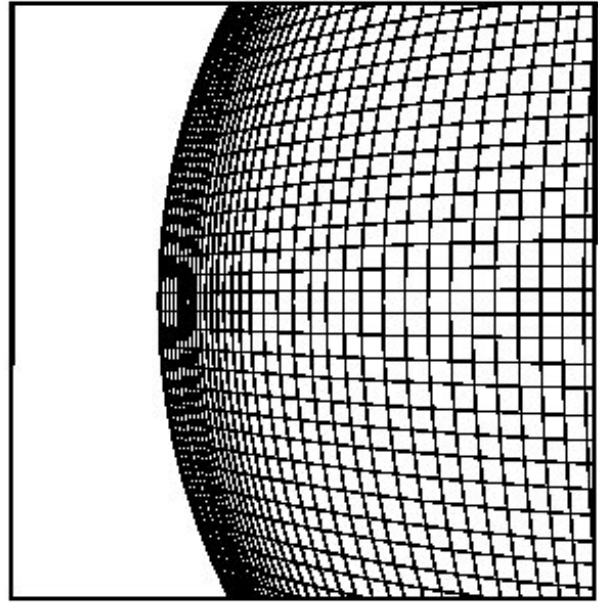
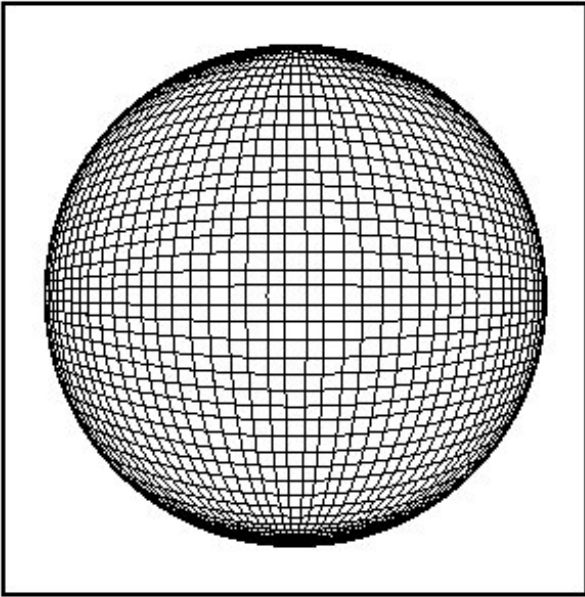
25°

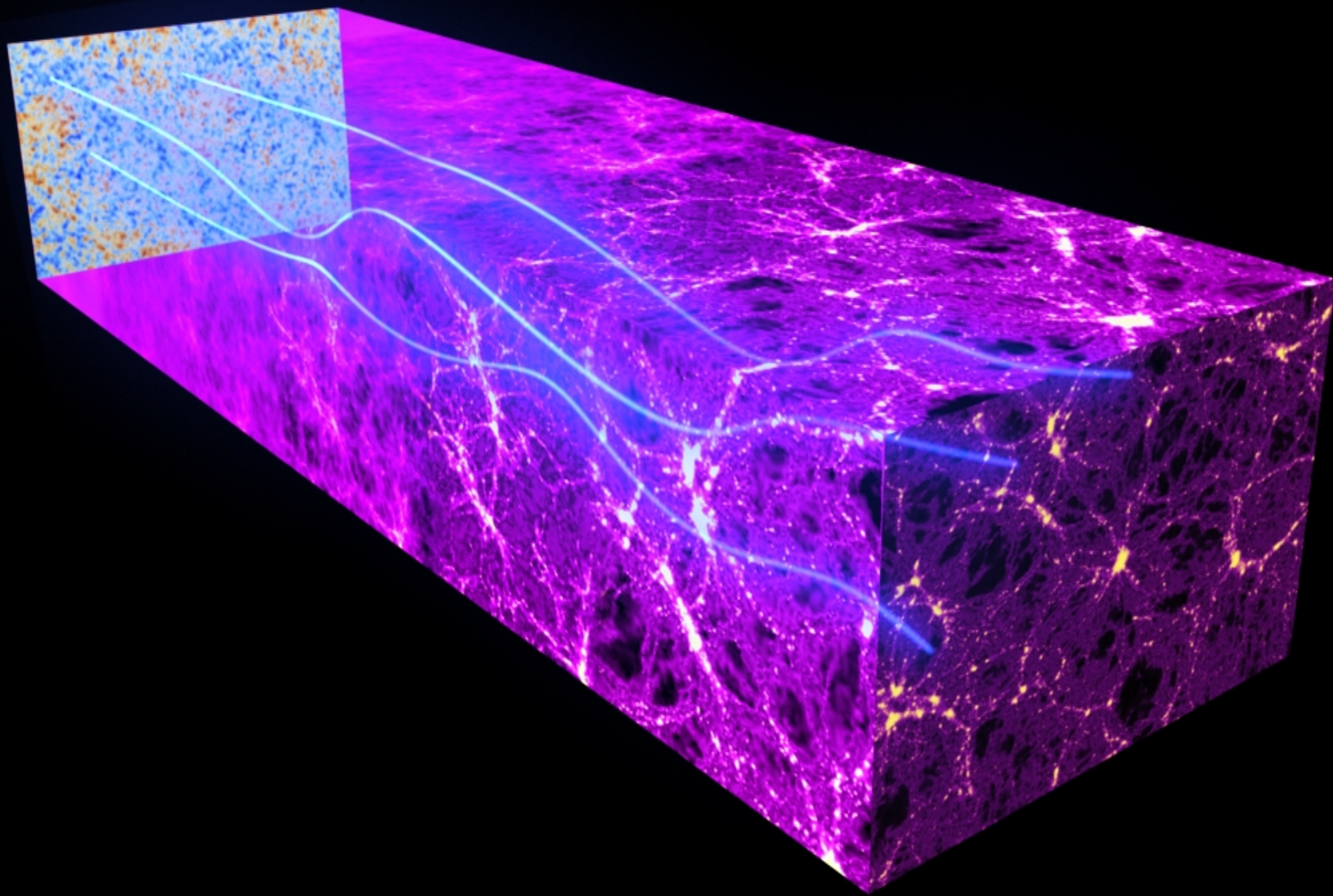


Spatial Curvature

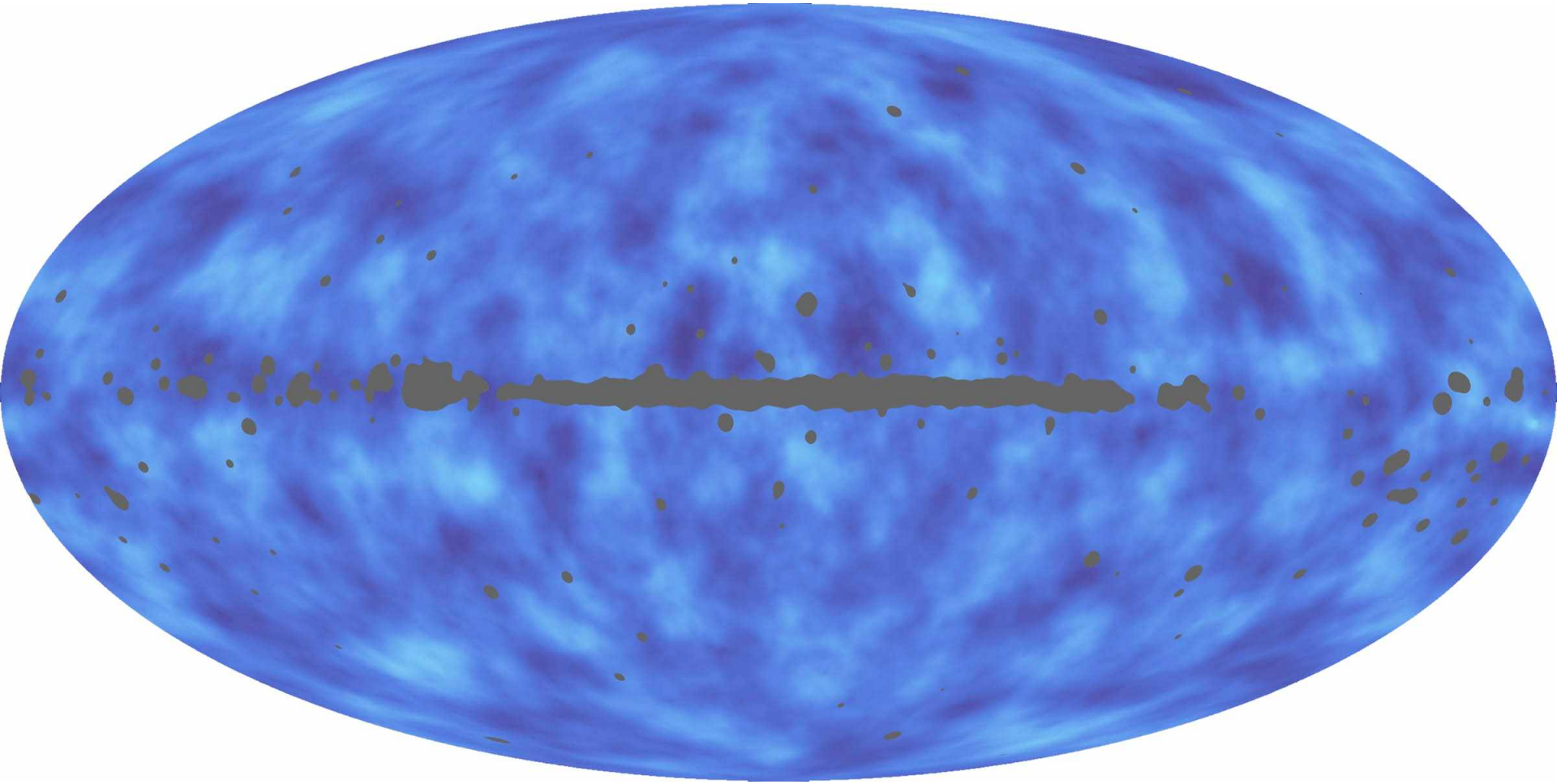


geodesics

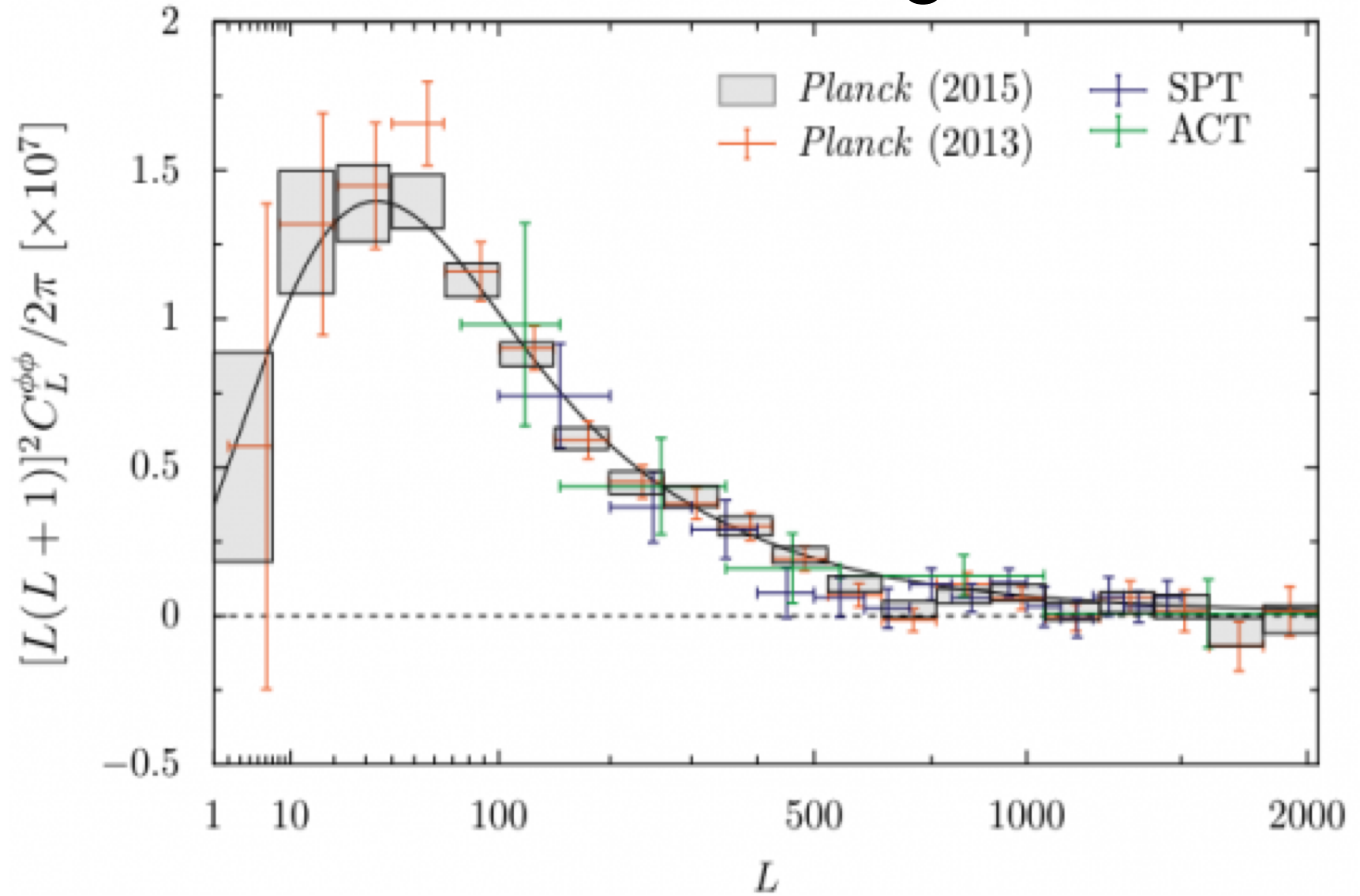




CMB lensing



CMB lensing



Planck Collaboration: Cosmological parameters

Parameter	<i>Planck</i> +WP		<i>Planck</i> +WP+highL		<i>Planck</i> +lensing+WP+highL		<i>Planck</i> +WP+highL+BAO	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022032	0.02205 ± 0.00028	0.022069	0.02207 ± 0.00027	0.022199	0.02218 ± 0.00026	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.12038	0.1199 ± 0.0027	0.12025	0.1198 ± 0.0026	0.11847	0.1186 ± 0.0022	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04119	1.04131 ± 0.00063	1.04130	1.04132 ± 0.00063	1.04146	1.04144 ± 0.00061	1.04148	1.04147 ± 0.00056
τ	0.0925	$0.089^{+0.012}_{-0.014}$	0.0927	$0.091^{+0.013}_{-0.014}$	0.0943	$0.090^{+0.013}_{-0.014}$	0.0952	0.092 ± 0.013
n_s	0.9619	0.9603 ± 0.0073	0.9582	0.9585 ± 0.0070	0.9624	0.9614 ± 0.0063	0.9611	0.9608 ± 0.0054
$\ln(10^{10} A_s)$	3.0980	$3.089^{+0.024}_{-0.027}$	3.0959	3.090 ± 0.025	3.0947	3.087 ± 0.024	3.0973	3.091 ± 0.025
A_{100}^{PS}	152	171 ± 60	209	212 ± 50	204	213 ± 50	204	212 ± 50
A_{143}^{PS}	63.3	54 ± 10	72.6	73 ± 8	72.2	72 ± 8	71.8	72.4 ± 8.0
A_{217}^{PS}	117.0	107^{+20}_{-10}	59.5	59 ± 10	60.2	58 ± 10	59.4	59 ± 10
A_{143}^{CIB}	0.0	< 10.7	3.57	3.24 ± 0.83	3.25	3.24 ± 0.83	3.30	3.25 ± 0.83
A_{217}^{CIB}	27.2	29^{+6}_{-9}	53.9	49.6 ± 5.0	52.3	50.0 ± 4.9	53.0	49.7 ± 5.0
A_{143}^{tSZ}	6.80	...	5.17	$2.54^{+1.1}_{-1.9}$	4.64	$2.51^{+1.2}_{-1.8}$	4.86	$2.54^{+1.2}_{-1.8}$
$r_{143 \times 217}^{PS}$	0.916	> 0.850	0.825	$0.823^{+0.069}_{-0.077}$	0.814	0.825 ± 0.071	0.824	0.823 ± 0.070
$r_{143 \times 217}^{CIB}$	0.406	0.42 ± 0.22	1.0000	> 0.930	1.0000	> 0.928	1.0000	> 0.930
γ^{CIB}	0.601	$0.53^{+0.13}_{-0.12}$	0.674	0.638 ± 0.081	0.656	0.643 ± 0.080	0.667	0.639 ± 0.081
$\xi^{tSZ \times CIB}$	0.03	...	0.000	< 0.409	0.000	< 0.389	0.000	< 0.410
A^{kSZ}	0.9	...	0.89	$5.34^{+2.8}_{-1.9}$	1.14	$4.74^{+2.6}_{-2.1}$	1.58	$5.34^{+2.8}_{-2.0}$
Ω_Λ	0.6817	$0.685^{+0.018}_{-0.016}$	0.6830	$0.685^{+0.017}_{-0.016}$	0.6939	0.693 ± 0.013	0.6914	0.692 ± 0.010
σ_8	0.8347	0.829 ± 0.012	0.8322	0.828 ± 0.012	0.8271	0.8233 ± 0.0097	0.8288	0.826 ± 0.012
z_{re}	11.37	11.1 ± 1.1	11.38	11.1 ± 1.1	11.42	11.1 ± 1.1	11.52	11.3 ± 1.1
H_0	67.04	67.3 ± 1.2	67.15	67.3 ± 1.2	67.94	67.9 ± 1.0	67.77	67.80 ± 0.77
Age/Gyr	13.8242	13.817 ± 0.048	13.8170	13.813 ± 0.047	13.7914	13.794 ± 0.044	13.7965	13.798 ± 0.037
$100\theta_*$	1.04136	1.04147 ± 0.00062	1.04146	1.04148 ± 0.00062	1.04161	1.04159 ± 0.00060	1.04163	1.04162 ± 0.00056
r_{drag}	147.36	147.49 ± 0.59	147.35	147.47 ± 0.59	147.68	147.67 ± 0.50	147.611	147.68 ± 0.45

Table 5. Best-fit values and 68% confidence limits for the base Λ CDM model. Beam and calibration parameters, and additional nuisance parameters for “highL” data sets are not listed for brevity but may be found in the Explanatory Supplement (Planck Collaboration ES 2013).

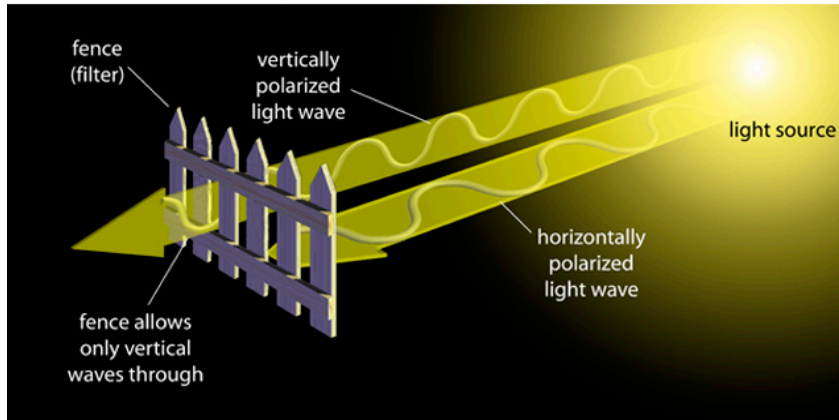
Parameter	<i>Planck</i> (CMB+lensing)		<i>Planck</i> +WP+highL+BAO	
	Best fit	68 % limits	Best fit	68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067	1.04148	1.04147 ± 0.00056
τ	0.0949	0.089 ± 0.032	0.0952	0.092 ± 0.013
n_s	0.9675	0.9635 ± 0.0094	0.9611	0.9608 ± 0.0054
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057	3.0973	3.091 ± 0.025
Ω_Λ	0.6964	0.693 ± 0.019	0.6914	0.692 ± 0.010
σ_8	0.8285	0.823 ± 0.018	0.8288	0.826 ± 0.012
z_{re}	11.45	$10.8^{+3.1}_{-2.5}$	11.52	11.3 ± 1.1
H_0	68.14	67.9 ± 1.5	67.77	67.80 ± 0.77
Age/Gyr	13.784	13.796 ± 0.058	13.7965	13.798 ± 0.037
$100\theta_*$	1.04164	1.04156 ± 0.00066	1.04163	1.04162 ± 0.00056
r_{drag}	147.74	147.70 ± 0.63	147.611	147.68 ± 0.45
$r_{drag}/D_V(0.57)$	0.07207	0.0719 ± 0.0011		

CMB

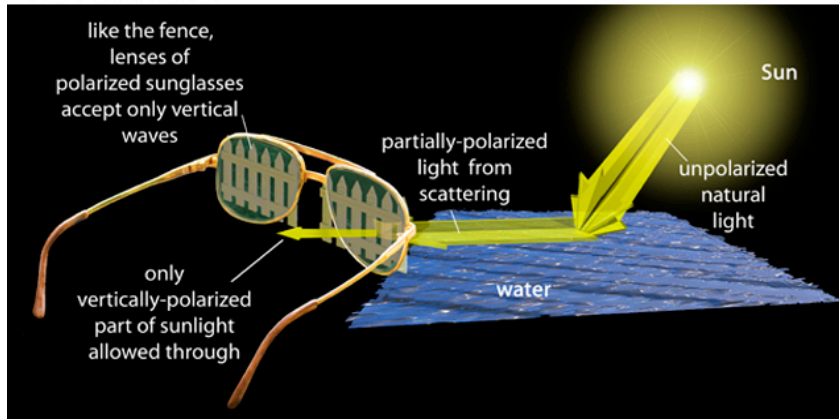
Polarization

Anisotropies

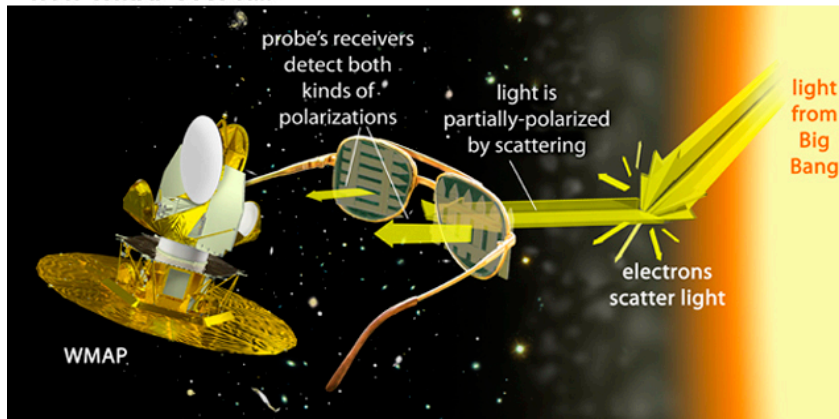
Polarization: How It Works



how we see it...



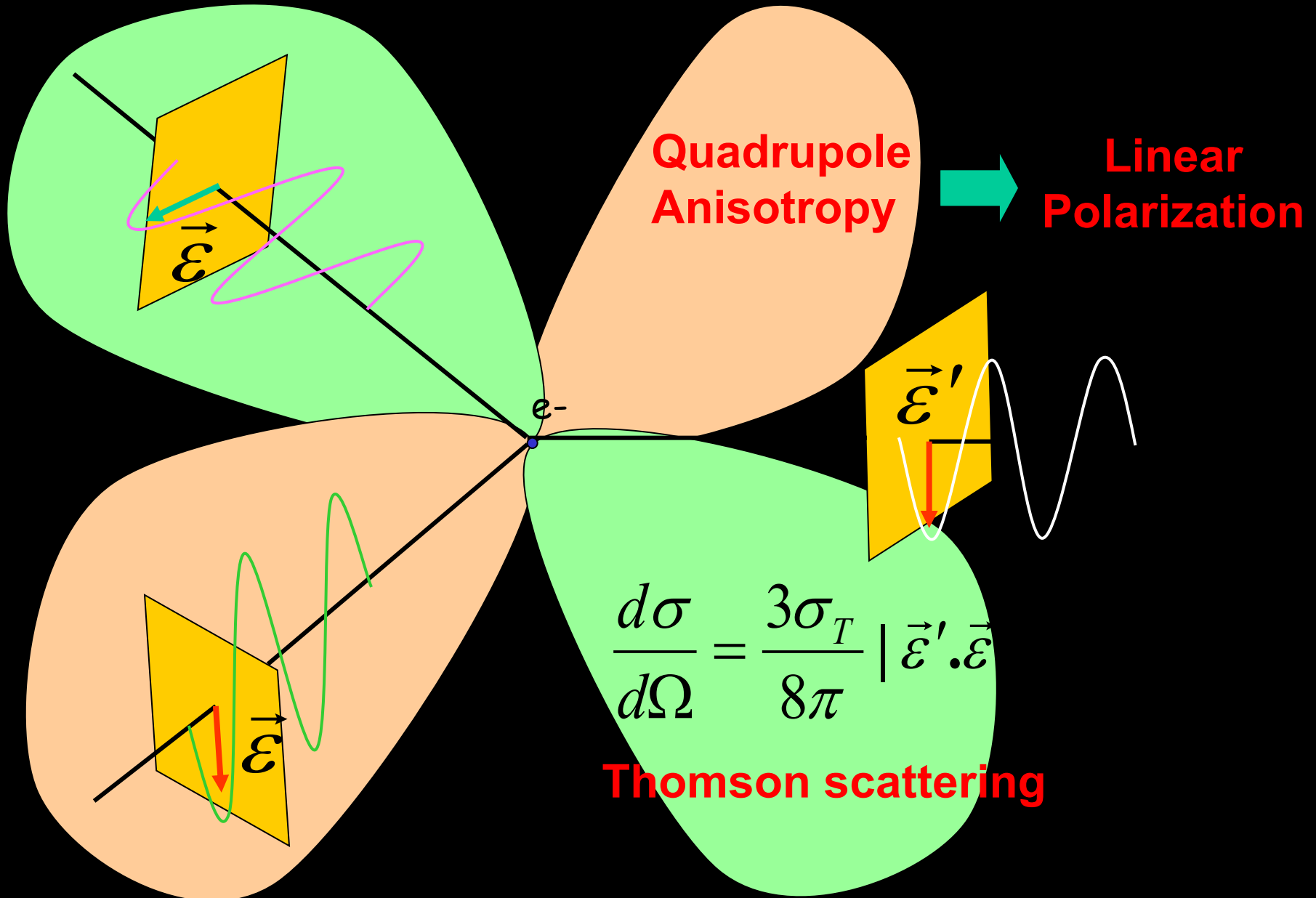
how WMAP sees it...



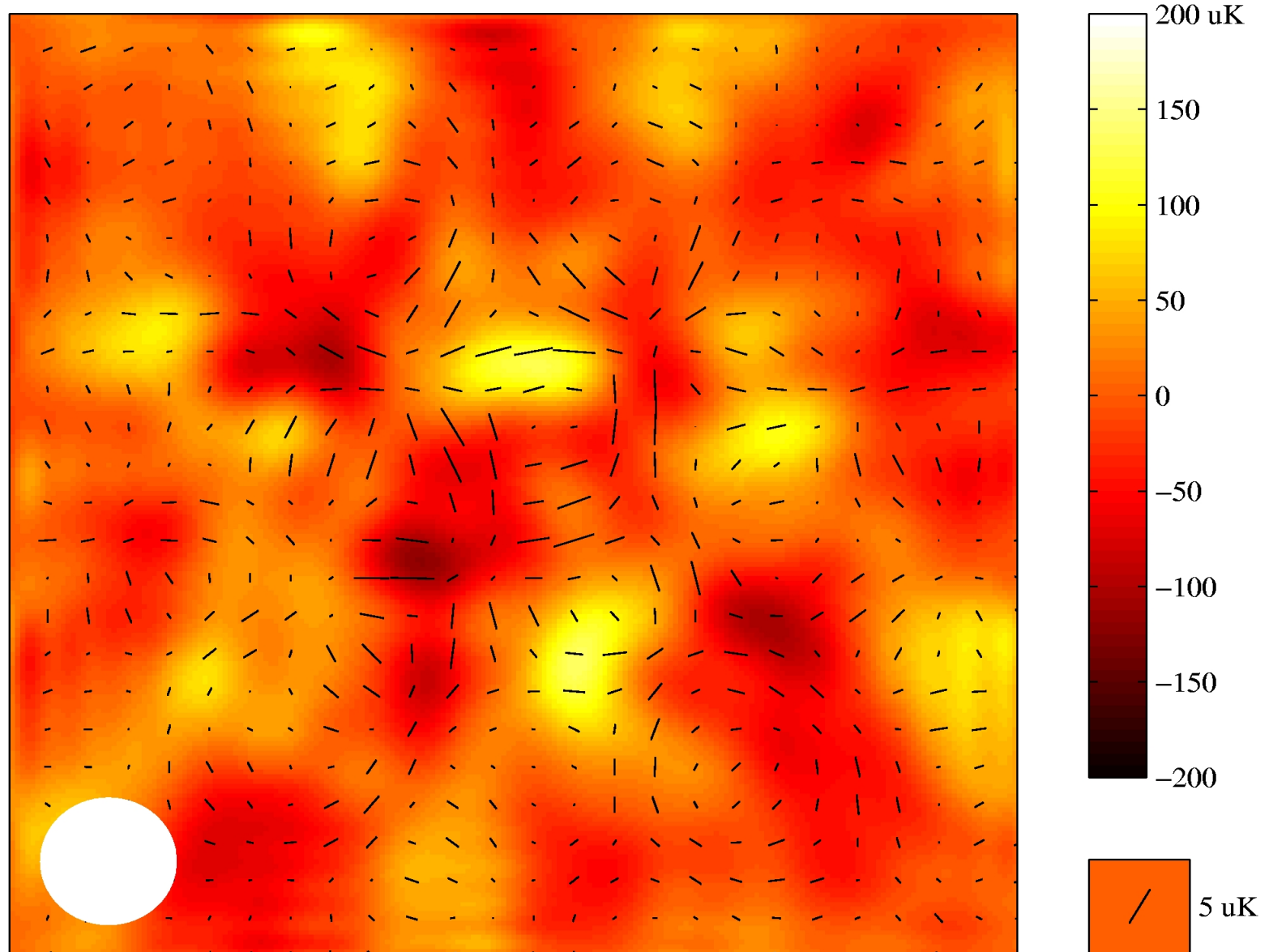
Light carries energy and polarization (vector field)

A vector field has two comp. gradient + curl ($E + B$)

Linear Polarization of CMB

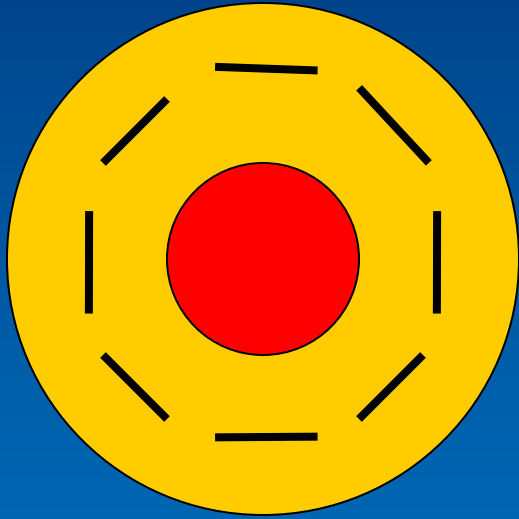


DASI: First measurement Polarization (2002)



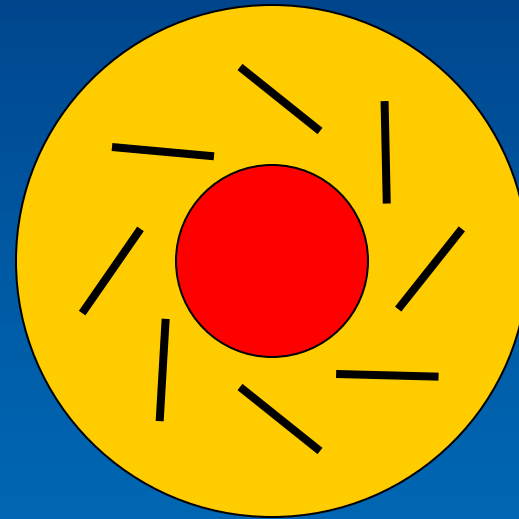
Map is 5 degrees square

Polarization around Hot spots



E Polarization

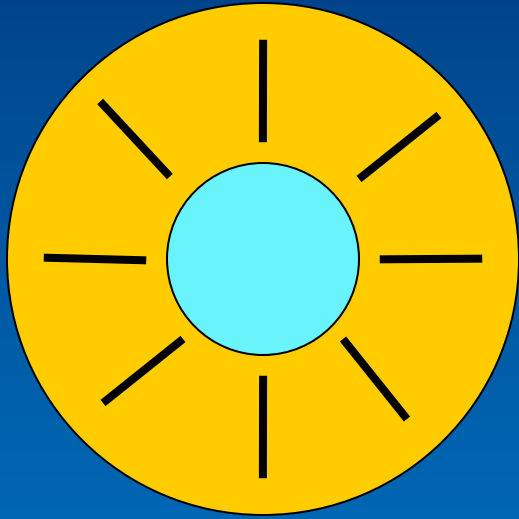
$$\nabla \times E = 0$$



B Polarization

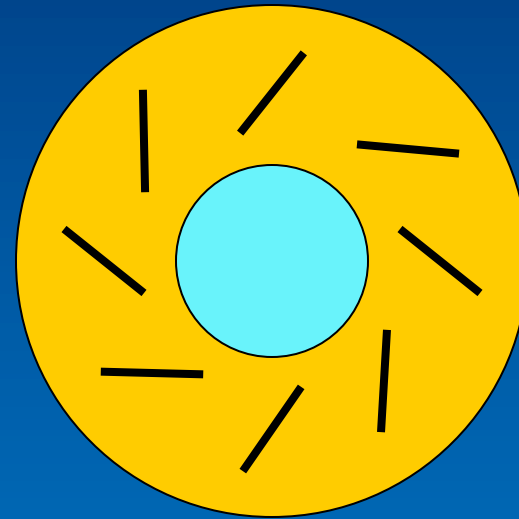
$$\nabla \cdot B = 0$$

Polarization around Cold spots



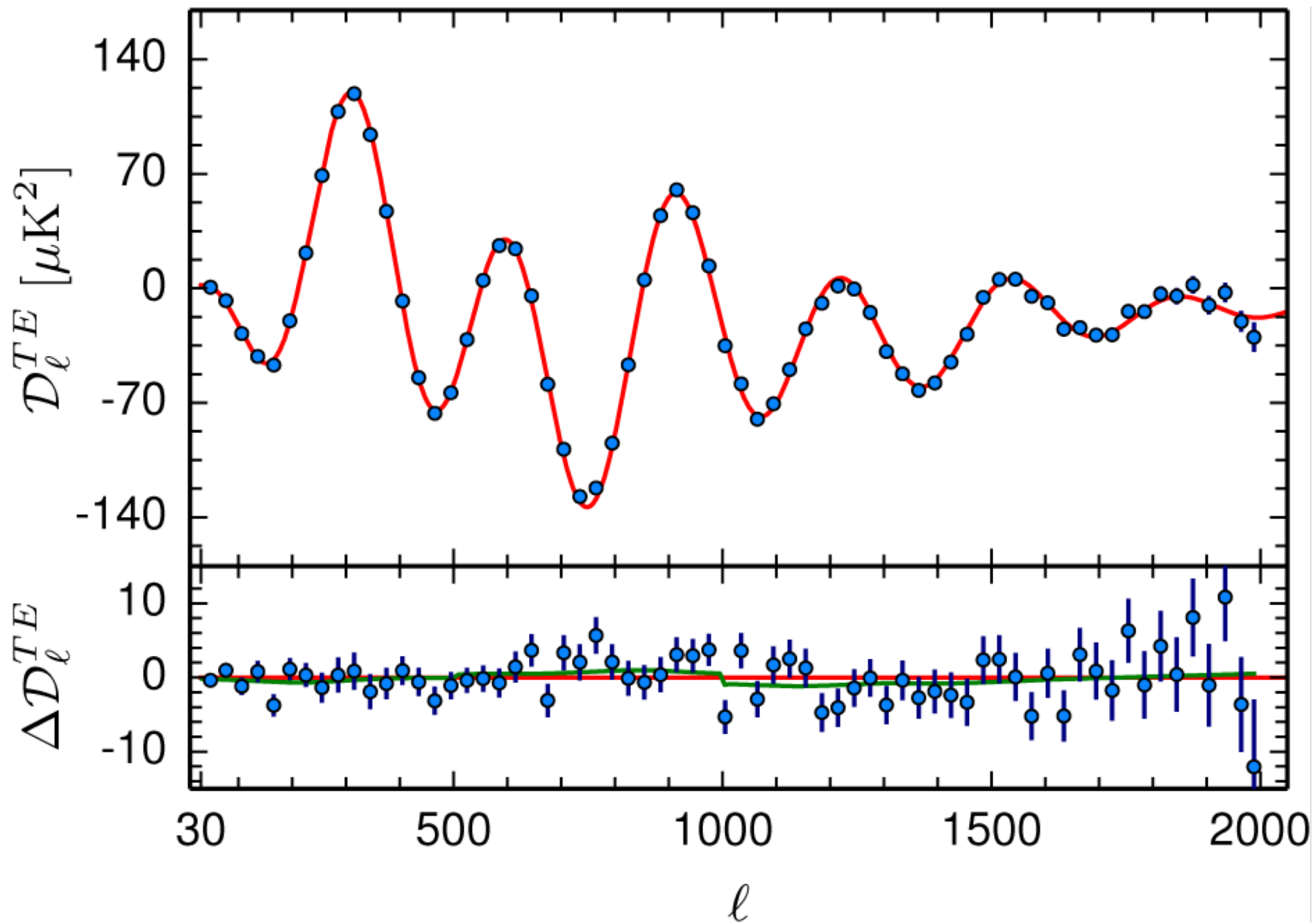
E Polarization

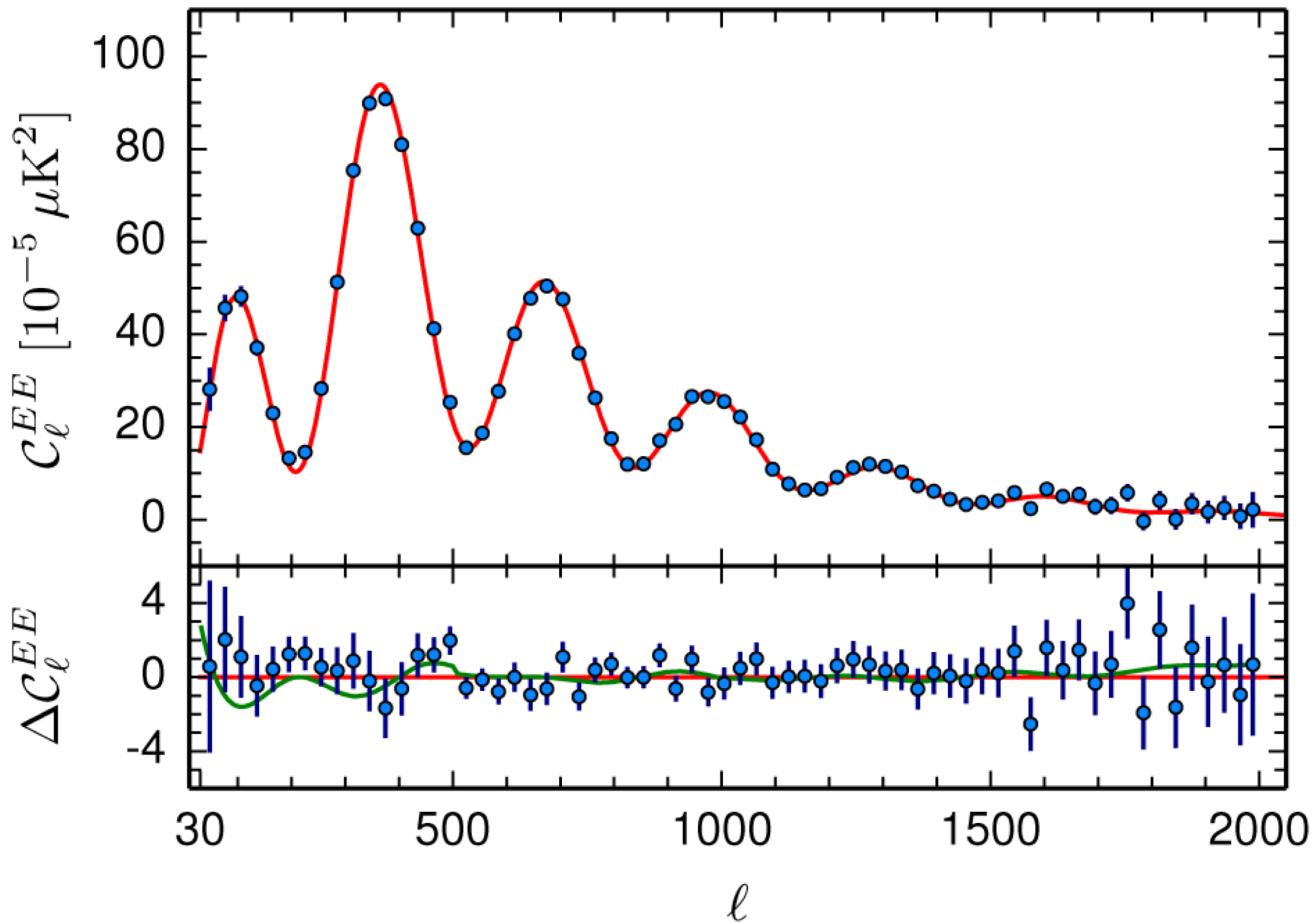
$$\nabla \times E = 0$$



B Polarization

$$\nabla \cdot B = 0$$





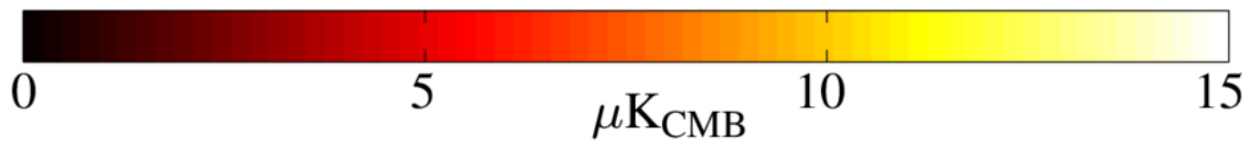
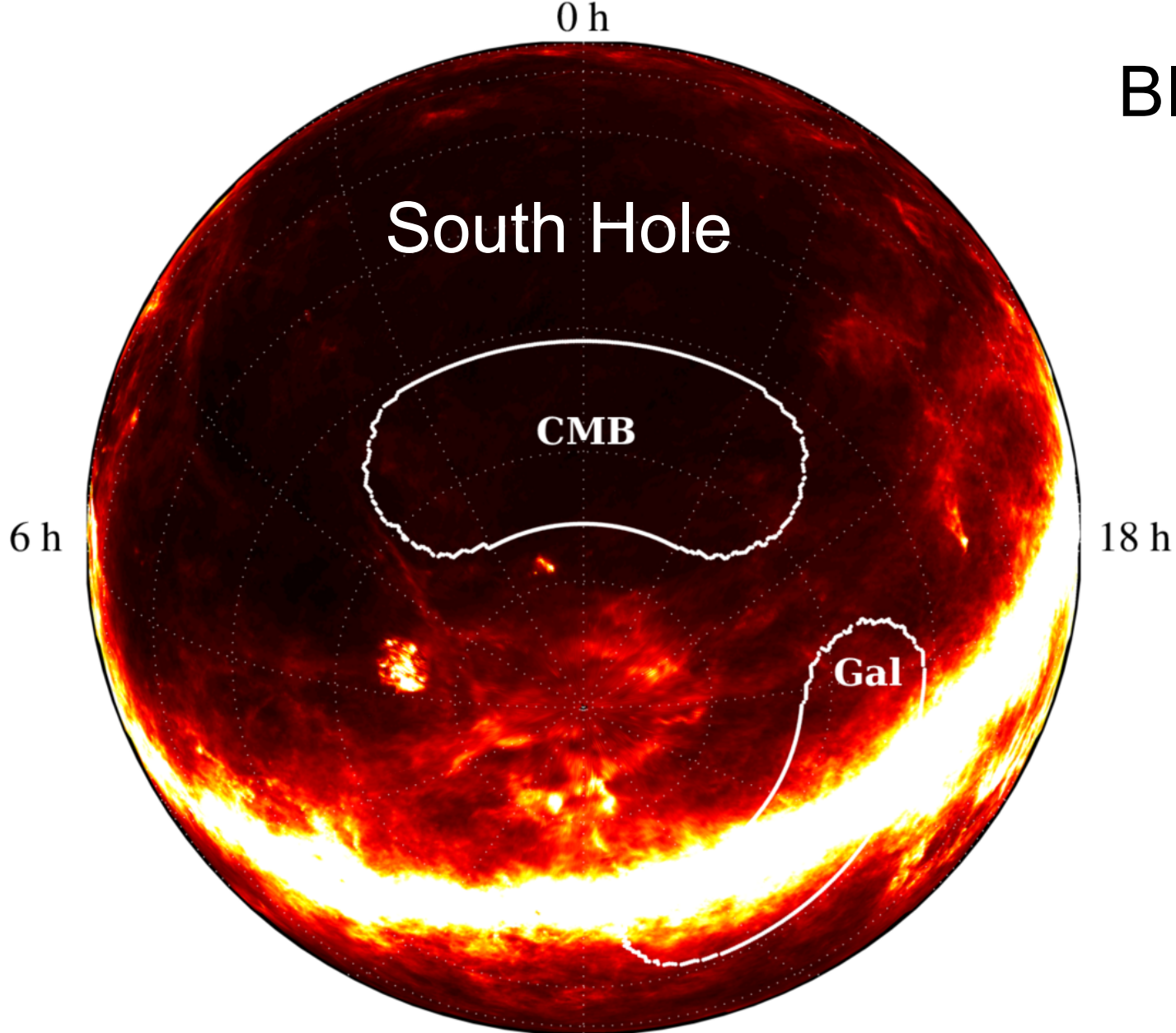
Scott-Amundsen South Pole Station

BICEP

South Pole Telescope



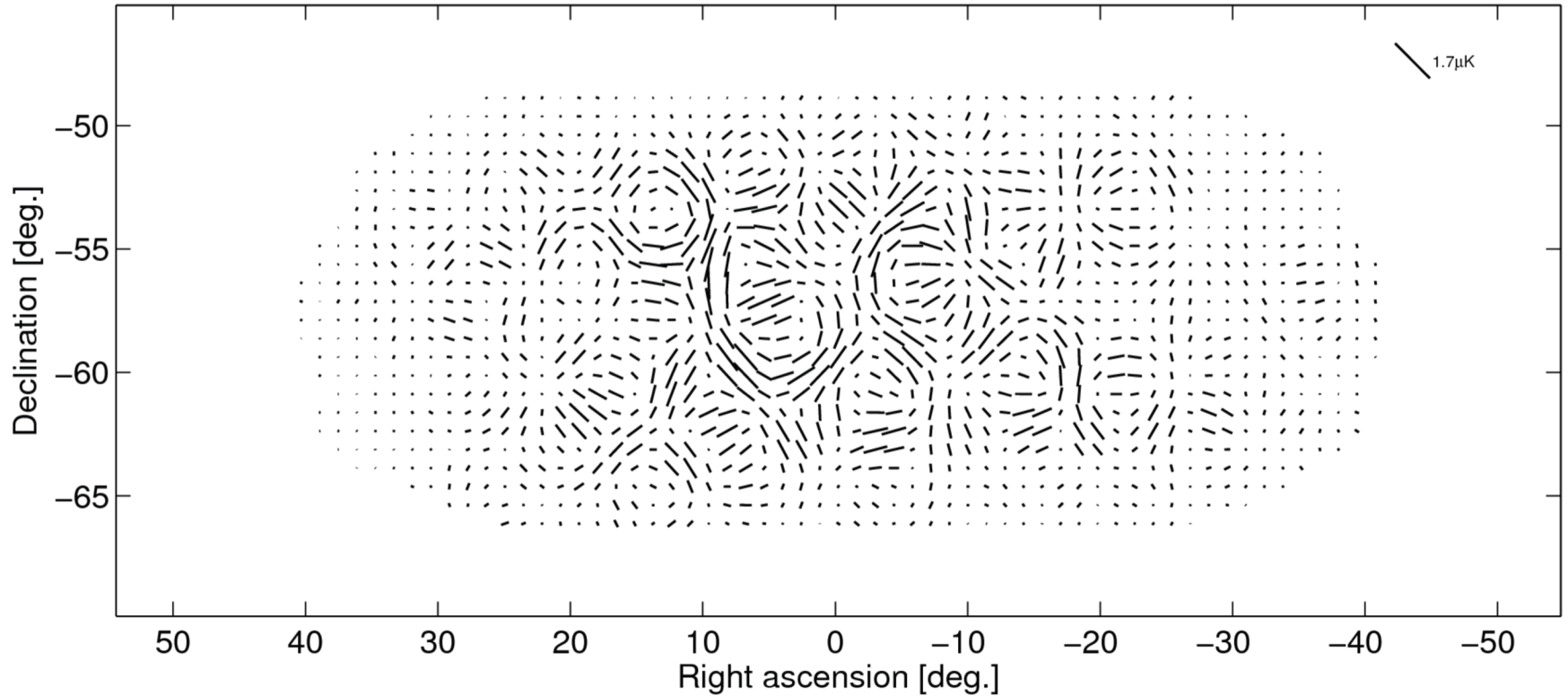
BICEP



Total Polarization

BICEP2 total polarization signal

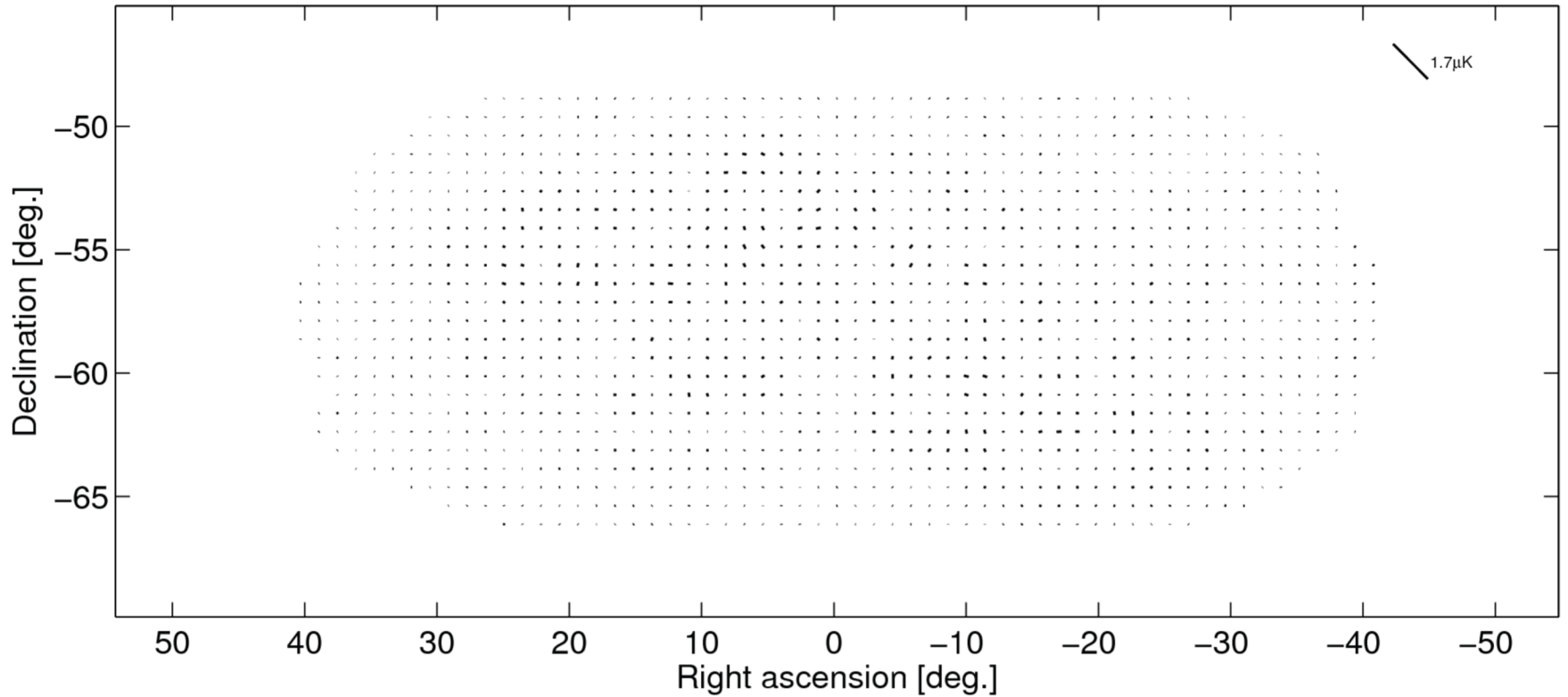
Scale: $1.7 \mu K$



B-mode Contribution

BICEP2 B-mode signal

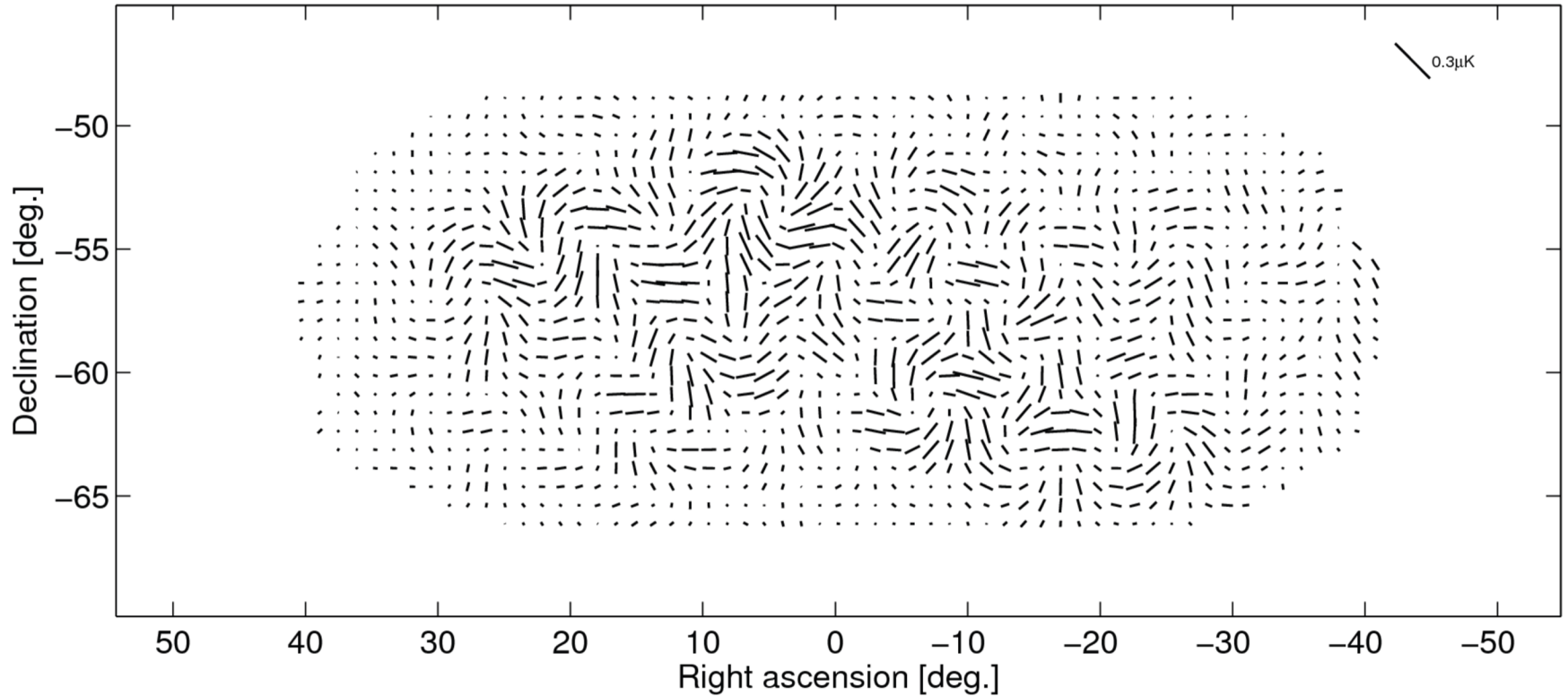
Scale: $1.7 \mu K$



B-mode Contribution

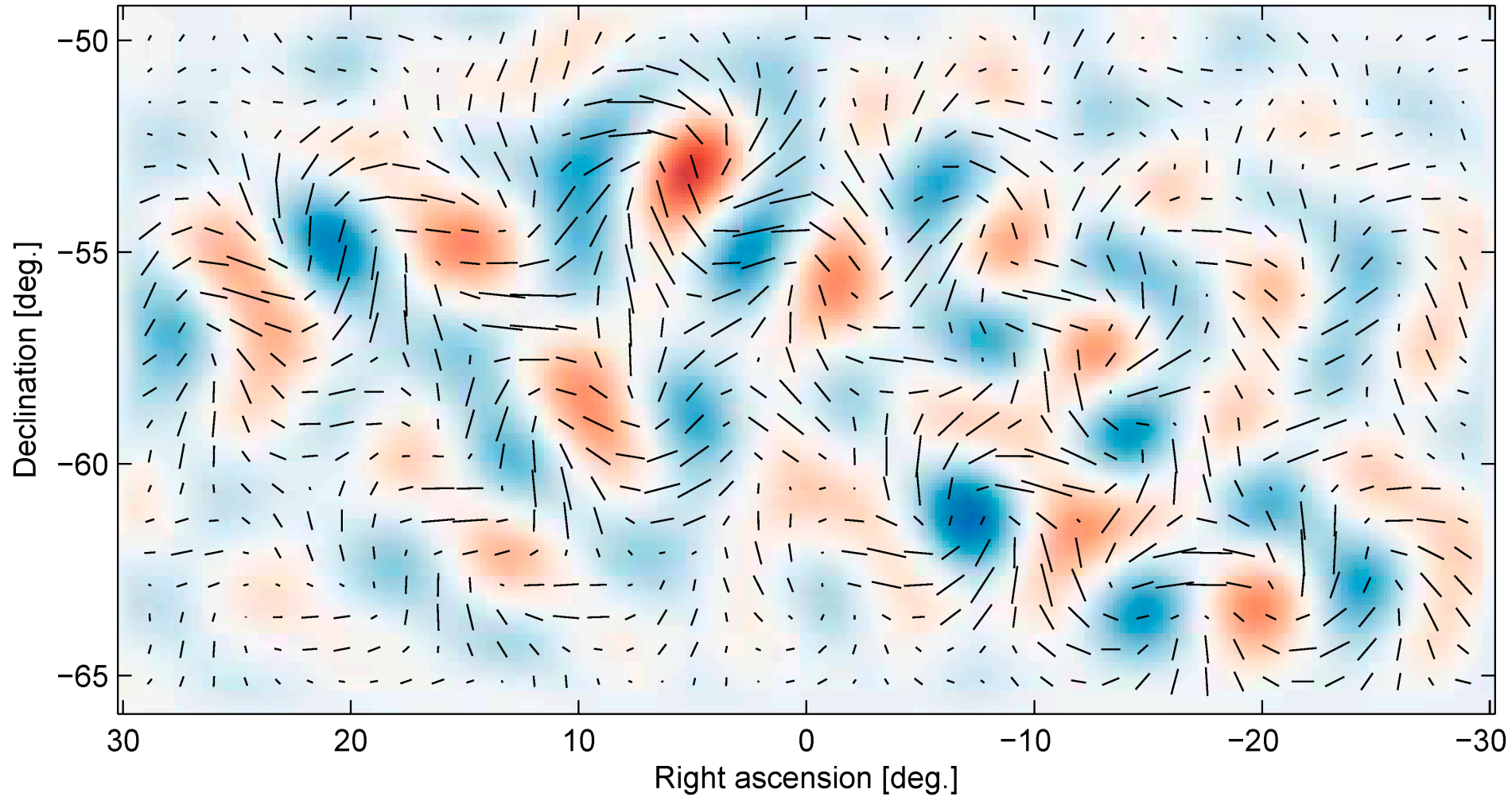
BICEP2 B-mode signal

Scale: $0.3 \mu K$

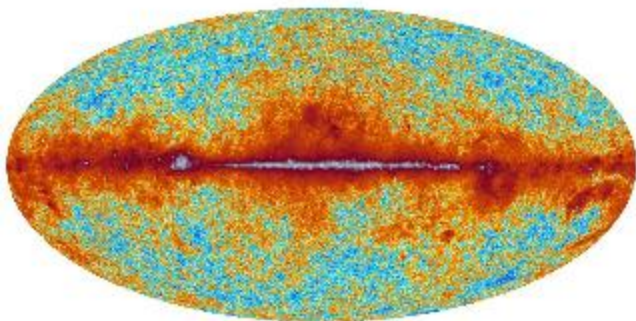


Measurements BICEP2

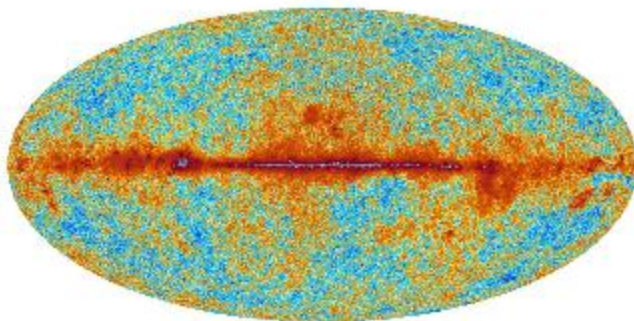
BICEP2 B-mode signal



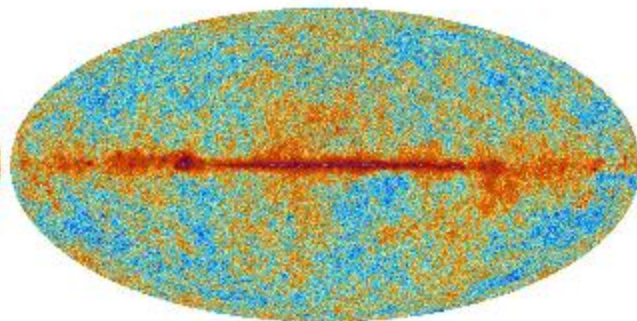
30 GHz



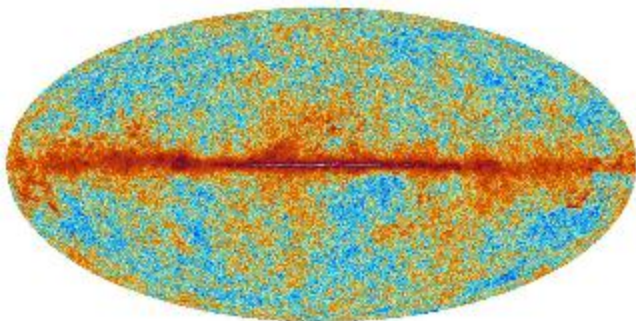
44 GHz



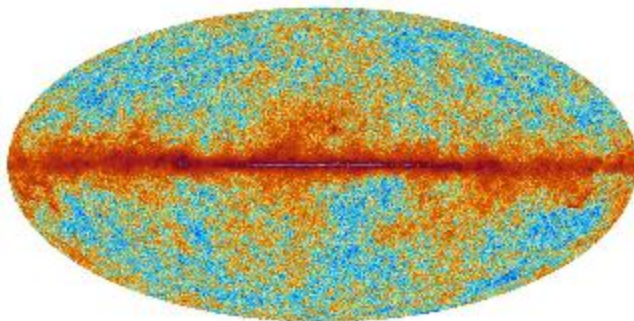
70 GHz



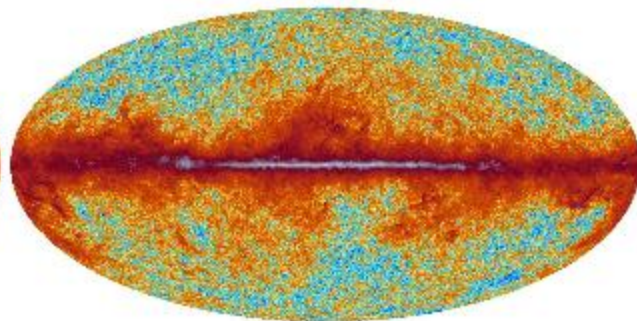
100 GHz



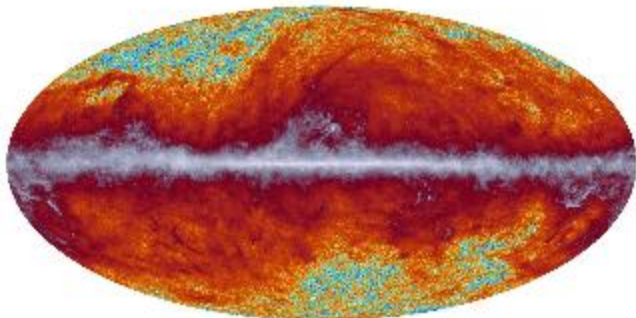
143 GHz



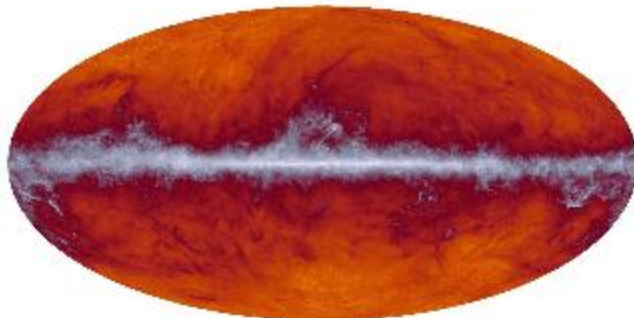
217 GHz



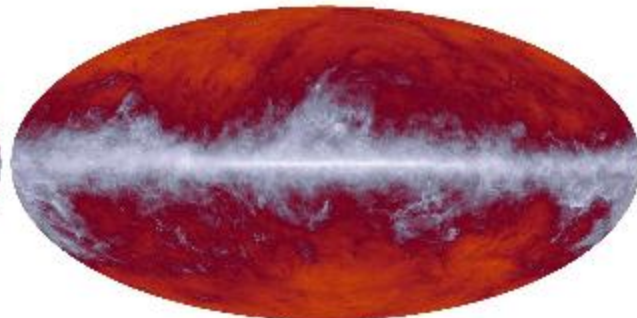
353 GHz



545 GHz



857 GHz



-10^3 -10^2 -10 -1 0 1 10 10^2 10^3 10^4 10^5 10^6

30–353 GHz: δT [μK_{CMB}]; 545 and 857 GHz: surface brightness [kJy/sr]

LA GALAAAAAXIA

37!

BRAZO DE SAGITARIO

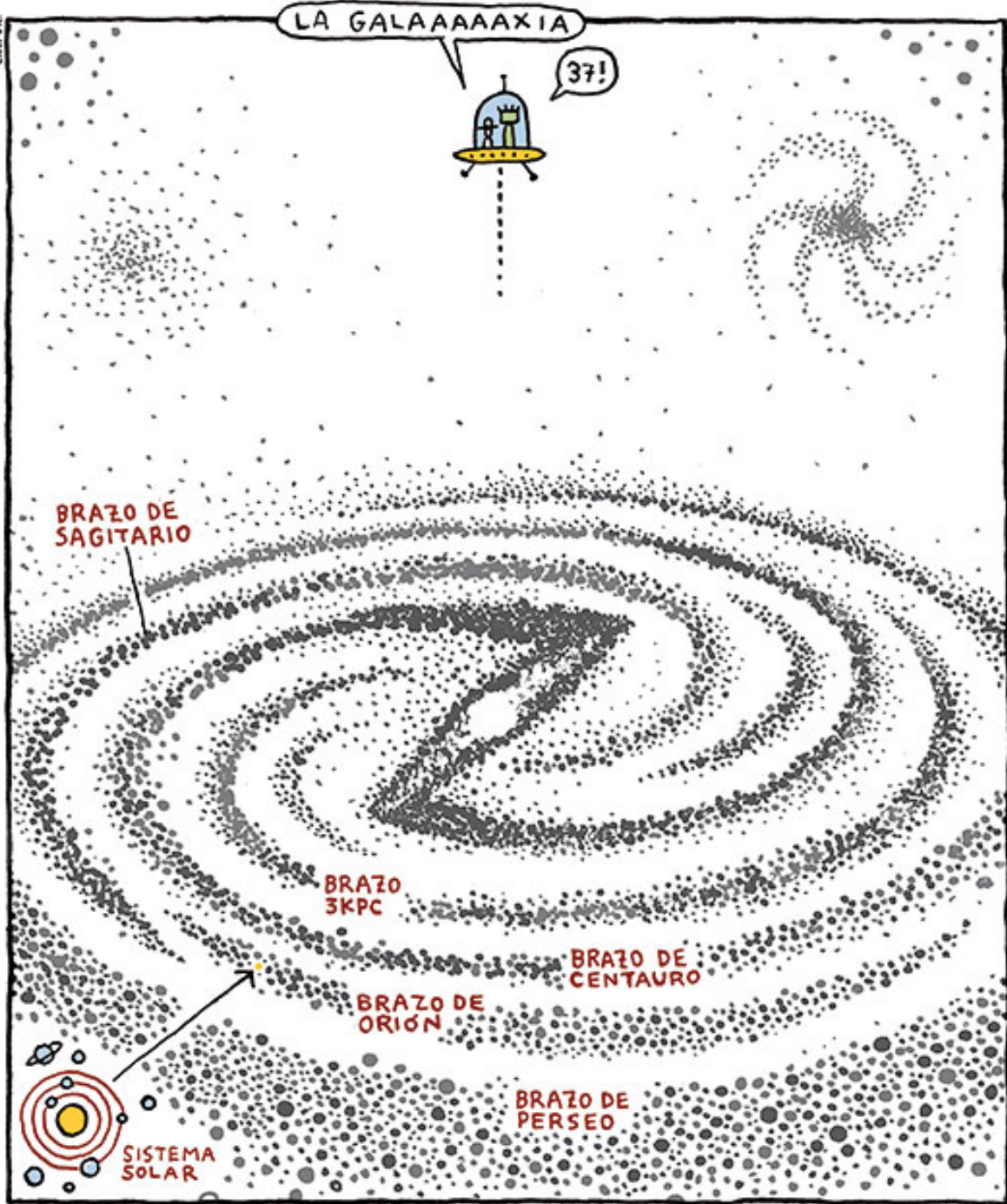
BRAZO 3KPC

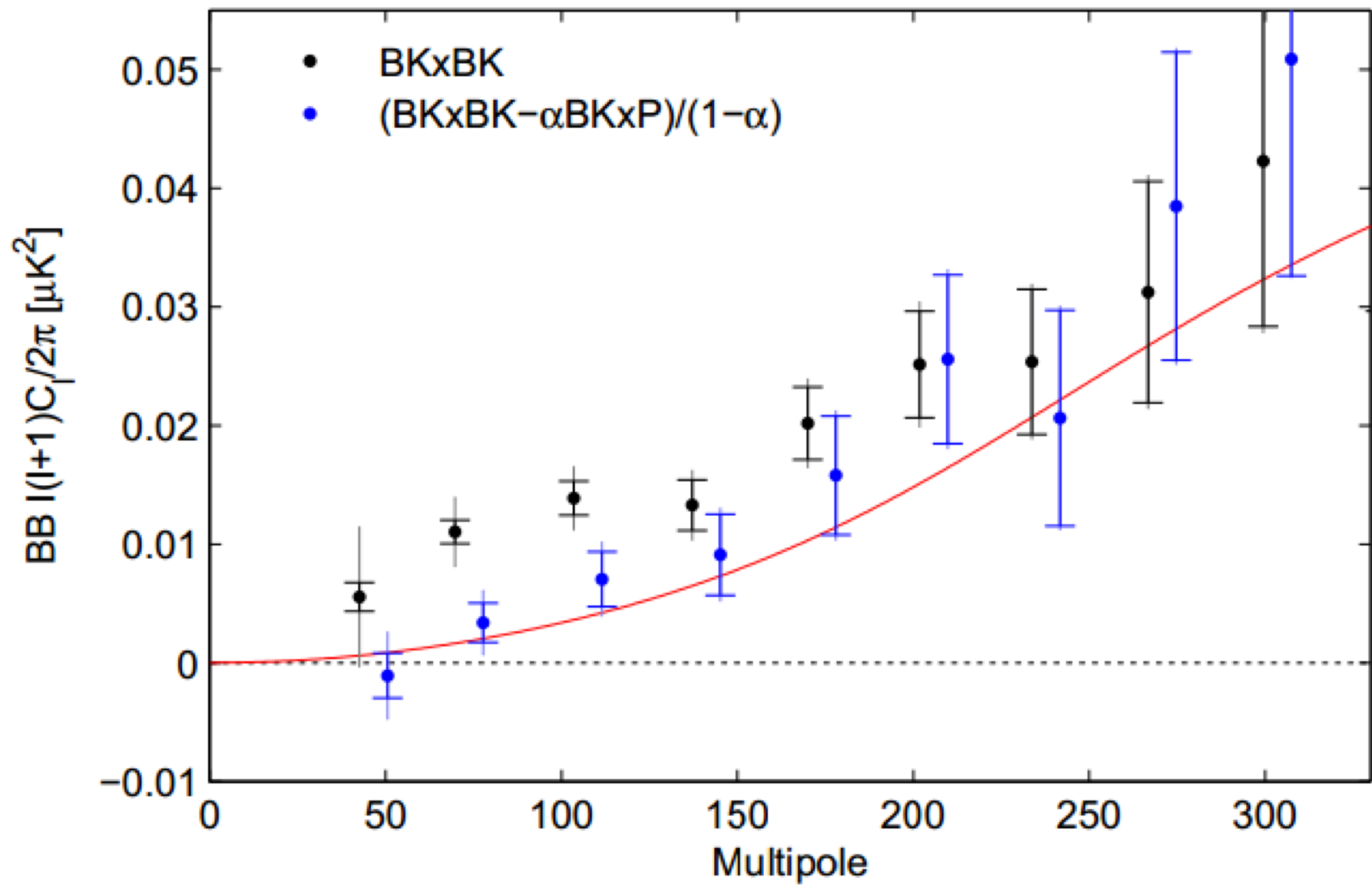
BRAZO DE CENTAURO

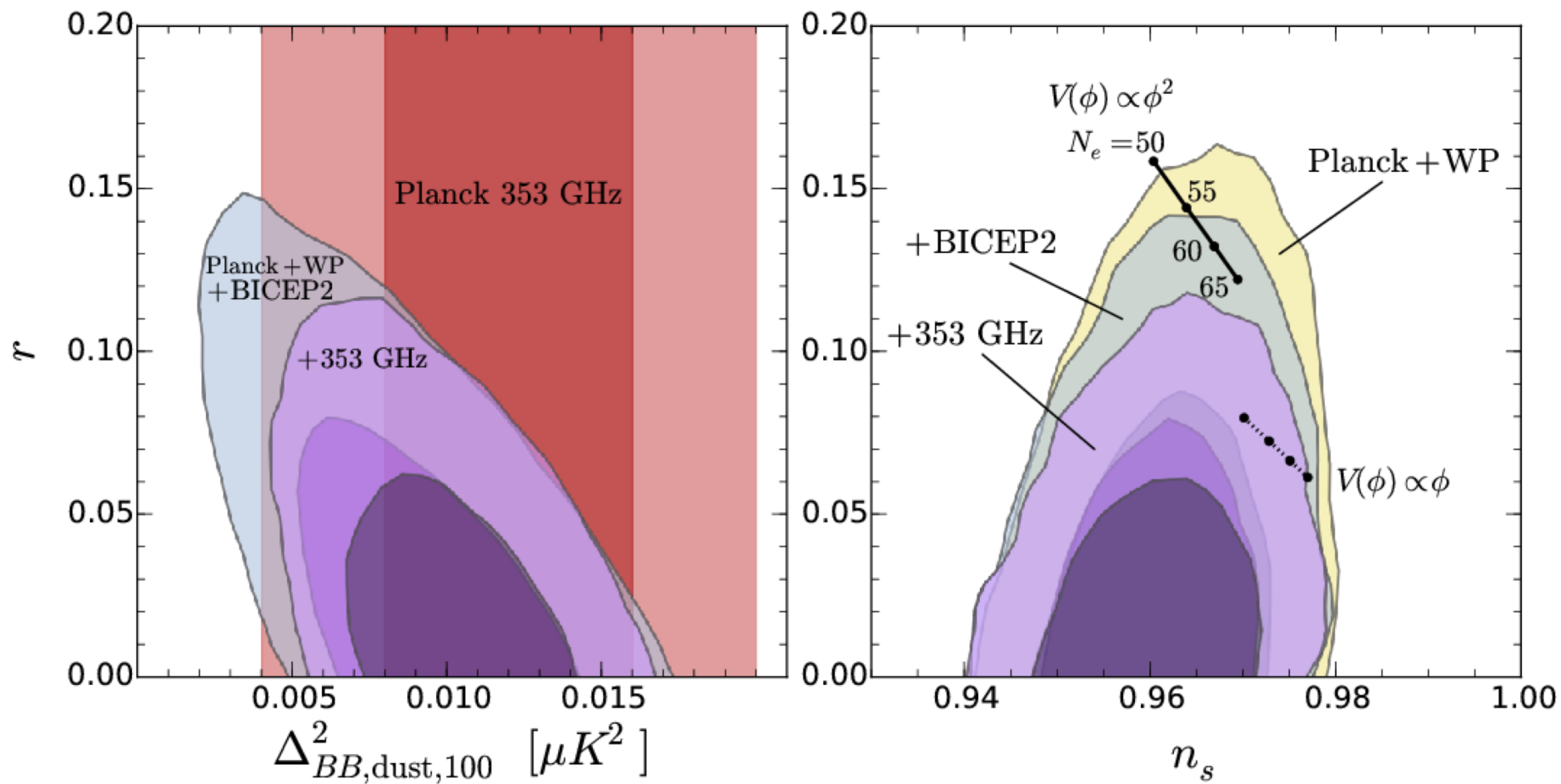
BRAZO DE ORION

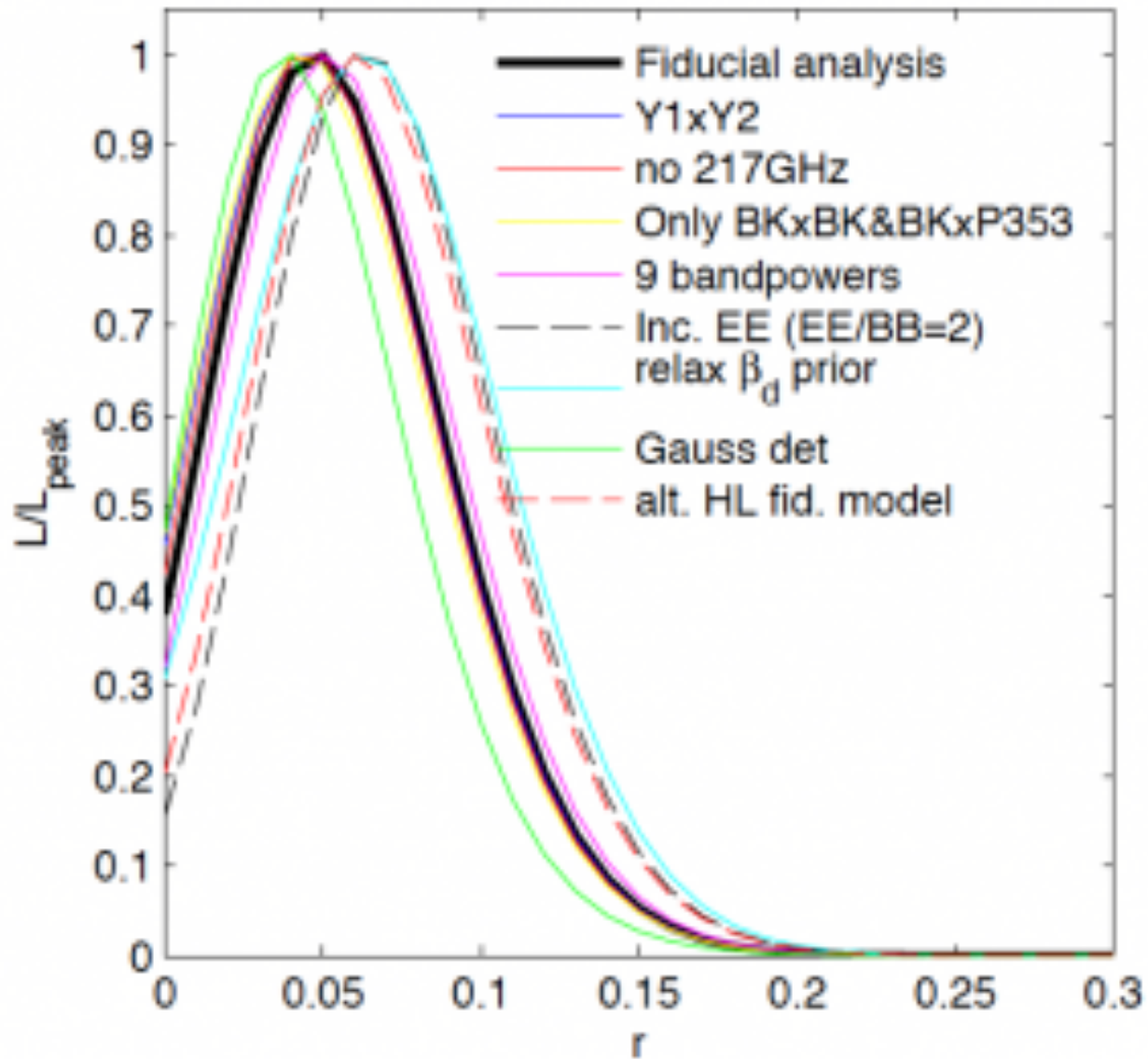
BRAZO DE PERSEO

SISTEMA SOLAR









All Dust
No Gravitati

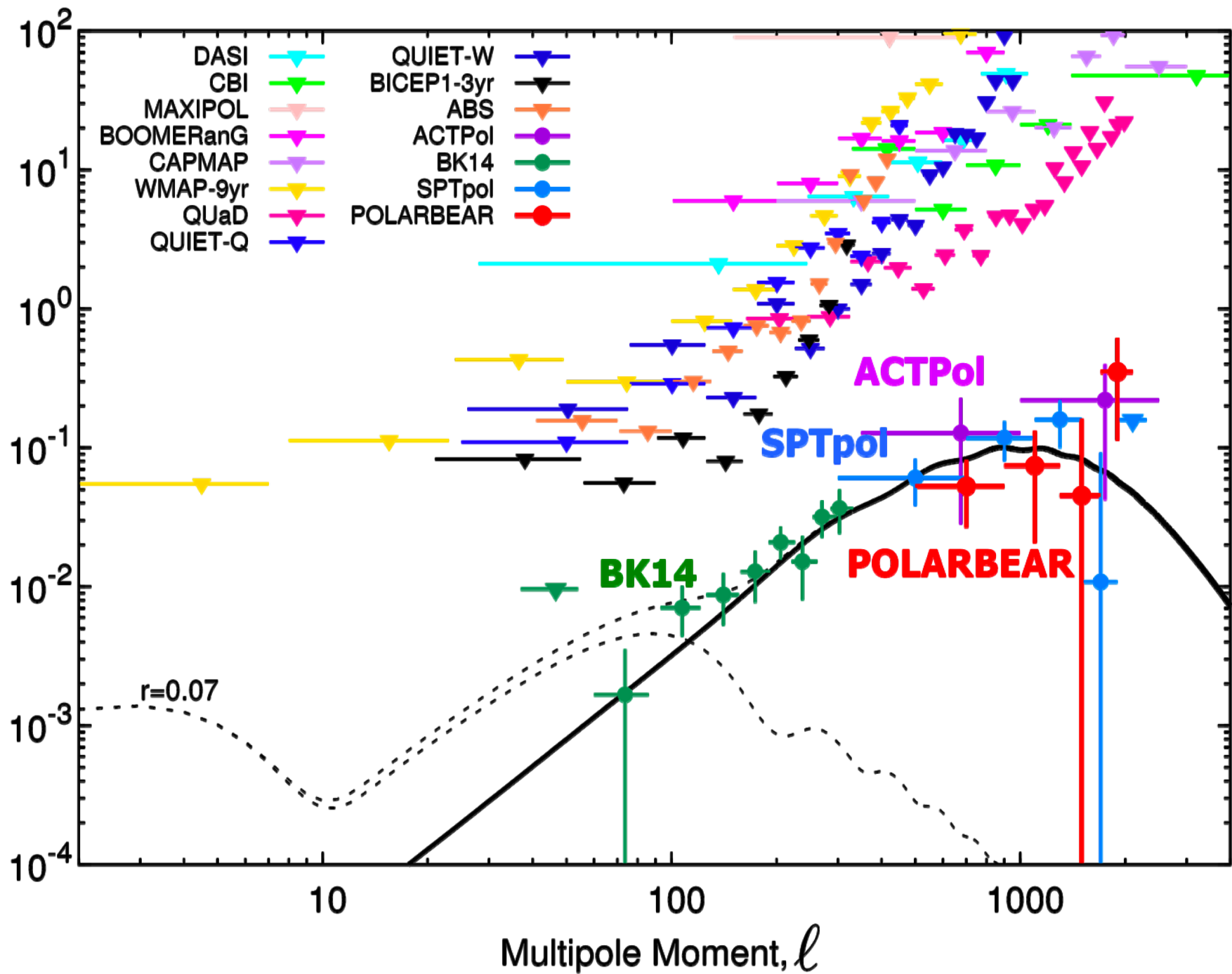
Waves

Other Experiments coming on line

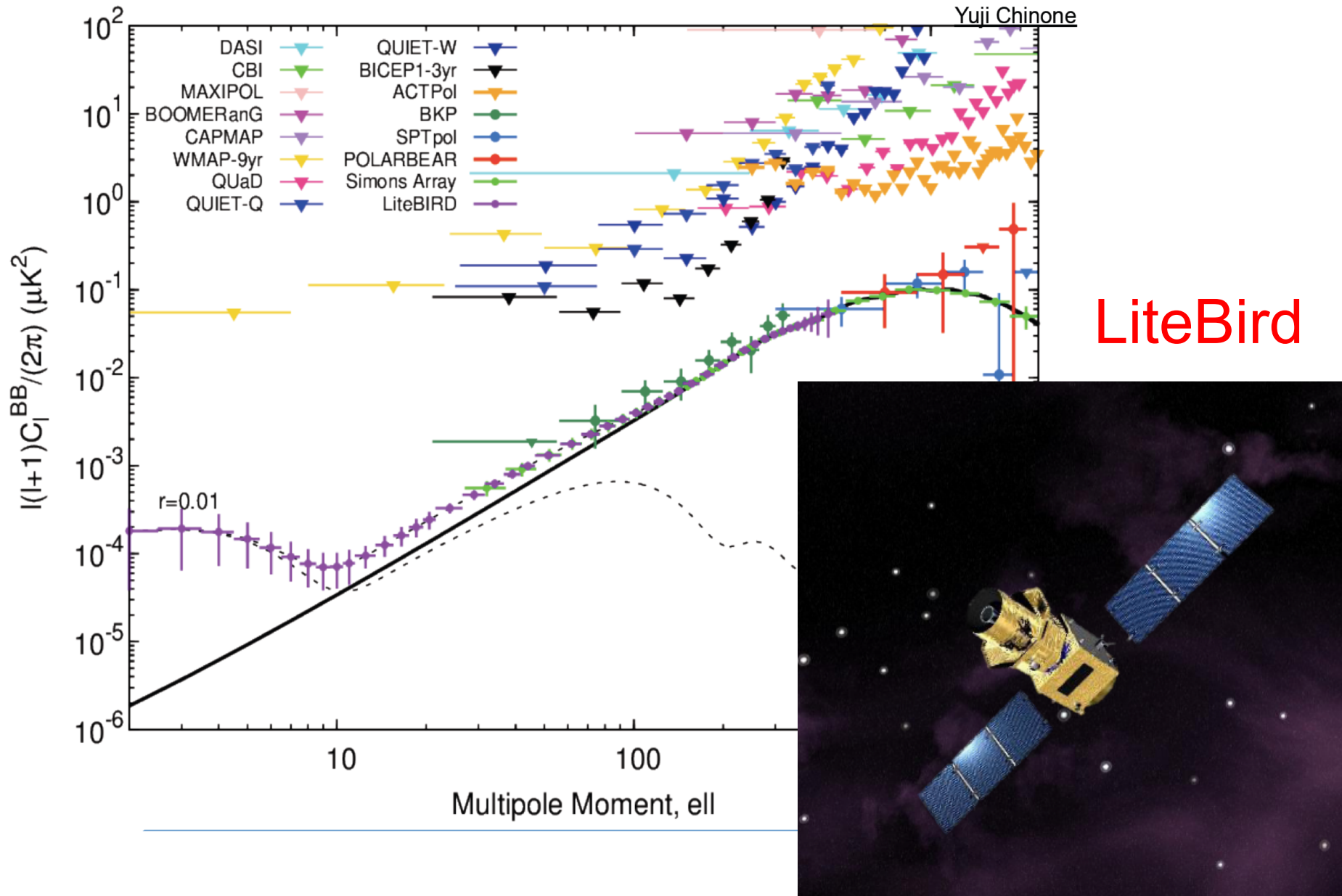
South Pole CMB telescopes



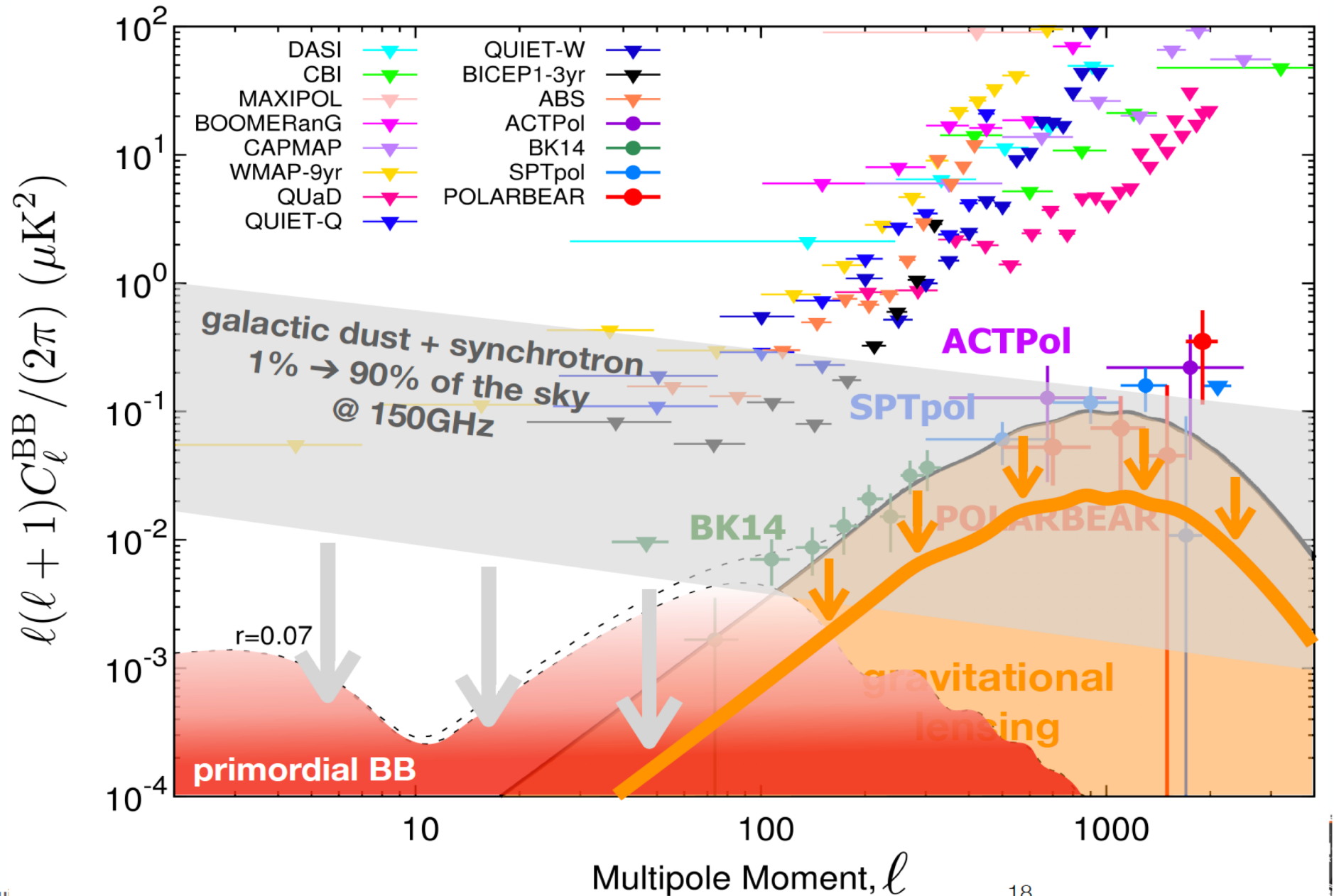
Chile(Atacama): PolarBear, ACTpol
Balloon @ South Pole: EBEX(6K)

$\ell(\ell + 1)C_\ell^{\text{BB}} / (2\pi) \ (\mu\text{K}^2)$ 

Future experiments that will search for B modes



The Simons Observatory science goals



Conclusions

- The Standard Model of Cosmology (SMC) is well founded in GR, QFT and Thermodynamics.
- Early Universe Inflation is a rich paradigm with confirmed predictions.
- The cosmic microwave background is the more complete source of information we have at the moment on the SCM.
- Soon we will have multiprobe access to SMC:
with CMB, LSS and GW.