Neutrino Course

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Outline

- Introduction to neutrino physics
- Neutrinos in the Standard Model
- Neutrino masses beyond the Standard Model
- Neutrino oscillations in vacuum and matter
- Three-flavour neutrino oscillations
- Neutrino oscillations beyond 3 flavours: sterile neutrinos
- The absolute scale of neutrino mass
- Neutrino physics beyond the Standard Model

Neutrino Course (Part I)

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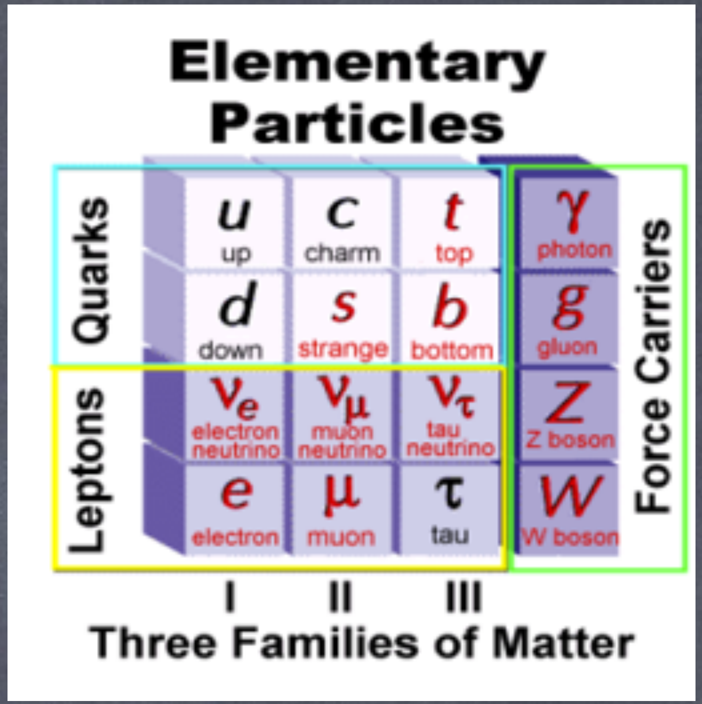
What is a neutrino?

- spin 1/2 particle
- massless particle (almost)
- neutral
- 3 flavors (mixing)



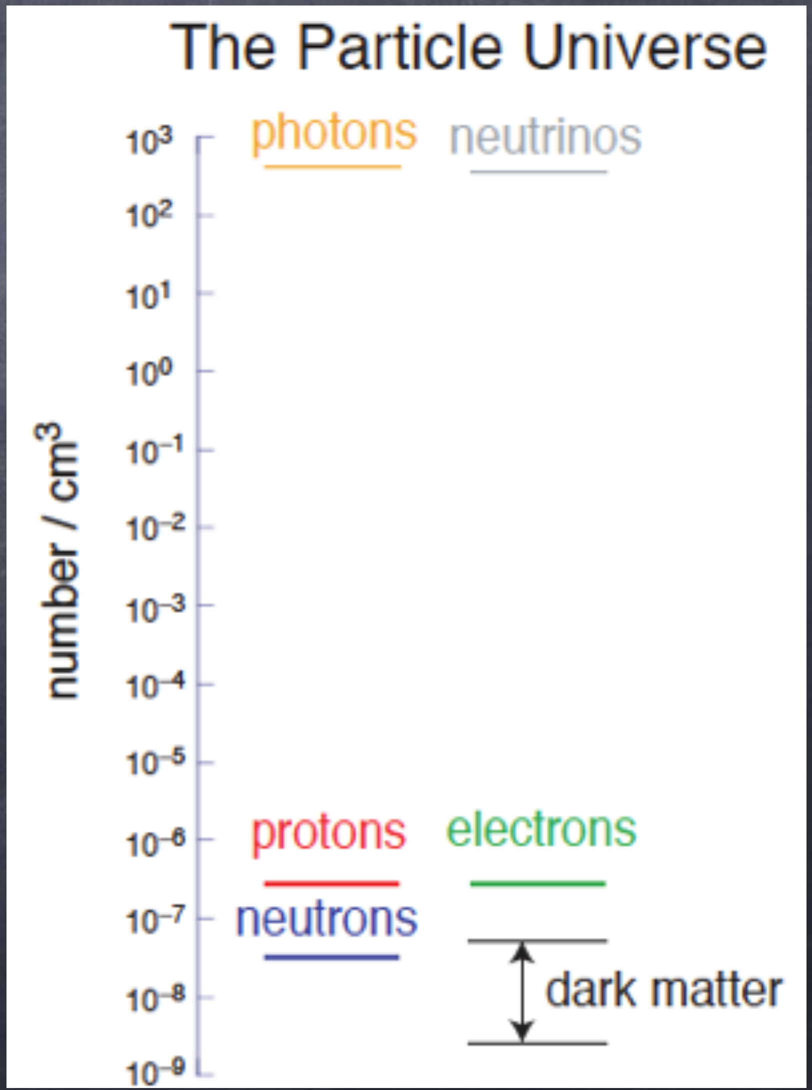
Anything else?

Every second we are traversed by:

- 400×10^{12} neutrinos from the Sun
- 50×10^{9} neutrinos from natural radioactivity
- 10×10^{9} neutrinos from nuclear power plants

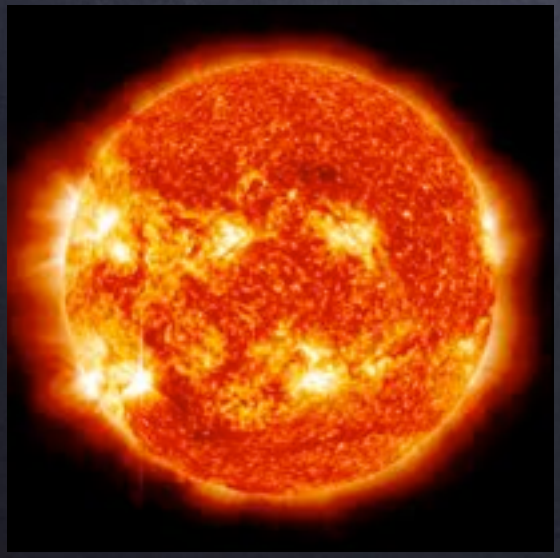
Moreover:

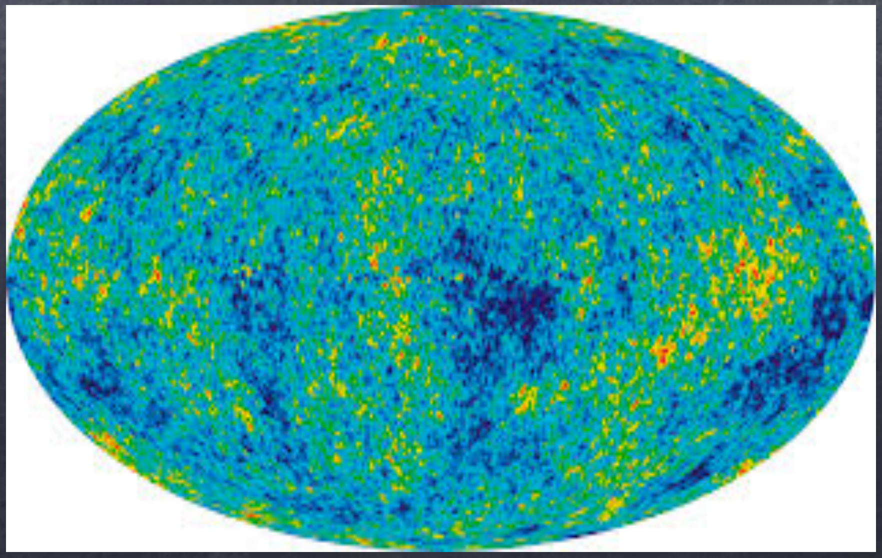
- our body emits 400 neutrinos/s (^{40}K decay)
- the Universe contains ~ 330 neutrinos/cm^3



Why neutrinos are so important?

- they can probe environments that other techniques cannot: SN explosions, core of the Sun,...
- their role is crucial for the evolution of the universe (Big Bang Nucleosynthesis, structure formation)
- they could help explaining the matter-antimatter asymmetry of the Universe (leptogenesis mechanism)
- they could be a component of the dark matter of the universe.

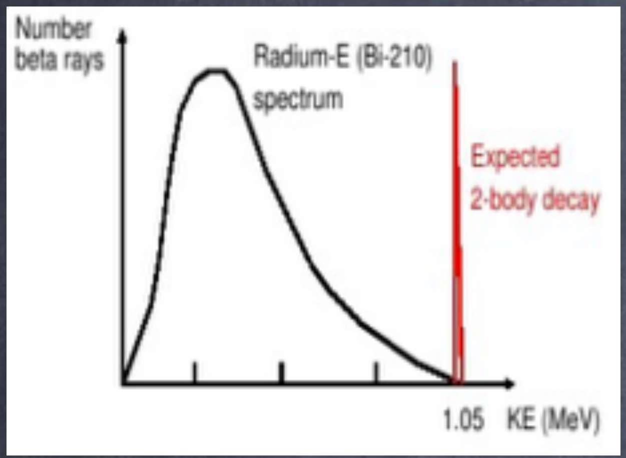


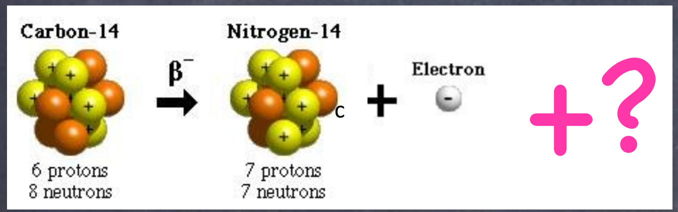



Historical introduction to neutrino physics

The proposal of the neutrino

- 1930: Pauli introduced the neutrino to explain continuous electron spectrum in nuclear beta decay.

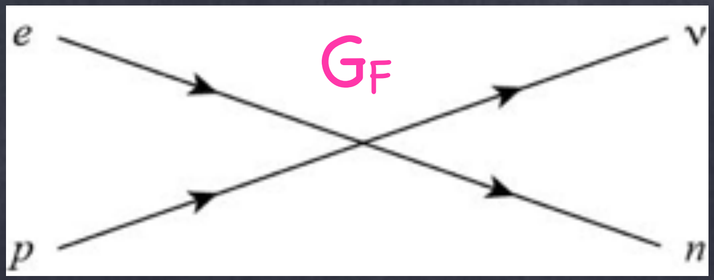


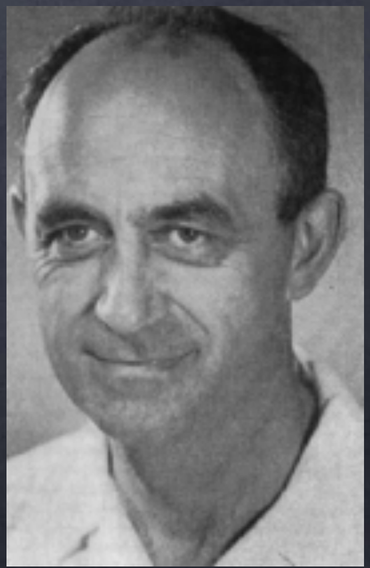


"Dear radioactive ladies and gentlemen,
I have come upon a desperate way out regarding ... [some fairly obscure data], as well as to the continuous β-spectrum, in order to save ... The energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles which I shall call neutrons, which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. ... The continuous β-spectrum would then become understandable from the assumption that in β-decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant."

- 1933: Fermi postulated the first theory of nuclear beta decay, the theory of weak interactions

$$n \to p + e^- + \bar{\nu}_e$$





➡ new name for particle: neutrino

Where was the neutrino?

- 1934: Bethe and Peierls calculated the cross section σ for the processes:

$$\nu + n \rightarrow p + e^-$$

$$\bar{\nu} + p \rightarrow n + e^+$$

Fermi theory predicted (for ν + p):

$$\sigma \approx 10^{-44}\mathrm{cm}^2 \left(\frac{\mathrm{E}_\nu}{\mathrm{m}_e \mathrm{c}^2}\right)^2$$

for $E_\nu \sim 2$ MeV: $\sigma \sim 10^{-43}$ cm^2

(to be compared with $\sigma_{\gamma p} \sim 10^{-25}$ cm^2) !!!

→ mean free path of neutrinos in water: $\lambda_{water} \approx 1.7\mathrm{x}10^{17}$ m $\sim$ 15 ly

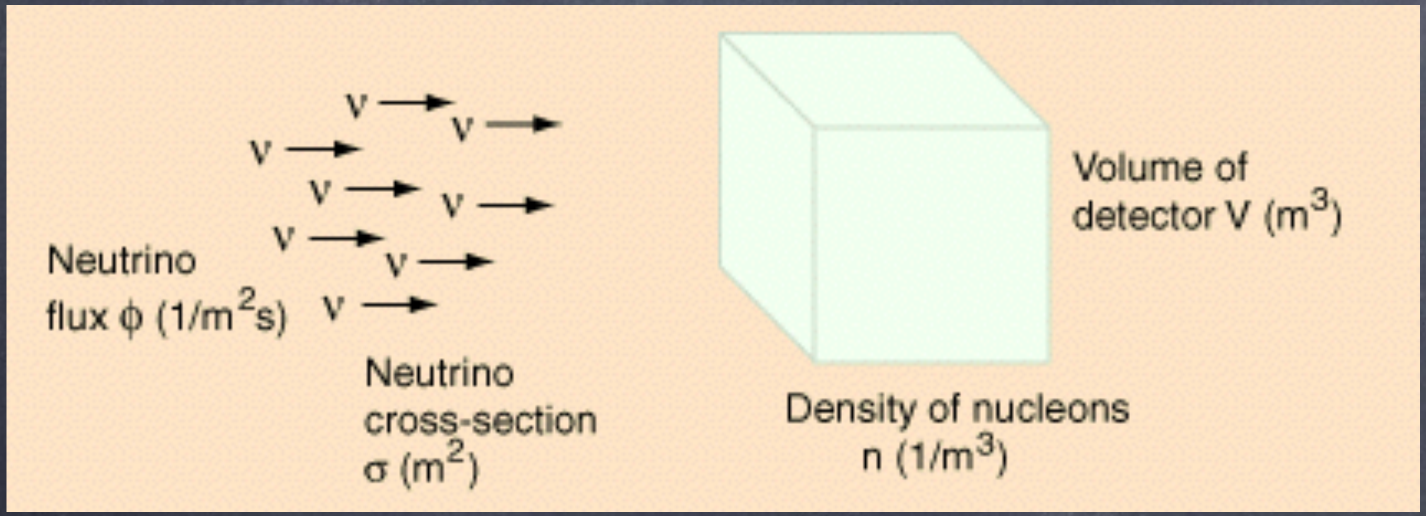
→ mean free path of neutrinos in lead: $\lambda_{lead} \approx 1.5\mathrm{x}10^{16}$ m $\sim$ 1.5 ly

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

Pauli, 1930

Event number in a neutrino experiment:



$$N = \varphi \; \sigma \; N_{targ} \; \Delta t$$

- with a 1000 kg detector and a flux of 10^{10} ν/s: few ν events/day

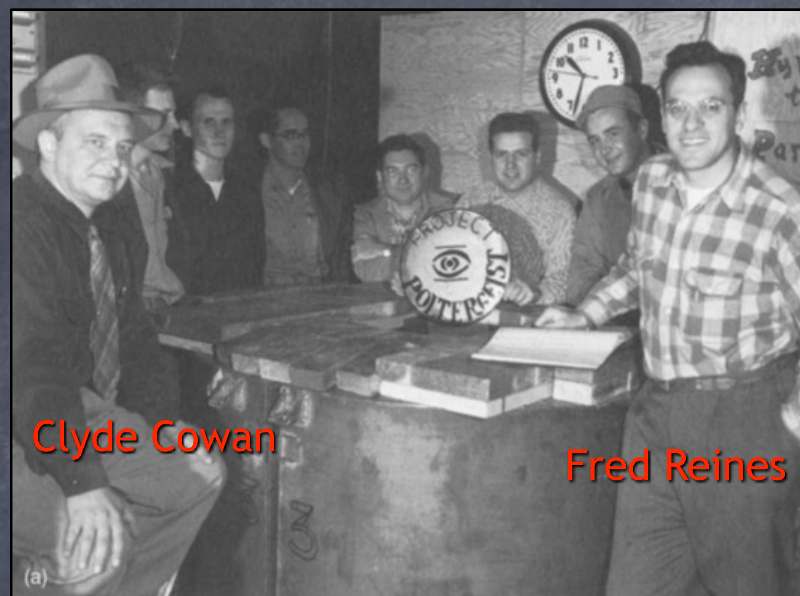
→ solar neutrino flux ~ 7×10^{10} $\nu/cm^2/s$

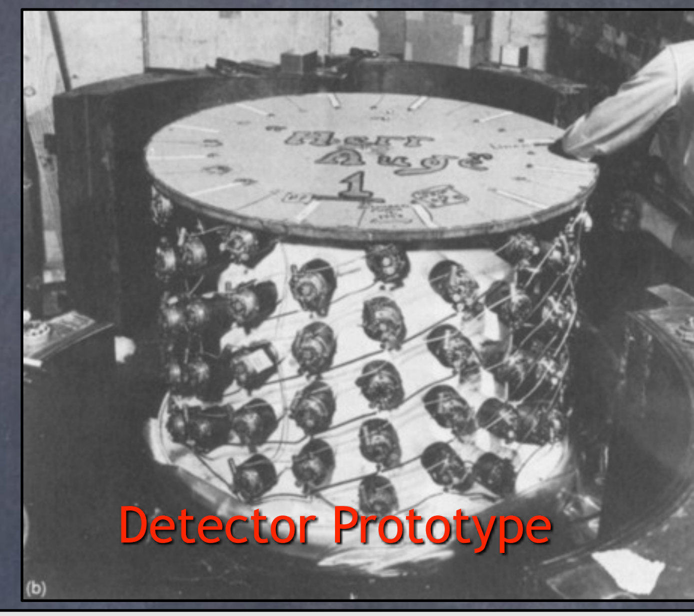
→ reactor neutrino flux ~ 10^{20} ν/s

Difficult but not impossible!

Discovery of the neutrino

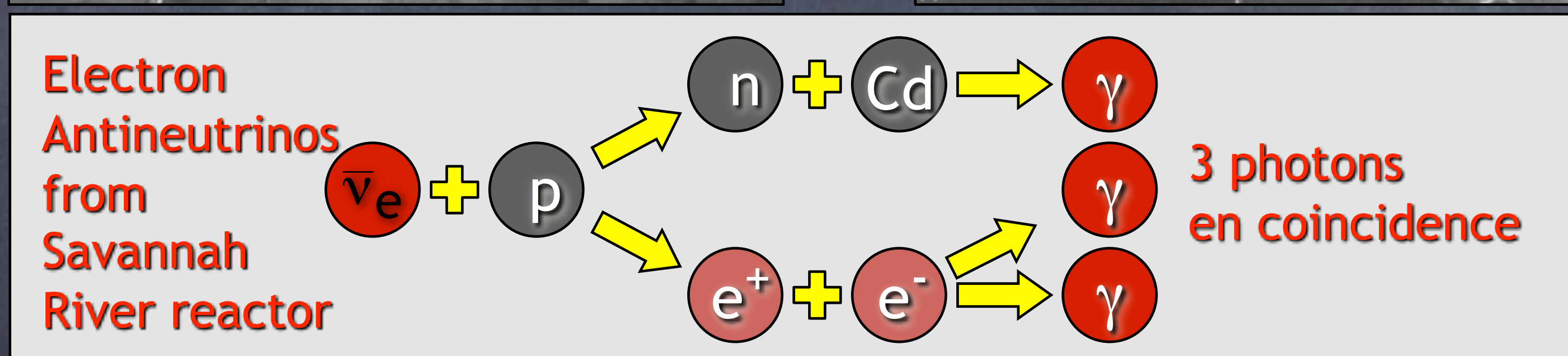
- 1956: First observation of reactor ν_e by Reines and Cowan.





2 tanks with
200 liters H_2O
+
40 kg $CdCl_2$

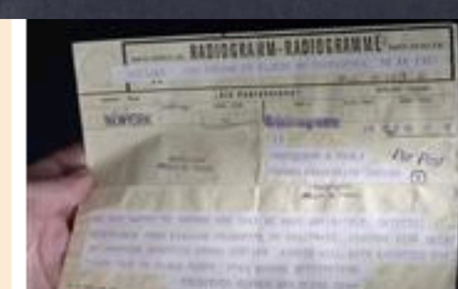
3 scintillator
layers with PMTs



1995 Nobel Prize
in Physics to
Reines

Telegram to Pauli on 12/06/1956

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected six times ten to minus forty-four square centimeters"

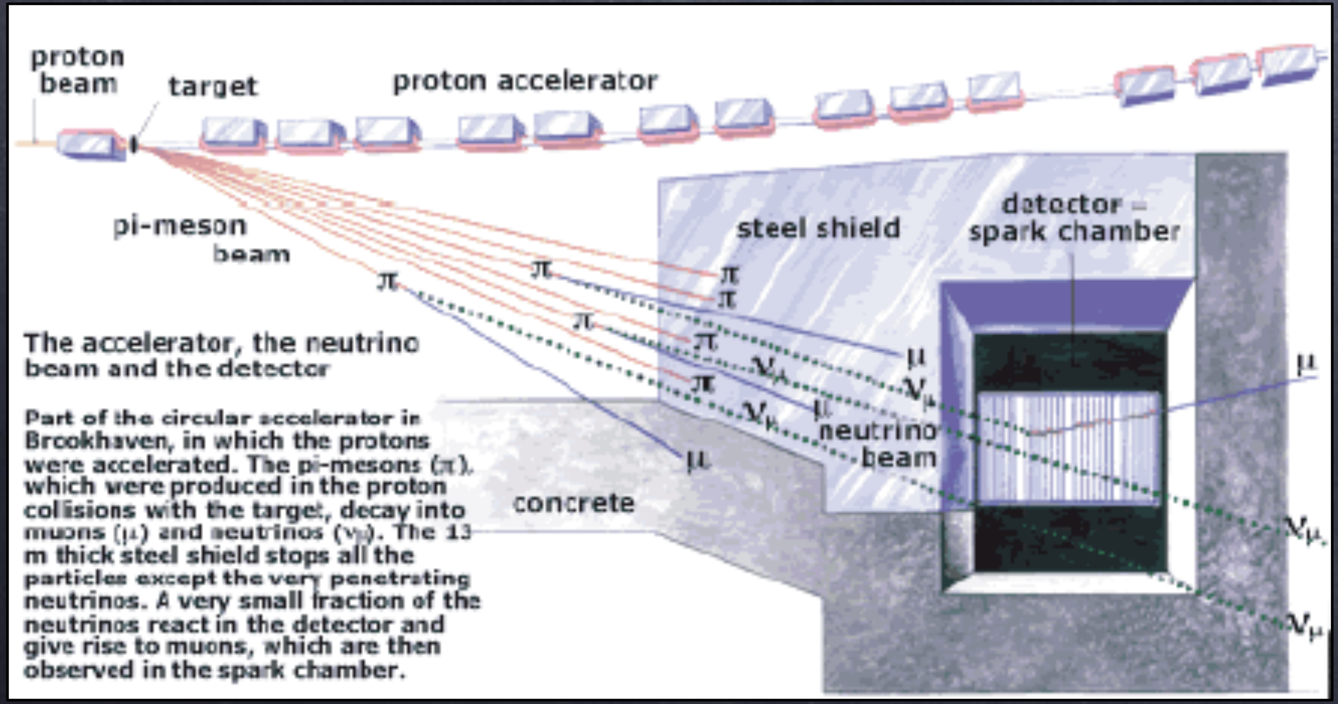


More than one neutrino flavour?

- 1959: Pontecorvo suggested the existence of a different neutrino, associated to muon decay and proposed an experiment to check it.

$$\nu_{\rm acc} + n \to p + (e^- \,{\rm or}\, \mu^- \,?)$$

- 1962: Discovery of ν_μ by Lederman, Schwartz and Steinberger



$$\pi^+ \to \mu^+ + \nu_\mu$$

not e^-

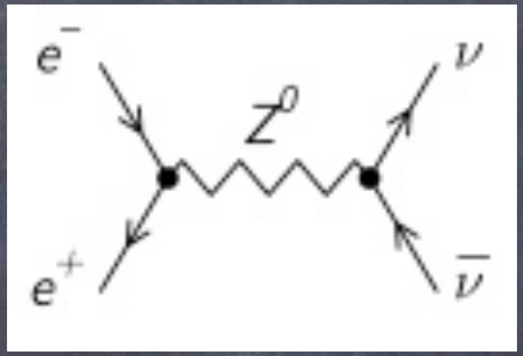
$$\nu_\mu + n \to p + \mu^-$$

1988 Nobel Prize in Physics

More than two neutrino flavours?

- 1978: Discovery of τ at SLAC $\rightarrow$ imbalance of energy in τ decay suggests existence of a third neutrino.

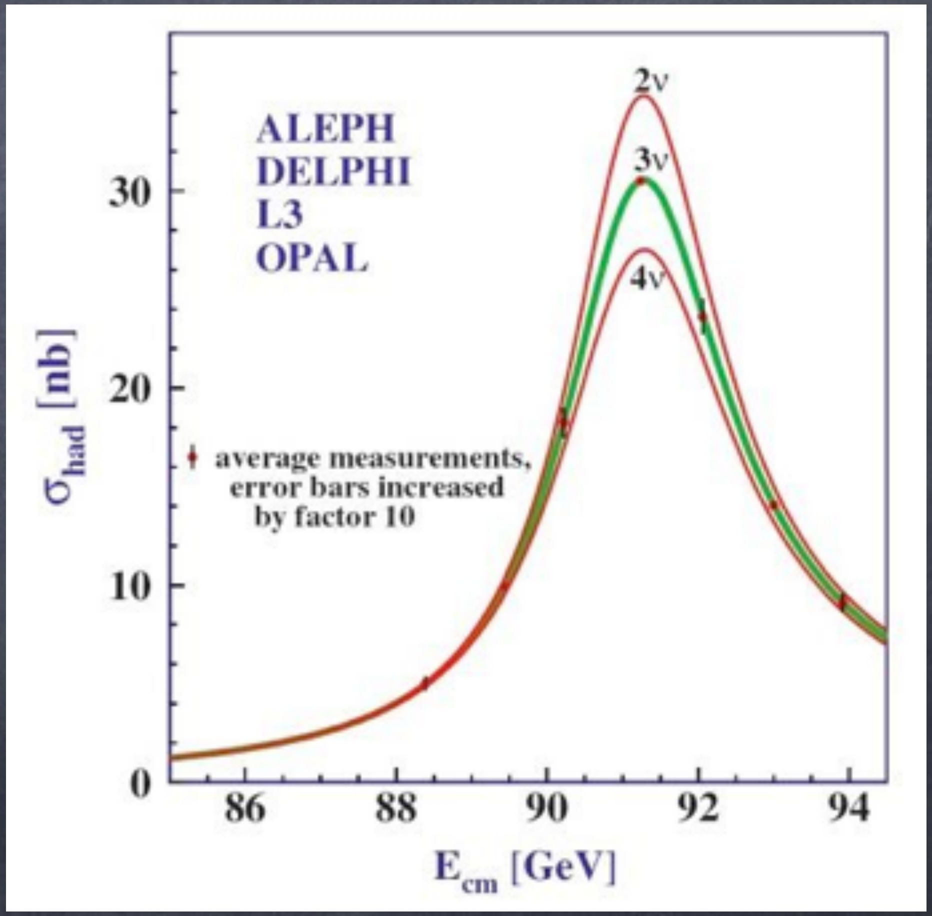
- 1989: LEP measurements of the invisible decay width of Z boson



$$\Gamma_{\rm inv} \equiv \Gamma_{\rm Z} - \Gamma_{\rm had} - 3\Gamma_{\rm lep}$$

$$N_\nu = \Gamma_{\rm inv}/\Gamma_{\rm SM}(Z \rightarrow \nu_i \bar{\nu}_i)$$

$\rightarrow N_\nu = 2.984 \pm 0.008$



- 2000: Discovery of ν_τ by the DONUT Collaboration.

800 GeV p $\rightarrow$ D_s meson ($\equiv c\bar{s}$) $\rightarrow$ ν_τ beam $\rightarrow$ τ detected

Neutrino oscillations

‣1957: Pontecorvo suggests oscillations between neutrinos & antineutrinos (only ν_e).

B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957)549.
B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.

‣1962: Maki, Nakagawa and Sakata proposed flavor neutrino oscillations.

$$\nu_1 = \nu_e \cos\delta + \nu_\mu \sin\delta,$$
$$\nu_2 = -\nu_e \sin\delta + \nu_\mu \cos\delta.$$

2ν mixing

true neutrinos

weak neutrinos

Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

‣1969: Gribov & Pontecorvo calculated the neutrino oscillation probability (in vacuum) for the first time

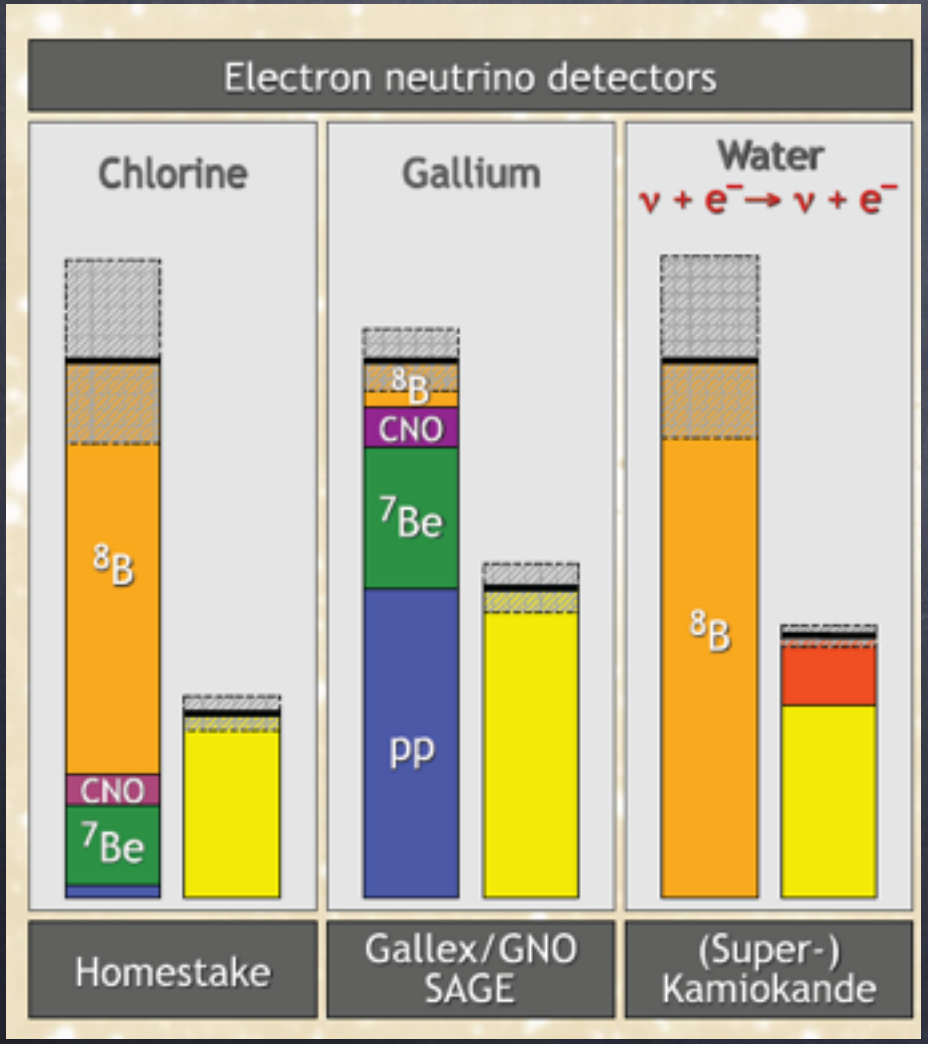
V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493.

First indication of ν oscillations

- 1968: First observation of solar neutrinos by R. Davis in Homestake.

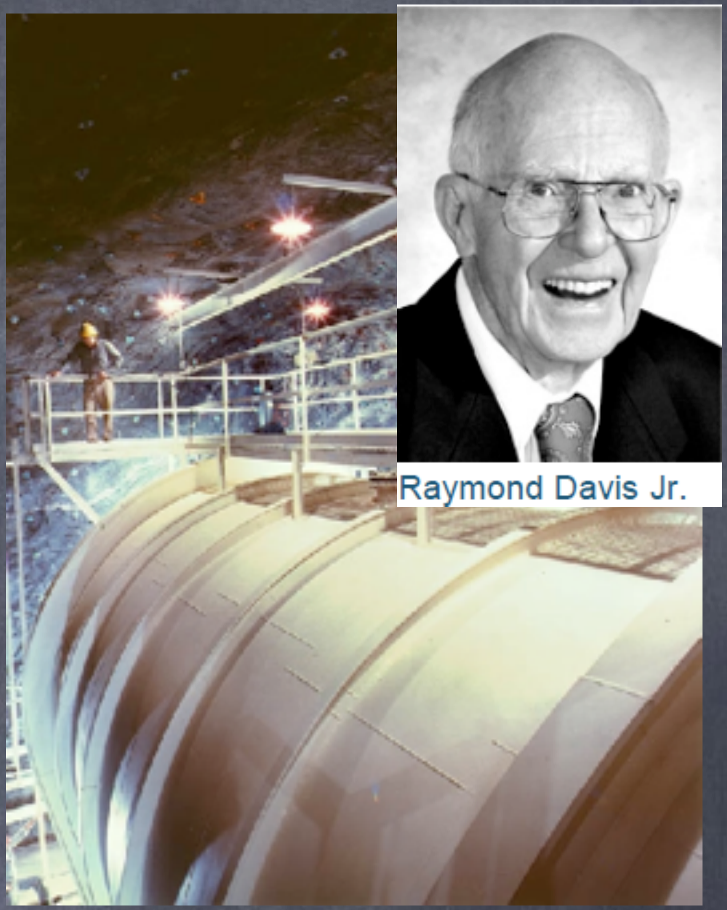
$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

➡ 1/3 of the Standard Solar Model prediction !!



~30% ~50% ~40%

→ confirmed by the following experiments



2002 Nobel Prize in Physics

Explanation?

→ theory (SM, SSM) was wrong

→ experiments were wrong (all of them?)

→ something was happening to neutrinos

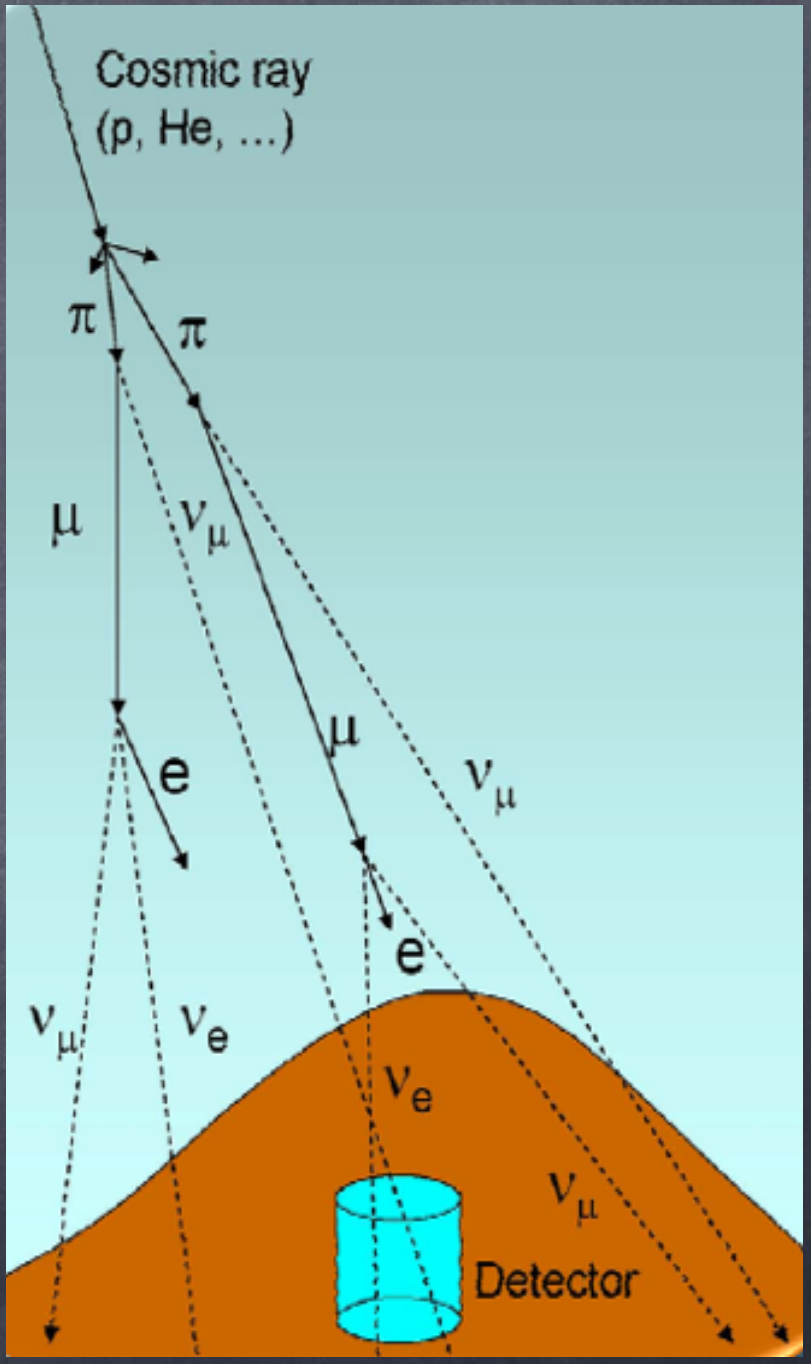
The atmospheric ν anomaly

‣1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.

‣1994: Kamiokande finds the ν_μ deficit depends on travelled by the neutrino.

‣1998: Discovery of atmospheric neutrino oscillations in Super-Kamiokande.

➡ first evidence for non-zero neutrino masses.



$\boxed{\text{oscillation channel } \nu_\mu \rightarrow \nu_\tau}$

Other important dates

M. Koshiba

2002 Nobel Prize in Physics

- 1987: Supernova neutrino detection from supernova 1987A in Kamiokande & IMB.

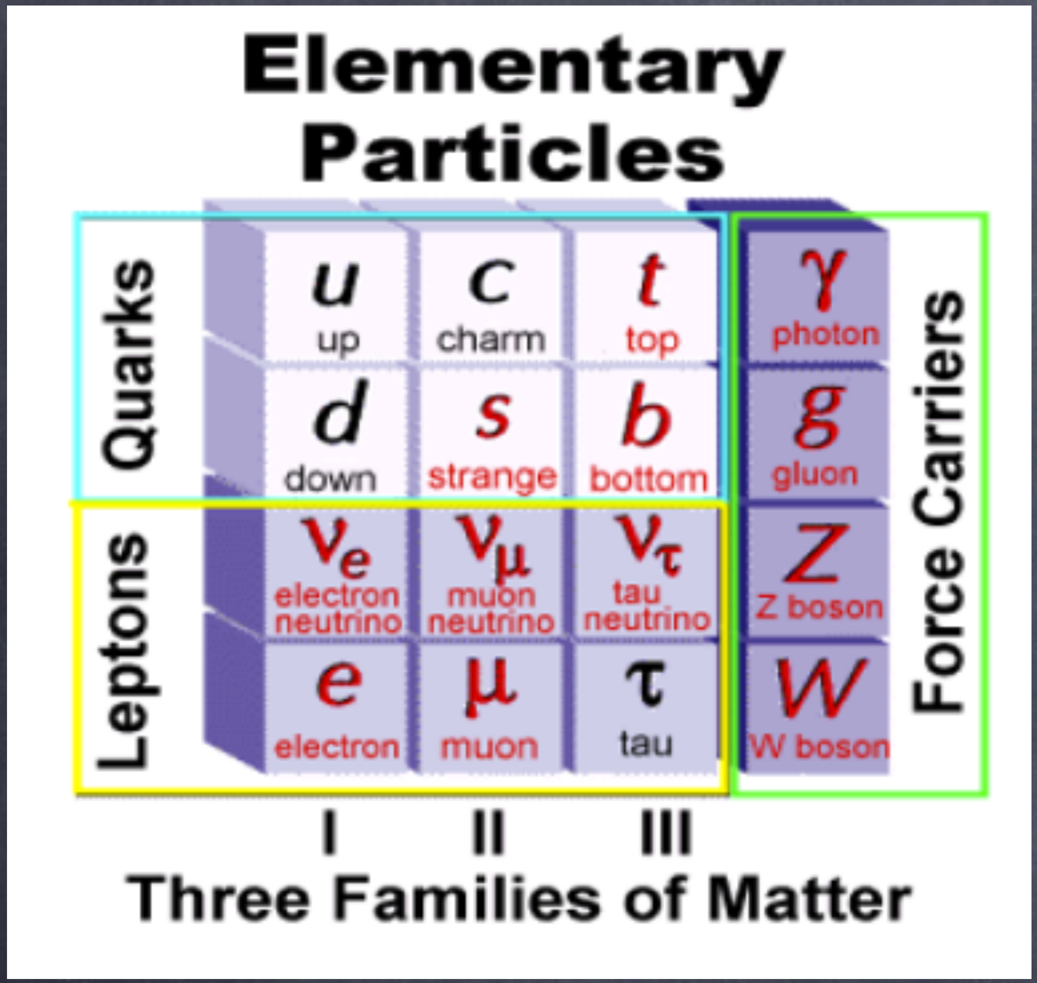
- 2001: Sudbury Neutrino Observatory (SNO) confirms a change of flavor in solar ν_e flux.

- 2002: KamLAND experiment confirms solar neutrino oscillations using neutrinos from nuclear reactors

- 2011-2012: neutrino oscillations observed in solar, atmospheric, reactor and accelerator neutrino experiments.

Neutrinos in the Standard Model

Neutrinos in the Standard Model



- neutrinos come in 3 flavours, corresponding to the charged lepton associated
- they belong to SU(2) lepton doublets

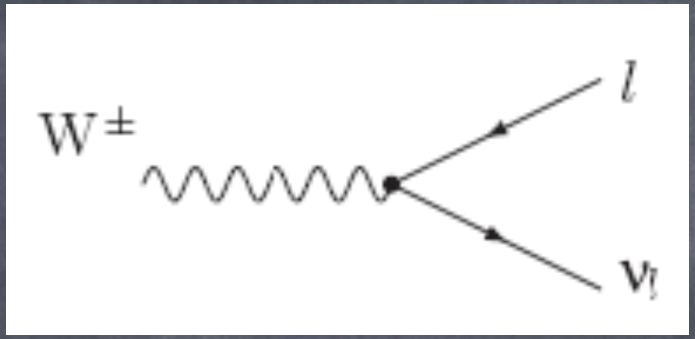
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

- In the SM, there are no SU(2) neutrino singlets (alike e_R, μ_R, τ_R)
- neutrinos interact only through weak force
- In the SM, neutrinos do not have mass

From ν oscillations, we know $m_\nu \neq 0$

Neutrino interactions in the SM

Charged Current (CC):



$$W^- \to l_\alpha^- + \bar{\nu}_\alpha$$

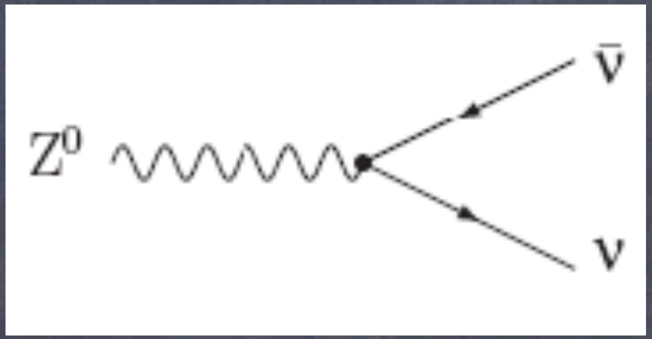
$$W^+ \to l_\alpha^+ + \nu_\alpha$$

$$\alpha = e, \mu, \tau$$

$$\mathcal{L}_{\rm int}^{\rm CC} = -\frac{g}{\sqrt{2}}\left(\sum_\alpha \bar{\nu}_{\alpha L}\gamma_\rho l_{\alpha L} W^\rho + \text{h.c.}\right)$$

in the SM, only LH neutrinos and RH antineutrinos participate in weak interactions

Neutral Current (NC):



$$Z^0 \to \nu_\alpha + \bar{\nu}_\alpha$$

$$\mathcal{L}_{\rm int}^{\rm NC} = -\frac{g}{4\cos\theta_W}\left(\sum_\alpha \bar{\nu}_\alpha\gamma_\rho(1-\gamma_5)\nu_\alpha Z^\rho + \text{h.c.}\right)$$

- interactions conserve total Lepton Number L:

$$L(l^-) = L(\nu) = -L(l^+) = -L(\bar{\nu}) = 1$$

- family lepton numbers L_e, L_μ, L_τ are also conserved (1998: nu oscill !!)

Neutrino mass in the Standard Model

- In the SM, fermion masses appears in the lagrangian in the term:

$$m\bar{\psi}\psi$$ $\rightarrow$ Dirac mass term

decomposing into its chiral states: $\psi = \nu \equiv \nu_L + N_R$

$$\boxed{-\mathcal{L}_D = m_D\bar{\nu}\nu = m_D(\overline{\nu_L} + \overline{N_R})(\nu_L + N_R) = m_D(\overline{\nu_L}N_R + \overline{N_R}\nu_L)}$$

$\rightarrow$ mass couples L and R chiral states of a particle: flips chirality

$\rightarrow$ OK for most of particles but SM neutrino has only a L-chiral state

$\rightarrow$ neutrinos are massless unless one adds a R-chiral state, N_R

Neutrino masses: Majorana neutrinos

- Other option: try to make a mass term from ν_L alone

Majorana, ~1930

→ a R-chiral field from a L-chiral field by charge conjugation:

$$\psi_R \equiv \psi_L^C = \hat{C}\overline{\psi_L}^T \qquad \hat{C} = i\gamma^2\gamma^0$$

→ the total neutrino field is: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^C$ 2 degrees of freedom

→ taking the charge conjugate $\psi^C = (\psi_L + \psi_L^C)^C = \psi_L^C + \psi_L = \psi$

$$\psi = \nu = \nu_L + \nu_L^C$$

neutrino = antineutrino

- Majorana mass term: $\boxed{-\mathcal{L}_\mathrm{M} = \frac{1}{2} m(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c)}$

However: mass term not invariant under weak isospin

Solution: coupling with Higgs triplet (not present in the SM) !!!!

Dirac mass term

$$-\mathcal{L}_D = m_D(\overline{\nu_L} N_R + \overline{N_R}\nu_L)$$

Majorana mass term

$$-\mathcal{L}_{\rm M} = \frac{1}{2} m(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c)$$

under U(1) transformation:

$$\psi \to e^{i\alpha}\psi, \qquad \overline{\psi} \to \overline{\psi} e^{-i\alpha}$$

invariant

not invariant

$\rightarrow$ conserves all charges (Q, L, B)

$\rightarrow$ breaks all charges in 2 units

1) charged particles must be Dirac OR only neutral particles can be Majorana

$\rightarrow$ neutrino, with Q(ν) =0, can be Majorana

2) if neutrinos are Majorana, total lepton number is not conserved

3) if neutrinos are Dirac, L conservation has to be imposed by hand

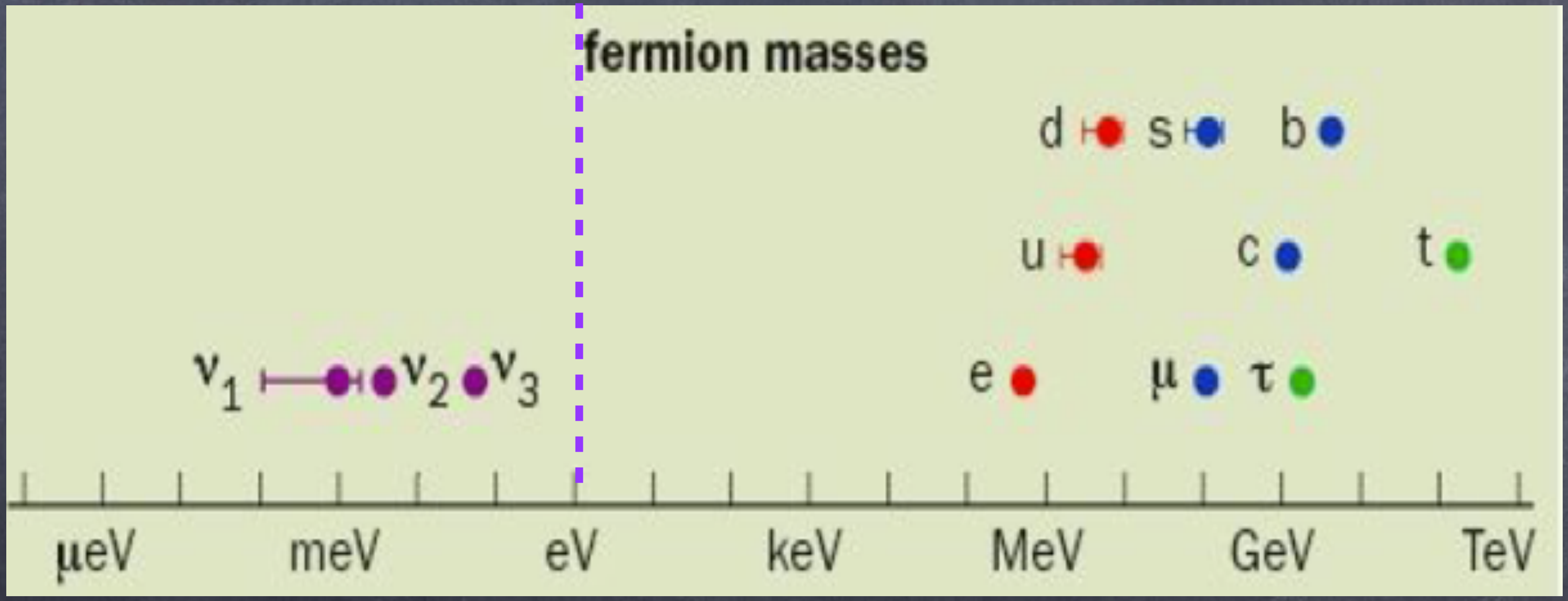
However: none of the terms can be constructed in SM

$\rightarrow$ no N_R in SM

$\rightarrow$ $\overline{\nu_L^c}\nu_L$ forbidden by weak isospin

Neutrinos are massless within the SM

But from oscillations we know neutrinos do have mass!!



$m_\nu \sim 10^{-3} - 1$ eV

Neutrino masses beyond the Standard Model

Dirac mass term

Minimal extension SM: add N_R $\rightarrow$ "sterile" neutrino

- 4 components Dirac neutrino: $\nu_L\ ,\ \overline{\nu_L}\ ,\ N_R\ ,\ \overline{N_R}$ 4 degrees of freedom

-> decomposing into its chiral states: $\psi = \nu \equiv \nu_L + N_R$

$$\boxed{-\mathcal{L}_D = m_D \bar{\nu}\nu = m_D(\overline{\nu_L} + \overline{N_R})(\nu_L + N_R) = m_D(\overline{\nu_L}N_R + \overline{N_R}\nu_L)}$$

- From ν oscill: $m_\nu \geq \sqrt{\Delta m^2_{31}} = 0.05\,\mathrm{eV}$

$$\mathcal{L}_{\mathrm{Yukawa}} = Y_\nu (\bar{\nu}_e\, \bar{e})_L \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} N_R + h.c.$$

-> after SSB: $<\phi>= \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$ $\quad m^D_\nu = Y_\nu \frac{v}{\sqrt{2}} \rightarrow Y_\nu \simeq 10^{-13}$

much smaller than other Yukawas: $Y_e \simeq 10^{-5}$

Minimal extension of SM for neutrino mass

- Add a right handed neutrino singlet under SU(2)xU(1):

$$\nu = \nu_L + \nu_L^C$$

$$N = N_R + N_R^C$$

SU(2) forbidden

- Most general mass term:

$$\mathcal{L} = \mathcal{L}_{\rm D} + \mathcal{L}_{\rm M} = \frac{1}{2}\left(\overline{\nu_L} \quad \overline{N_R^c}\right)\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

not mass eigenstates

$\rightarrow$ diagonalization:

$$\frac{1}{2}\left(\overline{\nu} \quad \overline{N}\right)\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix}$$

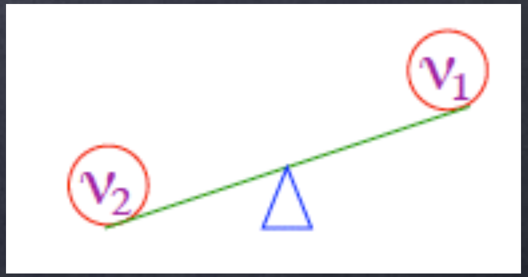
$$(m_D \simeq v Y_\nu)$$

for $M_R \gg m_D$:

$$M_1 \simeq \frac{m_D^2}{M_R}$$

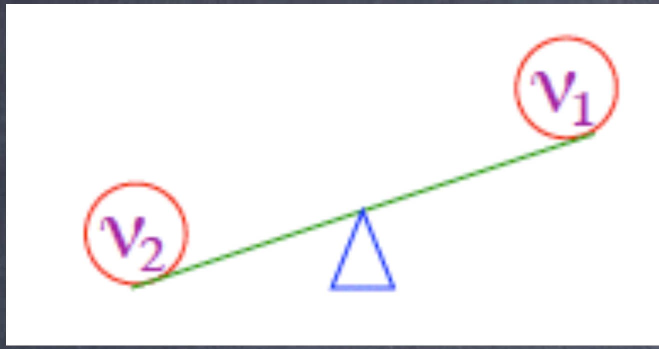
$$M_2 \simeq M_R$$

$\rightarrow$ seesaw mechanism



Seesaw mechanism for neutrino mass

- Provides a "natural" explanation for smallness of neutrino mass:



$$M_1 \simeq \frac{m_D^2}{M_R}, \quad M_2 \simeq M_R$$

for m_D~100 GeV and m_ν~0.01 eV

$\rightarrow$ M_R ~10^{15} GeV !!!

- Can explain baryon asymmetry of the Universe through leptogenesis:

if heavy neutrino decay violates CP: $\Gamma(N \to l + H) \neq \Gamma(N \to \bar{l} + \overline{H})$

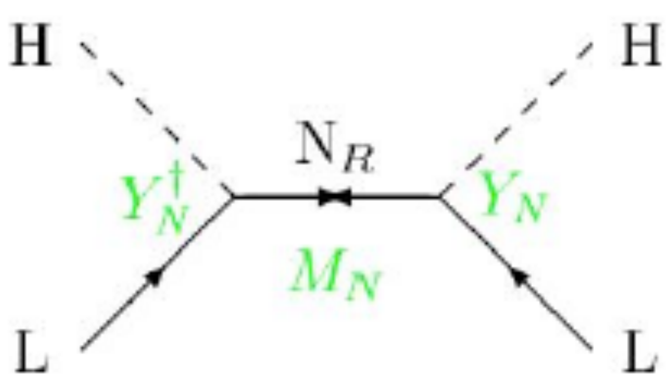
$\rightarrow$ thanks to (B-L) conservation, the lepton asymmetry generated L may be transformed in B asymmetry through "sphaleron processes":

$$B \neq \overline{B}$$

Seesaw mass models

⇒ neutrino masses are generated through their mixing with heavy particles

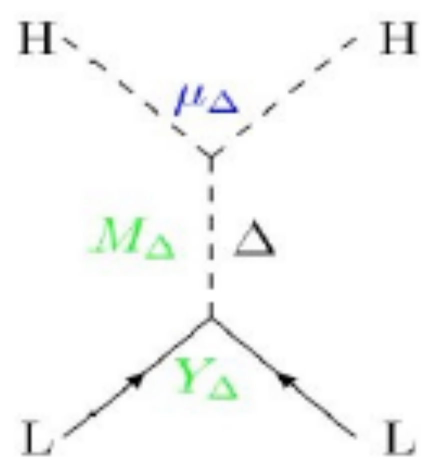
Right-handed singlet:
(type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

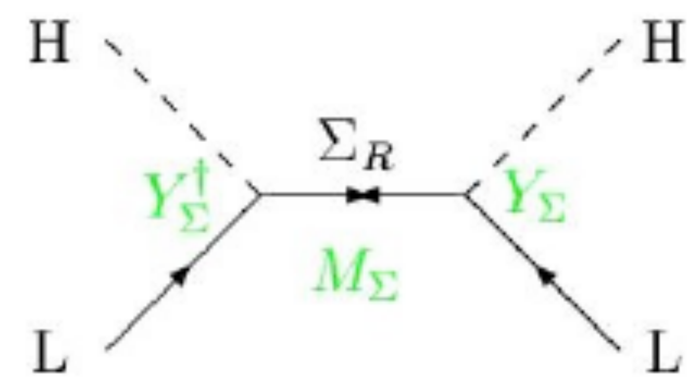
Scalar triplet:
(type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet:
(type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

Low energy seesaw models

Inverse seesaw model

Mohapatra and Valle, PRD 34 (1986) 1642

Extended lepton content: $(\nu, \underbrace{\nu^c, S}_{\text{SU(2) singlets}})$ L=(+1,-1,+1)

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \rightarrow m_\nu = M_D (M^T)^{-1} \mu \, M^{-1} M_D^T$$

- μ breaks L and generates neutrino mass (massless for μ=0)
- m_ν can be very light even if M is far below GUT scale:

with $\mu \sim$ keV and M$\sim 10^3$ GeV $\rightarrow$ $m_\nu \sim$eV

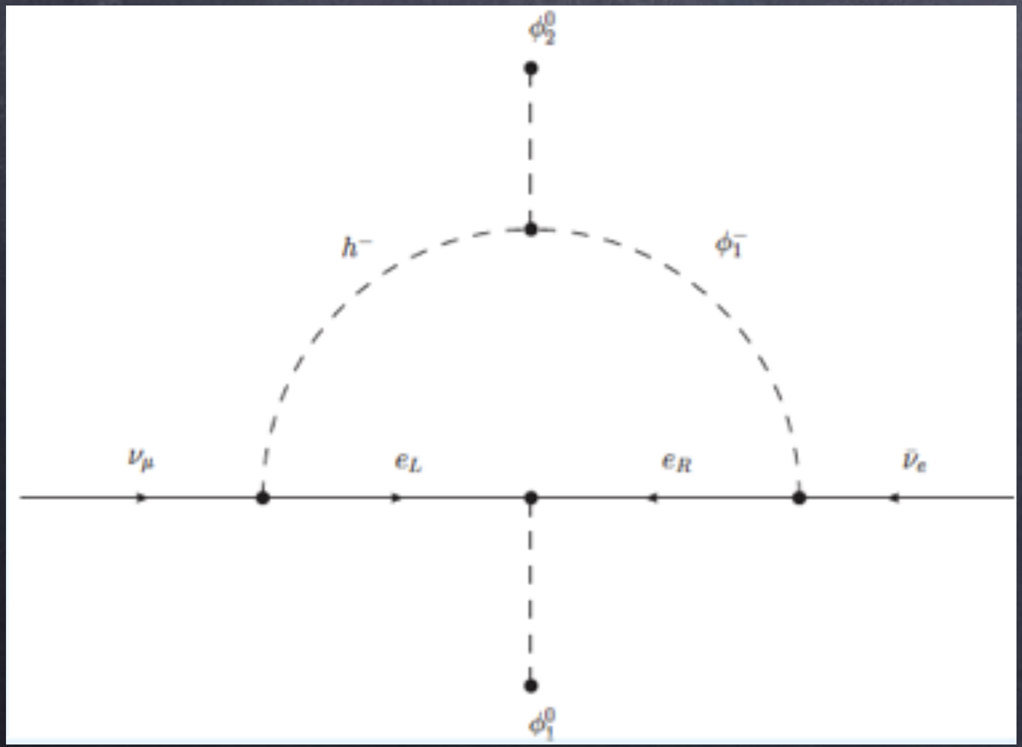
Radiative models of neutrino masses

- extension of scalar sector of the SM -> generate L violation
- neutrino masses can be generated through loops

  $\Rightarrow$ loop suppression accounts for the smallness of m_ν

Zee model

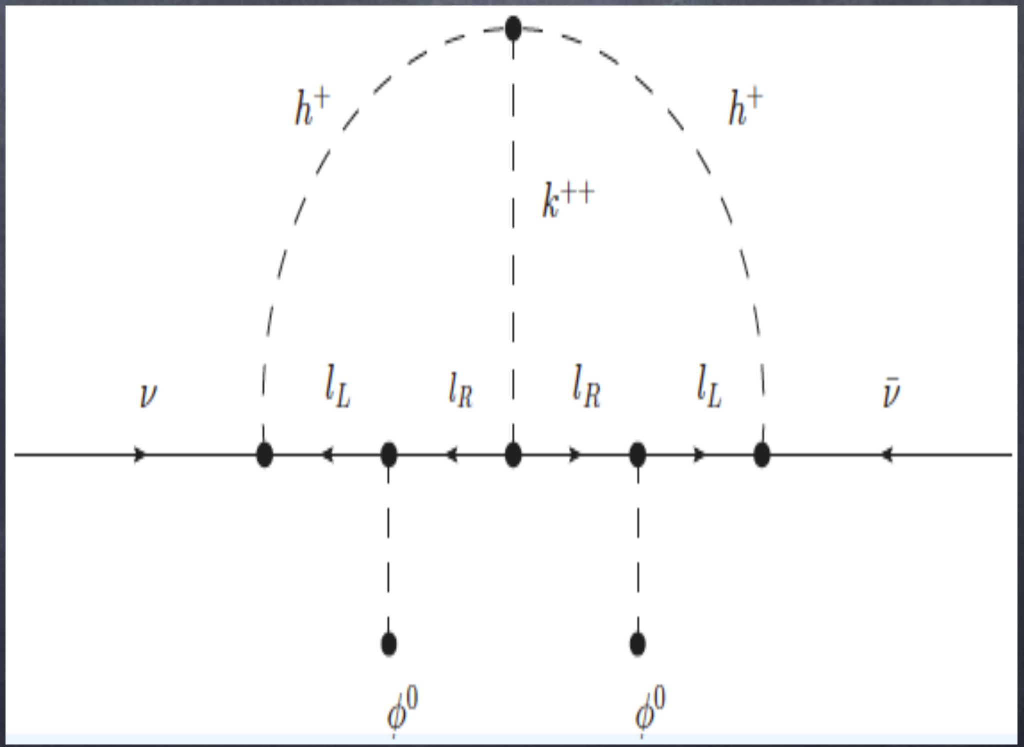
+ singlet scalar h^+
+ extra Higgs doublet H



Zee, PLB 93 (1980) 389

Zee-Babu model

+ singlet scalar h^+
+ singlet scalar k^{++}



Zee, NPB 264 (1986) 99; Babu, PLB 203 (1988) 132

The flavour problem

- seesaw models explain the smallness of neutrino masses

However, they can not explain:

- Why quark and lepton mixing and masses are so different?

$$U_{\rm CKM} = \begin{pmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.008 & 0.040 & 0.999 \end{pmatrix}$$

$$U_\nu = \begin{pmatrix} 0.80-0.83 & 0.36-0.45 & 0.15-0.18 \\ 0.46-0.54 & 0.47-0.59 & 0.63-0.75 \\ 0.24-0.36 & 0.59-0.69 & 0.65-0.76 \end{pmatrix}$$

$\theta_{12} \simeq 13^\circ$, $\theta_{13} \simeq 0.2^\circ$, $\theta_{23} \simeq 2.4^\circ$

$\theta_{12} \simeq 34^\circ$, $\theta_{13} \simeq 9^\circ$, $\theta_{23} \simeq 49^\circ$

- Why do fermion masses show these hierarchical relations?

$$\frac{m_2^u}{m_3^u} \approx \lambda_C^4 \qquad \frac{m_1^u}{m_2^u} \approx \lambda_C^3$$

$$\frac{m_2^{d,l}}{m_3^{d,l}} \approx \lambda_C^2 \qquad \frac{m_1^{d,l}}{m_2^{d,l}} \approx \lambda_C^2$$

$$\lambda_C < \frac{m_2^\nu}{m_3^\nu} < 1 \qquad 0 < \frac{m_1^\nu}{m_2^\nu} < 1$$

$$\lambda_C \sim 0.22$$

The flavour problem

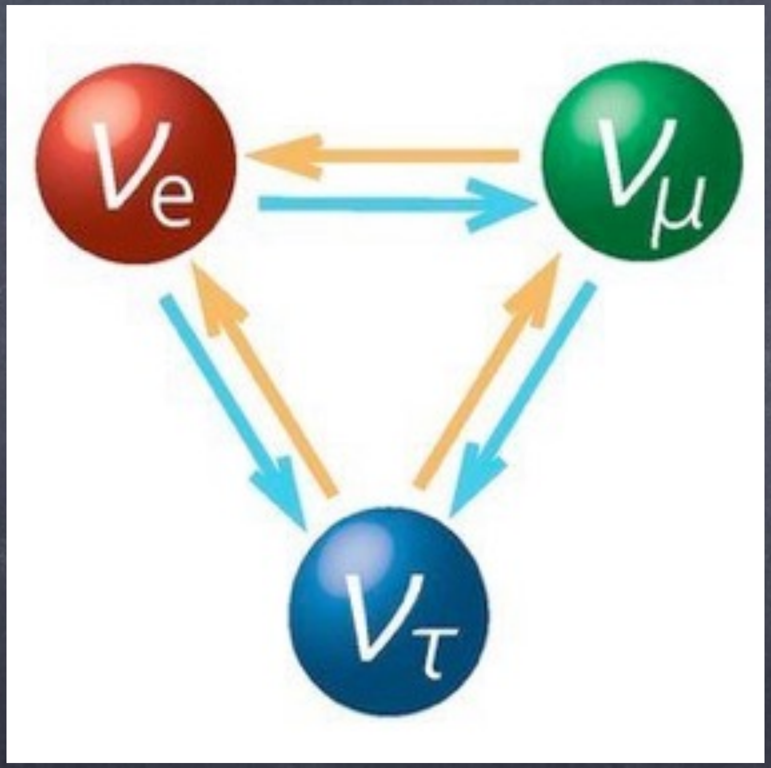
⇒ One can add new symmetries of leptons to Standard Model

$$SU_c(3) \times SU_L(2) \times U_Y(1) \times G_f$$

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	$1, 1', 2$	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, ...1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_7	14	$1, 1', 2, 2', 2''$	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	$1, 1', 1'', 3$	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$	$A^3 = B^2 = (BA)^5 = 1$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$	$A^3 = (AB)^3 = R^2 = 1, \ B^2 = R$
S_4	24	$1, 1', 2, 3, 3'$	$BM : A^4 = B^2 = (AB)^3 = 1$ $TB : A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, ...1_9, 3, \overline{3}$	
$PSL_2(7)$	168	$1, 3, \overline{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \overline{1'}, 3, \overline{3}$	$A^7 = B^3 = 1, \ AB = BA^4$

Altarelli & Feruglio, arXiv: 1002.0211

Neutrino oscillations



Neutrino mixing

- Mixing is described by the Maki-Nakagawa-Sakata (MNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha i} \nu_{kL}$$

$$U = U_l^\dagger U_\nu$$

- leptonic weak charged current:

$$j_\rho^{CC\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\alpha_L} \gamma_\rho \nu_{\alpha L} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\alpha_L} \gamma_\rho U_{\alpha k} \nu_{kL}$$

- NxN unitary matrix: NxN mixing parameters

  $\rightarrow$ N(N-1)/2 mixing angles + N(N+1)/2 phases

- Lagrangian invariant under global phase transformations of Dirac fields:

$$\alpha \rightarrow e^{i\theta_\alpha} \alpha, \; \nu_k \rightarrow e^{i\phi_k} \nu_k$$

$$j_\rho^{CC\dagger} \rightarrow 2 \sum_{\alpha,k} \overline{\alpha_L} \underbrace{e^{-i(\theta_e - \phi_1)}}_{1} \underbrace{e^{-i(\theta_\alpha - \theta_e)}}_{\text{N-1}} \gamma_\rho U_{\alpha k} \underbrace{e^{i(\phi_k - \phi_1)}}_{\text{N-1}} \nu_{kL}$$

$\rightarrow$ 2N-1 phases can be eliminated: (N-1)(N-2)/2 physical phases

Neutrino mixing

- For Majorana neutrinos, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \to e^{i\phi_k}\nu_k \qquad\qquad \nu^T_{kL}\mathcal{C}^\dagger\nu_{kL} \to e^{2i\phi_k}\nu^T_{kL}\mathcal{C}^\dagger\nu_{kL}$$

$\to$ only N phases can be eliminated by rephasing charged lepton fields:

$$j^{\mathrm{CC}\dagger}_\rho \to 2\sum_{\alpha,k}\overline{\alpha_L}\underset{\mathrm{N}}{e^{-i\theta_\alpha}}\gamma_\rho U_{\alpha k}\nu_{kL}$$

$\to$ N(N-1)/2 physical phases: (N-1)(N-2)/2 Dirac phases $\to$ effect in ν oscil.

(N-1) Majorana phases $\to$ relevant for $0\nu\beta\beta$

Neutrino mixing

- 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL measurements

reactor disapp + LBL appearance searches

solar + KamLAND measurements

Neutrino oscillations

- flavour states are admixtures of flavor eigenstates: $$\nu_{\alpha L} = \sum_k U_{\alpha i} \nu_{kL}$$
- Neutrino evolution equation: $$-i\frac{d}{dt}|\nu\rangle = H|\nu\rangle$$

in the 2-neutrino mass eigenstates basis ν_j : $$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$\longrightarrow \quad |\nu_j\rangle \rightarrow e^{-iE_j t}|\nu_j\rangle$$

since neutrinos are relativistic: t = L and $$E_j \simeq p + \frac{m_j^2}{2p} \simeq p + \frac{m_j^2}{2E}$$

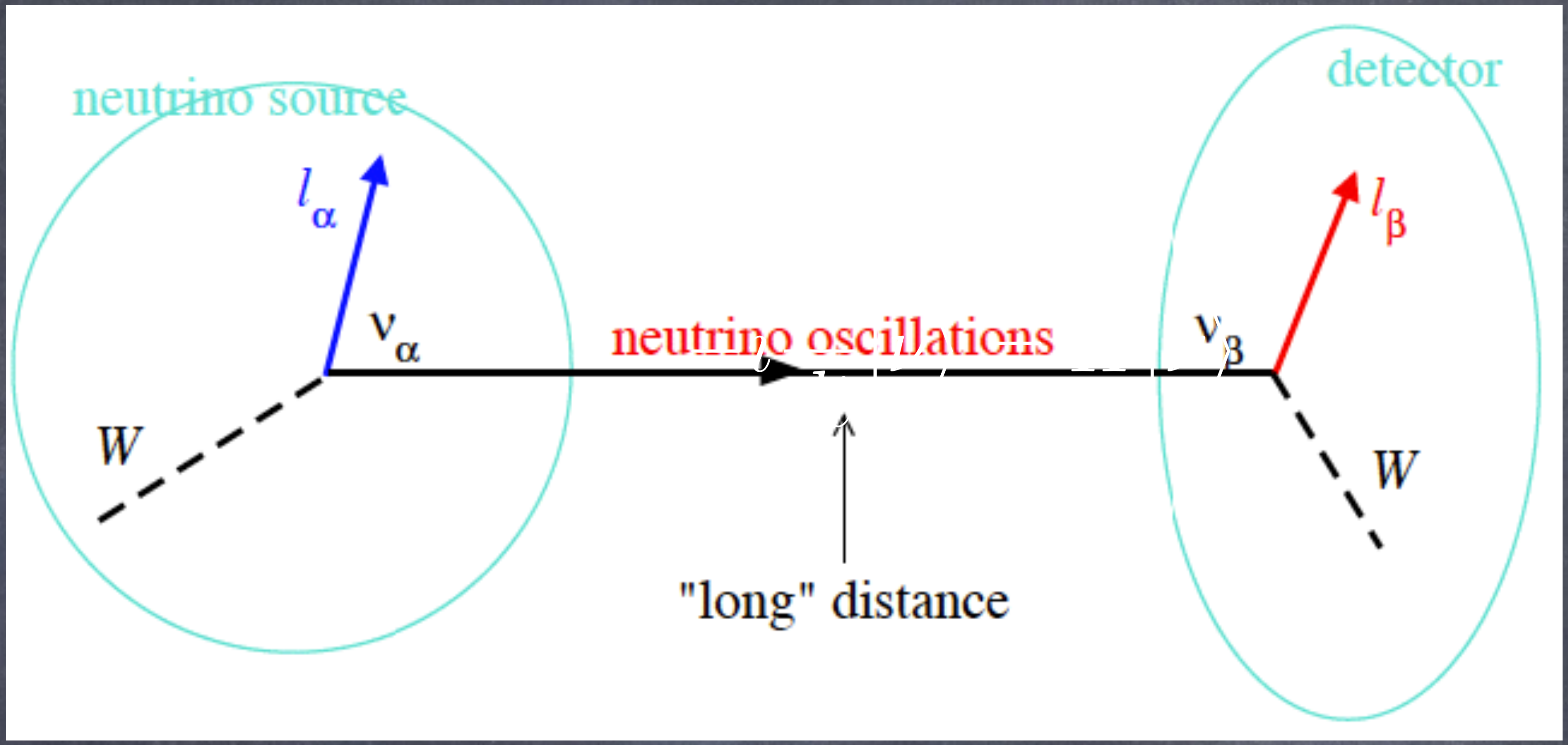
(equal momentum approach)

- Hamiltonian in the flavour basis:

$$H_{\text{flavour}} = U H_{\text{mass}} U^\dagger = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

with $\Delta m^2 = m_2^2 - m_1^2$

Neutrino oscillations picture



Production	Propagation	Detection
$\vert\nu_\alpha\rangle = \sum_j U^*_{\alpha j}\vert\nu_j\rangle$	$\nu_j : e^{-iE_j t}$	$\langle\nu_\beta\vert = \sum_j \langle\nu_j\vert U_{\beta j}$
coherent superposition of massive states	different propagation phases change ν_j composition	projection over flavour eigenstates

Neutrino oscillations

Neutrino oscillation amplitude:

detection production

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta(t) | \nu_\alpha(0) \rangle = \sum_j \langle \nu_\beta | \nu_j(t) \rangle \langle \nu_j(t) | \nu_j(0) \rangle \langle \nu_j(0) | \nu_\alpha \rangle$$

propagation

$$= \sum_j U_{\beta j} e^{-i \frac{m_j^2 L}{2E}} U^*_{\alpha j}$$

Neutrino oscillation probability:

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_j U_{\beta j} e^{-i \frac{m_j^2 L}{2E}} U^*_{\alpha j} \right|^2$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} Re(U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j}) \sin^2 \left(\frac{\Delta m^2_{ij} L}{4E} \right) +$$

$$+2 \sum_{i>j} Im(U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j}) \sin \left(\frac{\Delta m^2_{ij} L}{2E} \right)$$

General properties of neutrino oscillations

- Conservation of probability: $$\sum_{\beta} P(\nu_\alpha \to \nu_\beta) = 1$$

- For antineutrinos: U $\to$U*

- Neutrino oscillations violate flavour lepton number conservation (expected from mixing) but conserve total lepton number

- Complex phases in the mixing matrix induce CP violation:

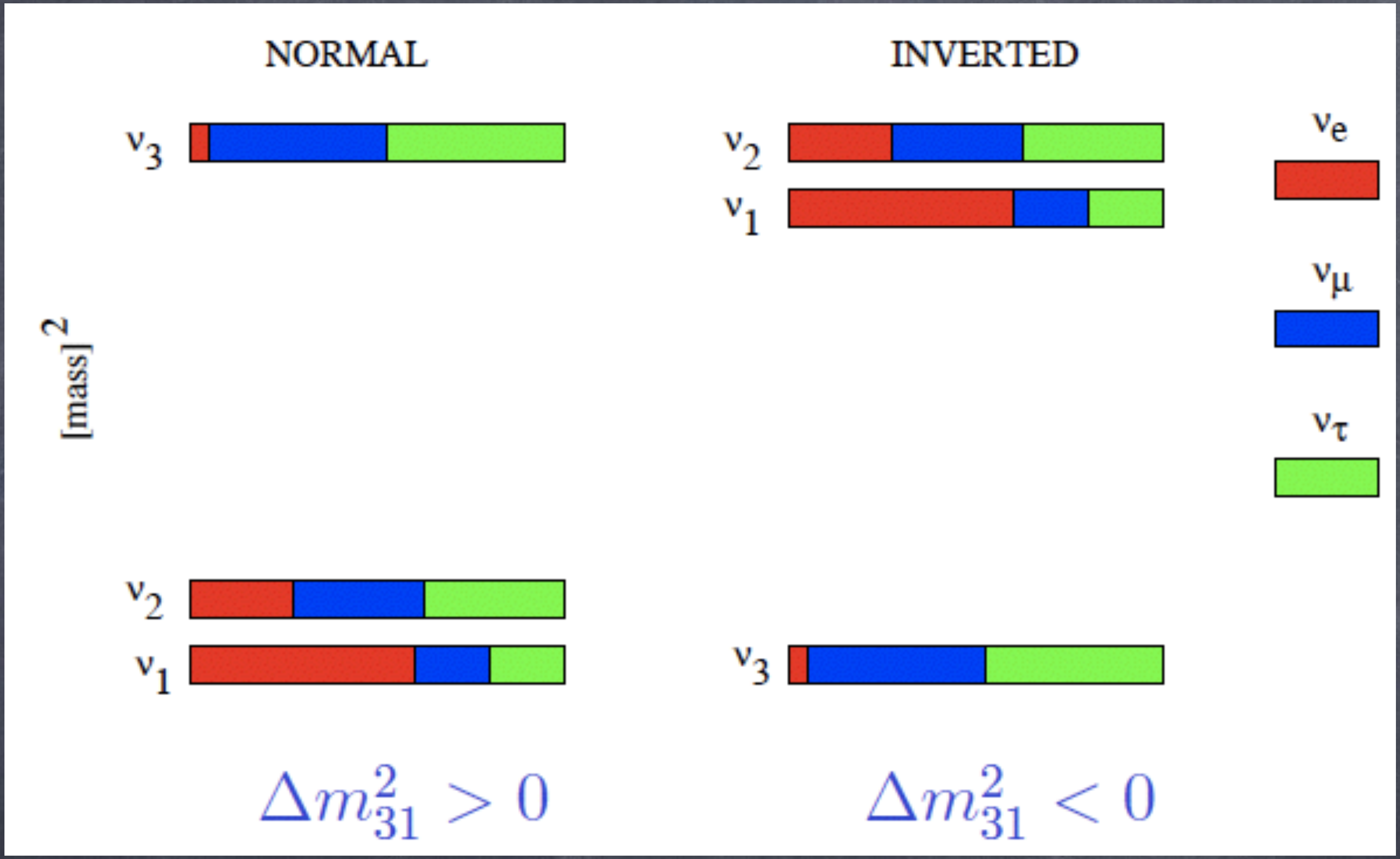
$$P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$$

- Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.

- Neutrino oscillations are sensitive only to mass squared differences:

$$\Delta m^2_{kj} = m^2_k - m^2_j$$

Two possible mass orderings:



- Δm^2_{31} : atmospheric + long-baseline
- Δm^2_{21} : solar + KamLAND (we know it is positive)

2-neutrino oscillations

- 2-neutrino mixing matrix:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- 2-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21} L}{2E}} \right|^2$$

$$= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

- The oscillation phase:

$$\phi = \frac{\Delta m^2_{21} L}{4E} = 1.27 \frac{\Delta m^2_{21}[eV^2] L[km]}{E[GeV]}$$

→ short distances, $\varphi \ll 1$: oscillations do not develop, $P_{\alpha\beta} = 0$

→ long distance, $\varphi \sim 1$: oscillations are observable

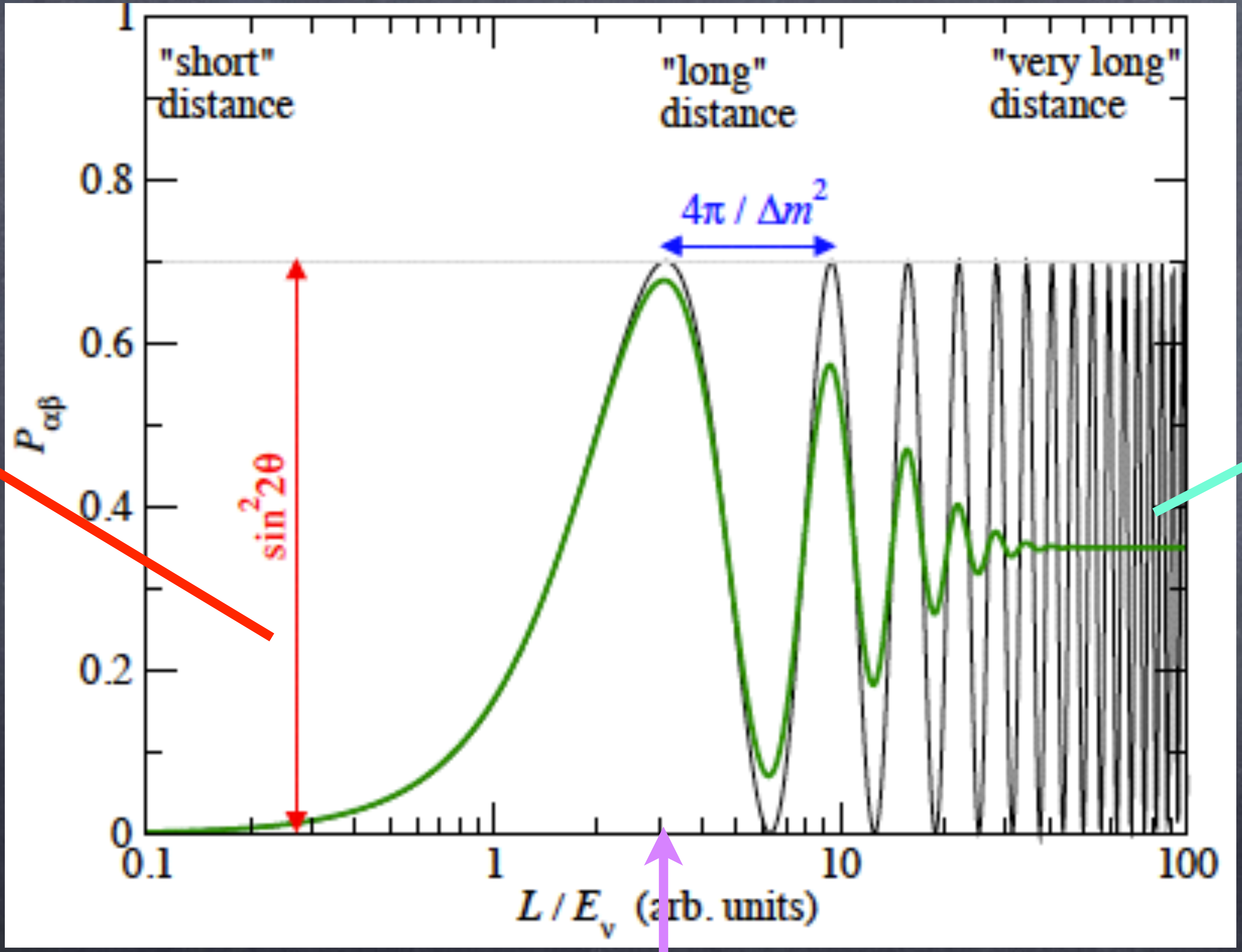
→ very long distances, $\varphi \gg 1$: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2(2\theta)$$

2-neutrino oscillation probability

$$P_{\alpha\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillation amplitude



averaged oscillations

first oscillation maximum:

oscillation length:

$$L_{osc} = \frac{4\pi E}{\Delta m^2}$$

Appearance vs disappearance experiments

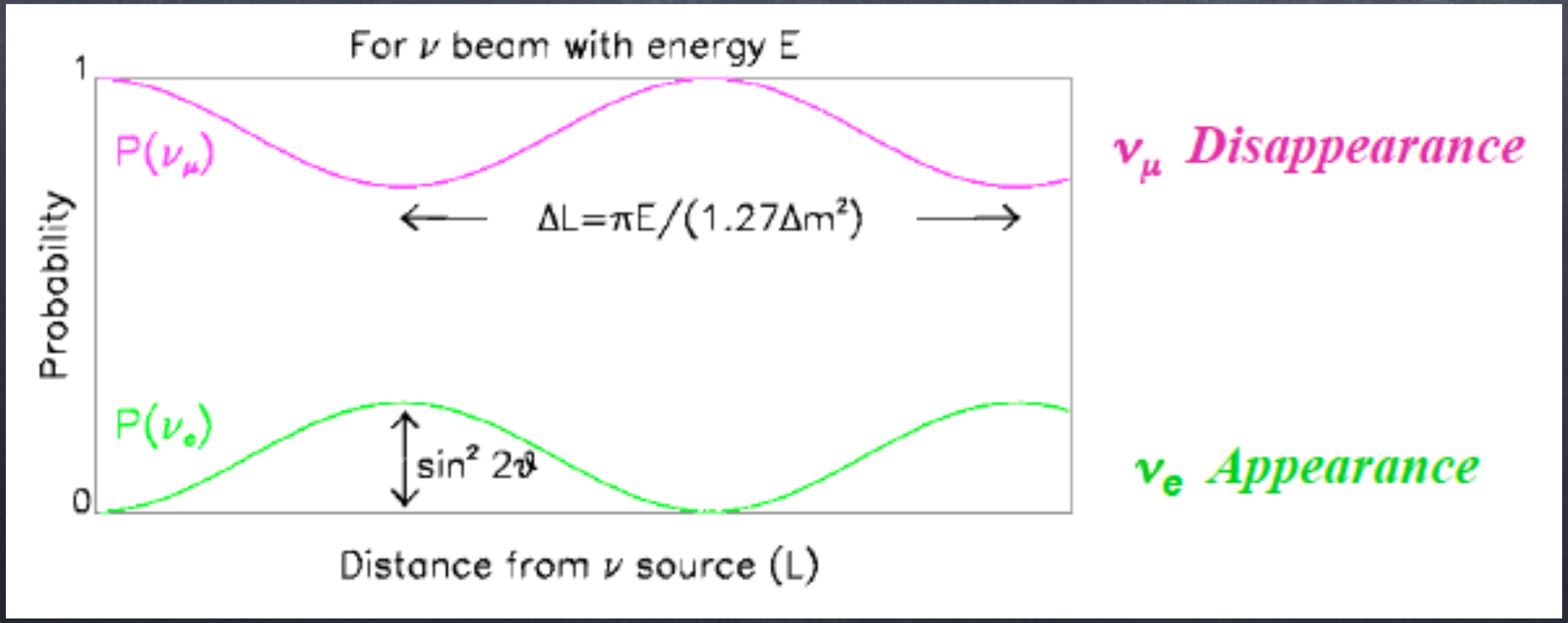
- appearance experiments: $P_{\alpha\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

  $\alpha \neq \beta$

$\rightarrow$ appearance of a neutrino of a new flavour β in a beam of ν_α

- disappearance experiments: $P_{\alpha\alpha} = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

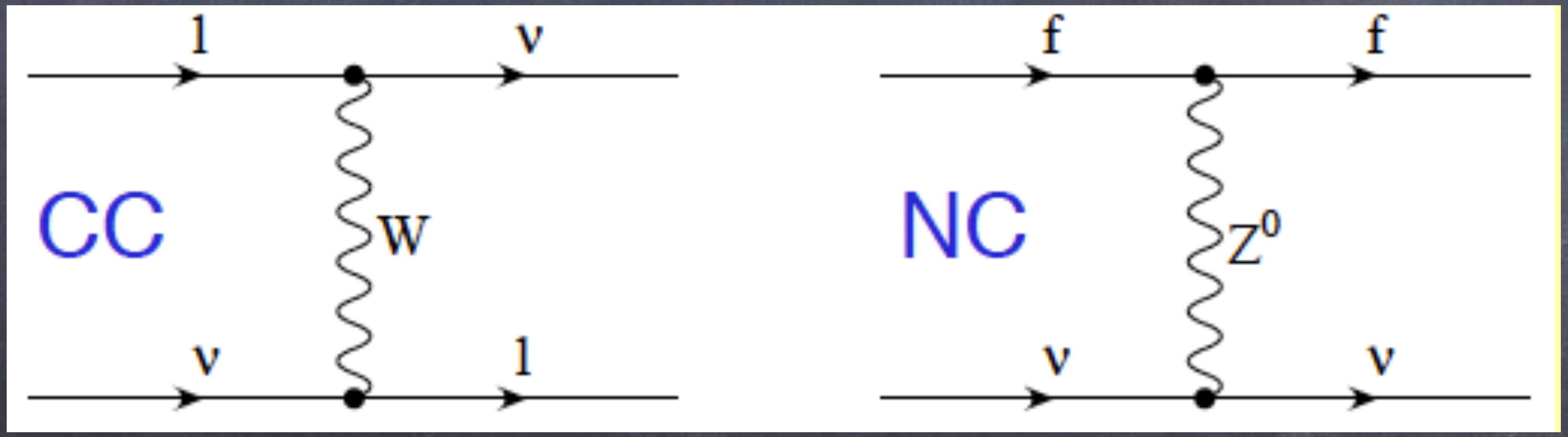
$\rightarrow$ measurement of the survival probability of a neutrino of given flavour



Matter effects on neutrino oscillations

- When neutrinos pass trough matter, the interactions with the particles in the medium induce an effective potential for the neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction for neutrinos. L. Wolfenstein, 1978]



→ modifies the mixing between flavor states and propagation states as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability with respect to vacuum oscillations.

Effective matter potential

- Effective four-fermion interaction Hamiltonian (CC+NC)

$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \overline{\nu_\alpha}\gamma_\mu(1-\gamma_5)\nu_\alpha \sum_j \underbrace{\overline{f}\gamma^\mu(g_V^{\alpha,f} - g_A^{\alpha,f}\gamma_5)f}_{J^{\mu\alpha}_{\rm matt}}$$

in ordinary matter: f=e⁻,p,n

To obtain the matter-induced potential we integrate over f-variables:

for a | non-relativistic / unpolarised / neutral | medium:

$$\langle \overline{f}\gamma^\mu f\rangle = \frac{1}{2} N_f \delta_{\mu,0}$$

$$\langle f\gamma_5\gamma^\mu f\rangle = 0$$

$$N_e = N_p$$

$$J^{\mu\alpha}_{\rm matt} = \frac{1}{2}\left[N_e(g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}\right]$$

Effective matter potential

$$J^{\mu\alpha}_{\rm matt} = \frac{1}{2}\left[N_e(g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}\right]$$

g_V	e^-	p	n
ν_e	$2\sin^2\Theta_W + \frac{1}{2}$	$-2\sin^2\Theta_W + \frac{1}{2}$	$-\frac{1}{2}$
$\nu_{\mu,\tau}$	$2\sin^2\Theta_W - \frac{1}{2}$	$-2\sin^2\Theta_W + \frac{1}{2}$	$-\frac{1}{2}$

$$J^{\mu\alpha}_{\rm matt} = (N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$$

$$\Rightarrow \quad V_{\rm matt} = \sqrt{2}G_F\,{\rm diag}(N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$$

- only νe are sensitive to CC (no μ,τ in ordinary matter)
- NC has the same effect for all flavours $\rightarrow$ it has no effect on evolution

  (however it can be important in presence of sterile neutrinos)
- for antineutrinos the potential has opposite sign

2-neutrino oscillations in matter

- 2-neutrino Hamiltonian in vacuum (mass basis): $$H^{\text{vac}} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

- In the flavour basis, where effective matter potential is diagonal:

$$H_f^{\text{matt}} = H_f^{\text{vac}} + V_{\text{eff}} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

$$V_{\text{CC}} = \sqrt{2} G_F N_e$$

Diagonalizing the Hamiltonian, we identify the mixing angle and mass splitting in matter:

$$H_f^{\text{matt}} = \frac{\Delta M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

In general: $N_e=N_e(x)$, so θ_M and ΔM^2 will be function of x as well

$\rightarrow$ however, in some cases analytical solutions can be obtained

2-ν oscillations in constant matter

- If N_e is constant (good approximation for oscillations in the Earth crust):

  $\rightarrow$ θ_M and ΔM^2 are constant as well

  $\rightarrow$ we can use vacuum expression for oscillation probability, replacing "vacuum" parameters by "matter" parameters:

$$\boxed{P_{\alpha\beta} = \sin^2(2\theta_M)\sin^2\left(\frac{\Delta M^2 L}{4E}\right)}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A = \frac{2EV}{\Delta m^2}$$

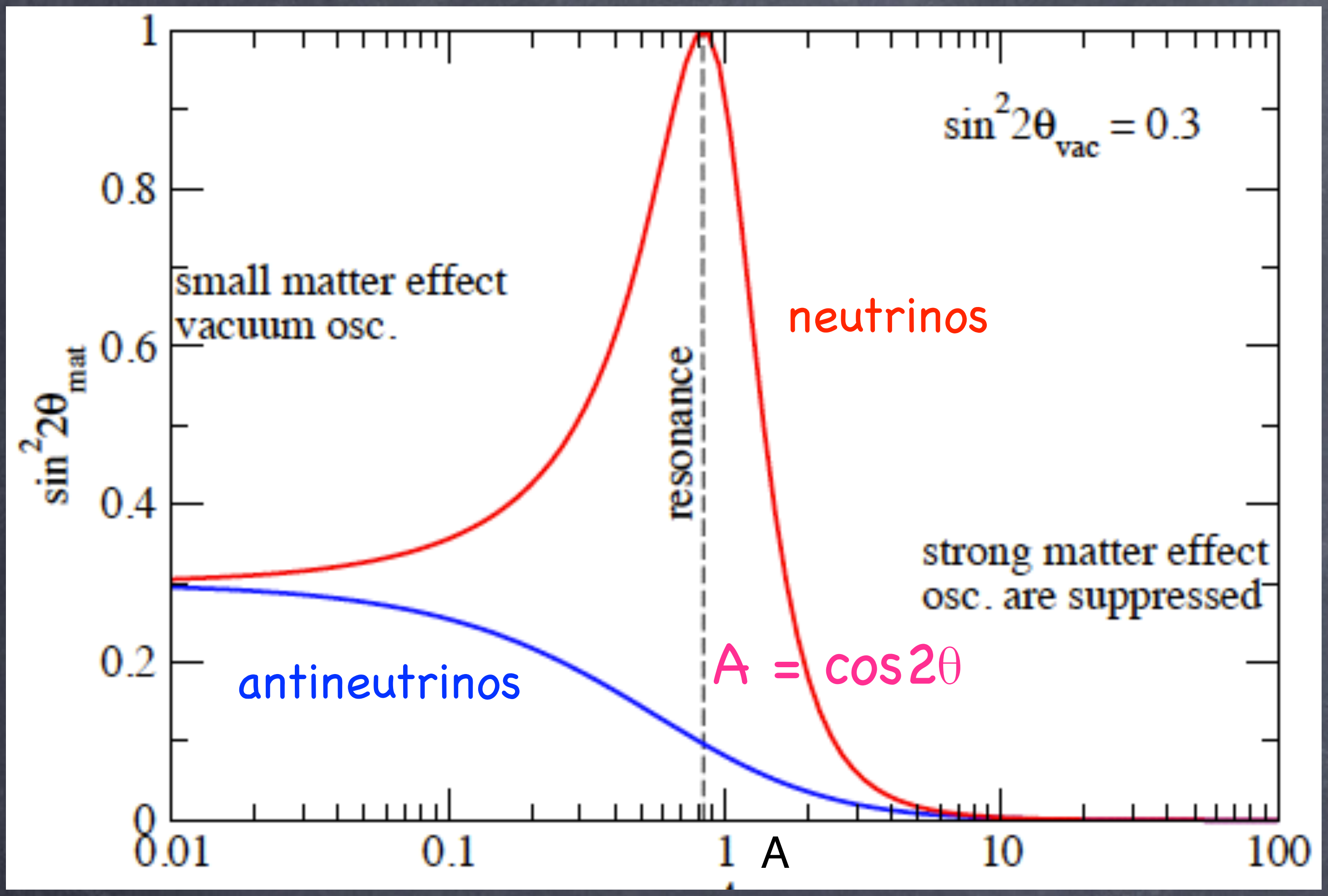
$$\Delta M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

There is a resonance effect for A = cos2θ $\rightarrow$ MSW effect

Wolfenstein, 1978

Mikheyev & Smirnov, 1986

2-ν oscillations in constant matter



mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A = \frac{2EV}{\Delta m^2}$$

- A << cos2θ, small matter effect → vacuum oscillations: $\theta_M = \theta$
- A >> cos2θ, matter effects dominate → oscillations are suppressed: $\theta_M \approx 0$
- A = cos2θ, resonance takes place→ maximal mixing $\theta_M \approx \pi/4$

→ resonance condition is satisfied for neutrinos for $\Delta m^2 > 0$

for antineutrinos for $\Delta m^2 < 0$

2-ν oscillations in varying matter

- If N_e varies with time (neutrino beam propagating through the Earth or the Sun)

 $\rightarrow$ we need to diagonalize the Hamiltonian at every instant to obtain the instantaneous values of θ_M and ΔM^2

 $\rightarrow$ evolution of the instantaneous ν eigenstates in matter ν_i^m:

$$i\frac{d}{dt}\nu_\alpha = i\frac{d}{dt}[U(\theta_M)\nu_i^m] = i\frac{d}{dt}U(\theta_M)\nu_i^m + U(\theta_M)i\frac{d}{dt}\nu_i^m$$

$$i\frac{d}{dt}\nu_\alpha = H_{\mathrm{f}}\,\nu_\alpha = U(\theta_M)H_{\mathrm{diag}}(\Delta M^2)U(\theta_M)^\dagger\nu_\alpha = U(\theta_M)H_{\mathrm{diag}}(\Delta M^2)\nu_i^m$$

$$\Rightarrow \quad i\frac{d}{dt}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} = \begin{pmatrix}-\Delta M^2/4E & -i\dot{\theta}_M\\ i\dot{\theta}_M & \Delta M^2/4E\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

the presence of off-diagonal terms induce the mixing of ν_i^m states

Adiabatic evolution

$$i\frac{d}{dt}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} = \begin{pmatrix}-\Delta M^2/4E & -i\dot{\theta}_M\\ i\dot{\theta}_M & \Delta M^2/4E\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

- For small off-diagonal terms: $|\dot{\theta}_M| \ll \Delta M^2/2E$

$\rightarrow$ the transitions between the instantaneous eigenstates ν_1^m and ν_2^m are suppressed: adiabatic approximation.

- adiabaticity condition:

$$\gamma^{-1} \equiv \frac{2\dot{\theta}_M}{\Delta m^2/2E} = \frac{\sin(2\theta)\frac{\Delta m^2}{2E}}{(\Delta M^2/2E)^3}|\dot{V}_{\rm CC}| << 1$$

adiabaticity parameter

from the instantaneous expression of θ_M

the typical value in the Sun: $\gamma^{-1} \sim \frac{\Delta m^2}{10^{-9} eV^2}\frac{\rm MeV}{E_\nu}$

$\rightarrow$ adiabaticity applies up to 10 GeV

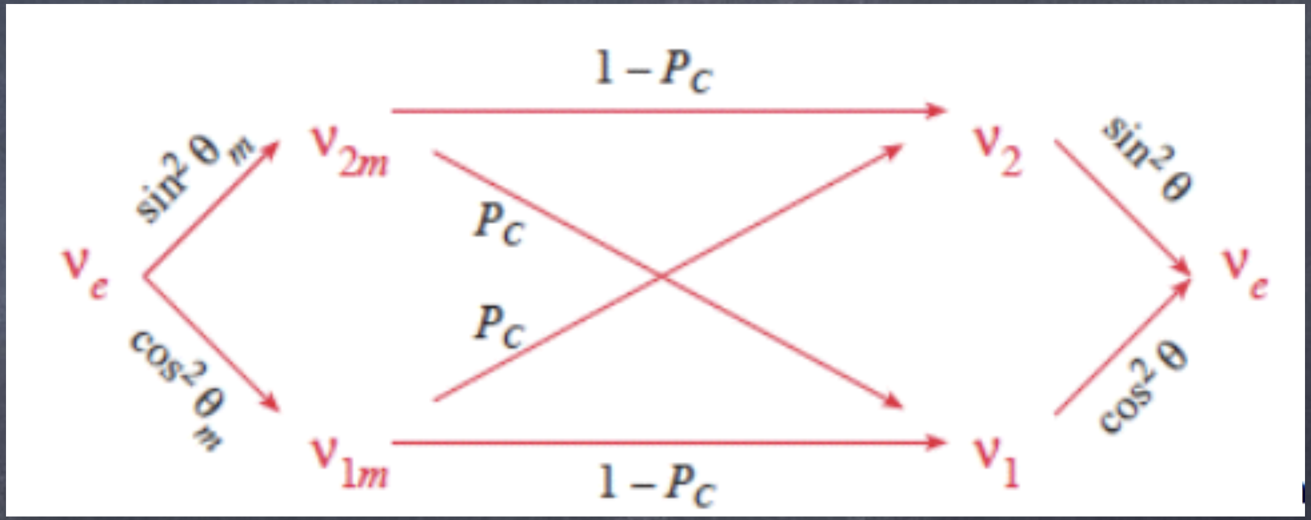
Beyond adiabaticity

- violations of adiabaticity can be described by the probability of jump between ν_1^m and ν_2^m

- for an exponential profile as in the Sun: $V_{\rm CC} \propto N_e \propto exp(-r/r_0)$

the "crossing probability" is given by: $P_{\rm C} = \dfrac{e^{\tilde{\gamma}\cos^2\theta} - 1}{e^{\tilde{\gamma}} - 1}$ $\quad \tilde{\gamma} = \dfrac{\pi r_0 \Delta m^2}{E_\nu}$

- neutrino evolution scheme:



$$P(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_C\right)\cos(2\theta)\cos(2\theta_M)$$

- neutrino propagation in the Sun is adiabatic: $P_C = 0$

Solar neutrinos: the MSW effect

- neutrino oscillations in matter were first discussed by Wolfenstein, Mikheyev and Smirnov (MSW effect)

- electron neutrino is born at the center of the Sun as:

$$|\nu_e\rangle = \cos\theta_M |\nu_1^m\rangle + \sin\theta_M |\nu_2^m\rangle$$

$\rightarrow$ ν_1^m and ν_2^m evolve adiabatically until the solar surface and propagate in vacuum from the Sun to the Earth:

$$P(\nu_e \to \nu_e) = P_{e1}^{\rm prod} P_{1e}^{\rm det} + P_{e2}^{\rm prod} P_{2e}^{\rm det}$$

$$P_{e1}^{\rm prod} = \cos^2\theta_M\,, \qquad P_{1e}^{\rm det} = \cos^2\theta$$

$$P_{e2}^{\rm prod} = \sin^2\theta_M\,, \qquad P_{2e}^{\rm det} = \sin^2\theta$$

$$\Rightarrow \quad \boxed{P_{ee} = \cos^2\theta_M \cos^2\theta + \sin^2\theta_M \sin^2\theta}$$

Solar neutrinos: the MSW effect

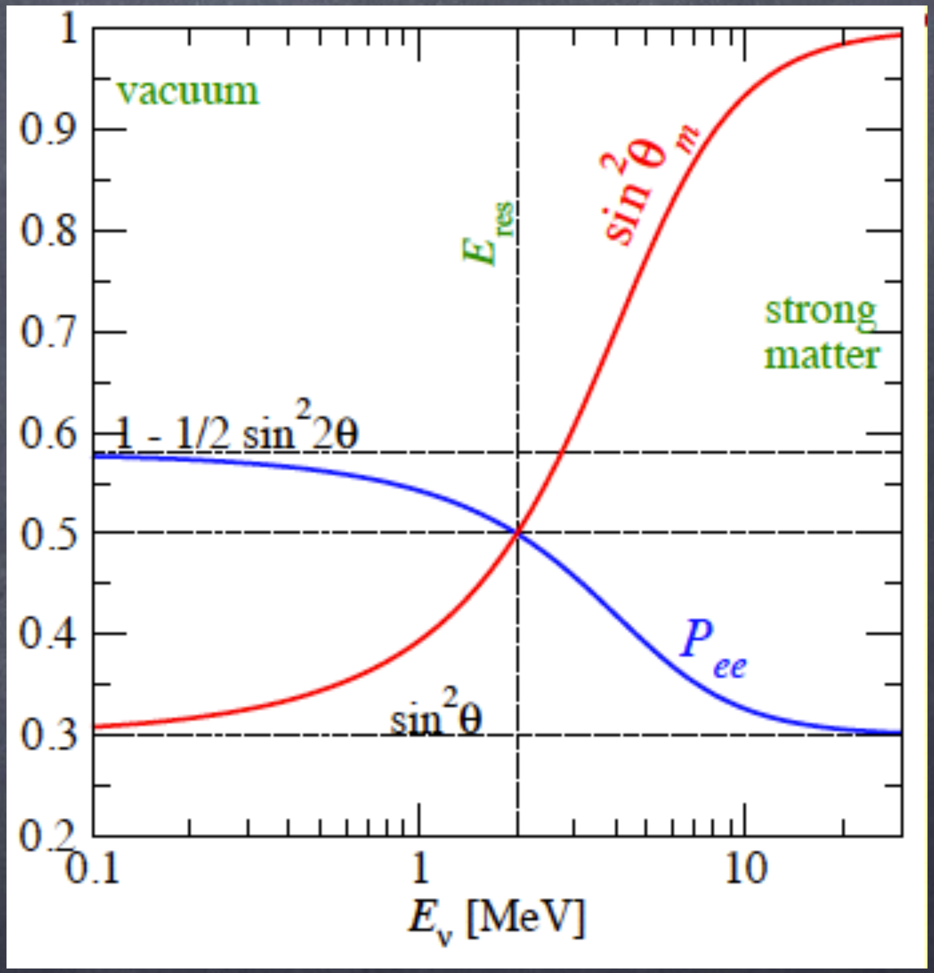
$$\boxed{P_{ee} = \cos^2\theta_M \cos^2\theta + \sin^2\theta_M \sin^2\theta}$$

- In the center of the Sun:

$$A = \frac{2EV}{\Delta m^2} \simeq 0.2 \left(\frac{E_\nu}{\mathrm{MeV}}\right)\left(\frac{8\times 10^{-5}\mathrm{eV}^2}{\Delta m^2}\right)$$

and resonance occurs for A = $\cos(2\theta)$ = 0.4

$\rightarrow E_{res} \approx$ 2 MeV



- For E < 2 MeV $\rightarrow$ vacuum osc: $\theta_M = \theta$

$$\boxed{P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta}$$

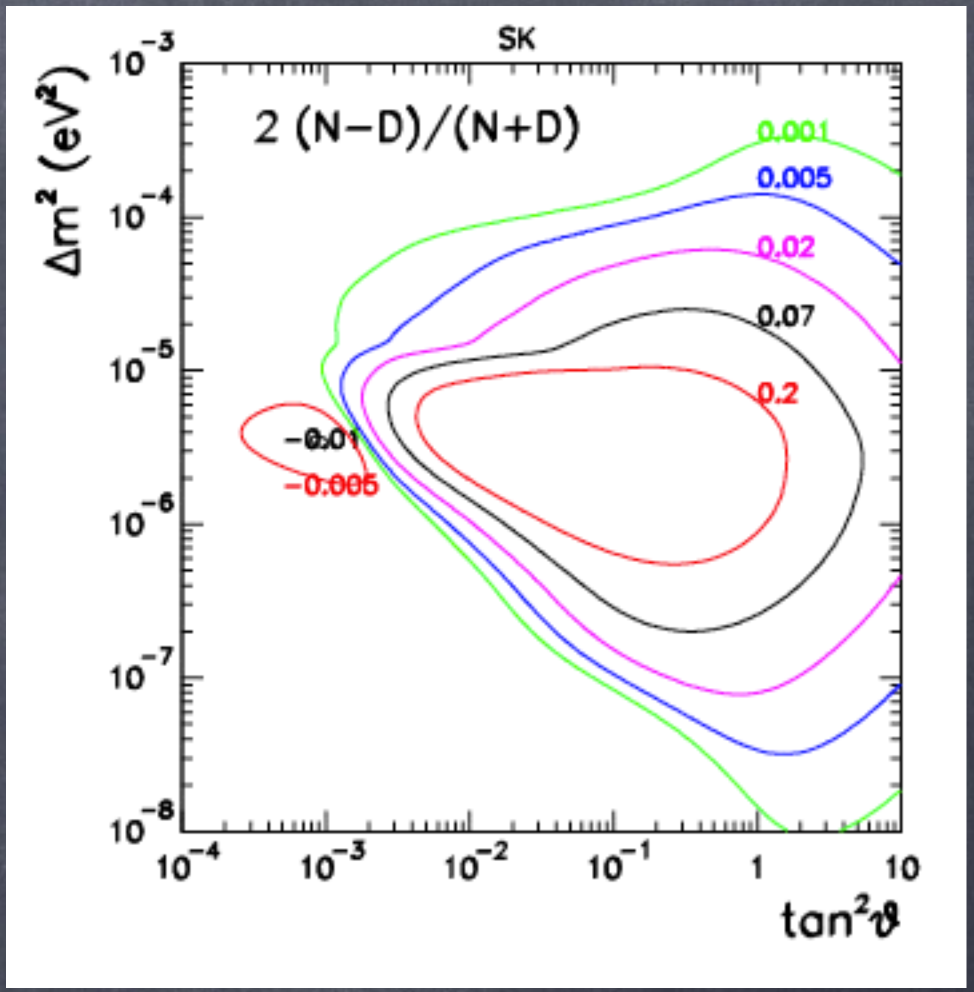
- For E > 2 MeV $\rightarrow$ strong matter effect: $\theta_M = \pi/2$ $\quad \boxed{P_{ee} = \sin^2\theta}$

$\rightarrow P_{ee}$ (E) will be crucial to understand solar neutrino data

Earth regeneration effect

- neutrinos observed during the night are also affected my Earth matter effects

- if neutrino cross only the Earth mantle, $P_{2e}^{\rm det}$ is well approximated by evolution of a constant potential:



$$P_{2e}^{\rm det} = \sin^2\theta + f_{\rm reg}$$

prob. during day

regeneration term

$$f_{\rm reg} = \frac{4EV_{\rm cc}}{\Delta m^2}\sin^2(2\theta_E)\sin^2\frac{\pi L}{L_{\rm osc}}$$

$$P_{ee}^{\rm Earth} = P_D - \cos 2\theta_M f_{\rm reg}$$

→ day-night asymmetry: $A_{\rm DN} \equiv 2\frac{(P_N - P_D)}{P_N + P_D}$

for the actual solar neutrino parameters $f_{\rm reg}$ ~ +1%