Injection and extraction

- Introductory slides:
  - Kickers, septa and normalised phase-space

Matthew Fraser, CERN (TE-ABT-BTP) based on lectures by Brennan Goddard

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- Injection methods
  - Single-turn hadron injection
  - Injection errors, filamentation and blow-up
  - Multi-turn hadron injection
  - Charge-exchange H- injection
  - Lepton injection

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  - Multi-turn (fast) extraction: mechanical and magnetic splitting
  - Resonant multi-turn (slow) extraction

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  - Resonant multi-turn (slow) extraction
- Extra slides:
  - One hour is rather short introduction… extra material for discussion (mainly HW)

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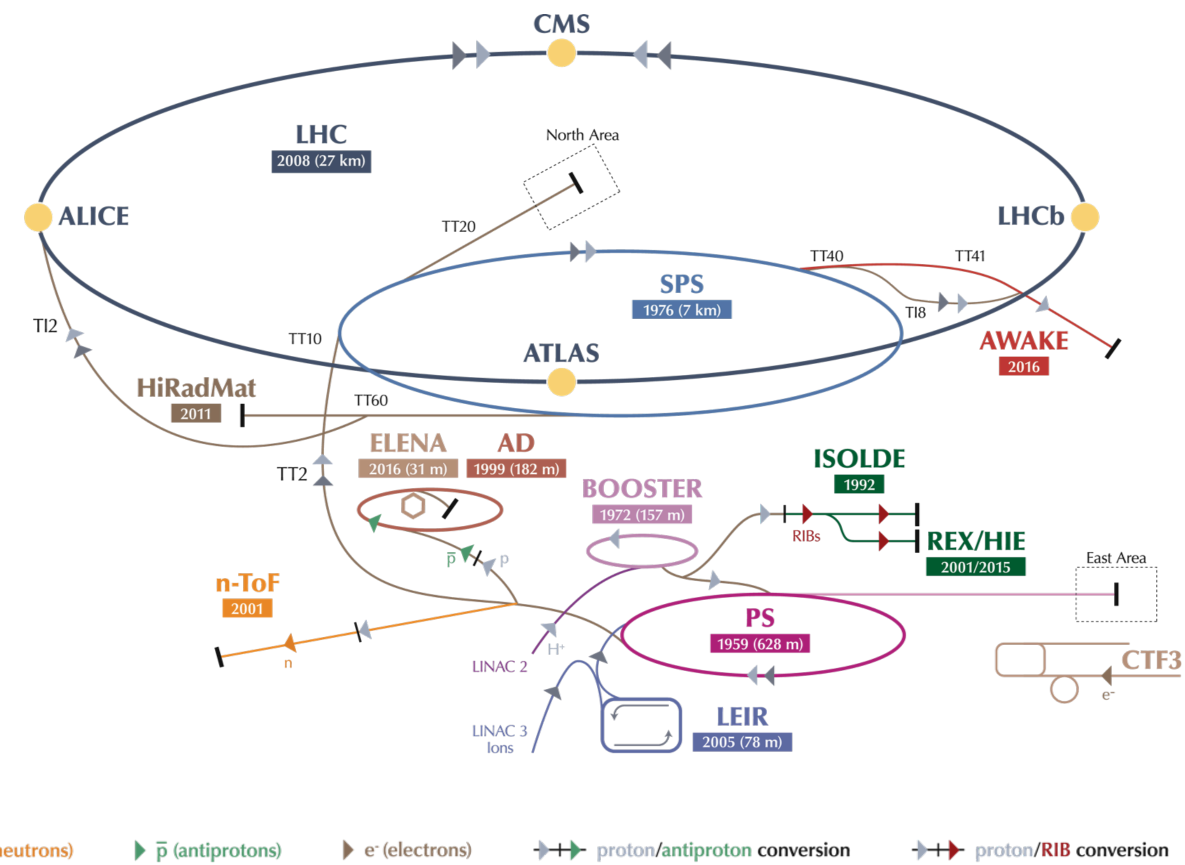
Further material

- This lecture is only intended as an introduction, giving a broad overview of a few important topics
- If videos don't work for you they can be download in different formats here:
  - http://cern.ch/mfraser/public/CAS/Oxford/Videos
- For a full overview of Injection, Extraction and Beam Transfer please refer to the material presented at:
  - CAS Beam Injection, Extraction and Transfer, Erice, Italy, 2017
    - https://indico.cern.ch/event/451905/timetable/
- For hand-outs relating specifically to the lecture material presented today please see:
  - CAS Introduction to Accelerator Physics, Budapest, Hungary, 2016
    - https://indico.cern.ch/event/532397/timetable/
    - Injection and extraction
    - Kickers, septa and transfer lines

Injection and extraction

- An accelerator has limited dynamic range
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External facilities and experiments:
  - e.g. ISOLDE, HIRADMAT, AWAKE…

CERN Accelerator Complex



LHC Large Hadron Collider SPS Super Proton Synchrotron PS Proton Synchrotron AD Antiproton Decelerator CTF3 Clic Test Facility

AWAKE Advanced WAKefield Experiment ISOLDE Isotope Separator OnLine REX/HIE Radioactive EXperiment/High Intensity and Energy ISOLDE

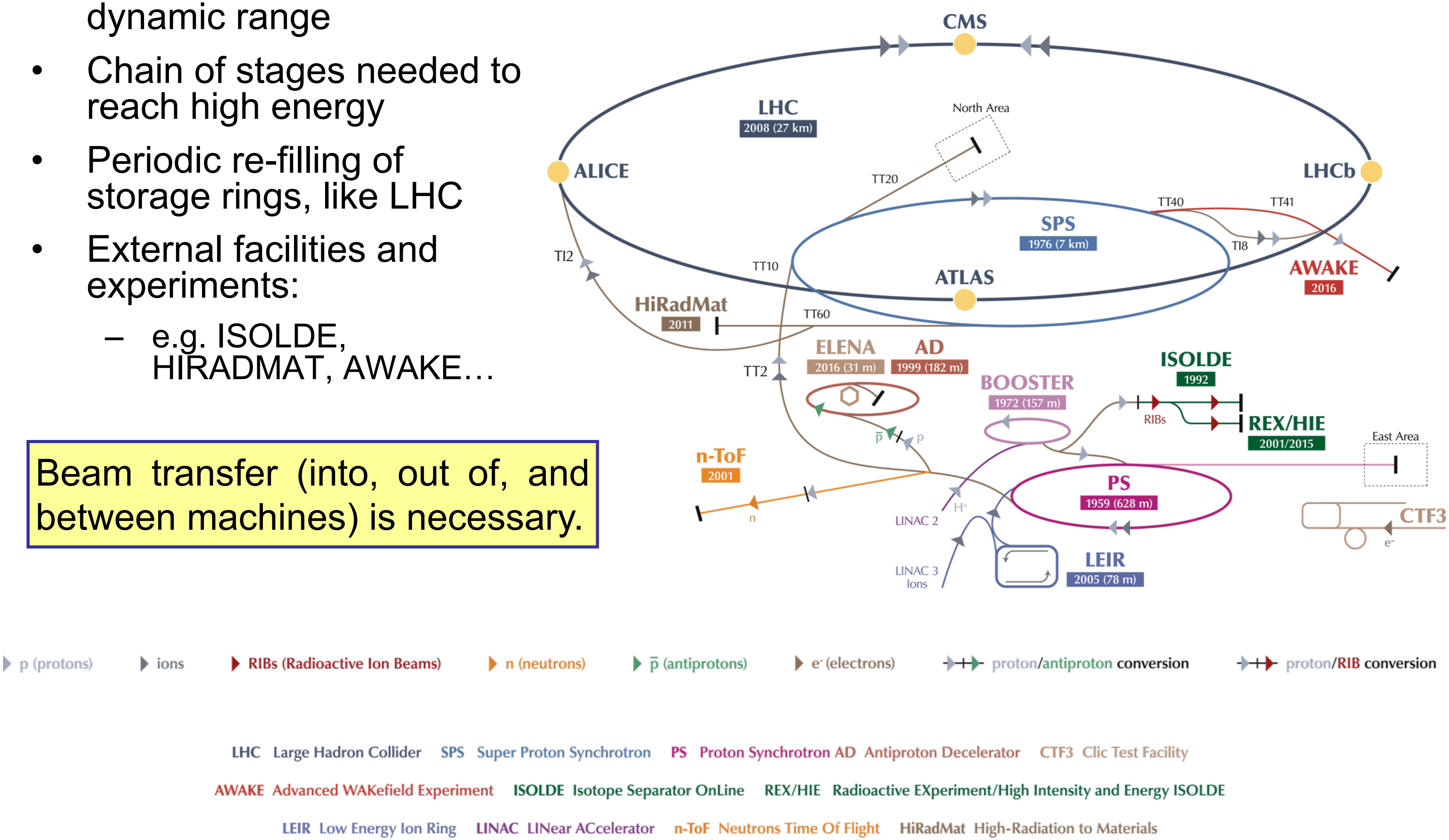
LEIR Low Energy Ion Ring LINAC LINear ACcelerator n-ToF Neutrons Time Of Flight HiRadMat High-Radiation to Materials

Injection and extraction

- An accelerator has limited dynamic range
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External facilities and experiments:
  - e.g. ISOLDE, HIRADMAT, AWAKE…

Beam transfer (into, out of, and between machines) is necessary.

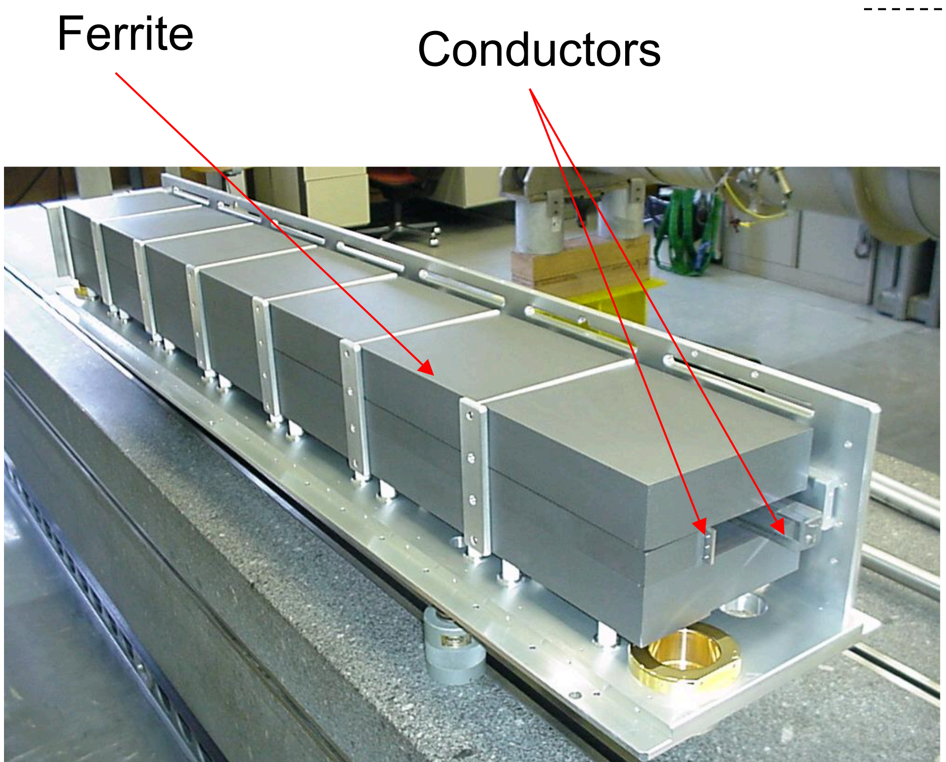
CERN Accelerator Complex

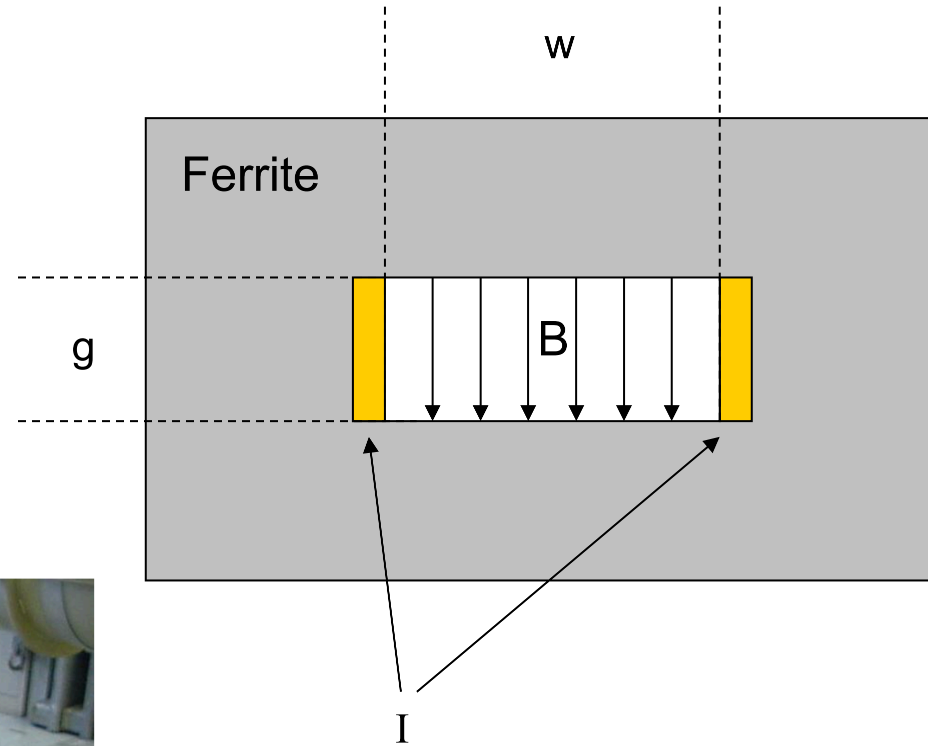


Kicker magnet

Pulsed magnet with very fast rise time (100 ns – few μs)

See extra slides for more details on fast-pulsed systems





$B = \mu_0 I / g$

L [per unit length] $= \mu_0 w / g$

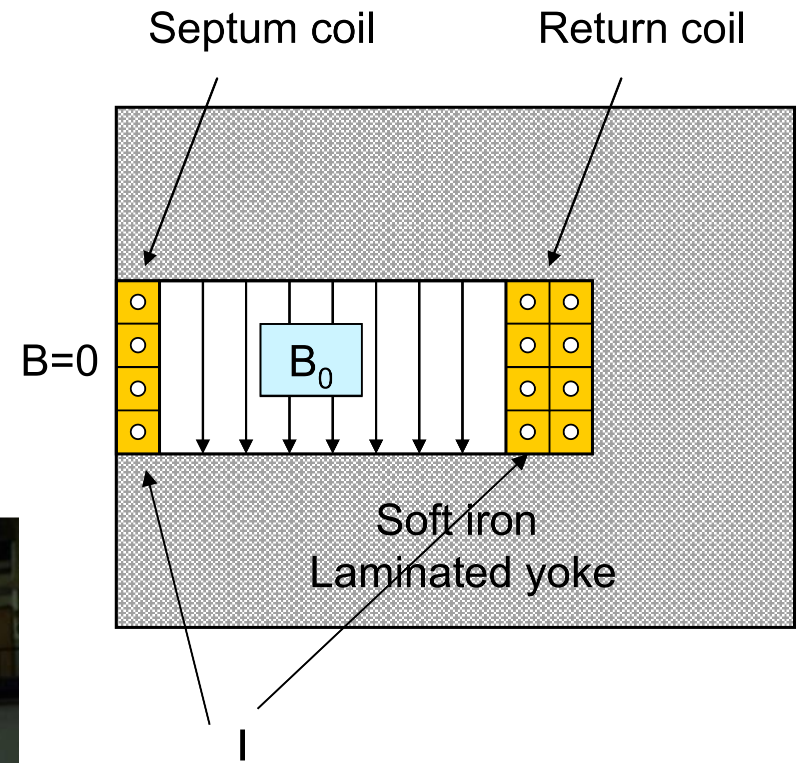
$dI/dt = V / L$

Typically 3 kA in 1 μs rise time

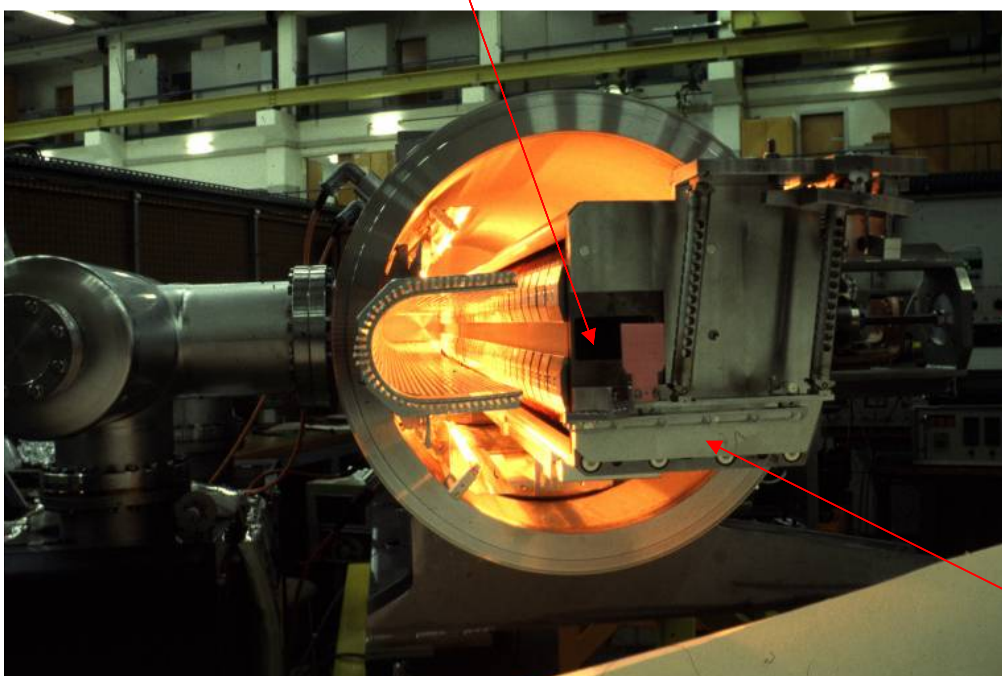
Magnetic septum

Pulsed or DC magnet with thin (2 – 20 mm) septum between zero field and high field region

Typically ~10x more deflection given by magnetic septa, compared to kickers



Septum coil



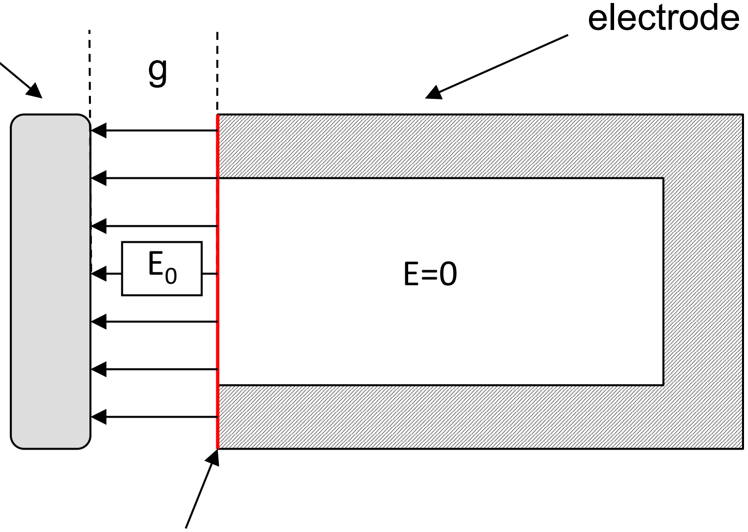
Yoke

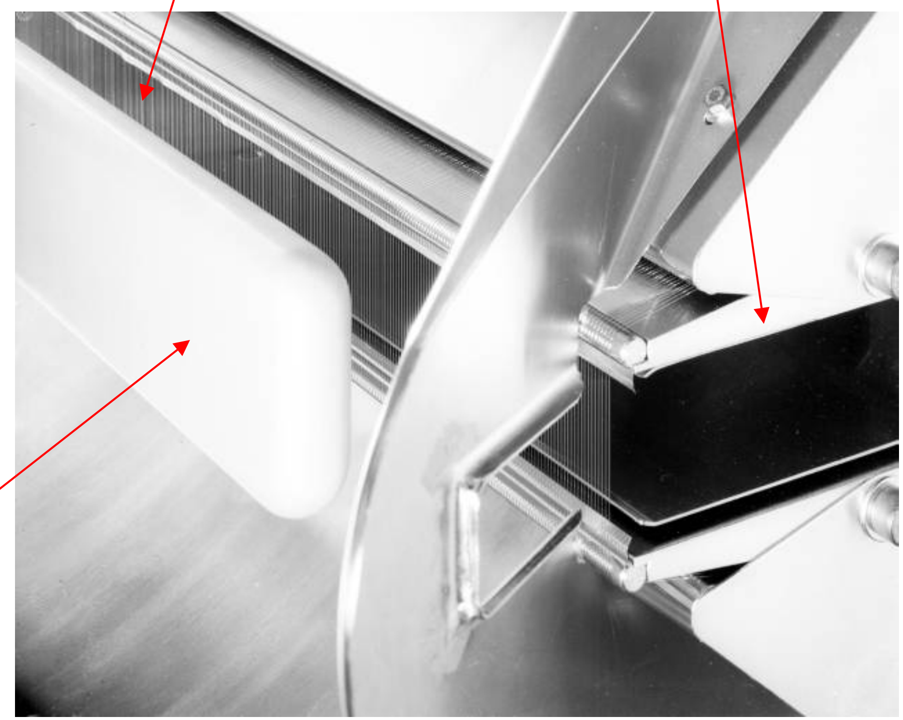
$B_o = \mu_0 I / g$

Typically I 5 - 25 kA

Electrostatic septum

DC electrostatic device with very thin septum between zero field and high field region





$E = V / g$

Typically $V = 200$ kV

$E = 100$ kV/cm

Normalised phase space

- Transform real transverse coordinates (*x, x', s)* to normalised co-ordinates $(\bar{X}, \bar{X}', \mu)$ where the independent variable becomes the phase advance *μ*:

$$\begin{bmatrix} \bar{X} \\ \bar{X}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta(s)}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

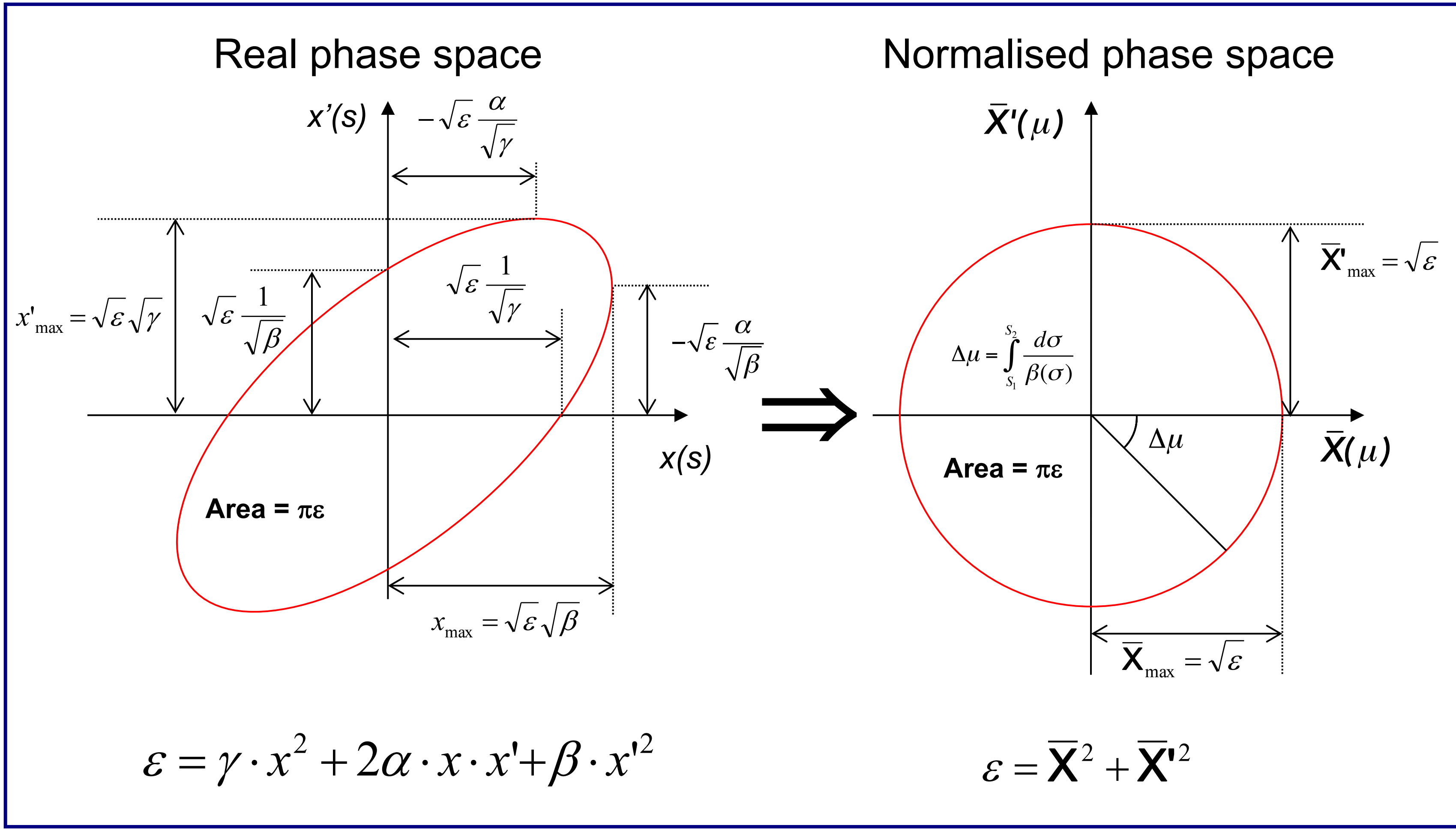
$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\left[\mu(s) + \mu_0\right]$$

$$\mu(s) = \int_0^s \frac{d\sigma}{\beta(\sigma)}$$

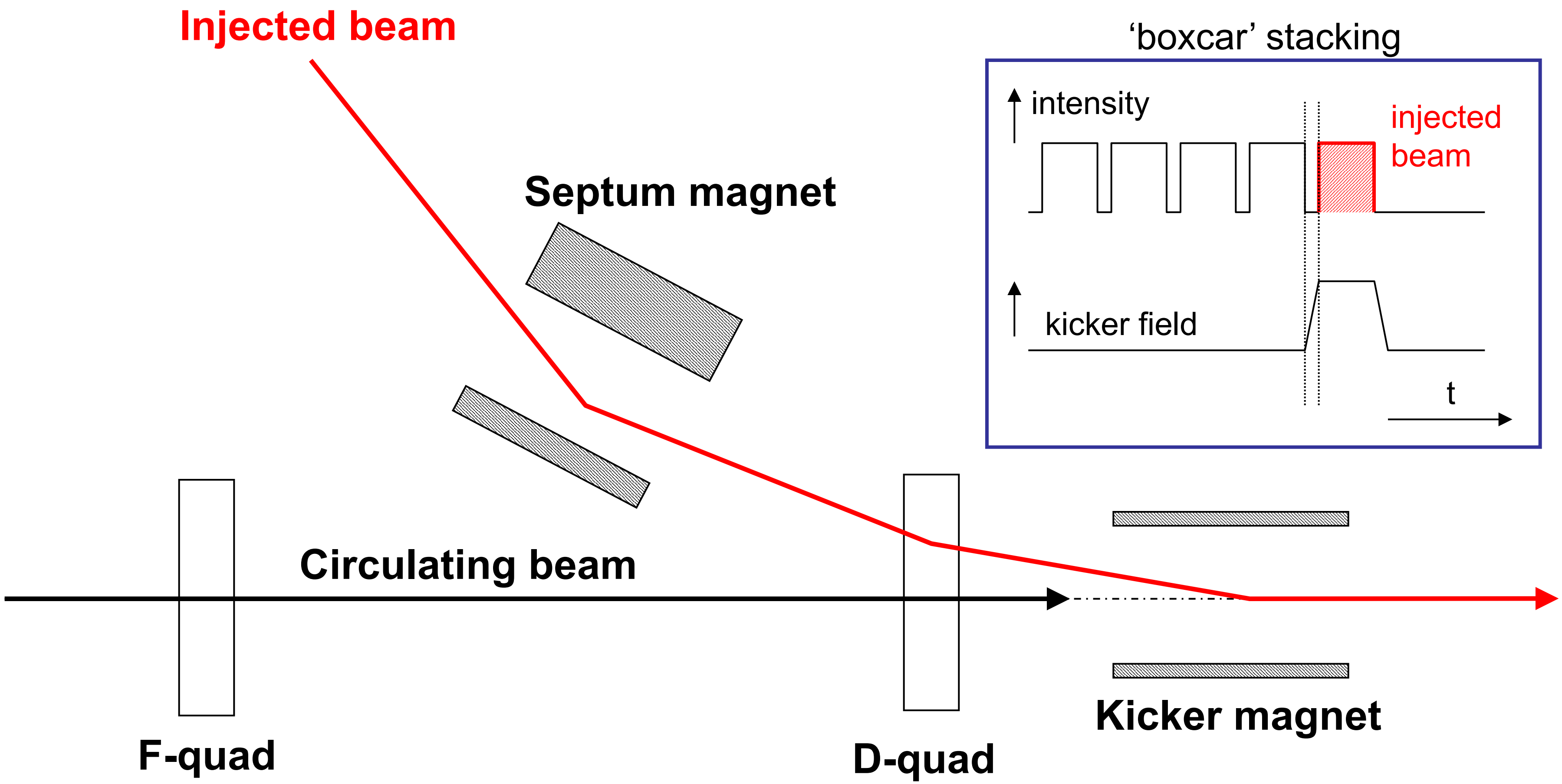
$$\bar{X}(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot x = \sqrt{\boldsymbol{\varepsilon}}\cos\left[\mu + \mu_0\right]$$

$$\bar{X}'(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot \alpha(s)x + \sqrt{\beta(s)}x' = -\sqrt{\boldsymbol{\varepsilon}}\sin\left[\mu + \mu_0\right] = \frac{d\bar{X}}{d\mu}$$

Normalised phase space



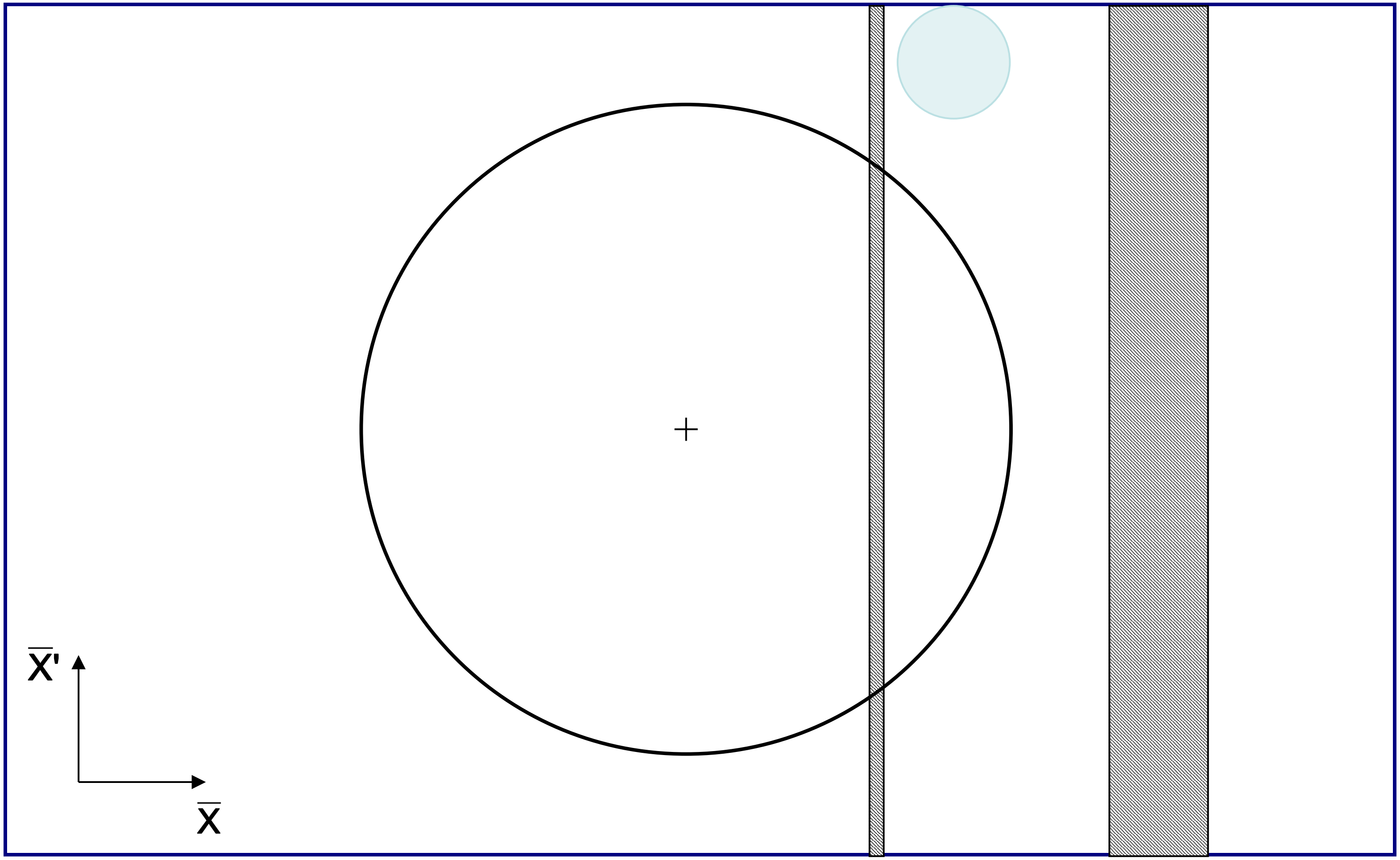
Single-turn (fast) injection



- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimise kicker strength

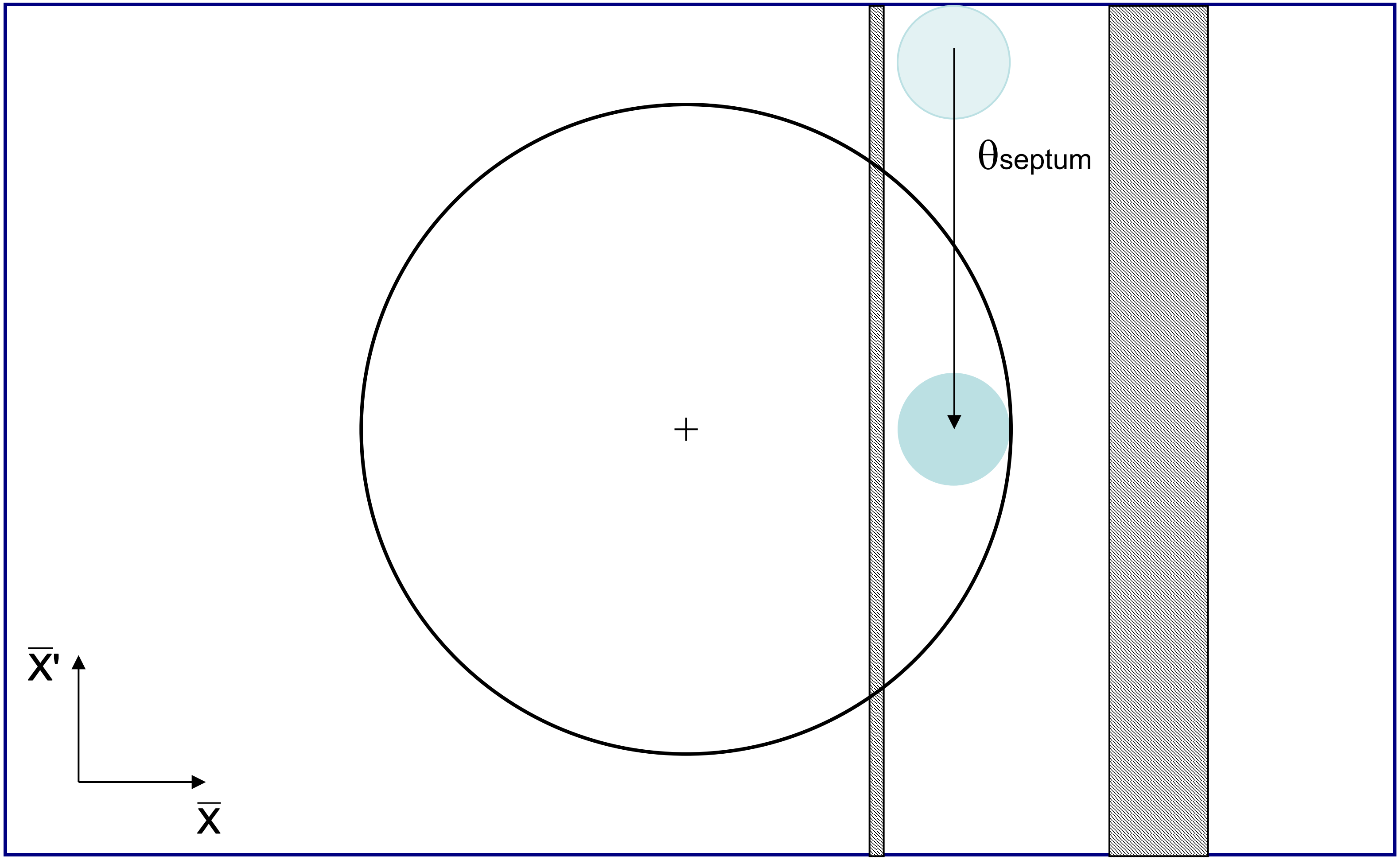
Single-turn injection

Normalised phase space at centre of idealised septum



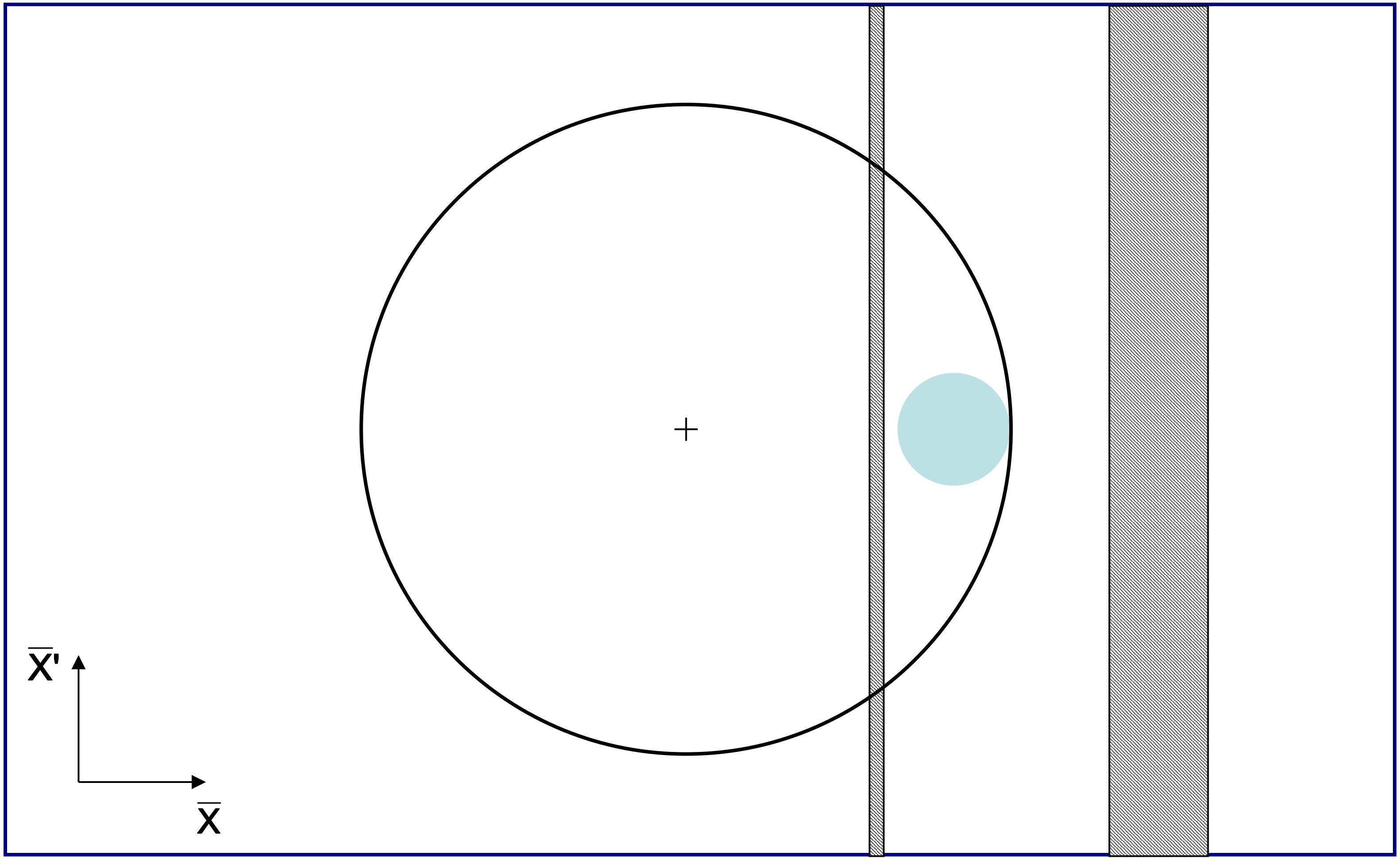
Single-turn injection

Normalised phase space at centre of idealised septum



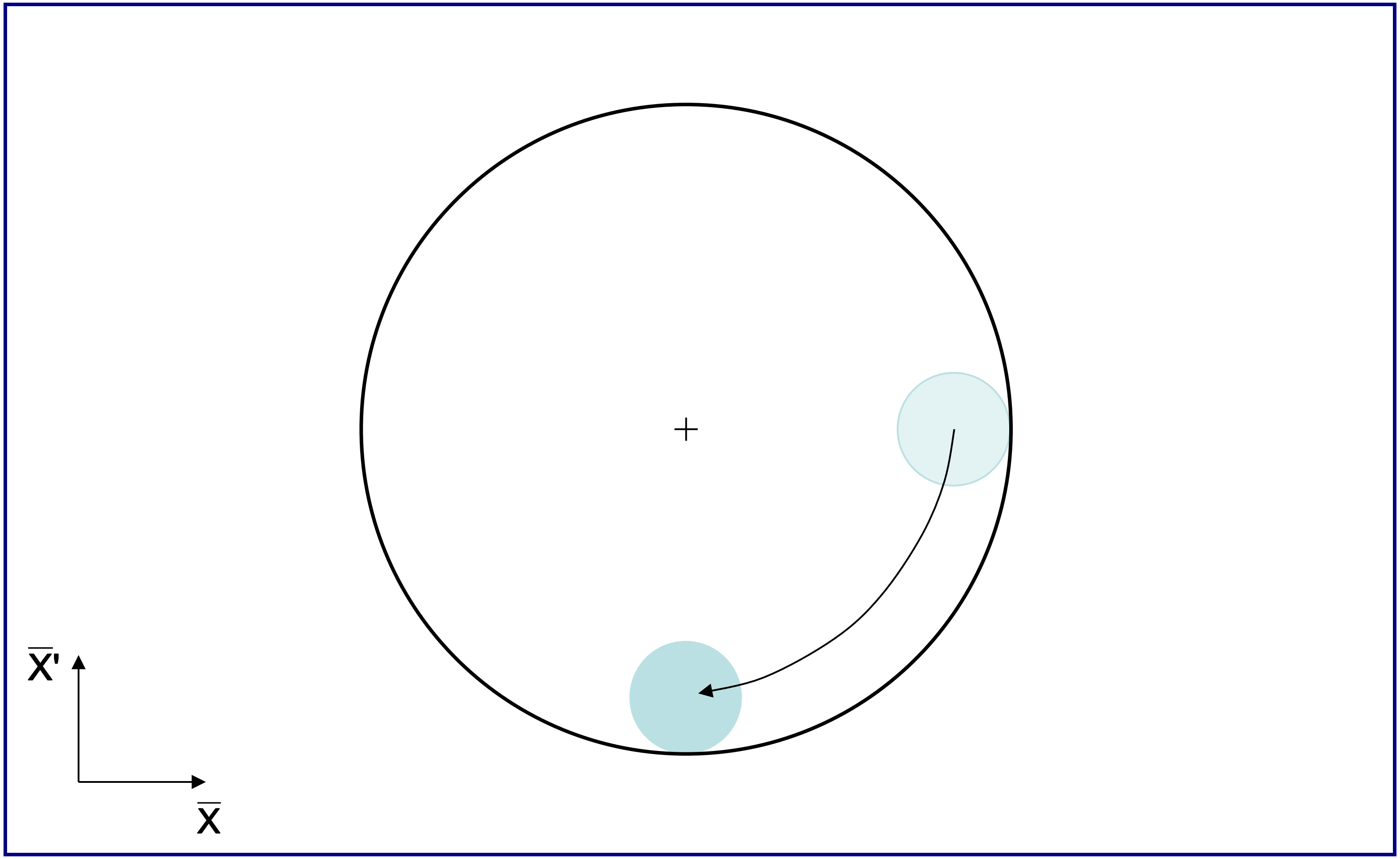
Single-turn injection

Normalised phase space at centre of idealised septum



Single-turn injection

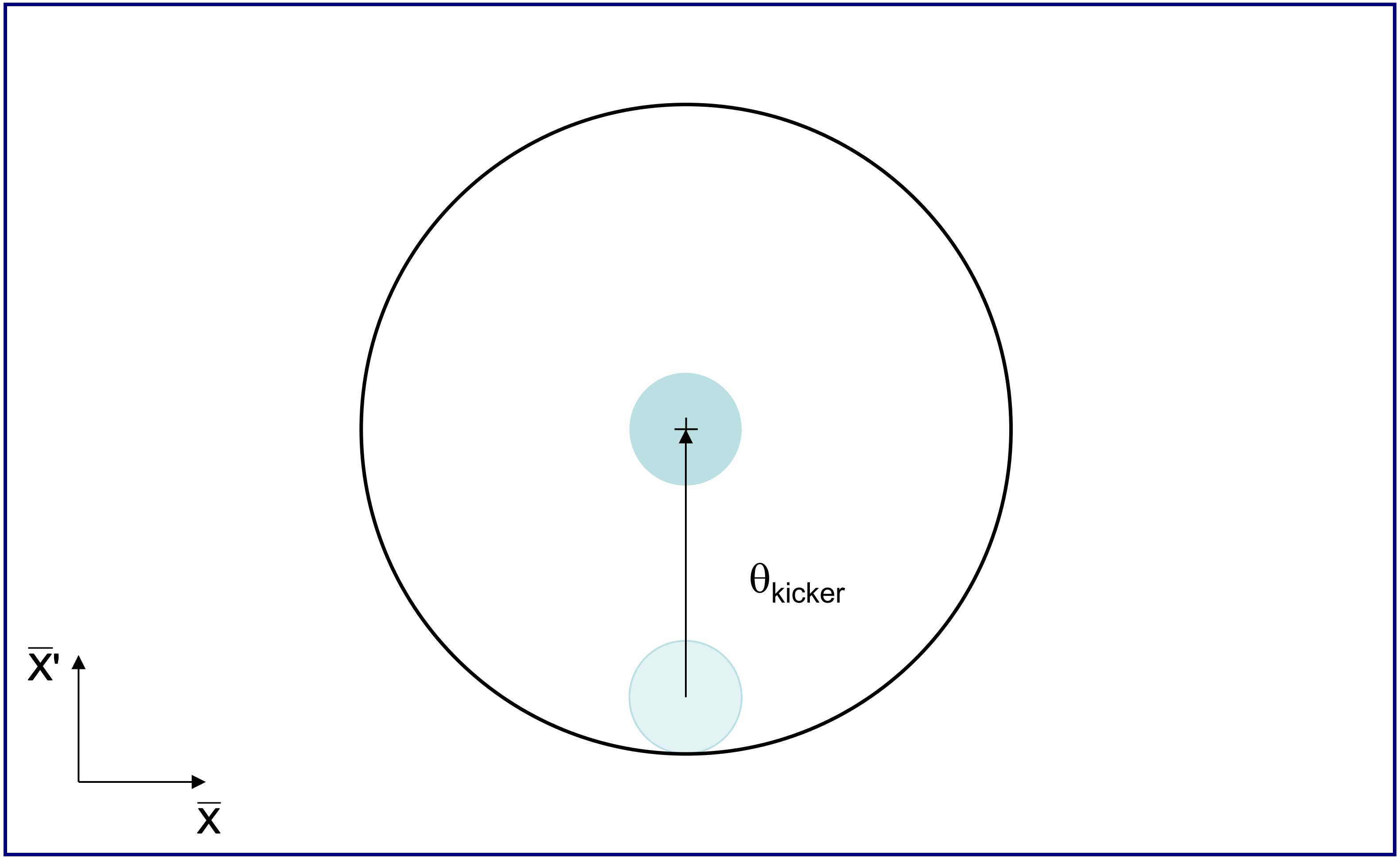
μ/2 phase advance to kicker location



Single-turn injection

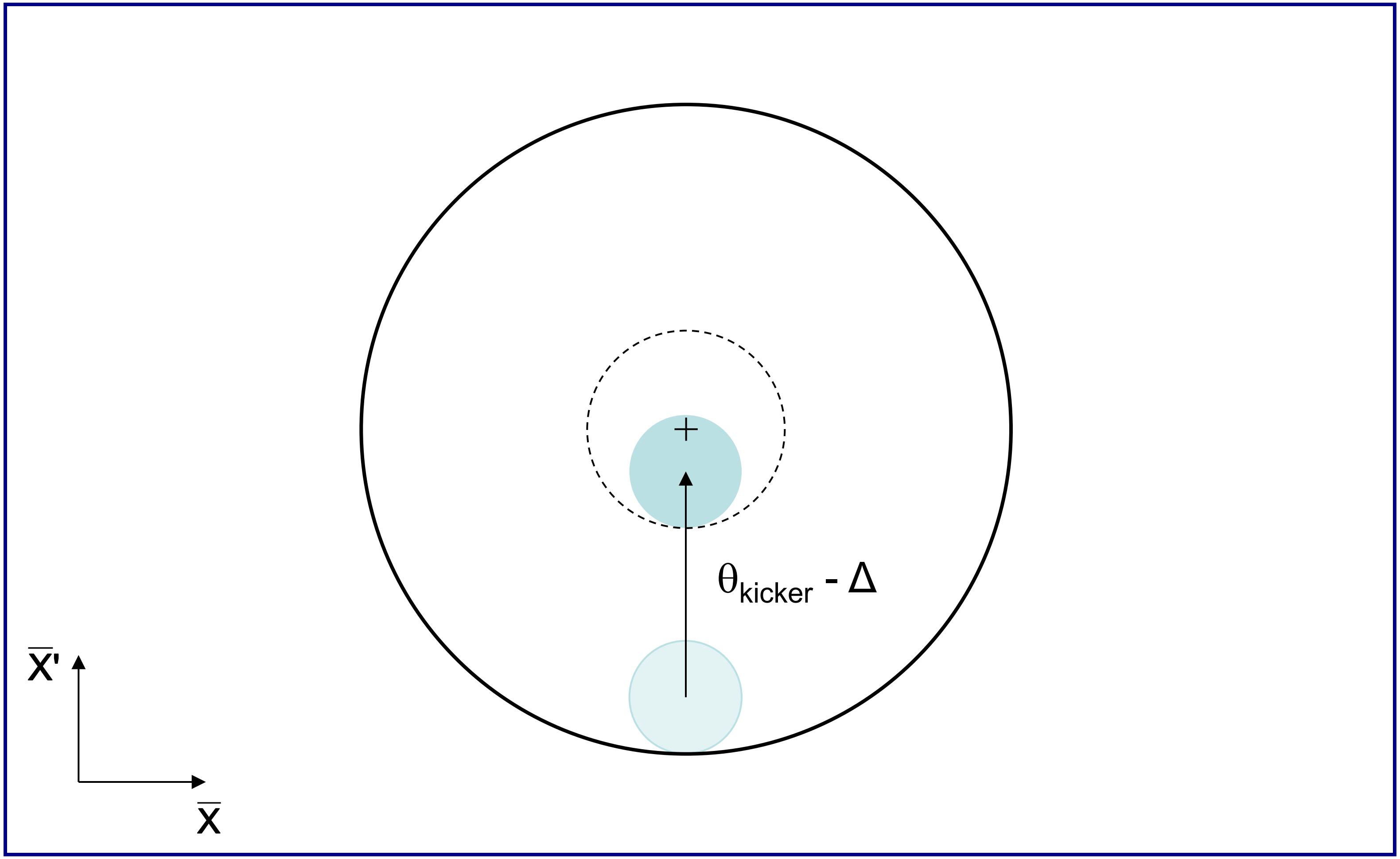
Normalised phase space at centre of idealised kicker

Kicker deflection places beam on central orbit:



Injection oscillations

For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:



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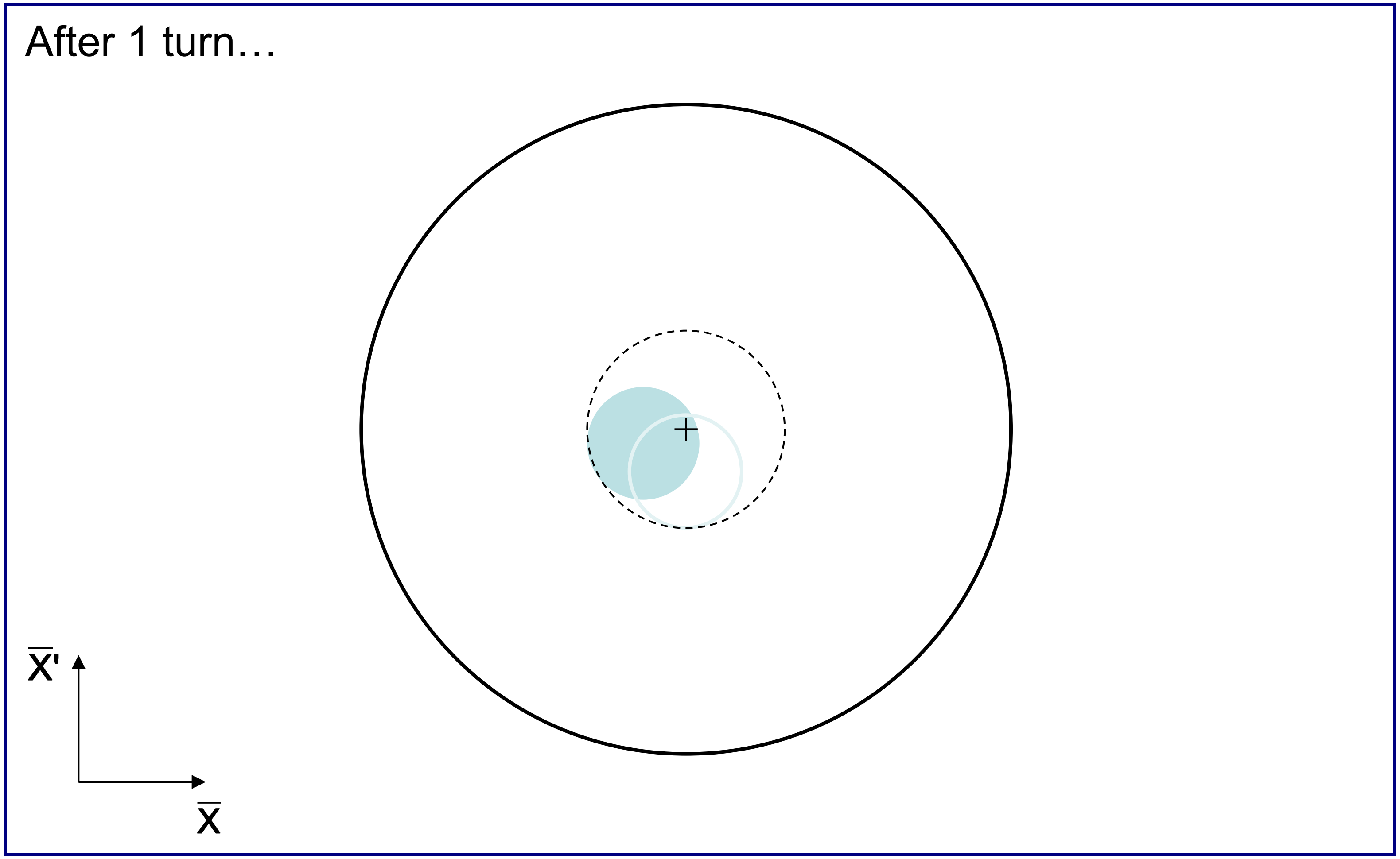

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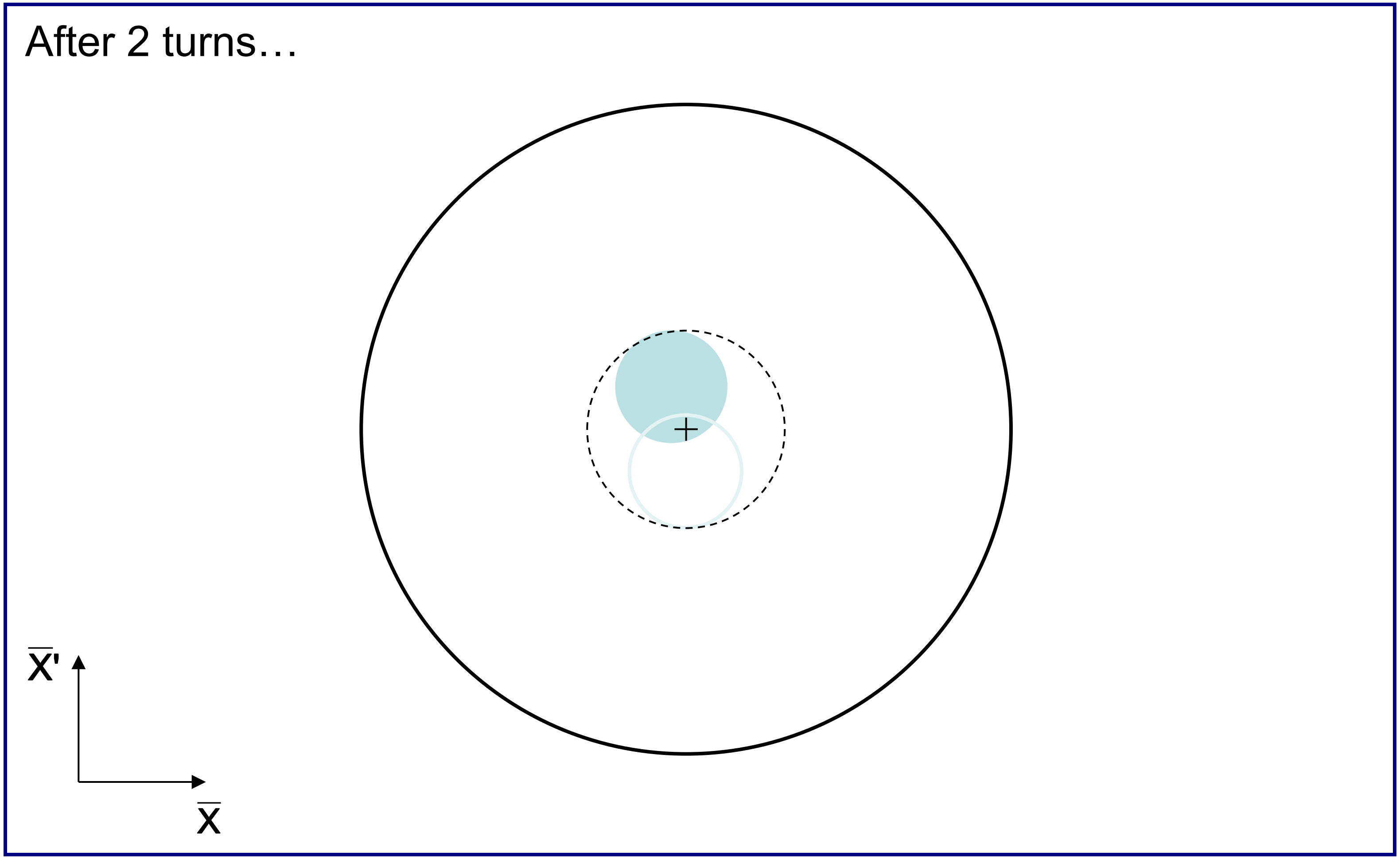

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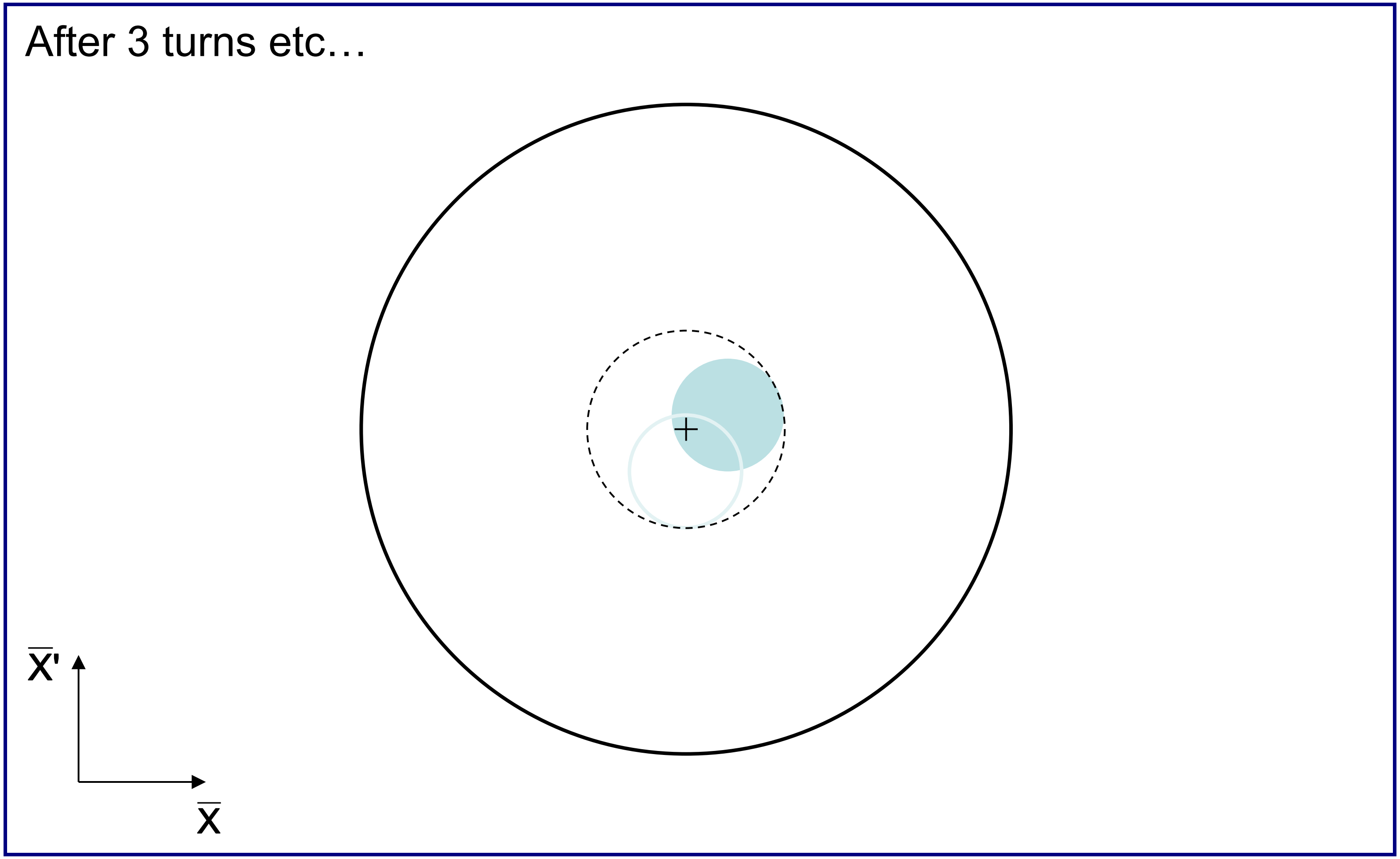
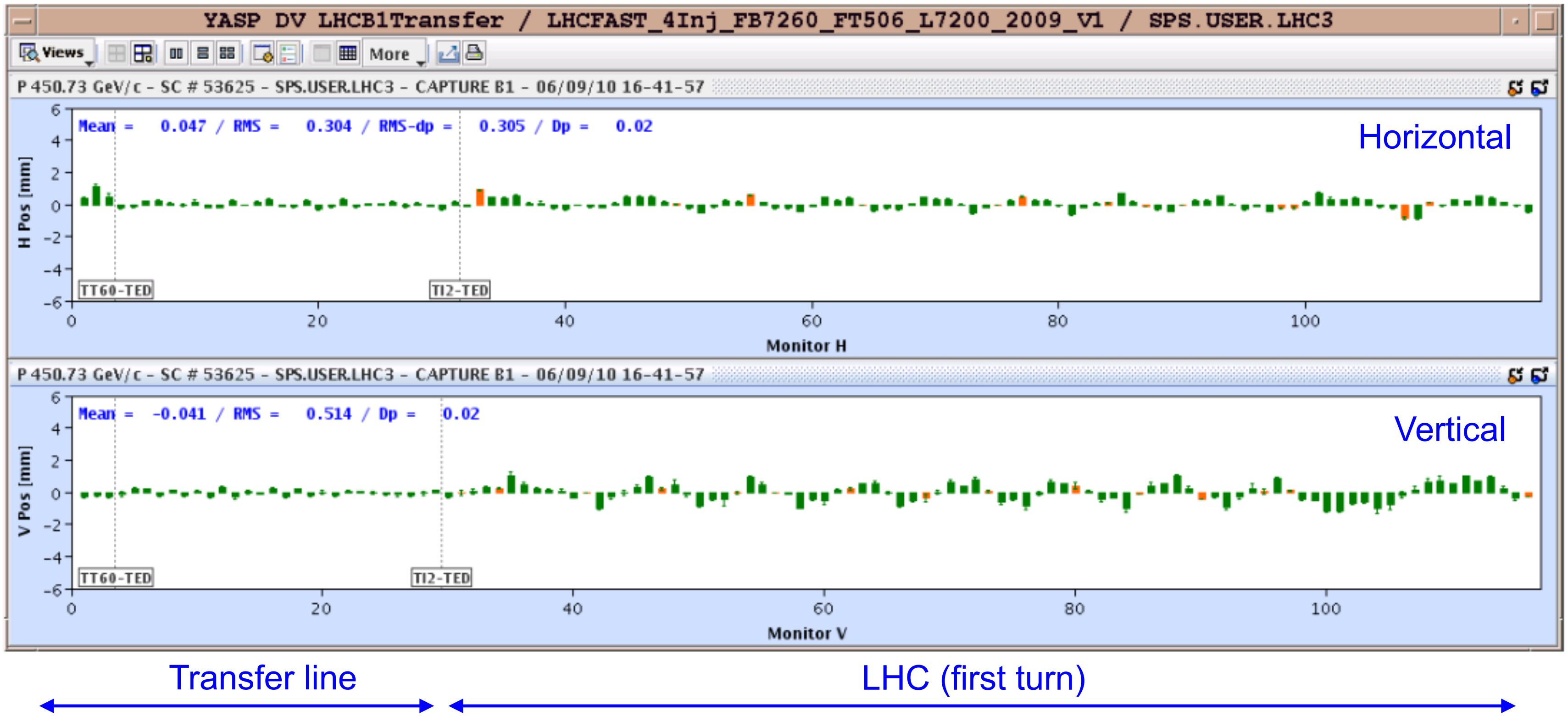
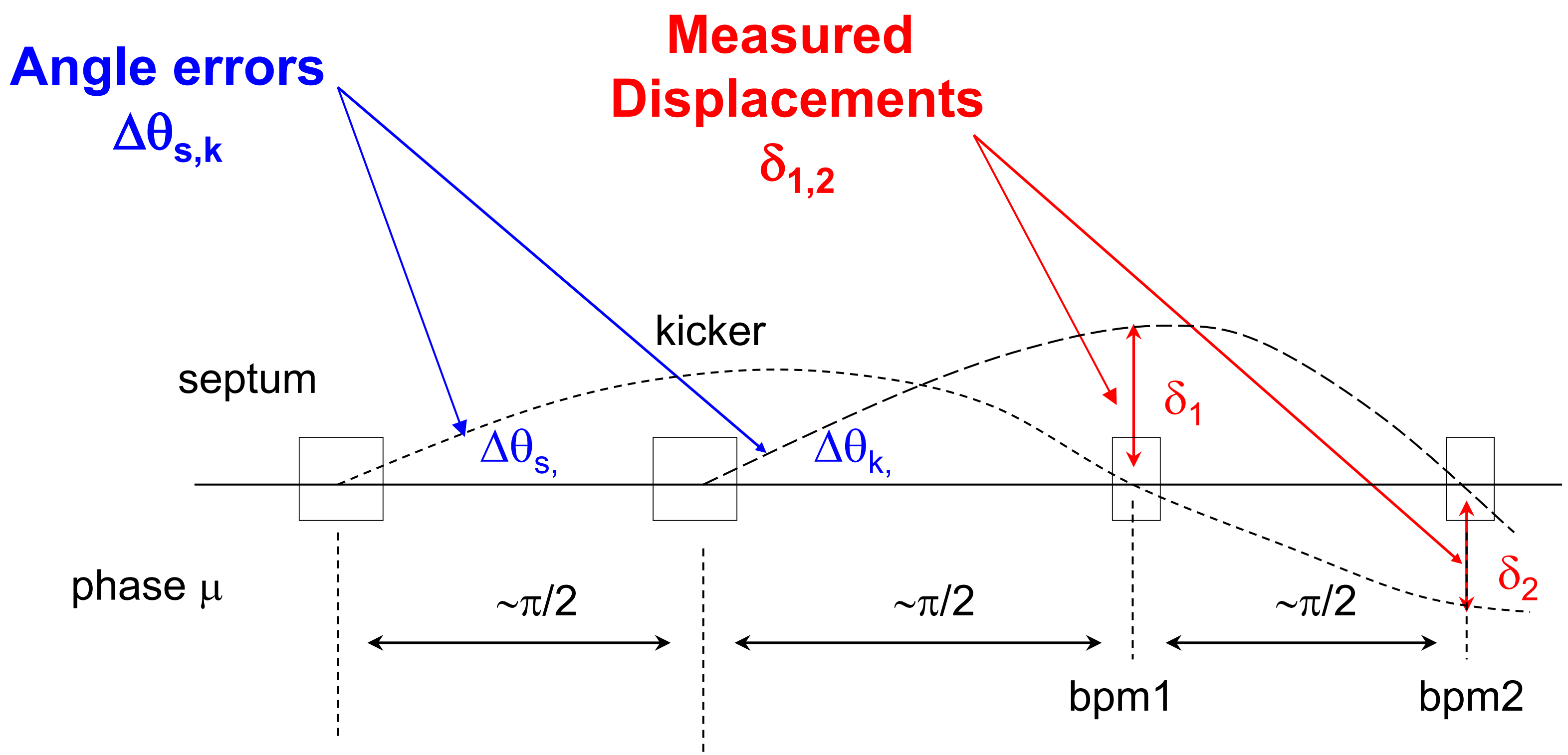


Injection oscillations

- Betatron oscillations with respect to the Closed Orbit:



Injection errors



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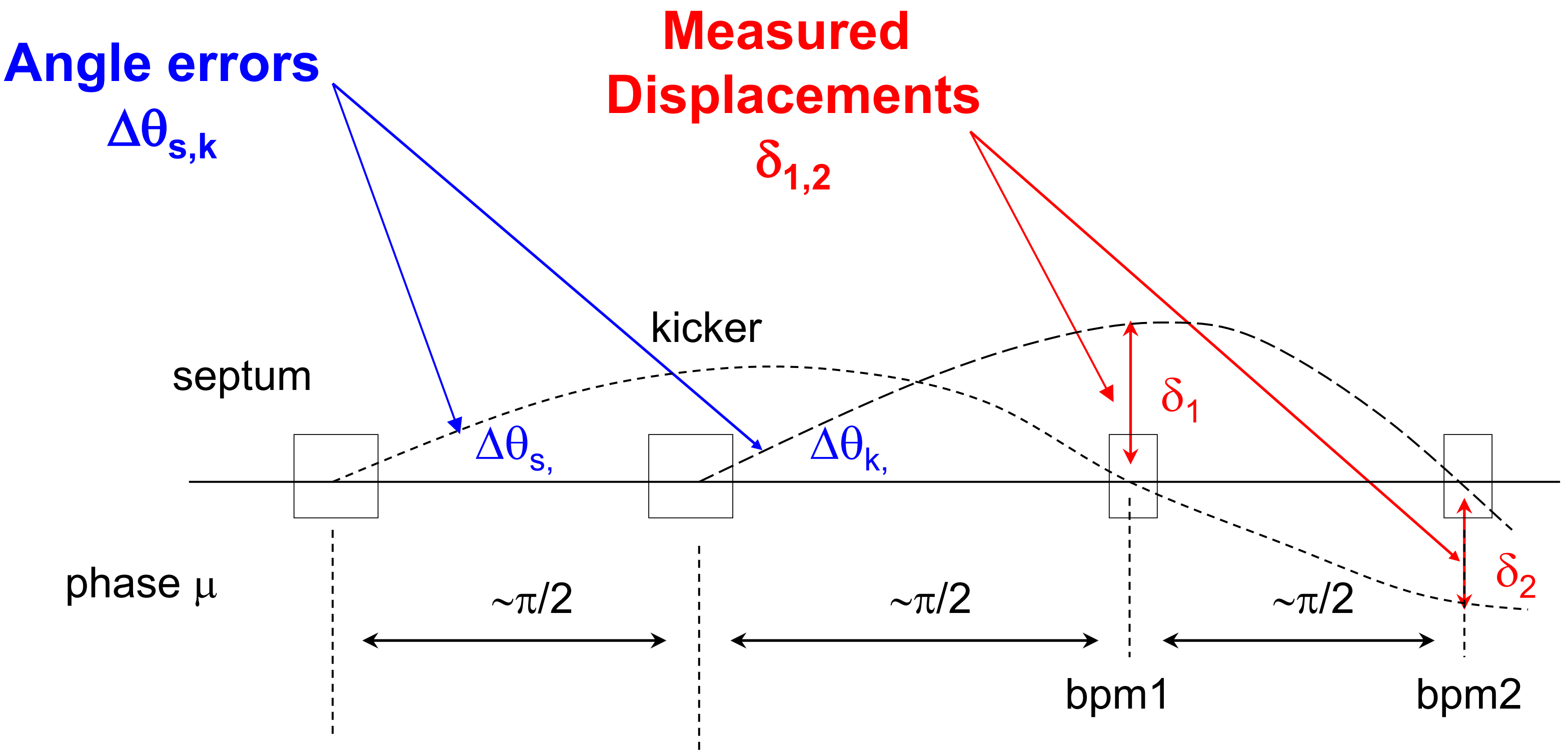
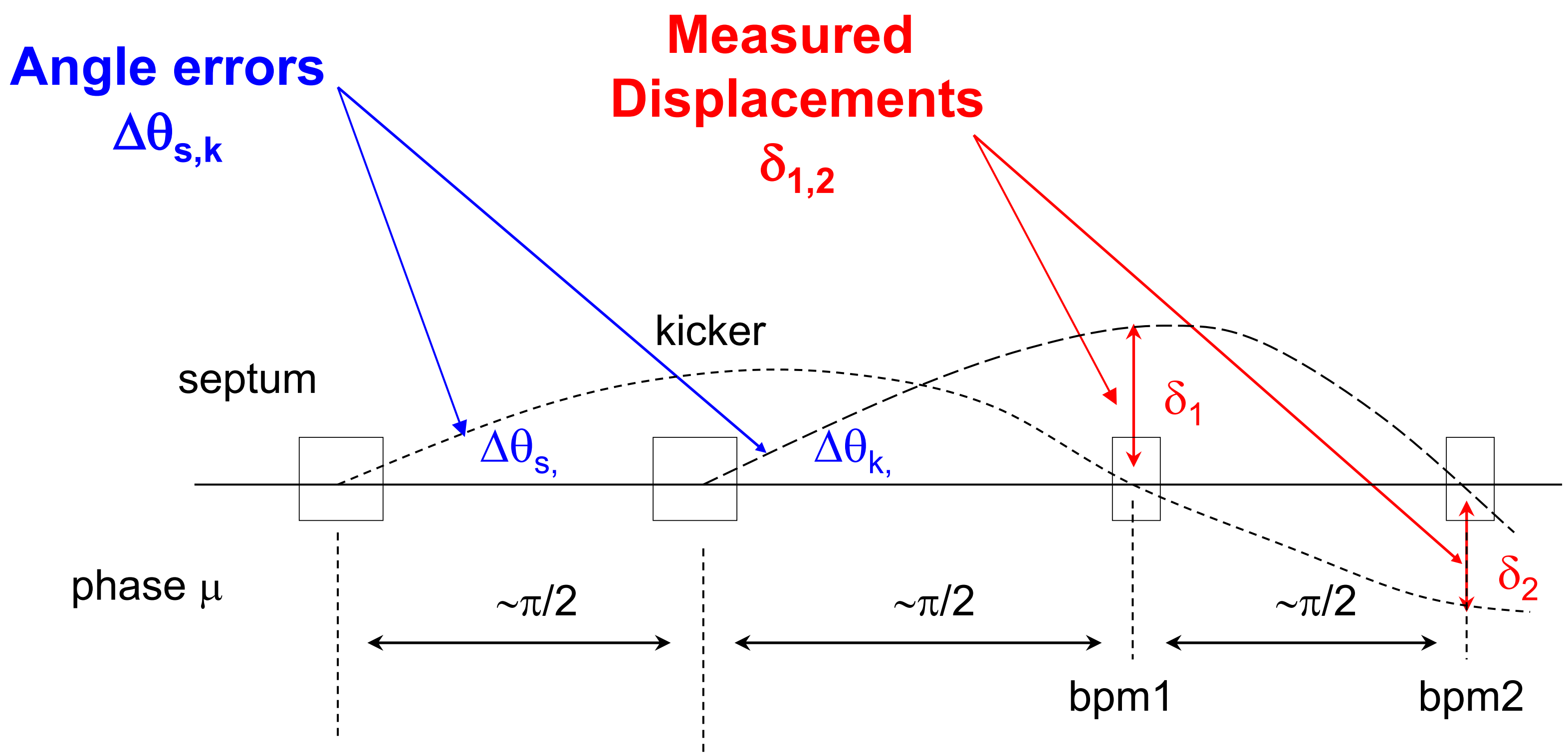


$$\delta_1 = \Delta\theta_s \sqrt{(\beta_s\beta_1)} \sin(\mu_1 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_1)} \sin(\mu_1 - \mu_k)$$
$$\approx \Delta\theta_k \sqrt{(\beta_k\beta_1)}$$

Injection errors



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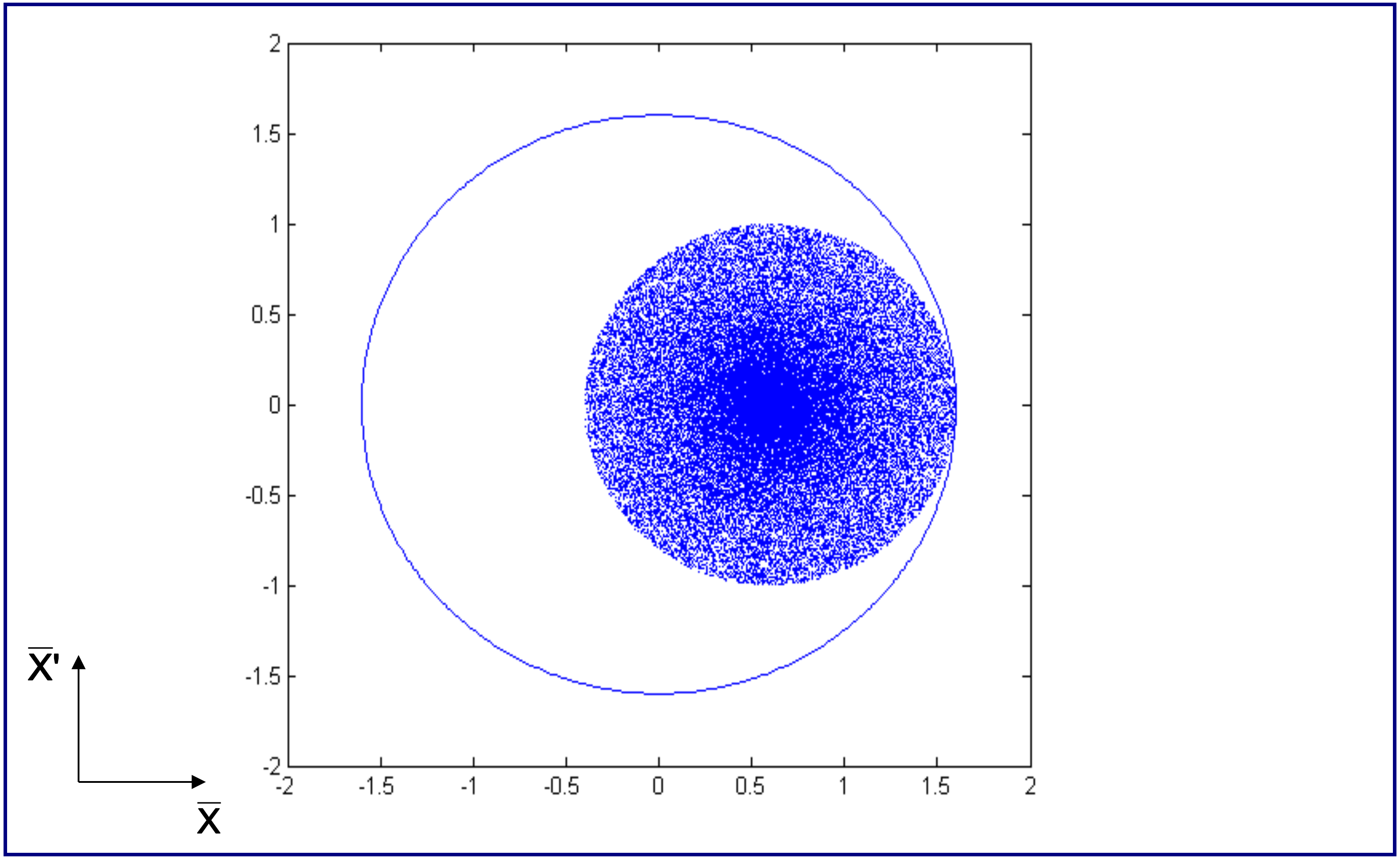
$$\approx \Delta\theta_k \sqrt{(\beta_k\beta_1)}$$

$$\delta_2 = \Delta\theta_s \sqrt{(\beta_s\beta_2)} \sin(\mu_2 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_2)} \sin(\mu_2 - \mu_k)$$

$$\approx -\Delta\theta_s \sqrt{(\beta_s\beta_2)}$$

Filamentation



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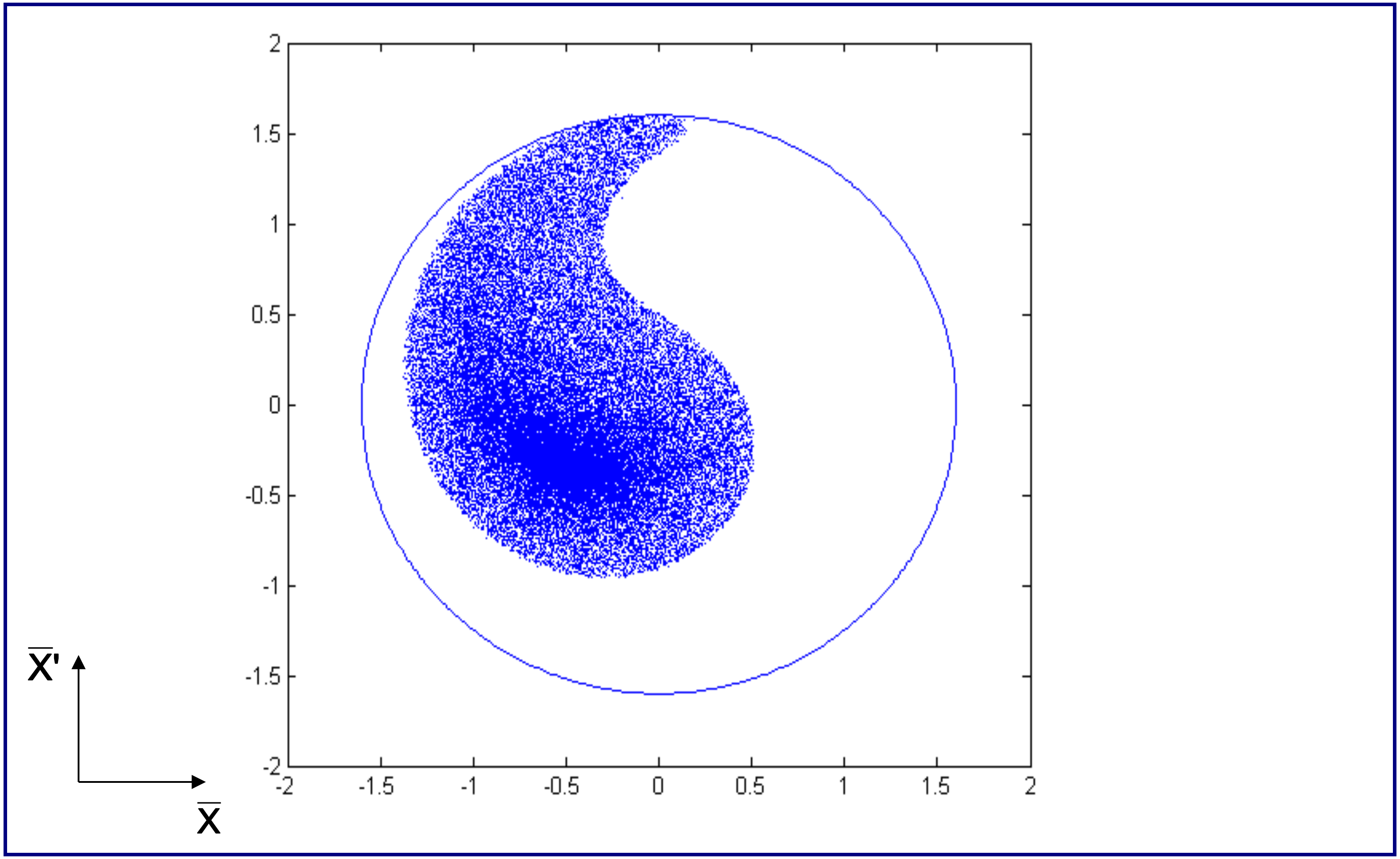

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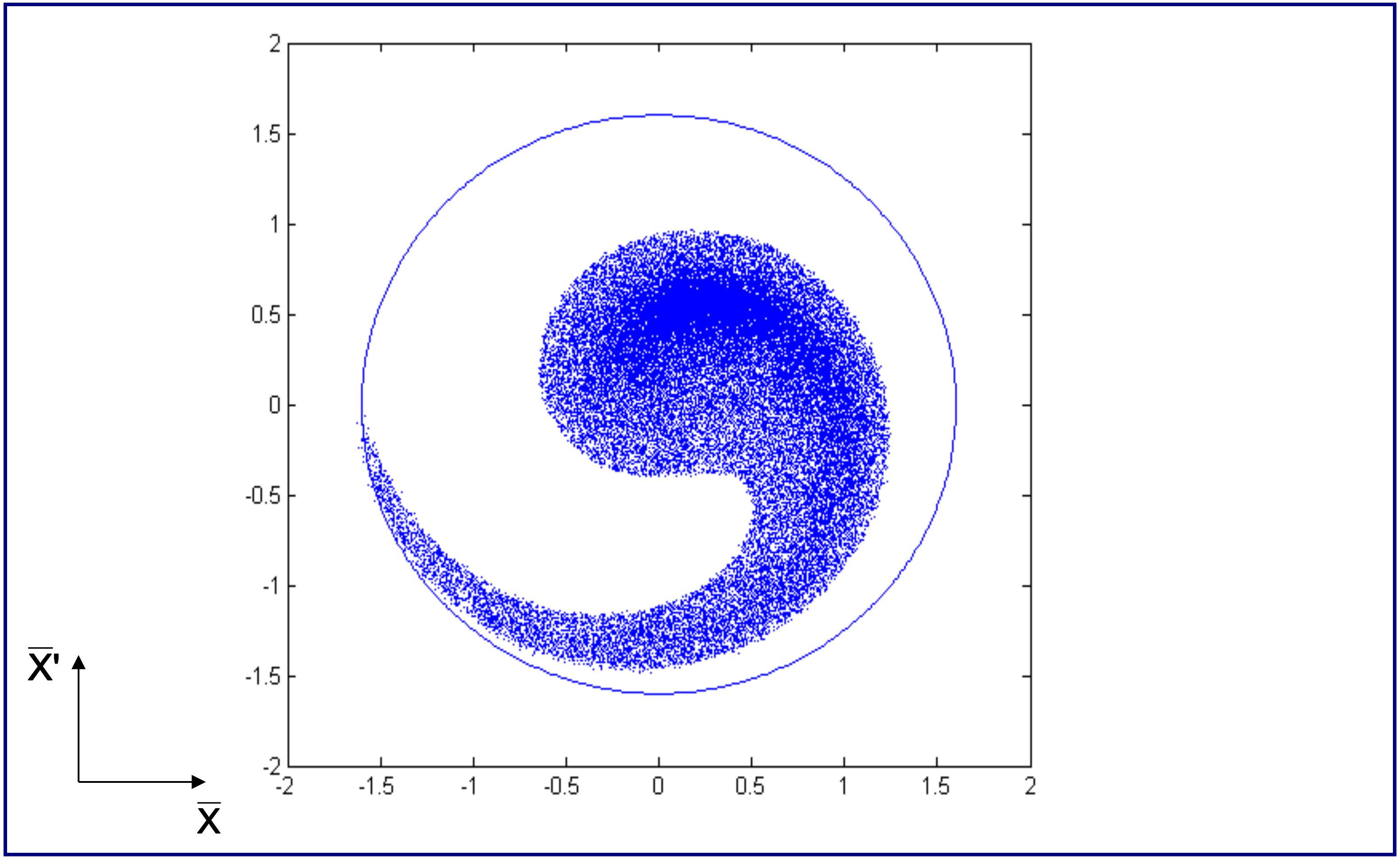
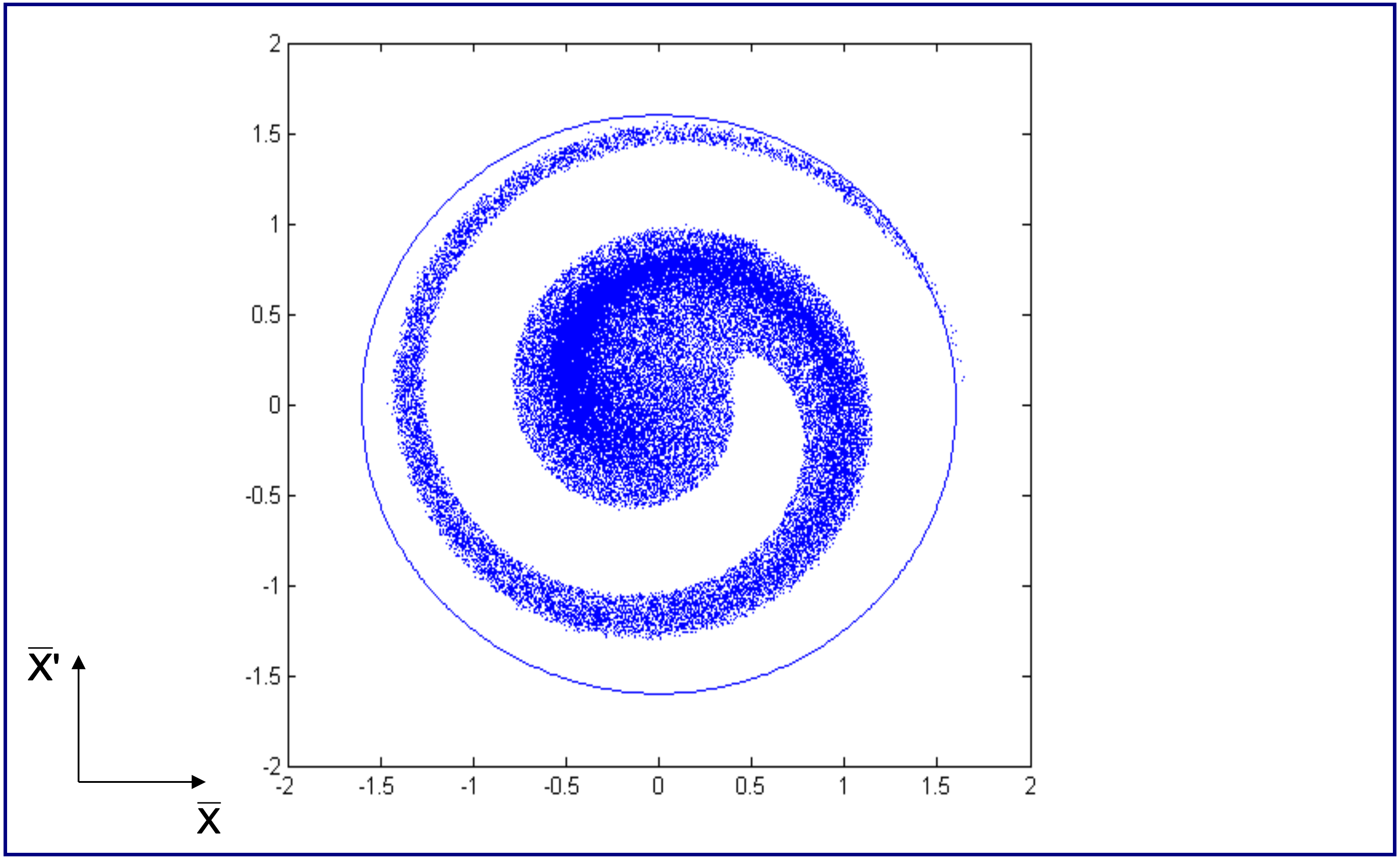
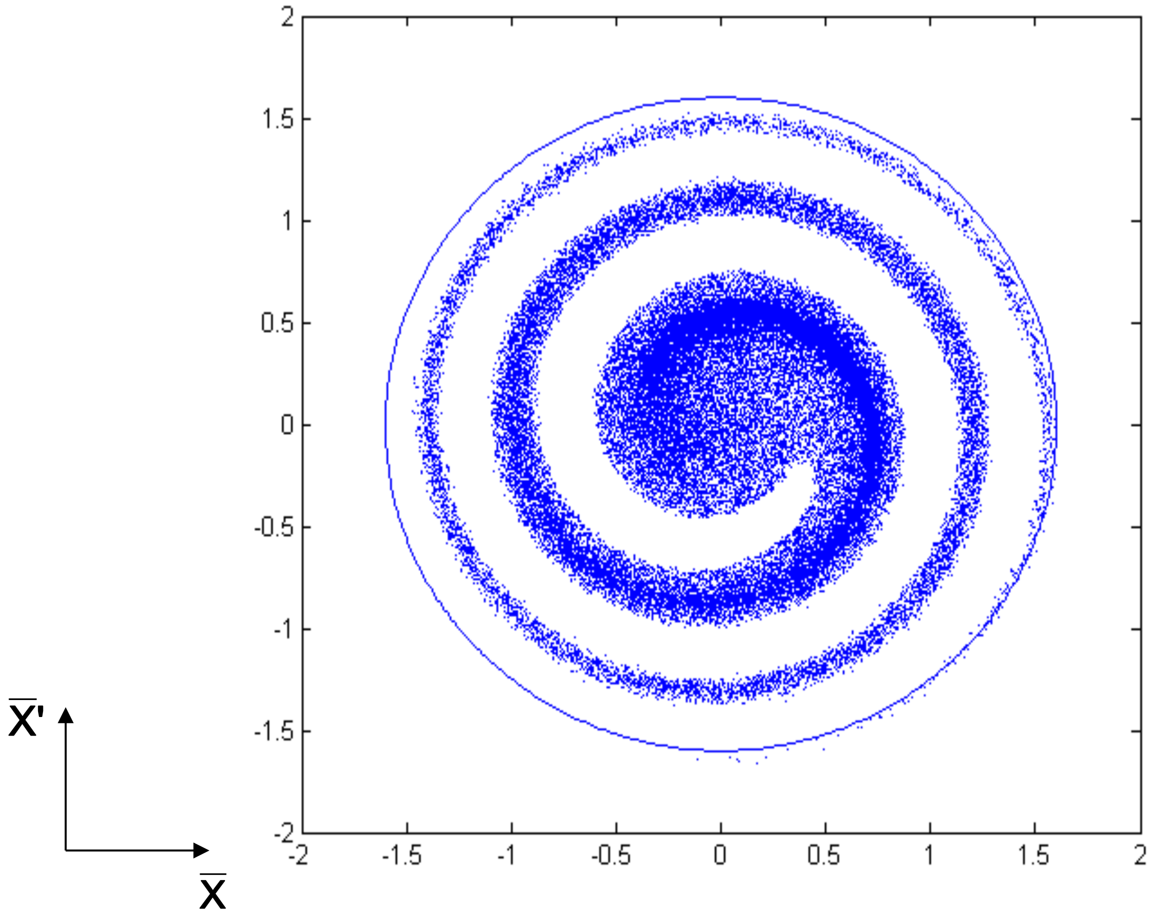


Filamentation



Filamentation



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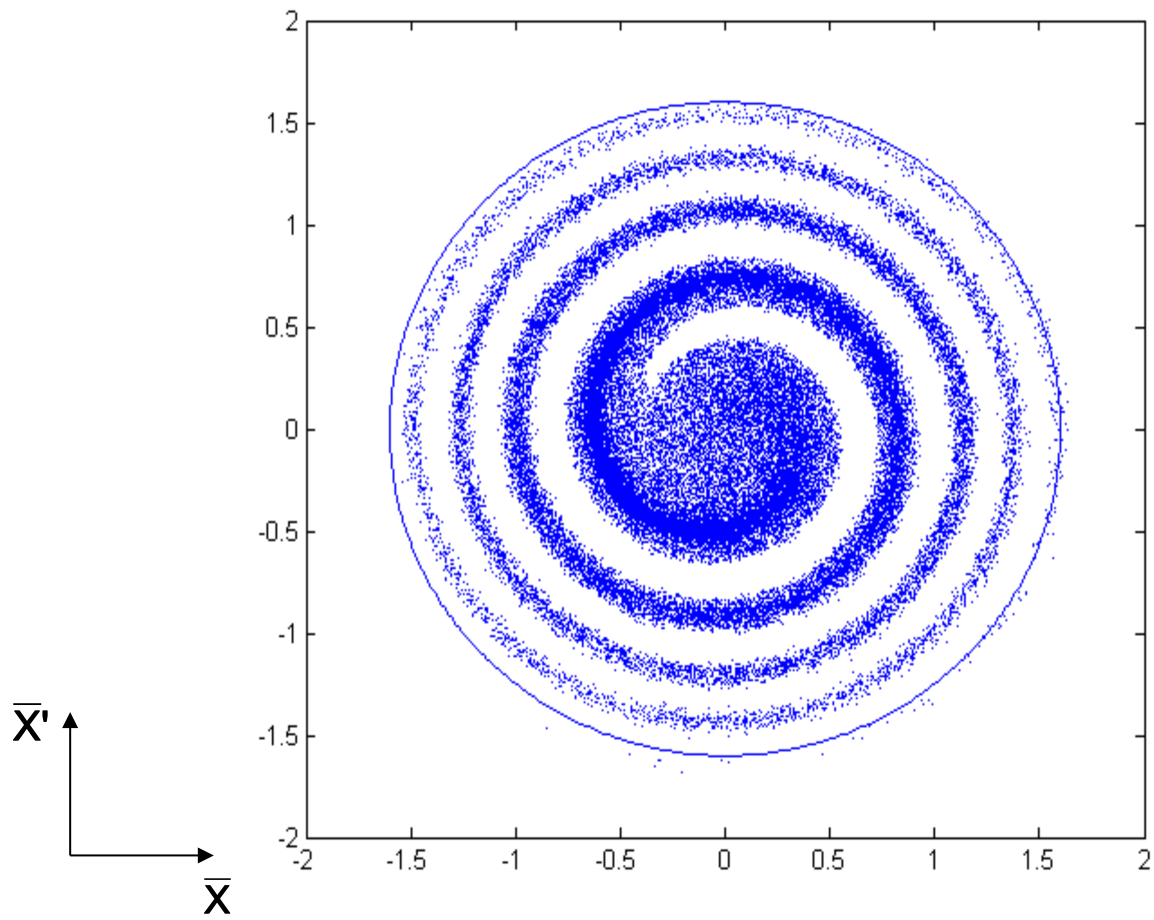

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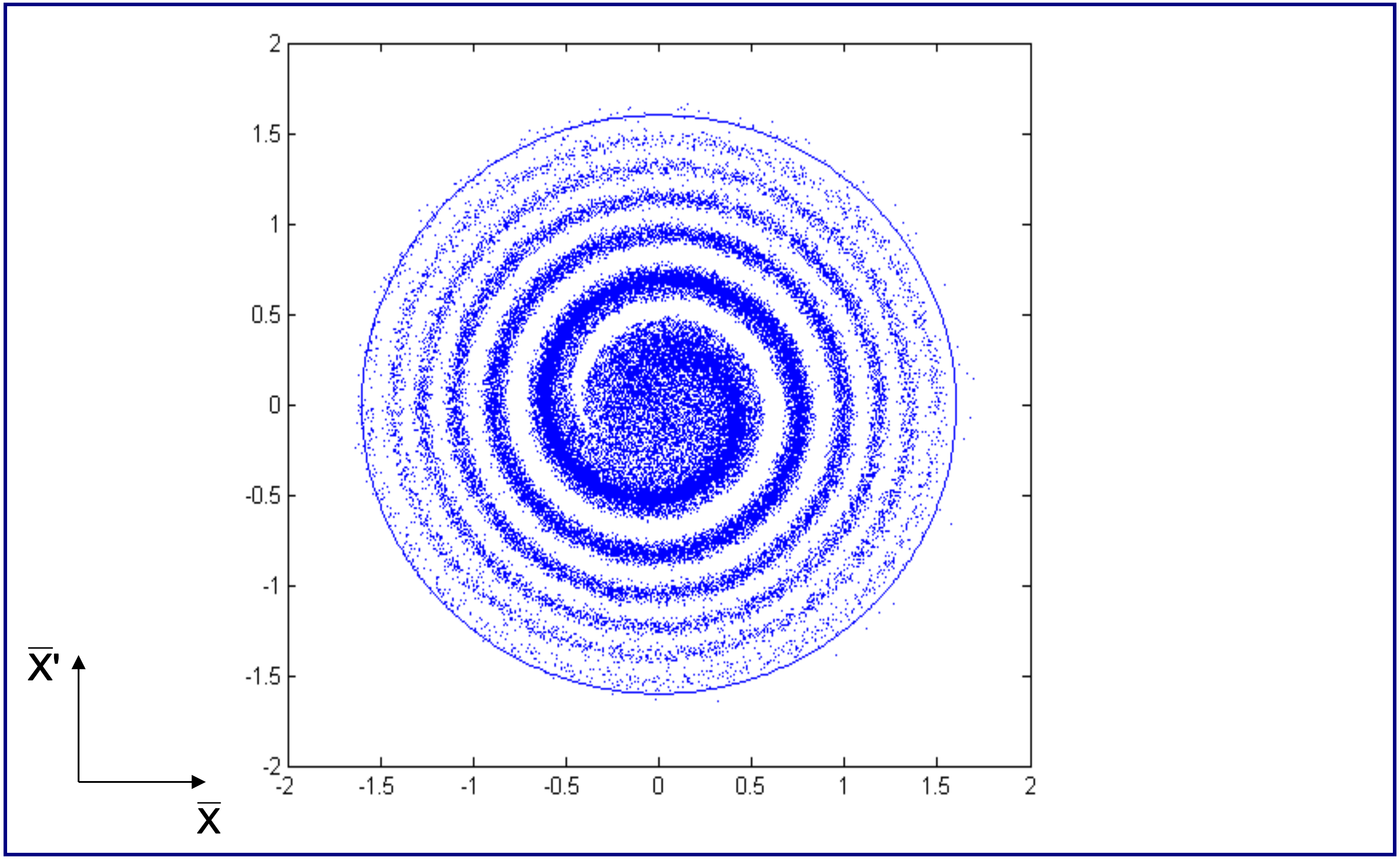

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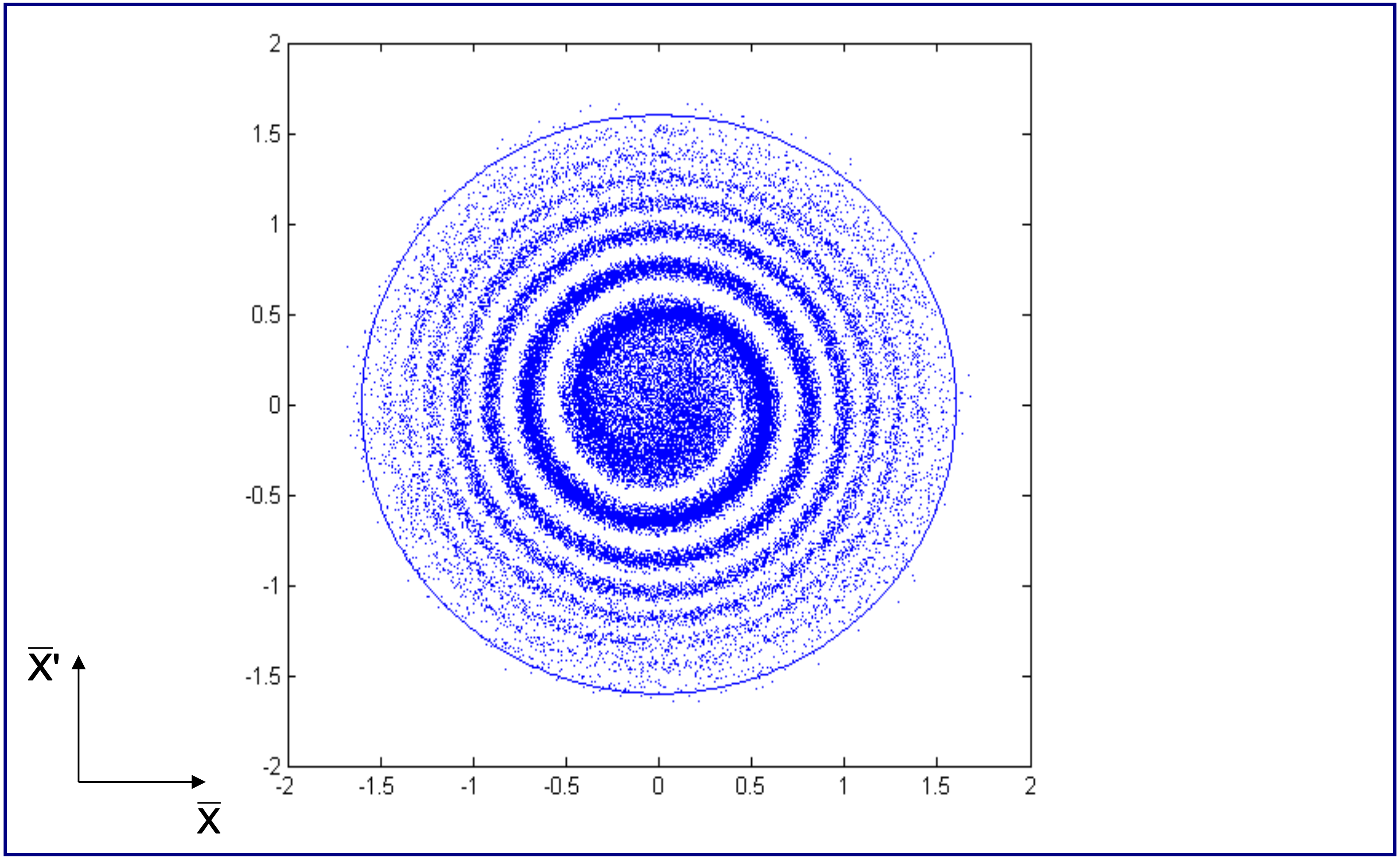

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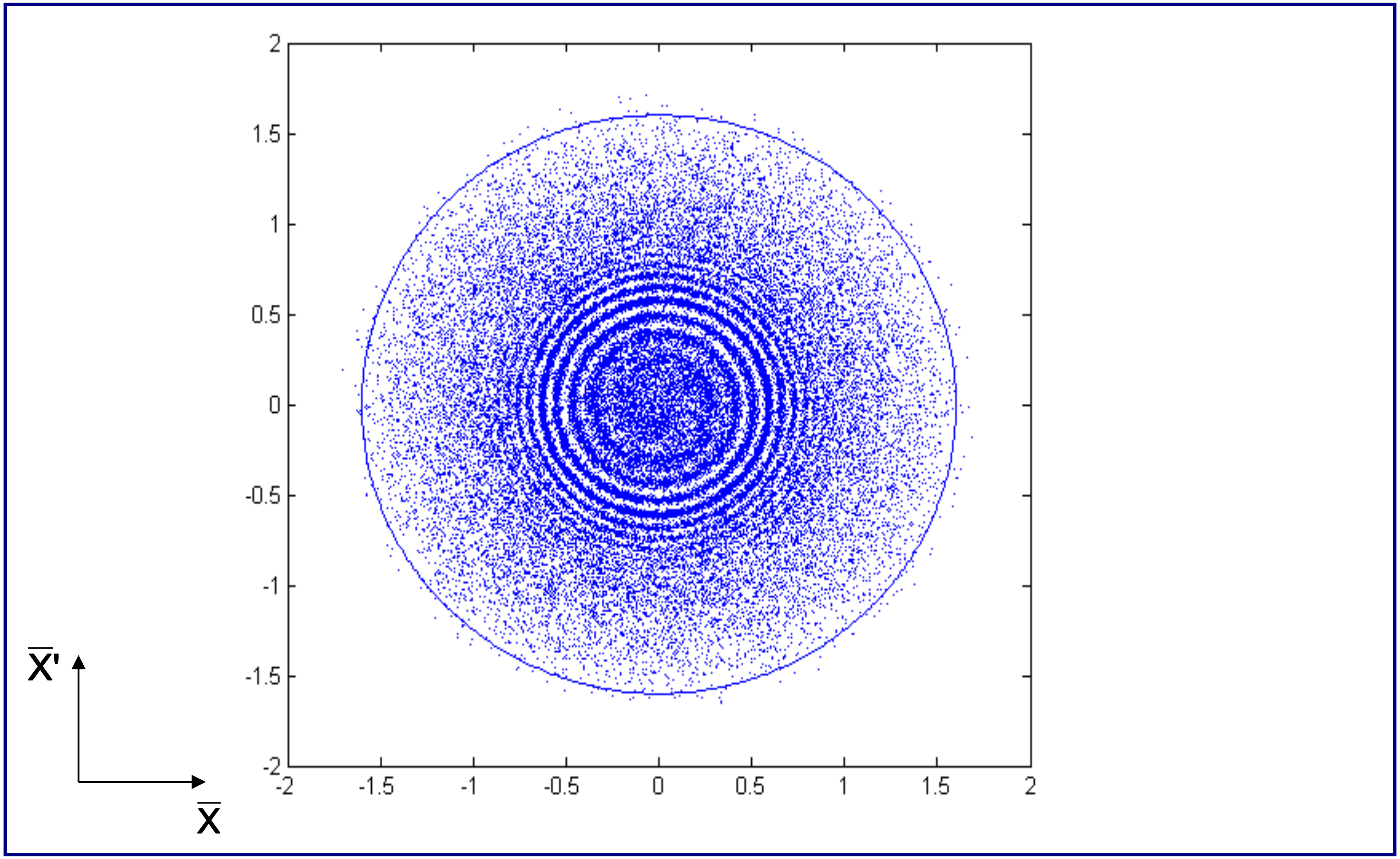
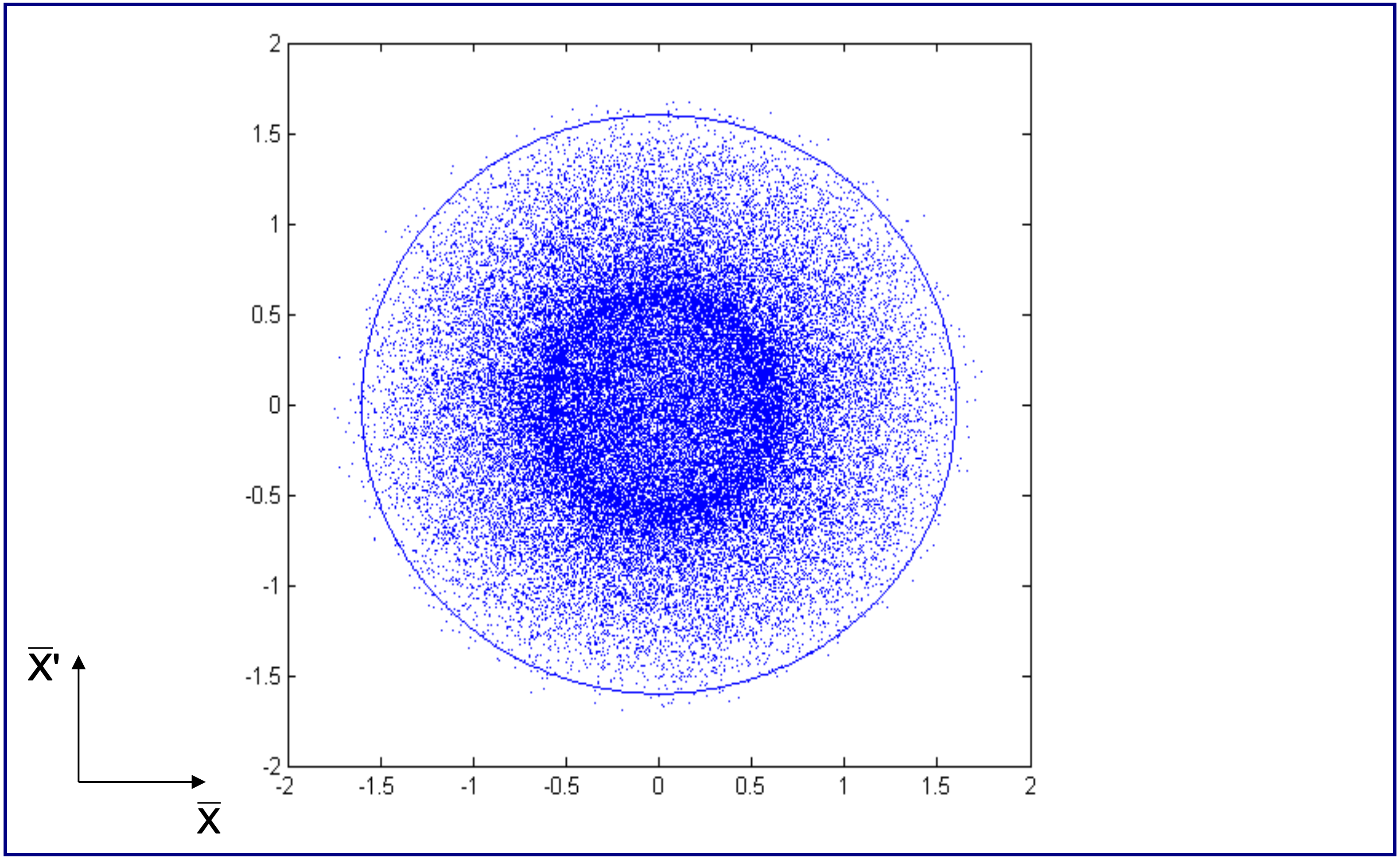
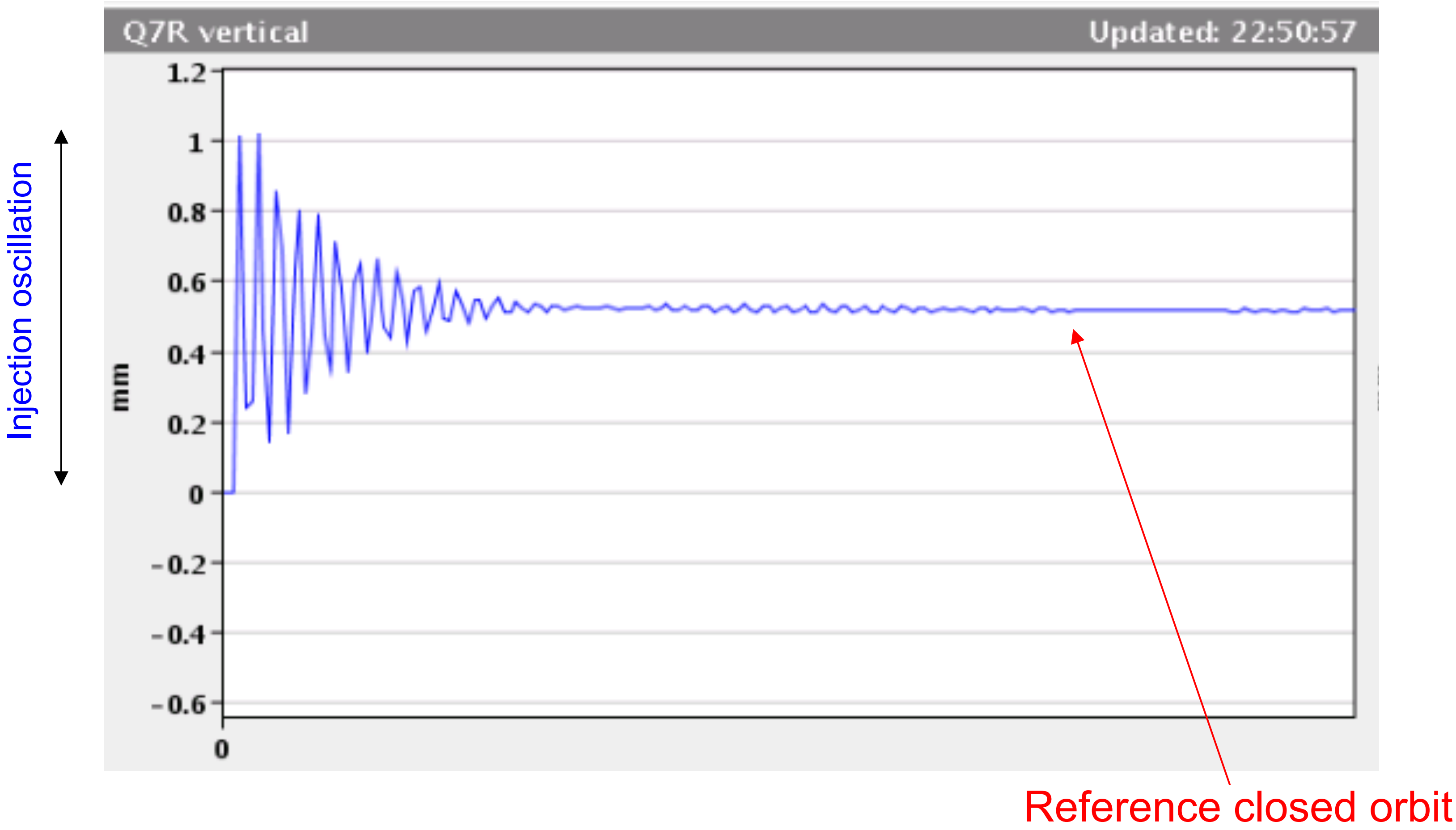


Filamentation



Filamentation

- Residual transverse oscillations lead to an *effective* emittance blow-up through filamentation:

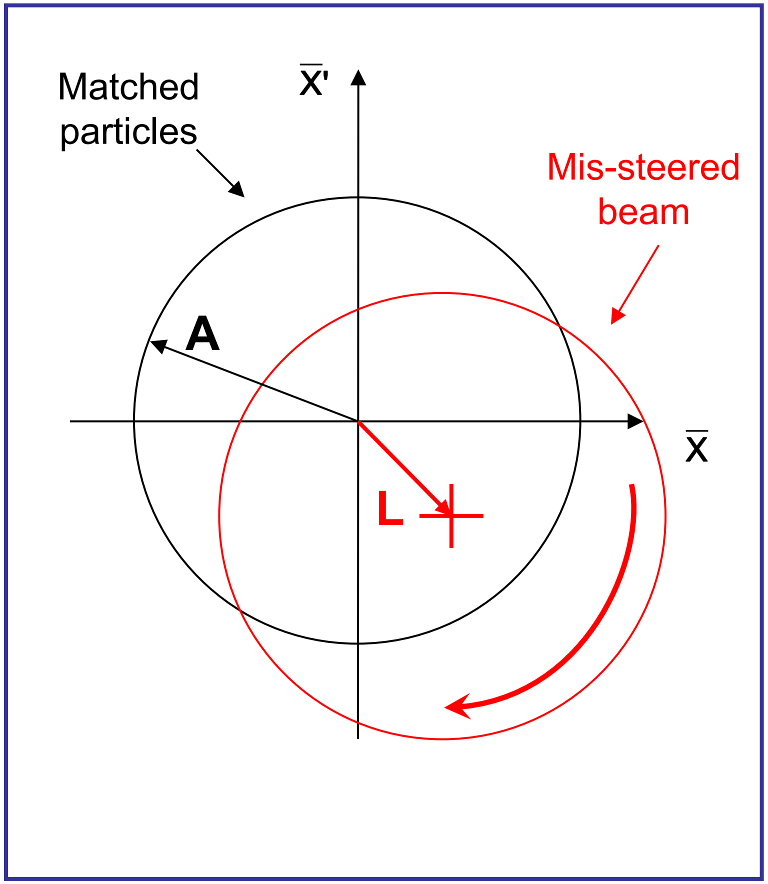


Filamentation

- Non-linear effects (e.g. higher-order field components) introduce amplitude-dependent effects into particle motion.
- Over many turns, a phase-space oscillation is transformed into an emittance increase.
- So any residual transverse oscillation will lead to an emittance blow-up through filamentation
  - Chromaticity coupled with a non-zero momentum spread at injection can also cause filmentation, often termed *chromatic decoherence*.
  - “Transverse damper” systems are used to damp injection oscillations - bunch position measured by a pick-up, which is linked to a kicker

Blow-up from steering error

- Consider a collection of particles with max. amplitudes A
- The beam can be injected with an error in angle and position.
- For an injection error Δa, in units of $\sigma = \sqrt{(\beta\varepsilon)}$, the mis-steered beam is offset in normalised phase space by an amplitude $L = \Delta a\sqrt{\varepsilon}$



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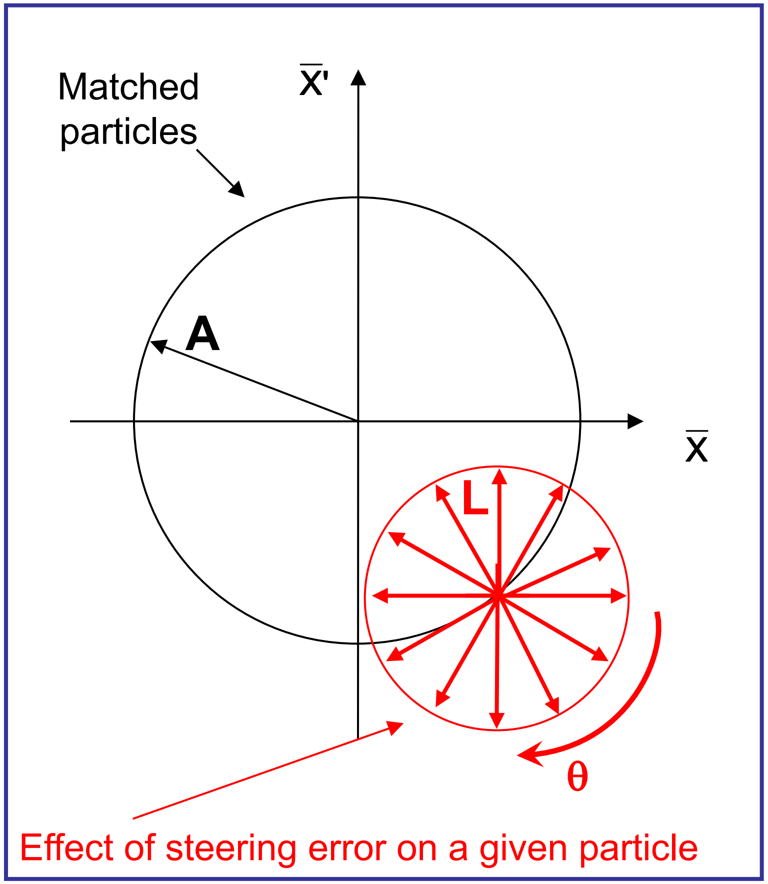
- Any given point on the matched ellipse is randomised over all phases after filamentation due to the steering error:



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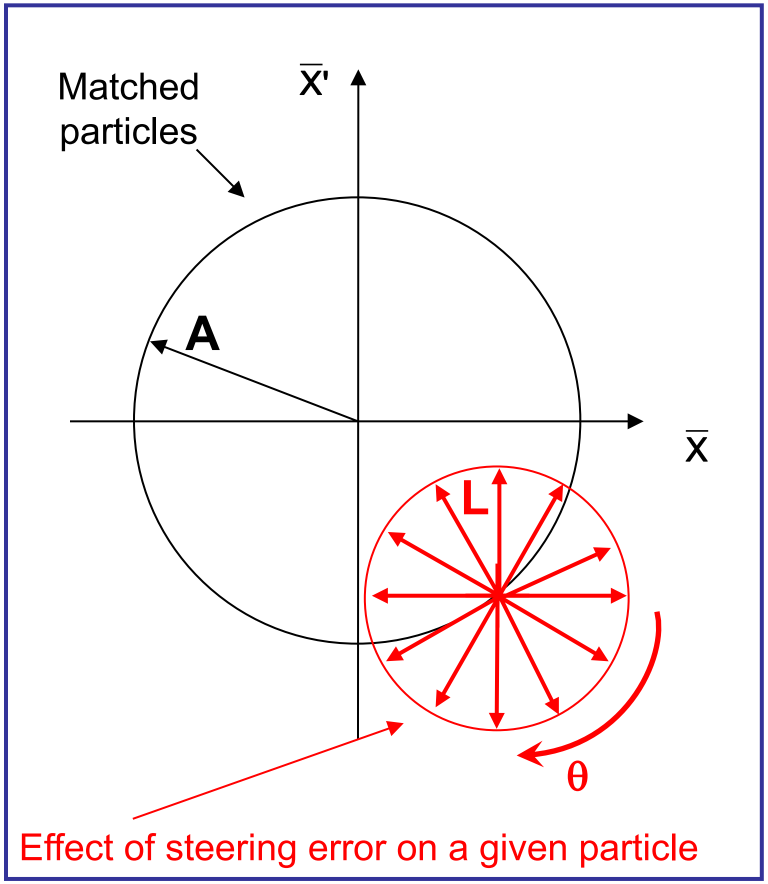
- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\varepsilon_{matched} = \left\langle A_i^2 \right\rangle / 2$$



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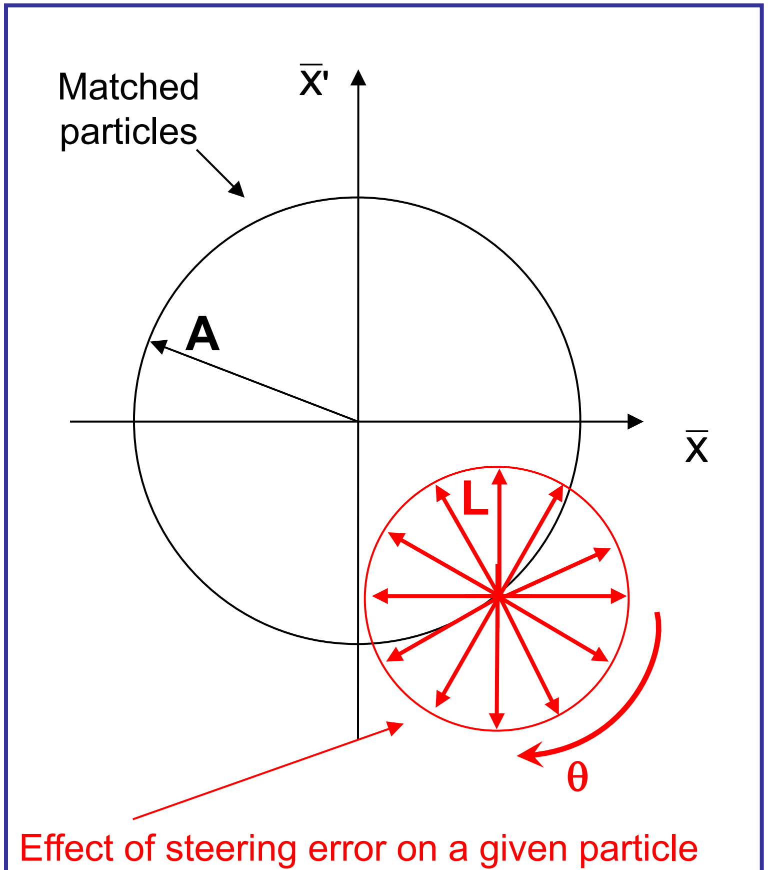
- After filamentation:

$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{L^2}{2}$$

See extra slides for derivation



Blow-up from steering error

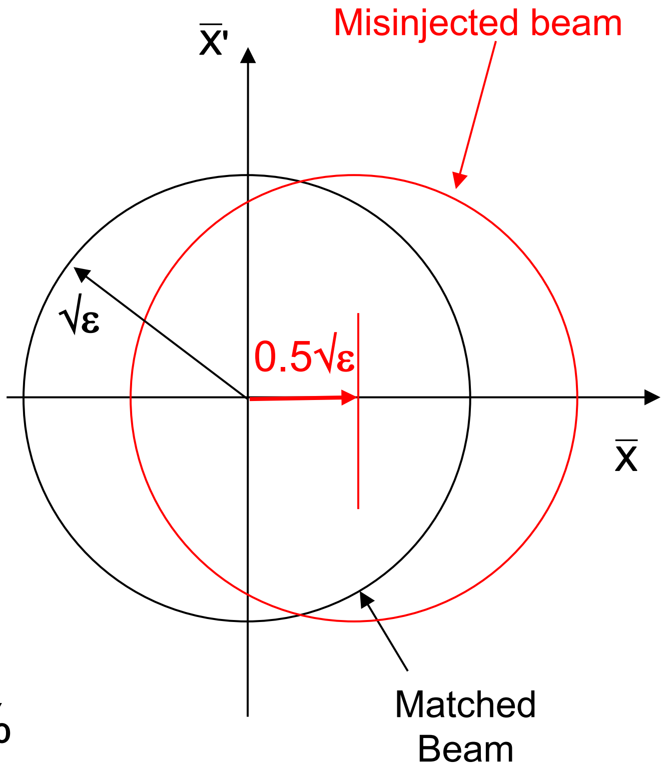
- A numerical example….
- Consider an offset Δa = 0.5σ for injected beam:

$$L = \Delta a \sqrt{\varepsilon_{matched}}$$

$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{L^2}{2}$$

$$= \varepsilon_{matched} \left[1 + \frac{\Delta a^2}{2} \right]$$

$$= \varepsilon_{matched} [1.125]$$



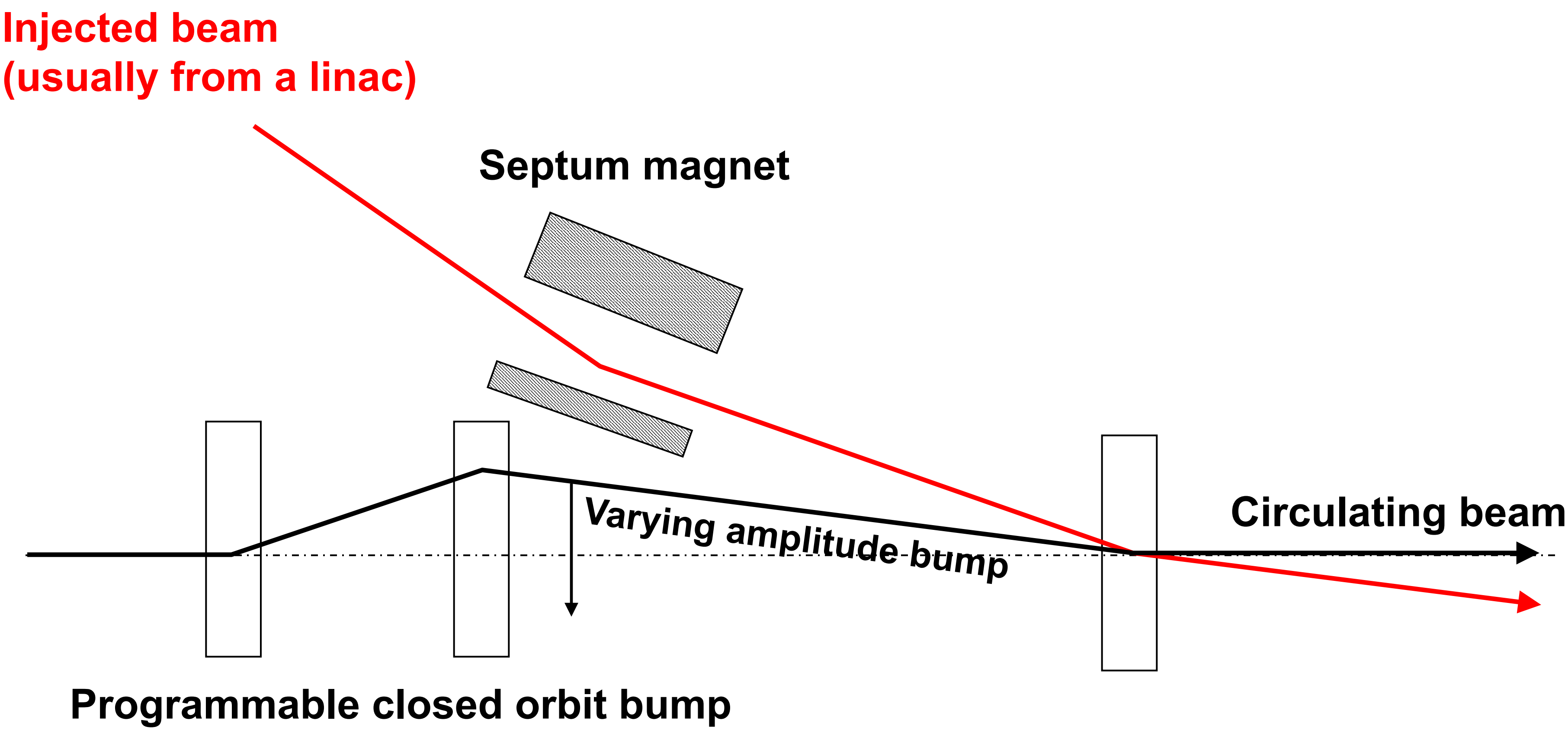
- For nominal LHC beam:

…allowed growth through LHC cycle ~10 %

Multi-turn injection

- For hadrons the beam density at injection can be limited either by space charge effects or by the injector capacity
- If we cannot increase charge density, we can sometimes fill the horizontal phase space to increase overall injected intensity.
  – If the acceptance of the receiving machine is larger than the delivered beam emittance we can accumulate intensity

Multi-turn injection for hadrons

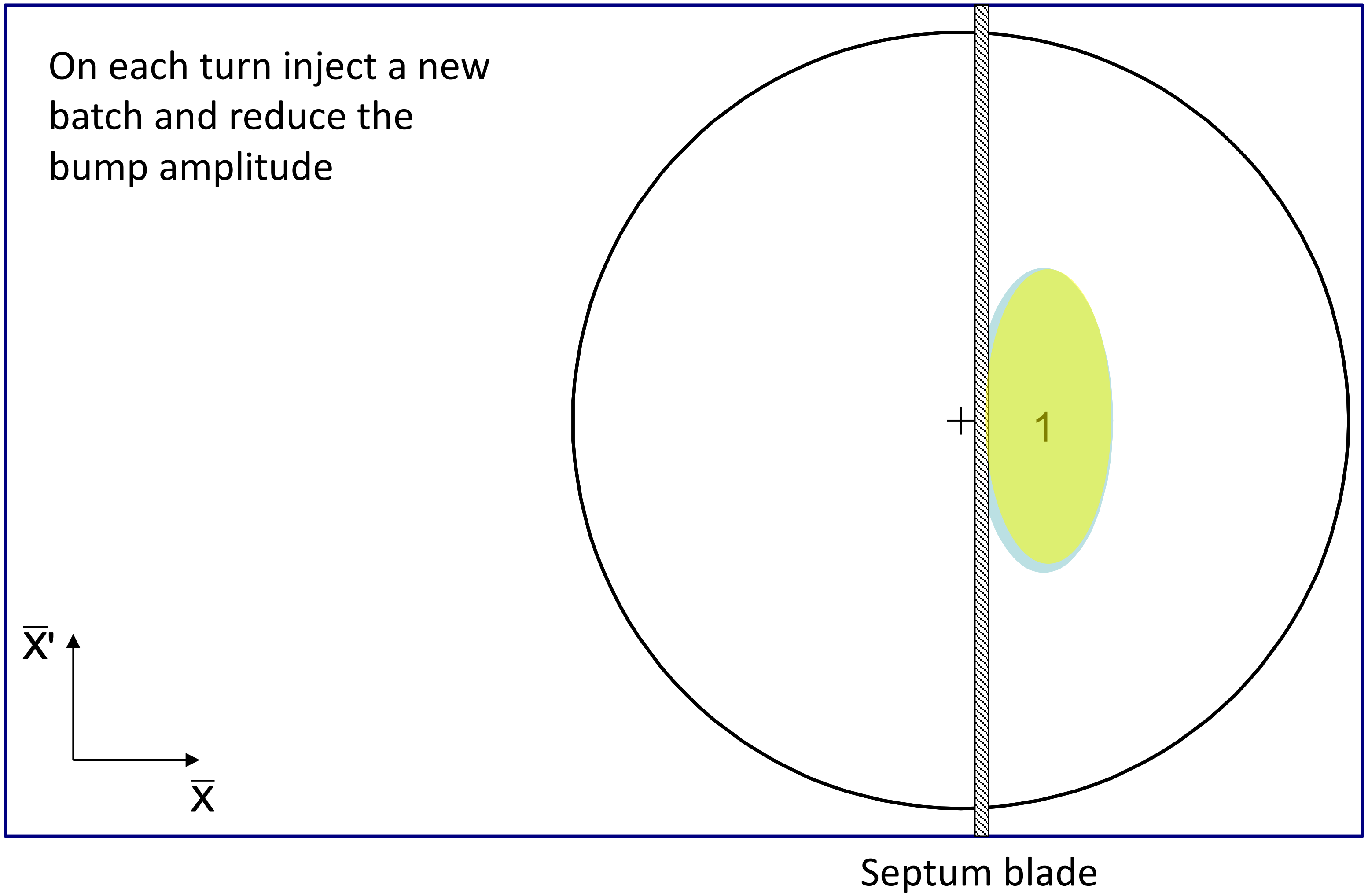


- No kicker but fast programmable bumpers
- Bump amplitude decreases and a new batch injected turn-by-turn
- Phase-space “painting”

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space

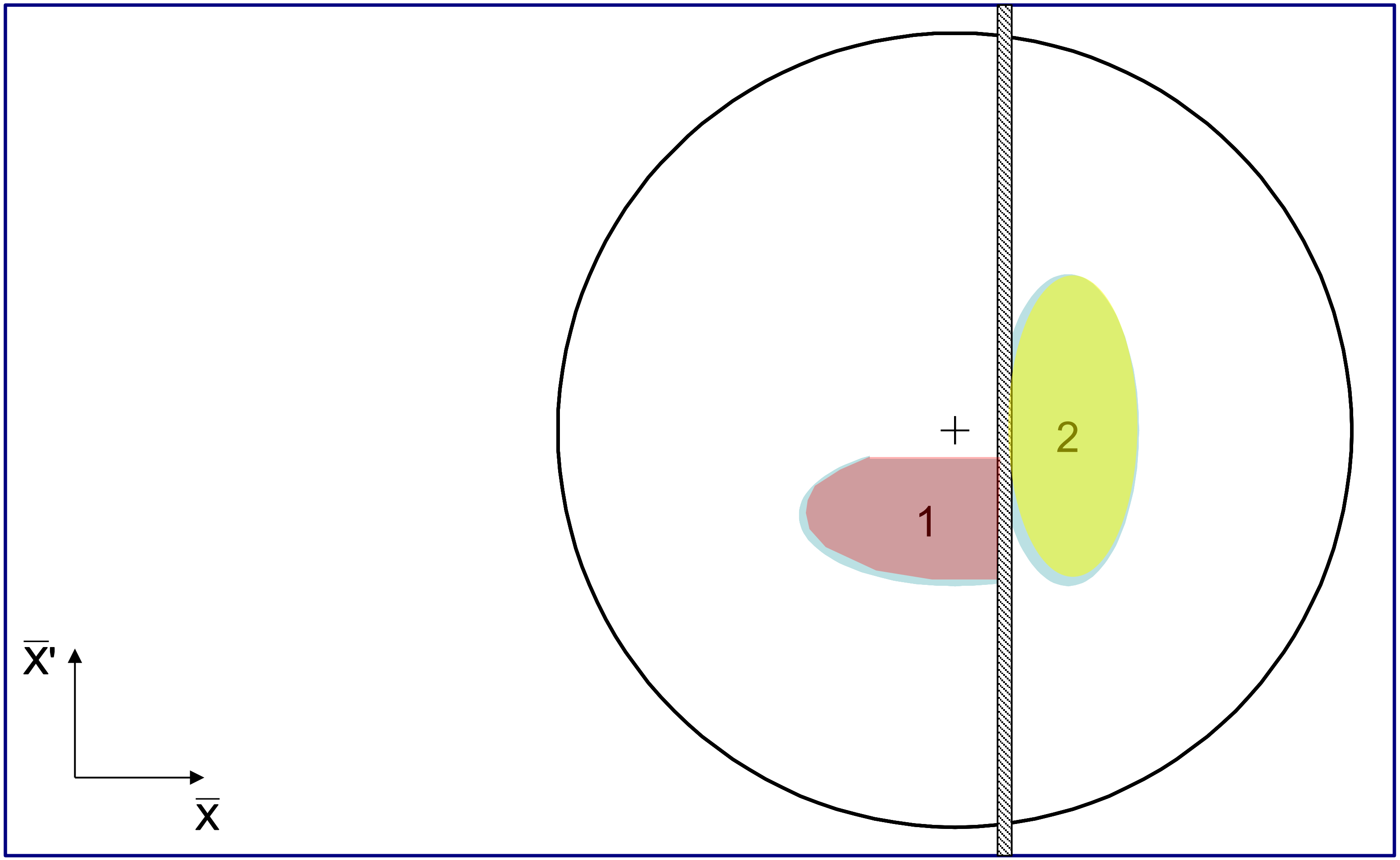
Turn 1



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates π/2 per turn in phase space

Turn 2

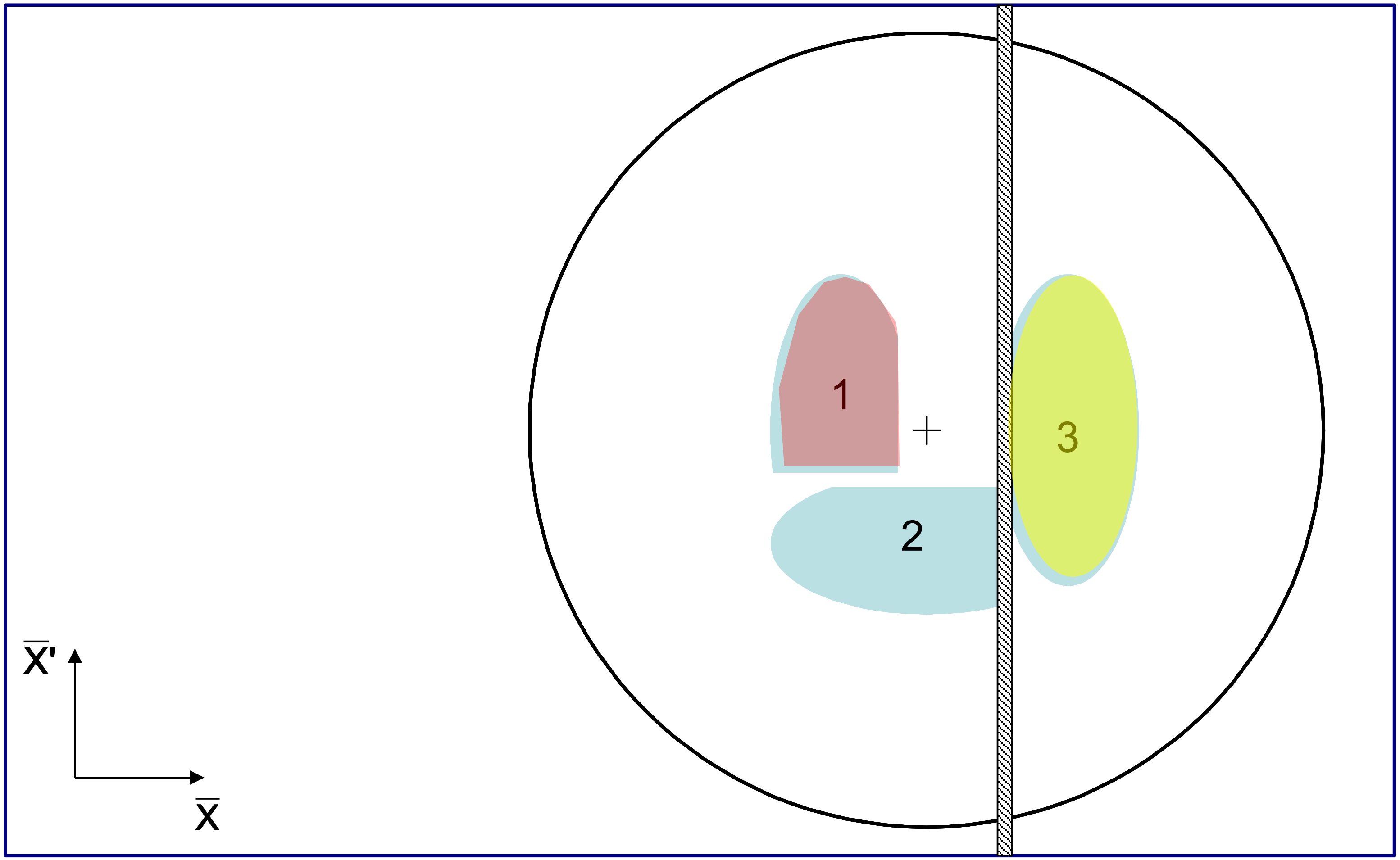


Septum blade

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates π/2 per turn in phase space

Turn 3



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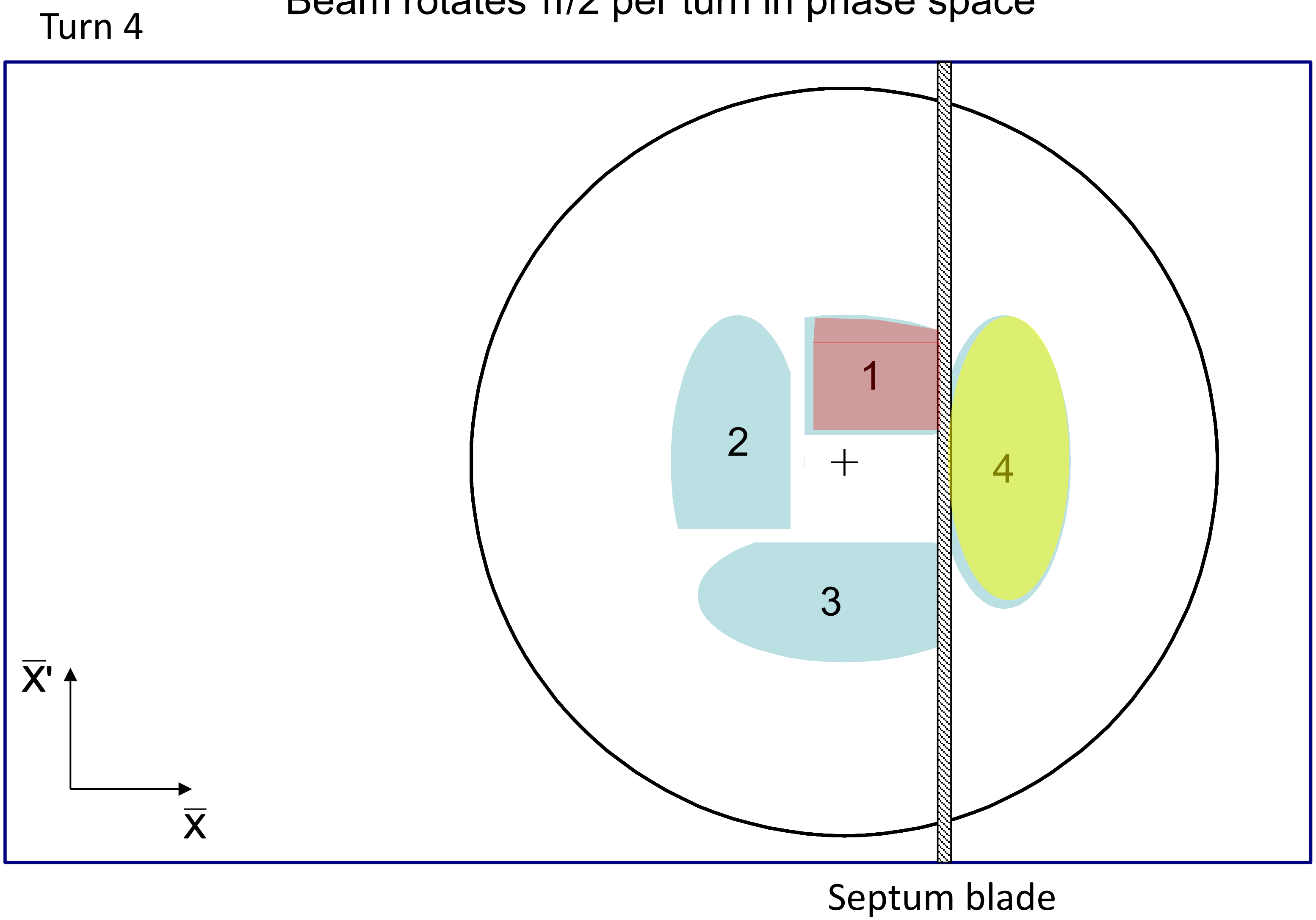
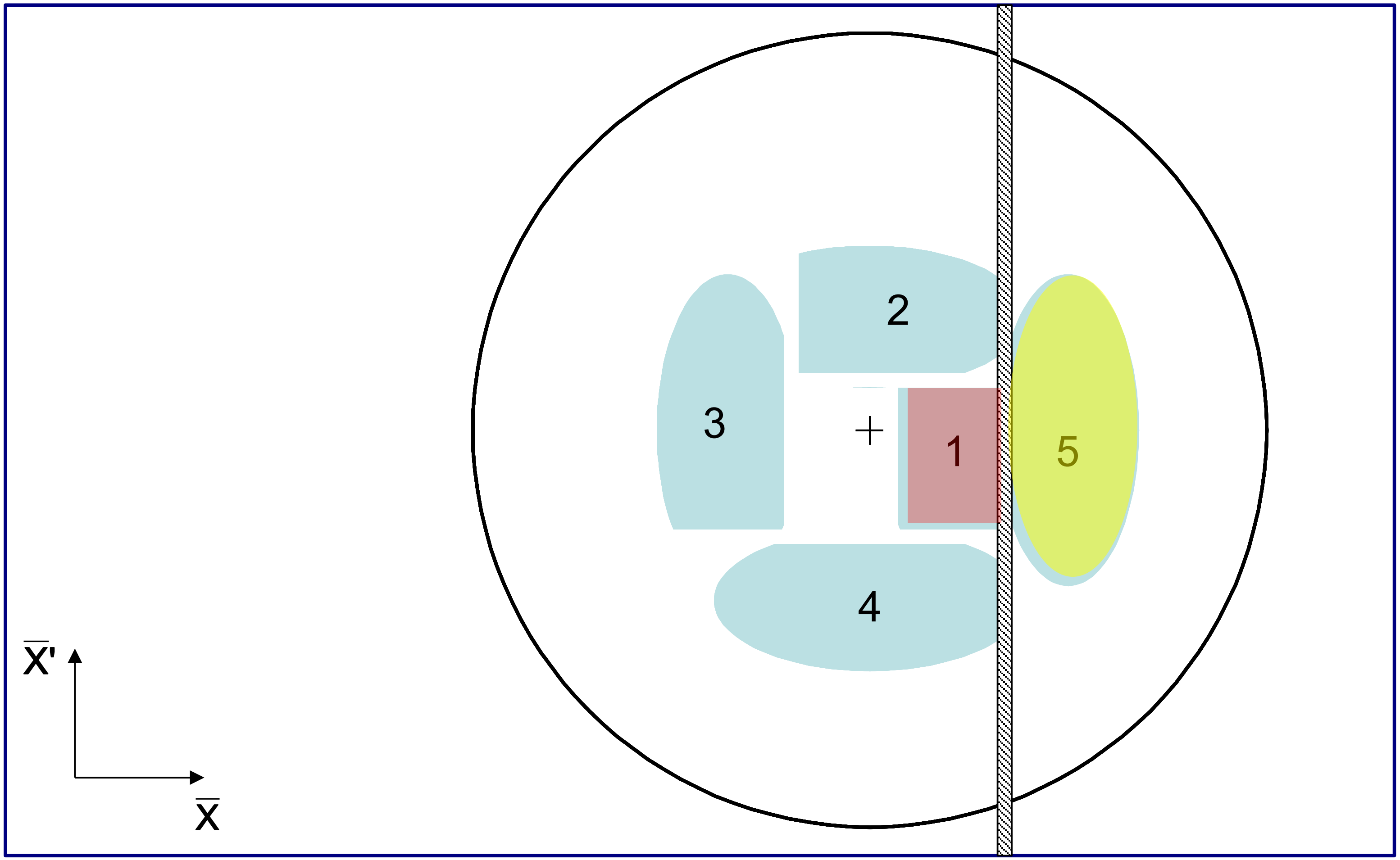


Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates π/2 per turn in phase space

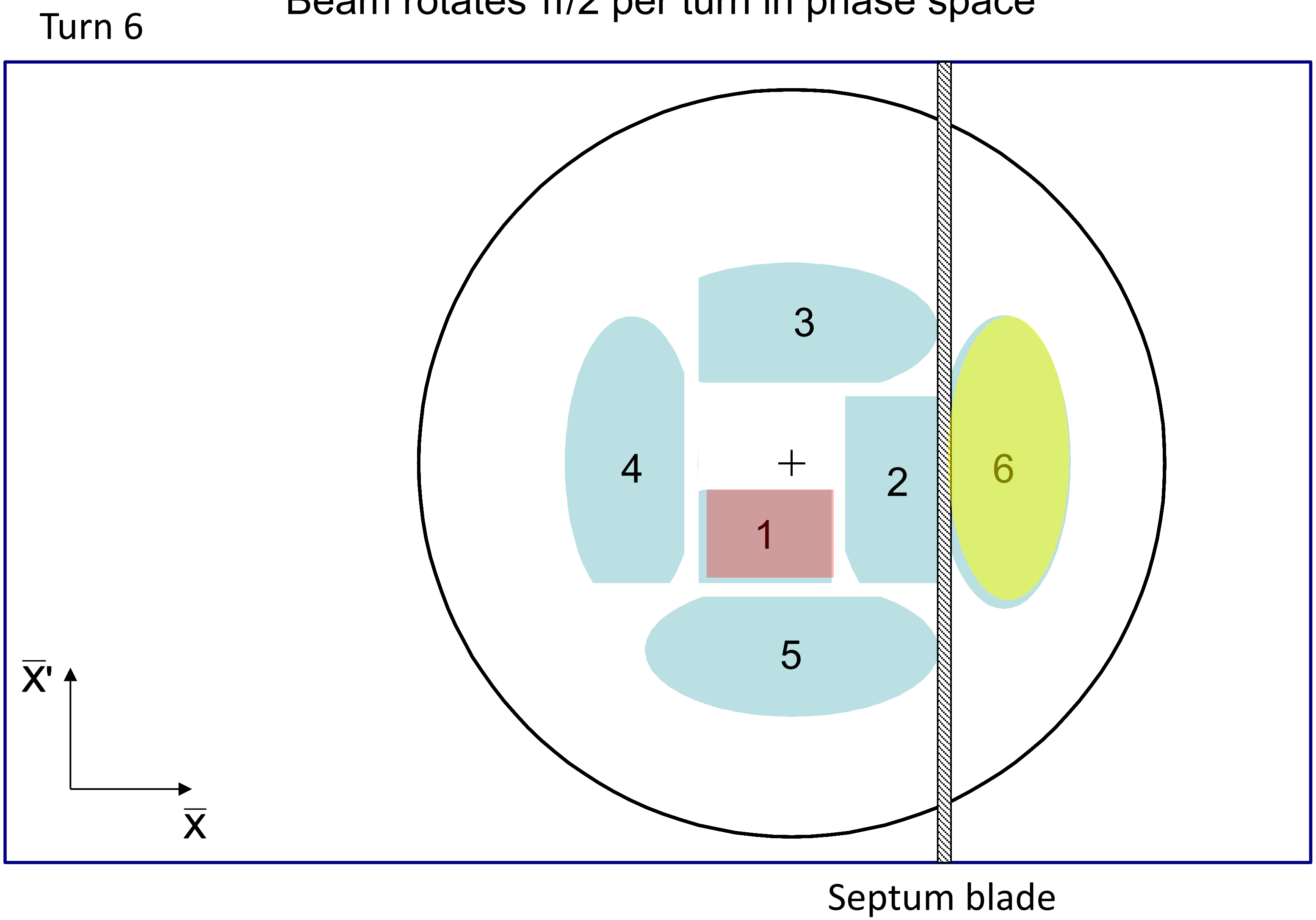
Turn 5



Septum blade

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates π/2 per turn in phase space



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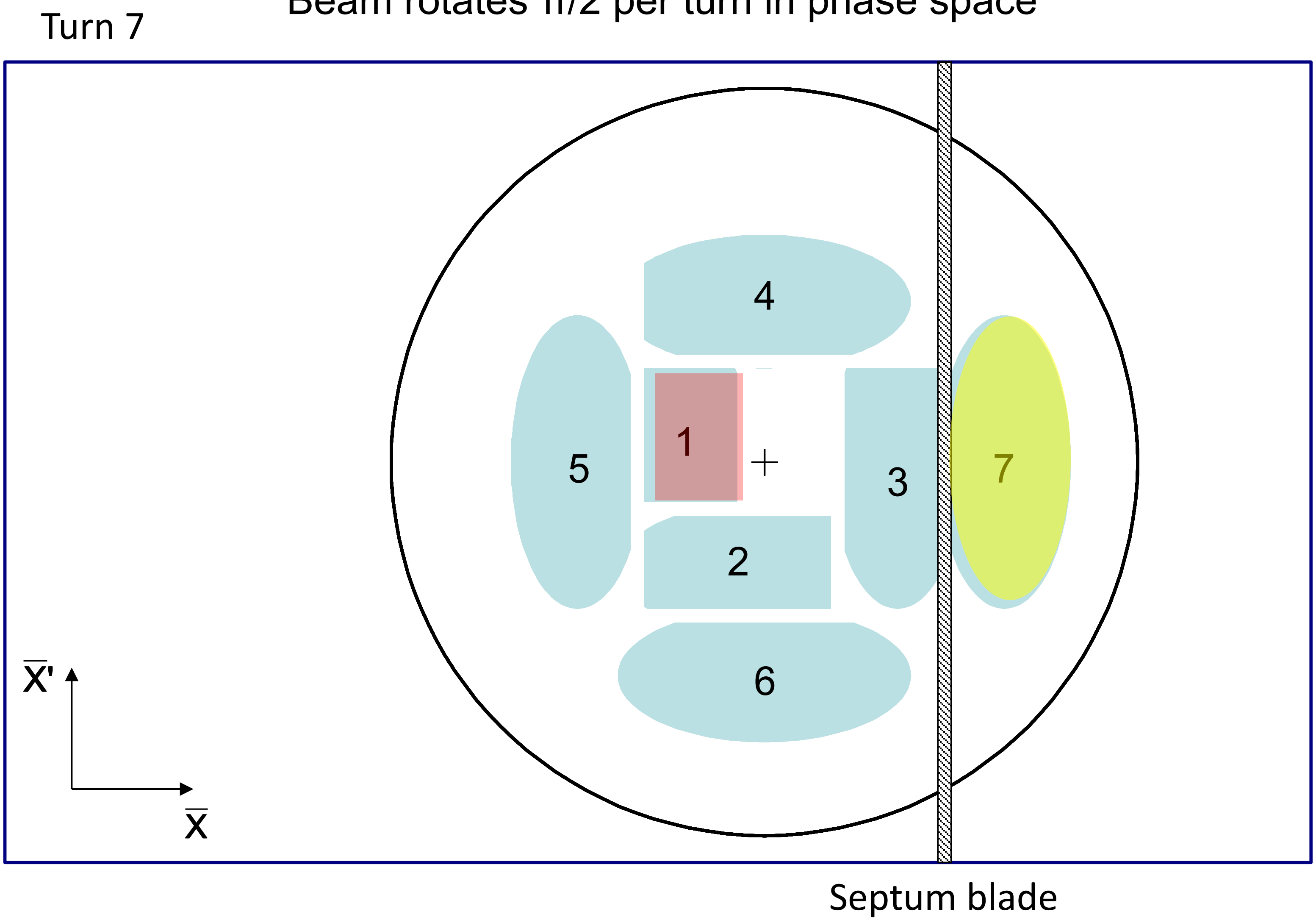

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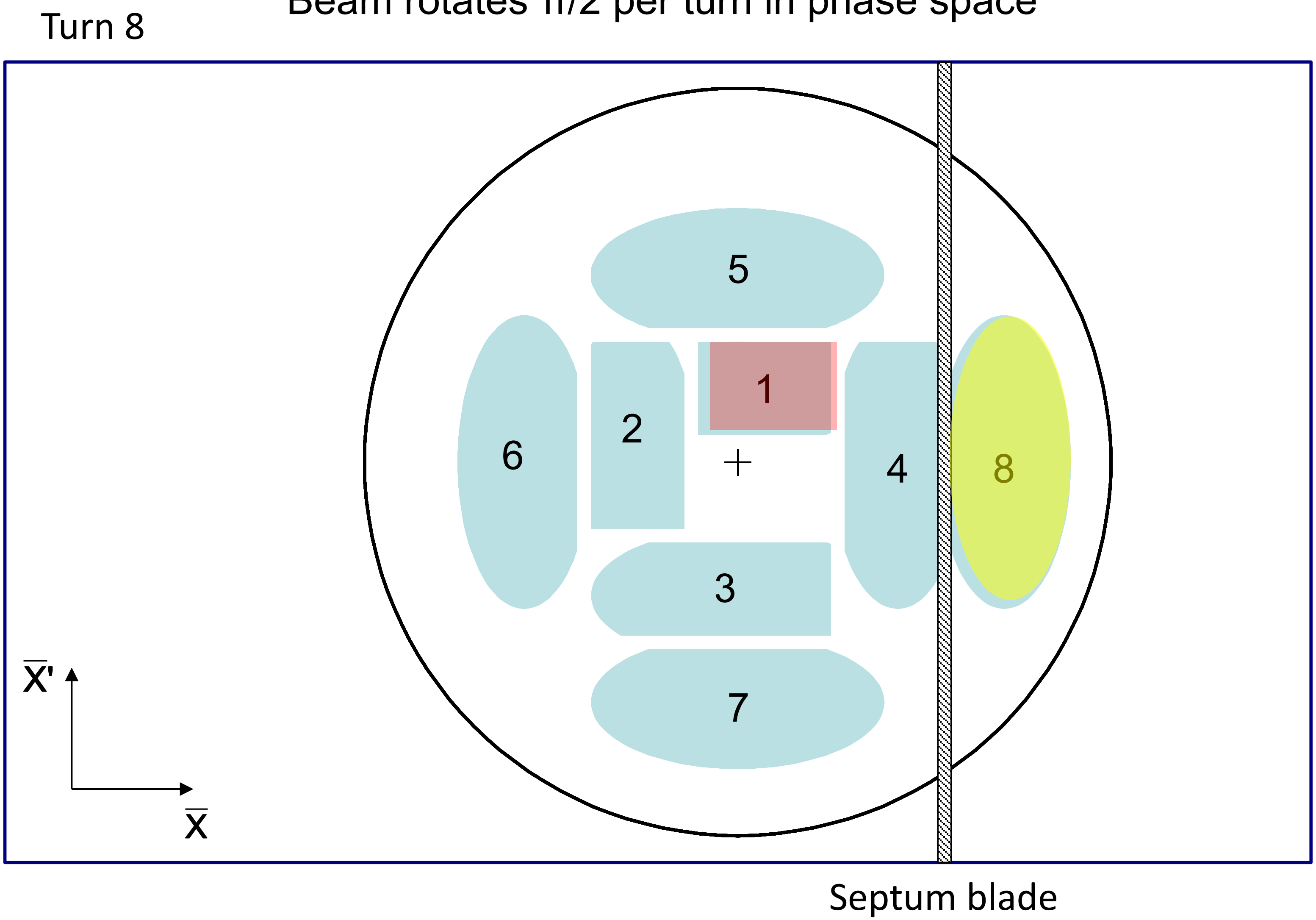

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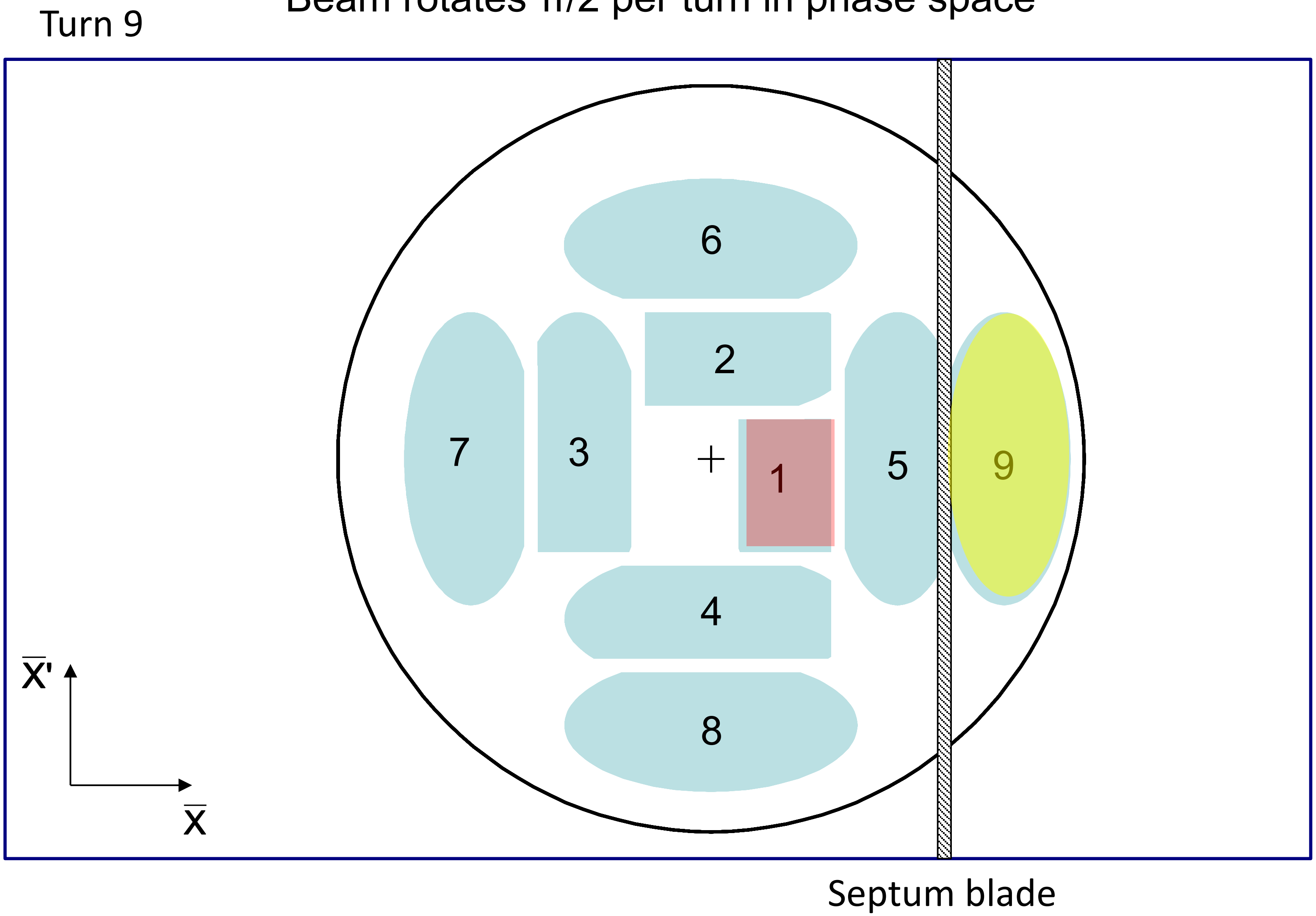

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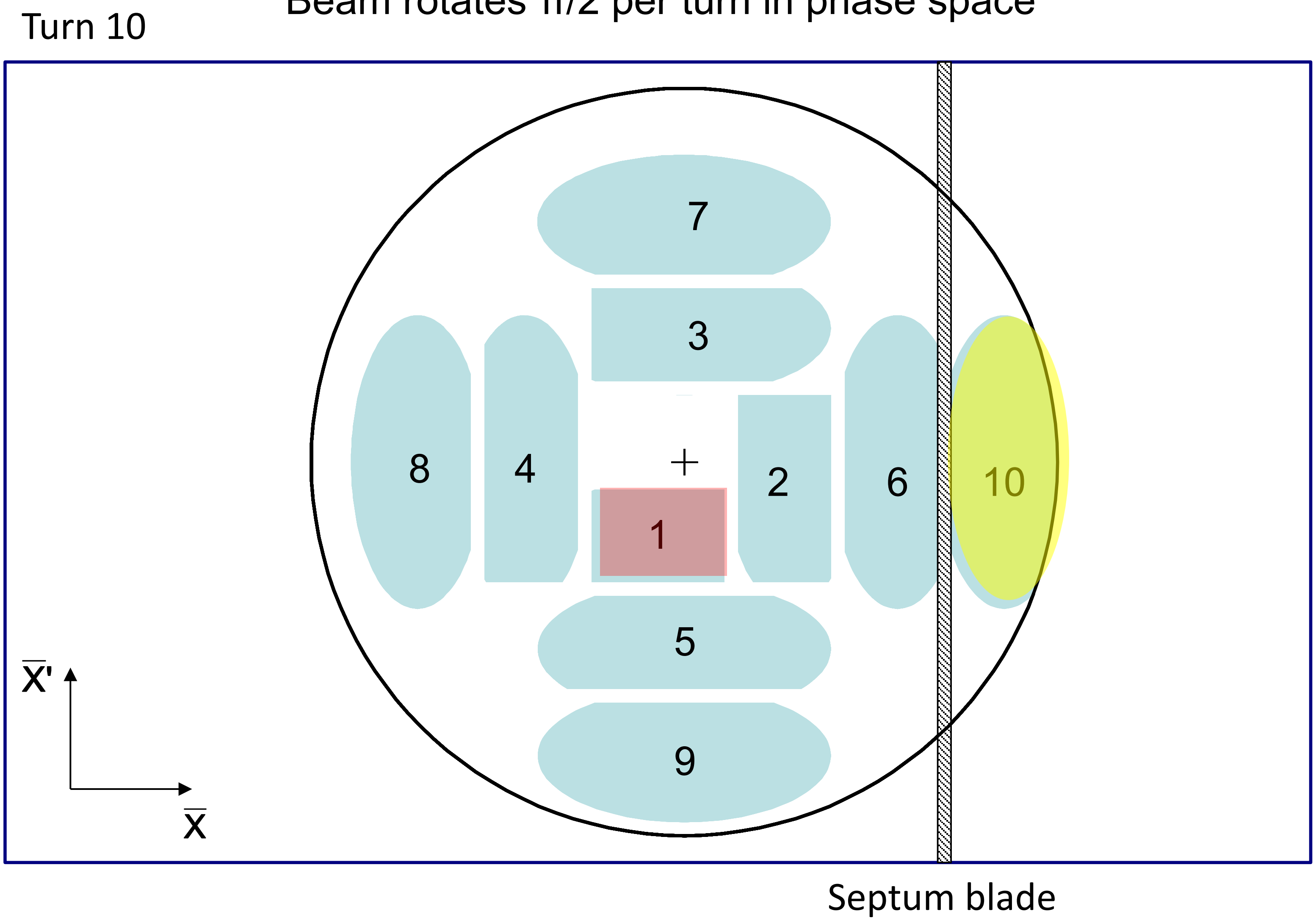

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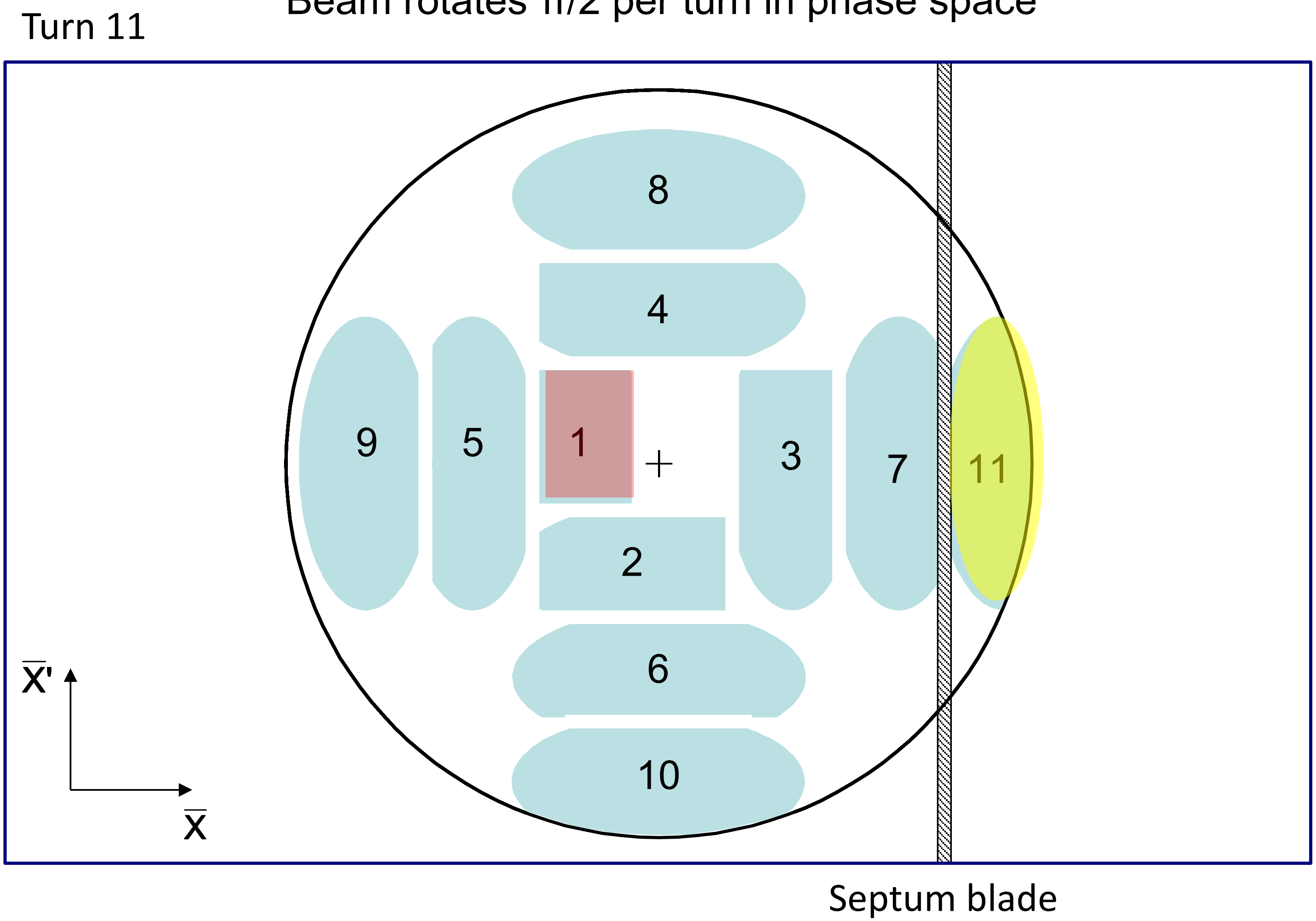

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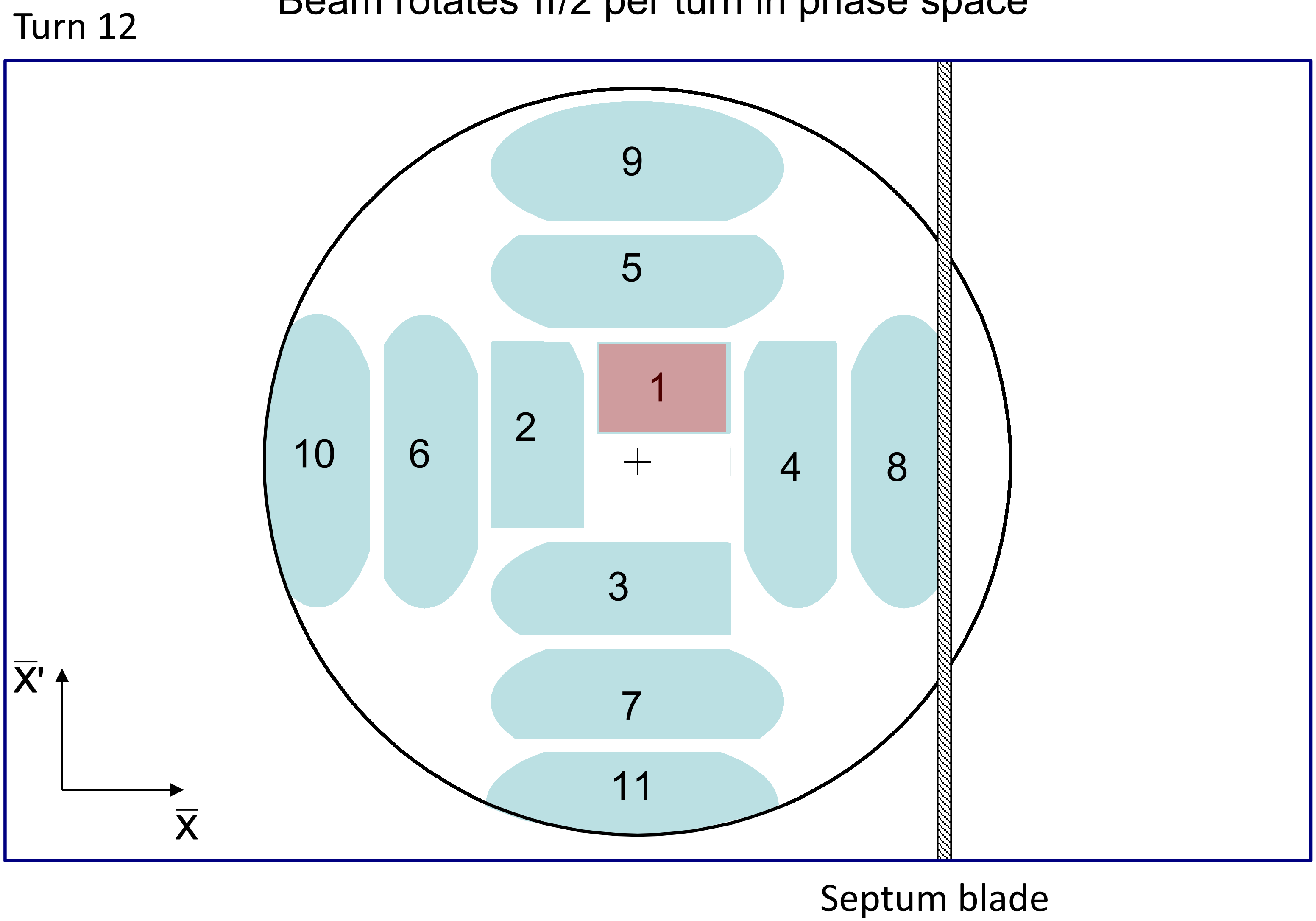

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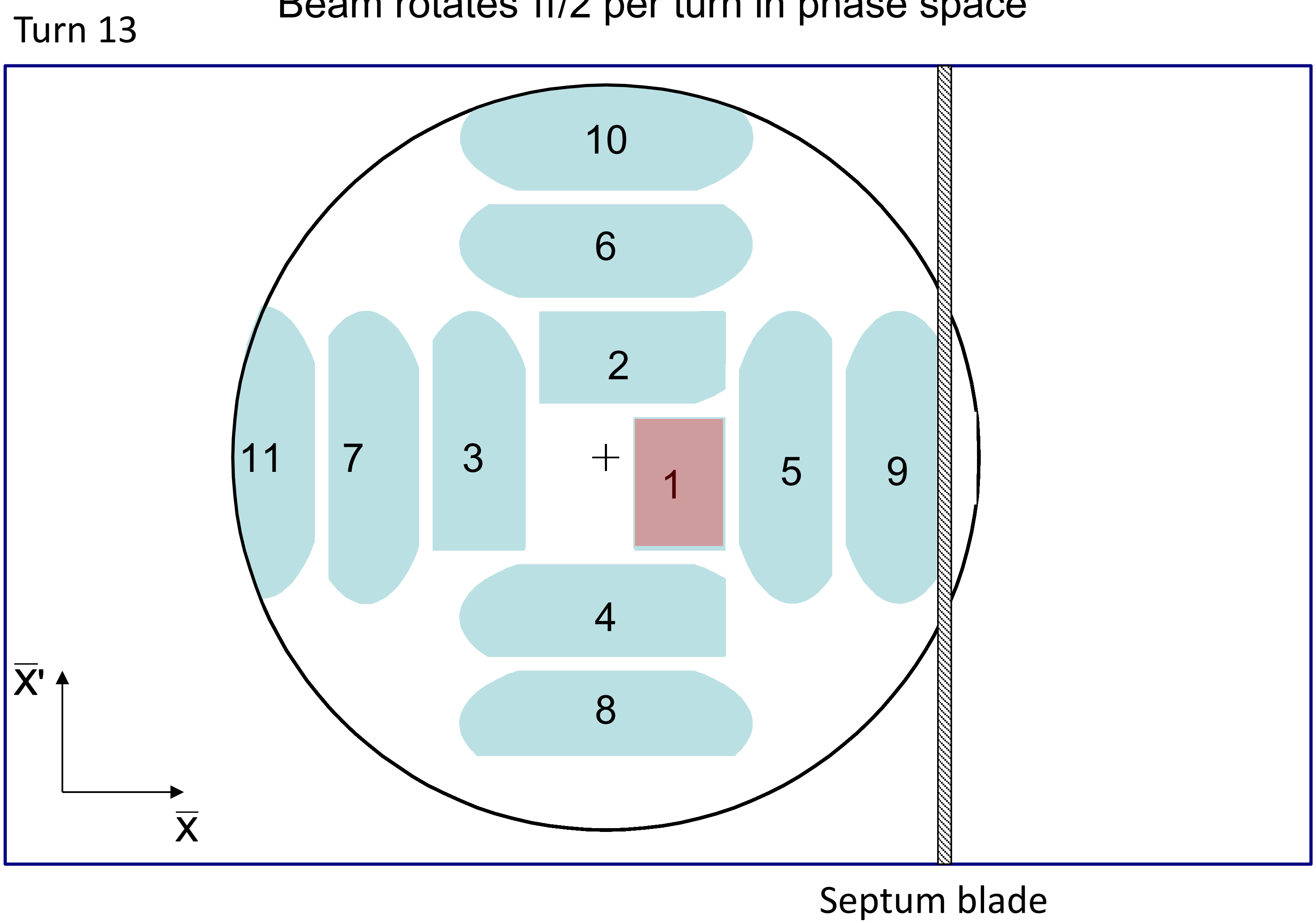
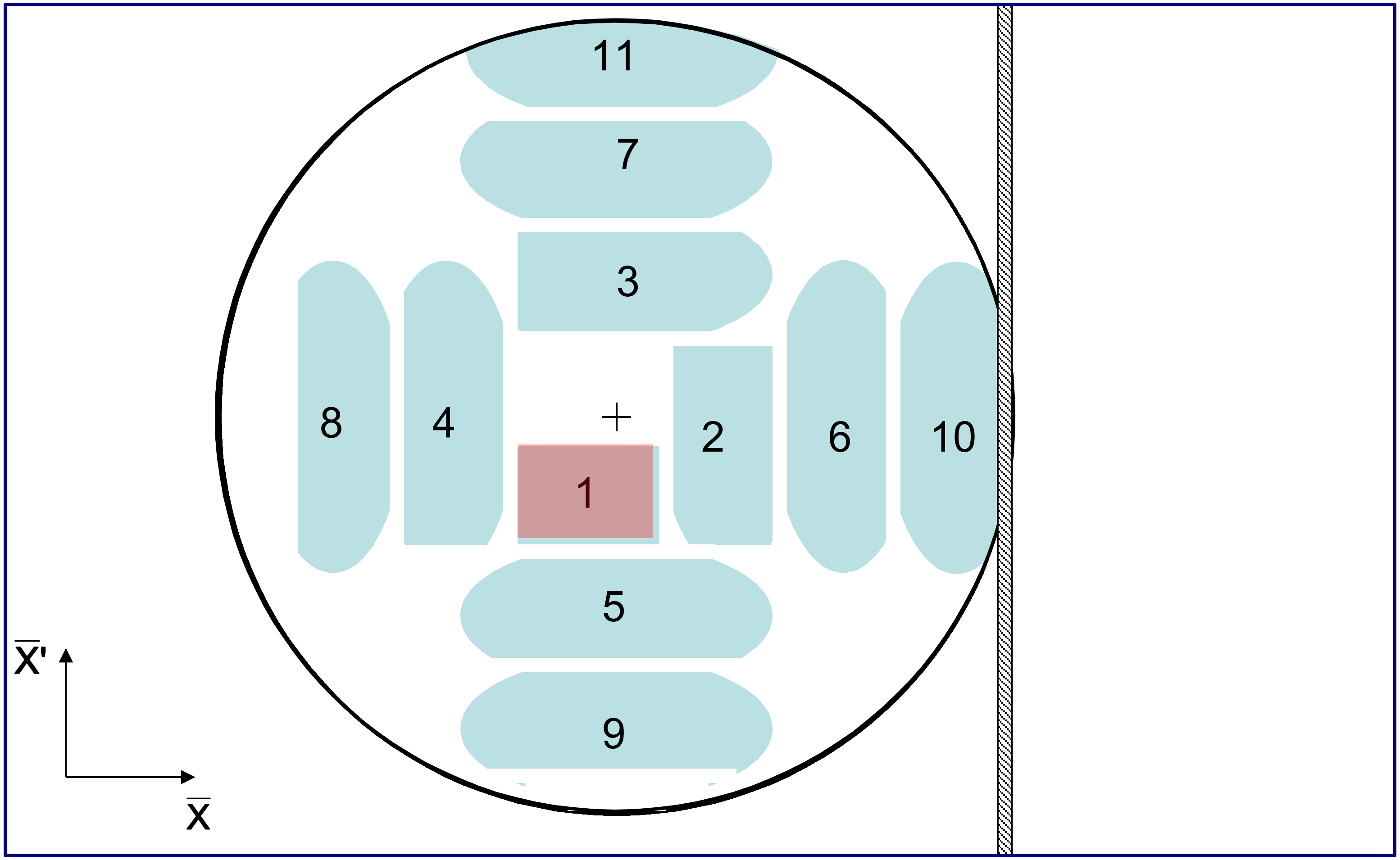


Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $q_{frac,h} \approx 0.25$
Beam rotates π/2 per turn in phase space

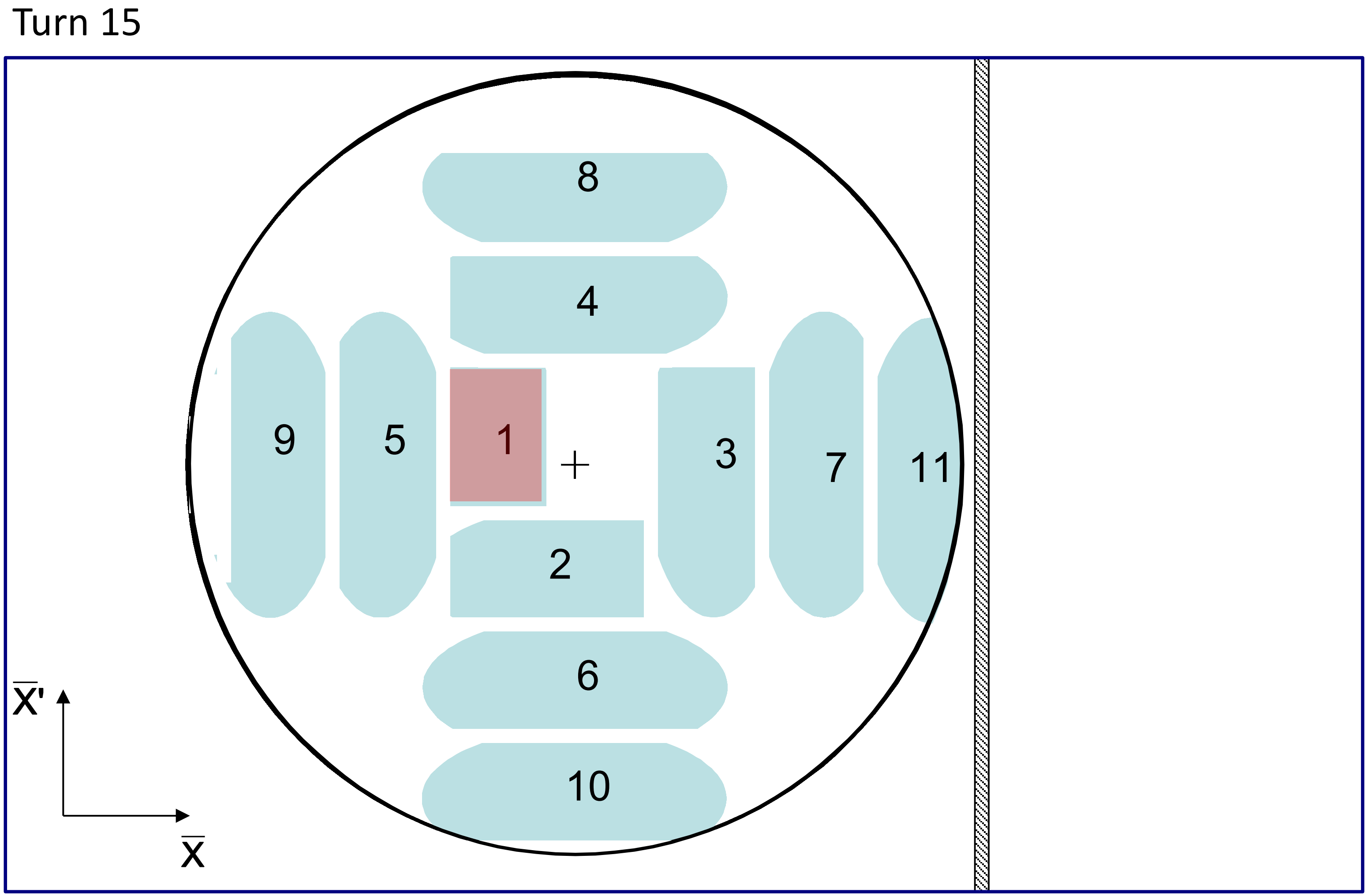
Turn 14



Septum blade

Multi-turn injection for hadrons

Phase space has been "**painted**"

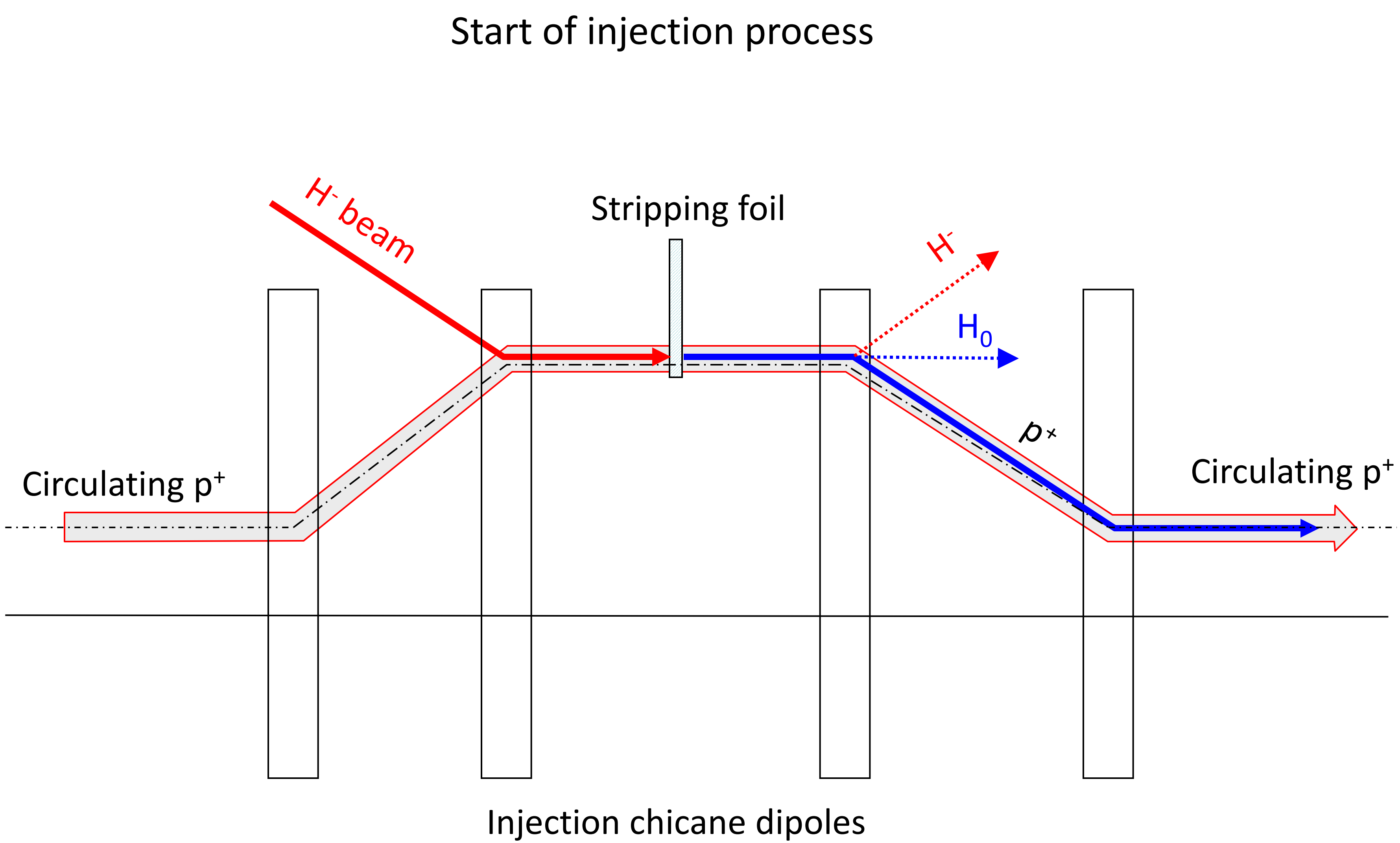


In reality, filamentation (often space-charge driven) occurs to produce a quasi-uniform beam

Charge exchange H- injection

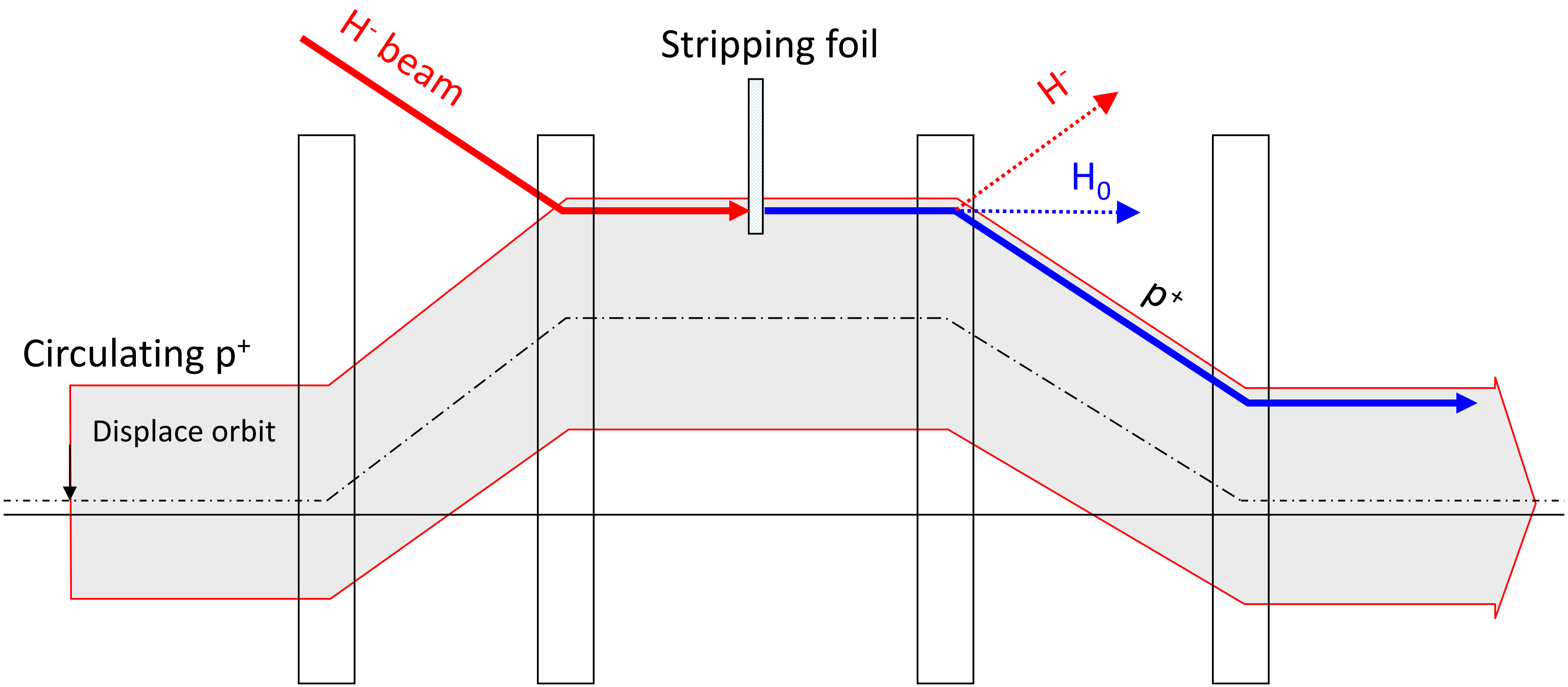
- Multi-turn injection is essential to accumulate high intensity
- Disadvantages inherent in using an injection septum:
  - Beam losses from circulating beam hitting septum:
    - typically 30 – 40 % for the CERN PSB injection at 50 MeV
  - Limits number of injected turns to 10 - 20
- Charge-exchange injection provides elegant alternative
  - Possible to “cheat” Liouville’s theorem, which says that emittance is conserved….
  - Convert H^- to p^+ using a thin stripping foil, allowing injection <u>into the same phase space area</u>

Charge exchange H- injection



Charge exchange H- injection

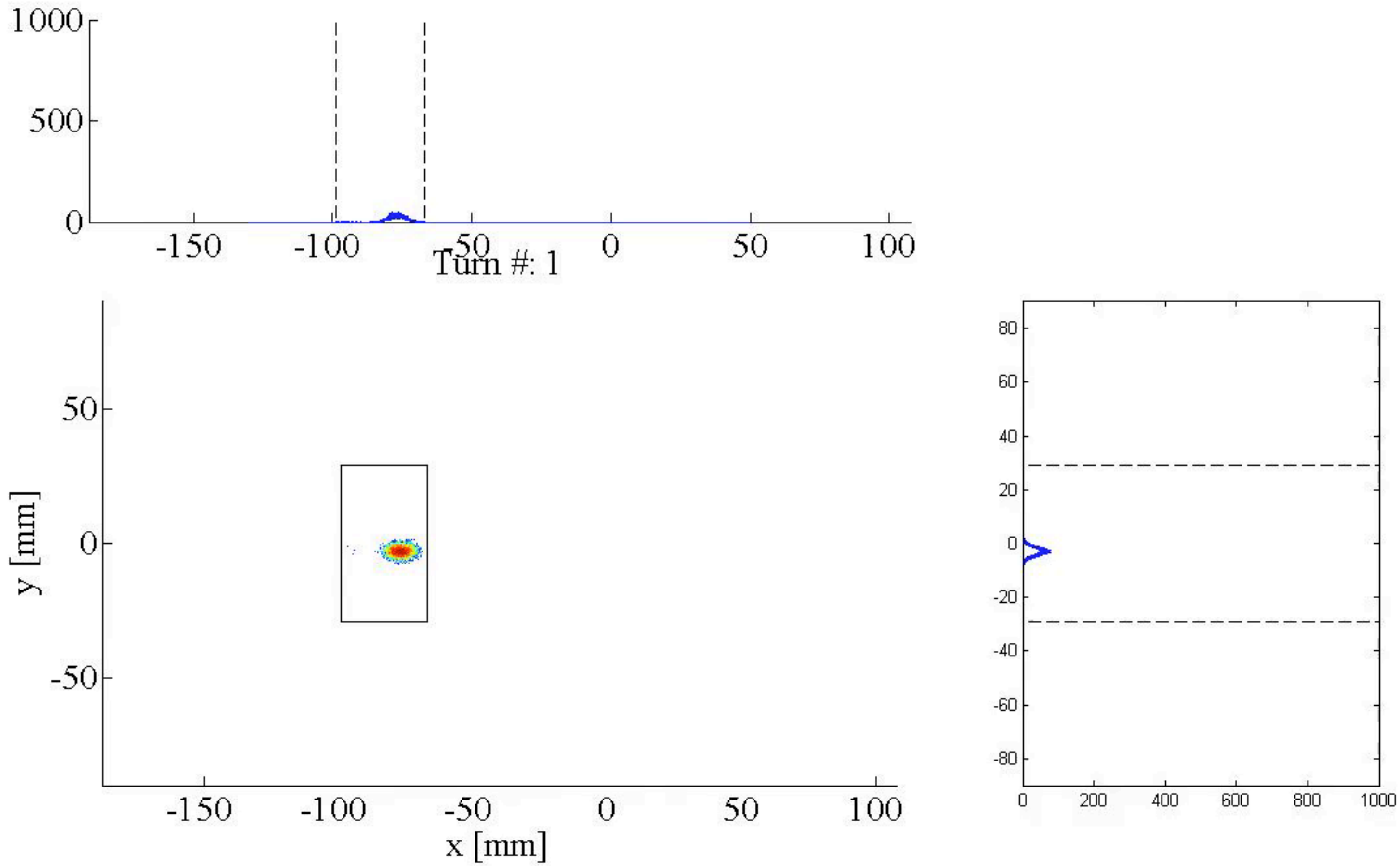
End of injection process with painting



Injection chicane dipoles

Accumulation process on foil

- Linac4 connection to the PS booster at 160 MeV:
  - H^- stripped to p^+ with an estimated efficiency ≈98 % with C foil 200 μg.cm^{-2}



V. Forte, Performance of the CERN PSB at 160 MeV with H- charge exchange injection, PhD thesis – CERN and Université Blaise Pascal

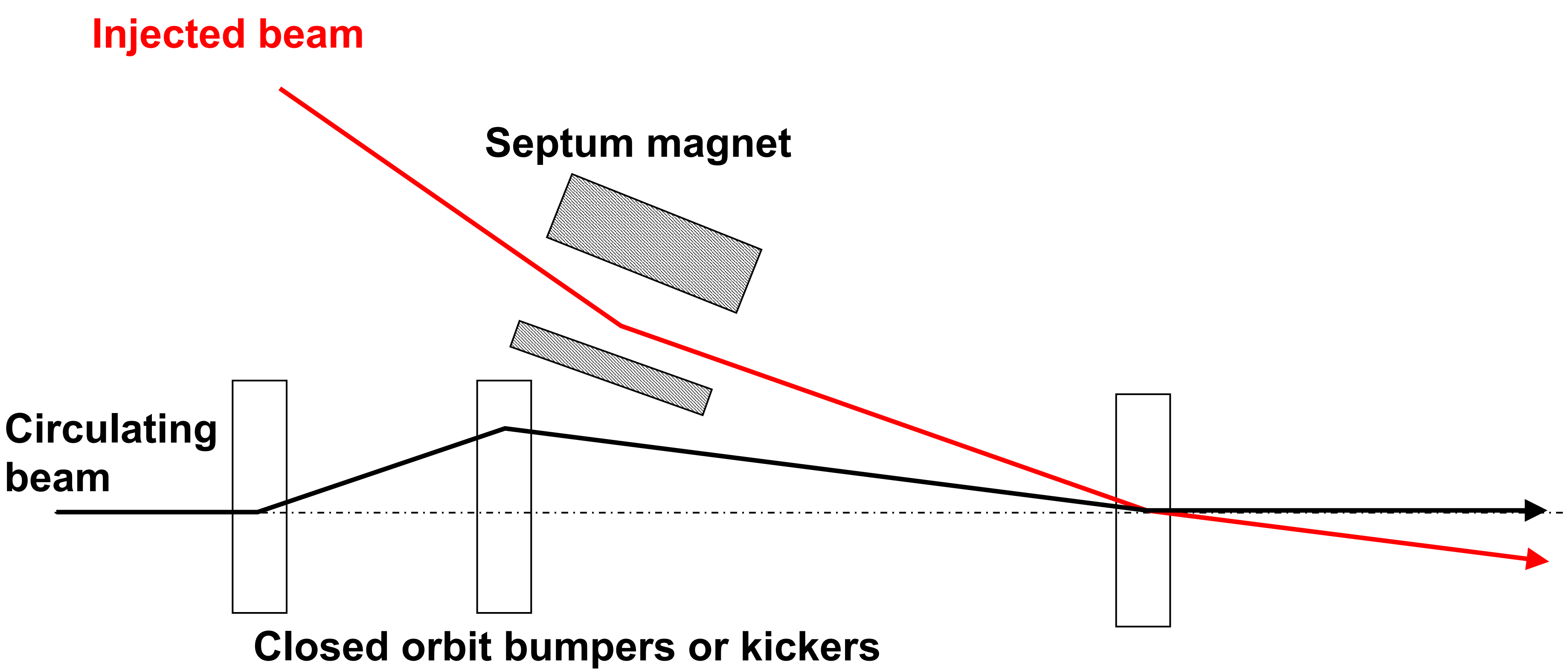
Charge exchange H- injection

- Paint uniform transverse phase space density by modifying closed orbit bump and steering injected beam
- Foil thickness calculated to double-strip most ions (≈ 99%)
  - 50 MeV – 50 μg.cm^{-2}
  - 800 MeV – 200 μg.cm^{-2} (≈ 1 μm of C!)
- Carbon foils generally used – very fragile
- Injection chicane reduced or switched off after injection, to avoid excessive foil heating and beam blow-up
- Longitudinal phase space can also be painted turn-by-turn:
  - Variation of the injected beam energy turn-by-turn (linac voltage scaled)
  - Chopper system in linac to match length of injected batch to bucket

Lepton injection

- Single-turn injection can be used as for hadrons; however, lepton motion is strongly damped (different with respect to proton or ion injection).
  - Synchrotron radiation:
    - see CERN Accelerator School lectures: *Electron Beam Dynamics by L. Rivkin*
- Can use transverse or longitudinal damping:
  - Transverse - Betatron accumulation
  - Longitudinal - Synchrotron accumulation

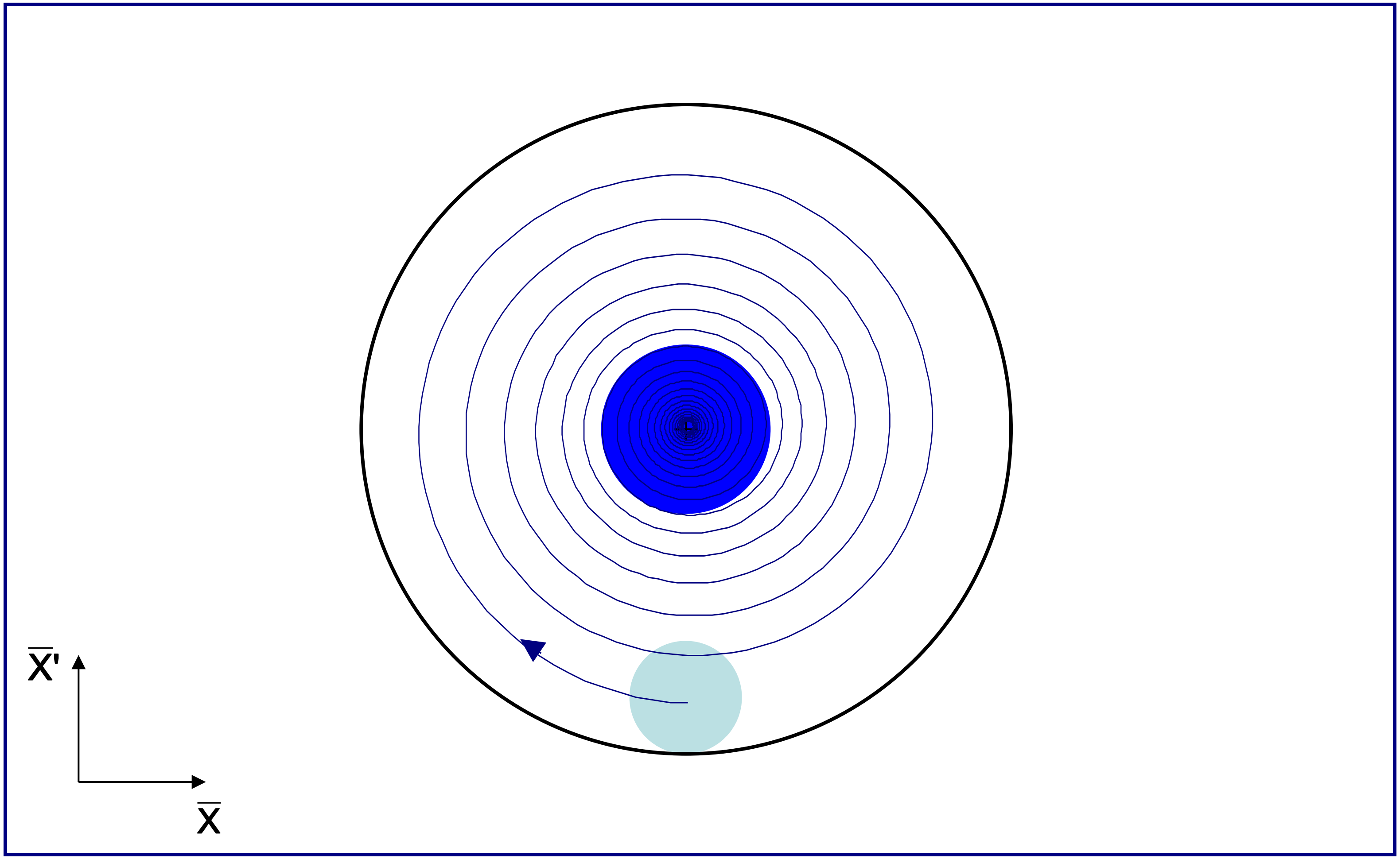
Betatron lepton injection



- Beam is injected with an angle with respect to the closed orbit
- Injected beam performs damped betatron oscillations about the closed orbit

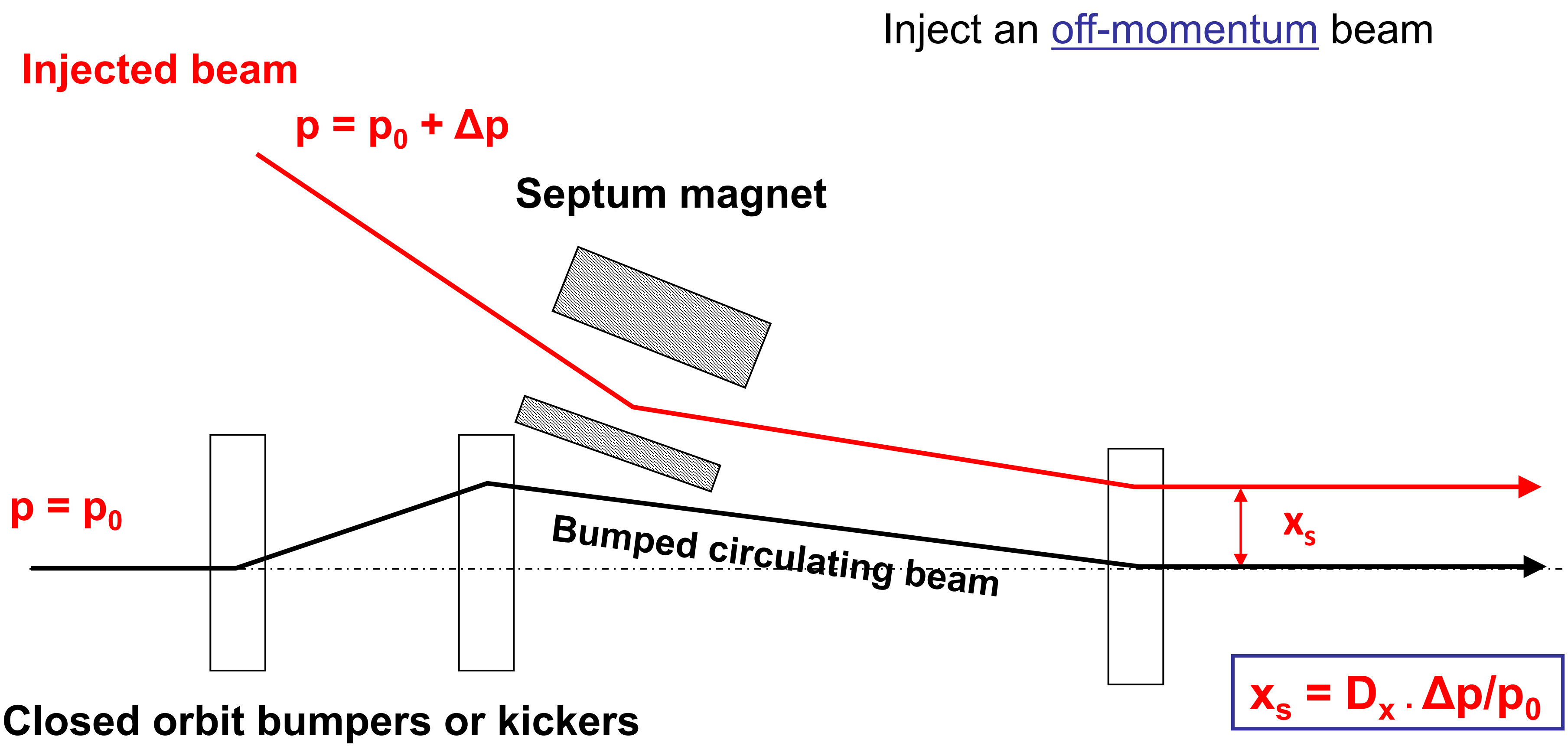
Betatron lepton injection

Injected bunch performs damped betatron oscillations



In LEP at 20 GeV, the damping time was about 6’000 turns (0.6 seconds)

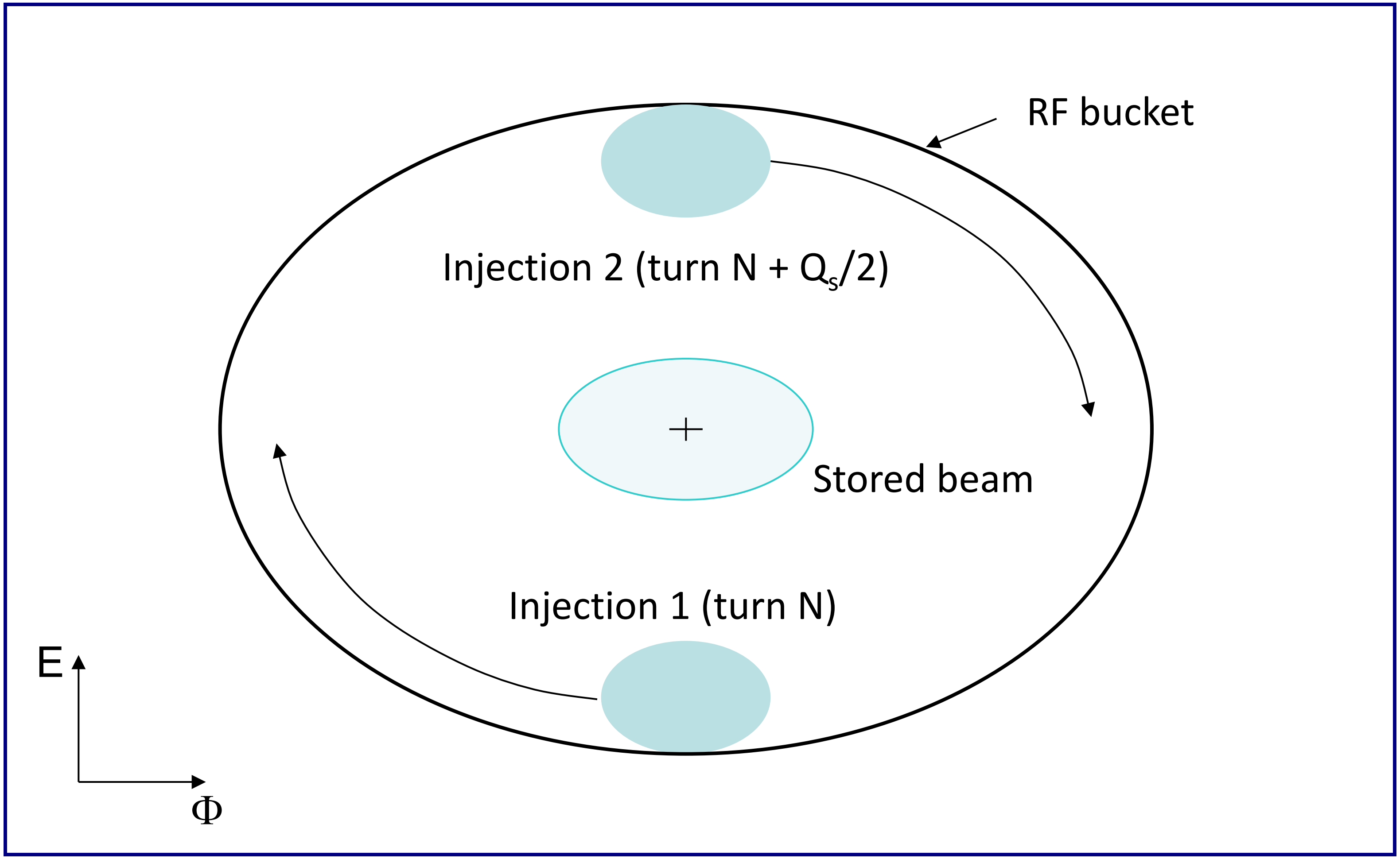
Synchrotron lepton injection



- Beam injected parallel to circulating beam, onto dispersion orbit of a particle having the same momentum offset $\Delta p/p$
- Injected beam makes damped synchrotron oscillations at Q_s but does not perform betatron oscillations

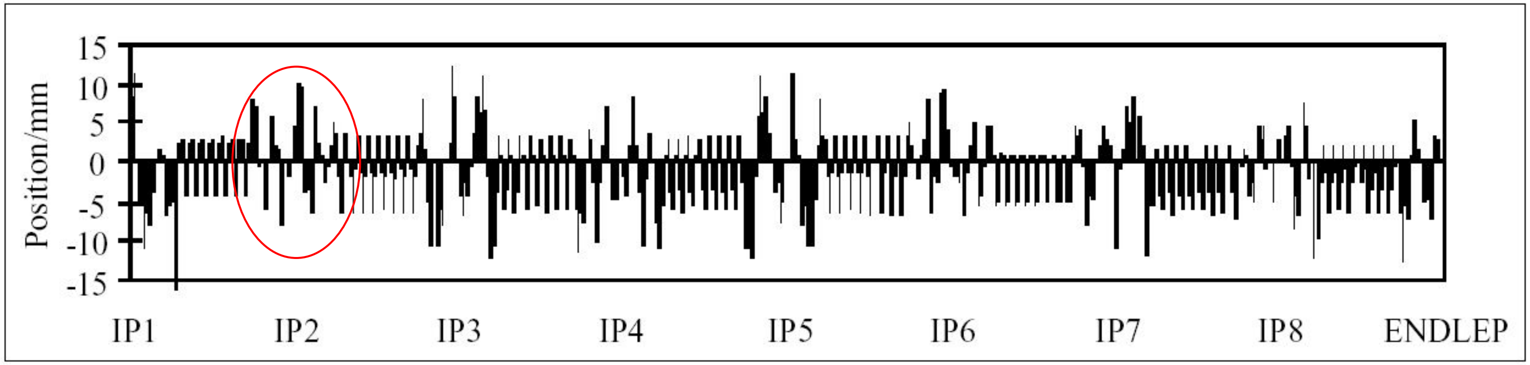
Synchrotron lepton injection

Double batch injection possible….

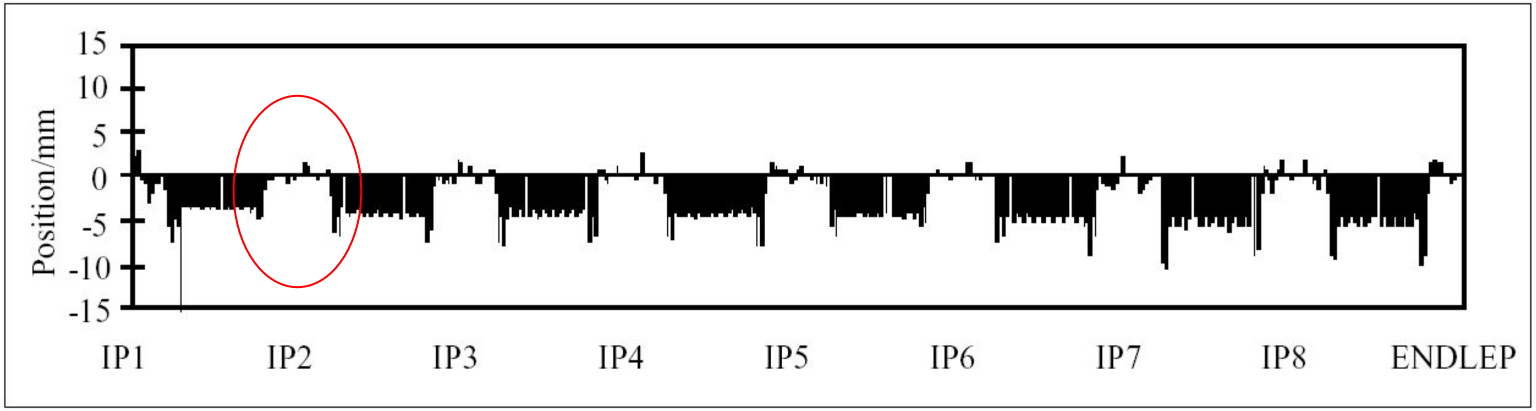


Longitudinal damping time in LEP was ~3’000 turns (2x faster than transverse)

Synchrotron lepton injection in LEP



Optimized Horizontal First Turn Trajectory for Betatron Injection of Positrons into LEP.



Optimized Horizontal First Turn Trajectory for Synchrotron Injection of Positrons with $\Delta P/P$ at -0.6%

Synchrotron injection in LEP gave improved background for LEP experiments due to small orbit offsets in zero dispersion straight sections

Injection - summary

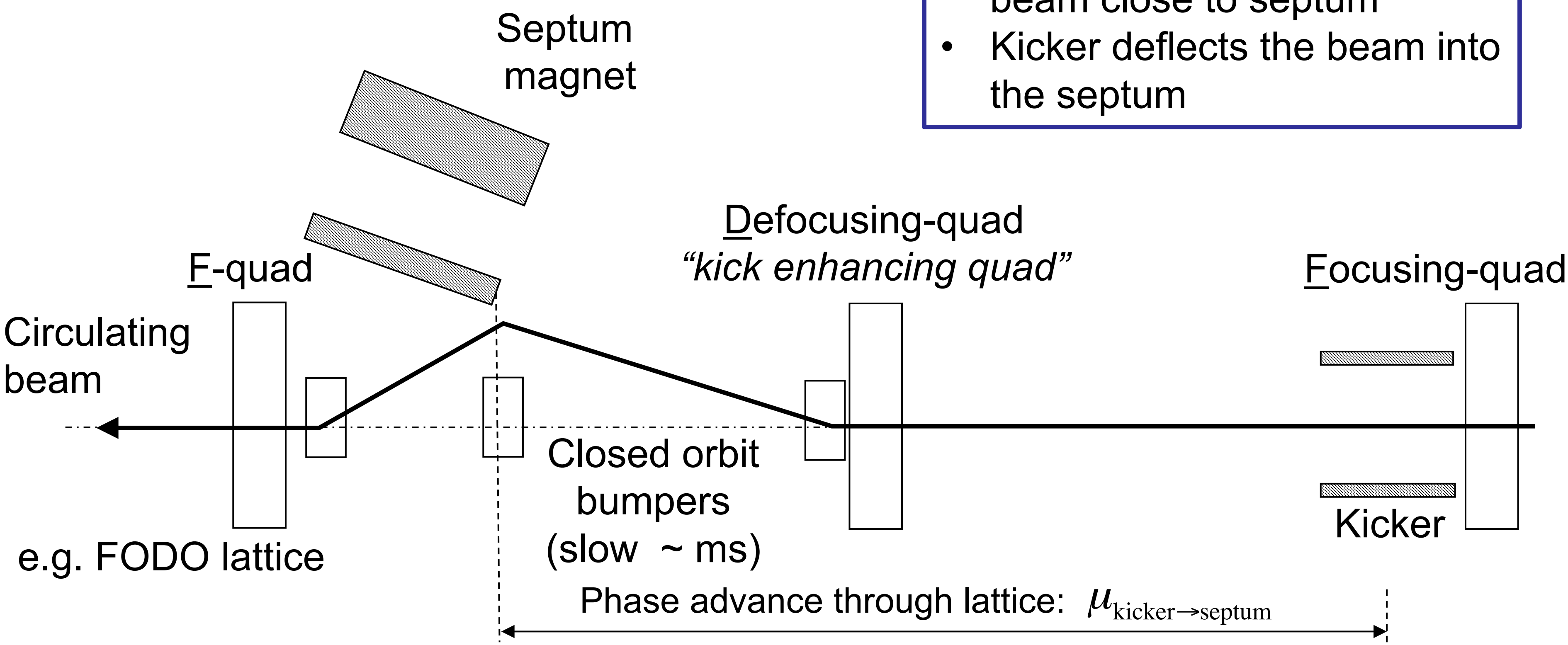
- Several different techniques using kickers, septa and bumpers:
  - Single-turn injection for hadrons
    - Boxcar stacking: transfer between machines in accelerator chain
    - Angle / position errors $\Rightarrow$ injection oscillations
    - Uncorrected errors $\Rightarrow$ filamentation $\Rightarrow$ emittance increase
  - Multi-turn injection for hadrons
    - Phase space painting to increase intensity
    - H- injection allows injection into same phase space area
  - Lepton injection: take advantage of damping
    - Less concerned about injection precision and matching

Extraction

- Different extraction techniques exist, depending on requirements
  - Fast extraction: ≤1 turn
  - Non-resonant multi-turn extraction (mechanical splitting): few turns
  - Resonant low-loss multi-turn extraction (magnetic splitting): few turns
  - Resonant multi-turn extraction: many thousands of turns
- Usually higher energy than injection $\Rightarrow$ stronger elements ($\int B.dl$)
  - At high energies many kicker and septum modules may be required
  - To reduce kicker strength, beam can be moved near to septum by closed orbit bump

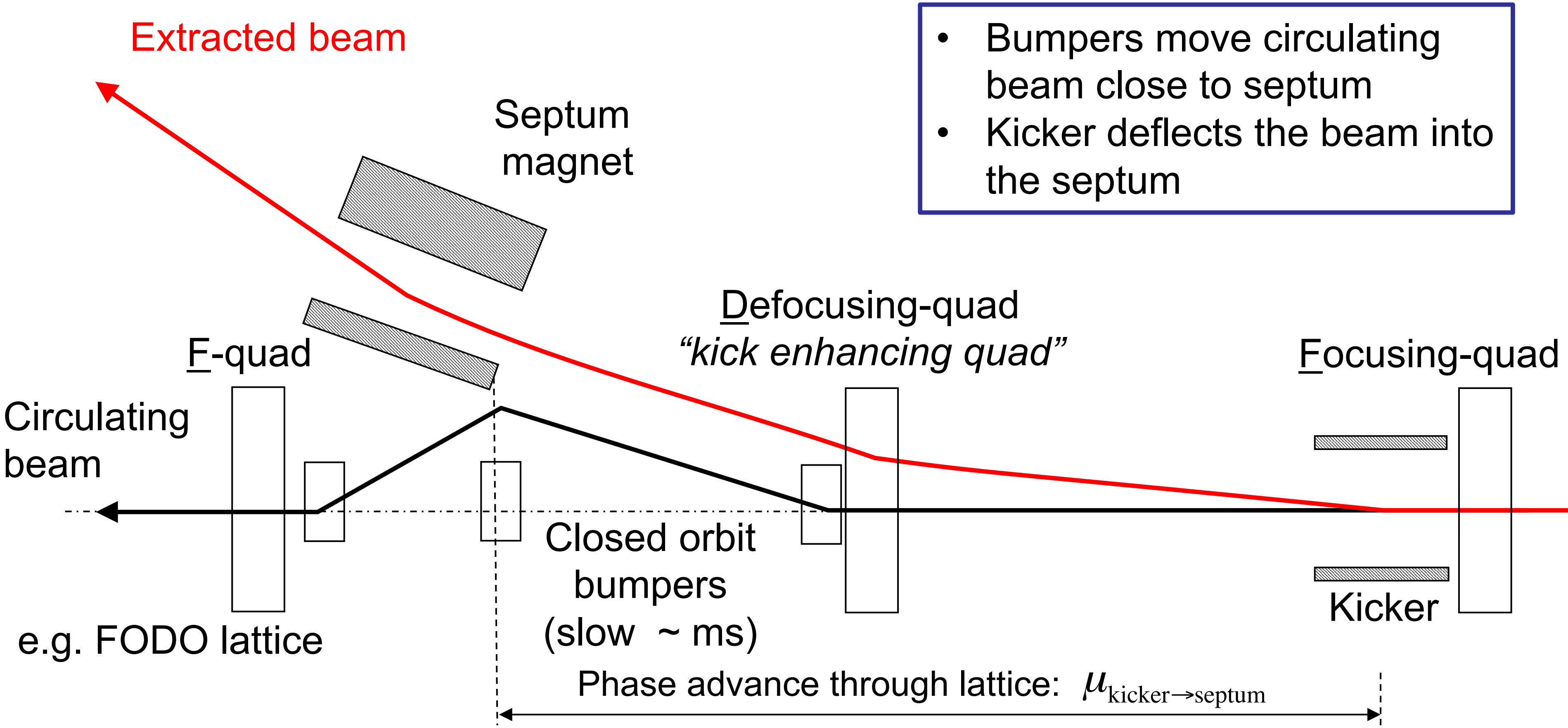
Fast extraction: spatial considerations

- Bumpers move circulating beam close to septum
- Kicker deflects the beam into the septum



- Important considerations:
  - optimum phase advance between kicker and septum, e.g. ≈ QD in between: β_x large at F-quads (near kicker and septum in this case)
  - aperture, e.g. inside quads, position of septum etc.
  - integration constraints, e.g. extracted beam trajectory

Fast extraction: spatial considerations



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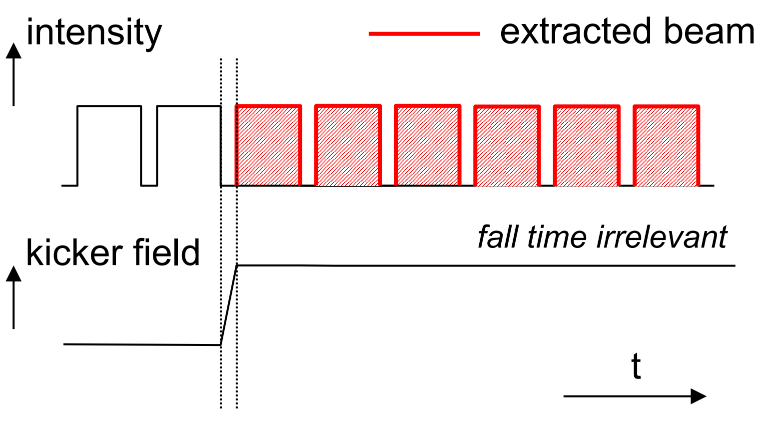
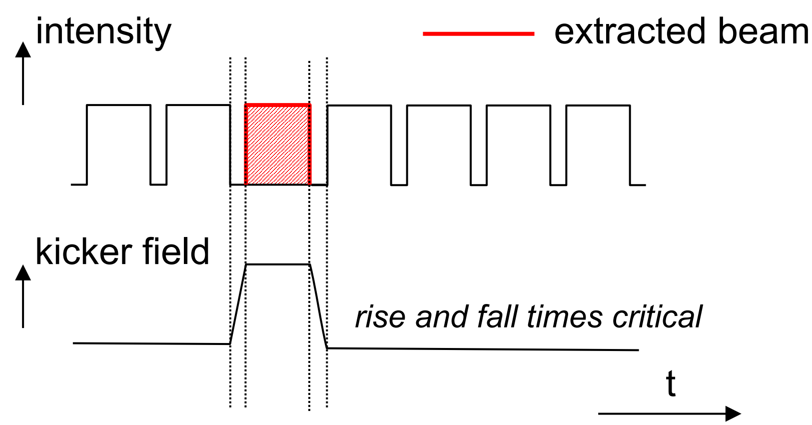
  - integration constraints, e.g. extracted beam trajectory

Fast extraction: temporal considerations

- For clean transfer, particle-free gaps in the circulating beam are essential:



single-turn extraction:
transfer between machines or fast-abort



multi-burst or multi-batch extraction:
kicker system must have time to re-charge

  - kicker field must have time to rise (and fall) before it is seen by the beam
  - gaps limit total intensity
  - repetition rate of kicker system: pulsed-power supply must have time to recharge, which typically takes many turns: $t_{recharge} >> t_{rev}$
    - continuous extraction over sequential turns (usually) requires transverse manipulation: *discussed later in this lecture (multi-turn extraction)*

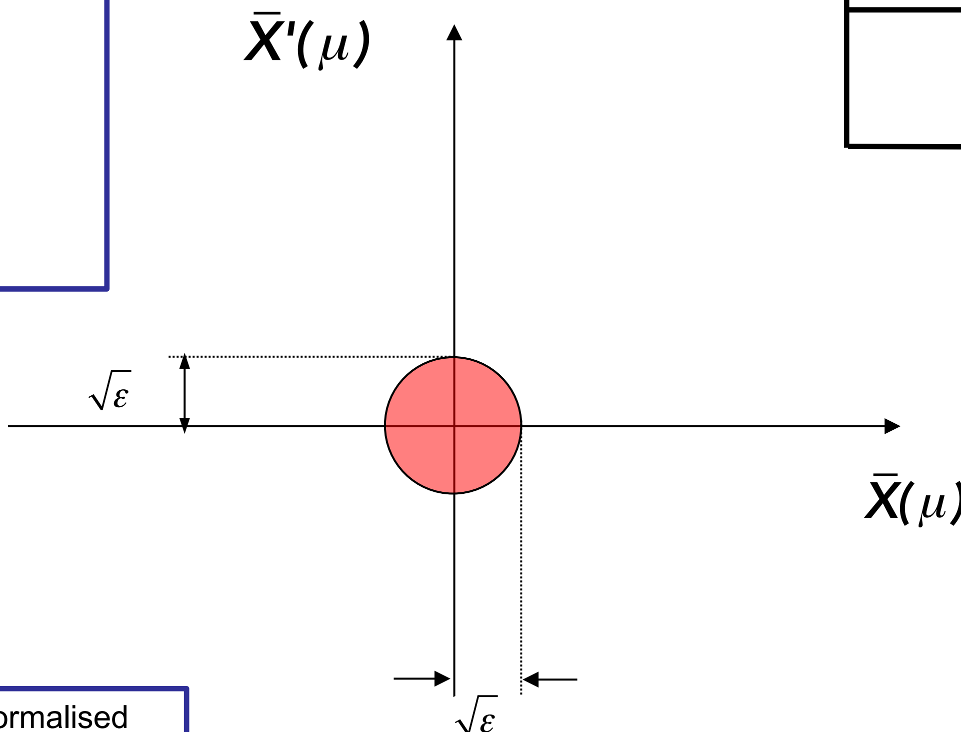
Kick dynamics

Normalised phase space at the kicker location:

$\Delta x'_{\text{kicker}}$	$\Delta \bar{X}'_{\text{kicker}}$
0	0

Kicker strength:

$$\Delta x'_{\text{kicker}} = 0$$



Reminder: transformation to normalised phase space:

$$\begin{bmatrix} \bar{X} \\ \bar{X}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta(s)}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

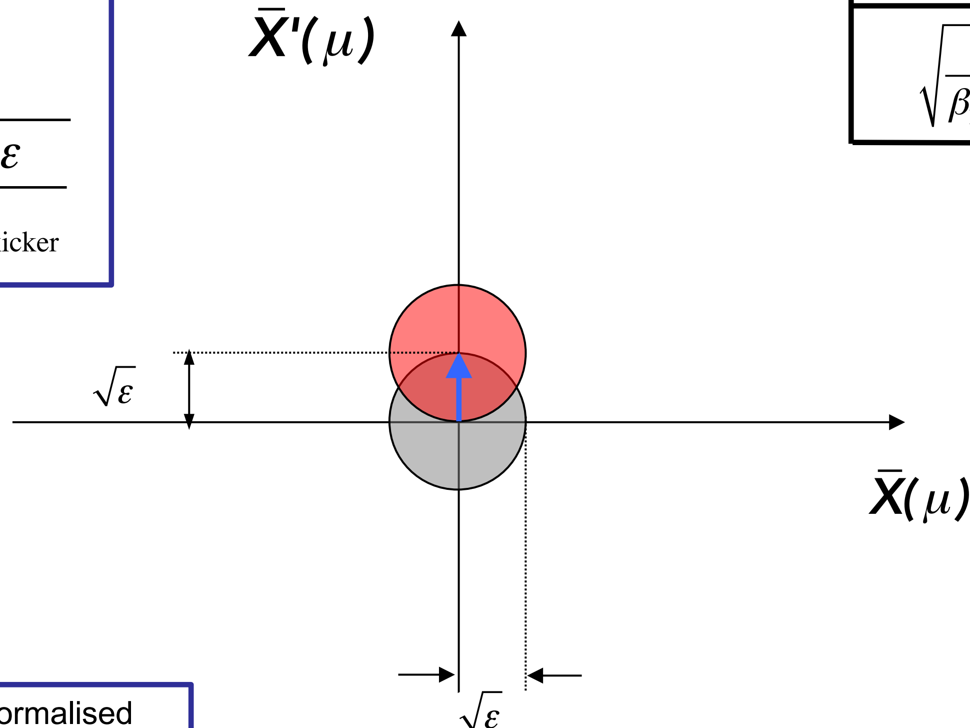
Kick dynamics

Normalised phase space at the kicker location:

$\Delta x'_{\text{kicker}}$	$\Delta \bar{X}'_{\text{kicker}}$
$\sqrt{\dfrac{\varepsilon}{\beta_{\text{kicker}}}}$	$\sqrt{\varepsilon}$

Kicker strength:

$$\Delta x'_{\text{kicker}}(+1\sigma) = \sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



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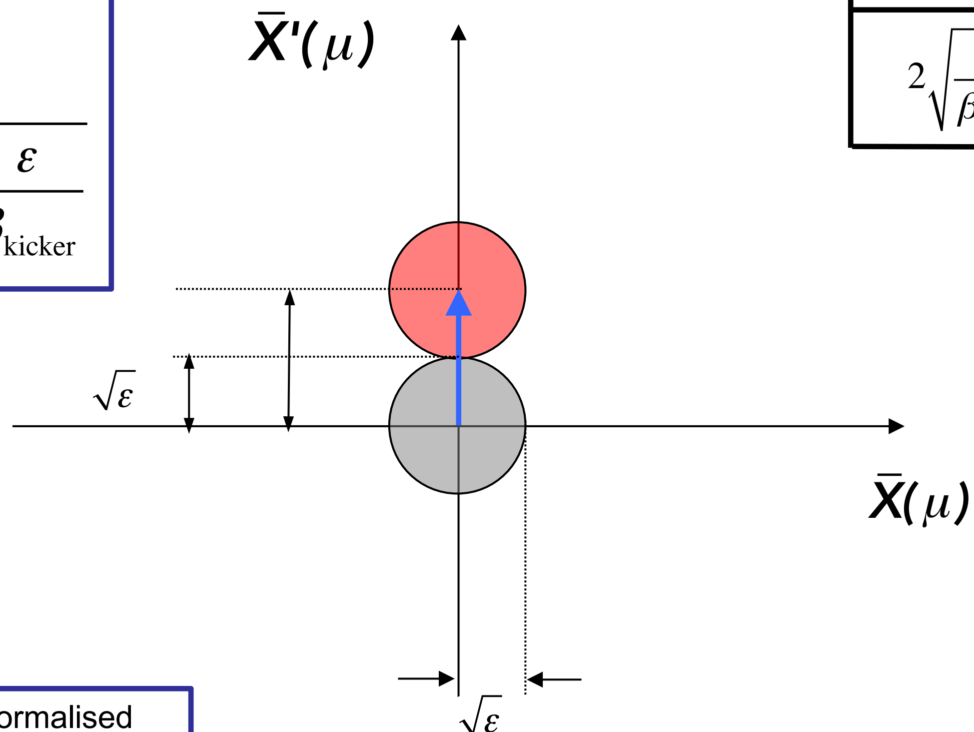
Kick dynamics

Normalised phase space at the kicker location:

$\Delta x'_{\text{kicker}}$	$\Delta \bar{X}'_{\text{kicker}}$
$2\sqrt{\dfrac{\varepsilon}{\beta_{\text{kicker}}}}$	$2\sqrt{\varepsilon}$

Kicker strength:

$$\Delta x'_{\text{kicker}}(+2\sigma) = 2\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



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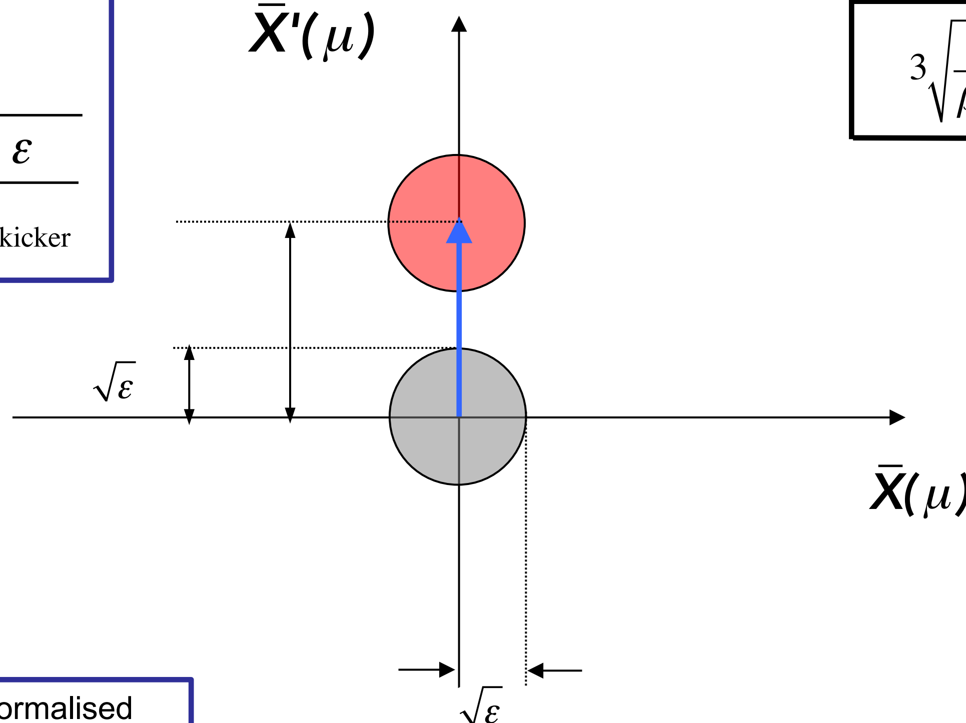
Kick dynamics

Normalised phase space at the kicker location:

$\Delta x'_{\text{kicker}}$	$\Delta \bar{X}'_{\text{kicker}}$
$3\sqrt{\dfrac{\varepsilon}{\beta_{\text{kicker}}}}$	$3\sqrt{\varepsilon}$

Kicker strength:

$$\Delta x'_{\text{kicker}}(+3\sigma) = 3\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



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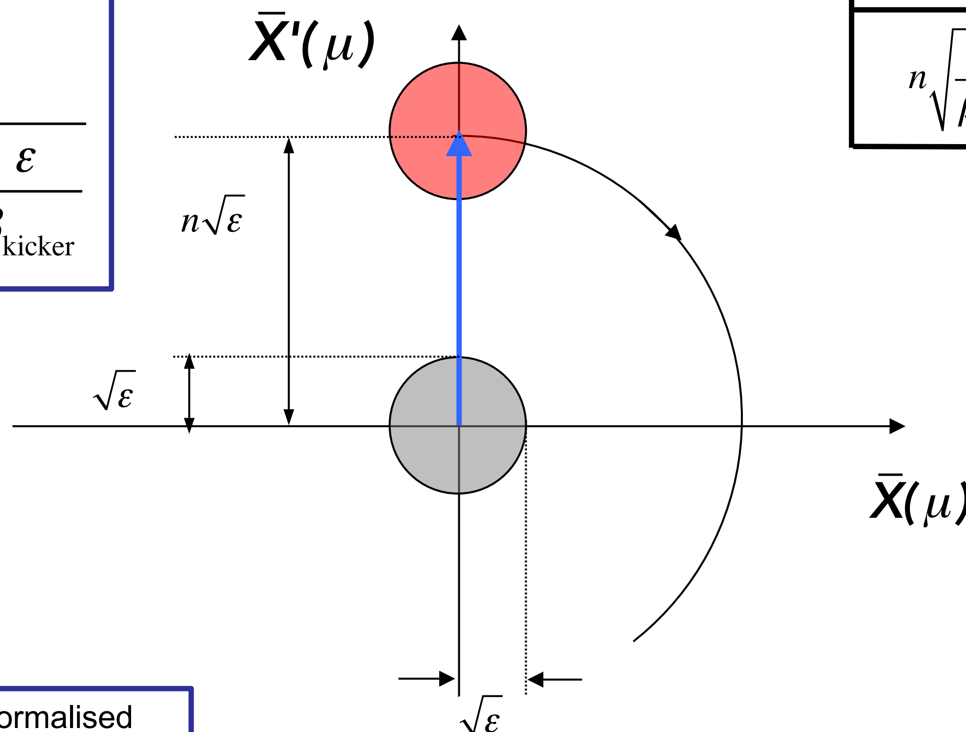
Kick dynamics

Normalised phase space at the kicker location:

$\Delta x'_{\text{kicker}}$	$\Delta \bar{X}'_{\text{kicker}}$
$n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$	$n\sqrt{\varepsilon}$

Kicker strength:

$$\Delta x'_{\text{kicker}}(+n\sigma) = n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



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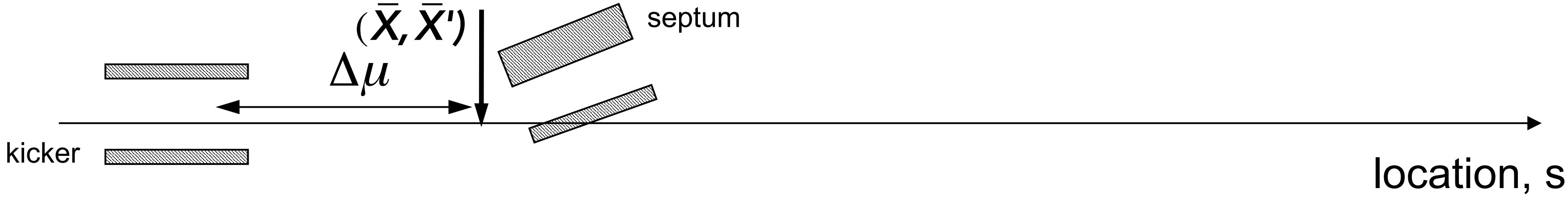
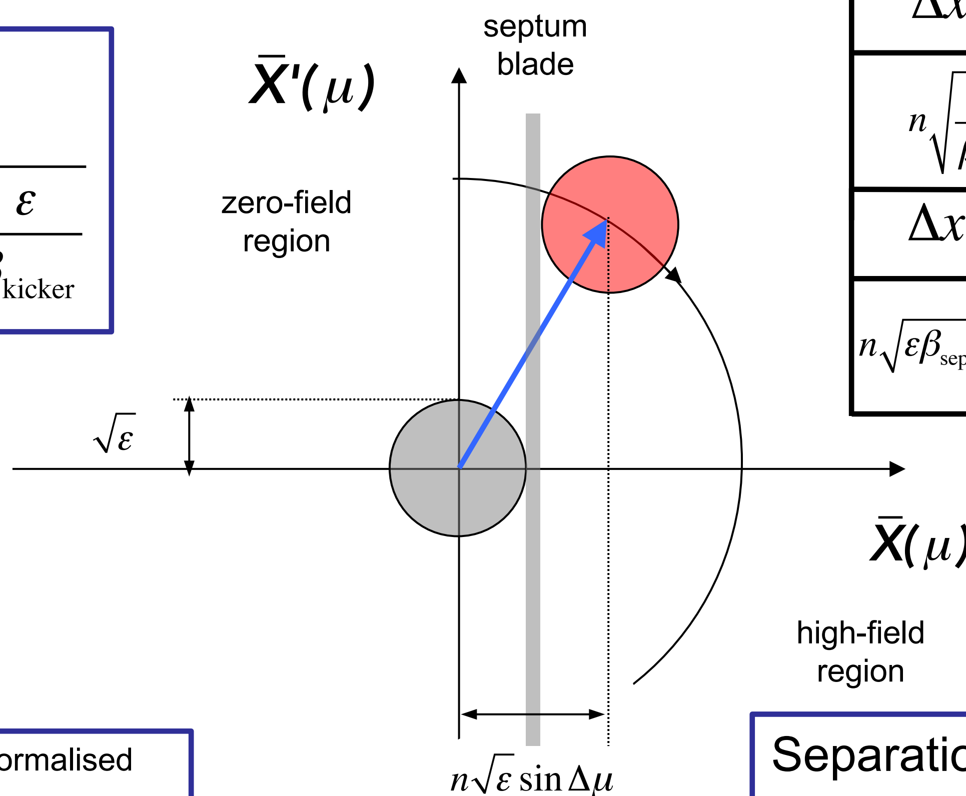
Kick dynamics



Normalised phase space at the septum location:

Kicker strength:

$$\Delta x'_{\text{kicker}}(+n\sigma) = n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



$\Delta x'_{\text{kicker}}$	$\Delta\bar{X}'_{\text{kicker}}$
$n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$	$n\sqrt{\varepsilon}$
Δx_{septum}	$\Delta\bar{X}_{\text{septum}}$
$n\sqrt{\varepsilon\beta_{\text{septum}}}\sin\Delta\mu$	$n\sqrt{\varepsilon}\sin\Delta\mu$

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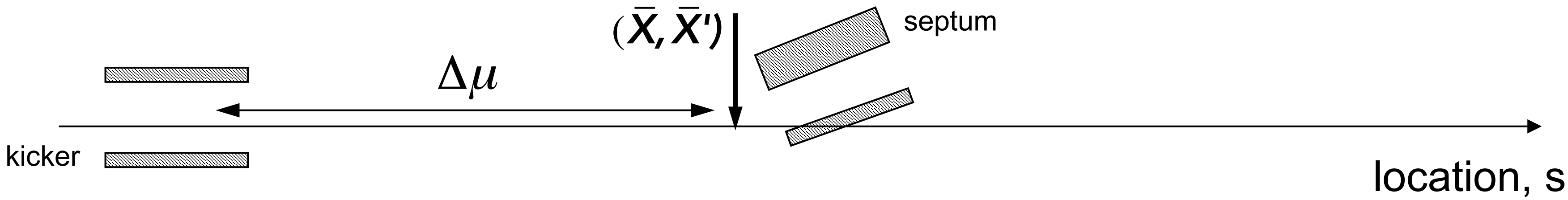
Separation at septum:

$$\Delta x_{\text{septum}} = \Delta x'_{\text{kicker}}\sqrt{\beta_{\text{kicker}}\beta_{\text{septum}}}\sin\Delta\mu$$

Kick dynamics



Normalised phase space at the septum location:

Kicker strength:

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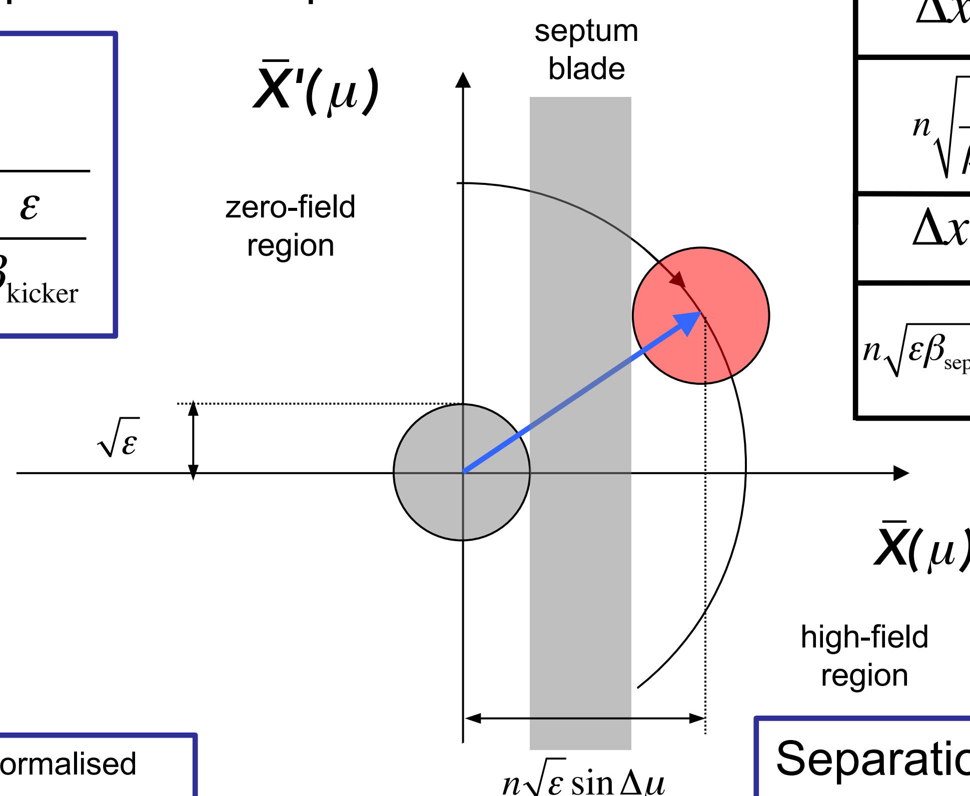

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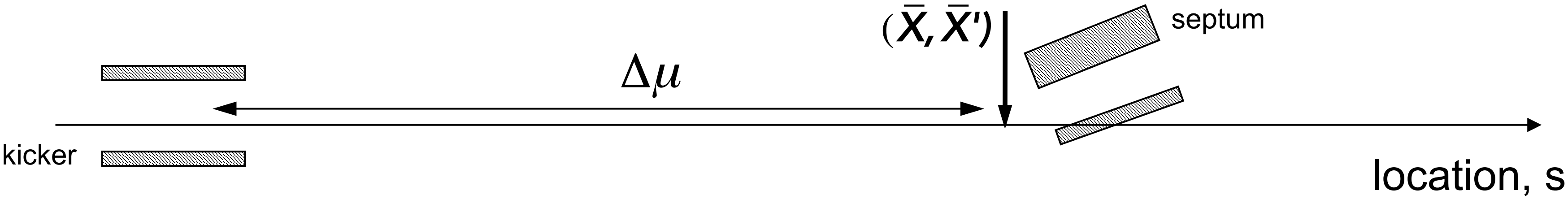
Kick dynamics



Normalised phase space at the septum location:

Kicker strength:

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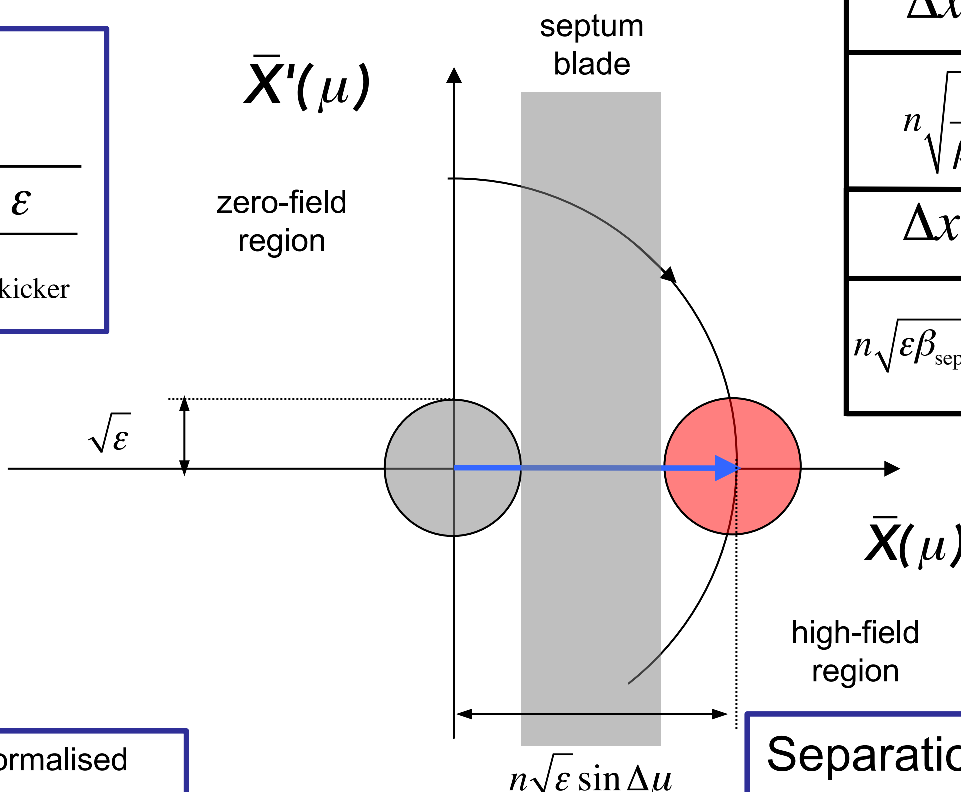

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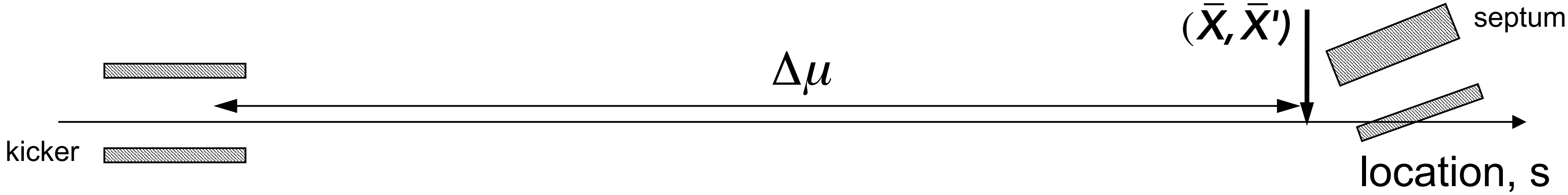
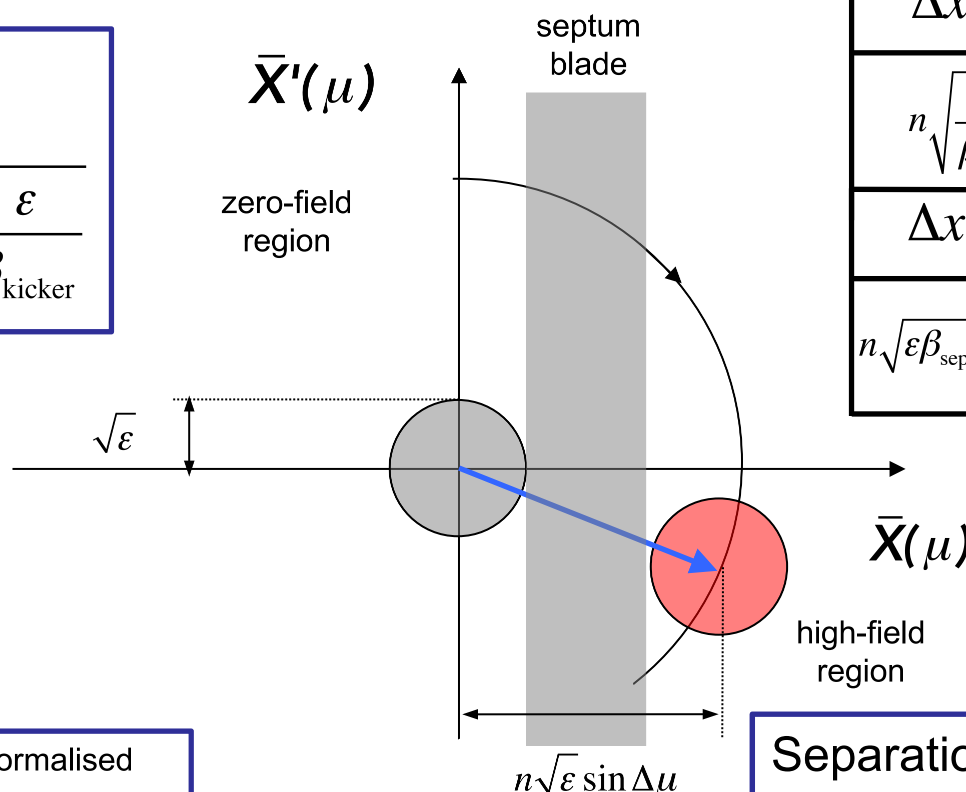
Kick dynamics



Normalised phase space at the septum location:

Kicker strength:

$$\Delta x'_{\text{kicker}}(+n\sigma) = n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



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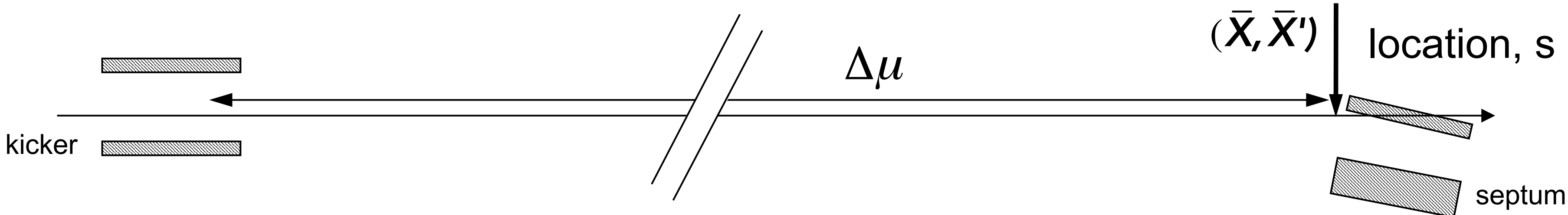
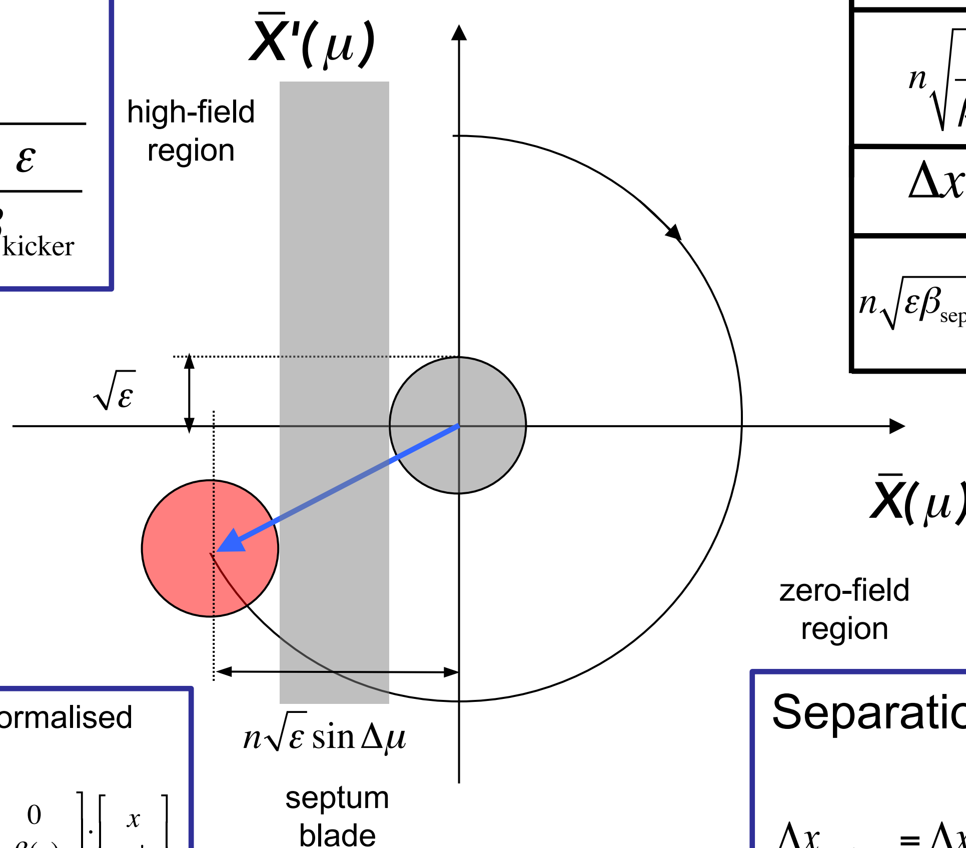
Kick dynamics



Normalised phase space at the septum location:

Kicker strength:

$$\Delta x'_{\text{kicker}}(+n\sigma) = n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$$



$\Delta x'_{\text{kicker}}$	$\Delta\bar{X}'_{\text{kicker}}$
$n\sqrt{\frac{\varepsilon}{\beta_{\text{kicker}}}}$	$n\sqrt{\varepsilon}$
Δx_{septum}	$\Delta\bar{X}_{\text{septum}}$
$n\sqrt{\varepsilon\beta_{\text{septum}}}\sin\Delta\mu$	$n\sqrt{\varepsilon}\sin\Delta\mu$

Separation at septum:

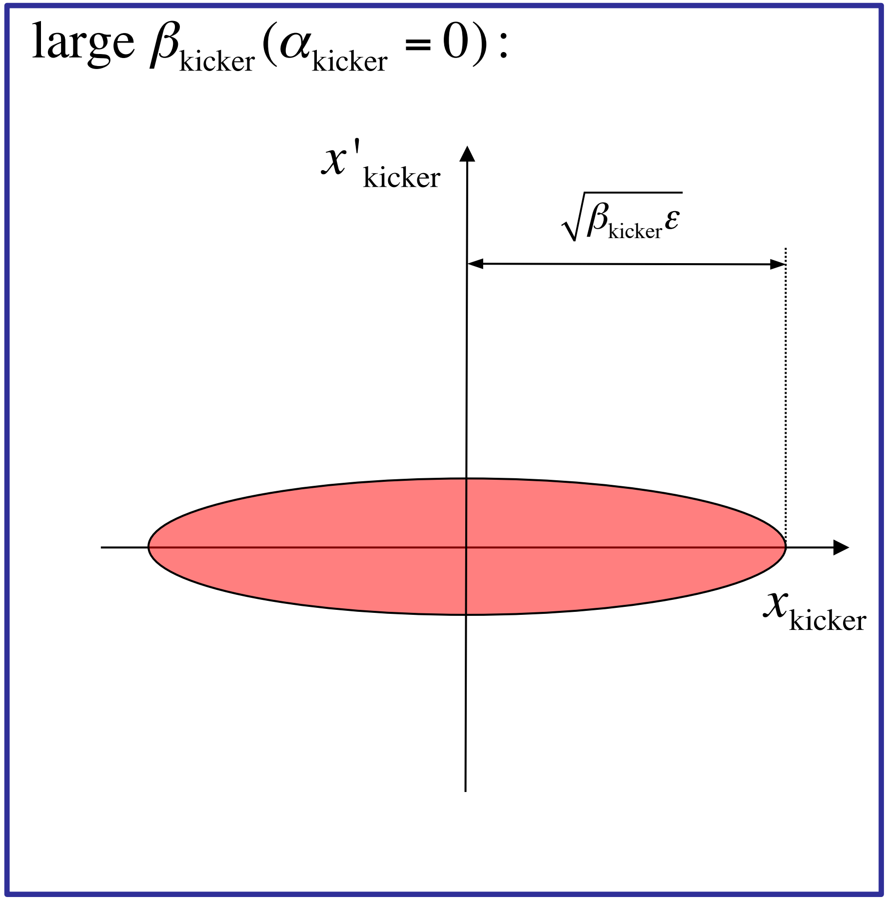
$$\Delta x_{\text{septum}} = \Delta x'_{\text{kicker}}\sqrt{\beta_{\text{kicker}}\beta_{\text{septum}}}\sin\Delta\mu$$

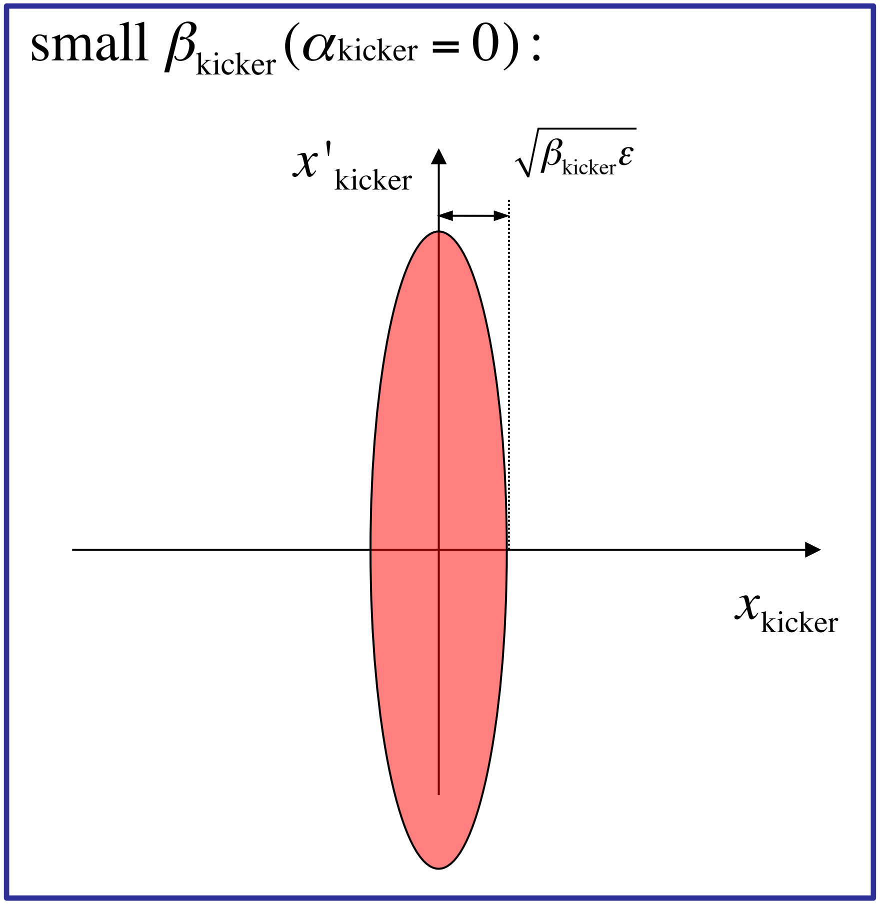
Reminder: transformation to normalised phase space:

$$\begin{bmatrix}\bar{X}\\ \bar{X}'\end{bmatrix} = \mathbf{N}\cdot\begin{bmatrix}x\\ x'\end{bmatrix} = \sqrt{\frac{1}{\beta(s)}}\cdot\begin{bmatrix}1 & 0\\ \alpha(s) & \beta(s)\end{bmatrix}\cdot\begin{bmatrix}x\\ x'\end{bmatrix}$$

Kick optimisation: β at the kicker

- Intuitively, we can see in **real** phase space why a large β-function at the kicker improves the separation between extracted and circulating beams:





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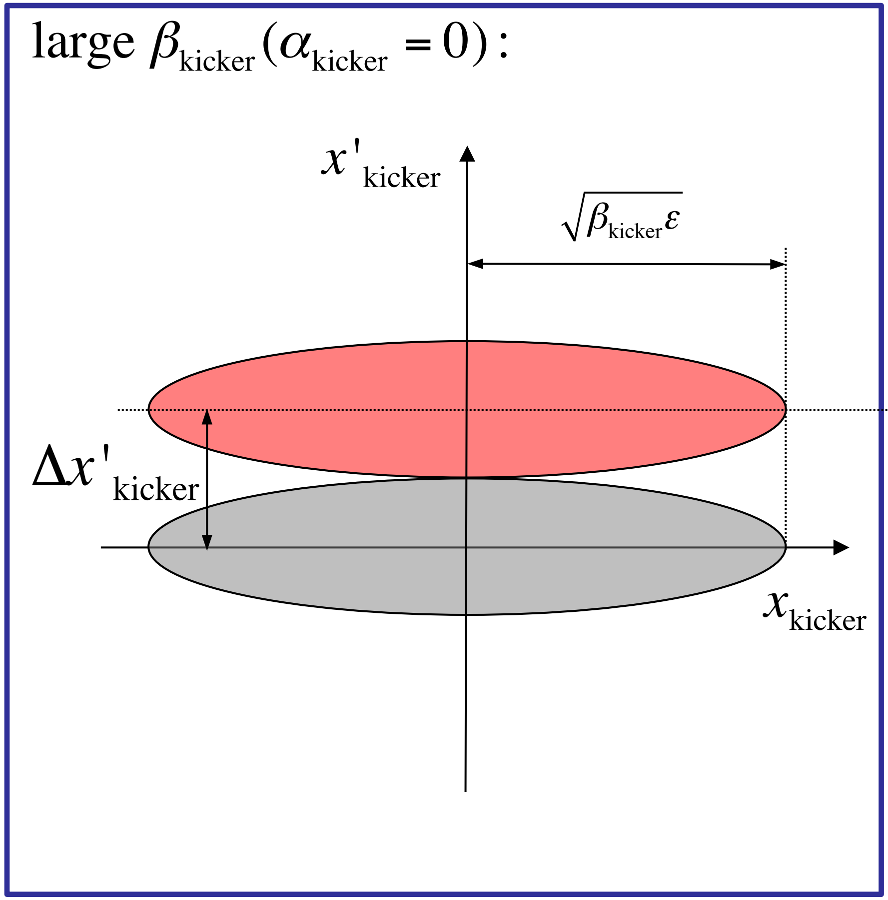

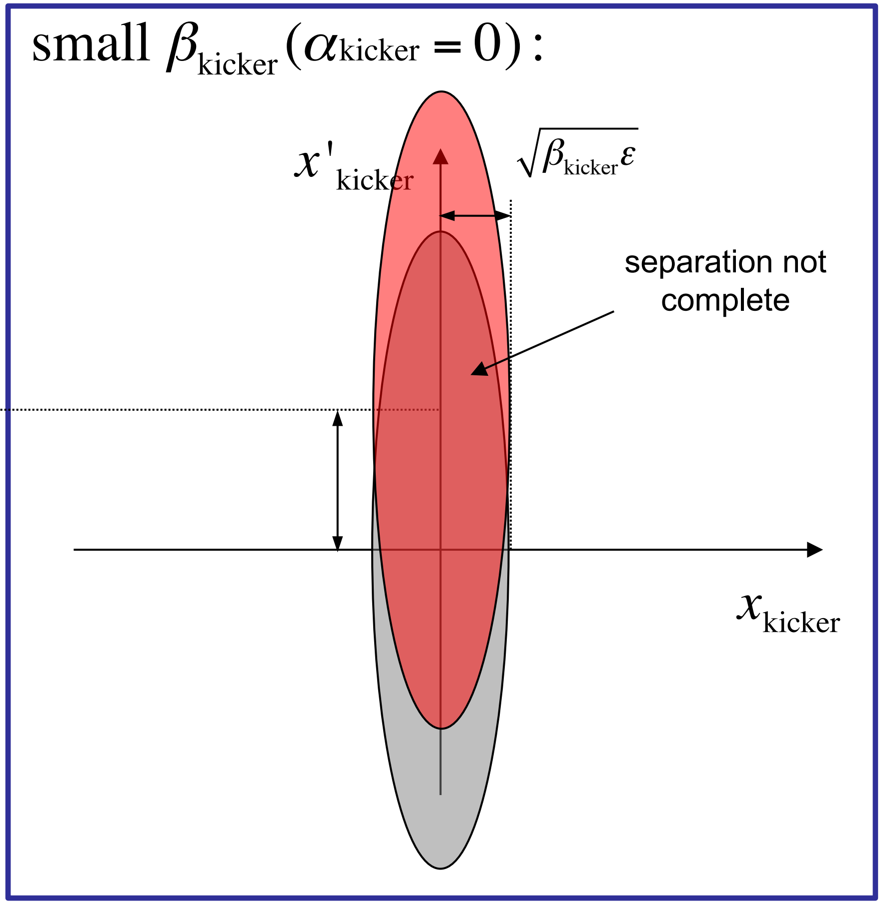

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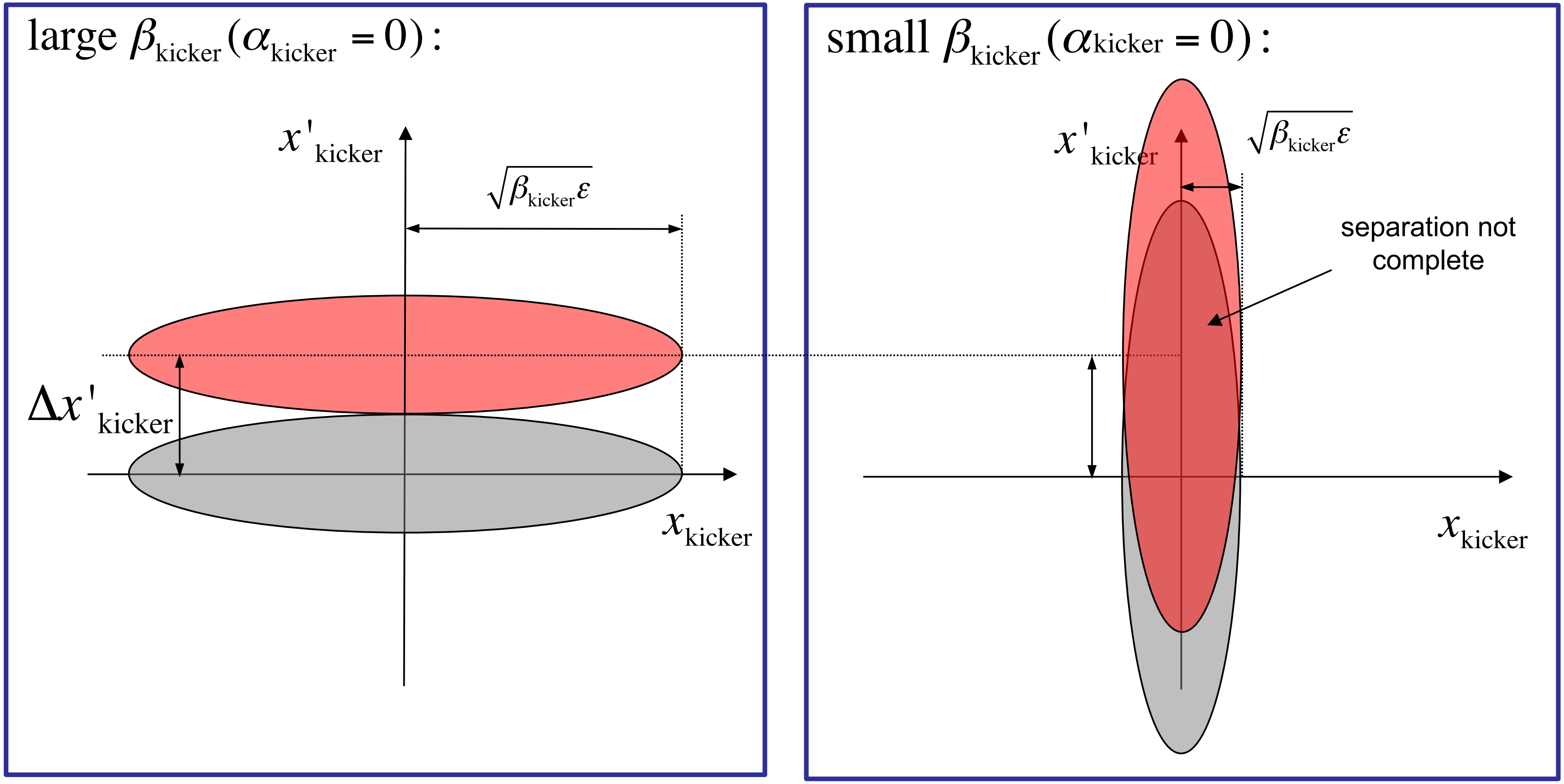
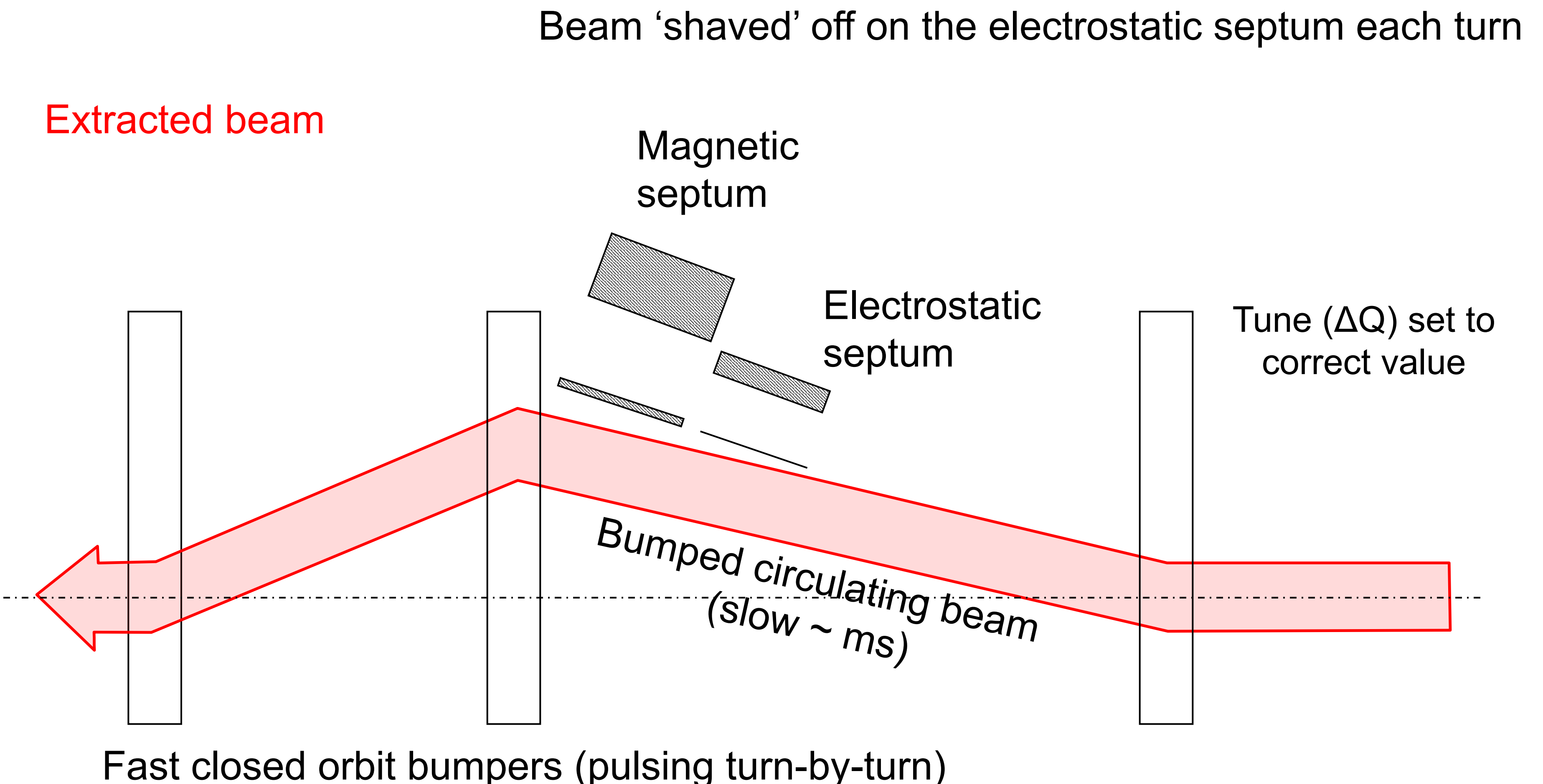


- When the beam divergence is small, we can easily “jump” outside the circulating beam

Multi-turn fast extraction



- Fast modulated bump deflects beam onto the septum, turn-by-turn
- The machine tune rotates the beam in phase space, turn-by-turn
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- Often combine thin electrostatic septa with magnetic septa ($\Delta\mu_{ES\text{->}MS} \neq 0$)

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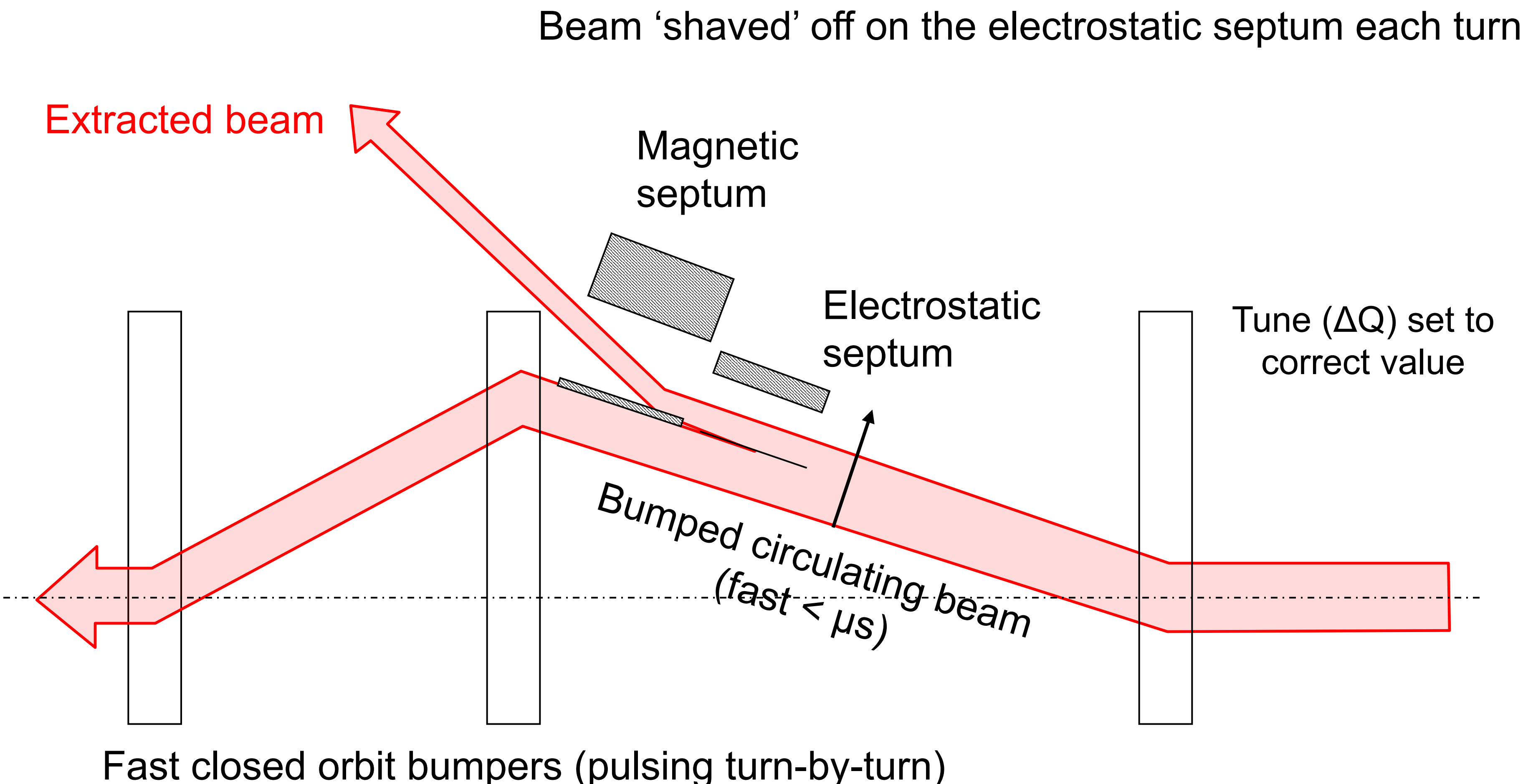

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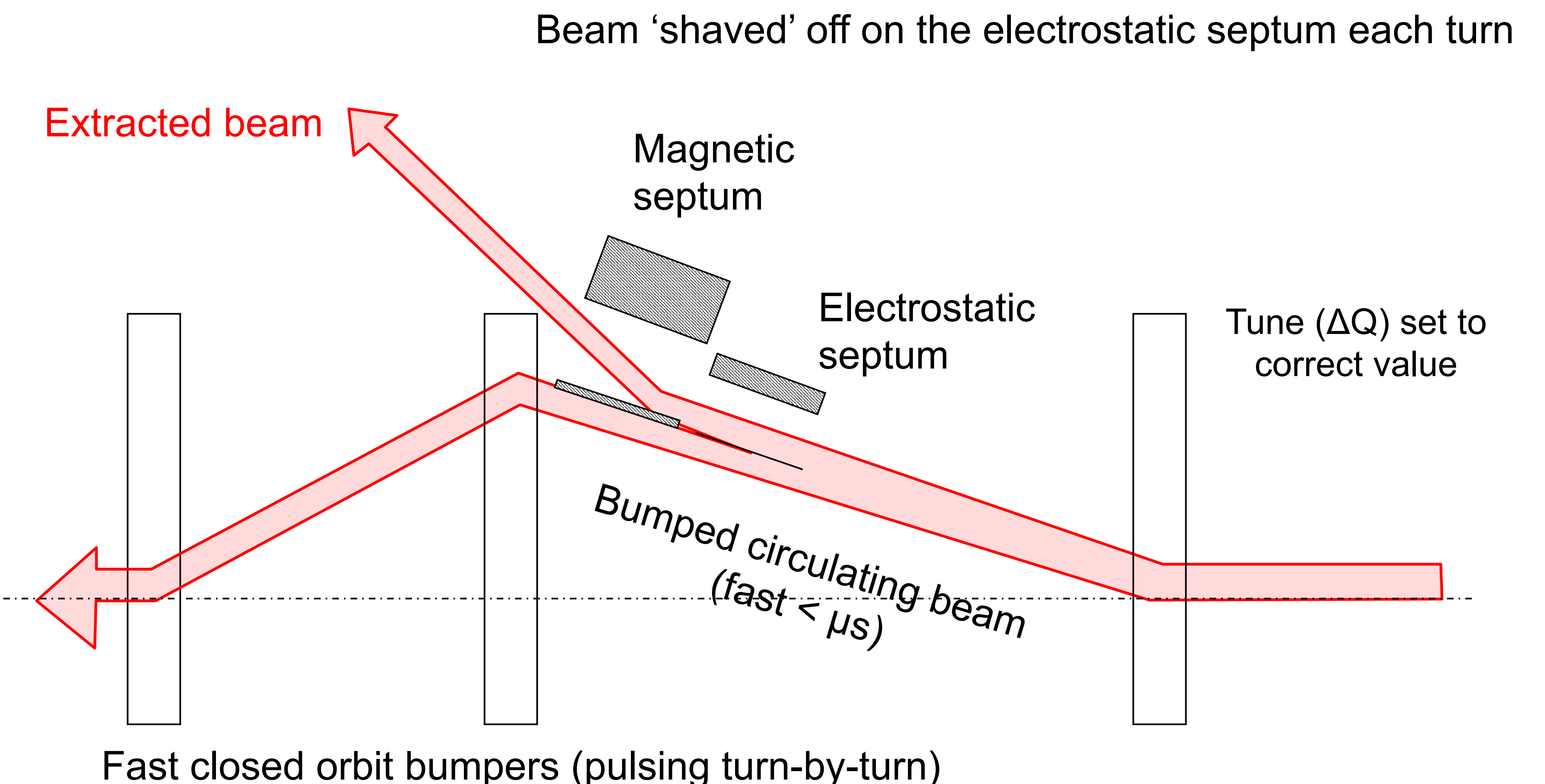

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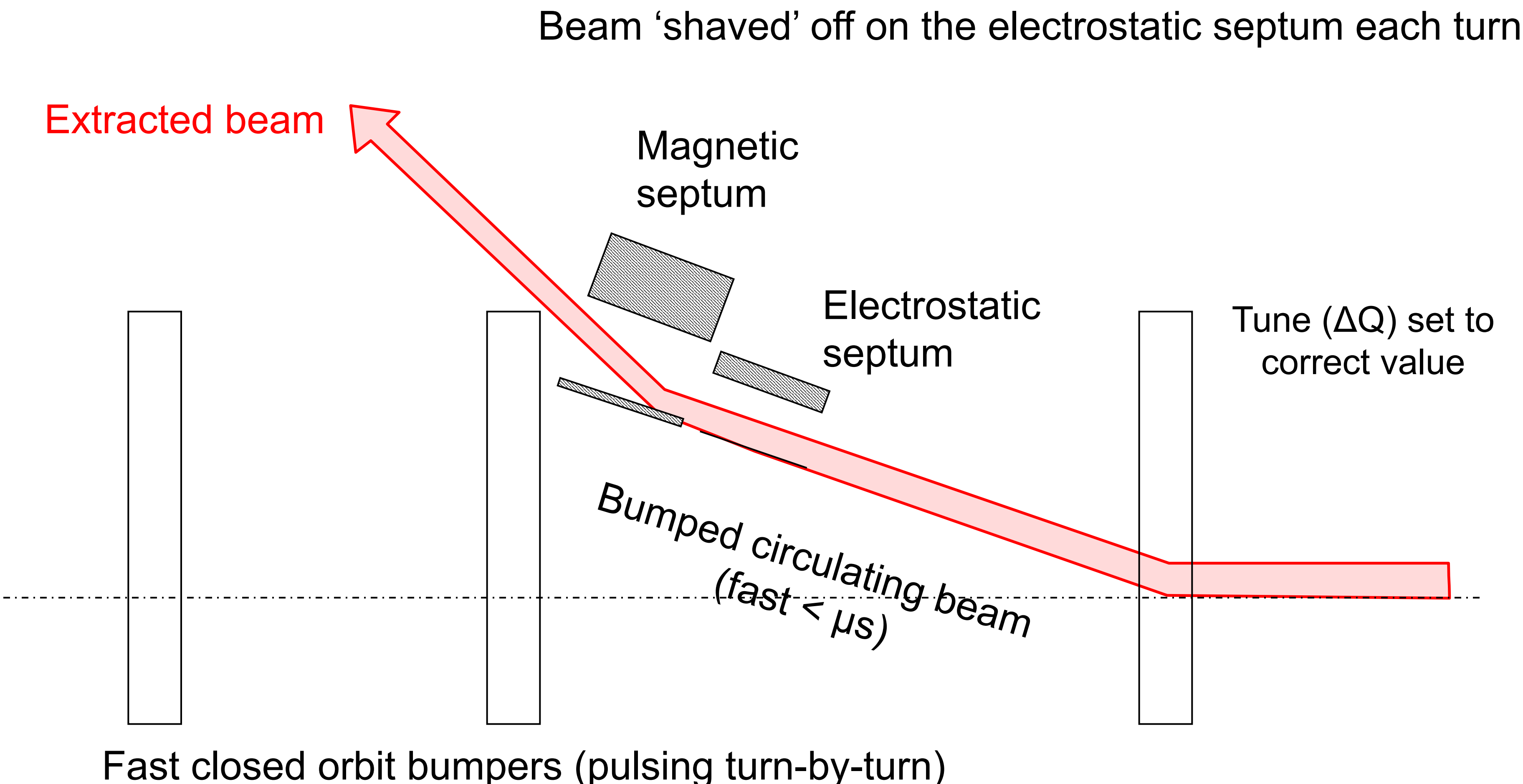

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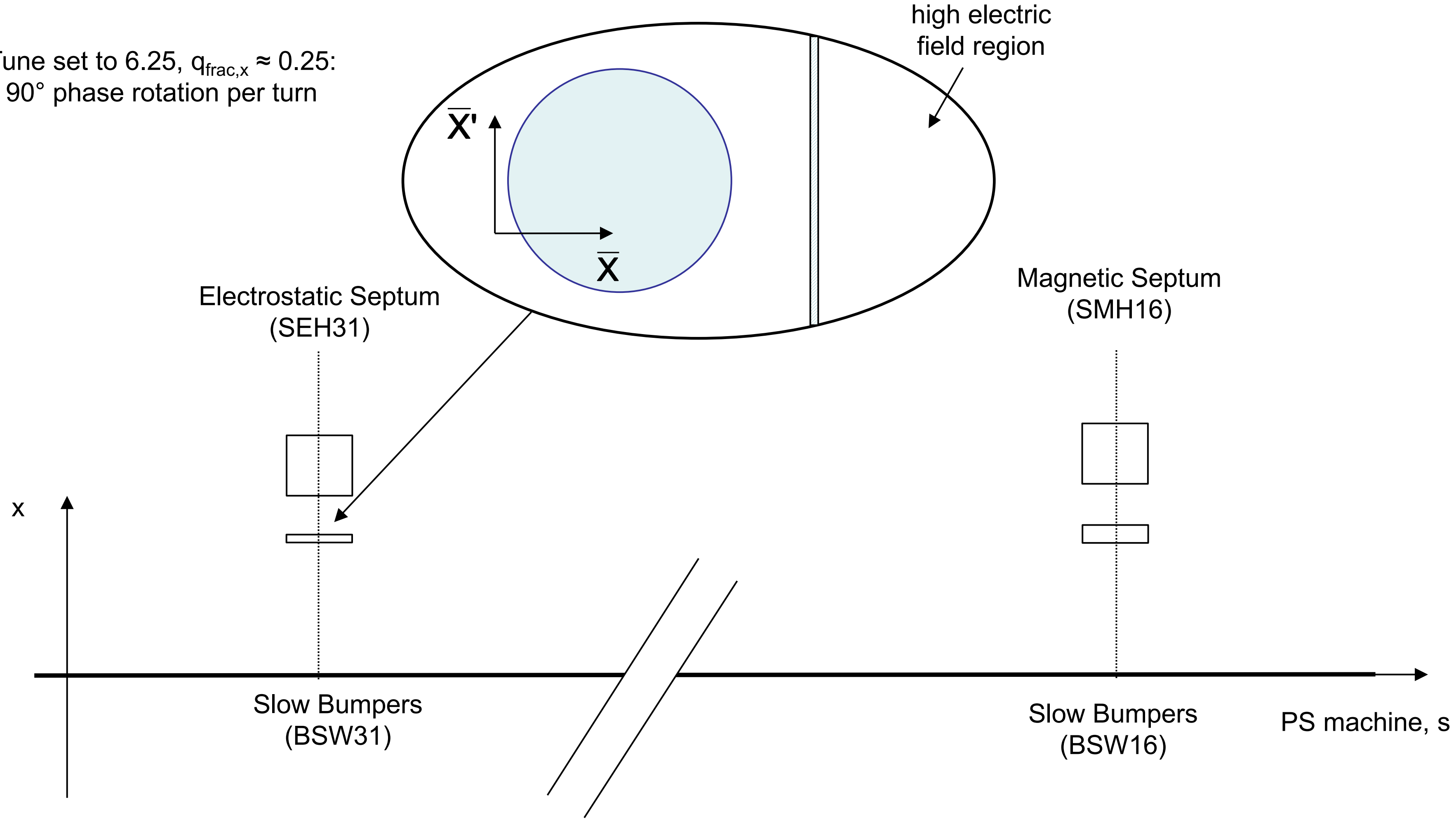
Operational implementation at the PS

Tune set to 6.25, $q_{frac,x} \approx 0.25$:
90° phase rotation per turn



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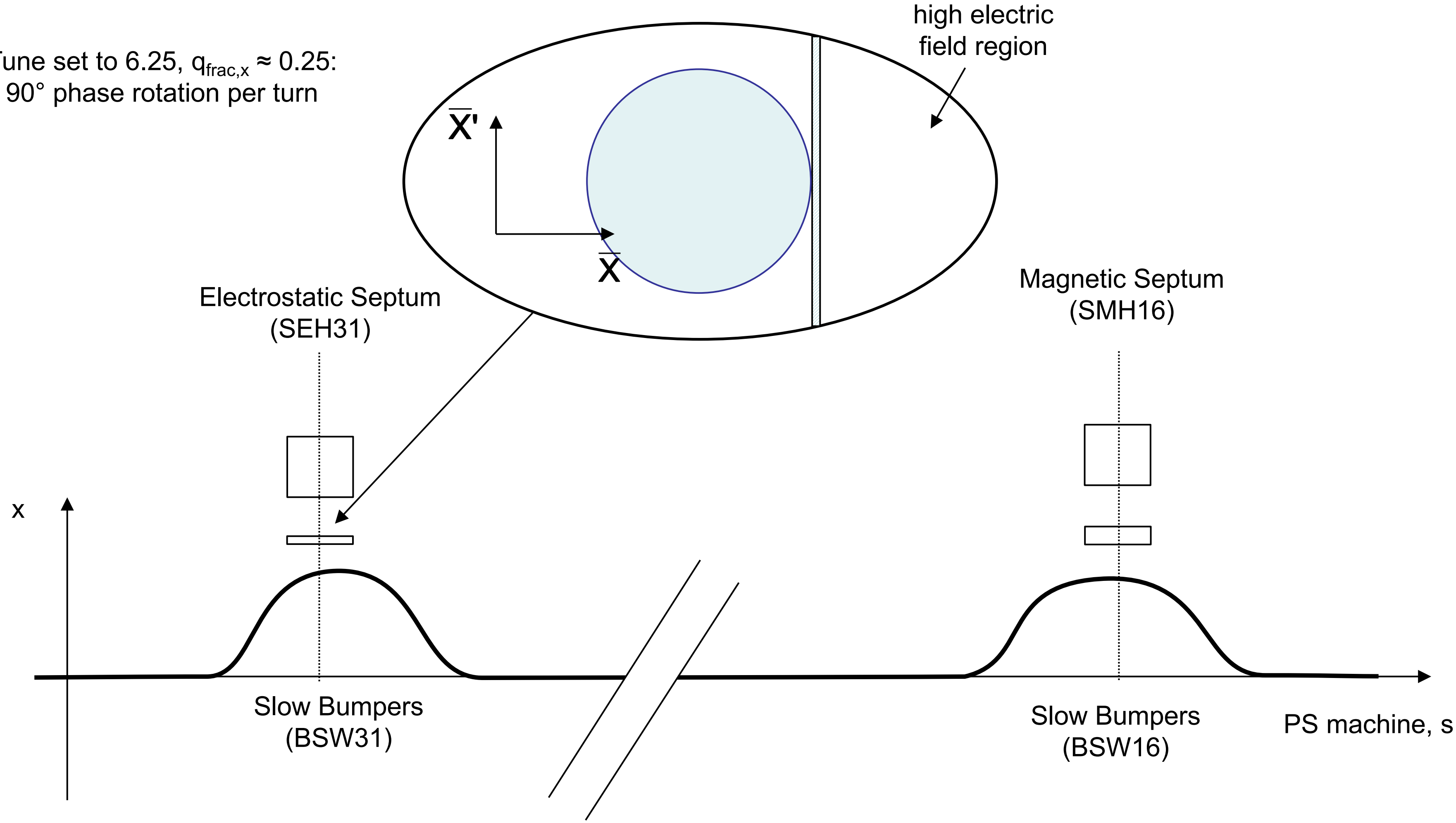
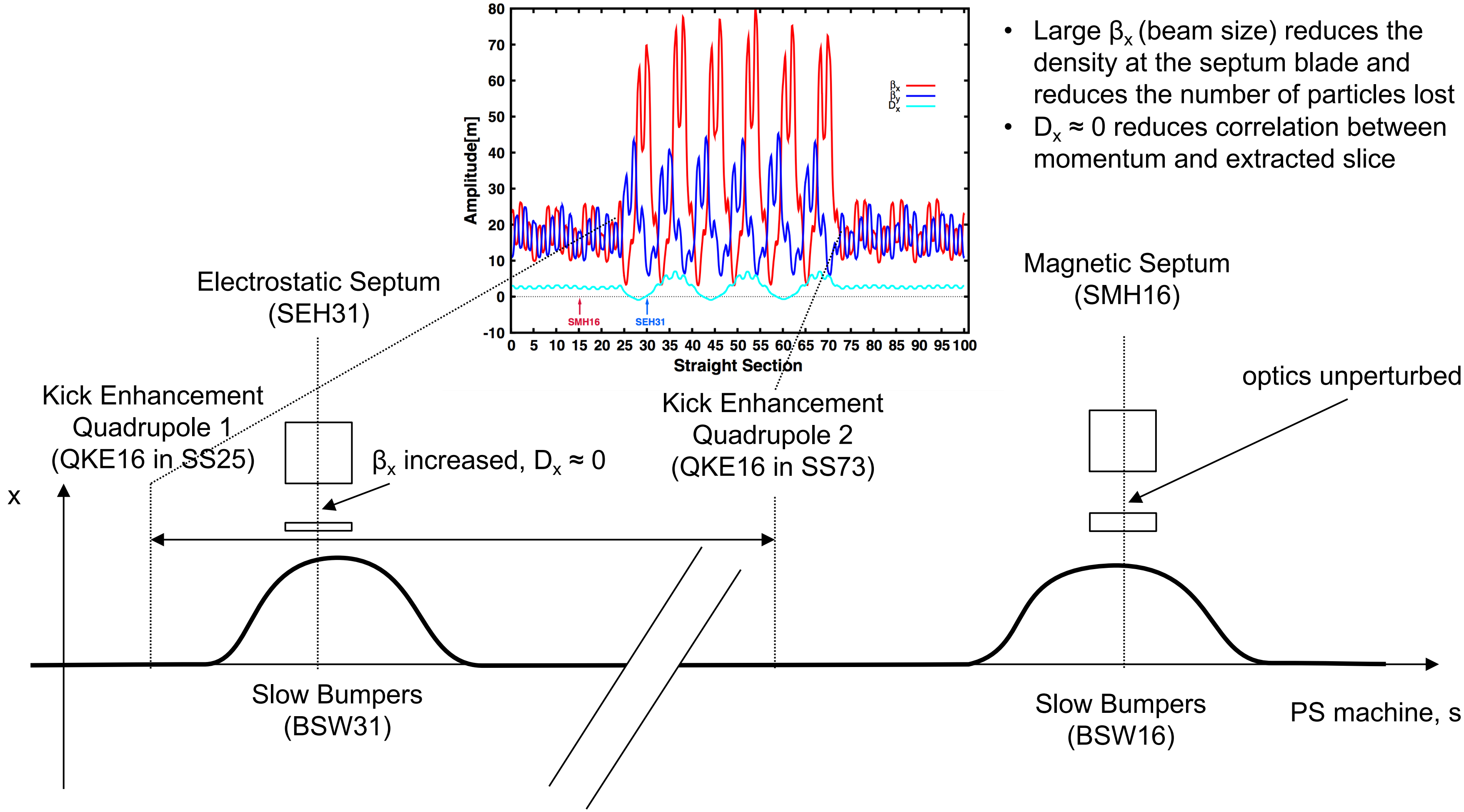


Operational implementation at the PS

- Large β_x (beam size) reduces the density at the septum blade and reduces the number of particles lost
- $D_x \approx 0$ reduces correlation between momentum and extracted slice



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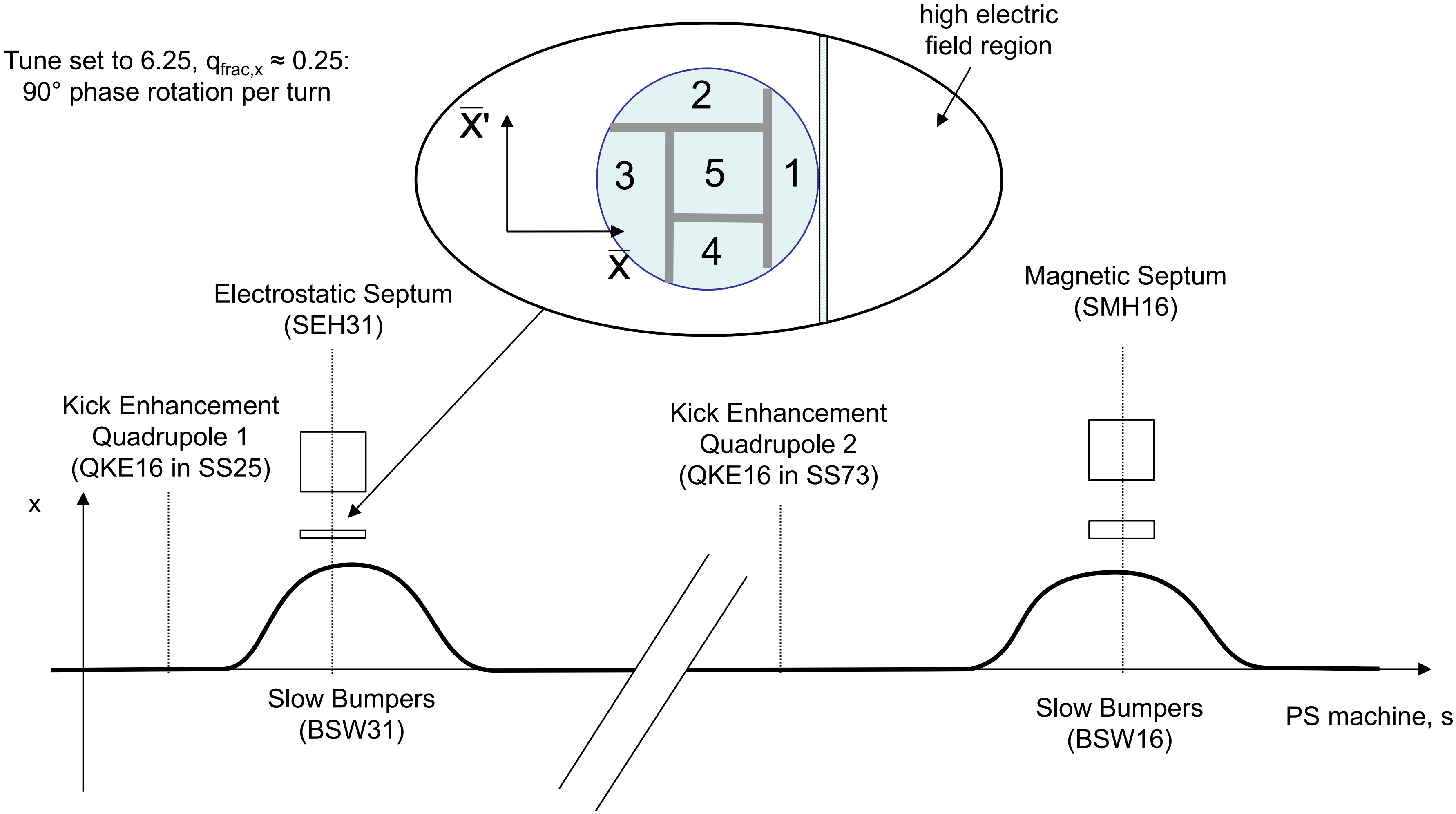

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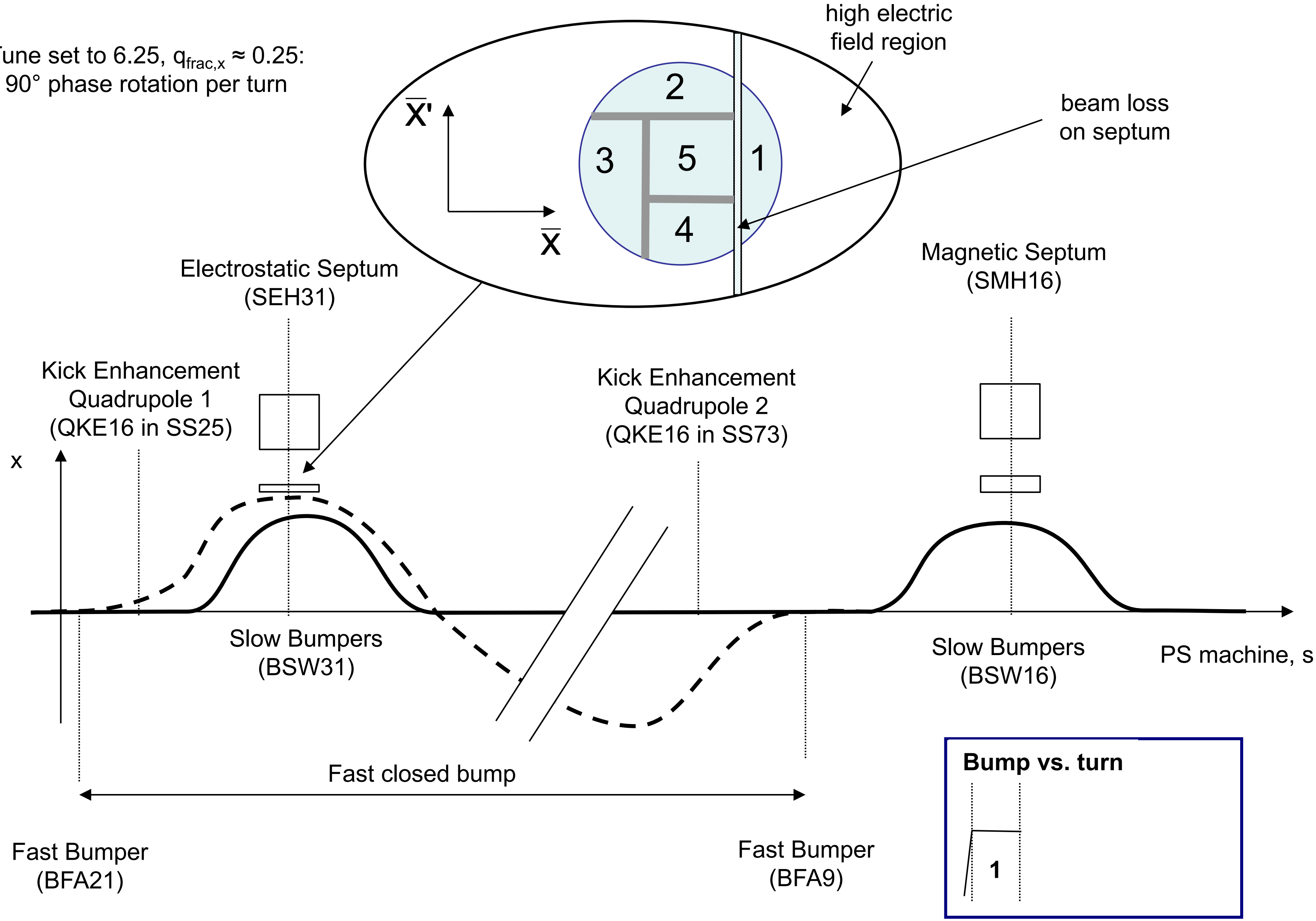

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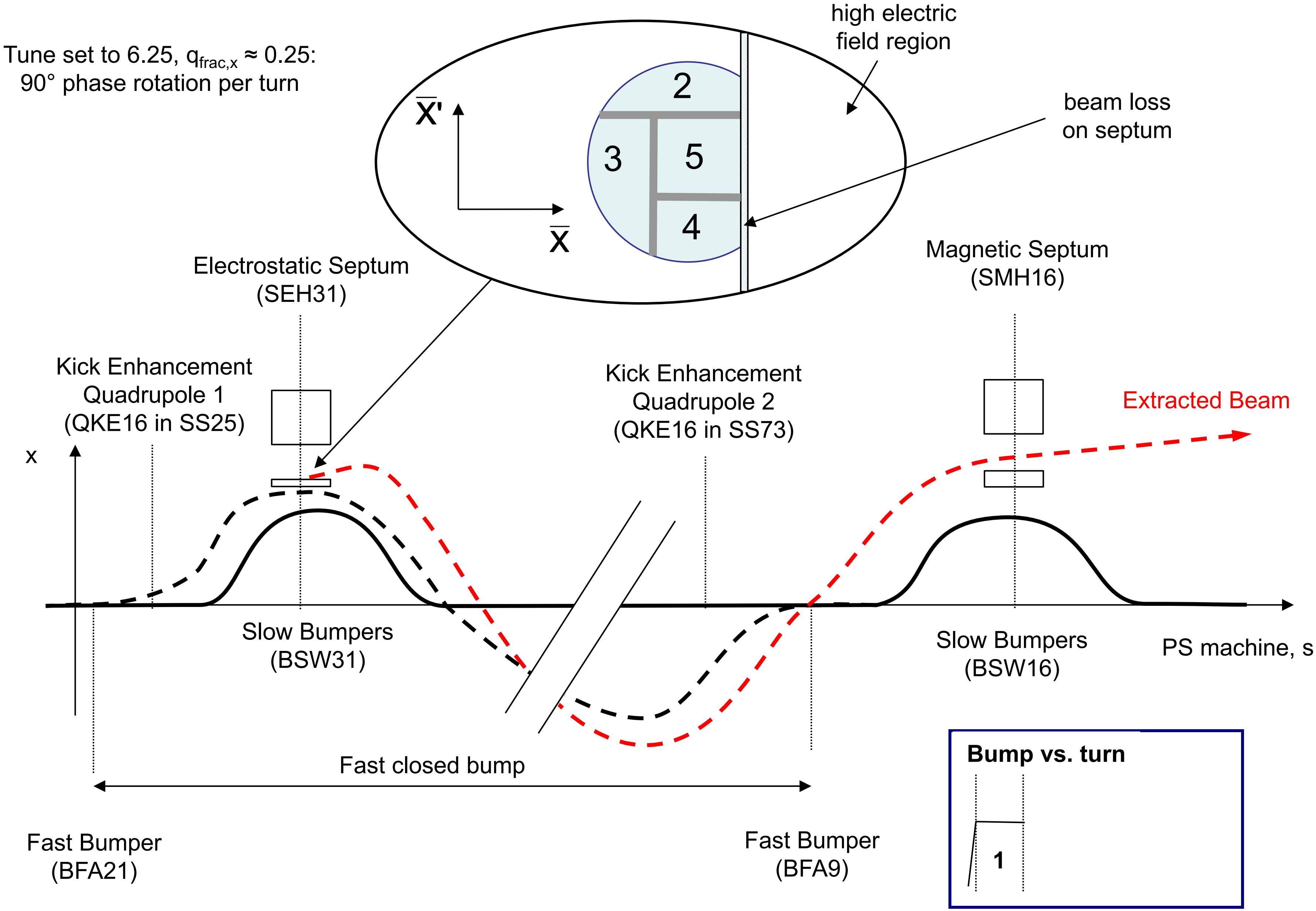

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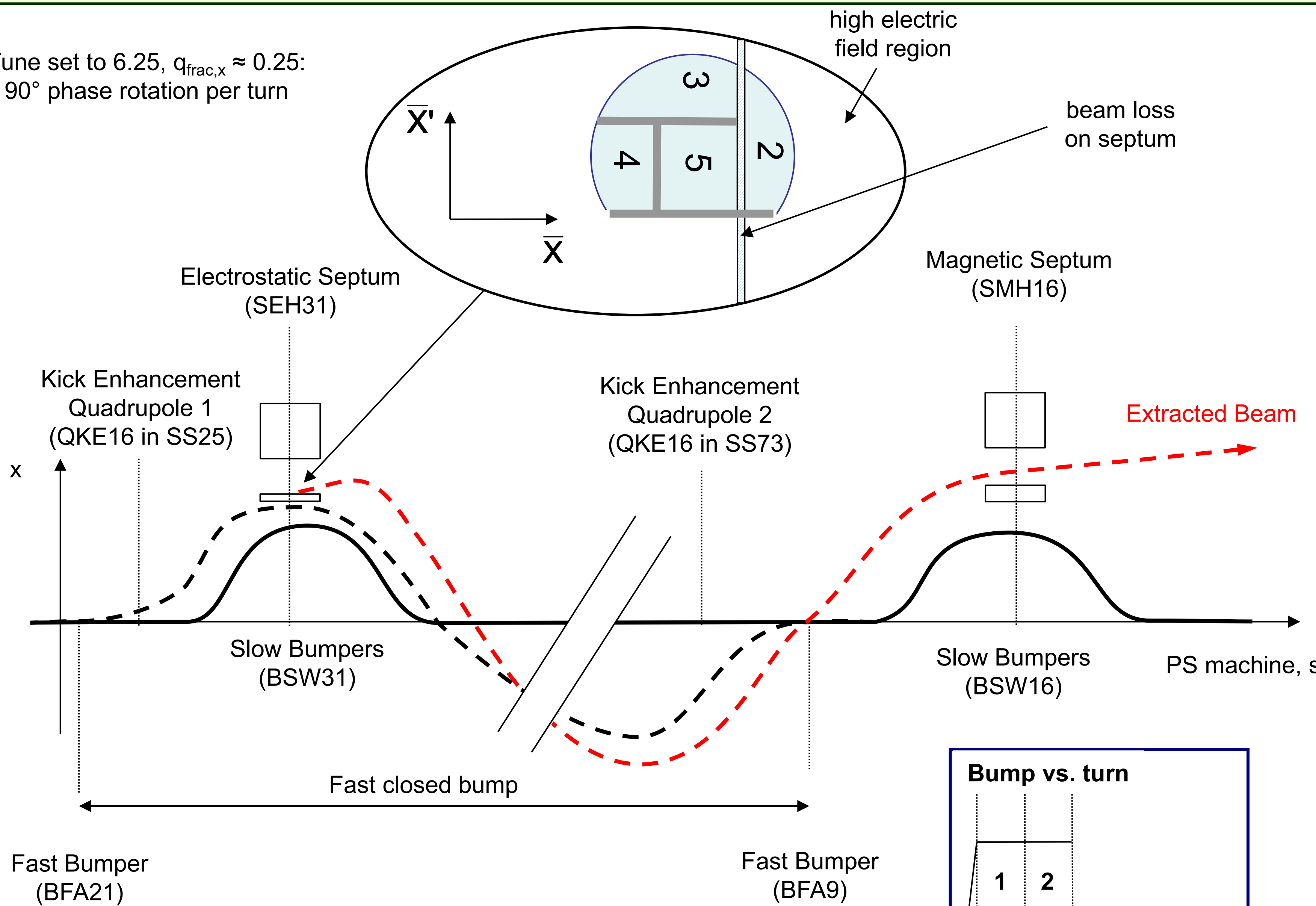

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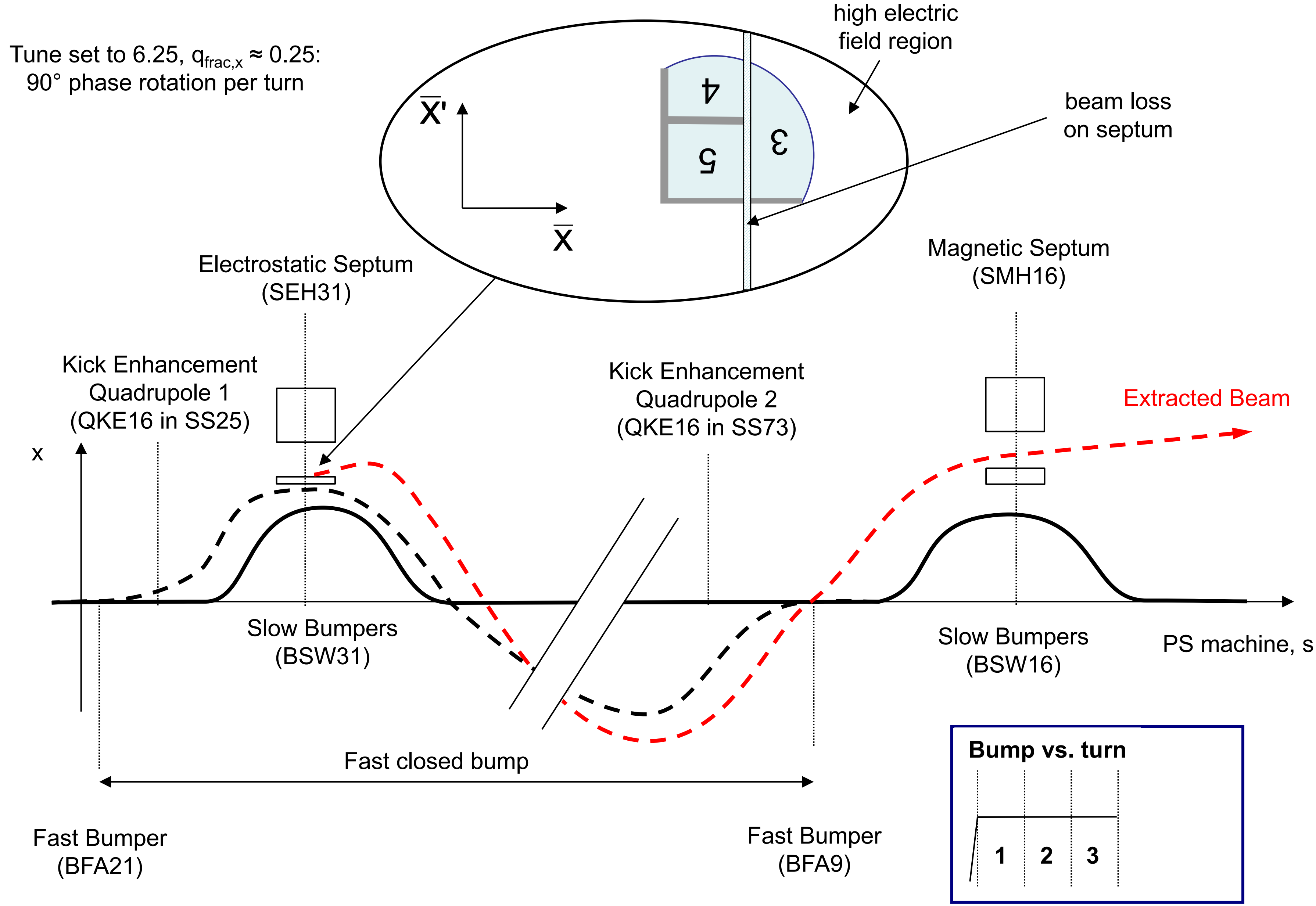

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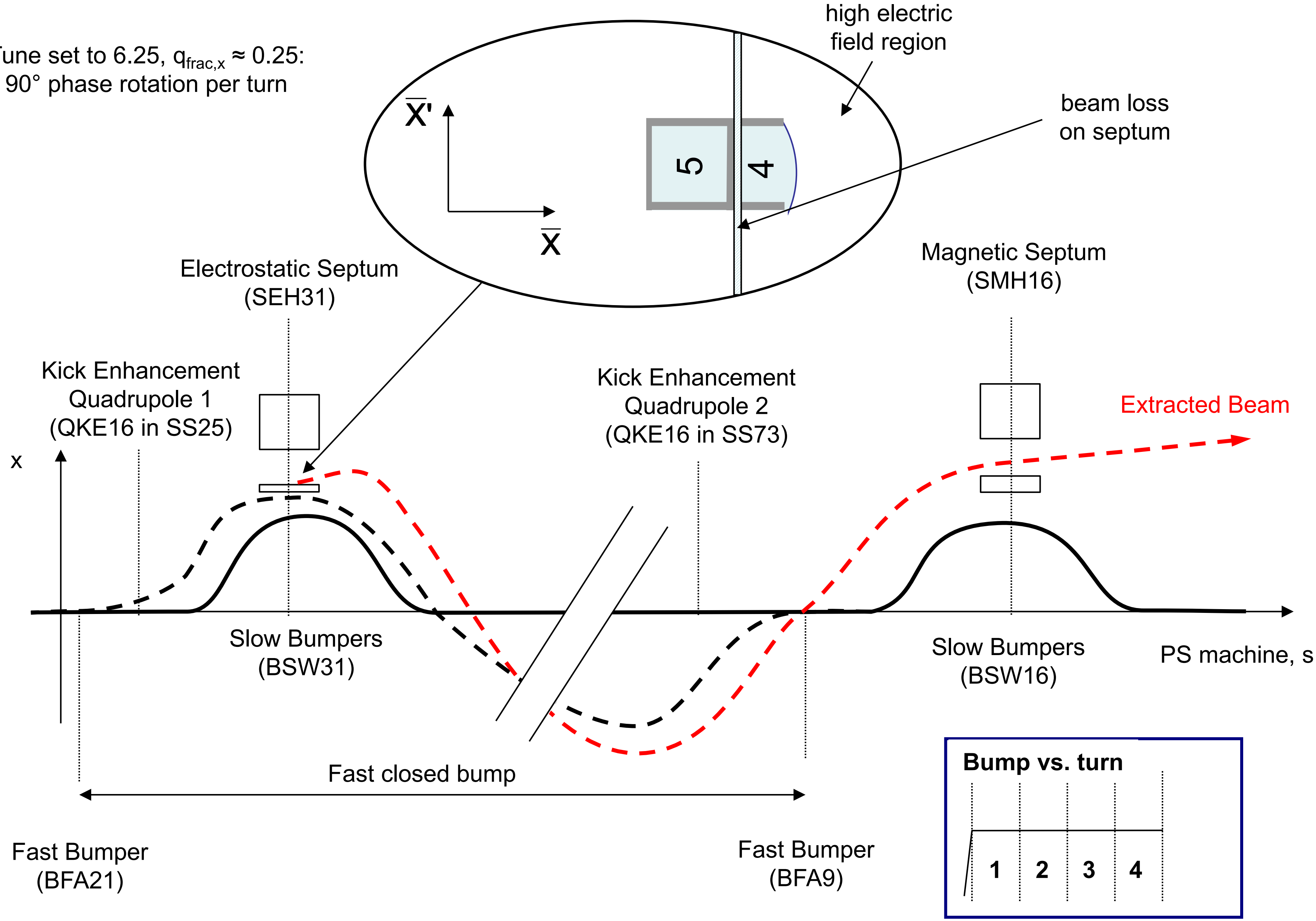

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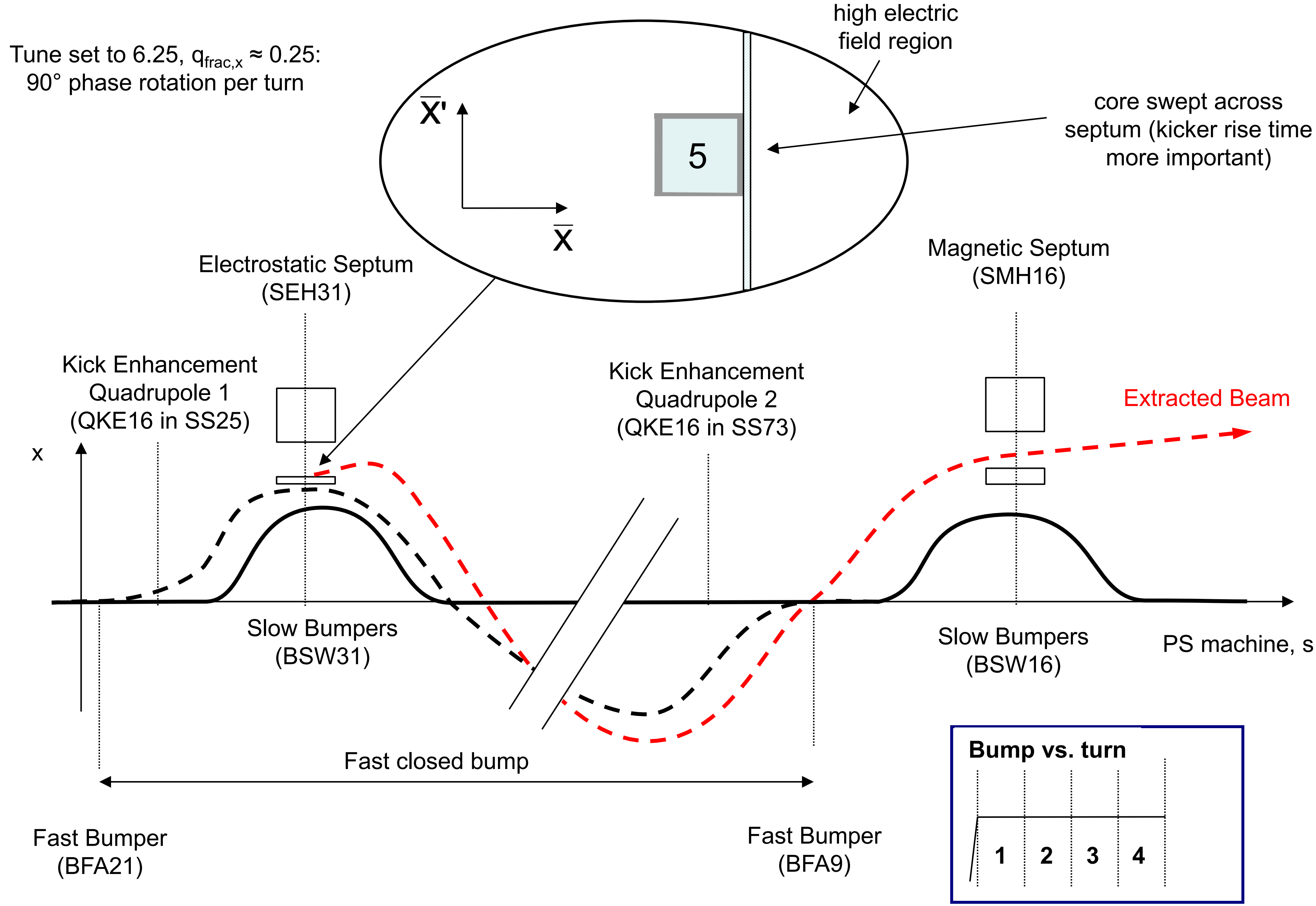

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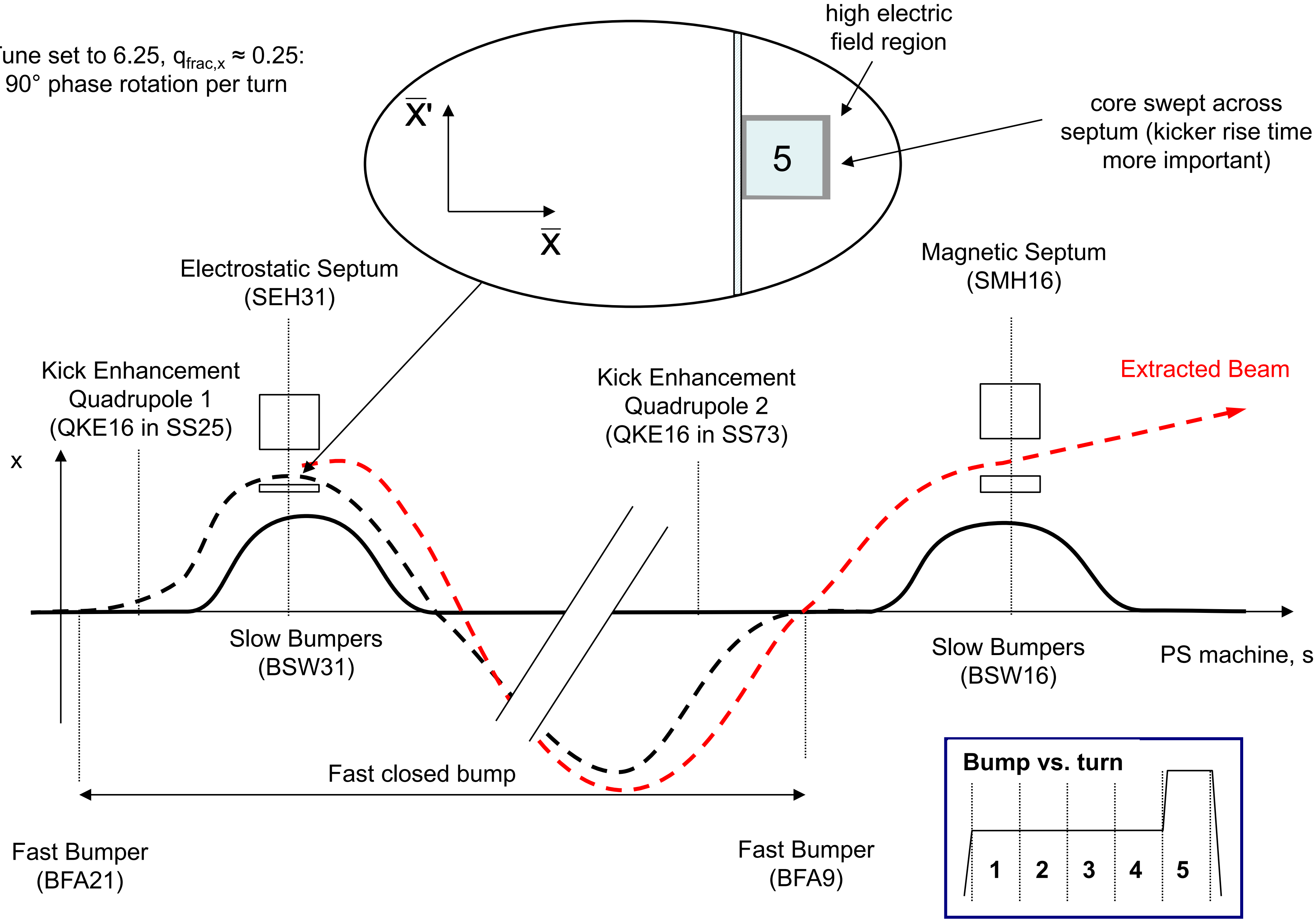

Continuous Transfer: performance aspects

- CT results in a **smaller emittance** in the plane that is “sliced”

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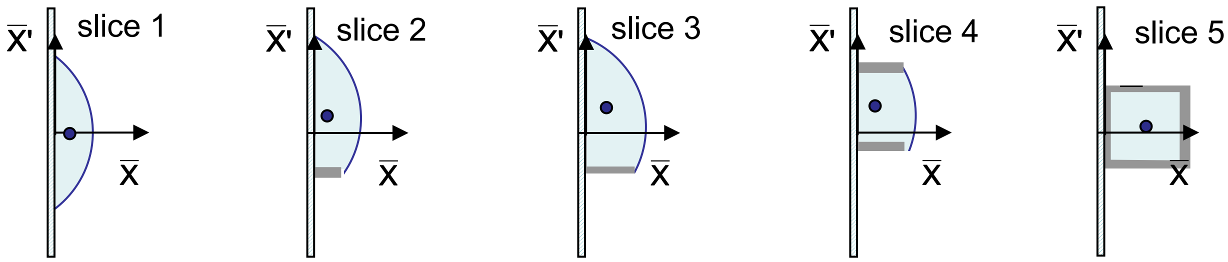
- Turn-by-turn **mismatch** causes emittance growth in receiving machine:
  - each slice has a **different emittance** and **optical parameters**
  - each slice has a **different centroid** and **trajectory error**
  - spills with **both uniform intensity** and **emittance are not possible**



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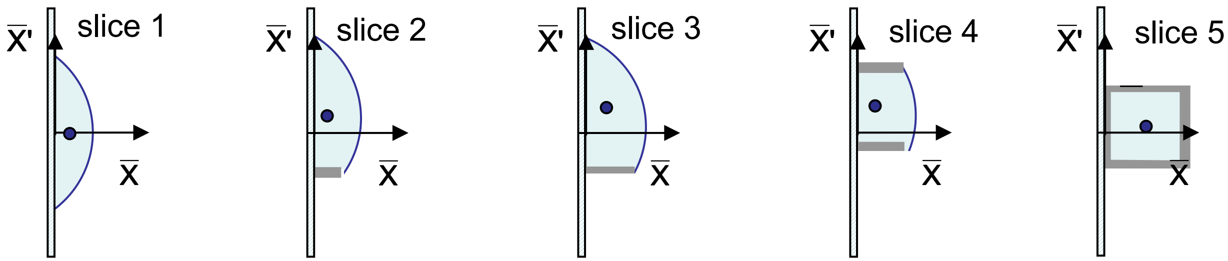


- Beam loss during extraction and unavoidable **induced radio-activation:**
  - particles impinging the septum are scattered around the machine aperture
  - electrostatic septum is irradiated making **hands-on maintenance** difficult
  - potential **limit for total intensity** throughput:
    - ≈ 40% of the all losses along the accelerator chain for the SPS FT physics programme occur at the PS electrostatic septum
    - e.g. for a future SPS Beam Dump Facility requesting 5×10^{19} p^+/yr, about 0.7×10^{19} p^+/yr would be lost on the PS electrostatic septum

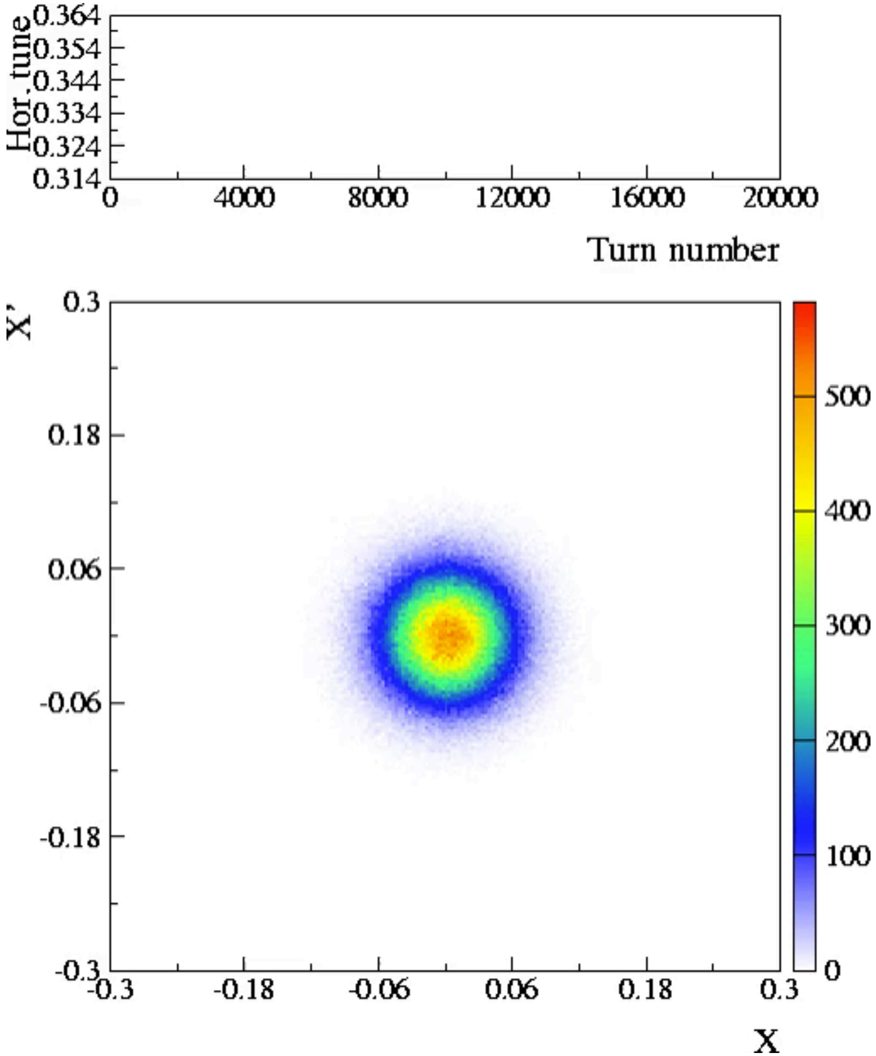
Magnetic splitting: motivation

- Split the beam by crossing a resonance condition in the presence of applied non-linear fields: sextupoles and octupoles
- Aim to do away with mechanical splitting, with several advantages:
  - Losses reduced significantly (no need for an electrostatic septum)
    - attractive for higher energy applications
  - Phase space matching improved with respect to mechanical shaving
    - ‘beamlets’ have same emittance and optical parameters at the extraction point

Magnetic splitting

- **Non-linear fields** can be used to split a beam in phase space:
  - **Sextupoles** and **octupoles** can be used to create **islands of stability** inside the circulating beam
  - A slow (adiabatic) tune variation across a resonance can **capture** particles into **separate islands**
  - Variation of the **tune** moves the islands to large amplitudes
- Pioneered over the last 20 years at CERN:
  - for further reading a **list of MTE references** is found at the end of the talk
  - see extra slides for measurement results carried out in the PS!

An example of splitting a beam into three stable islands $q_x \approx 0.33$



Non-linear beam dynamics (1)

- A vast subject (out of the scope of this lecture!) to solve the non-linear equation of motion (a driven simple harmonic oscillator):

$$\frac{d^2\overline{X}}{d\phi^2} + Q^2\overline{X} = -Q^2\beta^{3/2}\overbrace{\frac{\Delta B(\overline{X},\phi)}{(B\rho)}}^{\text{perturbing fields}}$$

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$$= -\frac{Q^2 B_0}{(B\rho)}\Big[\underbrace{\overbrace{(\beta^{3/2}b_0) + (\beta^{4/2}b_1)\bar{X}}^{\text{linear imperfections: integer and 1/2-integer}} + \overbrace{(\beta^{5/2}b_2)\bar{X}^2}^{\text{sextupole (1/3-integer)}} + \overbrace{(\beta^{6/2}b_3)\bar{X}^3}^{\text{octupole (1/4-integer)}} + \ldots}_{\ldots\text{these terms include harmonic functions of } \phi\text{, driving resonances}}\Big]$$

linear imperfections: integer and 1/2-integer

non-linear imperfections: sextupole (1/3-integer) and octupole (1/4-integer)

…these terms include harmonic functions of ϕ, driving resonances

- Many mathematical tools exist to help understand such dynamics:
  - the Hamiltonian
  - Taylor maps and Lie transformations
  - Perturbation theory, normal form analysis, etc.
- However, nowadays we can "cheat" and solve the equation of motion by integrating it numerically to gain insight:
  - one turn map + non-linear thin lens kick (sextupole and/or octupole)

Non-linear beam dynamics (2)

- We can learn a lot by tracking a few particles over a few 100 turns:

one-turn map, function of the machine tune

thin lens approximation of a sextupole and octupole at the same location in the ring

$$\begin{pmatrix} \bar{X} \\ \bar{X}' \end{pmatrix}_{n+1} = R(2\pi Q)\begin{pmatrix} \bar{X} \\ \bar{X}' + K_2\bar{X}^2 + K_3\bar{X}^3 \end{pmatrix}_n$$

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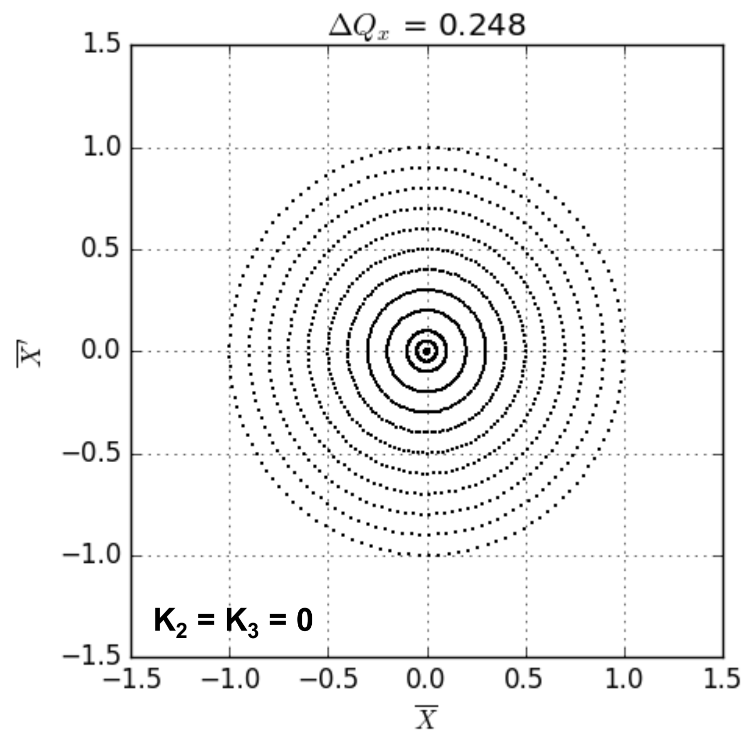
- Example:
  - Crossing **1/4 - integer resonance**
    - i.e. Q_x = integer + 0.25
  - **Sextupole OFF and octupole OFF:**
    - **$K_2 = K_3 = 0$**
  - Ramping tune from below resonance:
    - ΔQ_x = 0.248 to 0.252
  - 12 particles, 1000 turns



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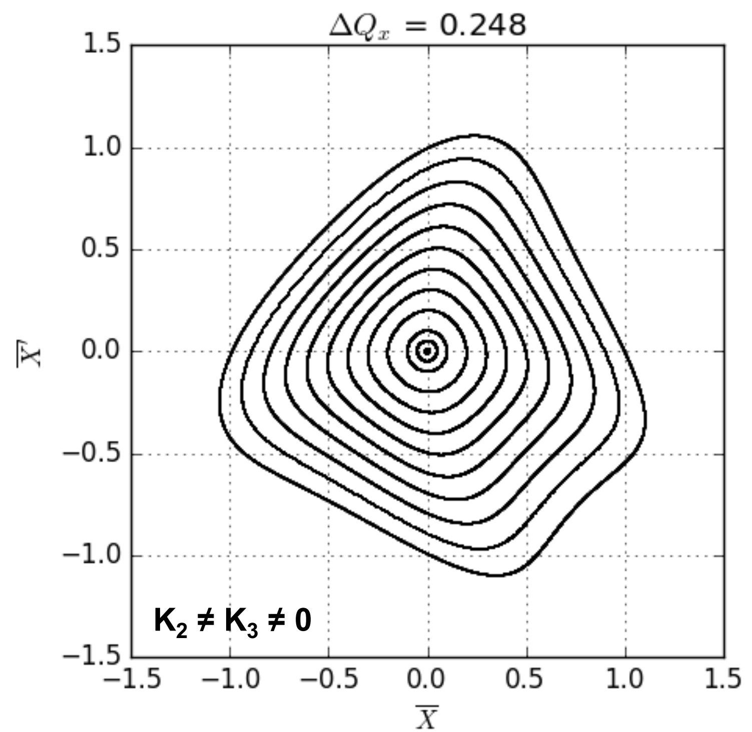

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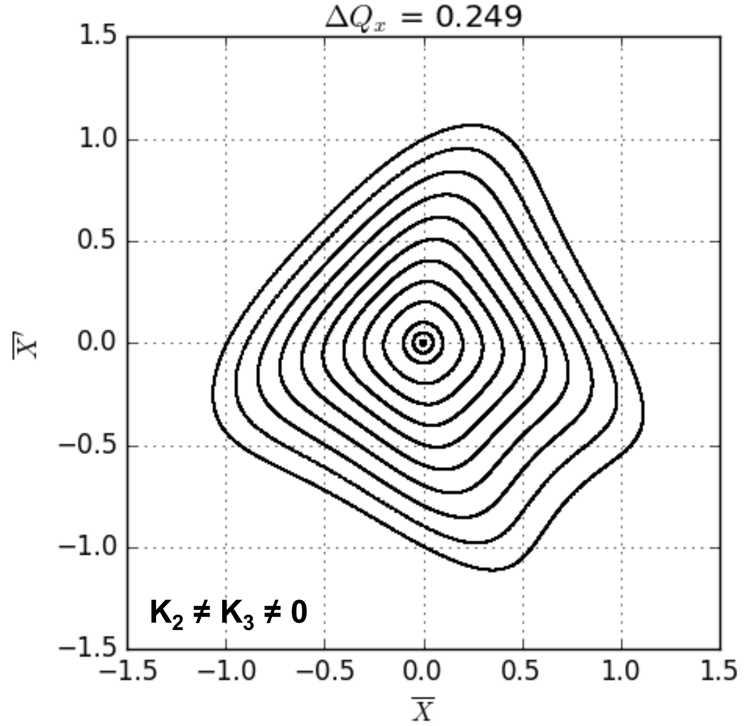

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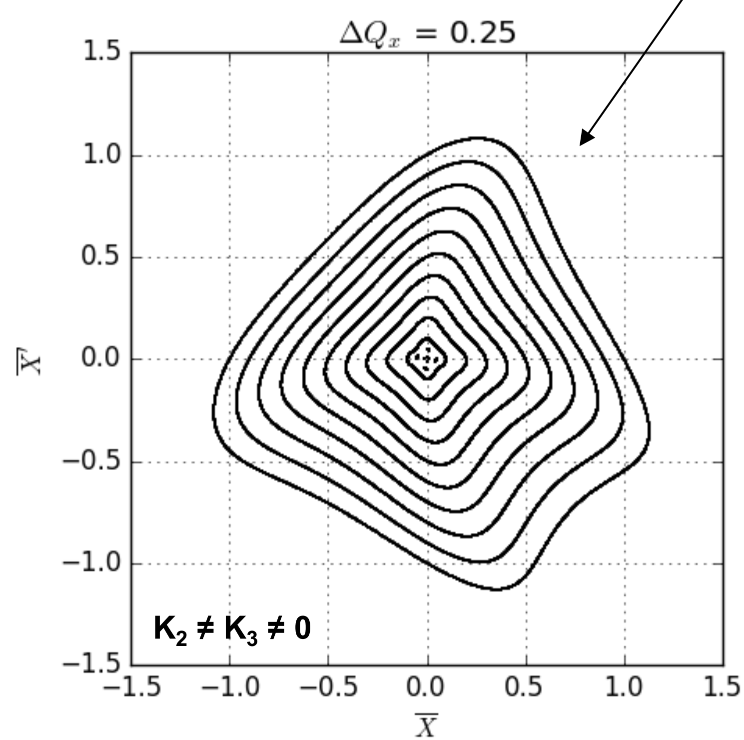

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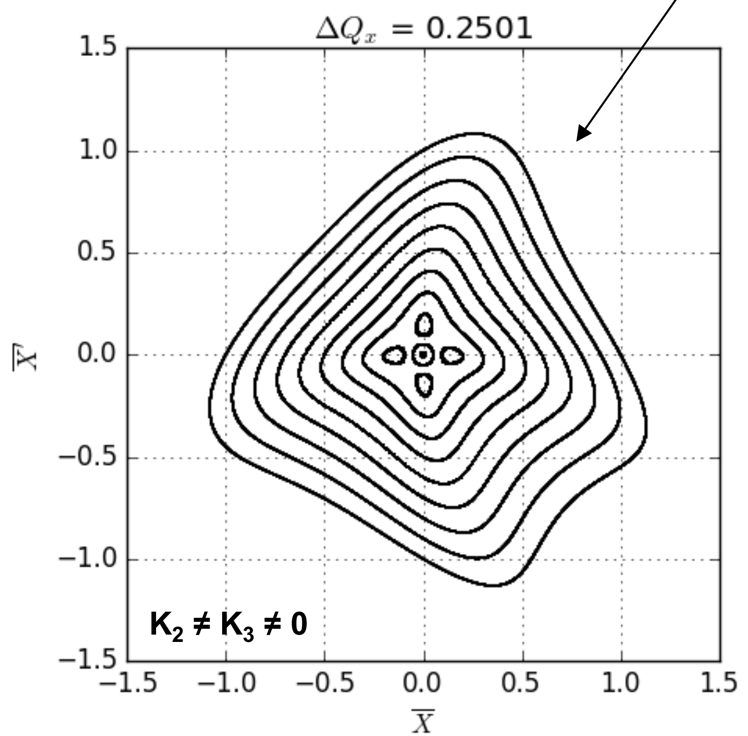

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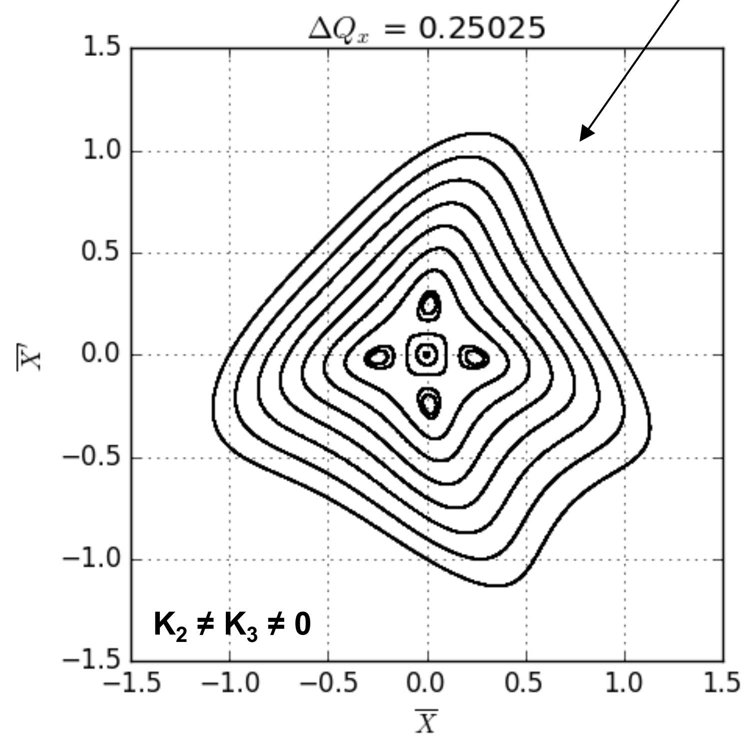

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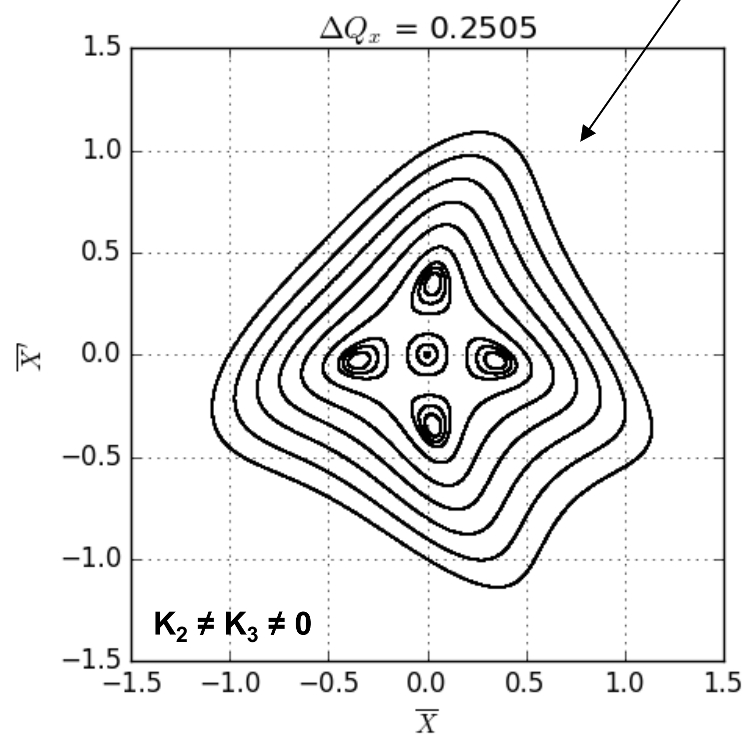

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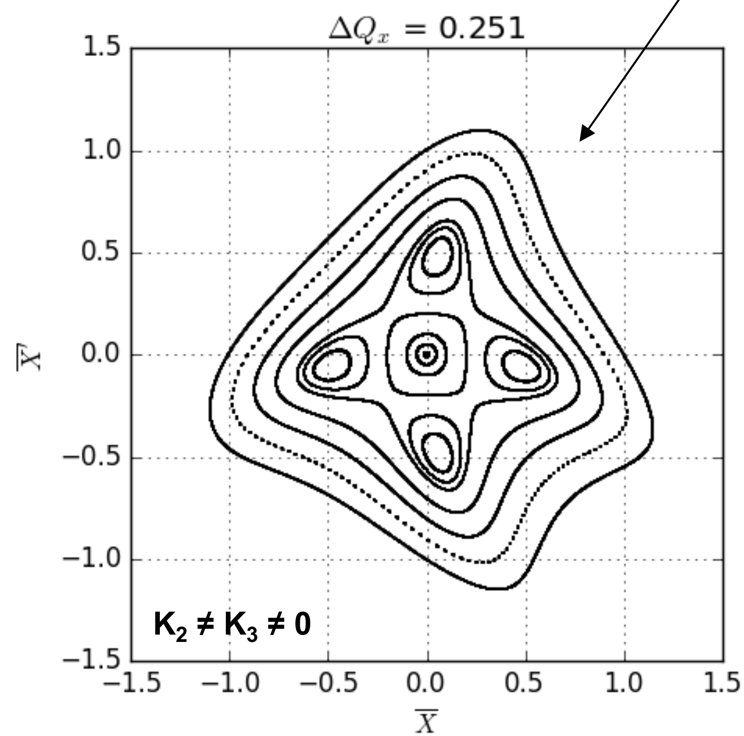

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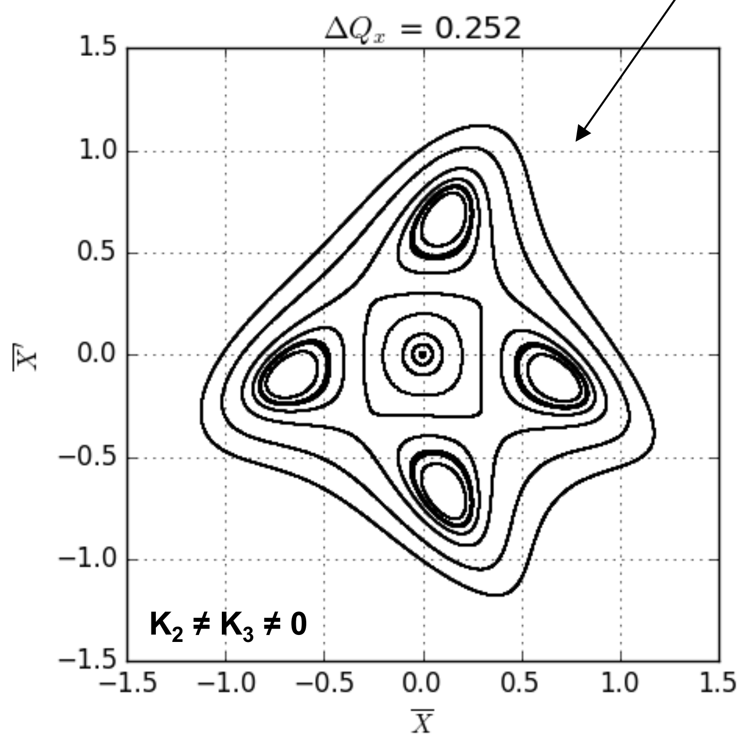
one-turn map, function of the machine tune

thin lens approximation of a sextupole and octupole at the same location in the ring

$$\begin{pmatrix} \bar{X} \\ \bar{X}' \end{pmatrix}_{n+1} = R(2\pi Q)\begin{pmatrix} \bar{X} \\ \bar{X}' + K_2\bar{X}^2 + K_3\bar{X}^3 \end{pmatrix}_n$$

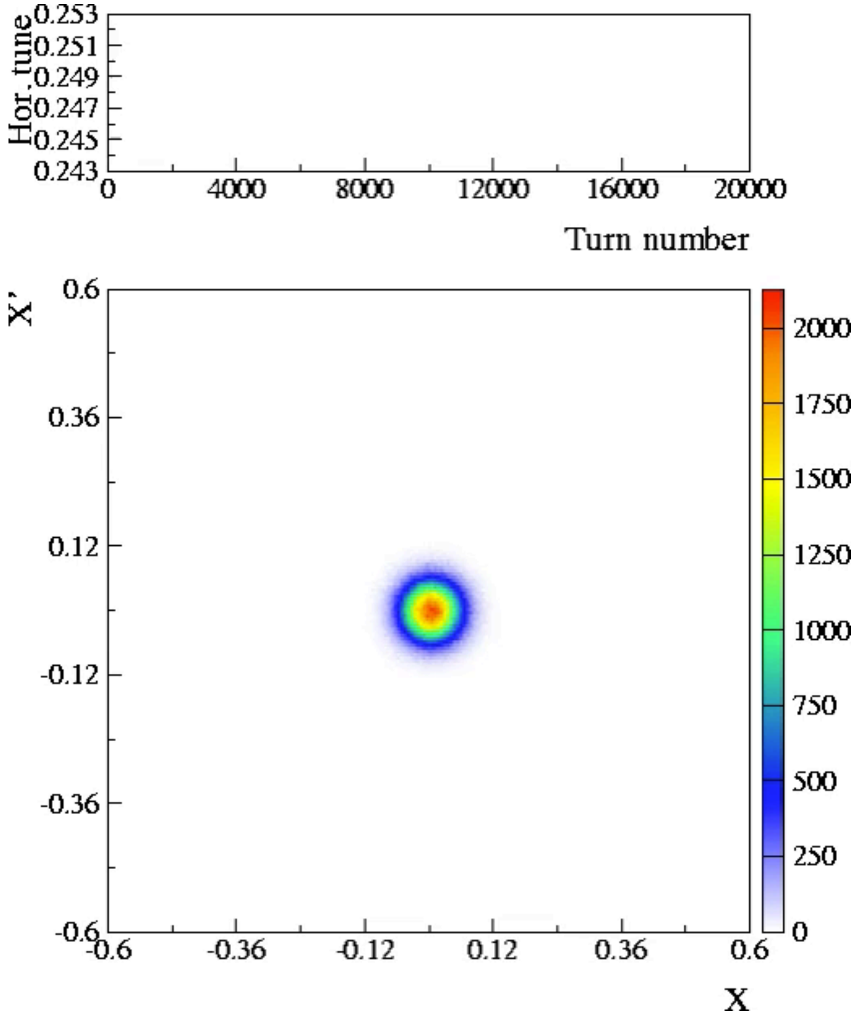
Ratio of K_2/K_3 can be used to tailor the phase space and size of the islands

- Example:
  - Crossing **1/4 - integer resonance**
    - i.e. Q_x = integer + 0.25
  - **Sextupole ON and octupole ON:**
    - $\mathbf{K_2 \neq K_3 \neq 0}$
  - Ramping tune from below resonance:
    - ΔQ_x = 0.248 to 0.252
  - 12 particles, 1000 turns



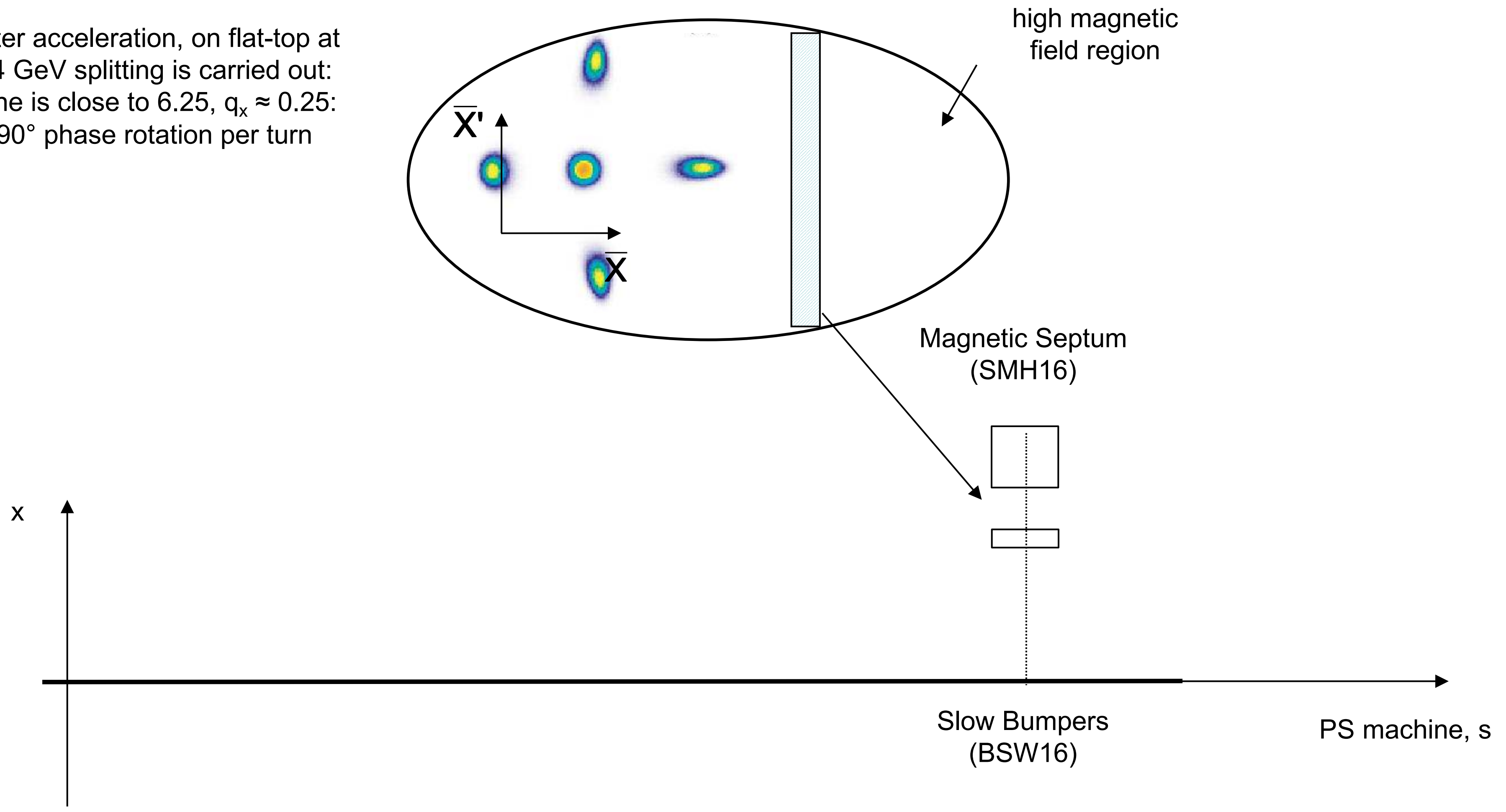
Multi-turn extraction suitable for the PS

- For an n^{th} order stable resonance $n + 1$ islands will be created:
  - the 4^{th} order resonance works for the CERN PS scenario:



MTE: operational implementation

After acceleration, on flat-top at 14 GeV splitting is carried out: tune is close to 6.25, $q_x \approx 0.25$: 90° phase rotation per turn



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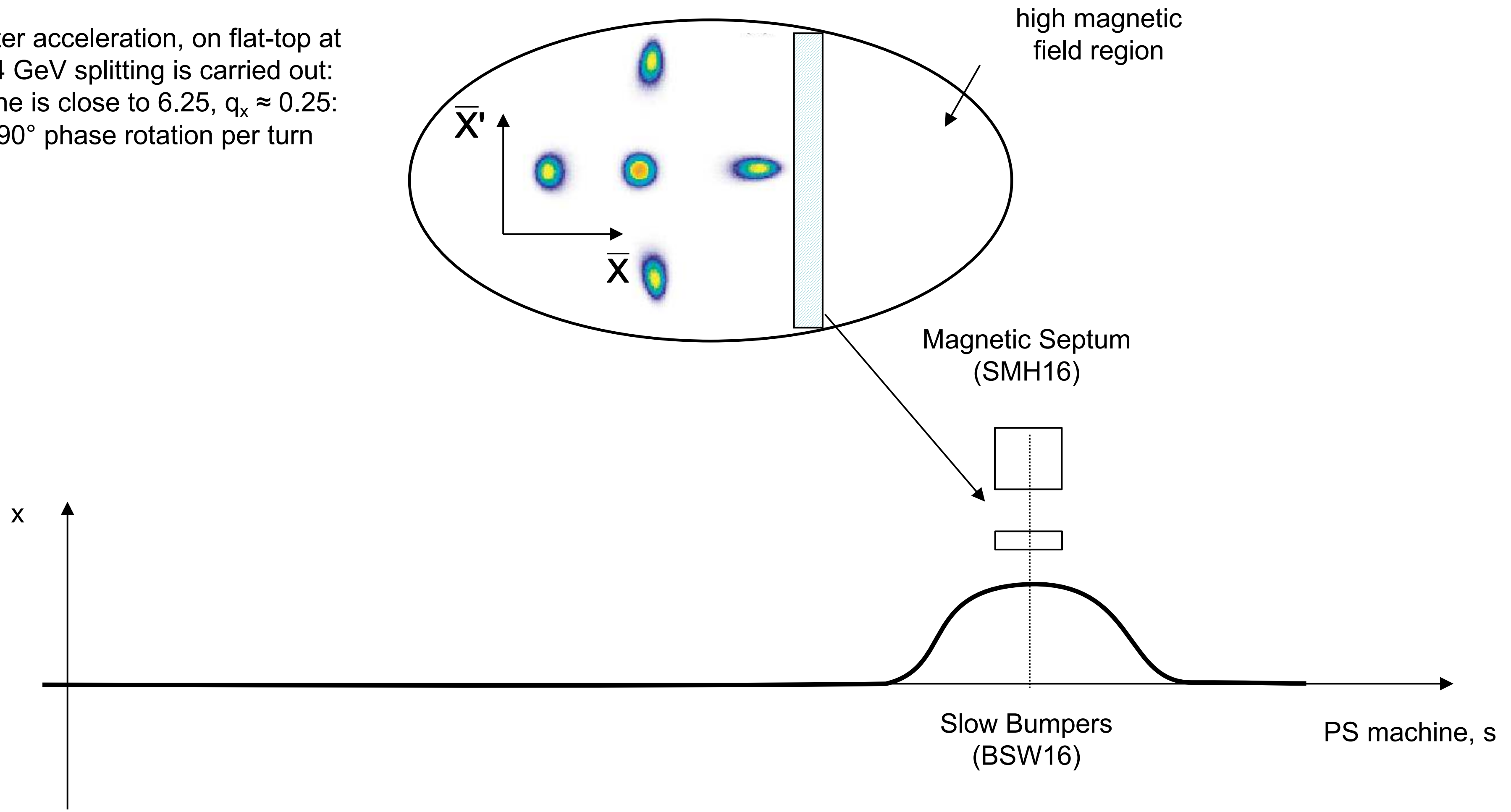

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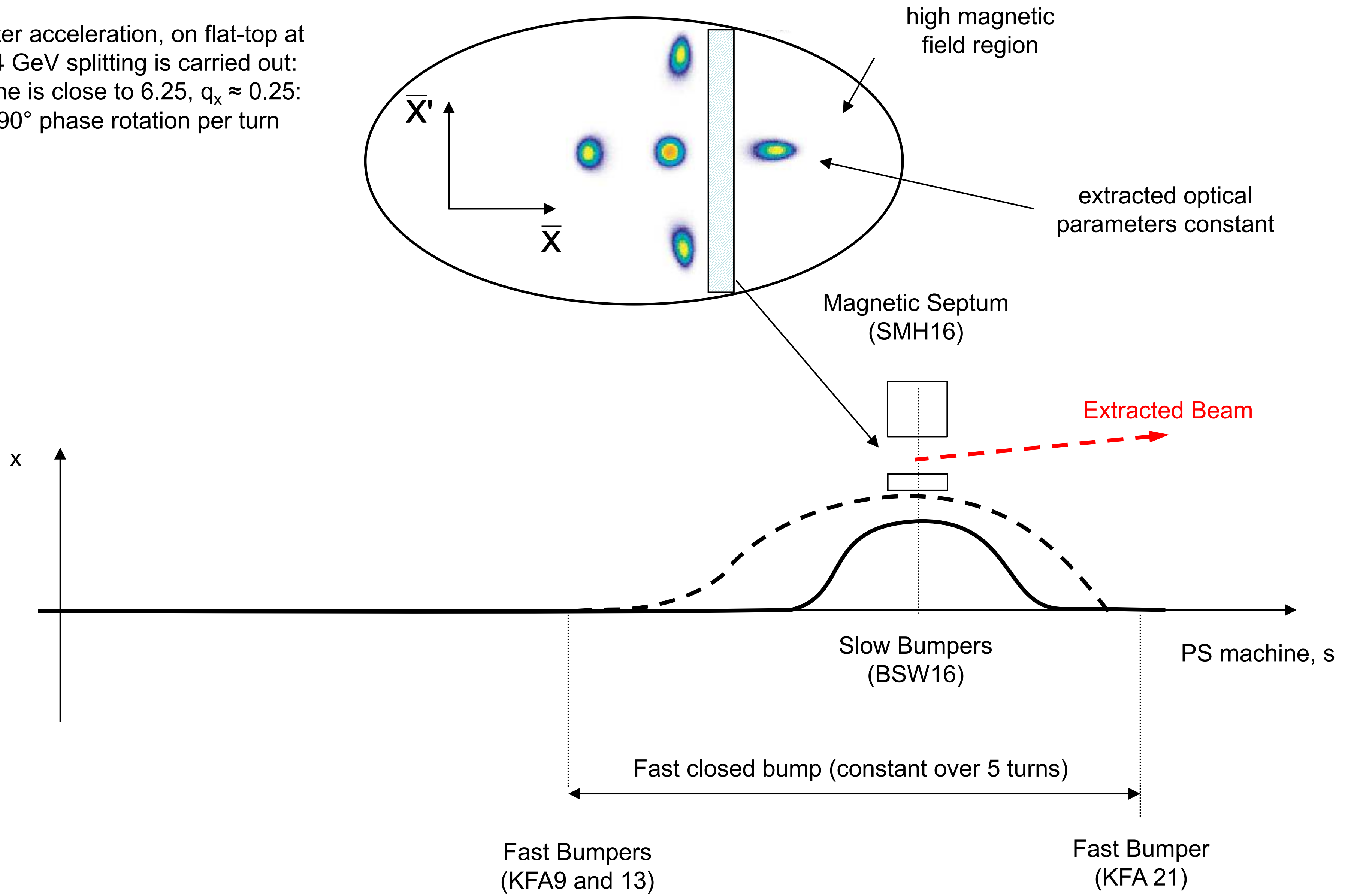

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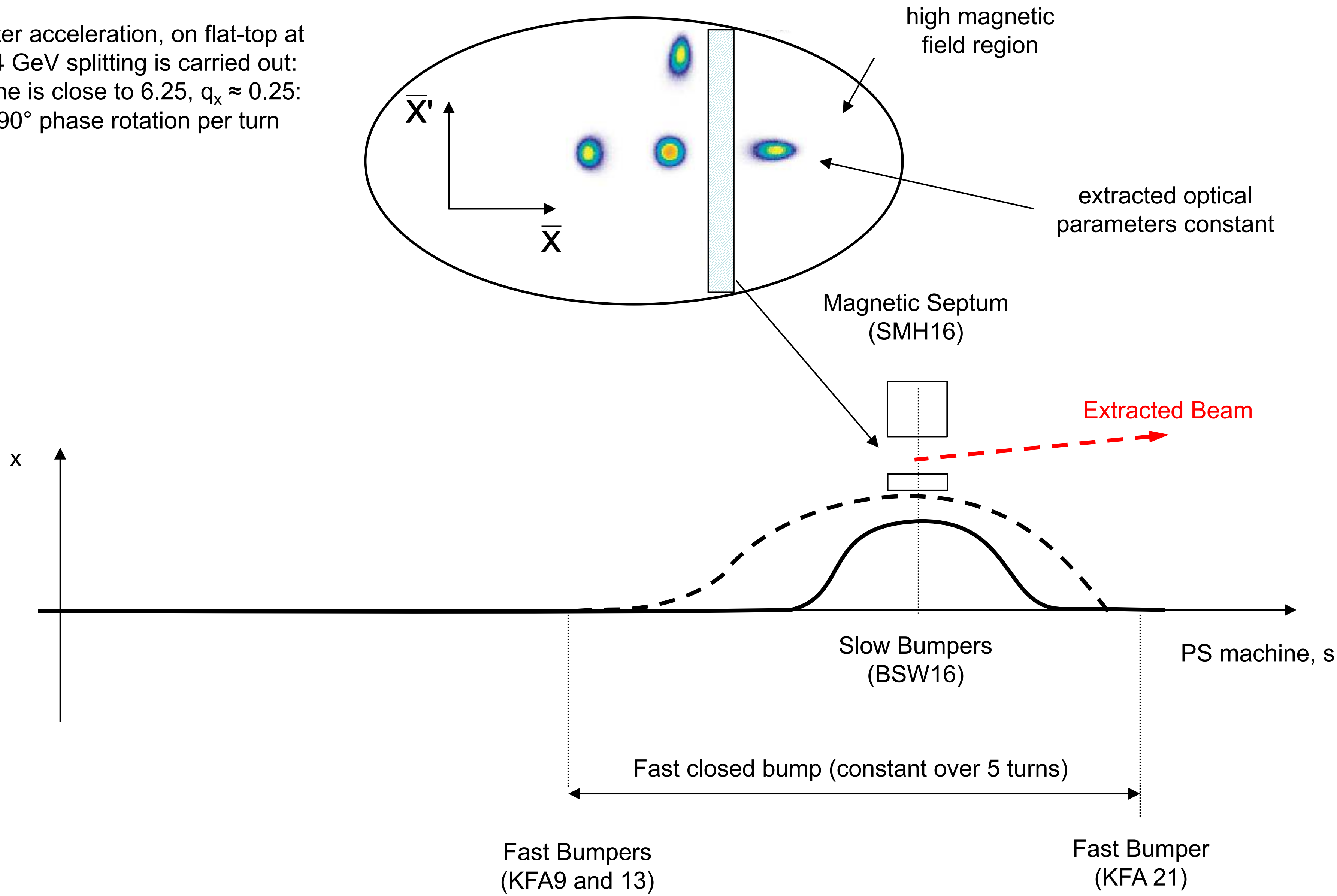

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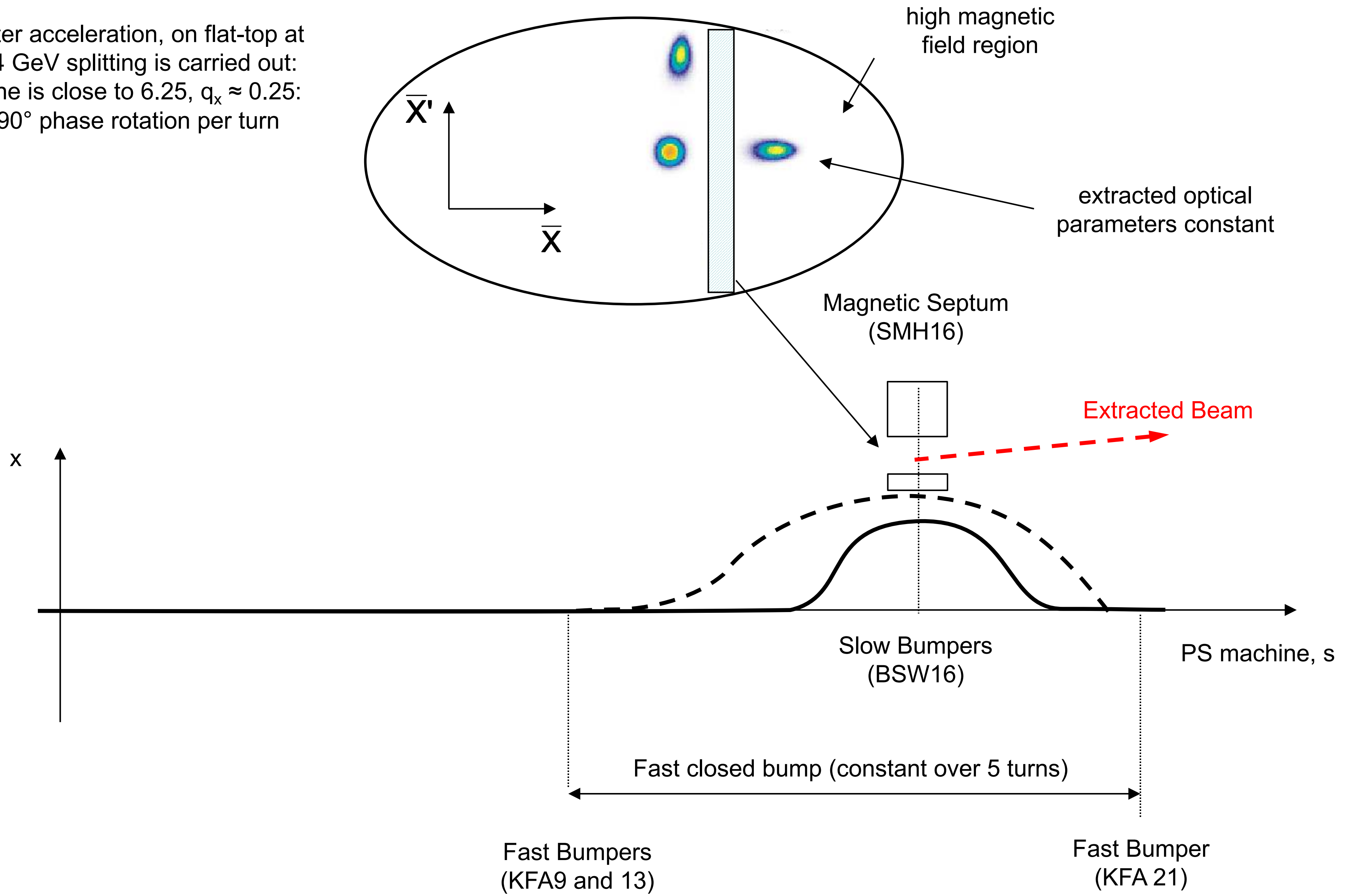

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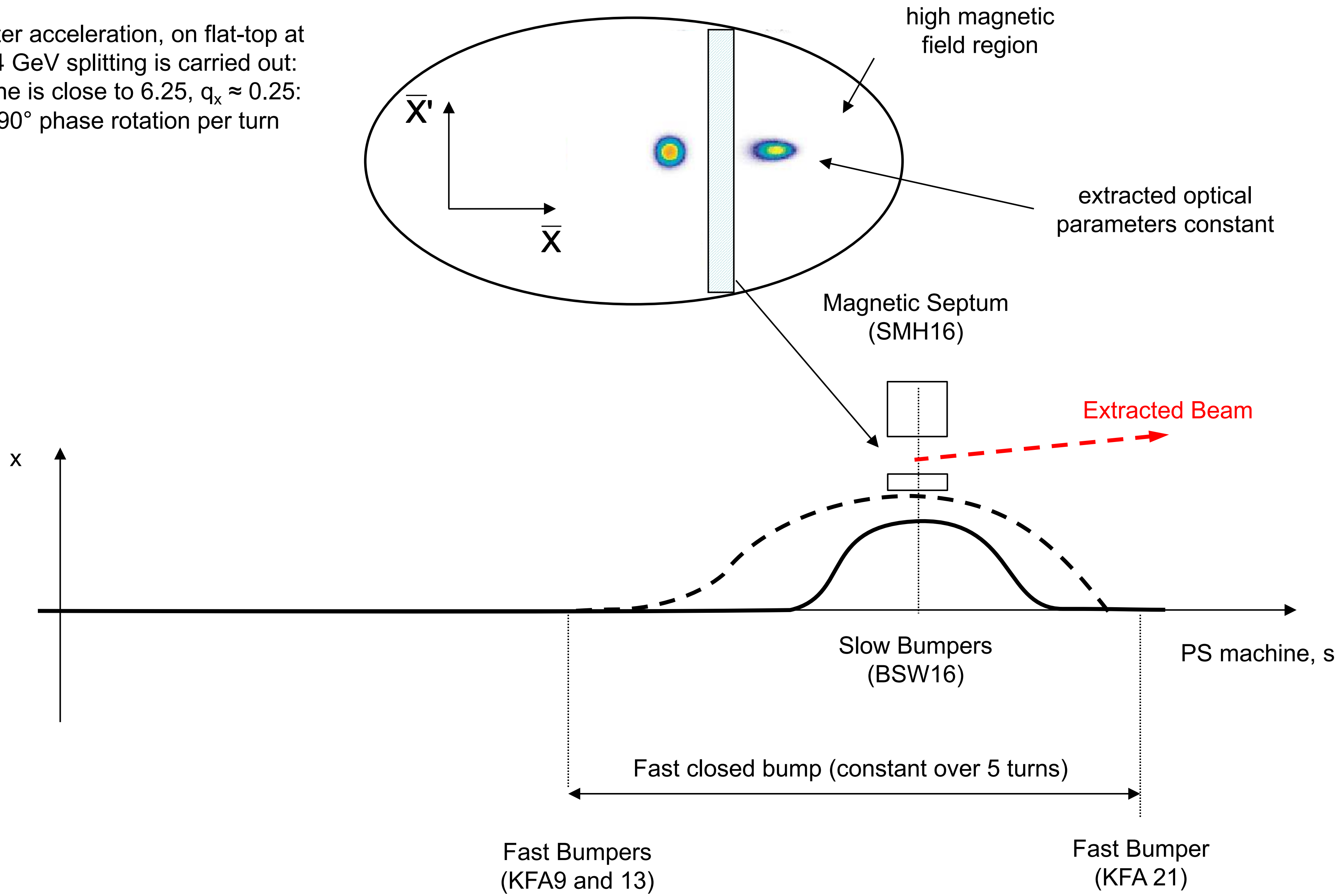

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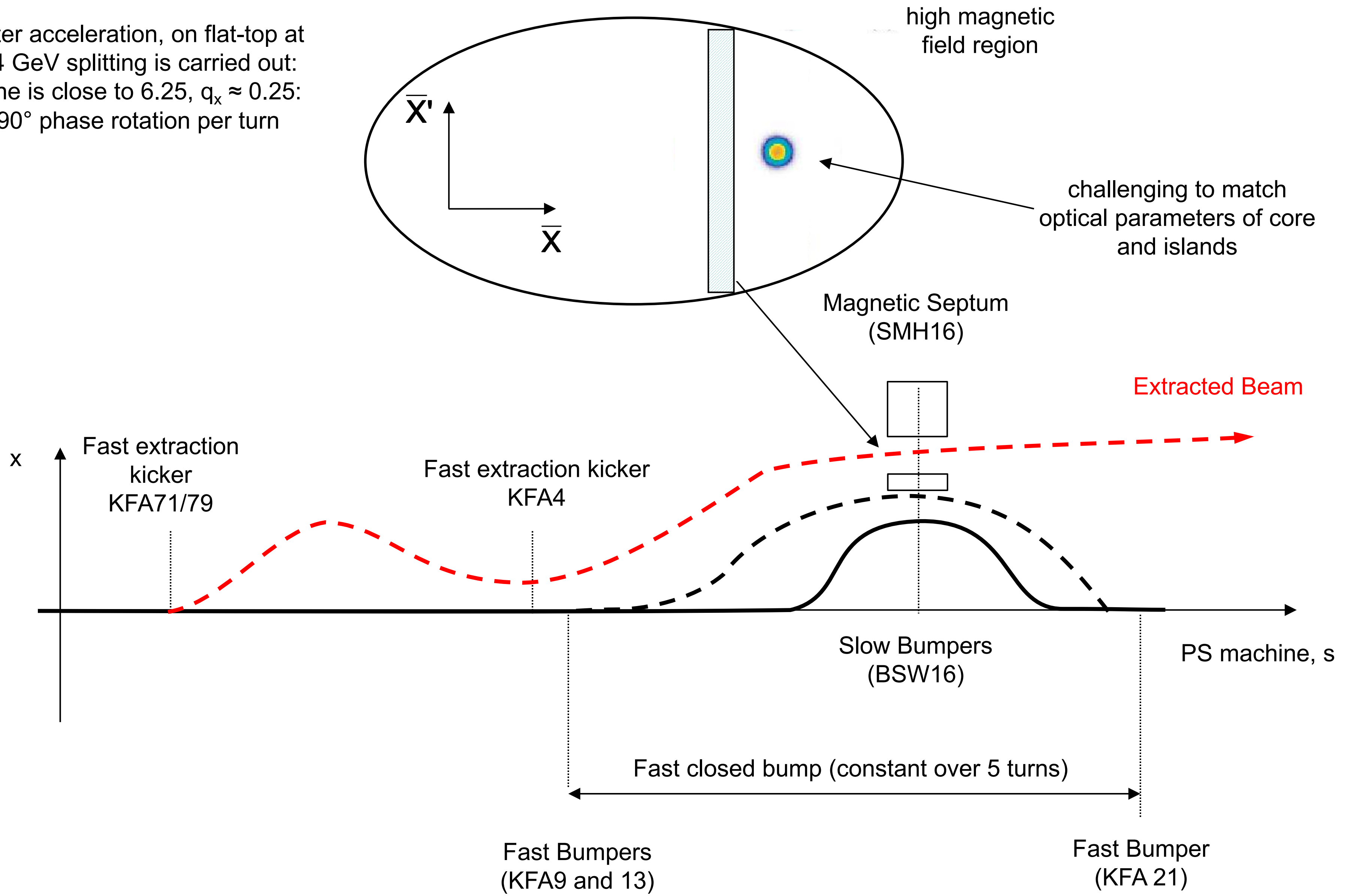
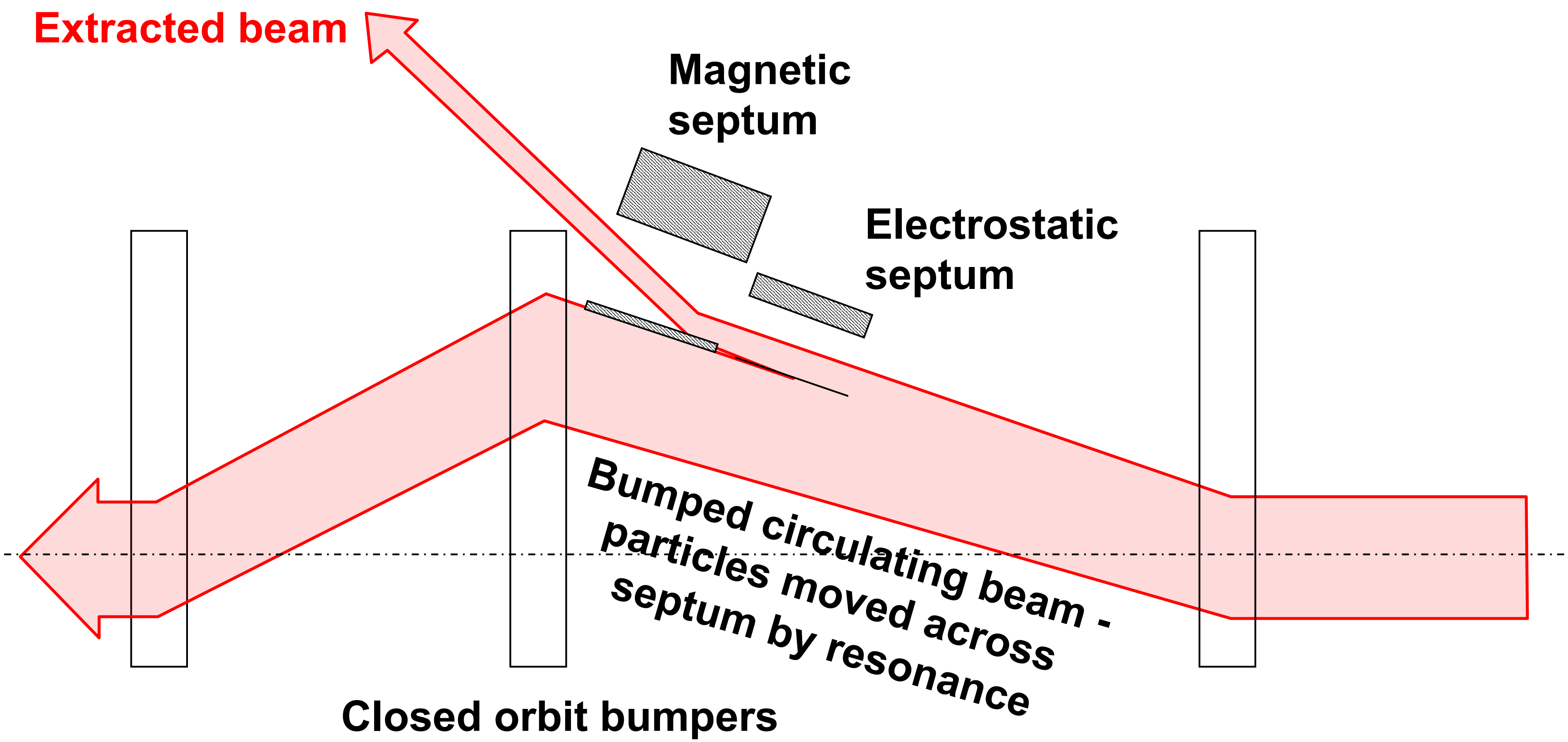


Resonant multi-turn extraction

Non-linear fields excite resonances that drive the beam slowly across the septum

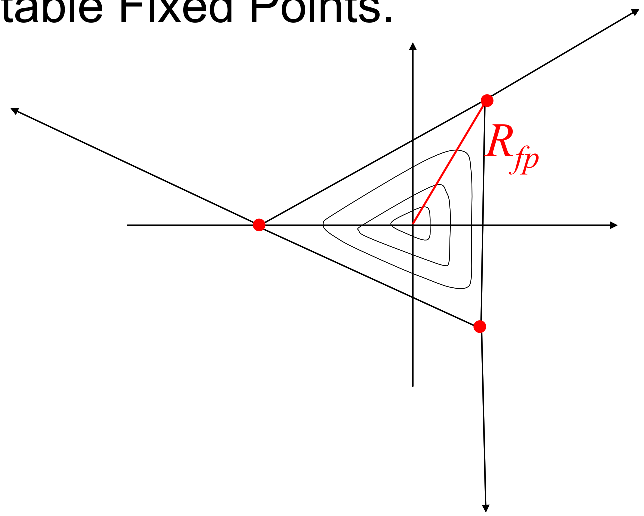


- Slow bumpers move the beam near the septum
- Tune adjusted close to n^{th} order betatron resonance
- Multipole magnets excited to define stable area in phase space, size depends on $\Delta Q = Q - Q_r$

Resonant multi-turn extraction

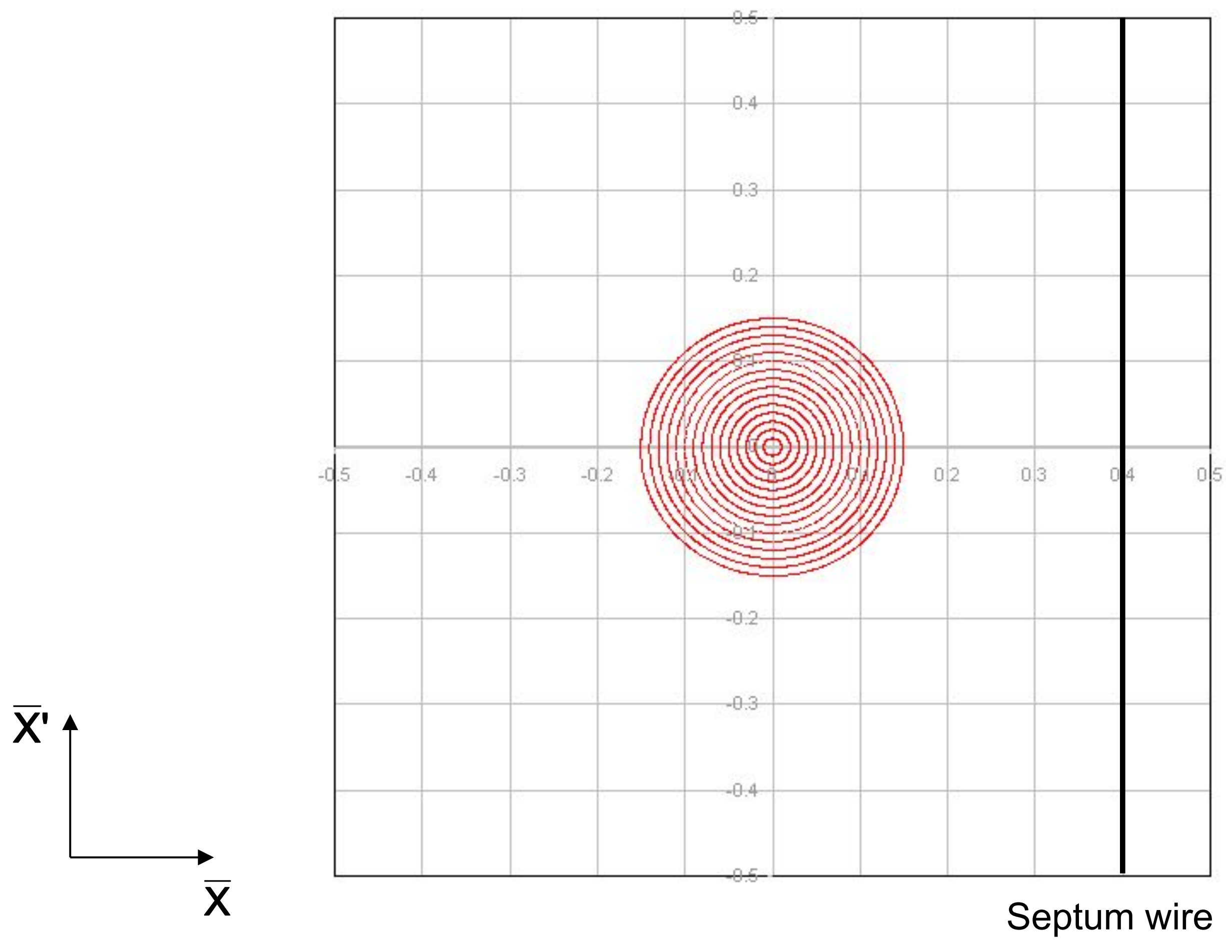
- Resonances in synchrotrons:
  - See CERN Accelerator school *lectures by A. Wolski*
  - Third-integer resonance: sextupole fields distort the circular normalised phase space particle trajectories.
  - Stable area defined, delimited by unstable Fixed Points.

$$R_{fp}^{\;1/2} \propto \Delta Q \cdot \frac{1}{k_2}$$



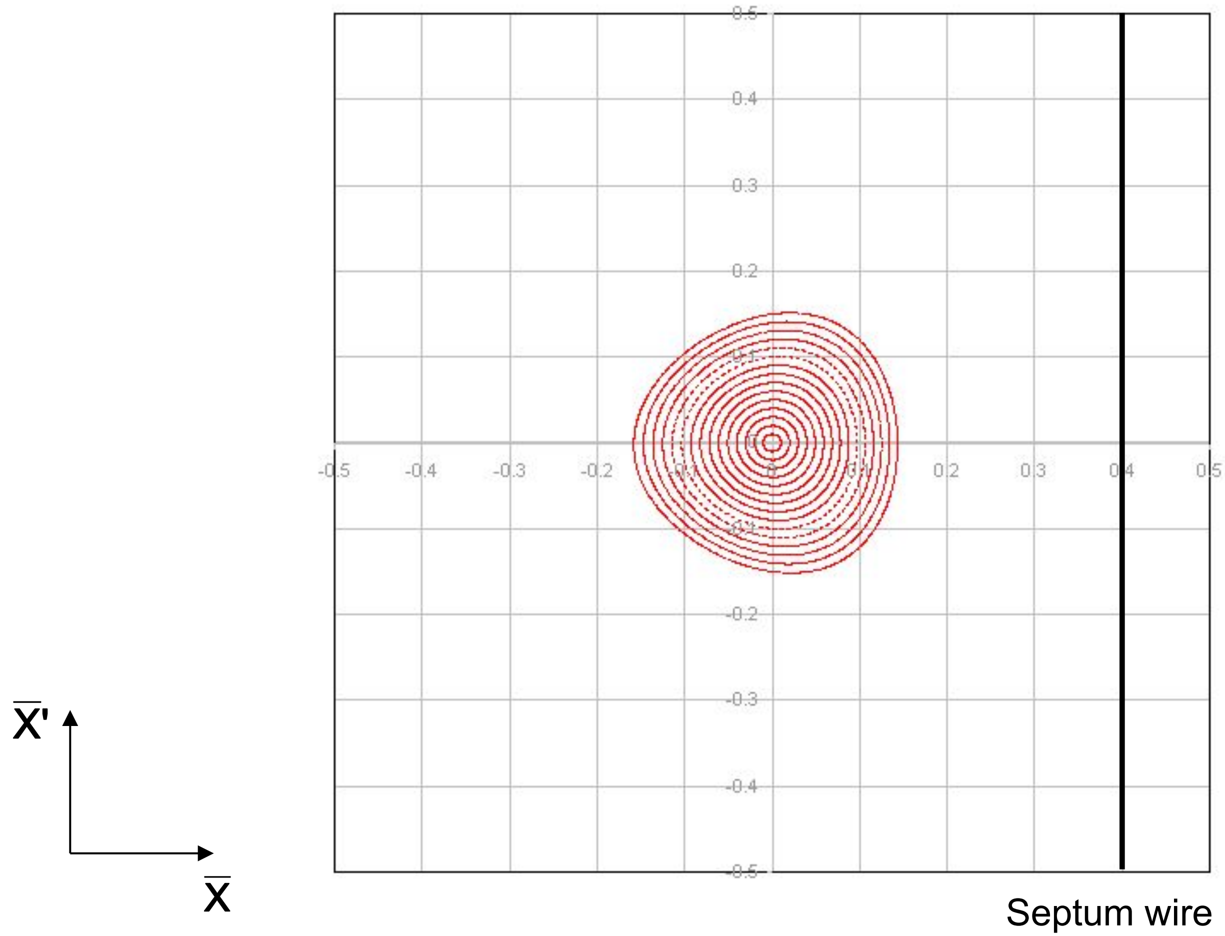
  - Sextupole magnets arranged to produce suitable phase space orientation of the stable triangle at thin electrostatic septum
  - Stable area can be reduced by…
    - Increasing the sextupole strength, or fixing the sextupole strength and scanning the machine tune through the tune spread of the beam
    - Large tune spread created with RF gymnastics (large momentum spread) and large chromaticity

Third-order resonant extraction



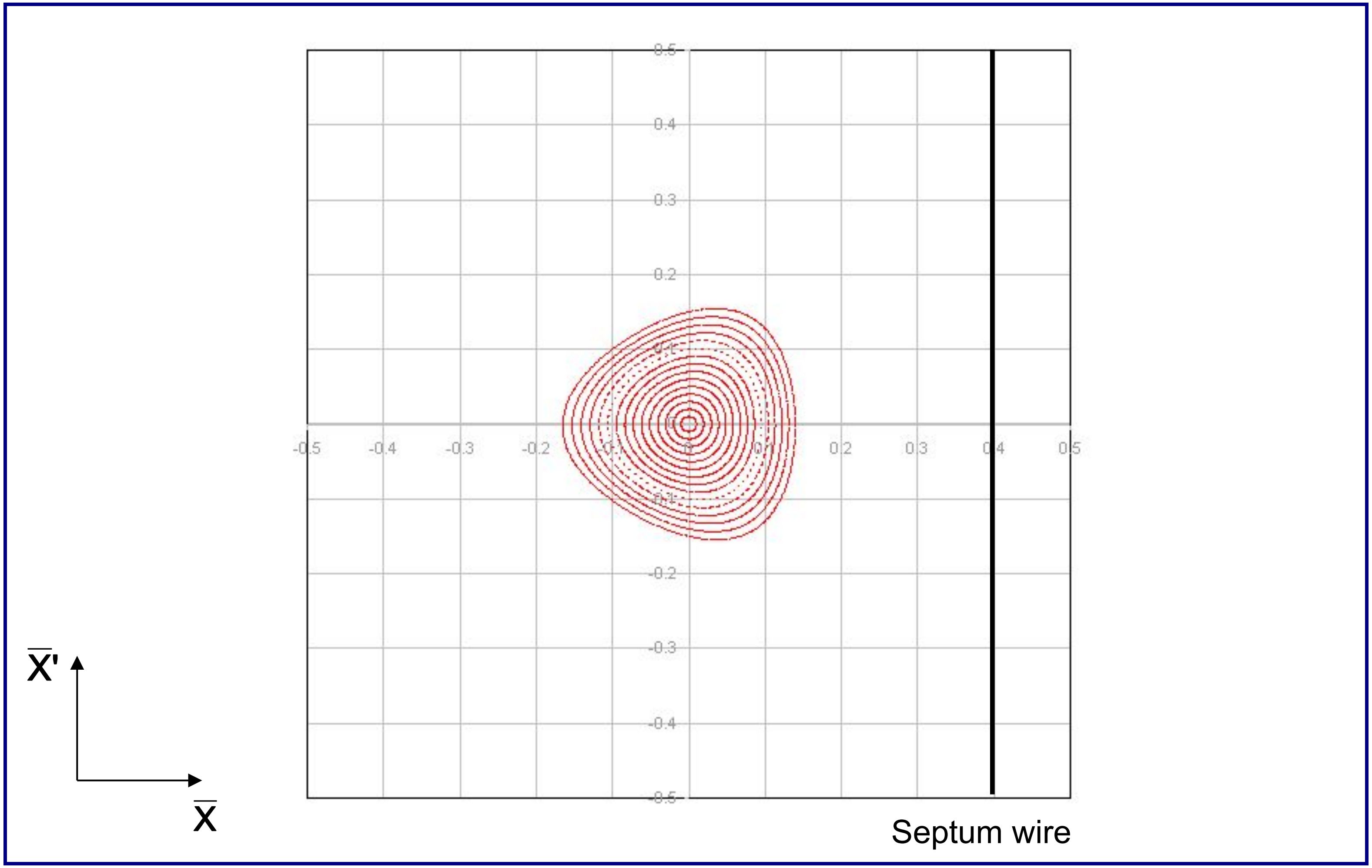
- Particles distributed on emittance contours
- ΔQ large – no phase space distortion

Third-order resonant extraction



- Sextupole magnets produce a triangular stable area in phase space
- ΔQ decreasing – phase space distortion for largest amplitudes

Third-order resonant extraction



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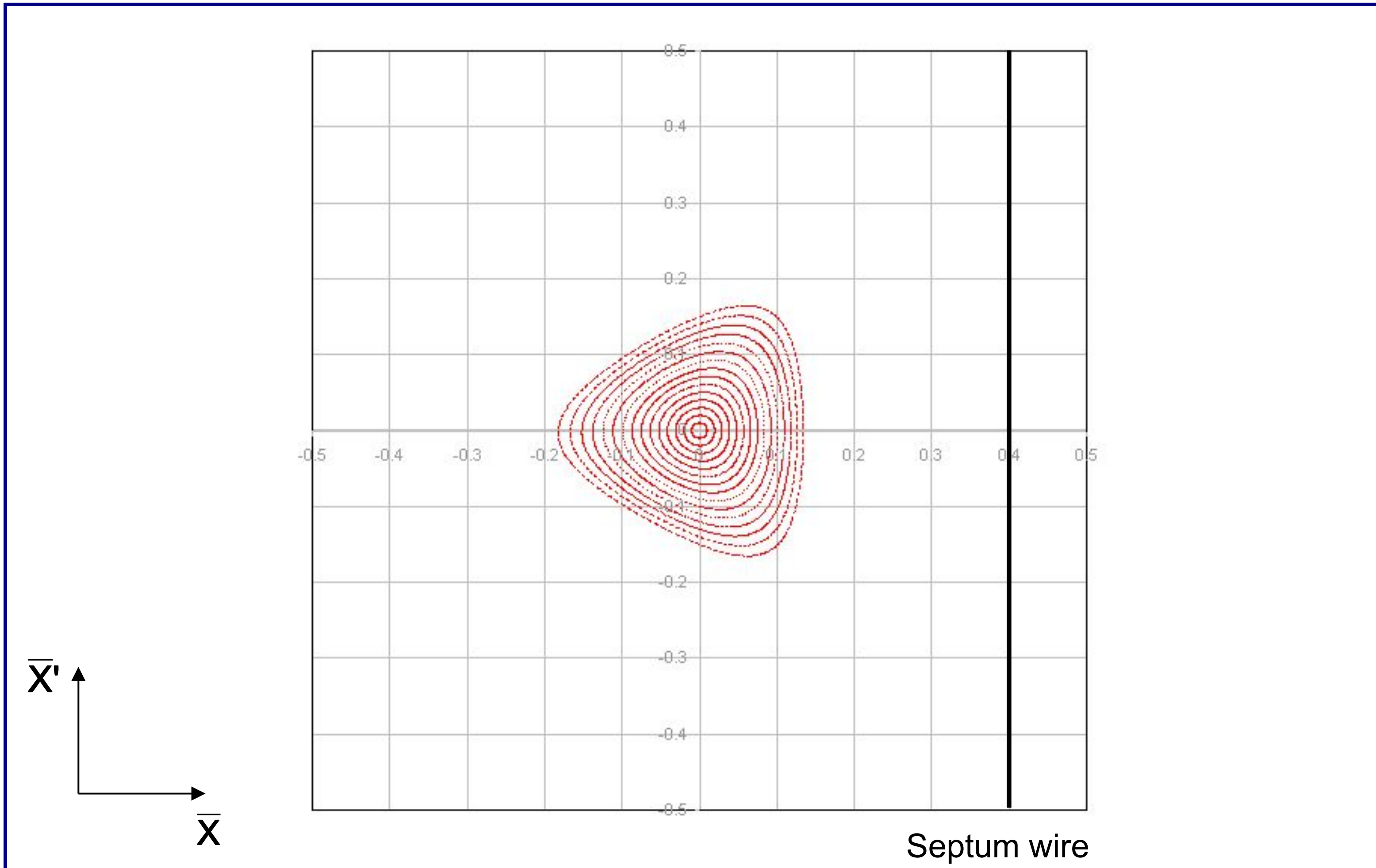

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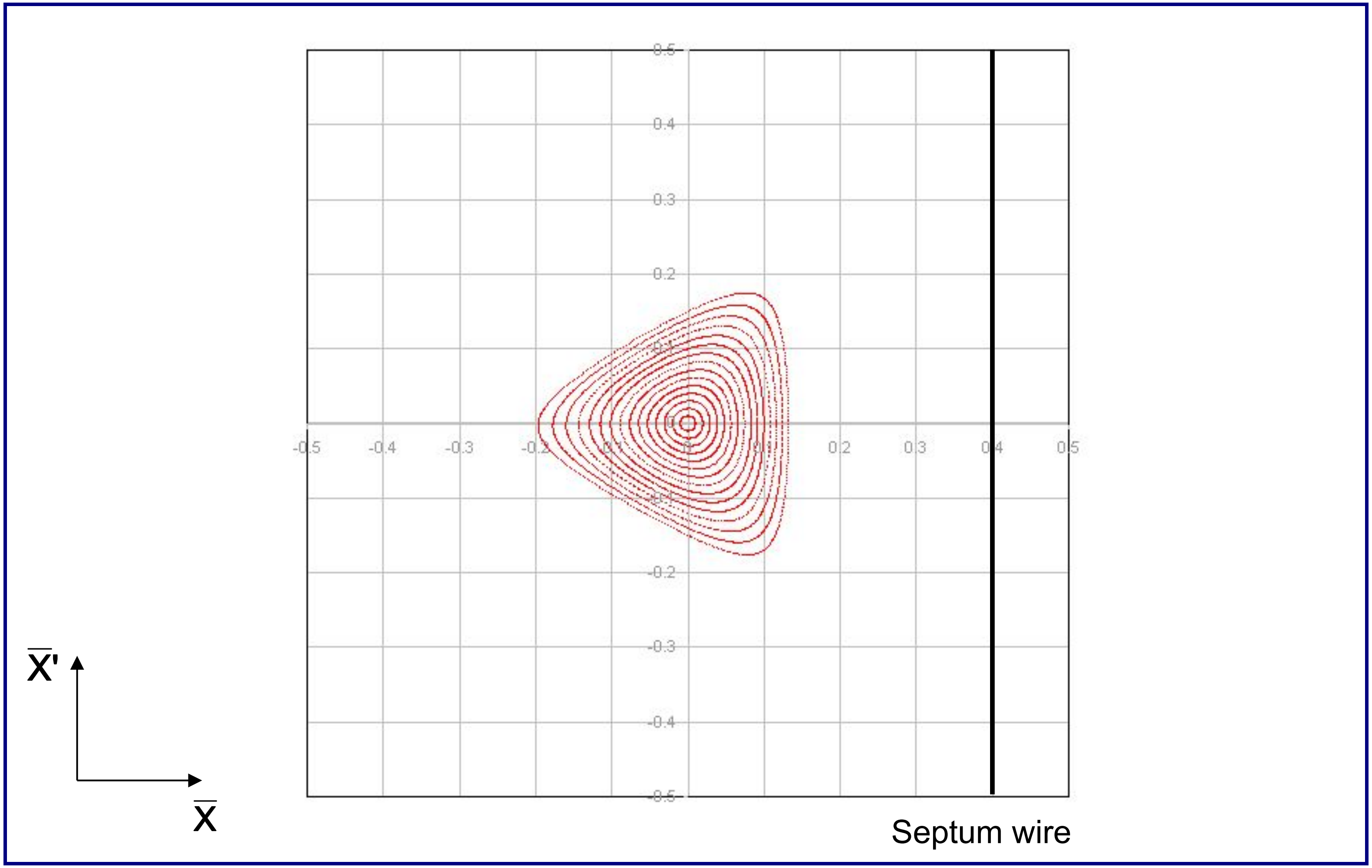

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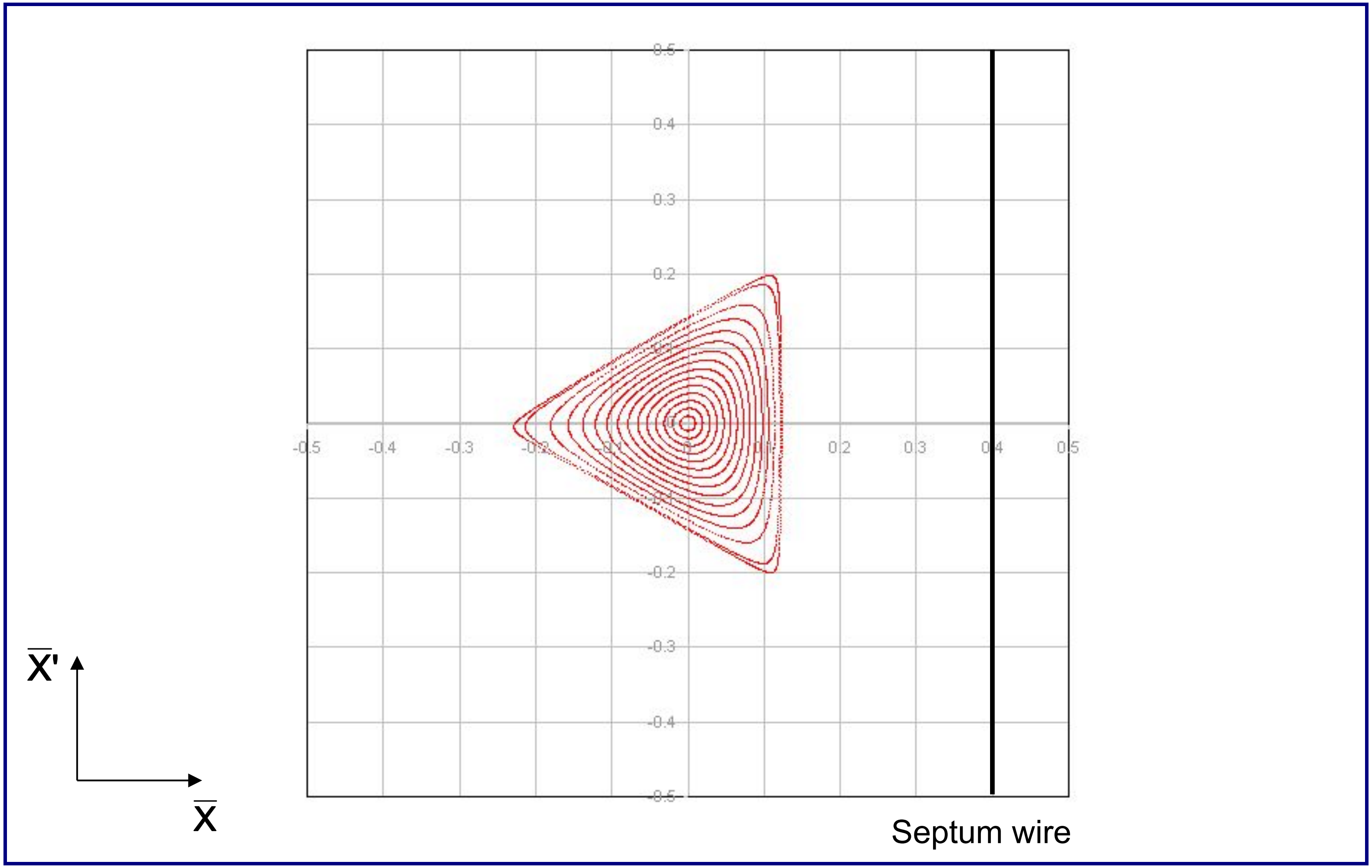
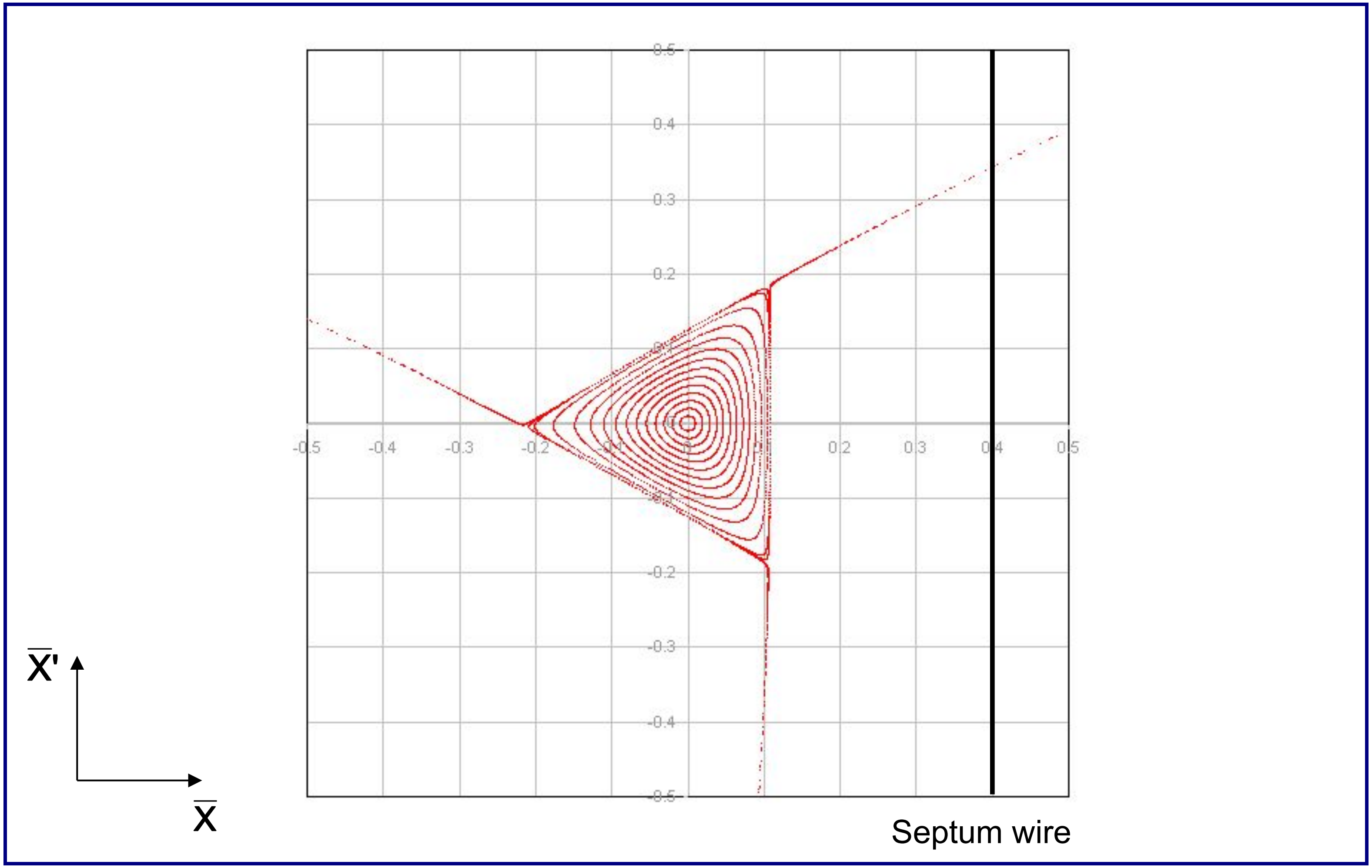


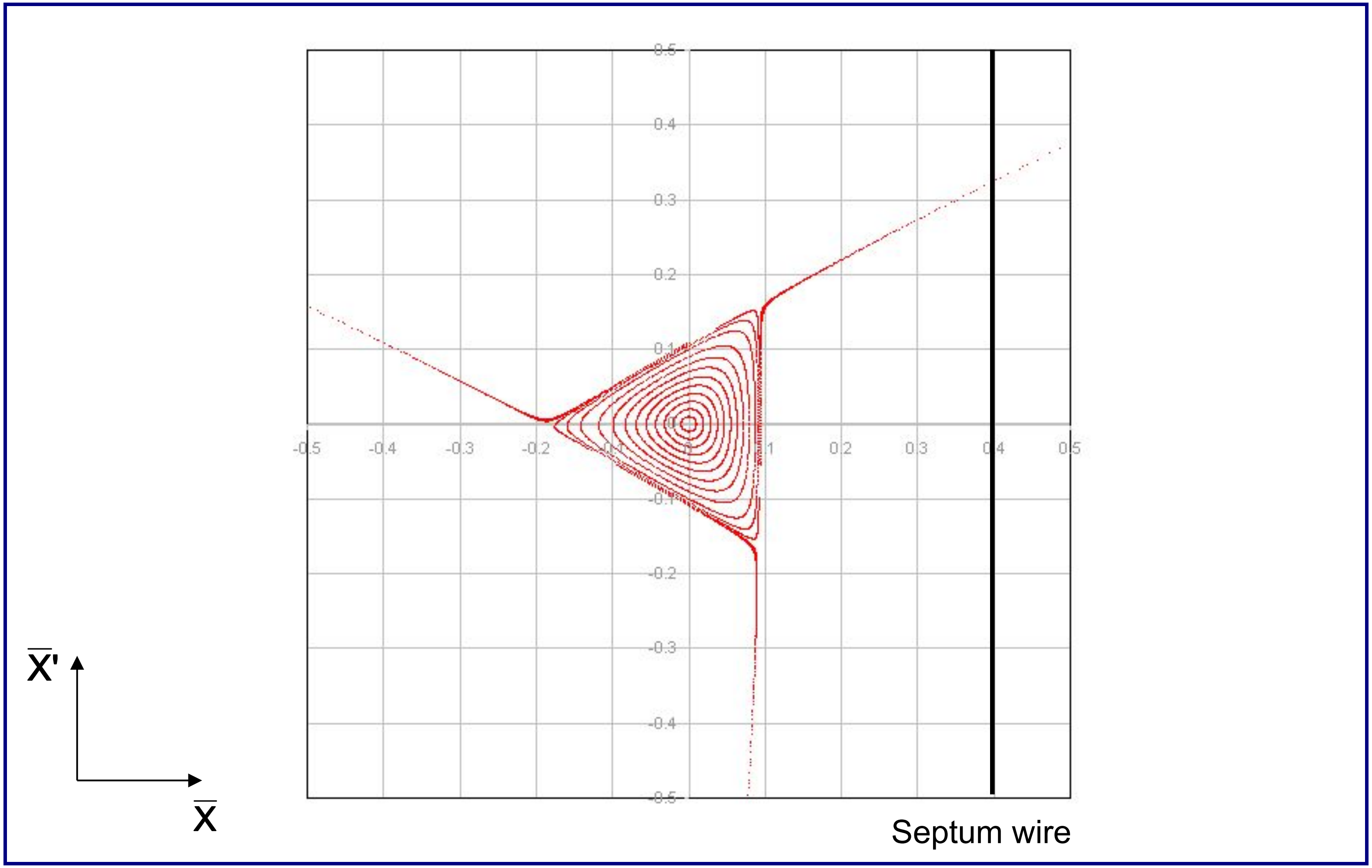
- Largest amplitude particle trajectories are significantly distorted
- Locations of fixed points discernable at extremities of phase space triangle

Third-order resonant extraction



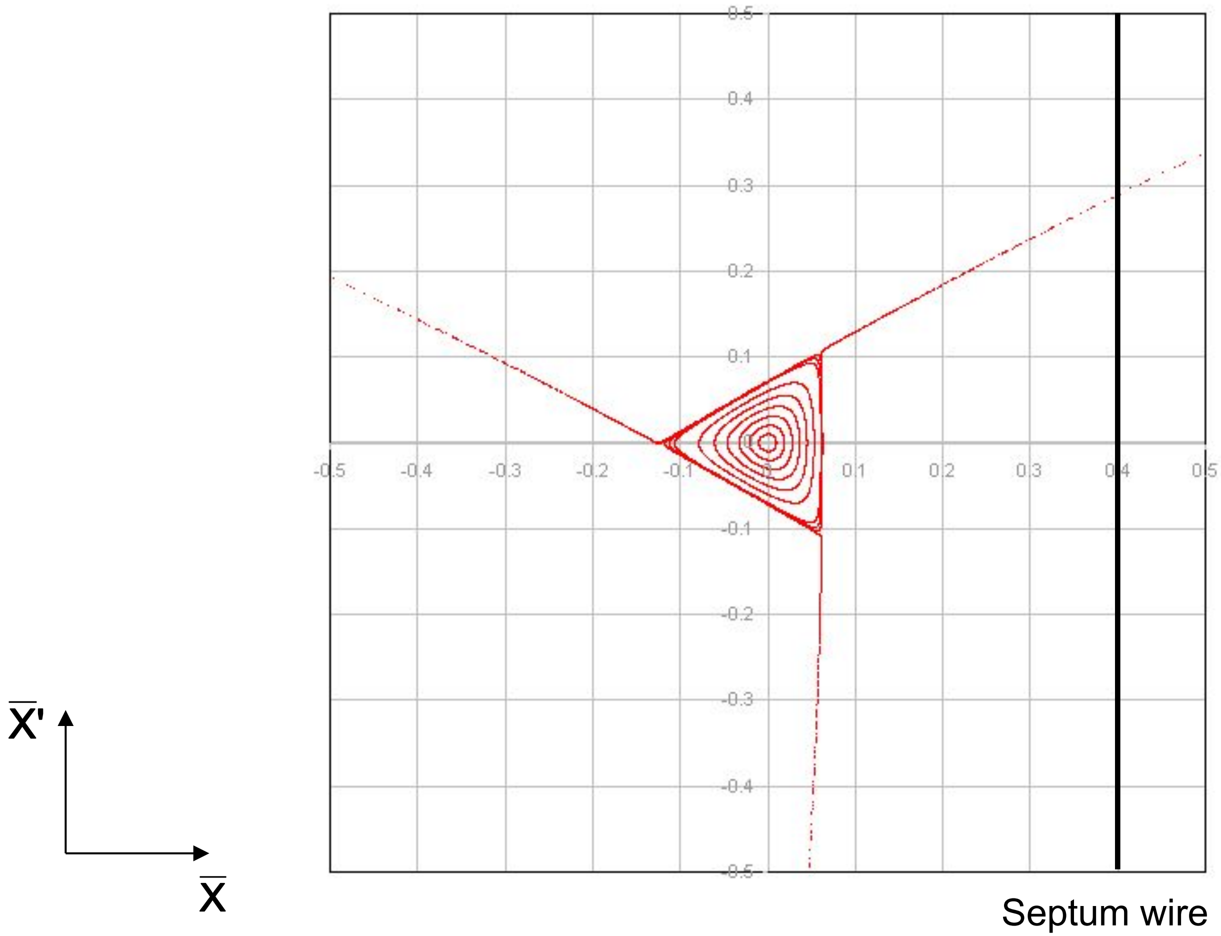
- ΔQ small enough that largest amplitude particle trajectories are unstable
- Unstable particles follow separatrix branches as they increase in amplitude

Third-order resonant extraction



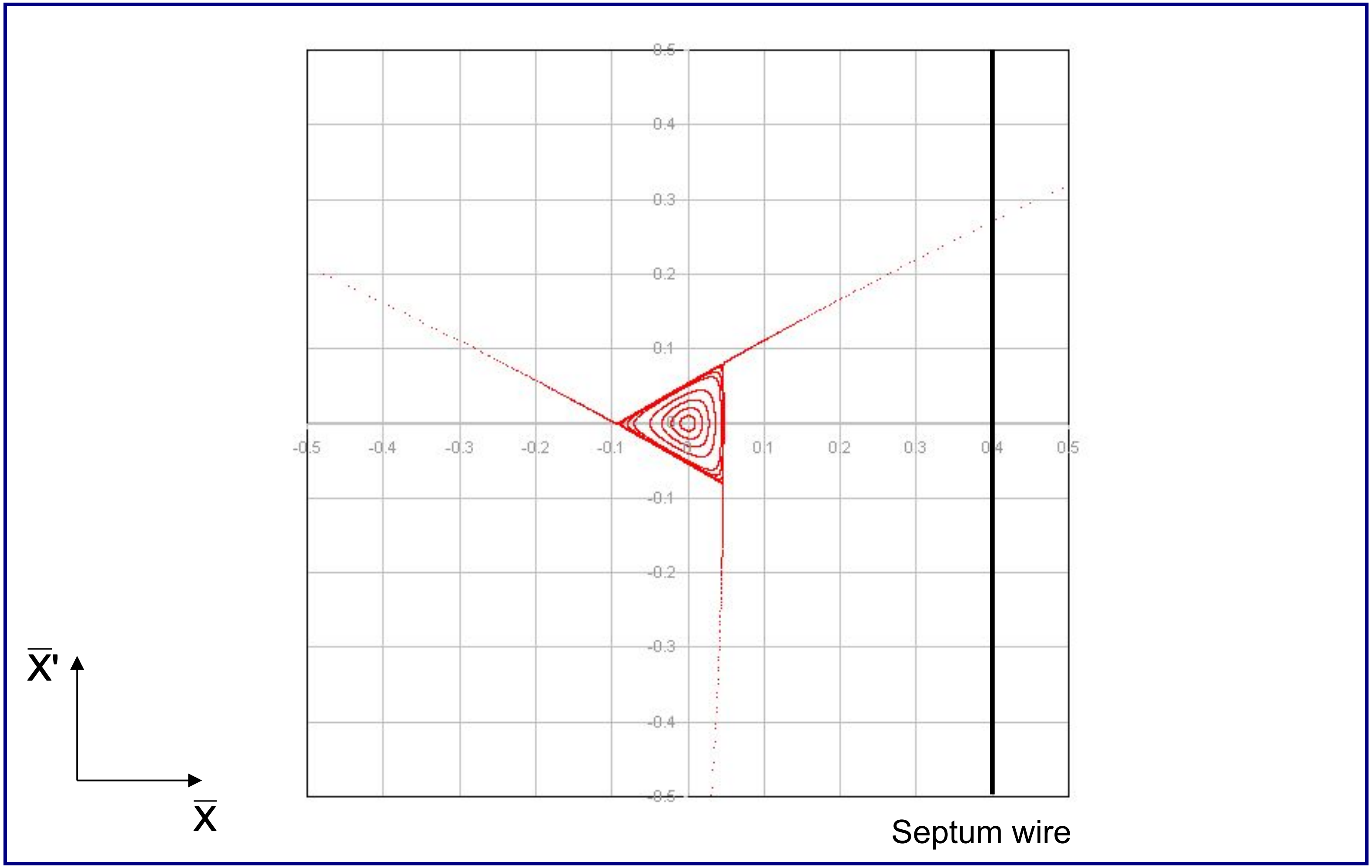
- Stable area shrinks as ΔQ becomes smaller

Third-order resonant extraction



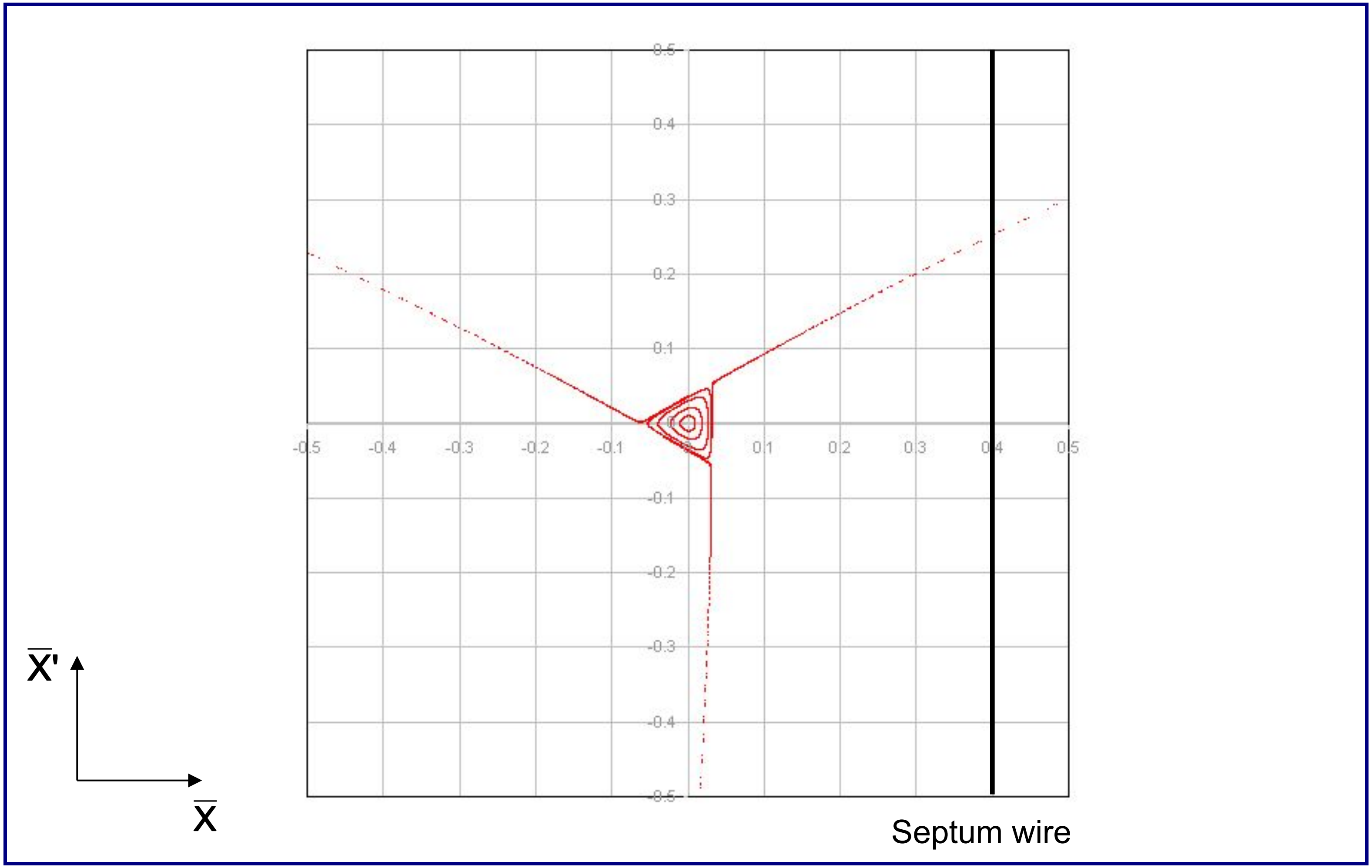
- Separatrix position in phase space shifts as the stable area shrinks

Third-order resonant extraction



- As the stable area shrinks, the circulating beam intensity drops since particles are being continuously extracted

Third-order resonant extraction



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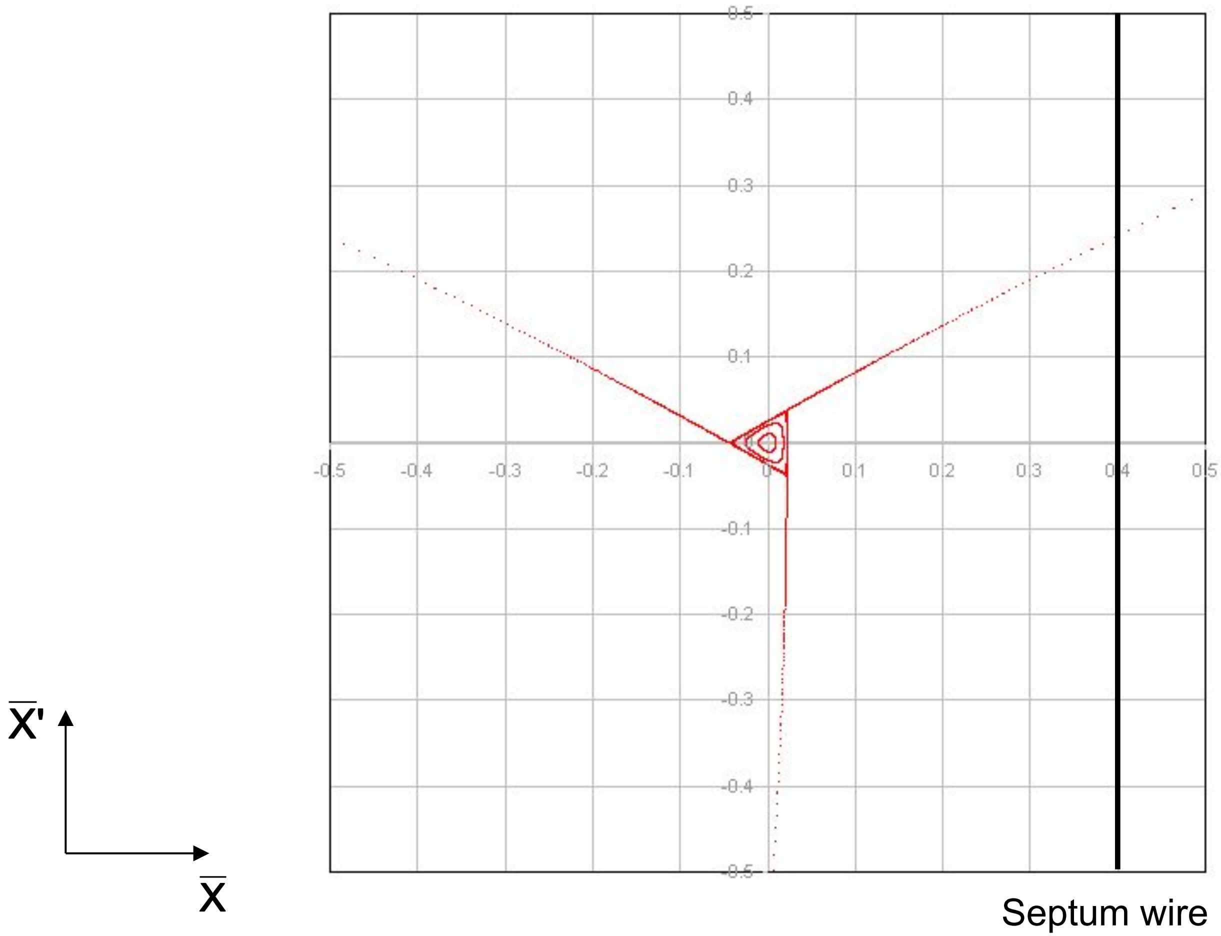

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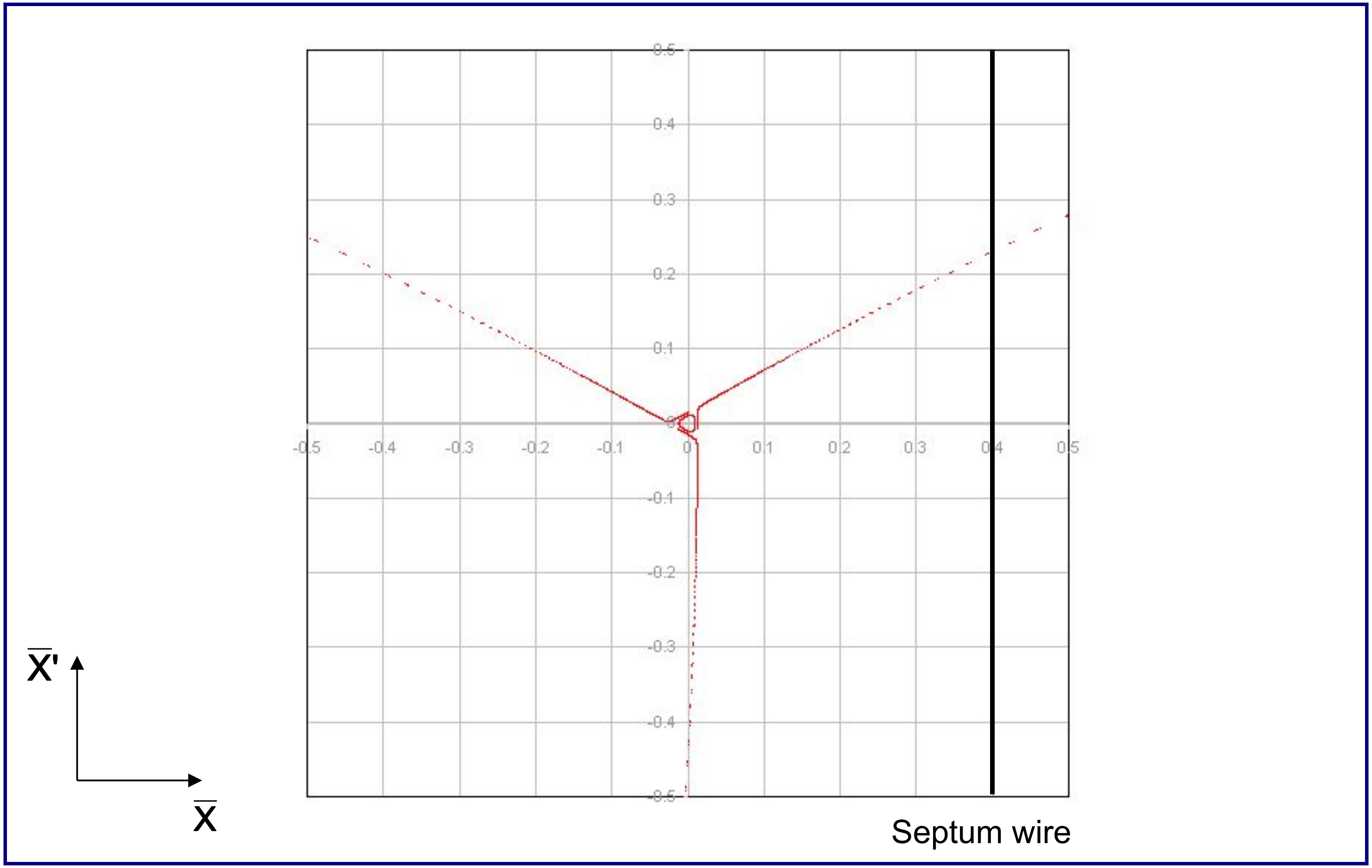

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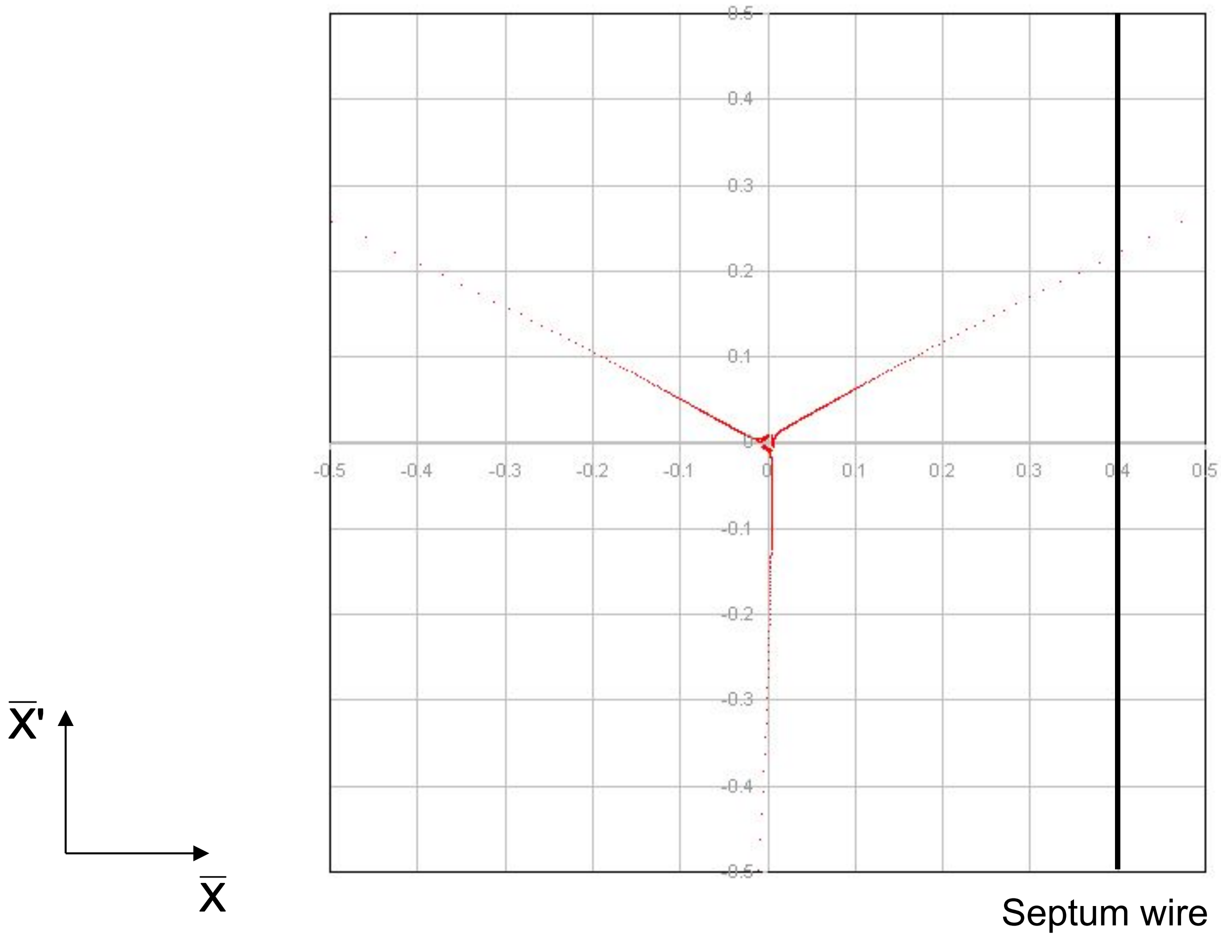
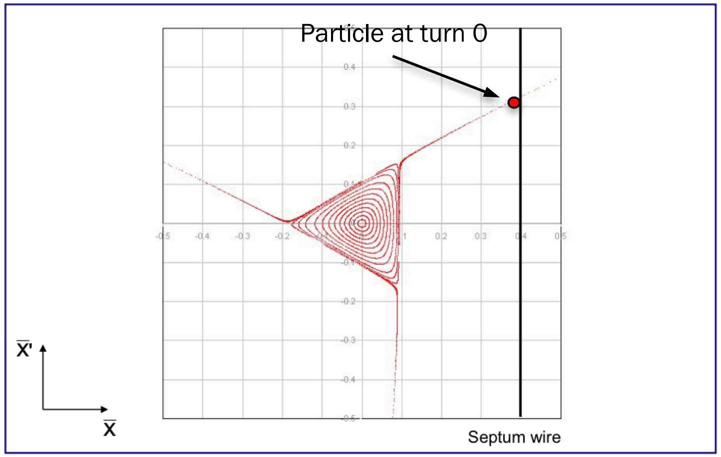


- As ΔQ approaches zero, the particles with very small amplitude are extracted

Third-order resonant extraction

- On resonance, sextupole kicks add-up driving particles over septum



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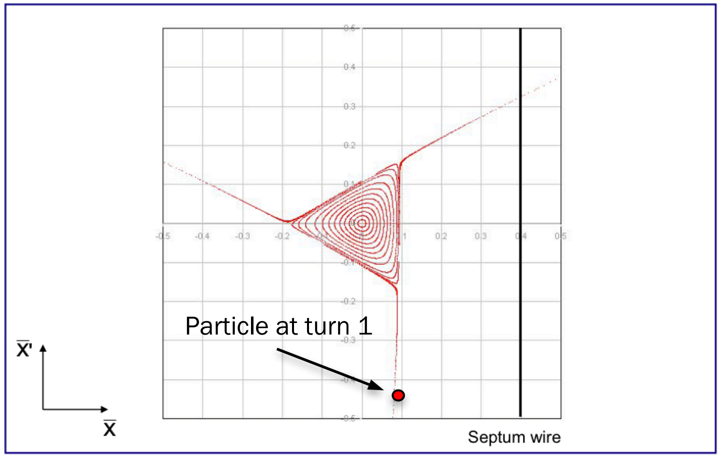

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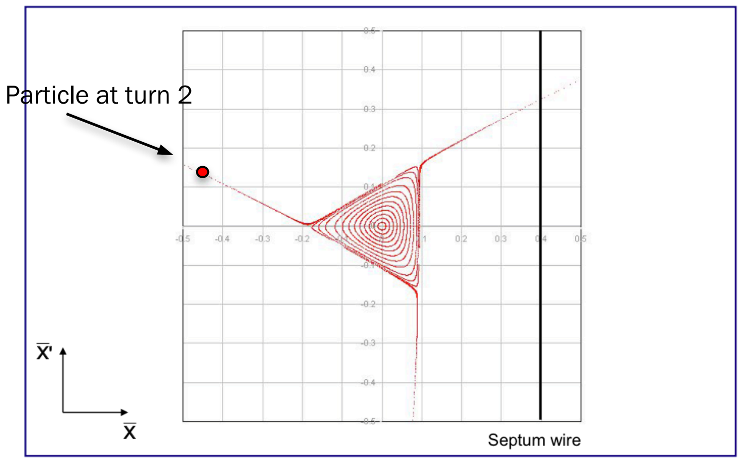

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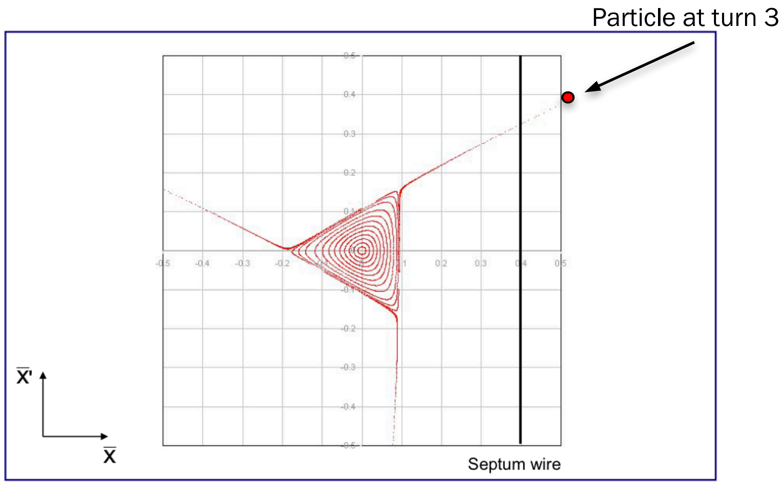
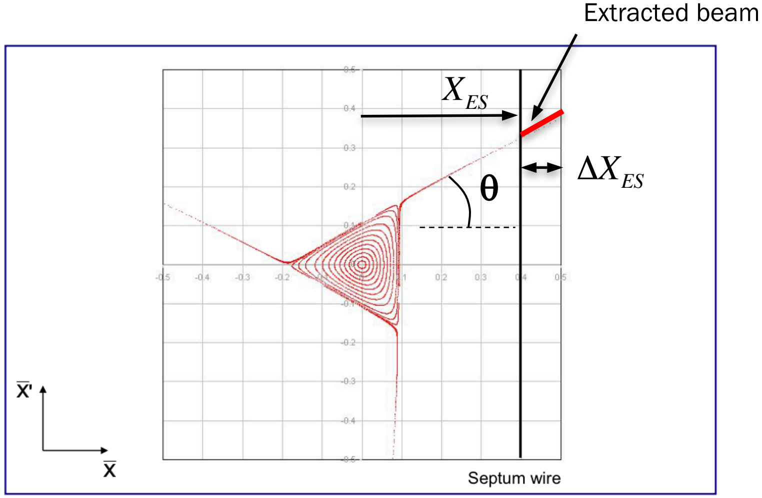


Third-order resonant extraction

- On resonance, sextupole kicks add-up driving particles over septum
  - Distance travelled in these final three turns is termed the "spiral step," ΔX_{ES}
  - Extraction bump trimmed in the machine to adjust the spiral step



$$\Delta X_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos\theta}$$

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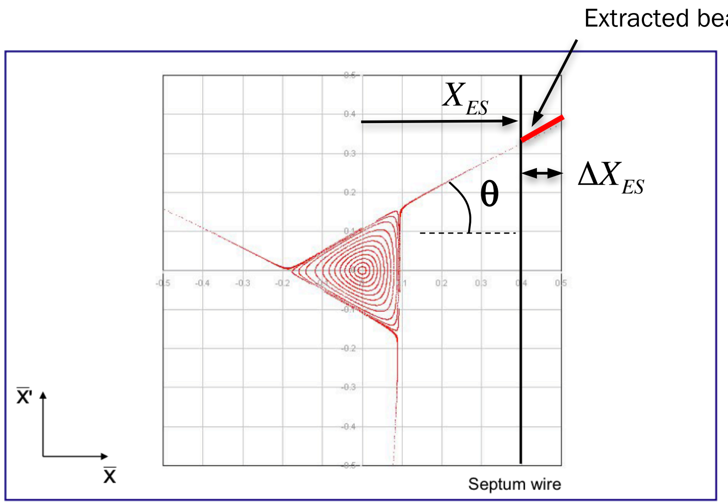
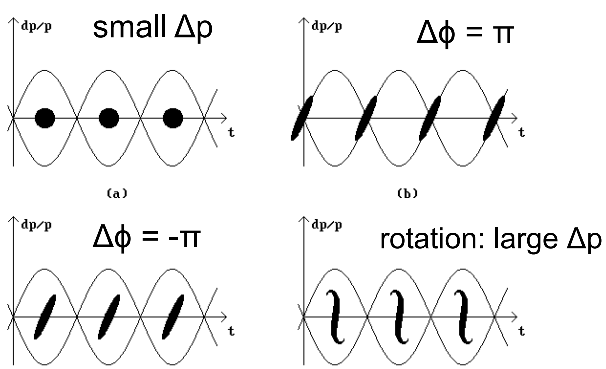
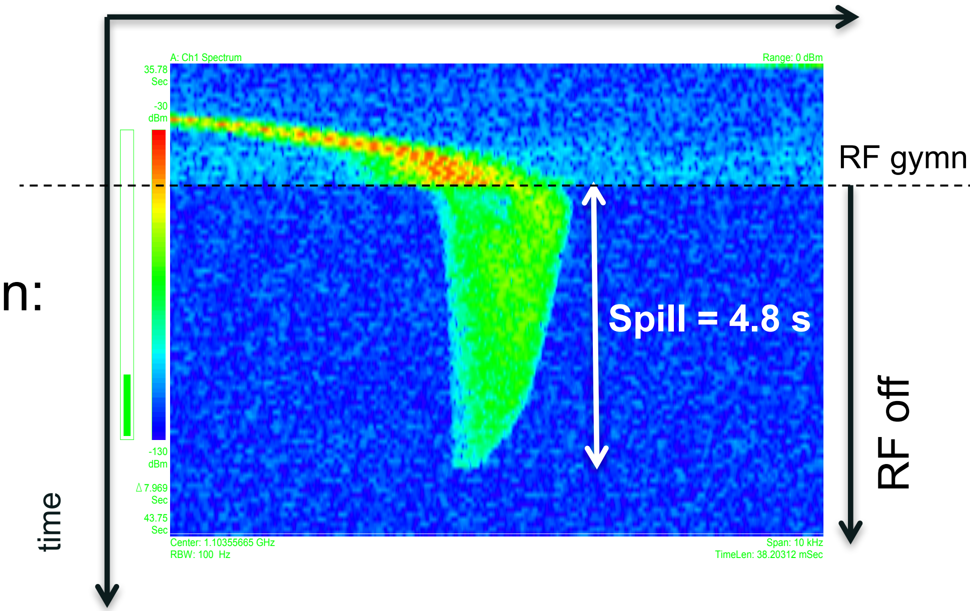


$$\Delta X_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos\theta}$$

- RF gymnastics before extraction:

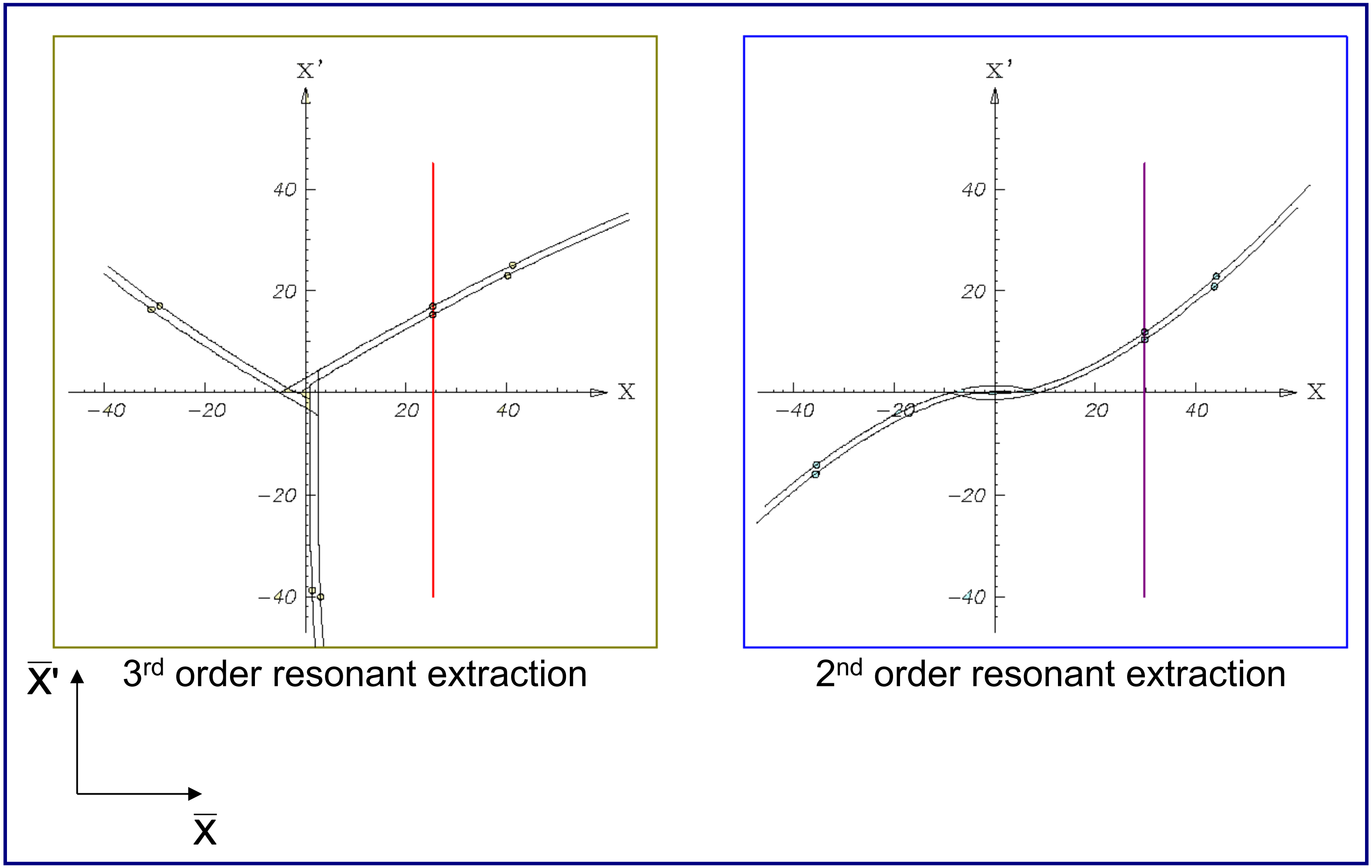


momentum spread, tune $\frac{\Delta p}{p} \propto -\Delta Q$



Schottky measurement during spill, courtesy of T. Bohl

Resonant extraction separatrices



- Amplitude growth for 2nd order resonance much faster than 3rd – shorter spills (≈milliseconds vs. seconds)
- Used where intense pulses are required on target – e.g. neutrino production

Extraction - summary

- Several different techniques:
  - Single-turn fast extraction:
    - for Boxcar stacking (transfer between machines in accelerator chain), beam abort
  - Non-resonant multi-turn extraction: mechanical splitting
    - slice beam into equal parts for transfer between machine over a few turns.
  - Resonant low-loss multi-turn extraction: magnetic splitting
    - create stable islands in phase space: slice off over a few turns.
  - Resonant multi-turn extraction
    - create stable area in phase space $\Rightarrow$ slowly drive particles into resonance $\Rightarrow$ long spill over many thousand turns.

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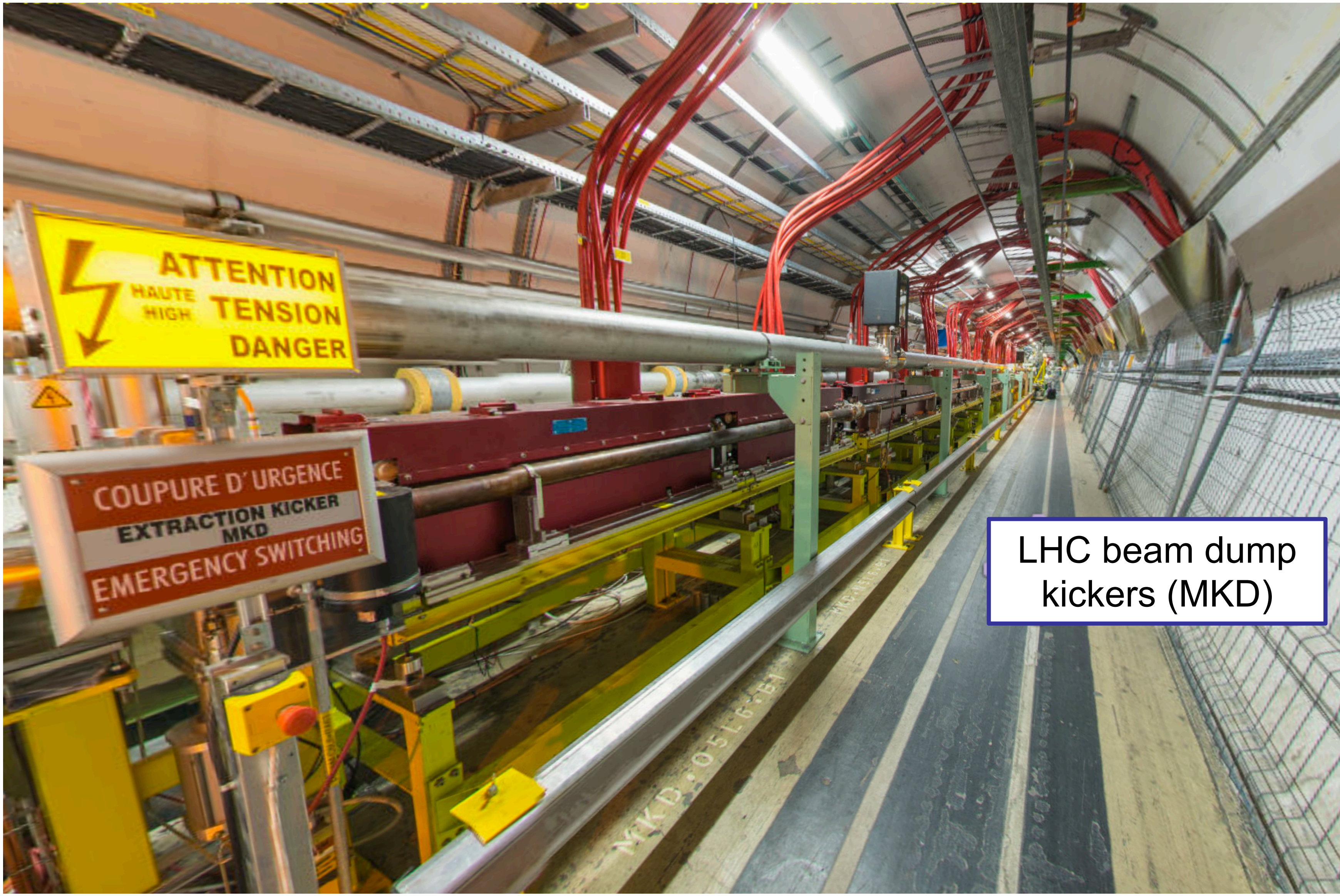
Thank you for your attention

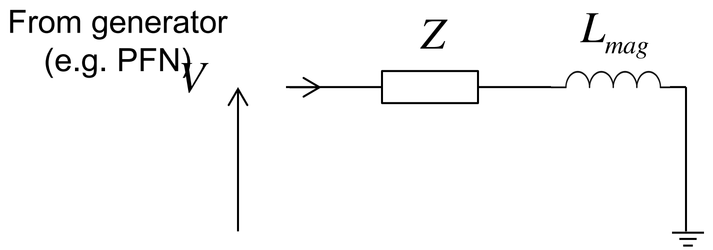
Extra slides

Kickers


ATTENTION
HAUTE
HIGH
TENSION
DANGER
COUPURE D'URGENCE
EXTRACTION KICKER
MKD
EMERGENCY SWITCHING
LHC beam dump kickers (MKD)

Magnets – design options

- Type: “lumped inductance”



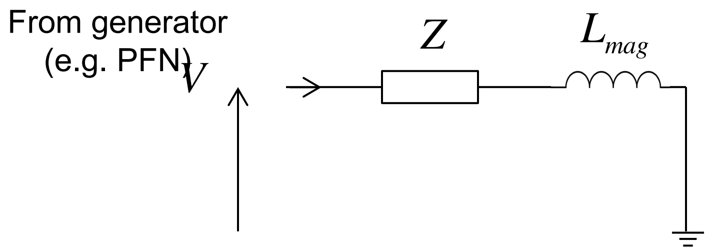
  - simple magnet design
  - magnet must be nearby the generator to minimise inductance
  - exponential field rise-time:

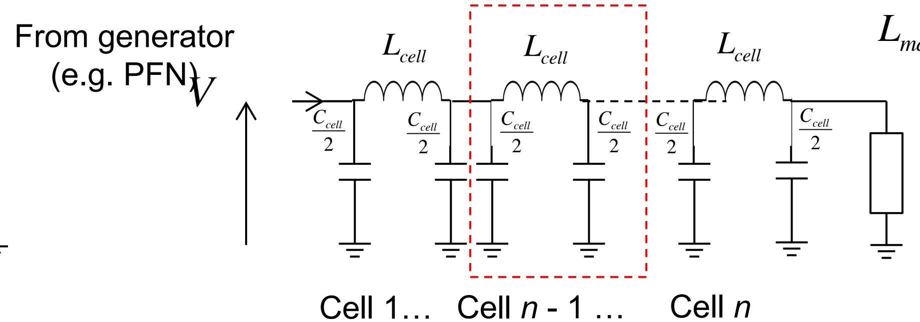
$$I = \frac{V}{Z}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L_{mag}}{Z}$$

  - slow: rise-times ~ 1 μs

Magnets – design options

- Type: “lumped inductance” or “distributed inductance” (**transmission line**)





$$L_{mag} = nL_{cell}$$

$$Z = \sqrt{\frac{L_{cell}}{C_{cell}}}$$

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$$I = \frac{V}{Z}(1 - e^{-t/\tau}) \qquad \tau = \frac{L_{mag}}{Z}$$

- slow: rise-times ~ 1 μs

- complicated magnet design
- impedance matching important
- field rise-time depends on propagation time of pulse through magnet:

$$\tau = n\sqrt{L_{cell} \cdot C_{cell}} = n\frac{L_{cell}}{Z} = \frac{L_{mag}}{Z}$$

- fast: rise-times << 1 μs

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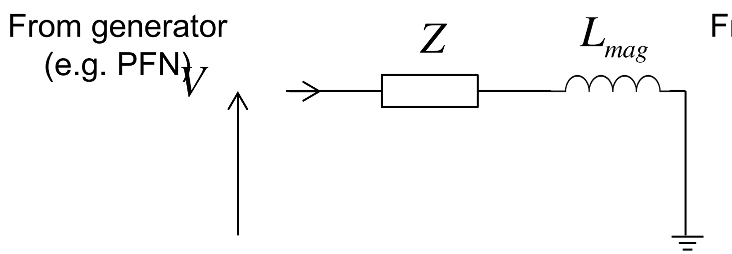

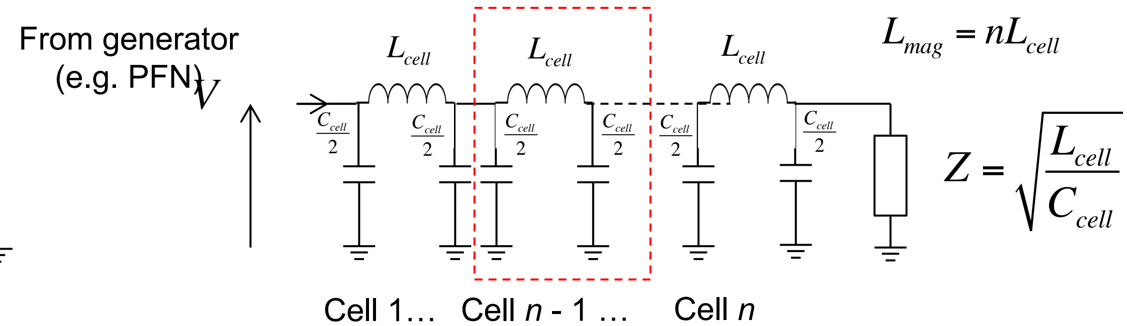

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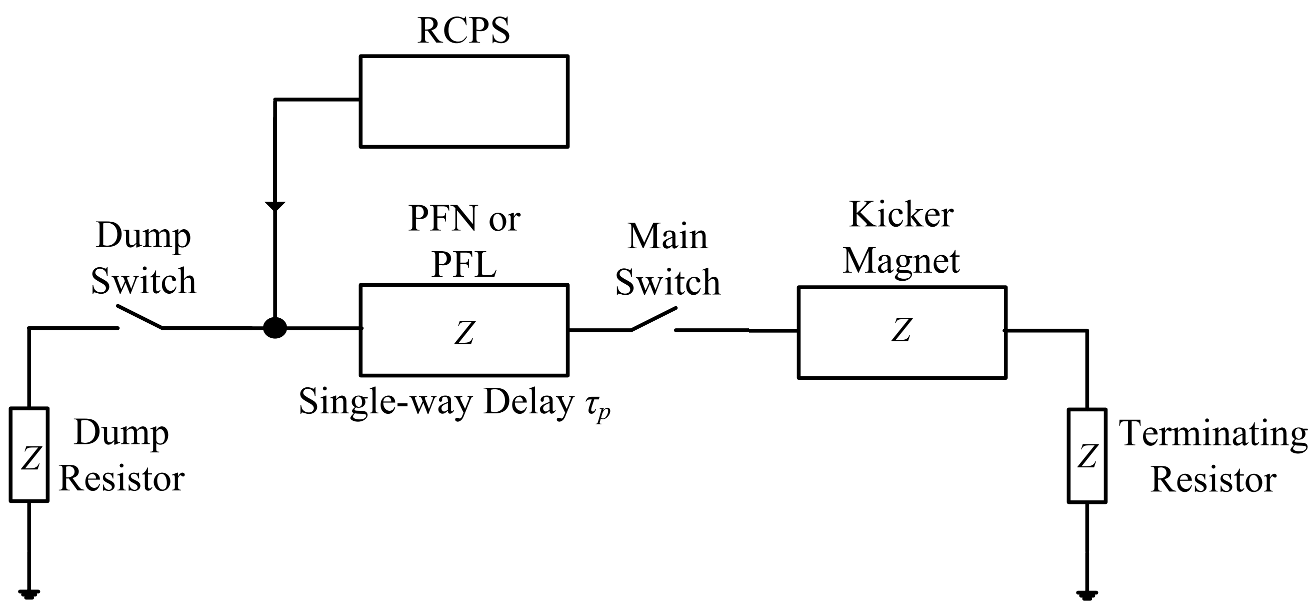
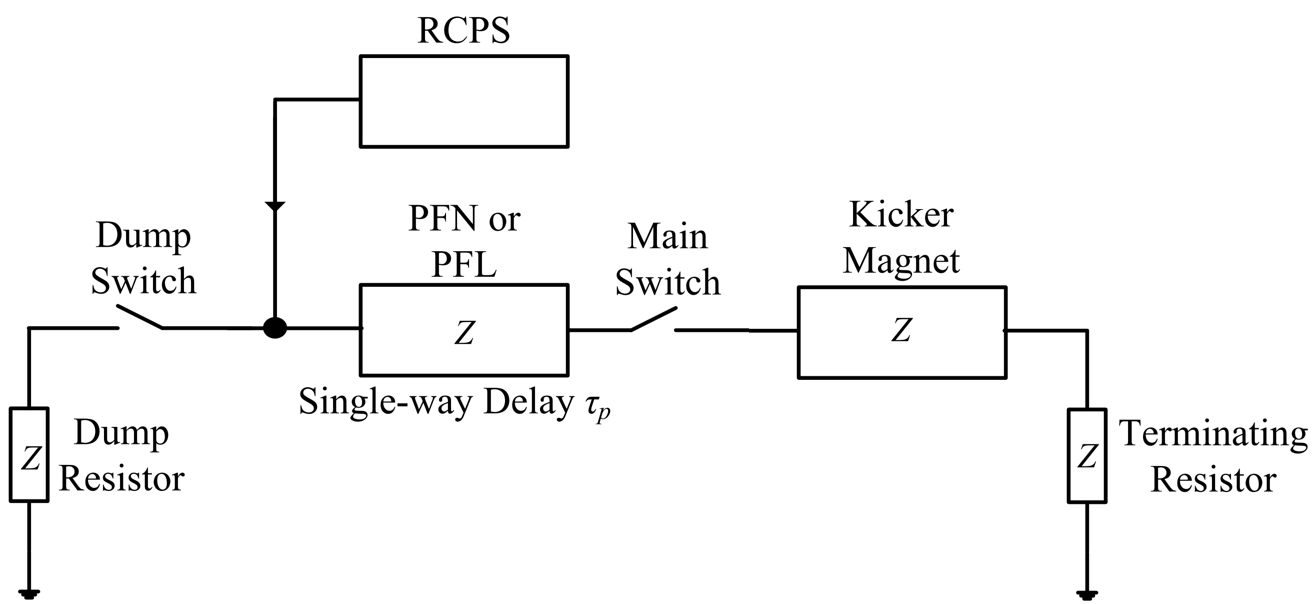
- Other considerations:
  - Machine vacuum: kicker in-vacuum or external
  - Aperture: geometry of ferrite core
  - Termination: matched impedance or short-circuit

Simplified kicker system schematic



- Main sub-systems ("components") of kicker system;
  - **RCPS** = Resonant Charging Power Supply
  - **PFL** = Pulse Forming Line (coaxial cable) or PFN = Pulse Forming Network (lumped elements)
  - Fast high power switch(es)
  - **Transmission line(s)**: coaxial cable(s)
  - **Kicker Magnet**
  - **Terminators** (resistive)

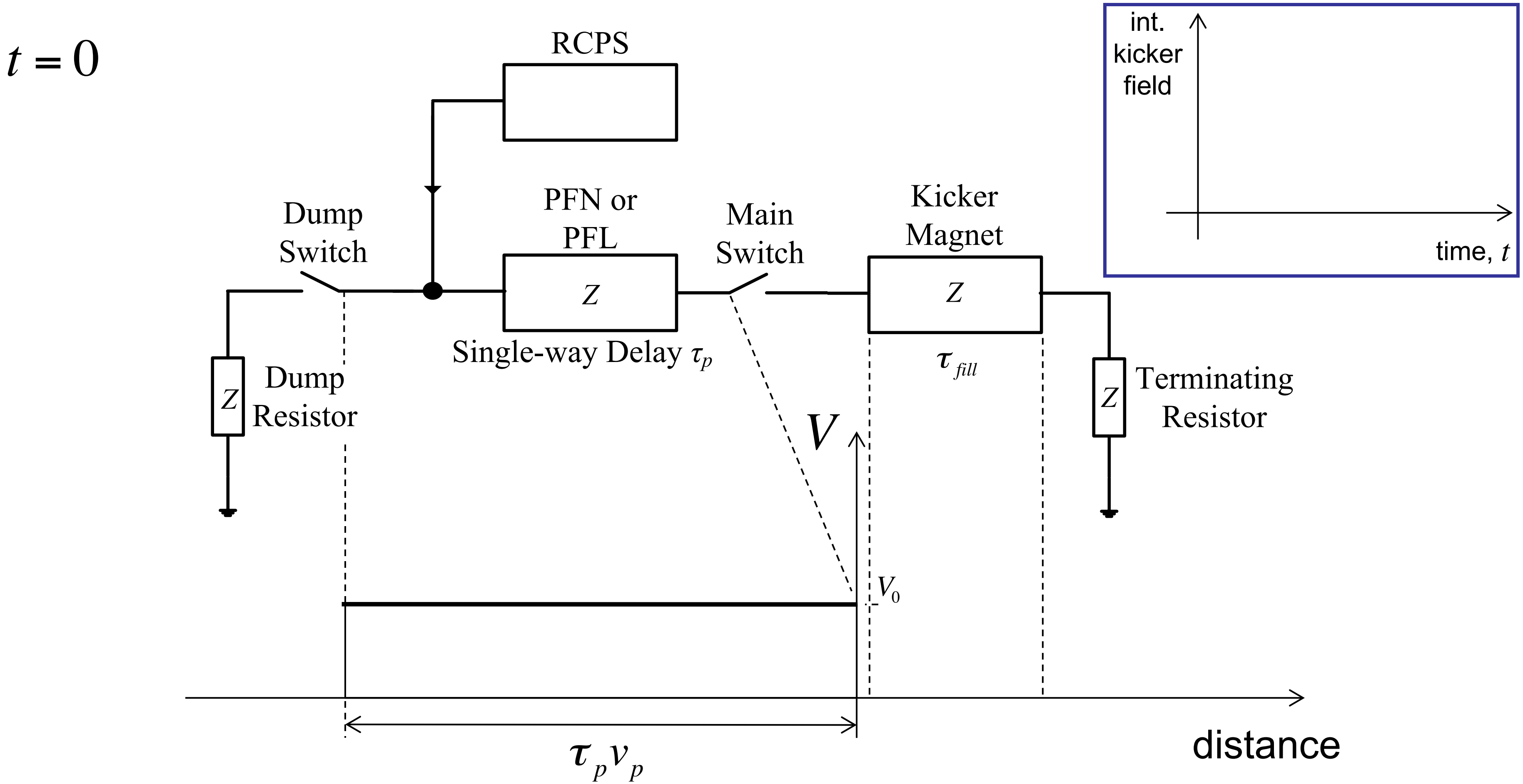
Simplified kicker system schematic



- PFL/PFN charged to voltage V_0 by the RCPS
- Main switch is closed…

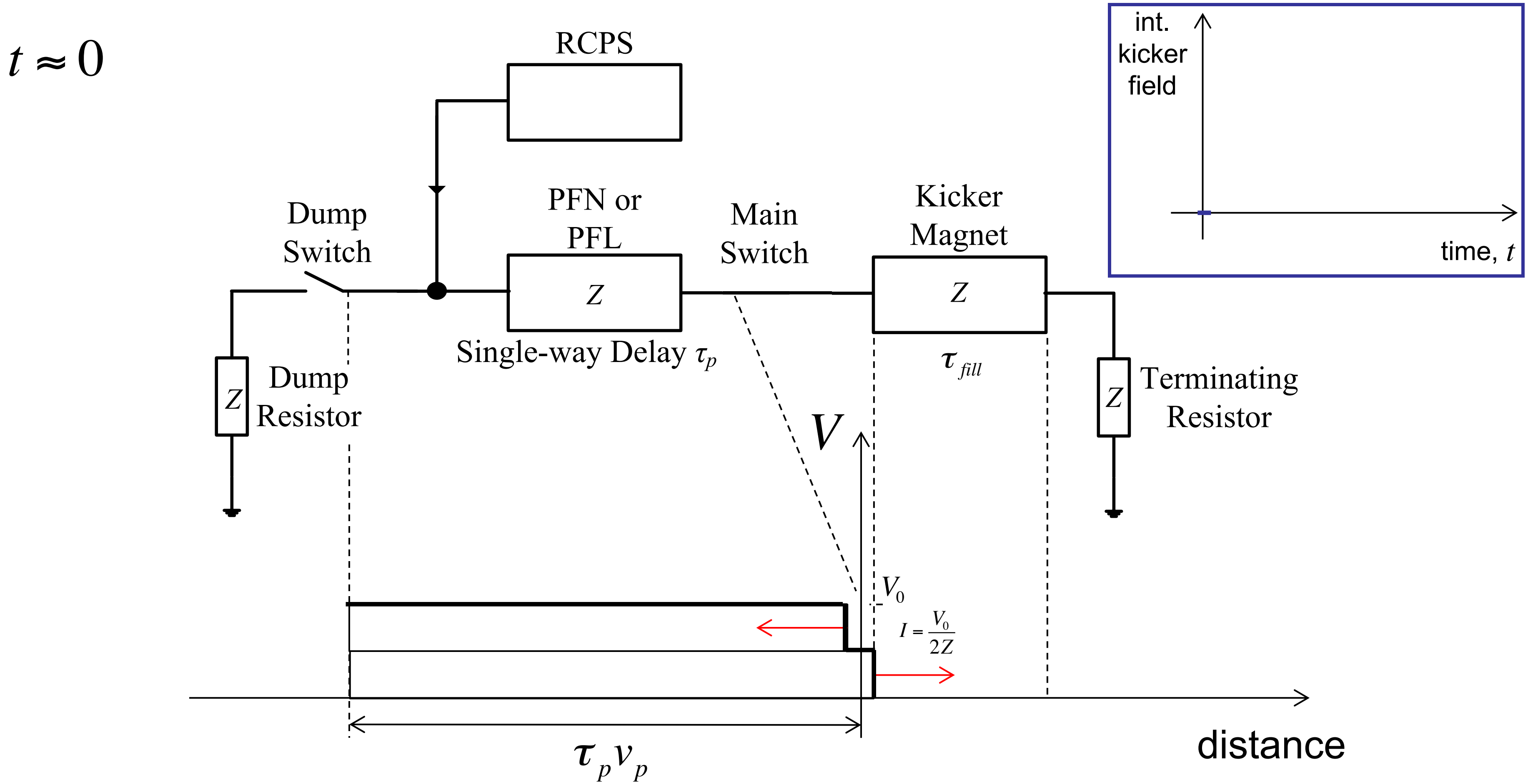
  …voltage pulse of $V_0/2$ flows through kicker
- Once the pulse reaches the (matched) terminating resistor full-field has been established in the kicker magnet
- Pulse length controlled between t = 0 and $2\tau_p$ with dump switch

Simplified kicker system schematic



- Pulse forming network or line (PFL/PFN) charged to voltage V_0 by the resonant charging power supply (RCPS)
  - RCPS is de-coupled from the system through a diode stack

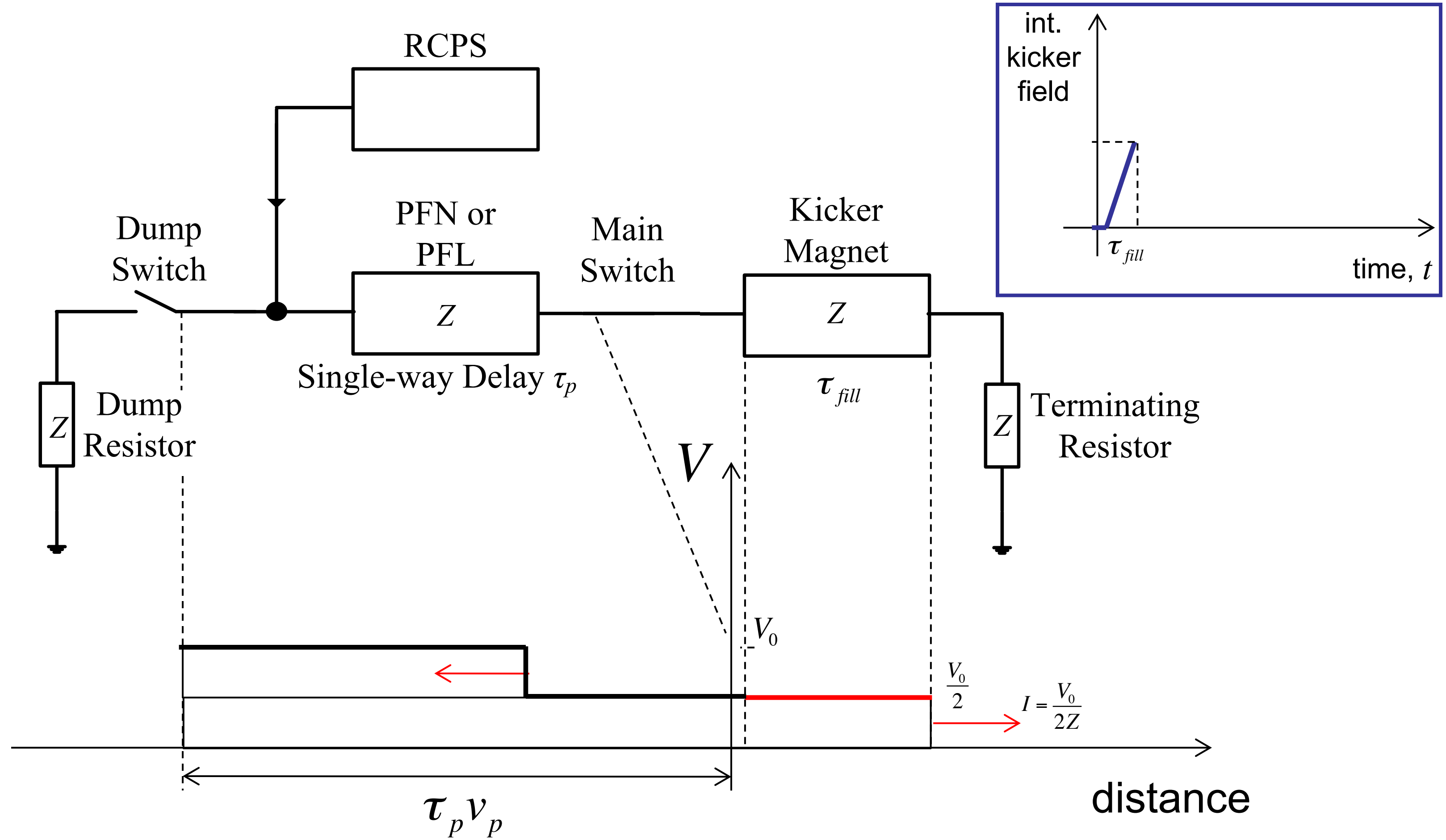
Simplified kicker system schematic



- Pulse forming network or line (PFL/PFN) charged to voltage V_0 by the resonant charging power supply (RCPS)
  - RCPS is de-coupled from the system through a diode stack
- At t = 0, main switch is closed and current starts to flow into the kicker

Simplified kicker system schematic

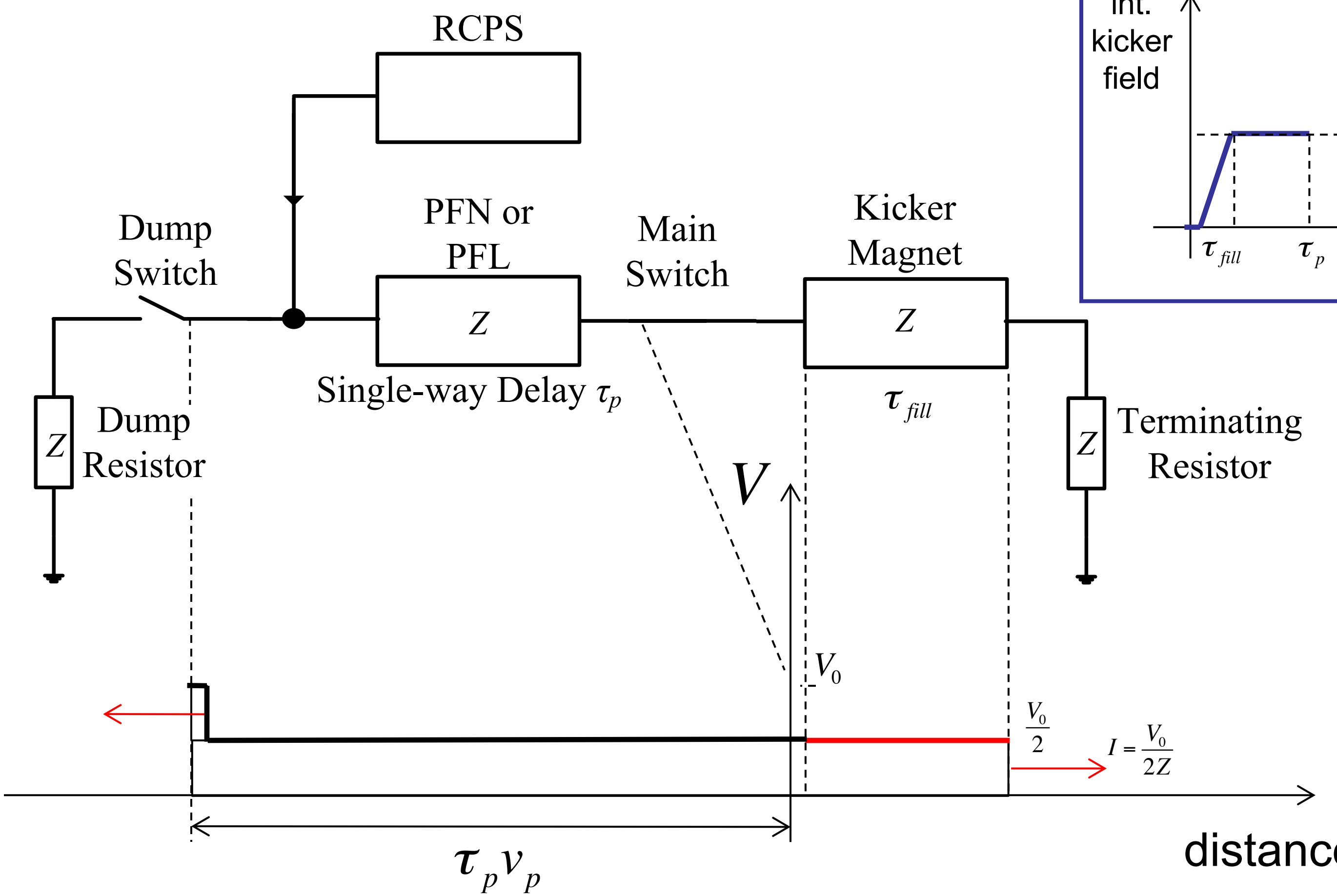
$t \approx \tau_{fill}$

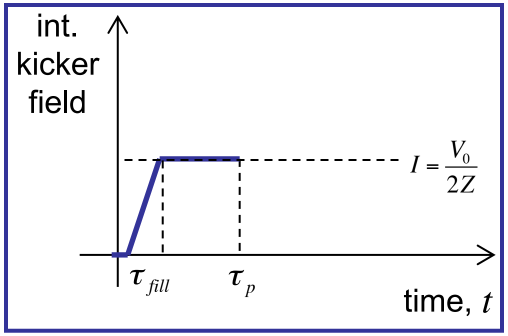


- At t = τ_{fill}, the voltage pulse of magnitude $V_0/2$ has propagated through the kicker and nominal field achieved with a current $V_0/2Z$
  - typically $\tau_p >> \tau_{fill}$ (schematic for illustration purposes)

Simplified kicker system schematic

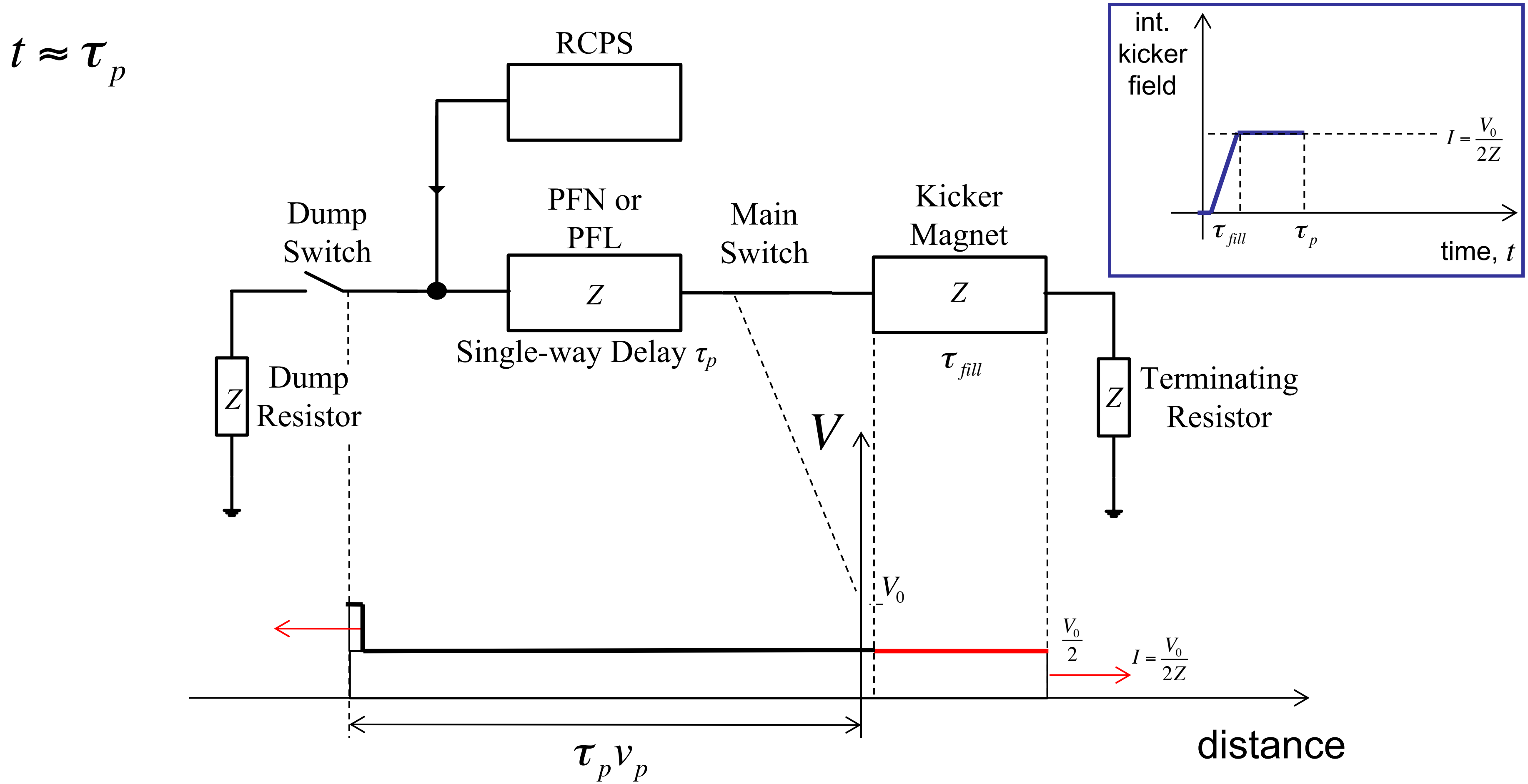
$t \approx \tau_p$





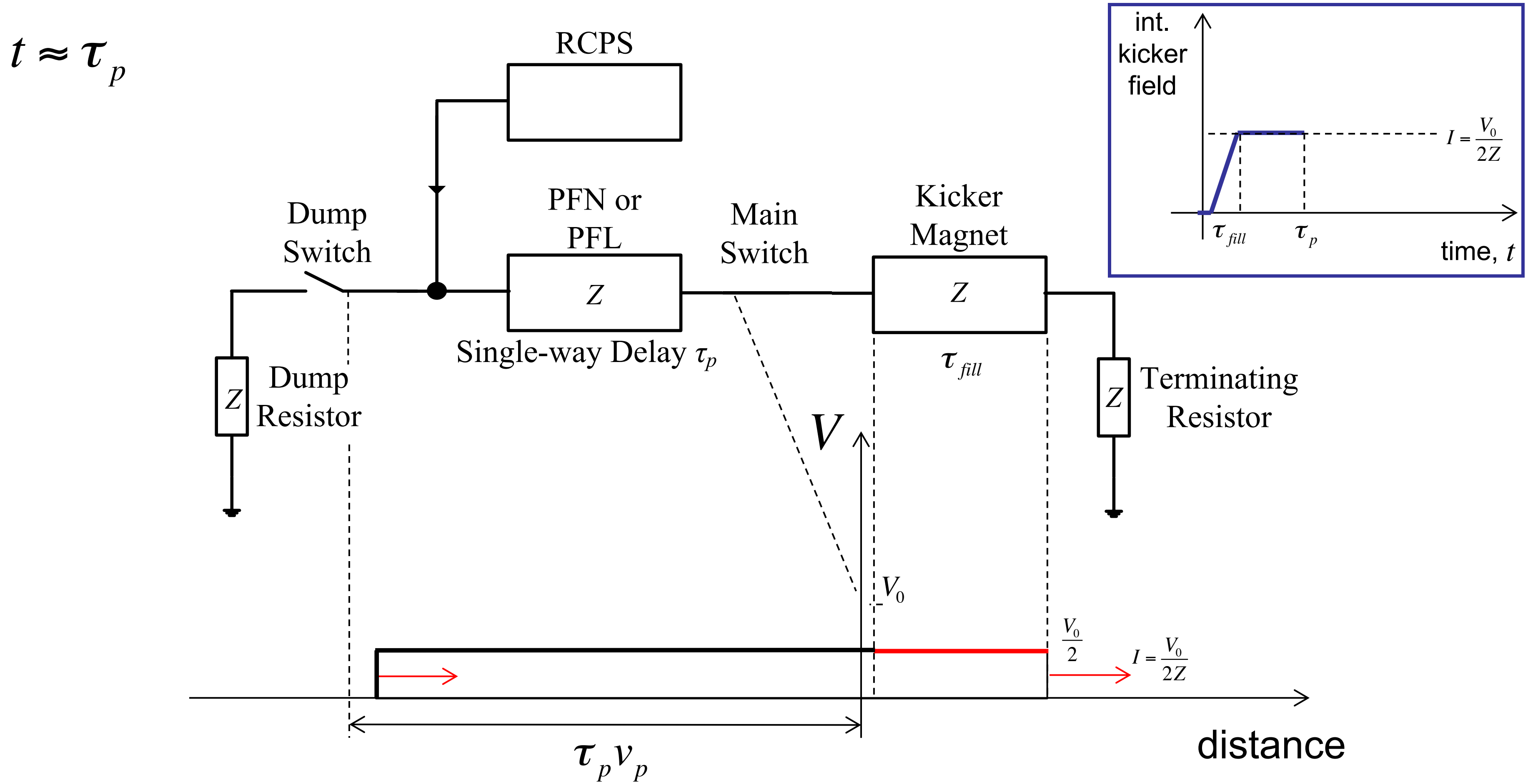
- PFN continues to discharge energy into kicker magnet and matched terminating resistor

Simplified kicker system schematic



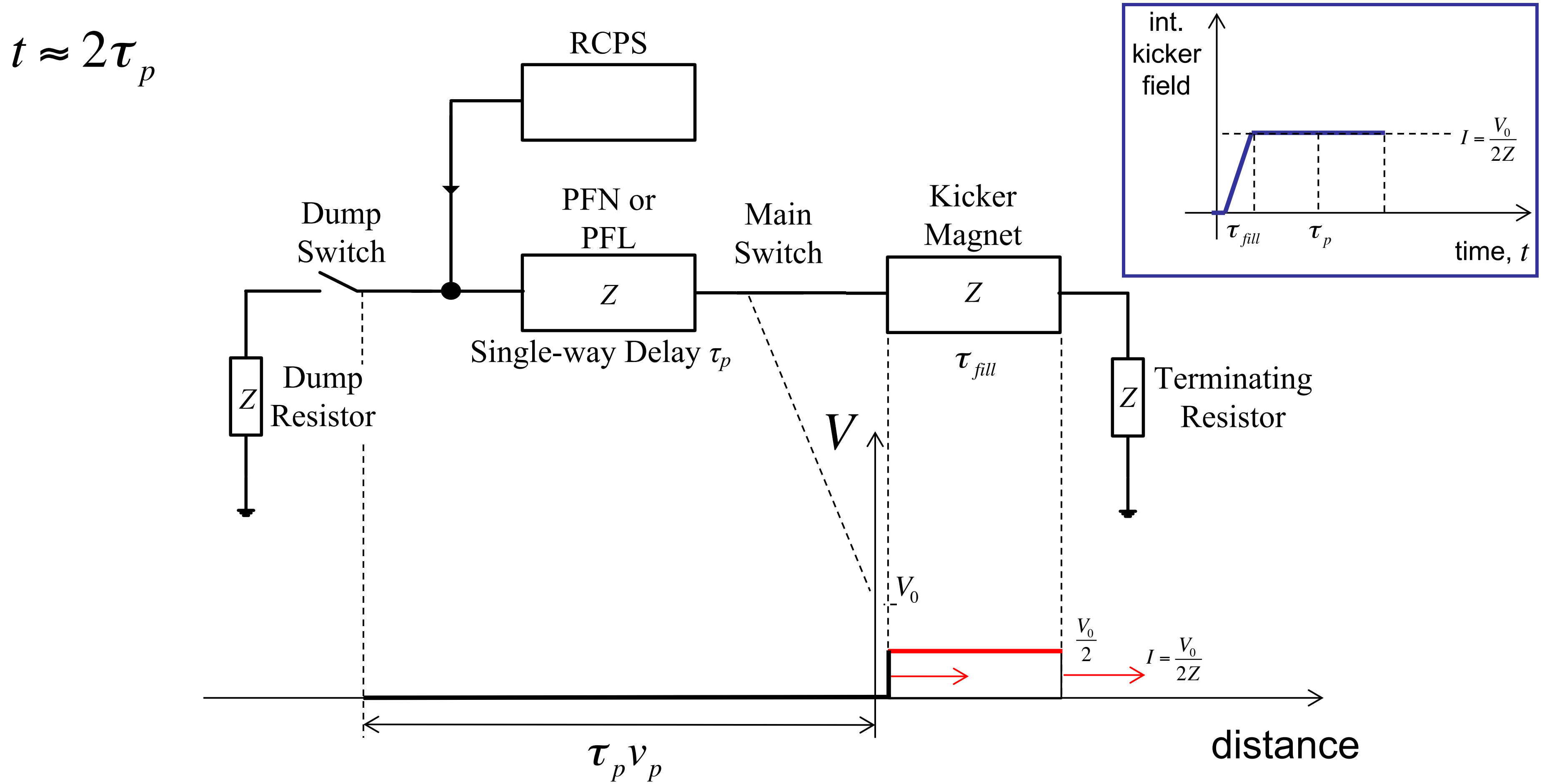
- PFN continues to discharge energy into kicker magnet and matched terminating resistor
- At $t \approx \tau_p$ the negative pulse reflects off the open end of the circuit (dump switch) and back towards the kicker

Simplified kicker system schematic



- PFN continues to discharge energy into matched terminating resistor
- At $t \approx \tau_p$ the negative pulse reflects off the open end of the circuit and back towards the kicker

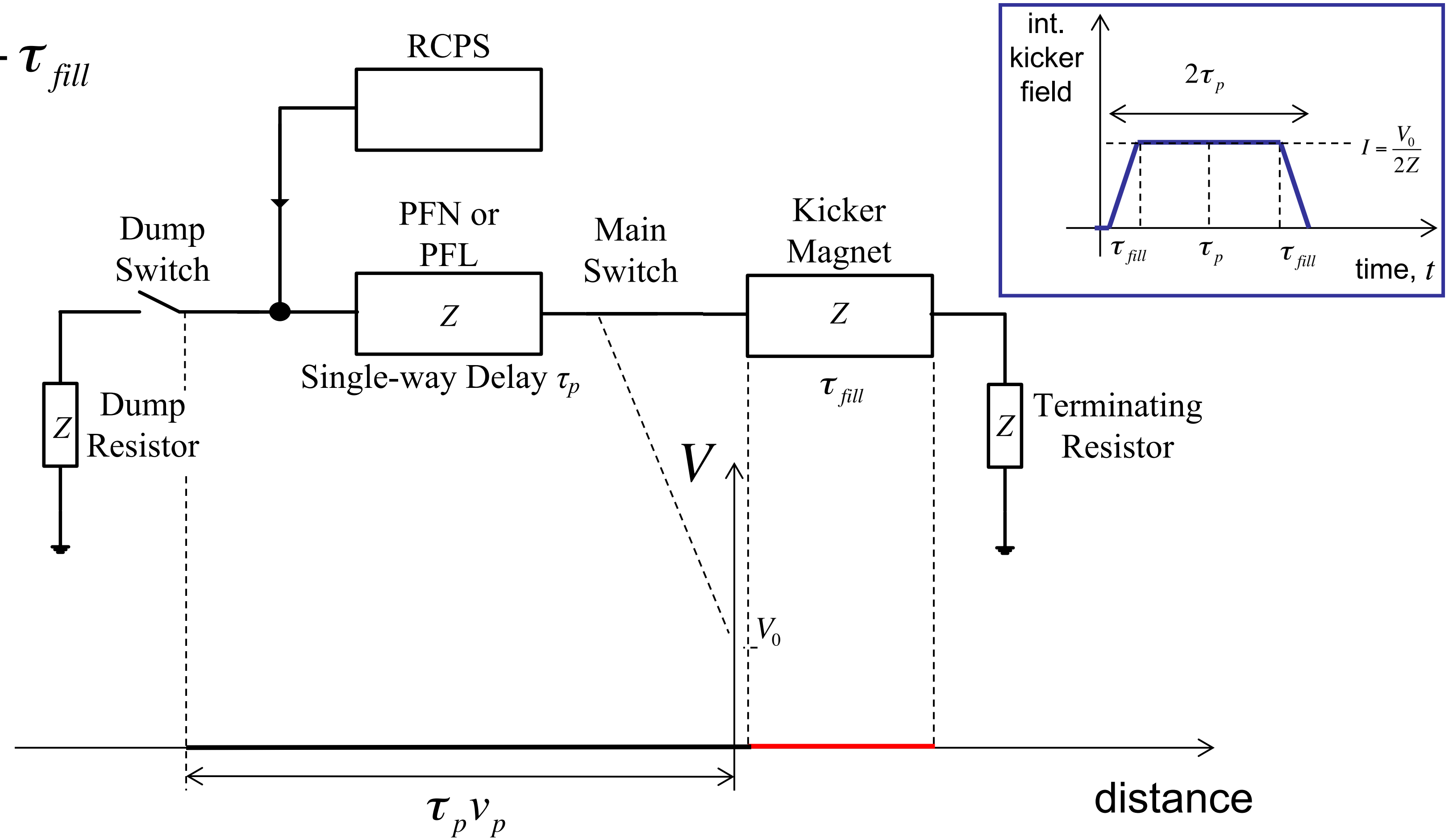
Simplified kicker system schematic



- At $t \approx 2\tau_p$ the pulse arrives at the kicker and field starts to decay

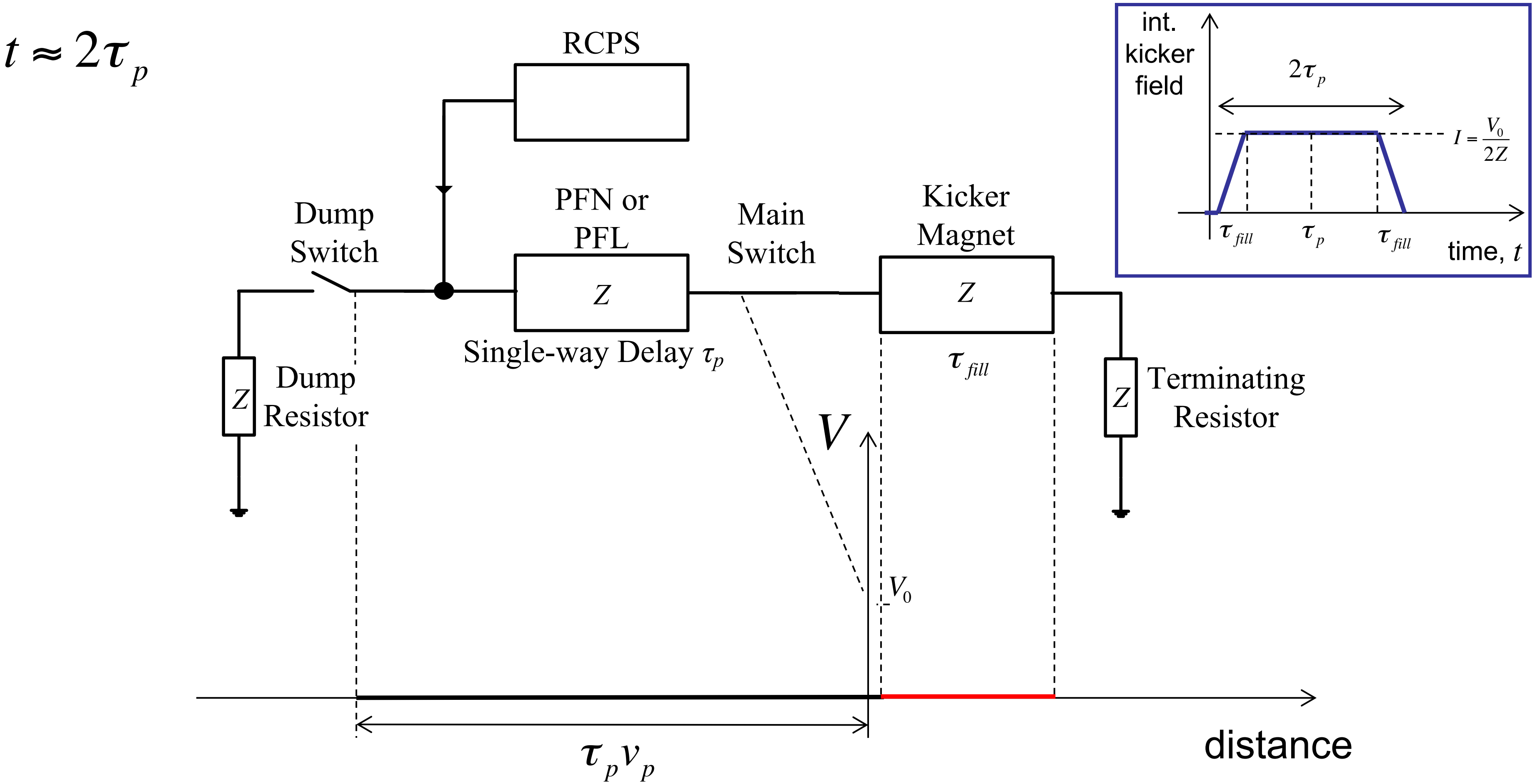
Simplified kicker system schematic

$$t = 2\tau_p + \tau_{fill}$$



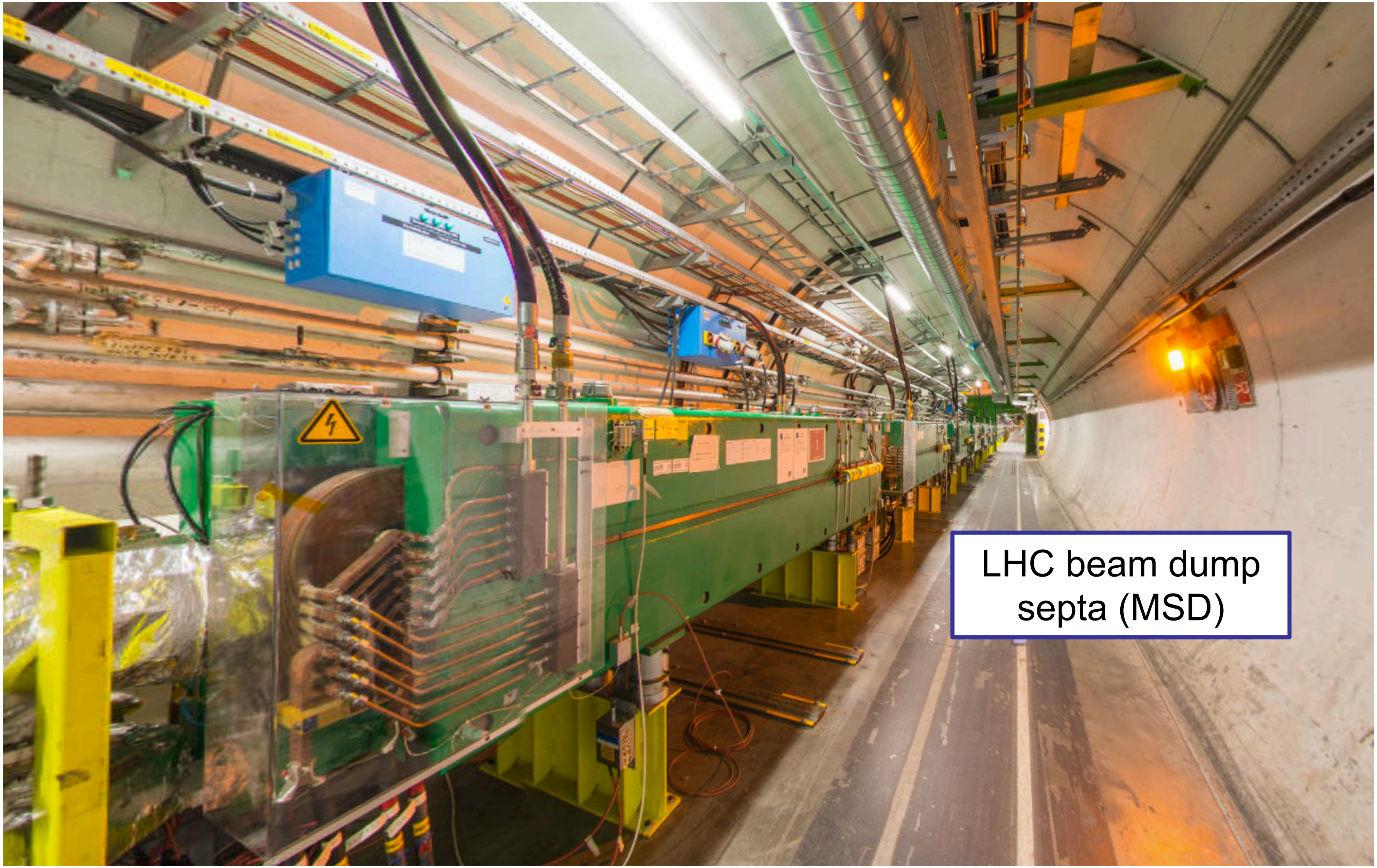
- A kicker pulse of approximately $2\tau_p$ is imparted on the beam and all energy has been emptied into the terminating resistor

Simplified kicker system schematic



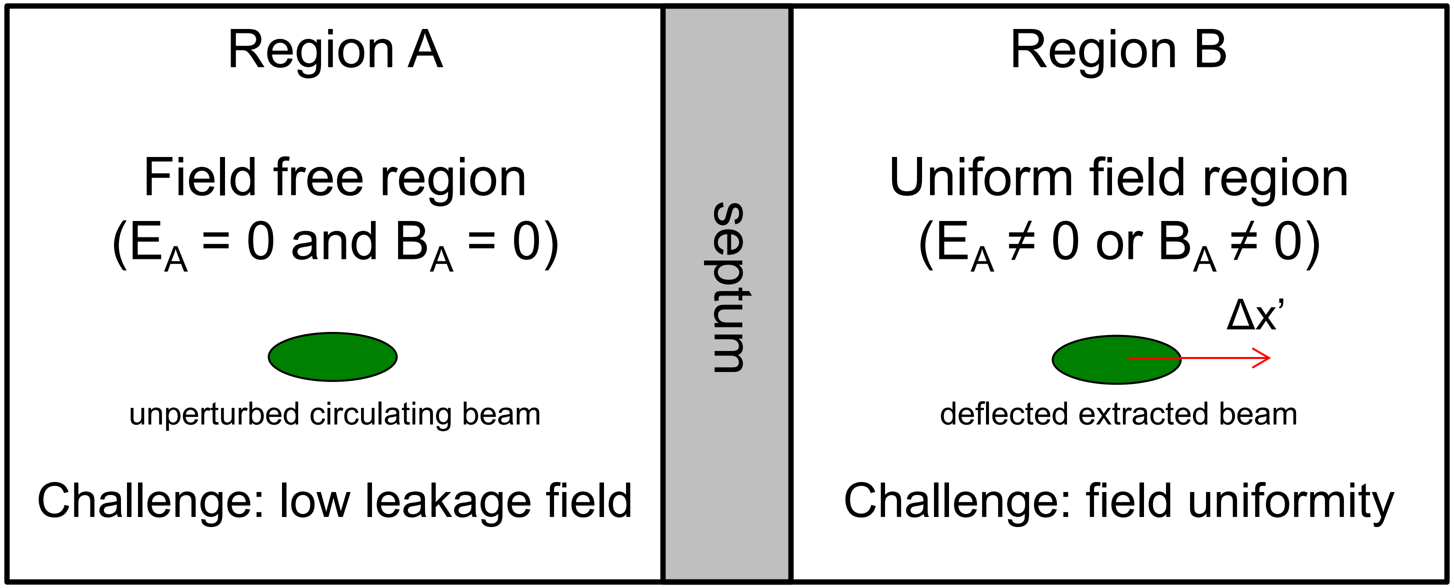
- Kicker pulse length can be changed by adjusting the relative timing of dump and main switches:
  - e.g. if the dump and main switches are fired simultaneously the pulse length will be halved and energy shared on dump and terminating resistors

Septa

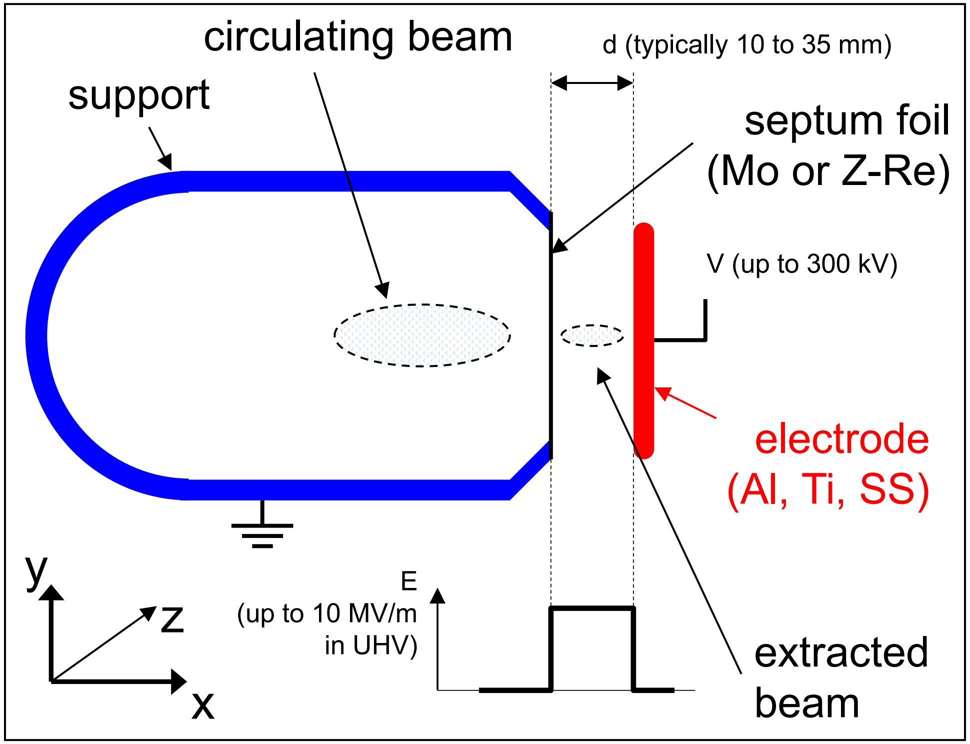


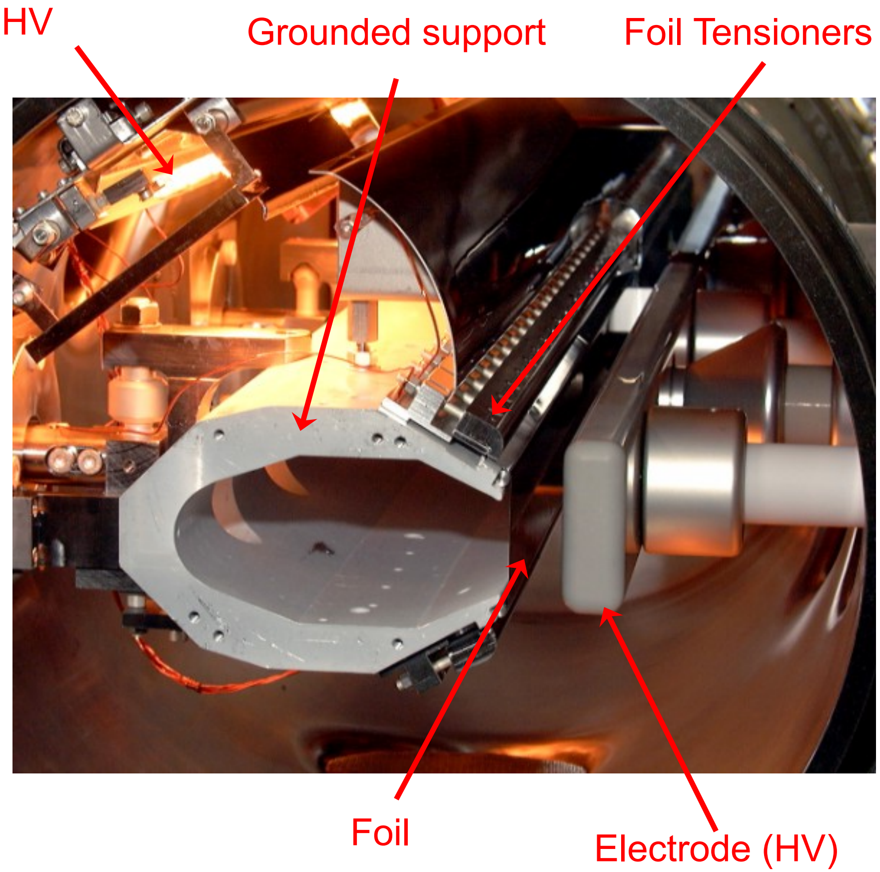
Septa

- Two main types:
  - Electrostatic septa (DC)
  - Magnetic septa (DC and pulsed):
    - Direct drive septum
    - Eddy current septum (pulsed only)
    - Lambertson septum (deflection parallel to septum)



Electrostatic septum



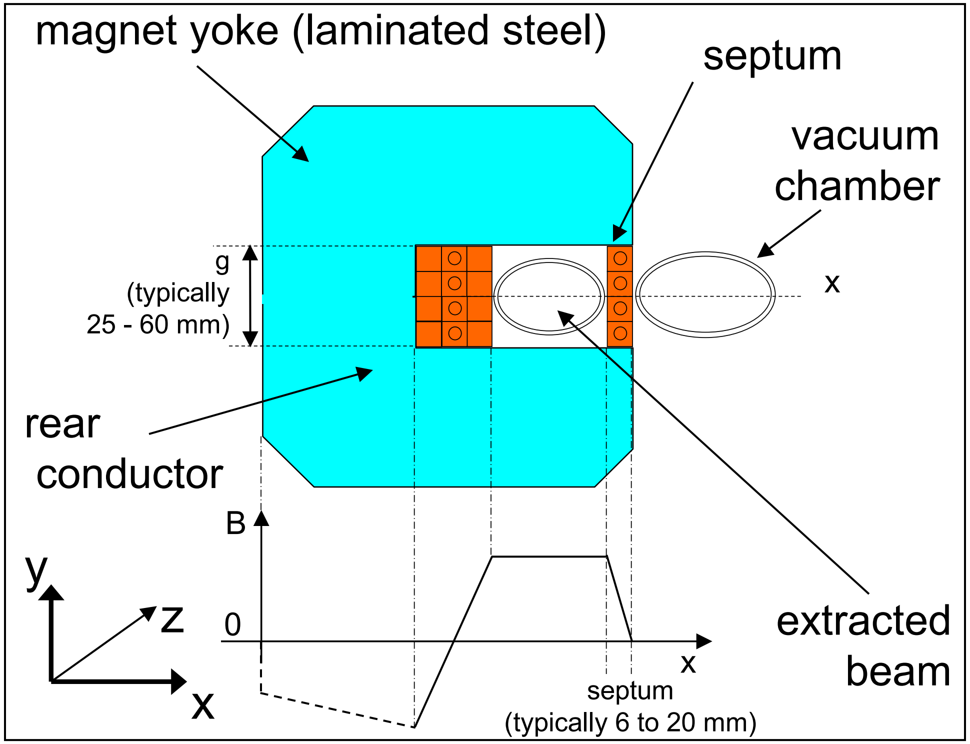


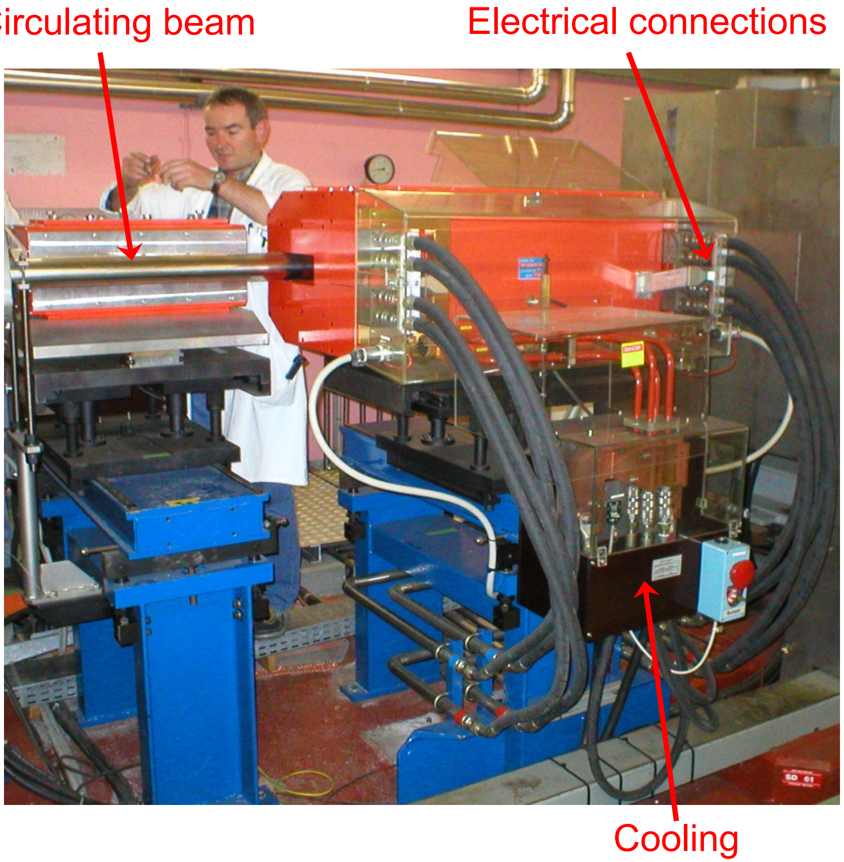
- Thin septum ~ 0.1 mm needed for high extraction efficiency:
  - Foils typically used
  - Stretched wire arrays provide thinner septa and lower effective density
- Challenges include conditioning and preparation of HV surfaces, vacuum in range of 10^{-9} – 10^{-12} mbar and in-vacuum precision position alignment

Electrostatic septum

- At SPS we slow-extract 400 GeV protons using approximately 15 m of septum split into 5 separate vacuum tanks each over 3 m long:
  - Alignment of the 60 - 100 μm wire array over 15 m is challenging!

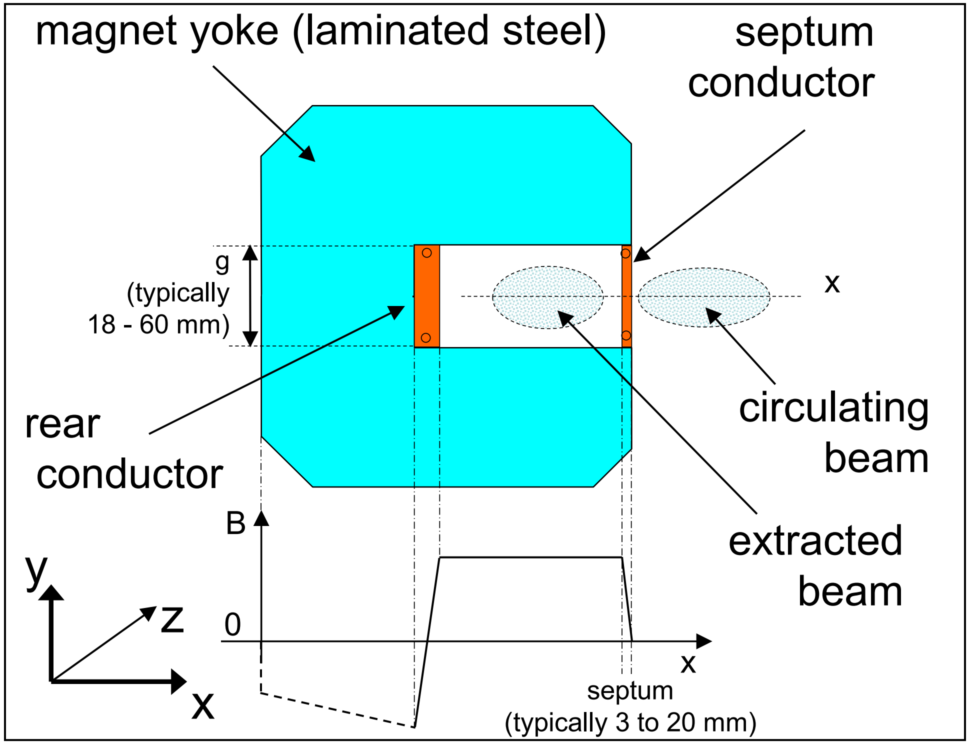
DC direct drive magnetic septum

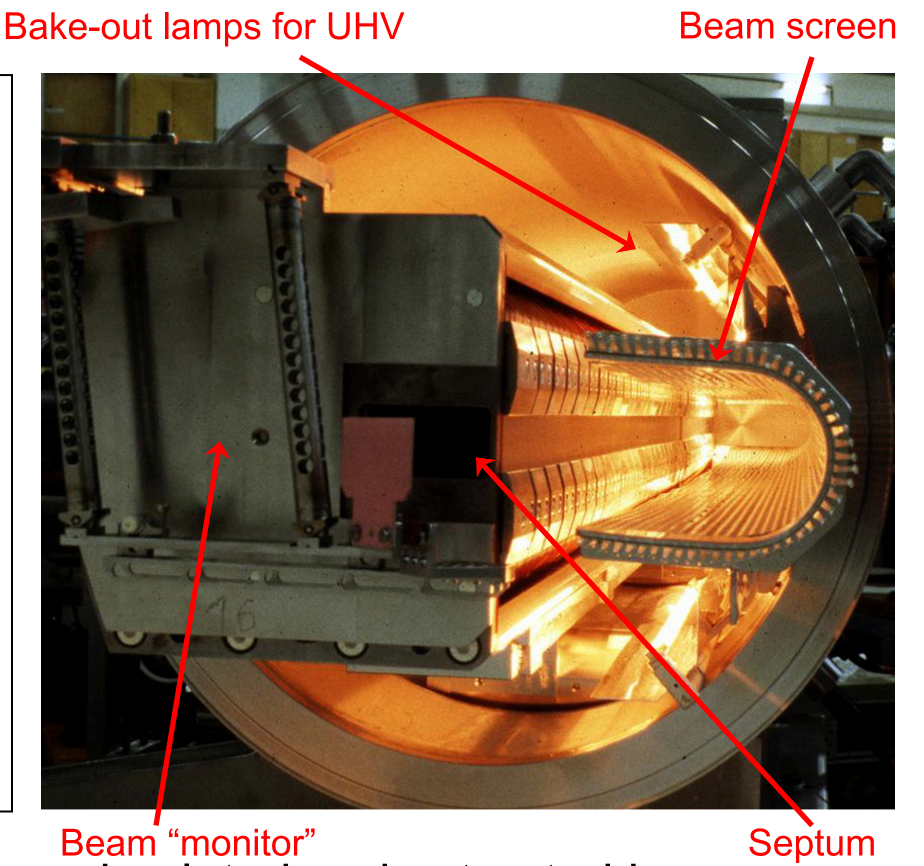




- Continuously powered, rarely under vacuum
- Multi-turn coil to reduce current needed but cooling still an issue:
  - Cooling water circuits flow rate typically at 12 – 60 l/min
  - Current can range from 0.5 to 4 kA and power consumption up to 100 kW!

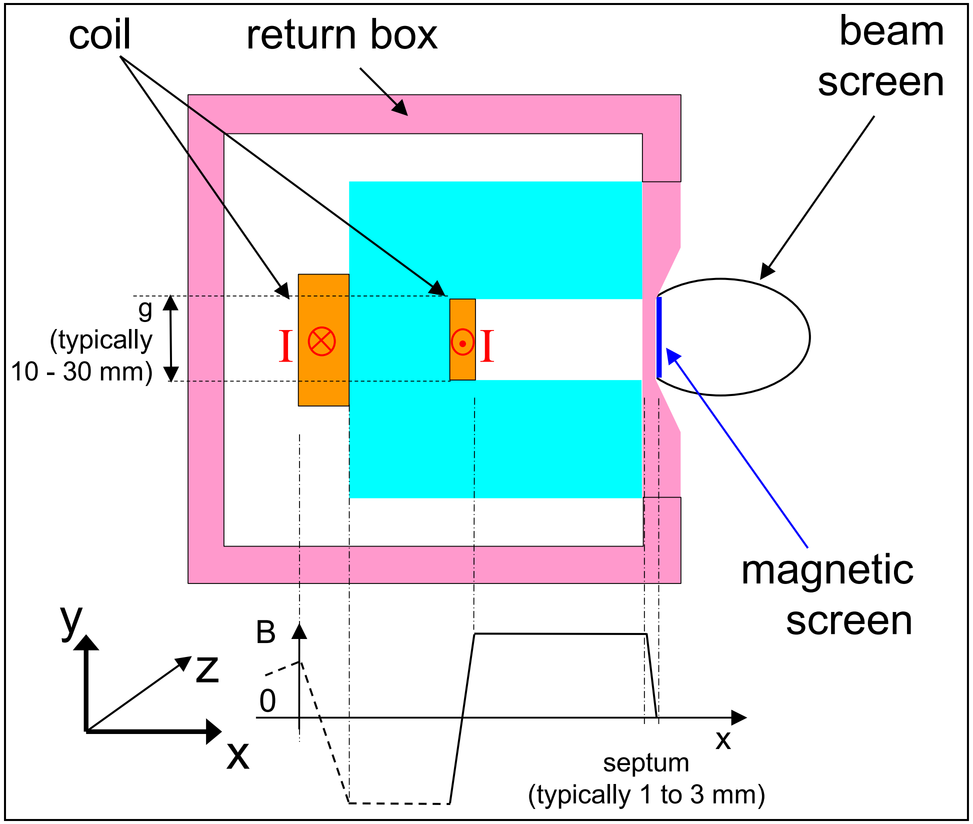
Direct drive **pulsed** magnetic septum





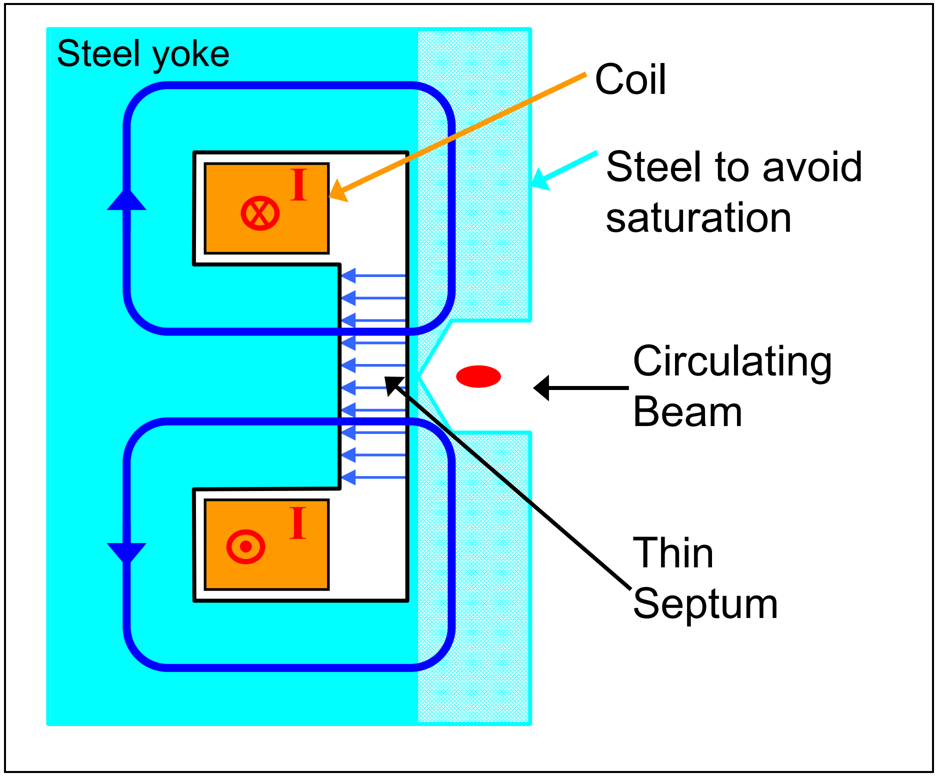
- In vacuum, to minimise distance between circulated and extracted beam
- Single-turn coil to minimise inductance, bake-out up to 200 °C (~10^{-9} mbar)
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
  - Current in range 7 – 40 kA with a few ms oscillation period
  - Cooling water circuits flow rate from 1 – 80 l/min

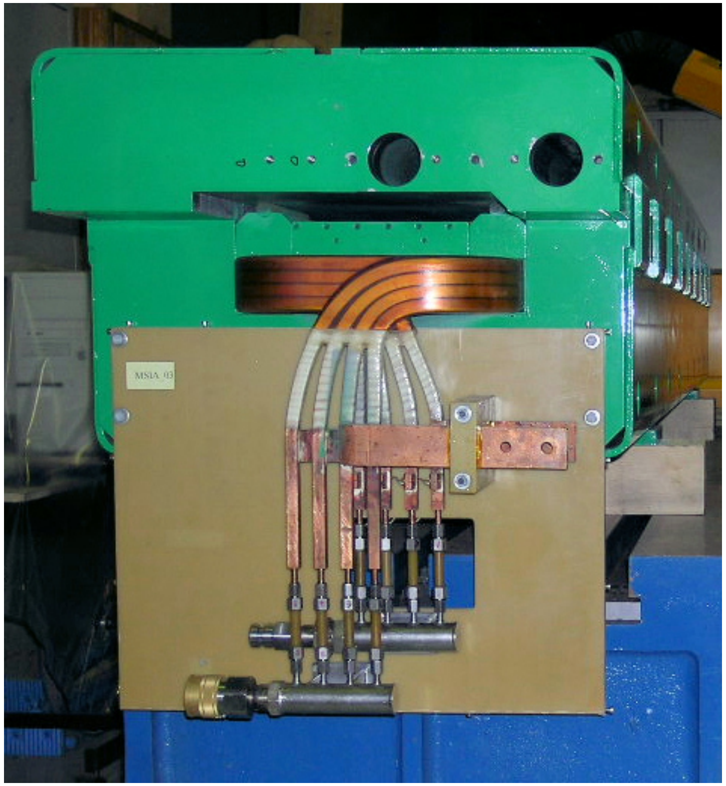
Eddy current septum



- Coil removed from septum and placed behind C-core yoke:
  - Coil dimension not critical
  - Very thin septum blade
- Magnetic field pulse induces eddy currents in septum blade
- Eddy currents shield the circulating beam from magnetic field
- Return box and magnetic screen reduce fringe field seen by circulating beam
- In or out of vacuum, single-turn coil
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
  - Current ~10 kA fast pulsed with ~ 50 μs oscillation period
  - Cooling water circuits flow rate from 1 – 10 l/min

Lambertson septum

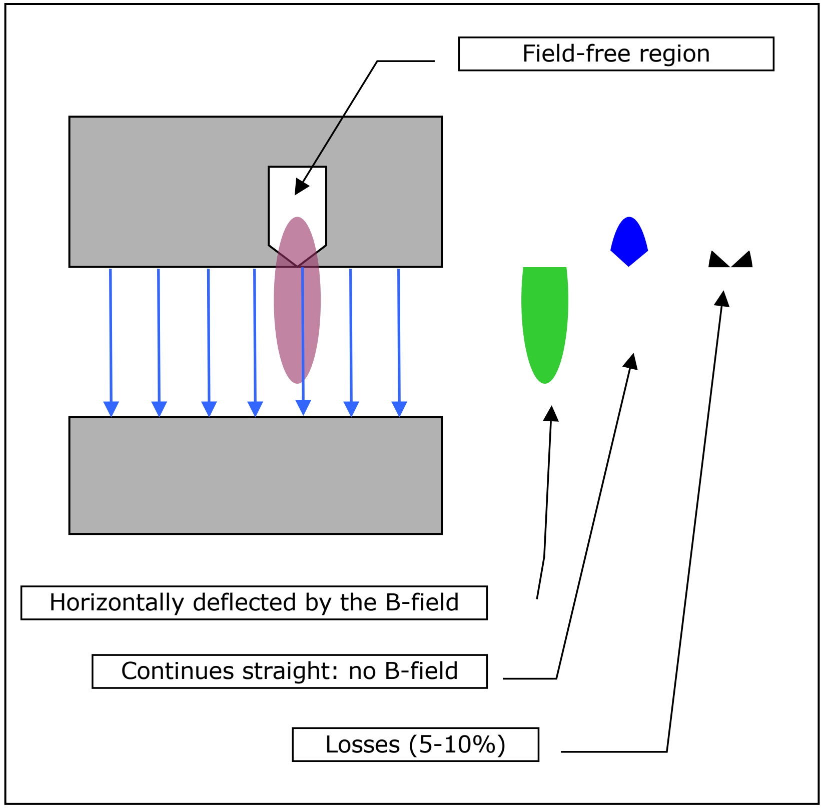




- Magnetic field in gap orthogonal to previous examples of septa:
  - Lambertson deflects beam orthogonal to kicker: dual plane injection/extraction
- Rugged design: conductors safely hidden away from the beam
- Thin steel yoke between aperture and circulating beam – however extra steel required to avoid saturation, magnetic shielding often added

Lambertson septum

- At SPS we use Lambertson septa to split the 400 GeV slow-extracted proton spill (~ seconds) to different target stations simultaneously:
  - These devices are radioactive: critical that coils are located away from the septum



Beam transfer lines

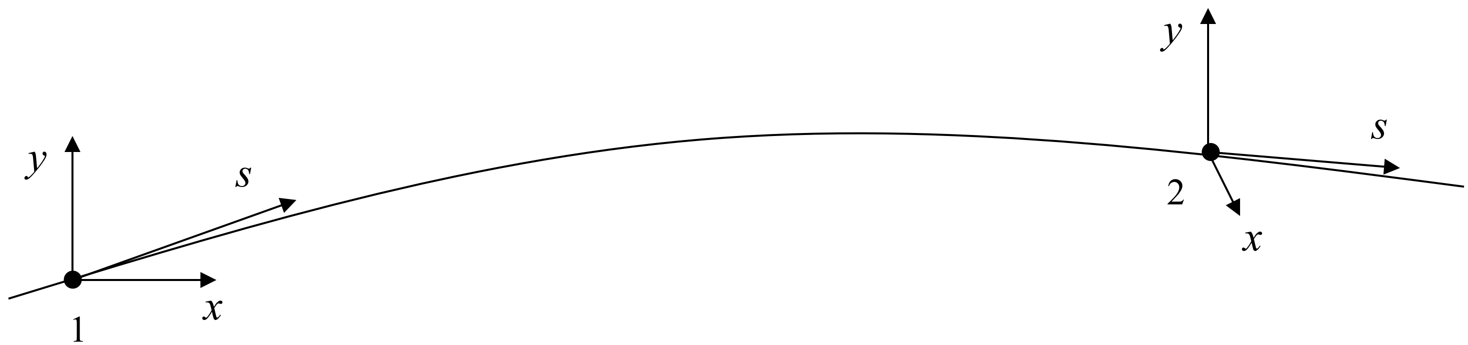
Beam transfer lines

Transfer lines transport beams between accelerators (extraction of one to injection of the next) and on to experimental targets and beam dumps

- Requirements:
  - Geometric link between machines/experiment
  - Match optics between machines/experiment
  - Preserve emittance
  - Change particles' charge state (stripping foils)
  - Measure beam parameters (measurement lines)
  - Protect downstream machine/experiment

General transport

Beam transport: moving from s_1 to s_2 through *n* elements, each with transfer matrix M_i



$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

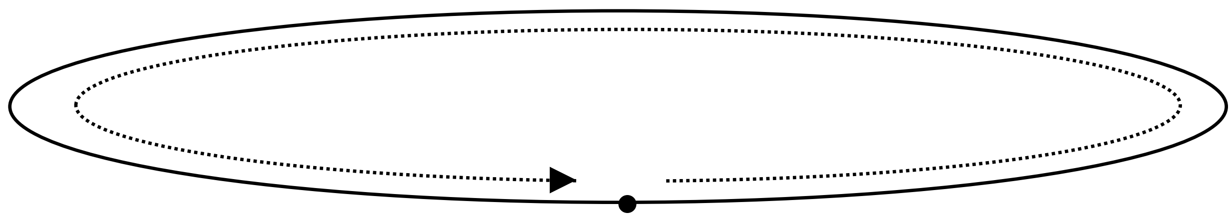
$$\mathbf{M}_{1\to 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

The transfer matrix (M_i) can be expressed using the Twiss formalism:

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[\left(\alpha_1 - \alpha_2\right)\cos\Delta\mu - \left(1 + \alpha_1\alpha_2\right)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

Circular machine

Circumference = L



One turn:

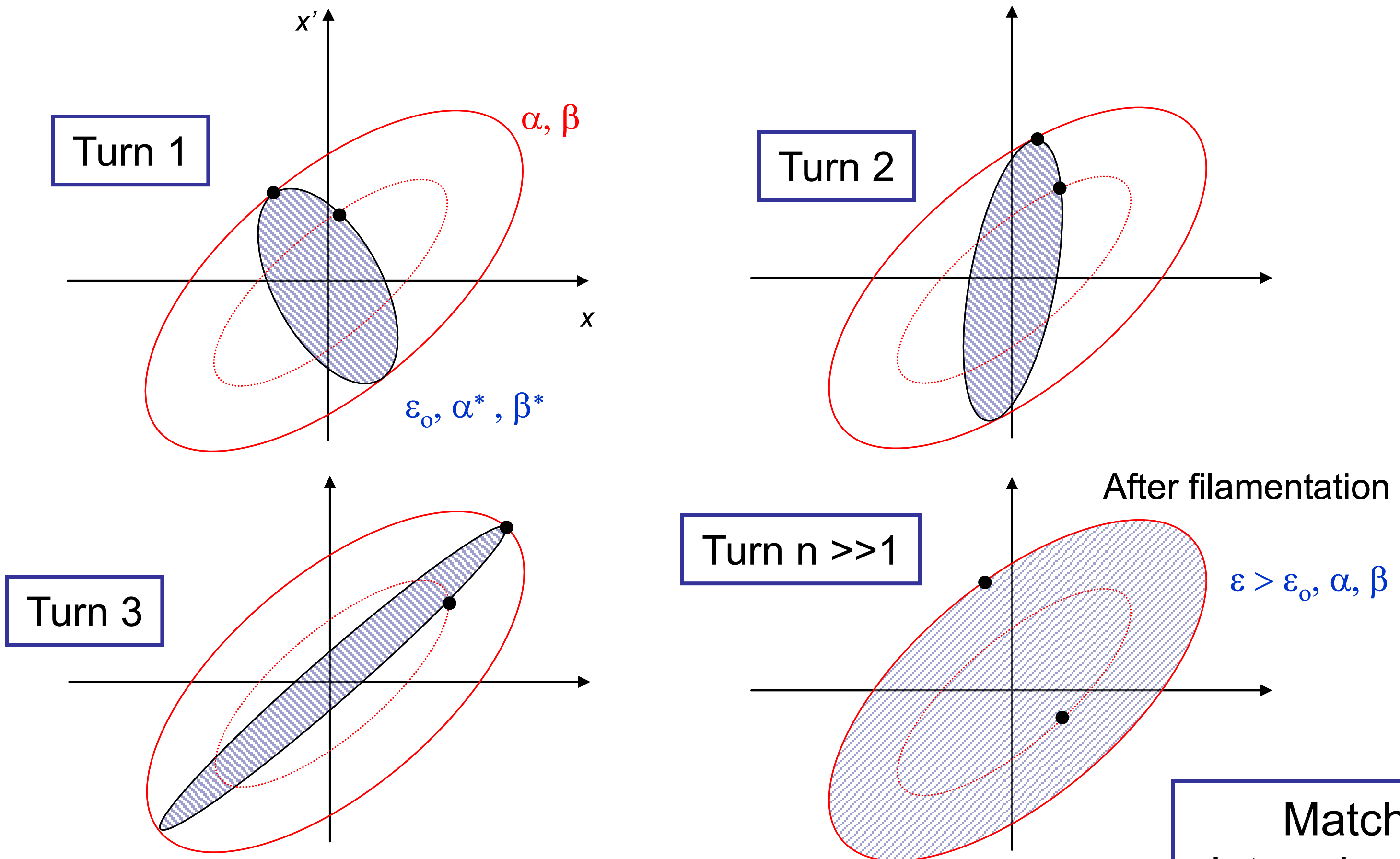
$\Delta\mu = 2\pi Q$

$$\mathbf{M}_{1\to 2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta \left(1+\alpha^2\right) \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $D_1 = D_2$
- This condition *uniquely* determines α(*s*), β(*s*), μ(*s*), D(s) around the whole ring
  - i.e. a single matched ellipse exists for each given location, *s*

Circular Machine

- At a location with matched ellipse (α, β) a mismatched injected beam (α^*, β^*) with emittance ε_0, generates (via filamentation) a larger ellipse with the matched α, β, but larger emittance: $\varepsilon > \varepsilon_0$

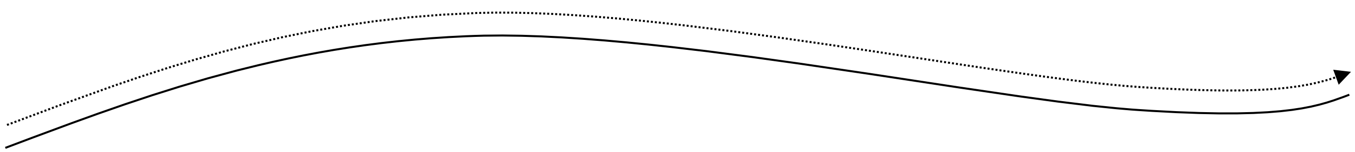


Matched ellipse determines beam shape

Transfer line

One pass: $$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$



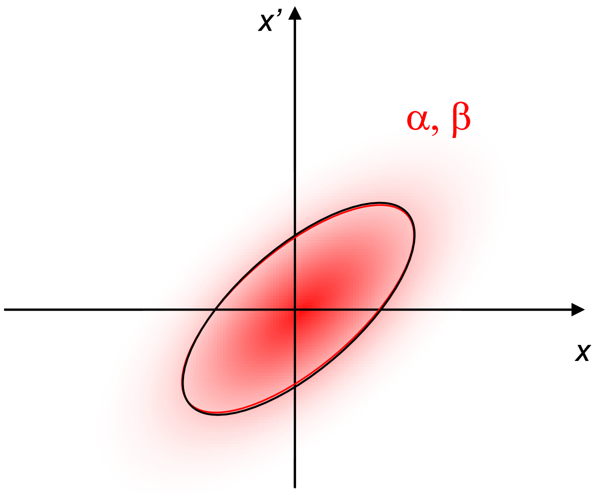
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

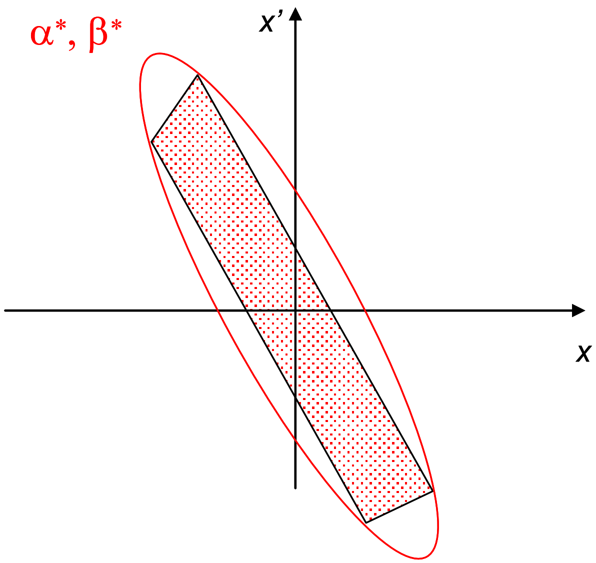
- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, α(s) β(s) are functions of α_1 and β_1

Transfer line

- Initial α, β are defined for a transfer line by the beam shape at the entrance



Gaussian beam

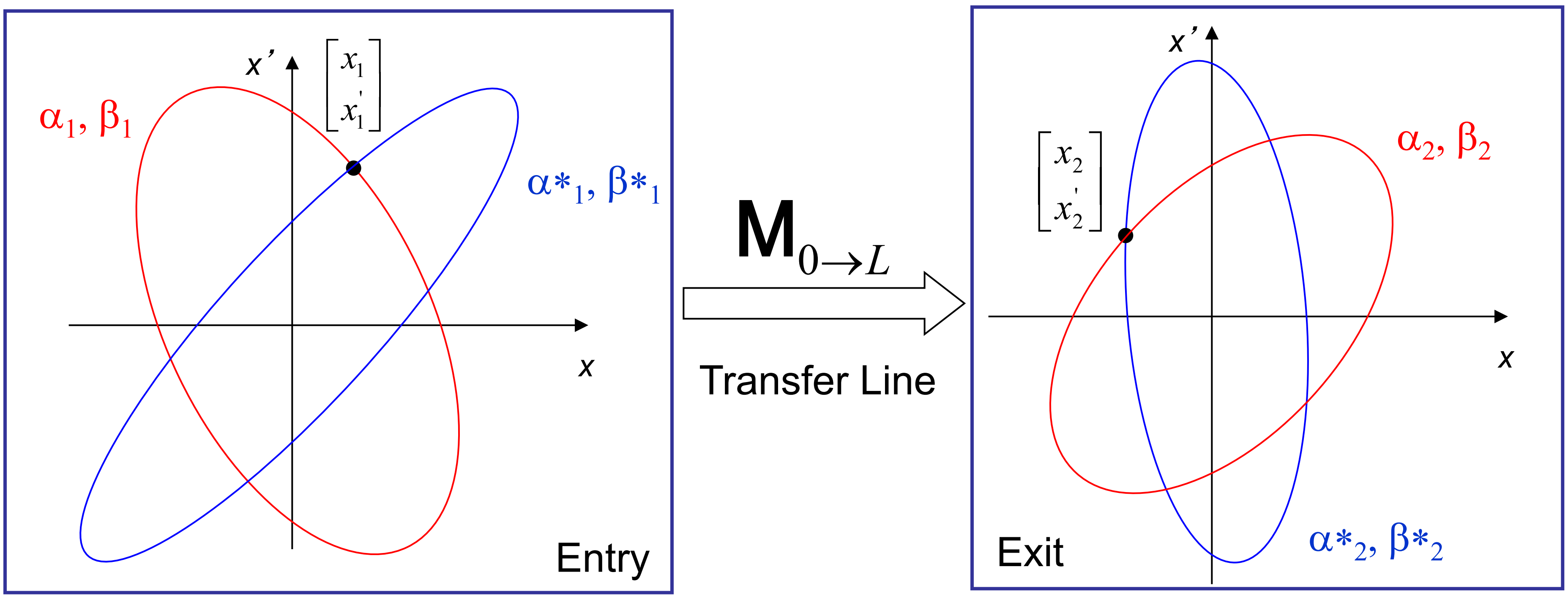


Non-Gaussian beam
(e.g. slow extracted)

- Propagation of this beam ellipse depends on the line
- <u>A transfer line optics is different for different input beams:</u>
  - Synchrotrons are often multi-purpose, accelerating different beams but extracting through a common line transfer line: optics must switch to match the input and output conditions for each beam type

Transfer line

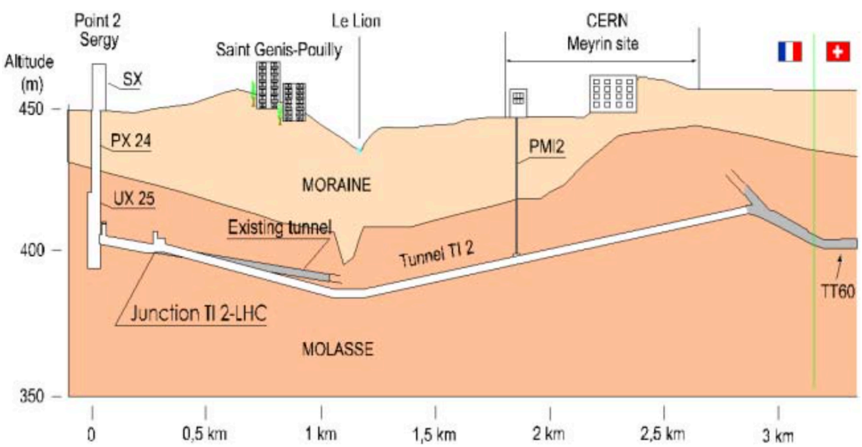
- On a single pass of a finite transfer line there is no regular motion from entrance to exit
  - Periodicity is not enforced: it's actually a design choice
  - Infinite number of possible starting ellipses are transported to an infinite number of final ellipses



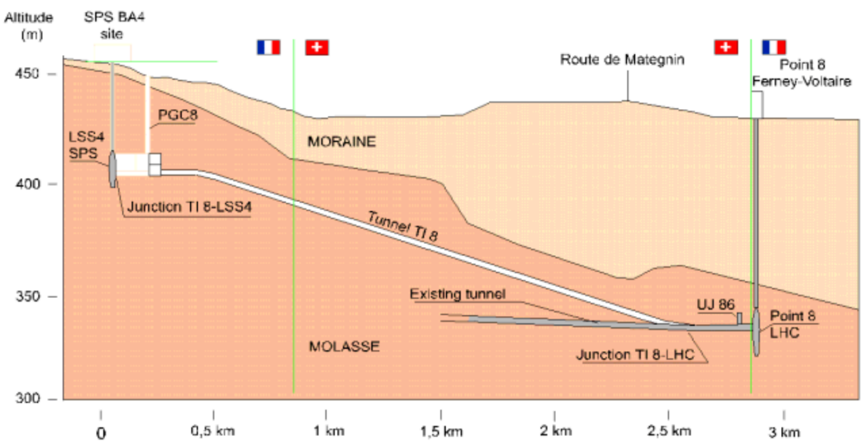
Linking machines

- Beams have to be transported from extraction of one machine to injection of the next machine:
  - Trajectory must be matched in all 6 geometric degrees of freedom (x,y,z,θ,Φ,ψ)

- Other important constraints can include:
  - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology or other obstacles, etc.



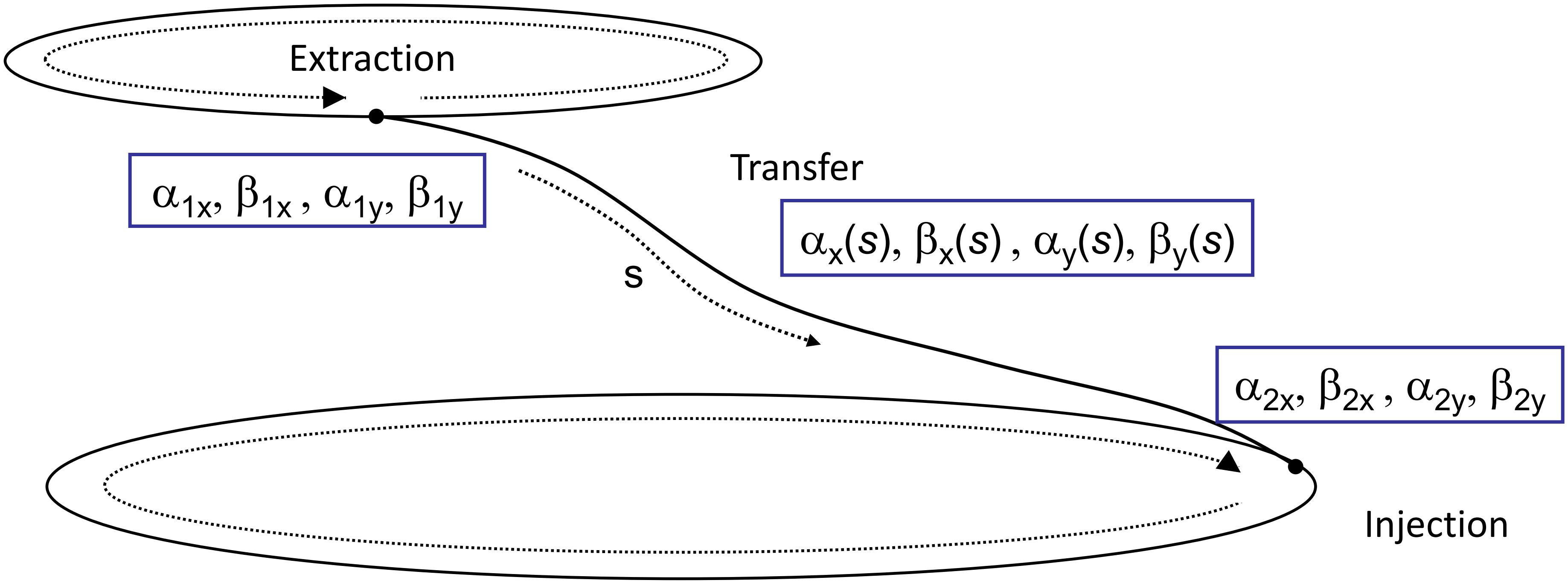
SPS to LHC transfer tunnel TI2 (Beam 1)



SPS to LHC transfer tunnel TI8 (Beam 2)

An example of how geology can influence transfer line design

Linking machines



The Twiss parameters can be propagated when the transfer matrix **M** is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

Linking machines

- Linking the optics is a complicated process:
  - Parameters at start of line have to be propagated to matched parameters at the end of the line (injection to another machine, fixed target etc.)
  - Need to "match" 8 variables (α_x, β_x, D_x, D'_x and α_y, β_y, D_y, D'_y)
  - Matching done with number of independently power ("matching") quadrupoles
  - Maximum β and D values are imposed by magnetic apertures
  - Other constraints exist:
    - Phase conditions for collimators
    - Insertions for special equipment like stripping foils
    - Low beam energy ($\beta \ll 1$) re-bunching cavities might be necessary, i.e. RF gymnastics in the transfer line

- Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error.

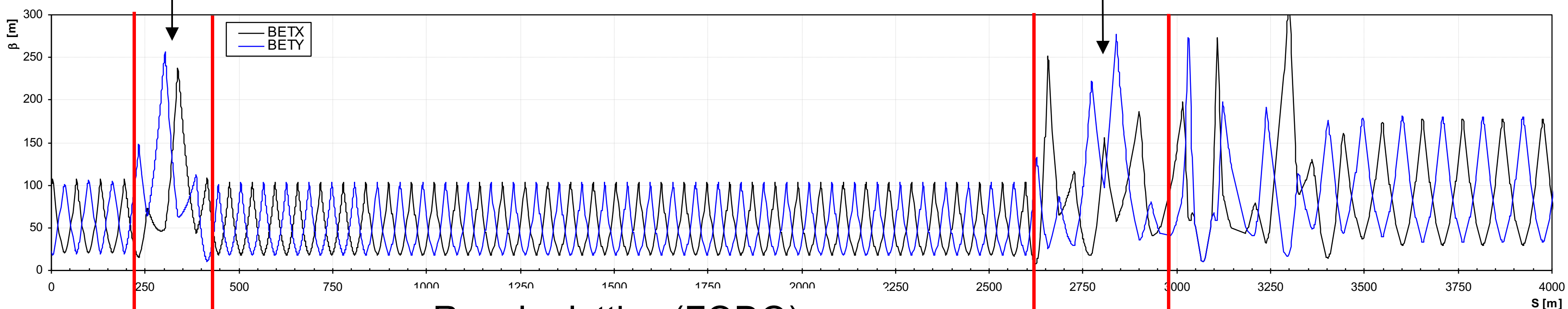
Optics matching

Initial matching section

Independently powered (tuneable) quadrupoles

Final matching section

Independently powered (tuneable) quadrupoles and (in this case) passive protection devices



Regular lattice (FODO)
(elements all powered in series
with same strengths)

SPS

LHC

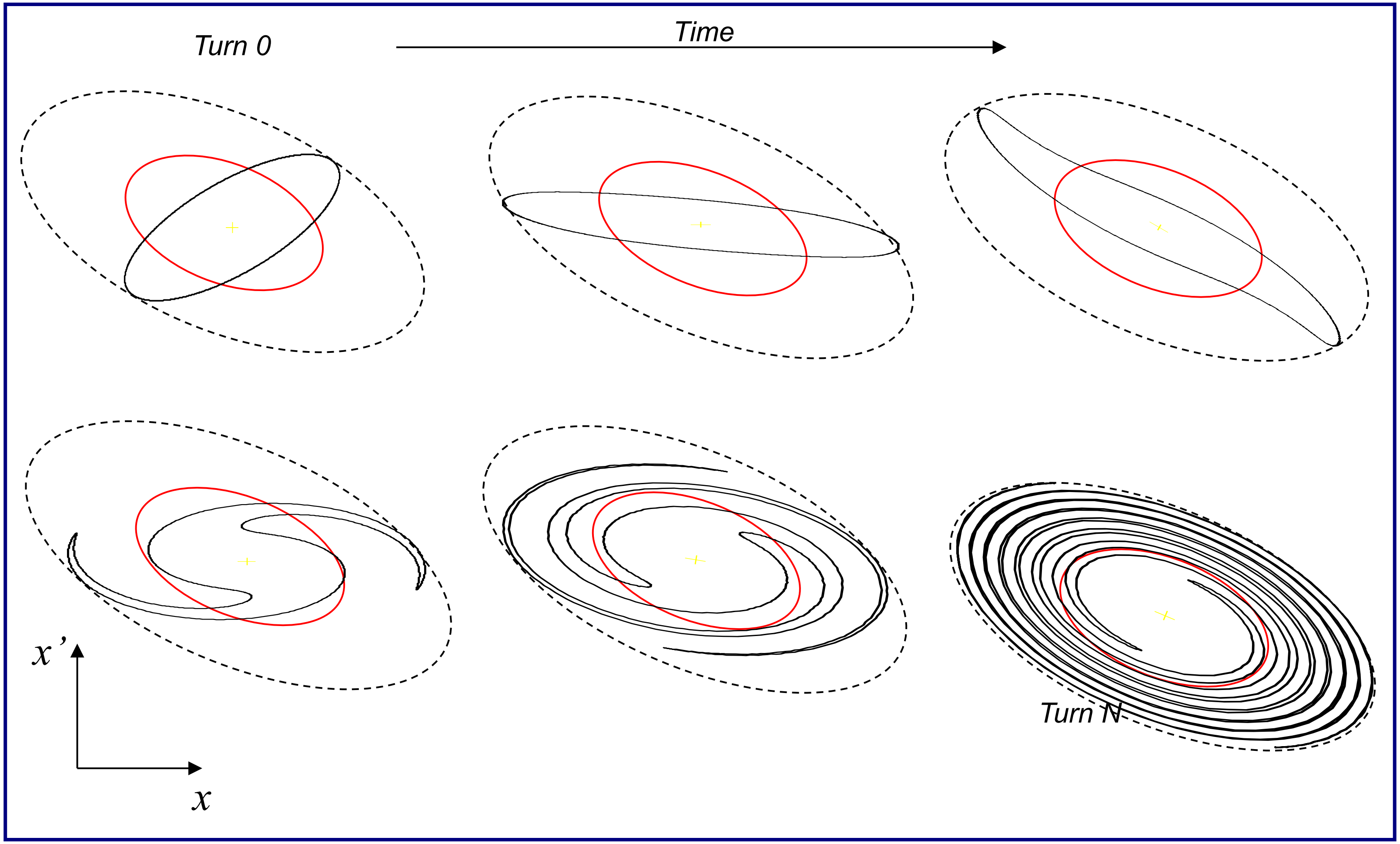
SPS to LHC Transfer Line (3 km)

Extraction point

Injection point

Optical mismatch at injection

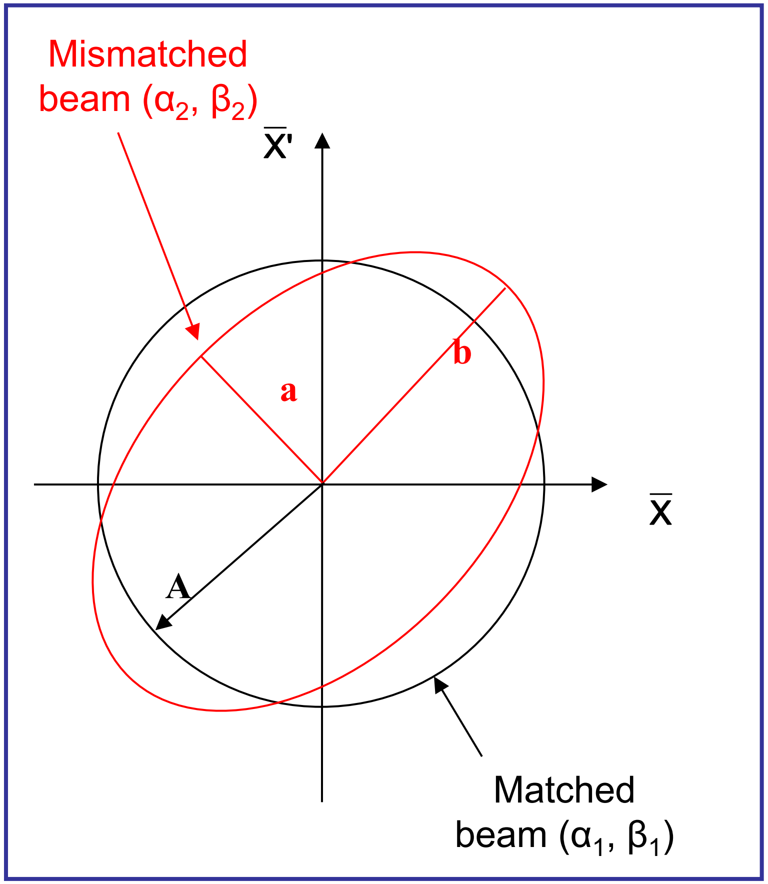
- Filamentation fills larger ellipse with same shape as matched ellipse



- Dispersion mismatch at injection will also cause emittance blow-up

Blow-up from betatron mismatch

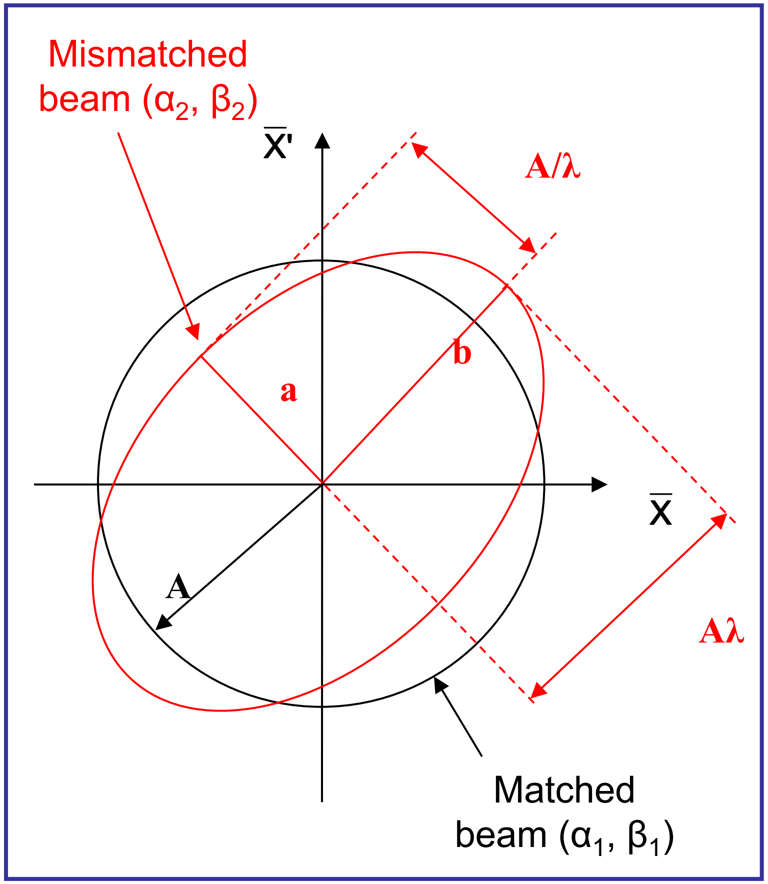
- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch
- Filamentation will produce an emittance increase
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse



Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch
- Filamentation will produce an emittance increase
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse
- The emittance after filamentation:

$$\varepsilon_{diluted} = \frac{\varepsilon_{matched}}{2}\left(\lambda^2 + \frac{1}{\lambda^2}\right) \quad \text{where} \quad \lambda = \sqrt{b/a}$$



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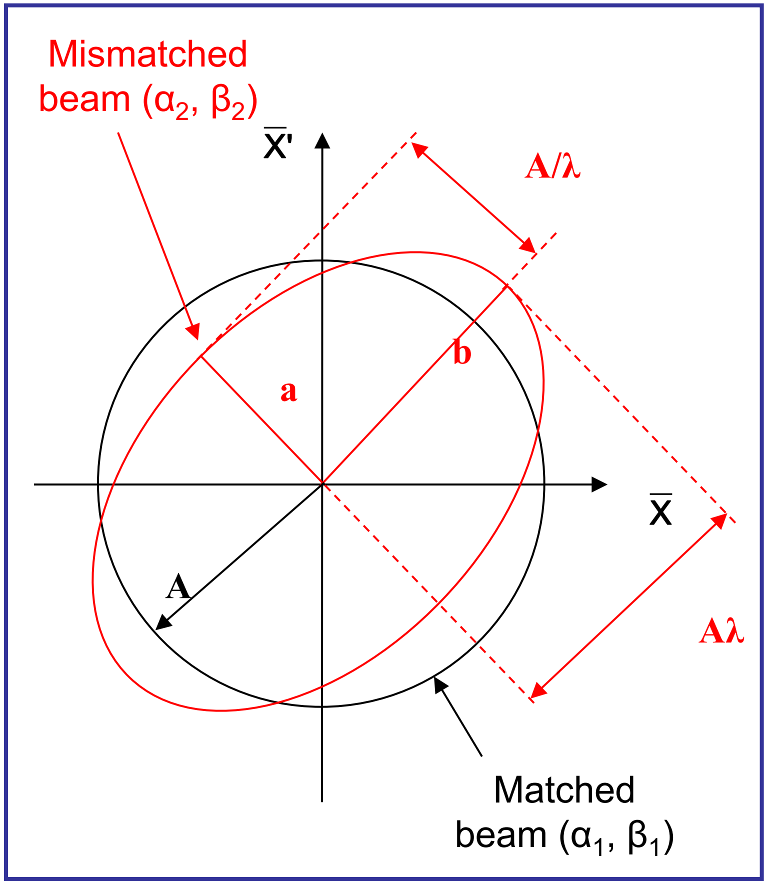
$$\varepsilon_{diluted} = \frac{\varepsilon_{matched}}{2}\left(\lambda^2 + \frac{1}{\lambda^2}\right) \quad \text{where} \quad \lambda = \sqrt{b/a}$$

- Writing λ as a function of the matched and mismatched Twiss parameters is an exercise in geometry:

$$\varepsilon_{diluted} = \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)\varepsilon_{matched}$$

See later slides for derivation



Blow-up from betatron mismatch

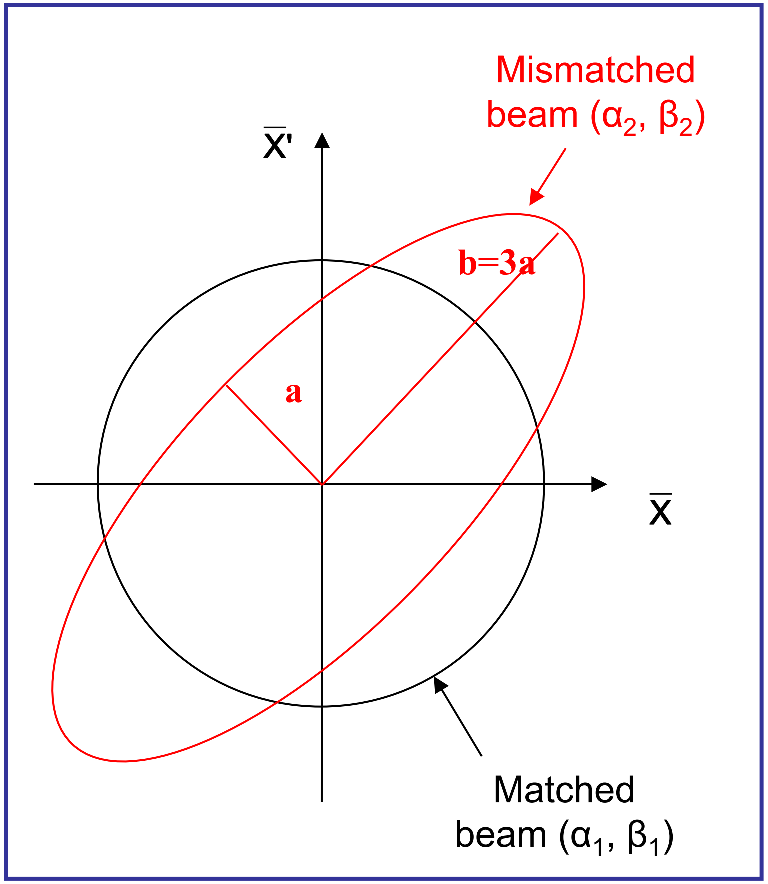
- A numerical example…

- Consider b = 3a for the mismatched ellipse:

$$\lambda = \sqrt{b/a} = \sqrt{3}$$

$$\varepsilon_{diluted} = \frac{\varepsilon_{matched}}{2}\left(\lambda^2 + \frac{1}{\lambda^2}\right)$$

$$= 1.67\varepsilon_{matched}$$



Blow-up from betatron mismatch

- General betatron motion:

$$x_2 = \sqrt{a_2\beta_2}\sin(\varphi+\varphi_o), \qquad x'_2 = \sqrt{a_2/\beta_2}\left[\cos(\varphi+\varphi_o) - \alpha_2\sin(\varphi+\varphi_o)\right]$$

- Applying the normalisation transformation for the matched beam…

$$\begin{bmatrix} \overline{\mathrm{X}}_2 \\ \overline{\mathrm{X}}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

…an ellipse is obtained in normalised phase space:

$$A^2 = \overline{\mathrm{X}}_2^2 \underbrace{\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right]}_{\gamma_{new}} + \overline{\mathrm{X}}'^2_2 \underbrace{\frac{\beta_2}{\beta_1}}_{\beta_{new}} - 2\overline{\mathrm{X}}_2\overline{\mathrm{X}}'_2 \underbrace{\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]}_{\alpha_{new}}$$

Blow-up from betatron mismatch

- From general ellipse properties one can write:

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right) \qquad \text{where} \qquad H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

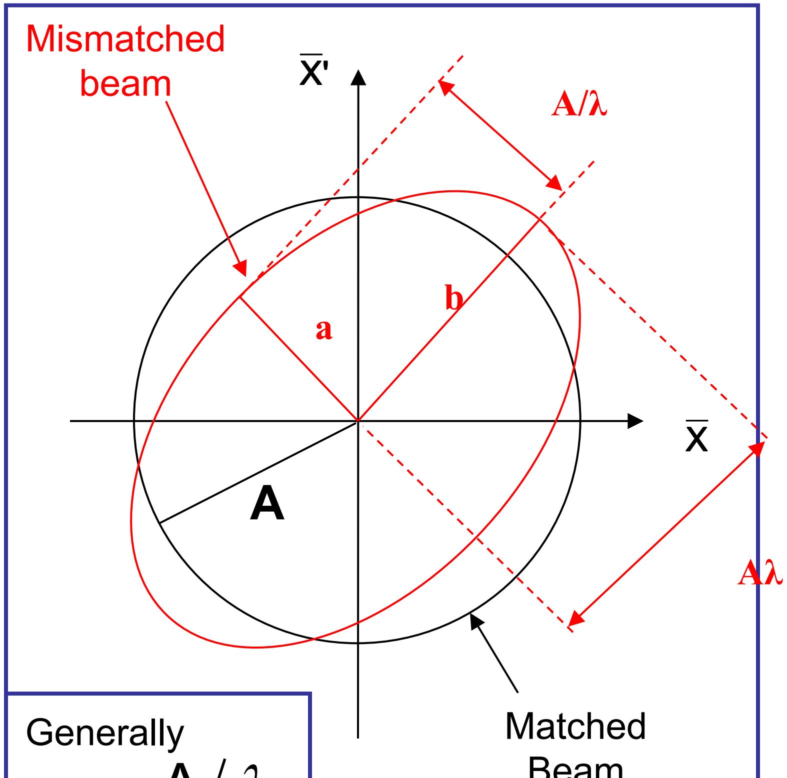
Giving:

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right),$$

$$\frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

- The co-ordinates of the mismatched beam can be expressed:

$$\overline{\mathbf{X}}_{new} = \lambda \cdot \mathbf{A}\sin(\phi + \phi_1), \qquad \overline{\mathbf{X}}'_{new} = \frac{1}{\lambda}\mathbf{A}\cos(\phi + \phi_1)$$



Generally

$$a = A/\lambda$$
$$b = A \cdot \lambda$$

Blow-up from betatron mismatch

- We can evaluate the square of the distance of a particle from the origin as:

$$\mathbf{A}_{new}^{2} = \overline{\mathbf{X}}_{new}^{2} + \overline{\mathbf{X}}'^{2}_{new} = \lambda^{2} \cdot \mathbf{A}_{0}^{2} \sin^{2}(\phi + \phi_{1}) + \frac{1}{\lambda^{2}} \mathbf{A}_{0}^{2} \cos^{2}(\phi + \phi_{1})$$

- The new emittance is the average for all particles with positions Ai over all phases:

$$\varepsilon_{diluted} = \frac{1}{2}\left\langle A_{new}^{2} \right\rangle = \frac{1}{2}\left(\lambda^{2} \left\langle A_{0}^{2} \sin^{2}(\varphi + \varphi_{1}) \right\rangle + \frac{1}{\lambda^{2}} \left\langle A_{0}^{2} \cos^{2}(\varphi + \varphi_{1}) \right\rangle \right)$$

$$= \frac{1}{2}\left\langle A_{0}^{2} \right\rangle \left(\lambda^{2} \left\langle \cancelto{1/2}{\sin^{2}(\varphi + \varphi_{1})} \right\rangle + \frac{1}{\lambda^{2}} \left\langle \cancelto{1/2}{\cos^{2}(\varphi + \varphi_{1})} \right\rangle \right) = \frac{1}{2}\varepsilon_{0}\left(\lambda^{2} + \frac{1}{\lambda^{2}} \right)$$

- If we're feeling diligent, we can substitute back for λ:

$$\varepsilon_{diluted} = \frac{1}{2}\varepsilon_{matched}\left(\lambda^{2} + \frac{1}{\lambda^{2}} \right) = H\varepsilon_{matched} = \frac{1}{2}\varepsilon_{matched}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^{2} + \frac{\beta_2}{\beta_1} \right)$$

where subscript 1 refers to the matched and 2 refers to mismatched cases

Blow-up from dispersion mismatch

- Dispersion mismatch will also introduce emittance blow-up through filamentation much like optical mismatch
- Introducing normalised dispersion: $D_n = \frac{D}{\sqrt{\beta}}$ $\quad D'_n = \frac{\alpha}{\sqrt{\beta}} D + \sqrt{\beta} D'$
- With a momentum error of $\delta = \frac{\Delta p}{p}$ the mismatch is:

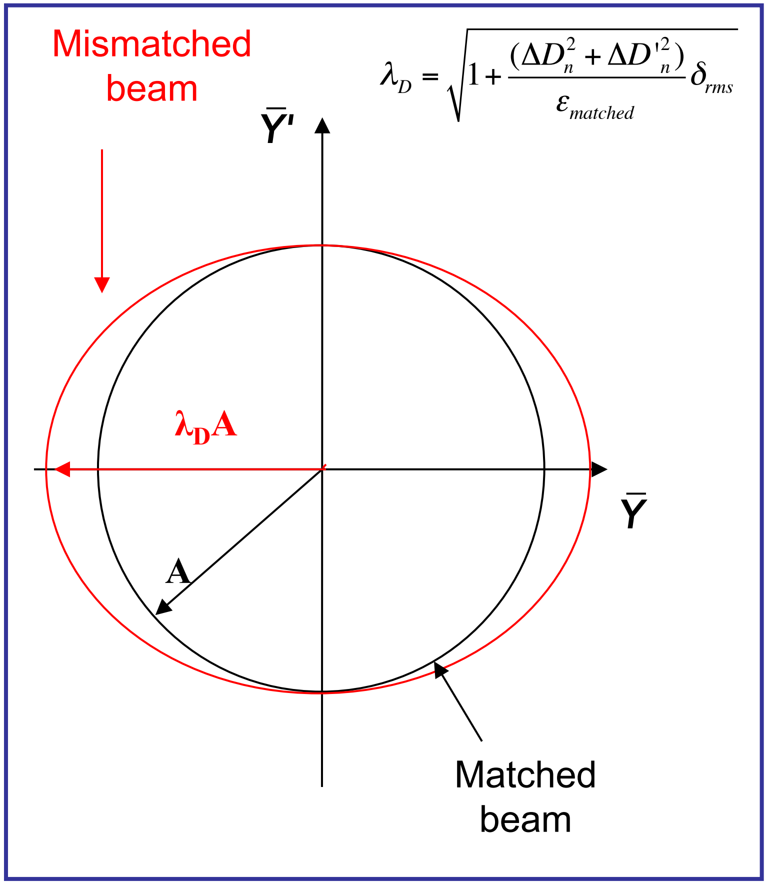
$$\bar{X} = \bar{X} + \Delta D_n \delta \qquad \bar{X}' = \bar{X}' + \Delta D'_n \delta$$

- Rotating the reference frame to a convenient reference (see plot):

$$\bar{Y} = \bar{Y} + \sqrt{\Delta D_n^2 + \Delta D'^2_n}\,\delta \qquad \bar{Y}' = \bar{Y}'$$

- And averaging over a distribution of particles, one can write the emittance blow-up as:

$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{\Delta D_n^2 + \Delta D'^2_n}{2} \delta_{rms}^2$$



Blow-up from steering error

- The new particle coordinates in normalised phase space are:

$$\bar{X}_{error} = \bar{X}_0 + L\cos\theta$$

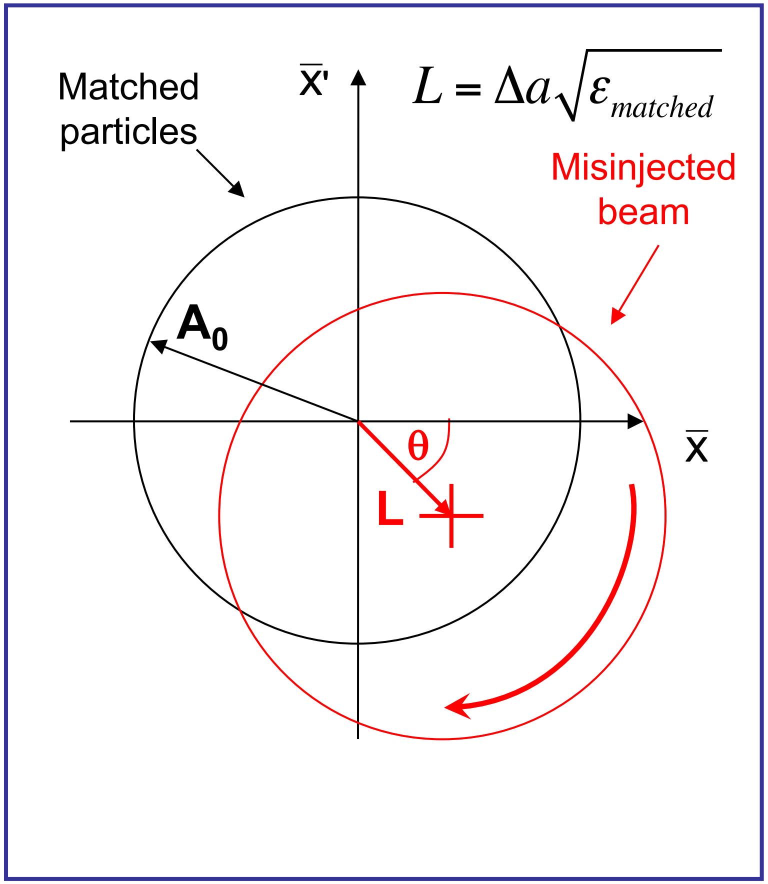
$$\bar{X}'_{error} = \bar{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\boldsymbol{A}_i^2 = \bar{X}_{0,i}^2 + \bar{X}'^2_{0,i}$$

- The emittance of the distribution is:

$$\varepsilon_{matched} = \langle \boldsymbol{A}_i^2 \rangle / 2$$



Blow-up from steering error

- So we plug in the new coordinates:

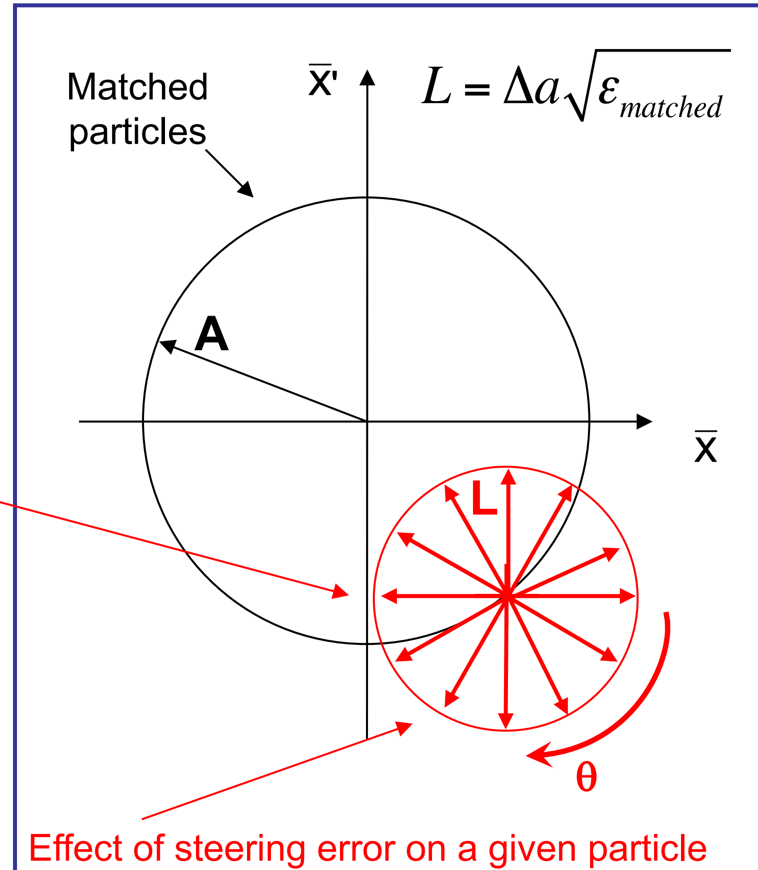
$$\boldsymbol{A}^2_{error} = \bar{X}^2_{error} + \bar{X}'^2_{error}$$
$$= (\bar{X}_0 + L\cos\theta)^2 + (\bar{X}'_0 + L\sin\theta)^2$$
$$= \bar{X}_0^2 + \bar{X}'^2_0 + 2L(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta) + \overbrace{L^2}^{\cos^2\theta + \sin^2\theta = 1}$$

- Taking the average over distribution:

$$\langle \boldsymbol{A}^2_{error} \rangle = \langle \boldsymbol{A}^2_0 \rangle + 2L(\underbrace{\langle \bar{X}_0\cos\theta \rangle}_{0} + \underbrace{\langle \bar{X}'_0\sin\theta \rangle}_{0}) + \langle L^2 \rangle$$
$$= 2\varepsilon_{matched} + L^2$$

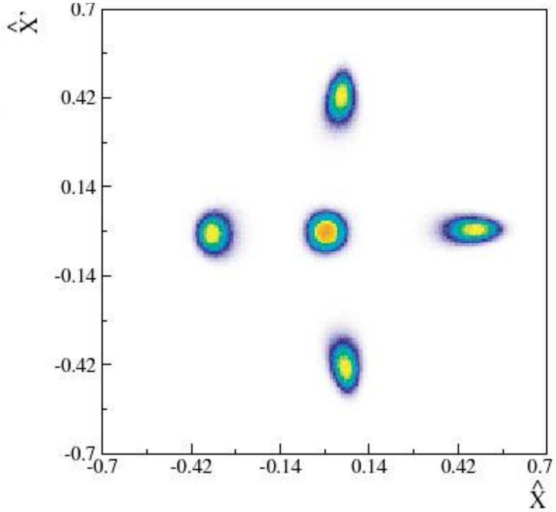
- Giving the diluted emittance as:

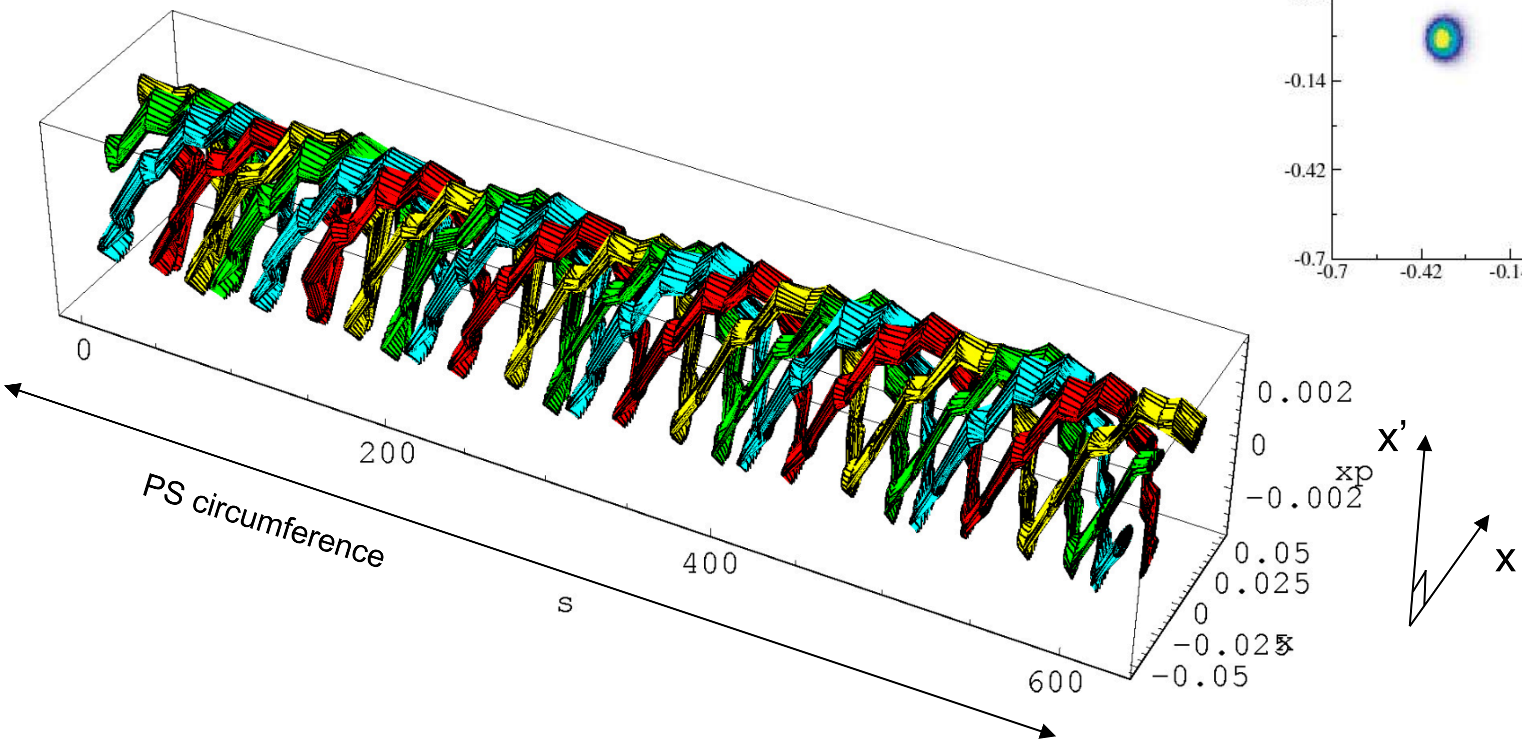
$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{L^2}{2}$$
$$= \varepsilon_{matched}\left[1 + \frac{\Delta a^2}{2}\right]$$



How many beams ?!

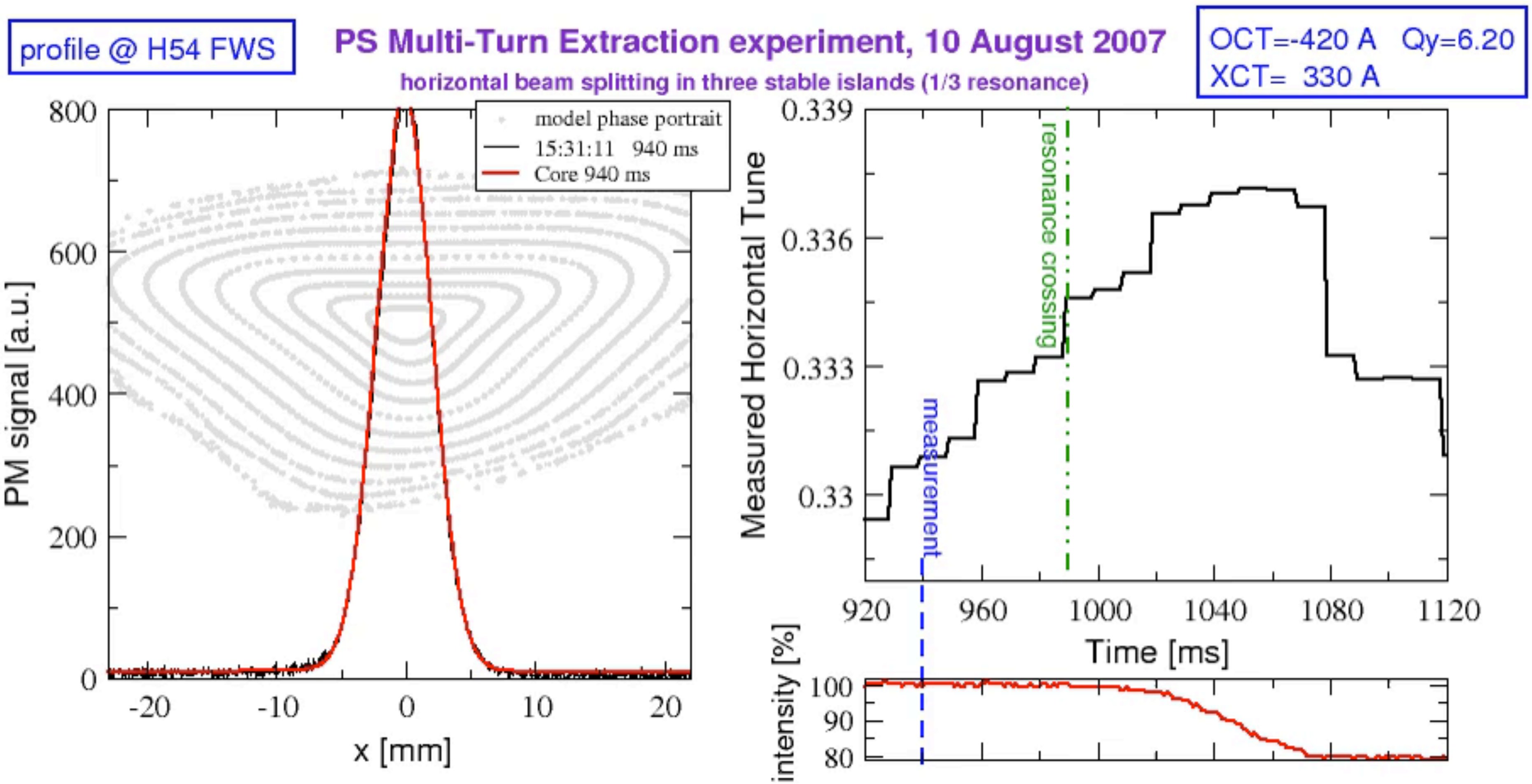
- In the PS case we end up with two beams circulating on distinct closed orbits in the machine (in the horizontal plane):
  - the islands are a separate, continuous entity (if de-bunched) wrapped around the machine circumference 4 times
  - the core circulates as usual
- Two fast "kicks" (islands + core) to extract





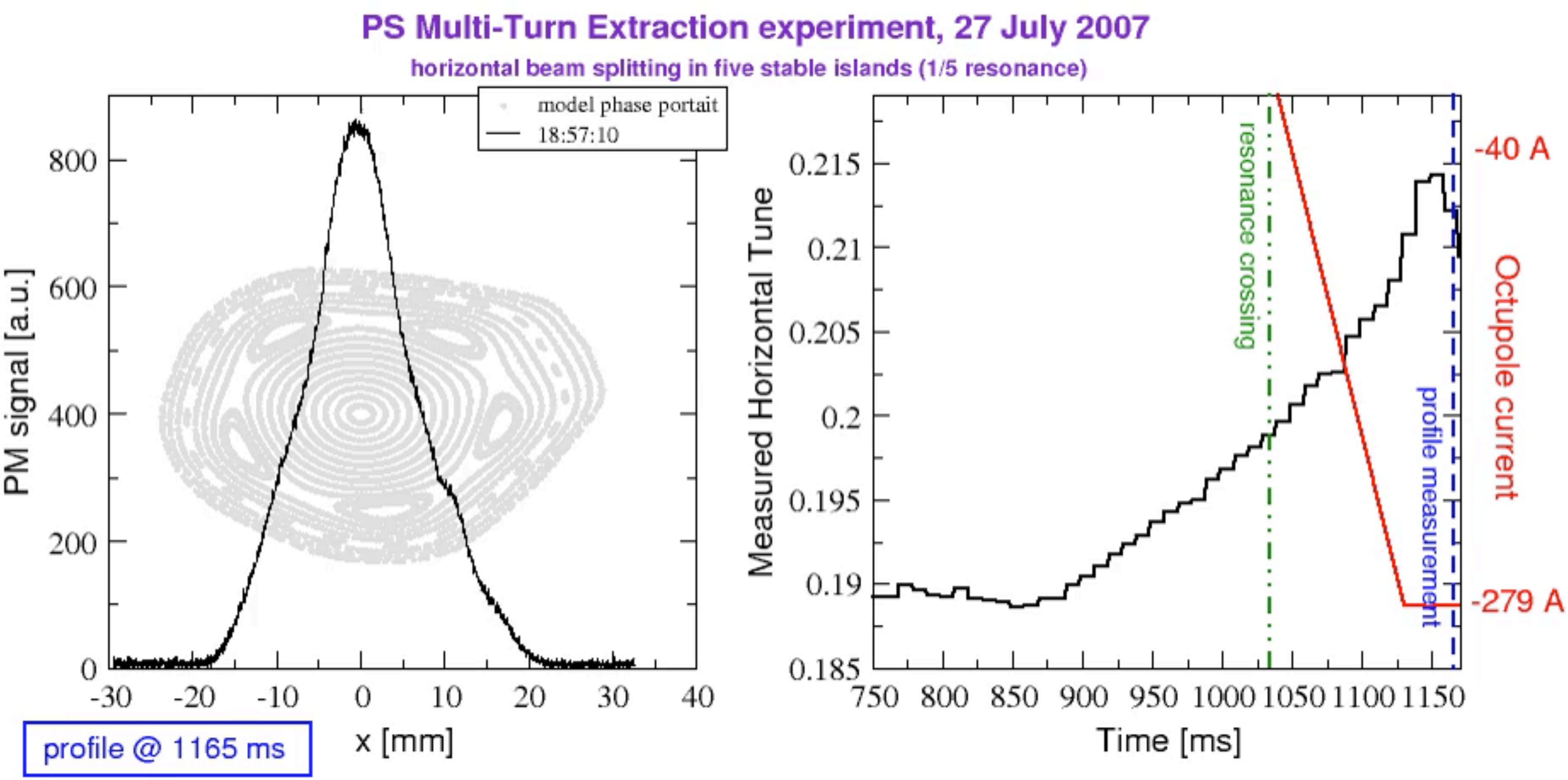
PS test: splitting in three stable islands

Exciting the unstable 1/3rd resonance the central island (beam core) is depleted. In the movie the evolution of the beam profile is shown. It was measured at a single machine section by means of horizontal flying wire installed in section 54 of the CERN Proton Synchrotron. Essentially no losses are observed for a moderate separation of the beamlets. No optimization of the working point was performed due to problems with the beam instrumentation. The beam used is a **single-bunch**, **medium-intensity** (about 2.6×10^{12}) proton beam.



PS test: splitting into six stable islands

The 1/5th stable resonance was also crossed. No beam losses were observed. The beam used is a **single-bunch**, **medium-intensity** (about 2.6x10^{12}) proton beam. The movie shows a superposition of different measurements in terms of the octupole settings during the trapping process.



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