

An introduction to Magnets for Accelerators

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John Adams Institute
Accelerator Course

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This is an introduction to magnets as building blocks of synchrotrons / transfer lines

```
//  
// MADX Example 2: FODO cell with dipoles  
// Author: V. Ziemann, Uppsala University  
// Date: 060911
```

```
TITLE, 'Example 2: FODO2.MADX';
```

```
BEAM, PARTICLE=ELECTRON, PC=3.0;
```

```
DEGREE:=PI/180.0;
```

```
QF: QUADRUPOLE, L=0.5, K1=0.2;
```

```
QD: QUADRUPOLE, L=1.0, K1=-0.2;
```

```
B: SBEND, L=1.0, ANGLE=15.0*DEGREE;
```

```
FODO: SEQUENCE, REFER=ENTRY, L=12.0;
```

```
  QF1:    QF,          AT=0.0;
```

```
  B1:     B,           AT=2.5;
```

```
  QD1:    QD,          AT=5.5;
```

```
  B2:     B,           AT=8.5;
```

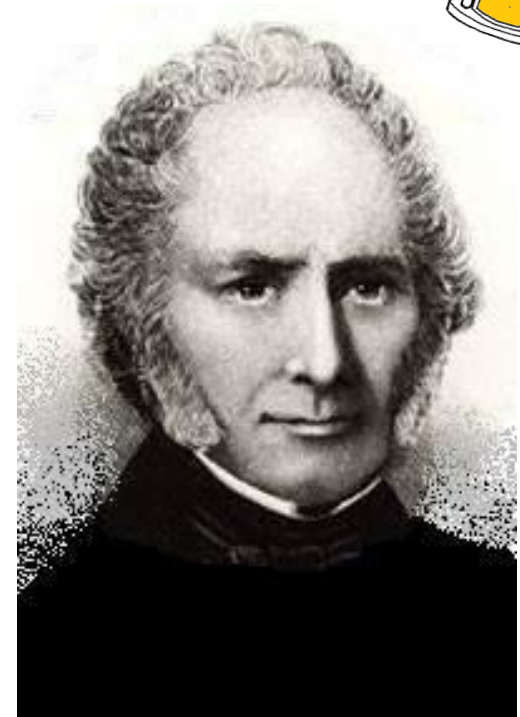
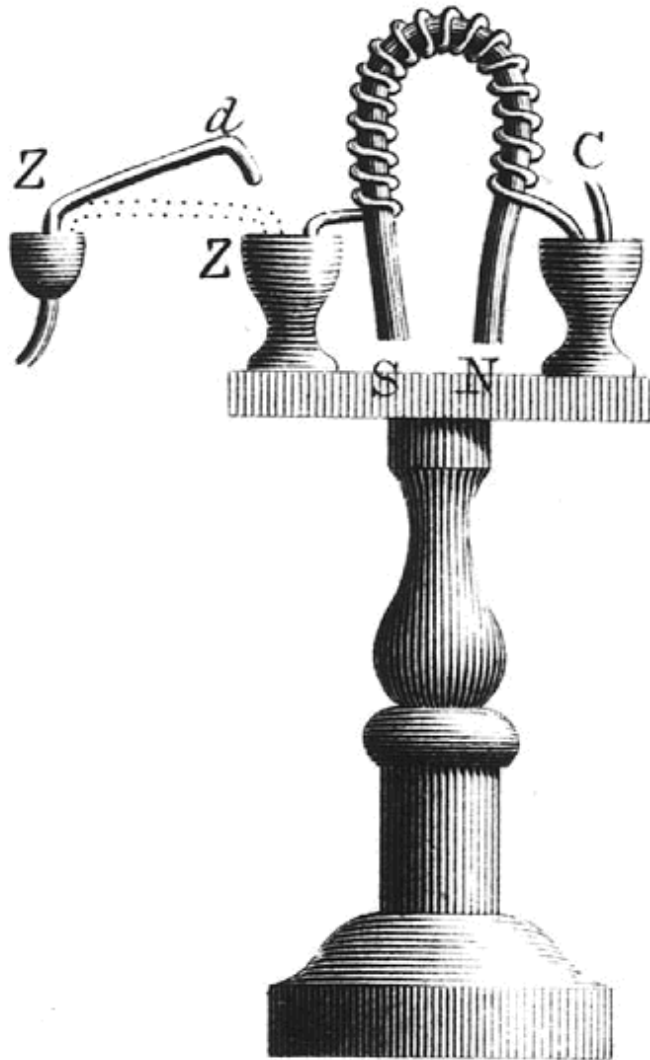
```
  QF2:    QF,          AT=11.5;
```

```
ENDSEQUENCE;
```

If you want to know more...

1. N. Marks, Magnets for Accelerators, J.A.I. Jan. 2015
2. D. Tommasini, Practical Definitions & Formulae for Normal Conducting Magnets
3. Lectures about magnets in CERN Accelerator Schools
4. Special CAS edition on magnets, Bruges, Jun. 2009
5. Superconducting magnets for particle accelerators in U.S. Particle Accelerator Schools
6. J. Tanabe, Iron Dominated Electromagnets
7. P. Campbell, Permanent Magnet Materials and their Application
8. K.-H. Mess, P. Schmüser, S. Wolff, Superconducting Accelerator Magnets
9. M. N. Wilson, Superconducting Magnets
10. A. Devred, Practical Low-Temperature Superconductors for Electromagnets
11. L. Rossi and E. Todesco, Electromagnetic design of superconducting dipoles based on sector coils

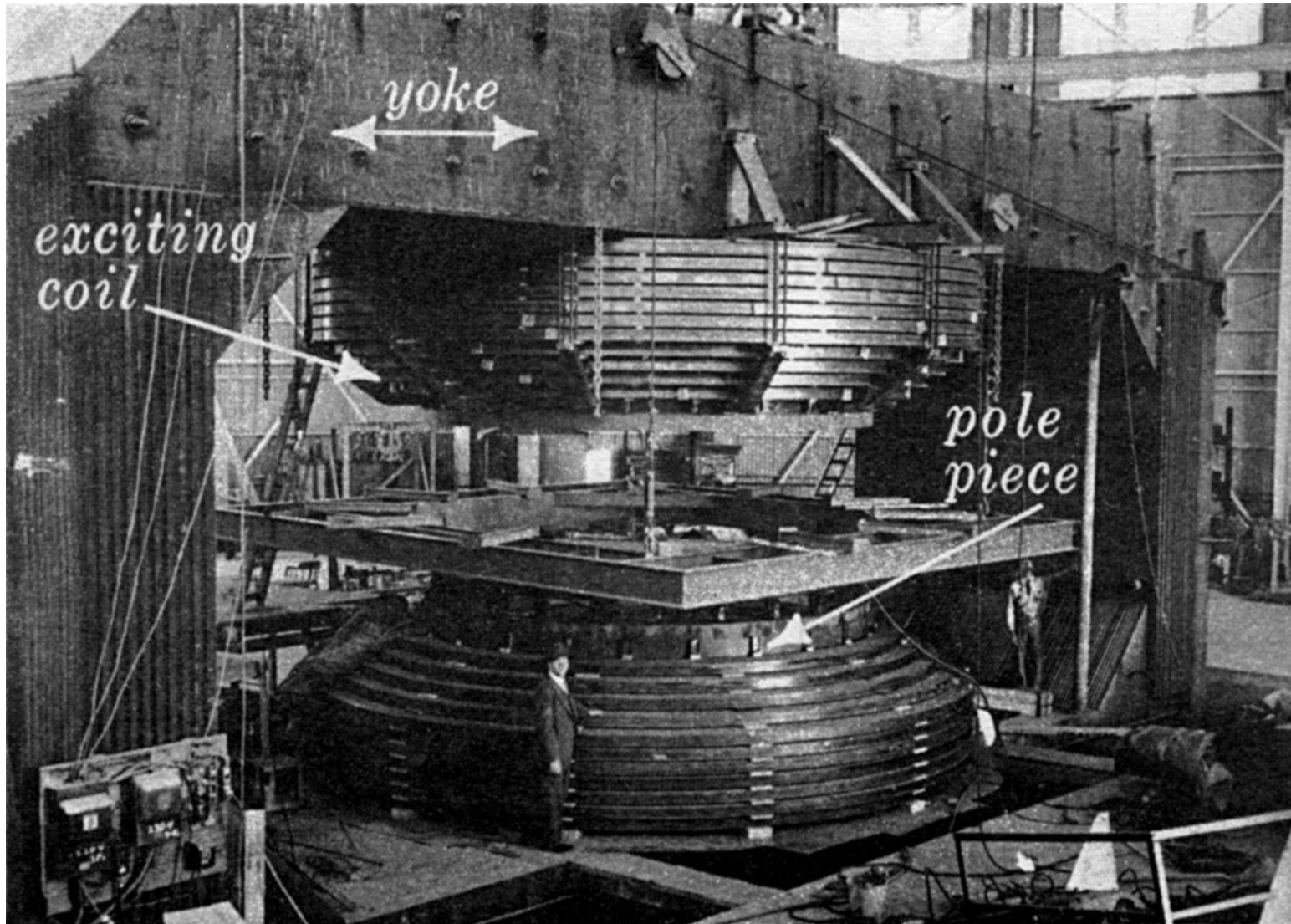
According to history, the first electromagnet (not for an accelerator) was built in England in 1824 by William Sturgeon



William Sturgeon



The working principle is the same as this large magnet, of the 184'' (4.7 m) cyclotron at Berkeley (picture taken in 1942)



This short course is organized in several blocks

1. Introduction, jargon, general concepts and formulae
2. Resistive magnets
3. Superconducting magnets
4. Tutorial with OPERA-2D

This is a classification of magnets based on their geometry / what they do to the beam

dipole

solenoid

quadrupole

combined function
bending

sextupole

corrector

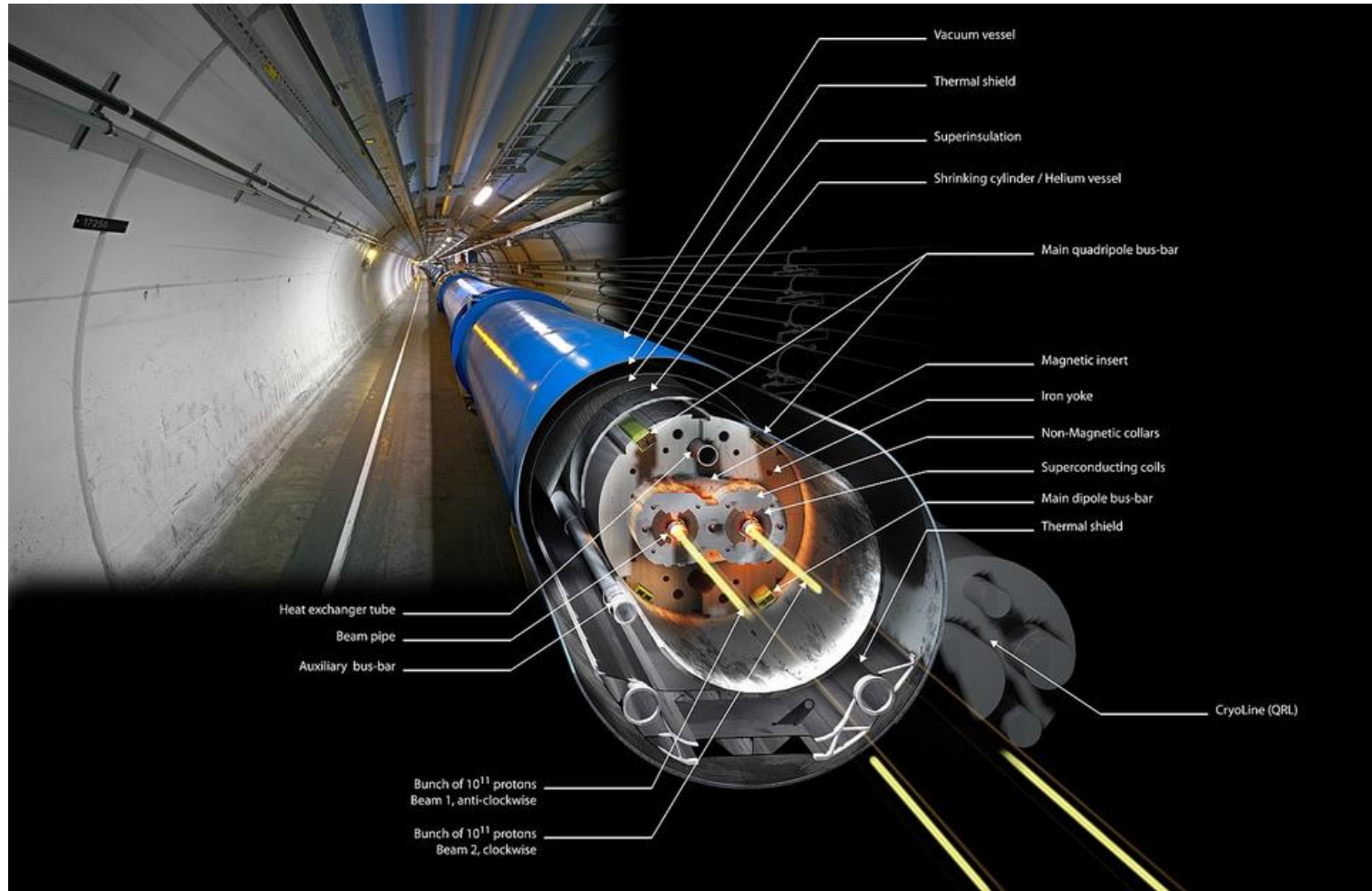
octupole

skew magnet

kicker

undulator / wiggler

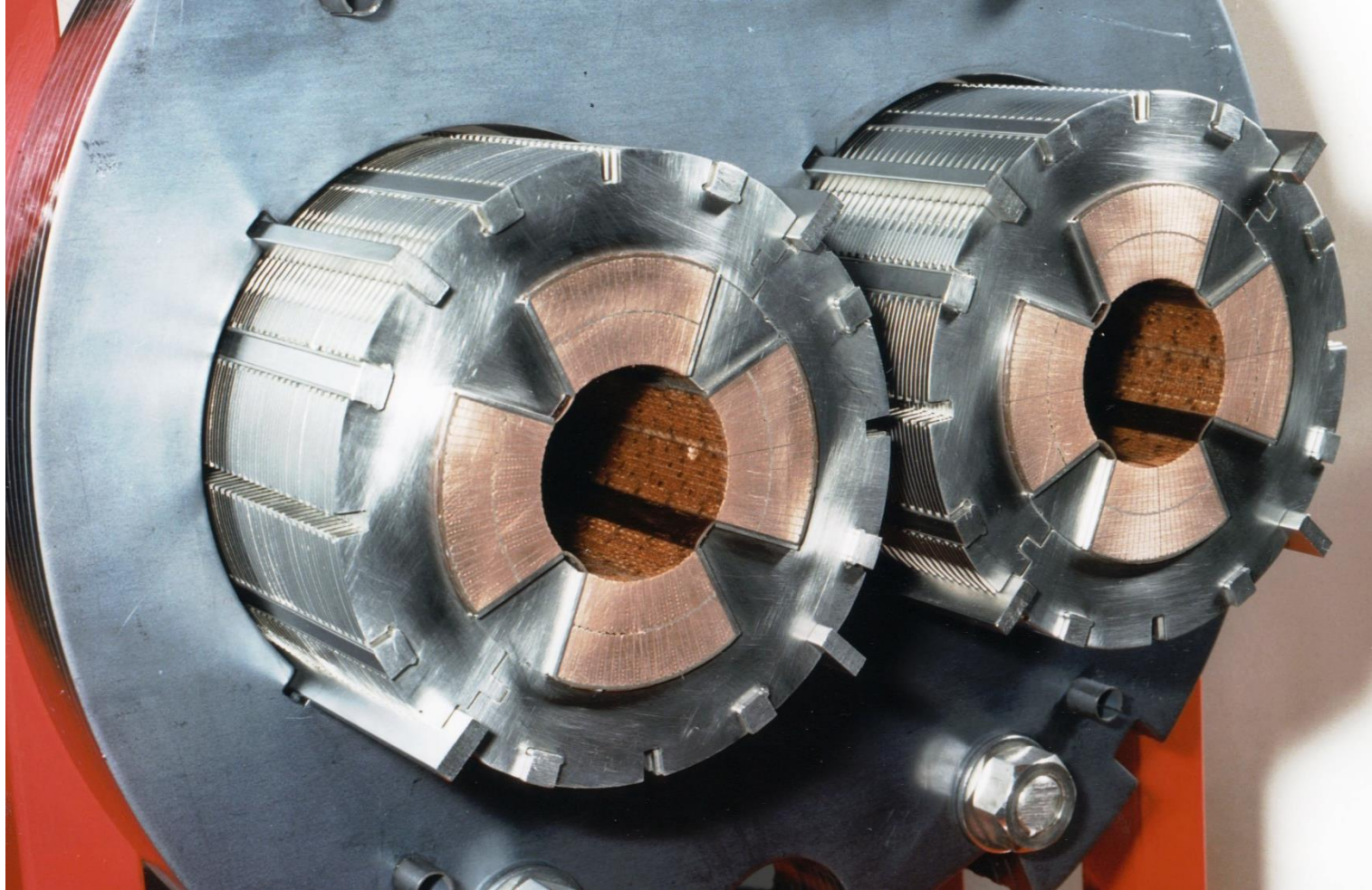
This is a main dipole of the LHC at CERN: $8.3\text{ T} \times 14.3\text{ m}$



These are main dipoles of the SPS at CERN: $2.0\text{ T} \times 6.3\text{ m}$



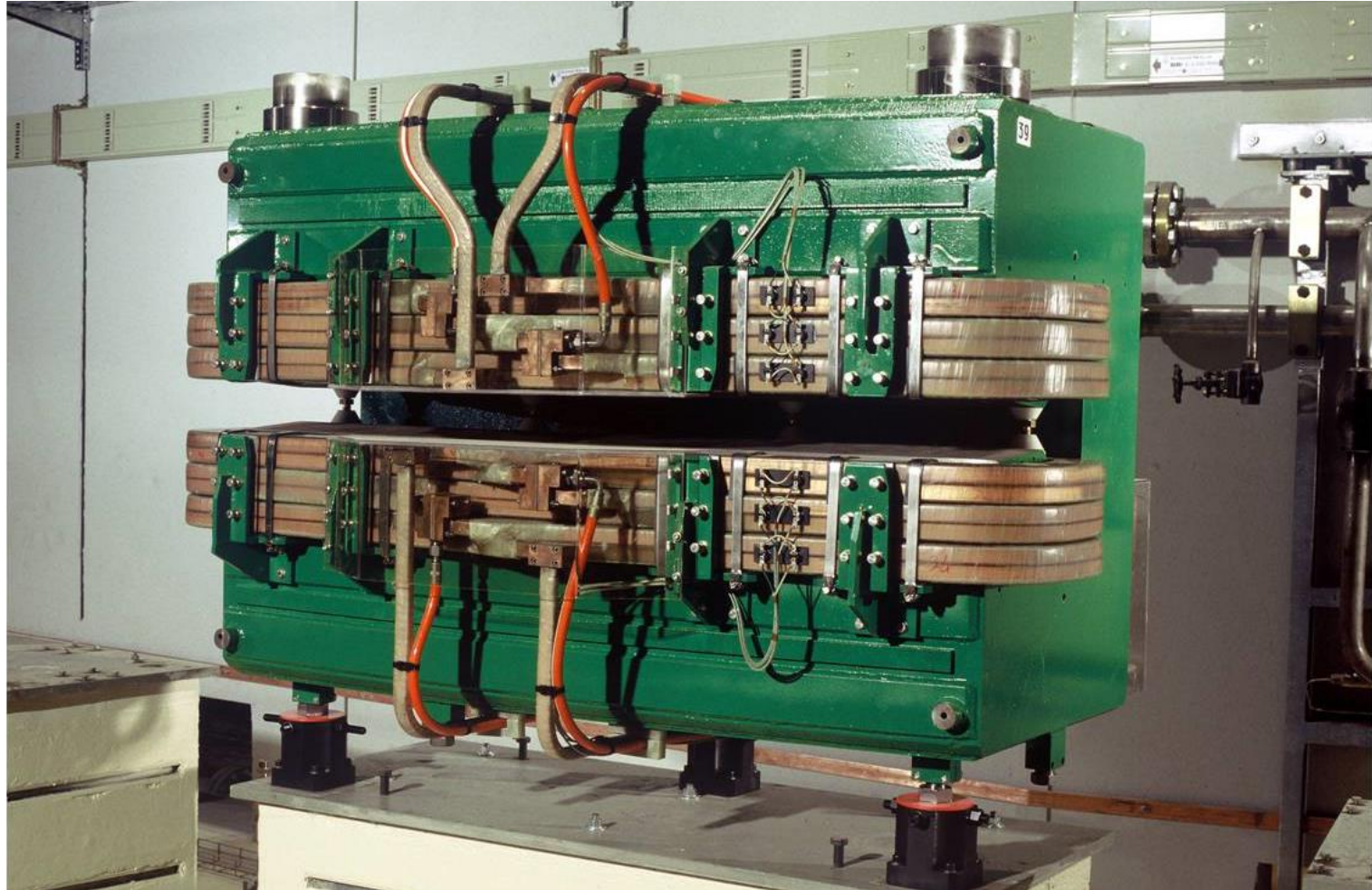
This is a cross section of a main quadrupole of the LHC at CERN:
 $223 \text{ T/m} \times 3.2 \text{ m}$



These are main quadrupoles of the SPS at CERN: 22 T/m \times 3.2 m



This is a combined function bending magnet of the ELETTRA light source



These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



Different classifications of magnets are also possible, for example based on technology

electromagnet

permanent magnet

iron dominated

coil dominated

normal conducting
(resistive)

superconducting

static

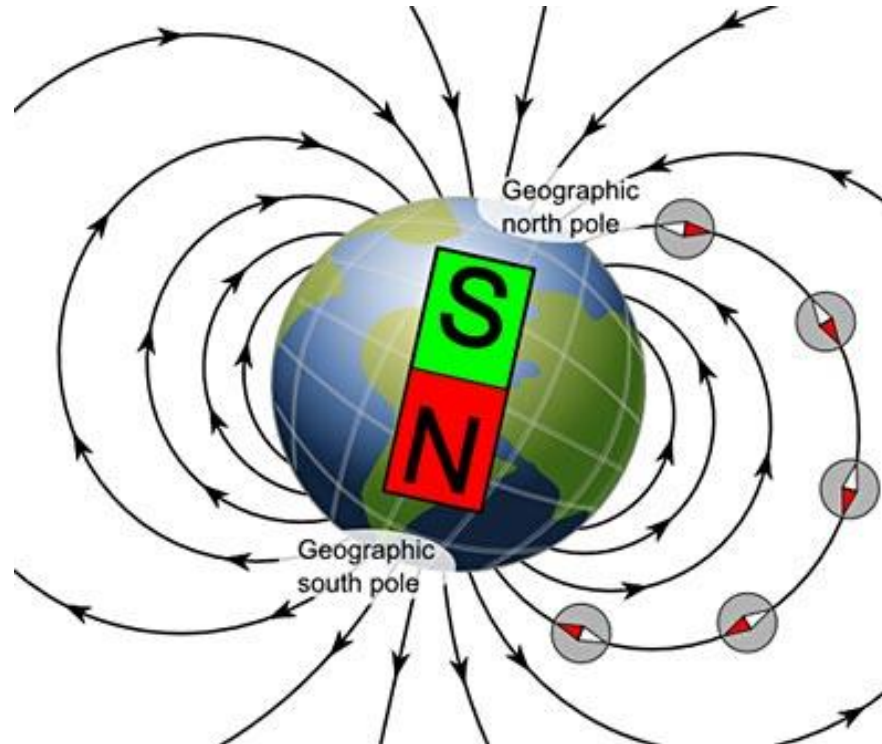
cycled / ramped
slow pulsed

fast pulsed

Nomenclature

B	magnetic field B field magnetic flux density magnetic induction	T (Tesla)
H	H field magnetic field strength magnetic field	A/m (Ampere/m)
μ_0	permeability of vacuum	$4\pi \cdot 10^{-7}$ H/m (Henry/m)
μ_r	relative permeability	dimensionless
μ	permeability, $\mu = \mu_0 \mu_r$	H/m

The polarity comes from the direction of the flux lines, that go from a North to a South pole



in Oxford, on 25/01/2017

$$|B| = 48728 \text{ nT} = 0.048728 \text{ mT} = 0.000048728 \text{ T}$$

Magnetostatic fields are described by Maxwell's equations, coupled with a law describing the material

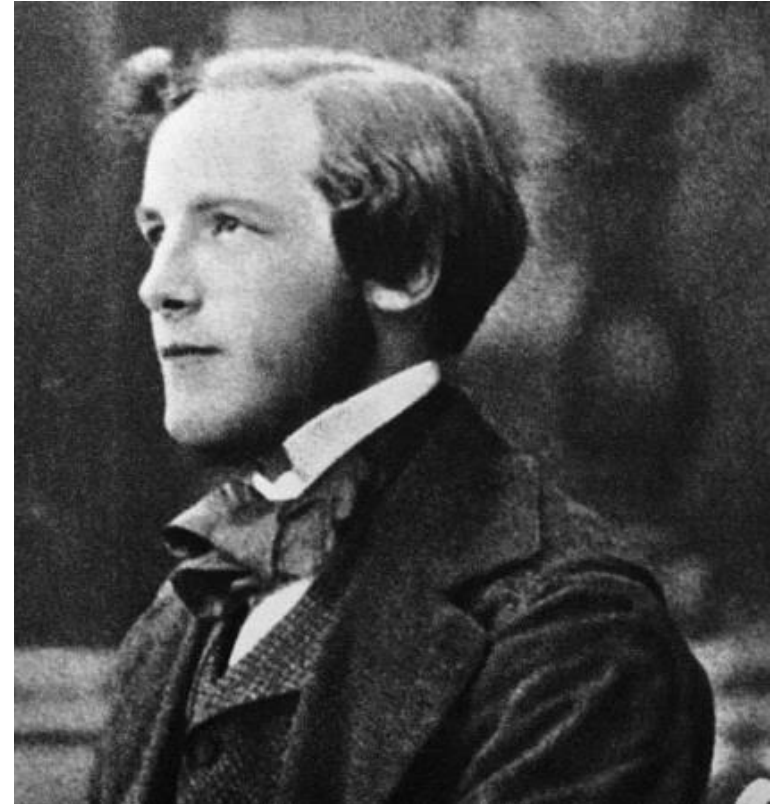
$$\operatorname{div} \vec{B} = 0$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\operatorname{rot} \vec{H} = \vec{j}$$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{j} \cdot d\vec{S} = NI$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$



James Clerk Maxwell

The Lorentz force is the main link between electromagnetism and mechanics

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

for charged beams

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

for conductors



Oliver Heaviside



Hendrik Lorentz

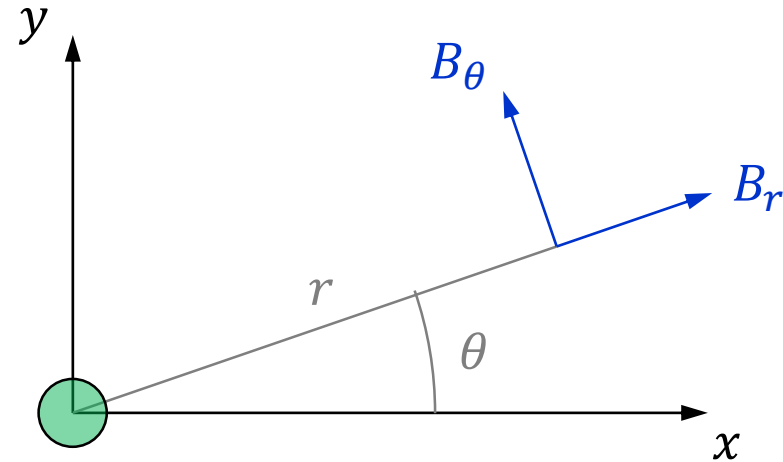


Pierre-Simon, marquis de Laplace

In synchrotrons / transfer lines magnets the B field as seen from the beam is usually expressed as a series of multipoles

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$



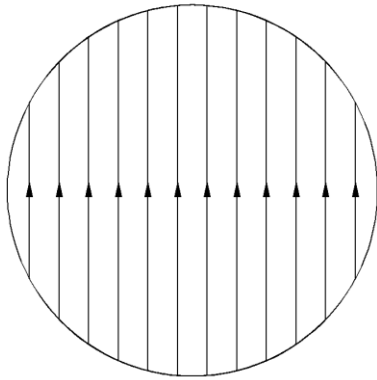
direction of the beam
(orthogonal to plane)

$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R}\right)^{n-1}$$

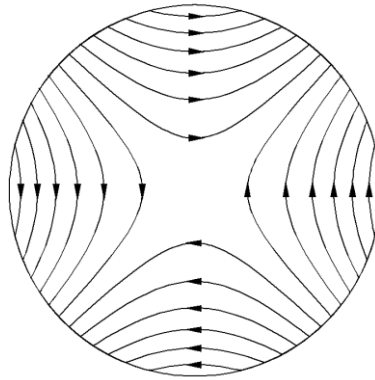
$$z = x + iy = re^{i\theta}$$

Each multipole term corresponds to a field distribution; they can be added up

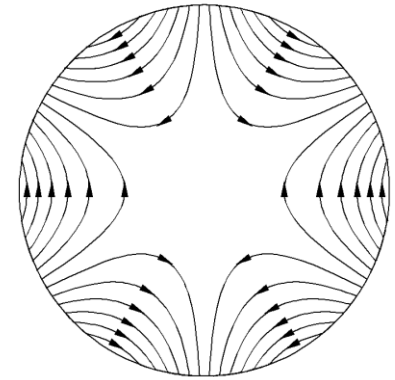
B_1 : normal dipole



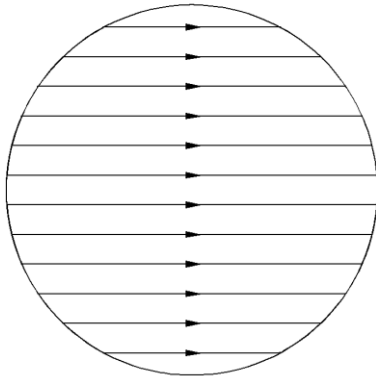
B_2 : normal quadrupole



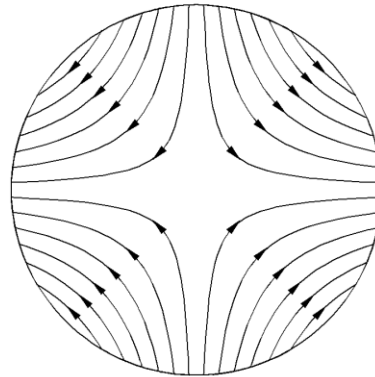
B_3 : normal sextupole



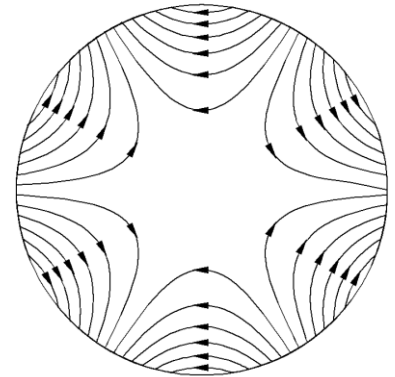
A_1 : skew dipole



A_2 : skew quadrupole

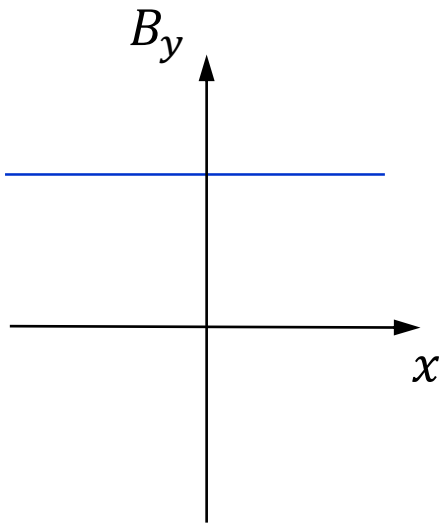


A_3 : skew sextupole

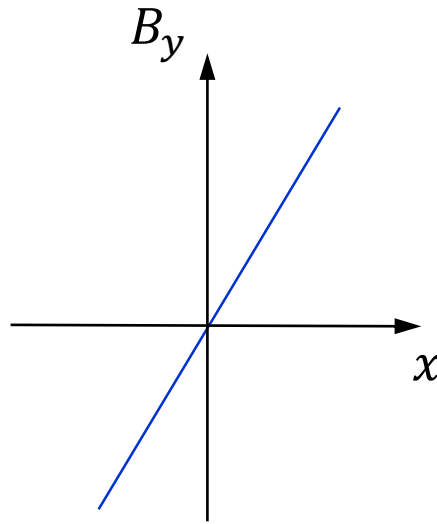


The field profile in the horizontal plane follows a polynomial expansion

$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R}\right)^{n-1} = B_1 + B_2 \frac{x}{R} + B_3 \frac{x^2}{R^2} + \dots$$

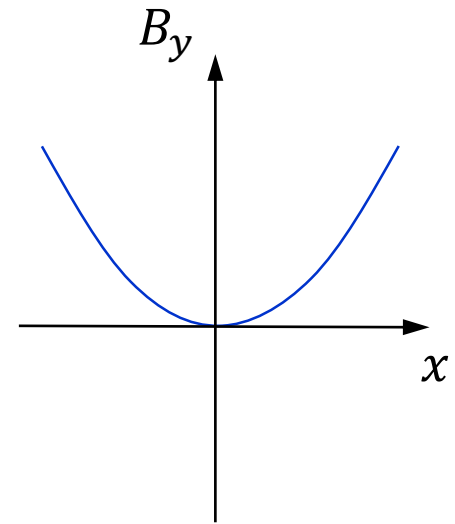


B_1 : dipole



B_2 : quadrupole

$$G = \frac{B_2}{R} = \frac{\partial B_y}{\partial x}$$



B_3 : sextupole

$$B'' = \frac{2B_3}{R^2}$$

For optics calculation, usually the field or multipole component is given, together with the (magnetic) length: ex. from MAD-X

Dipole

bend angle α [rad] & length L [m]

k_0 [1/m] & length L [m]

$$k_0 = B / (B\rho)$$

obsolete

$$B = B_1$$



Quadrupole

quadrupole coefficient k_1 [1/m²] \times length L [m]

$$k_1 = (dB_y/dx) / (B\rho)$$

$$G = dB_y/dx = B_2/R$$

Sextupole

sextupole coefficient k_2 [1/m³] \times length L [m]

$$k_2 = (d^2B_y/dx^2) / (B\rho)$$

$$(d^2B_y/dx^2)/2! = B_3/R^2$$

Here is how to compute magnetic quantities from MAD-X entries, and vice versa



```
BEAM, PARTICLE=ELECTRON, PC=3.0;  
DEGREE:=PI/180.0;  
QF: QUADRUPOLE, L=0.5, K1=0.2;  
QD: QUADRUPOLE, L=1.0, K1=-0.2;  
B: SBEND, L=1.0, ANGLE=15.0*DEGREE;
```

$$(B\rho) = 10^9/c*PC = 10^9/299792485*3.0 = 10.01 \text{ Tm}$$

dipole (SBEND)

$$B = |\text{ANGLE}|/L*(B\rho) = (15*\pi/180)/1.0*10.01 = 2.62 \text{ T}$$

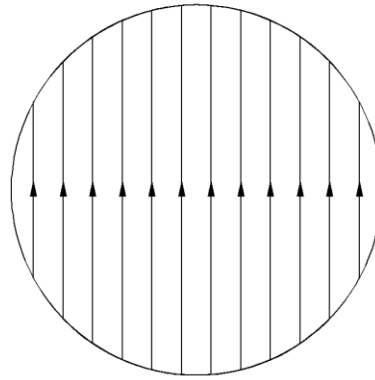
quadrupole

$$G = |K1|*(B\rho) = 0.2*10.01 = 2.00 \text{ T/m}$$

The harmonic decomposition is used also to describe the field quality (or field homogeneity), that is, the deviations of the actual B with respect to the ideal one



(normal) dipole



$$\vec{B}_{id}(x, y) = B_1 \vec{j}$$

$$B_y(z) + iB_x(z) =$$

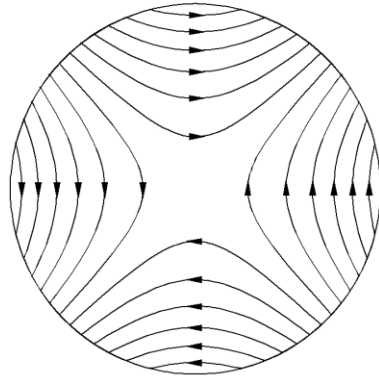
$$= B_1 + \frac{B_1}{10000} \left[ia_1 + (b_2 + ia_2) \left(\frac{z}{R} \right) + (b_3 + ia_3) \left(\frac{z}{R} \right)^2 + (b_4 + ia_4) \left(\frac{z}{R} \right)^3 + \dots \right]$$

$$b_2 = 10000 \frac{B_2}{B_1} \quad b_3 = 10000 \frac{B_3}{B_1} \quad a_1 = 10000 \frac{A_1}{B_1} \quad a_2 = 10000 \frac{A_2}{B_1} \quad \dots$$

The same expression can be written for a quadrupole



(normal) quadrupole



$$\vec{B}_{id}(x, y) = B_2[x\vec{j} + y\vec{i}] \frac{1}{R}$$

$$\begin{aligned} B_y(z) + iB_x(z) &= \\ &= B_2 \frac{z}{R} + \frac{B_2}{10000} \left[i a_2 \left(\frac{z}{R} \right) + (b_3 + i a_3) \left(\frac{z}{R} \right)^2 + (b_4 + i a_4) \left(\frac{z}{R} \right)^3 + \dots \right] \end{aligned}$$

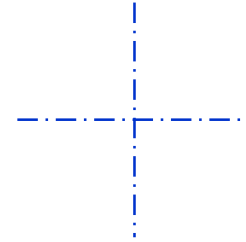
$$b_3 = 10000 \frac{B_3}{B_2} \quad b_4 = 10000 \frac{B_4}{B_2} \quad a_2 = 10000 \frac{A_2}{B_2} \quad \dots$$

The *allowed / not-allowed* harmonics refer to some terms that shall / shall not cancel out thanks to design symmetries

fully symmetric dipoles

allowed: B_1, b_3, b_5, b_7, b_9 , etc.

not-allowed: all the others



half symmetric dipoles

allowed: B_1, b_2, b_3, b_4, b_5 , etc.

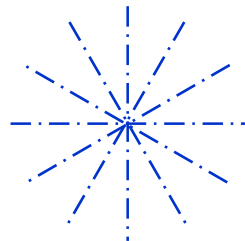
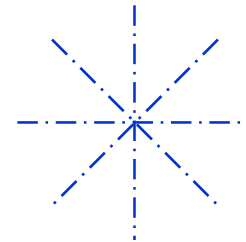
not-allowed: all the others



fully symmetric quadrupoles

allowed: $B_2, b_6, b_{10}, b_{14}, b_{18}$, etc.

not-allowed: all the others



fully symmetric sextupoles

allowed: B_3, b_9, b_{15}, b_{21} , etc.

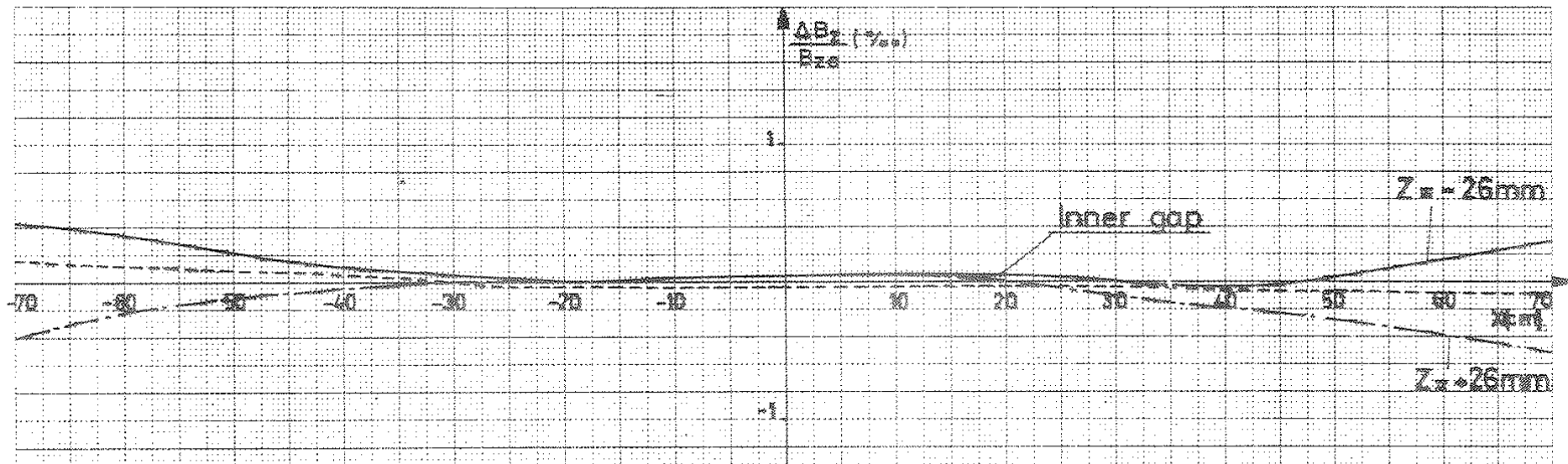
not-allowed: all the others

The field quality is often also shown with a $\Delta B/B$ plot



$$\frac{\Delta B}{B} = \frac{B(x, y) - B_{id}(x, y)}{B_{id}(x, y)}$$

done on one component,
usually B_y for a dipole



$\Delta B/B$ can (at least locally) be expressed from the harmonics:
this is the expansion for a dipole



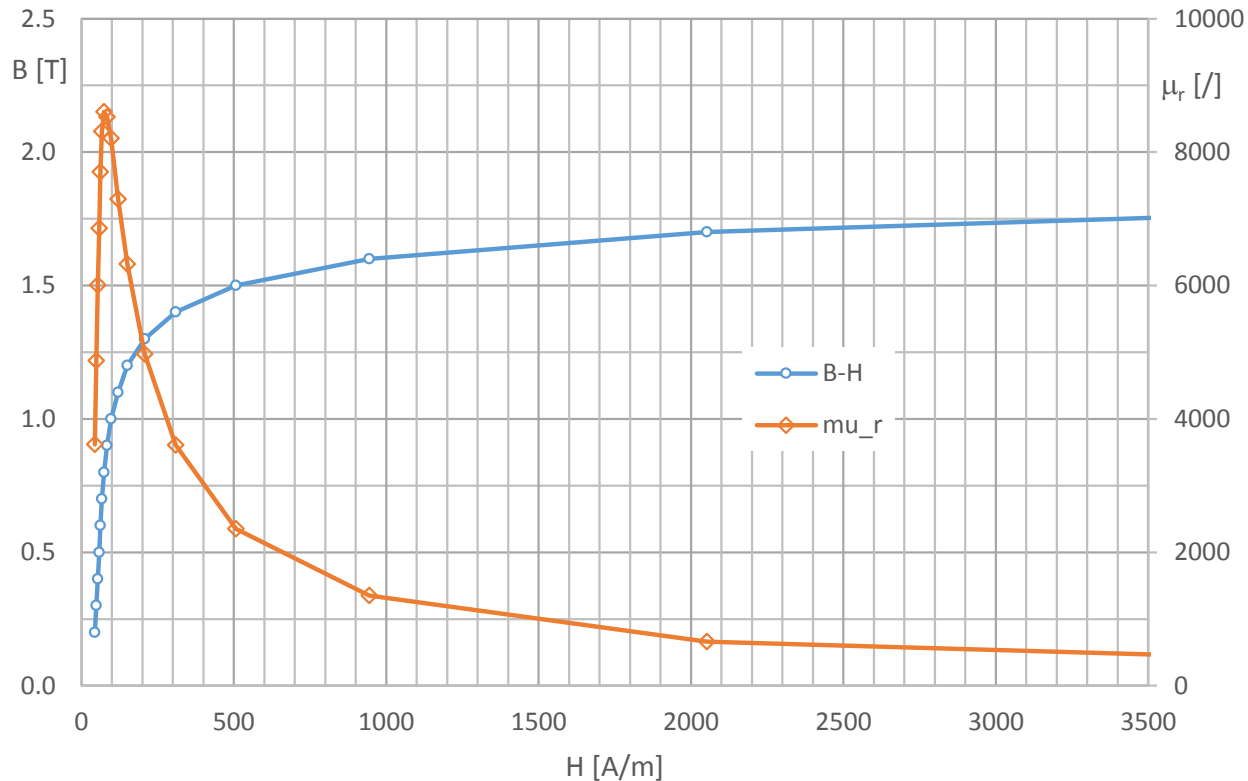
$$B_{y,id}(x) = B_1$$

$$B_y(x) = B_1 + \frac{B_1}{10000} \left[b_2 \left(\frac{x}{R} \right) + b_3 \left(\frac{x}{R} \right)^2 + b_4 \left(\frac{x}{R} \right)^3 + \dots \right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10000} \left[b_2 \left(\frac{x}{R} \right) + b_3 \left(\frac{x}{R} \right)^2 + b_4 \left(\frac{x}{R} \right)^3 + \dots \right]$$

1. Introduction, jargon, general concepts and formulae
2. Resistive magnets
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Resistive magnets are in most cases “iron-dominated” magnets:
the BH response of the yoke material is important



curves for typical M1200-100 A electrical steel

These are typical fields for resistive dipoles and quadrupole, taken from machines at CERN

PS @ 26 GeV

combined function bending $B = 1.5 \text{ T}$

SPS @ 450 GeV

bending $B = 2.0 \text{ T}$

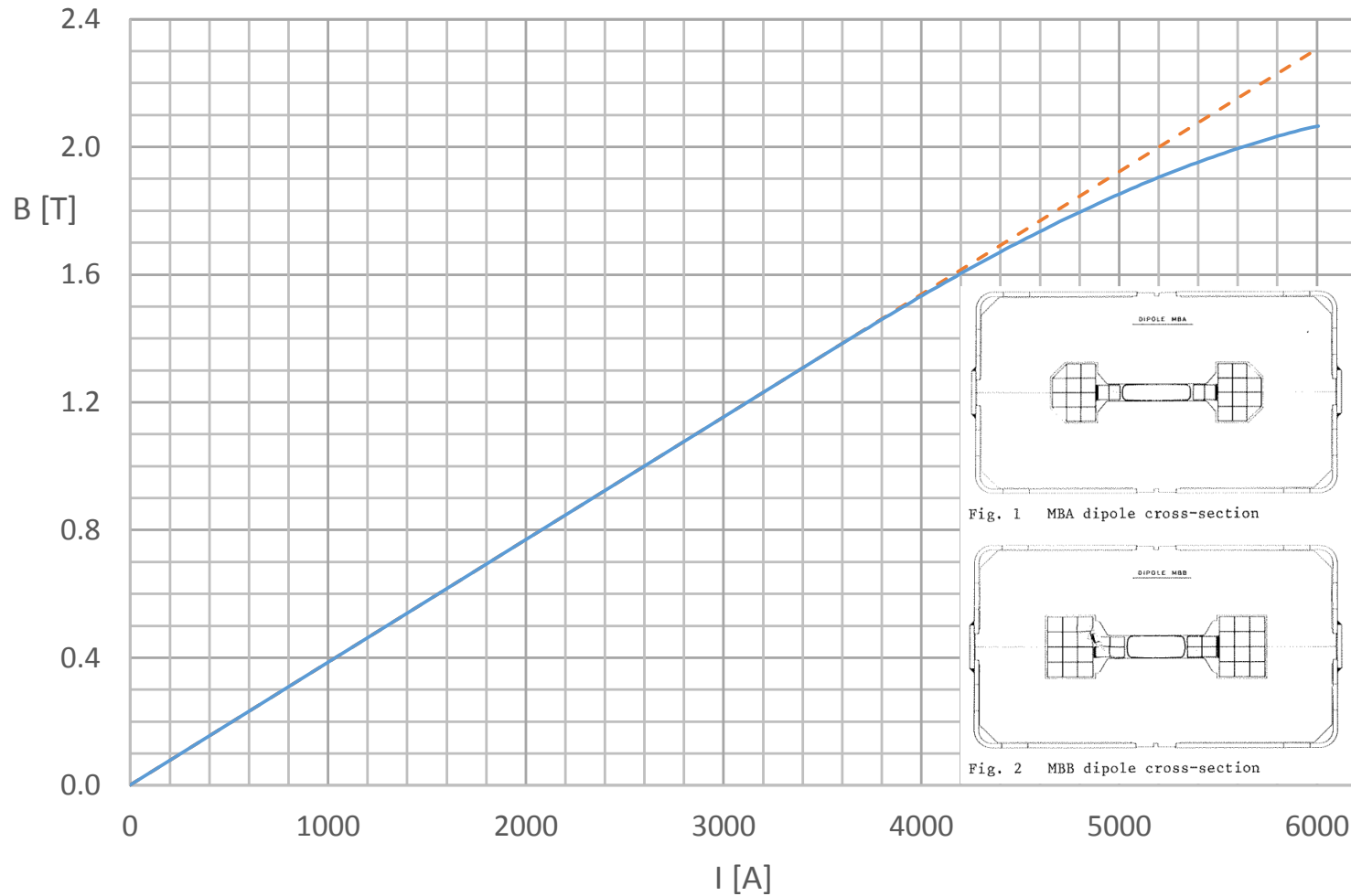
quadrupole $B_{\text{pole}} = 21.7 * 0.044 = 0.95 \text{ T}$

TI2 / TI8 (transfer lines SPS to LHC, @ 450 GeV)

bending $B = 1.8 \text{ T}$

quadrupole $B_{\text{pole}} = 53.5 * 0.016 = 0.86 \text{ T}$

This is the transfer function field B vs. current I for the SPS main dipoles



If the magnet is not dc, then an rms power / current is taken, considering the duty cycle



$$P_{rms} = RI_{rms}^2 = \frac{1}{T} \int_0^T R[I(t)]^2 dt$$

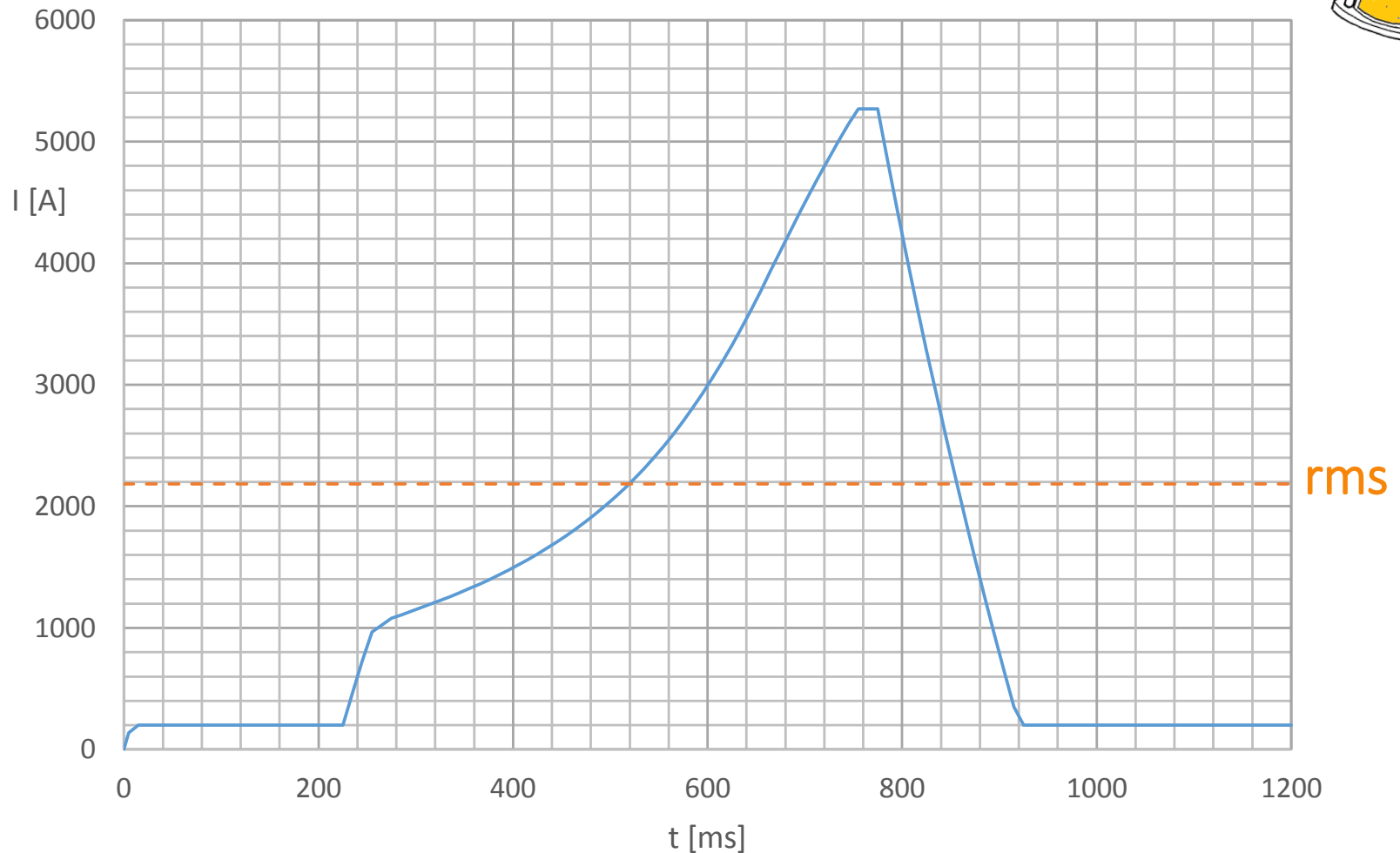
for a pure sine wave

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$

for a linear ramp from 0

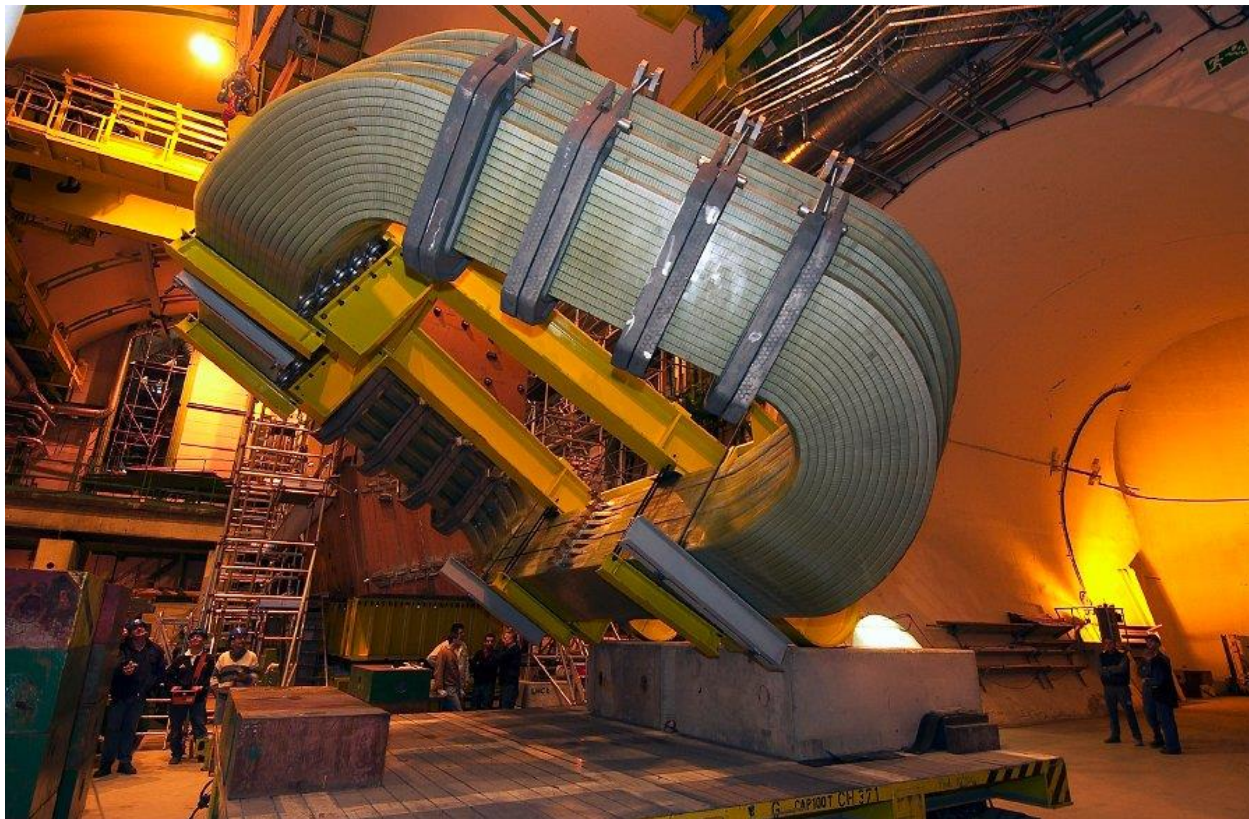
$$I_{rms} = \frac{I_{peak}}{\sqrt{3}}$$

This will be a cycle to 2.0 GeV of the PSB at CERN after the upgrade planned from 2019-2020



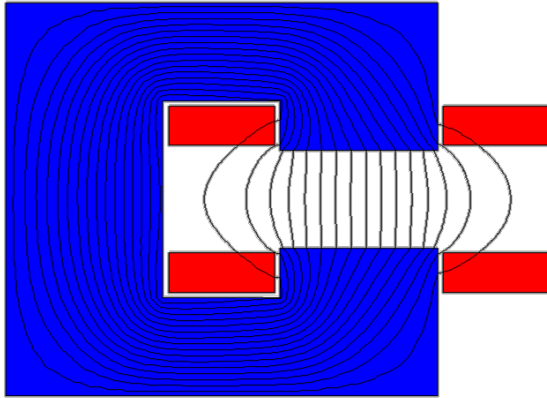
The material of the coils is most often copper, sometimes aluminum

	Cu	Al
raw metal price	$\approx 5000 \text{ \$/ton}$	$\approx 1500 \text{ \$/ton}$
electrical resistivity	$1.72 \cdot 10^{-8} \text{ } \Omega/\text{m}$	$2.65 \cdot 10^{-8} \text{ } \Omega/\text{m}$
density	8.9 kg/dm^3	2.7 kg/dm^3

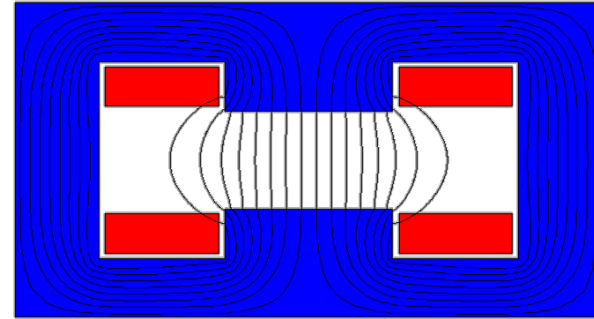


LHCb detector dipole
Al coils
coil mass $2 \times 25 \text{ t}$
power $2 \times 2.1 \text{ MW}$

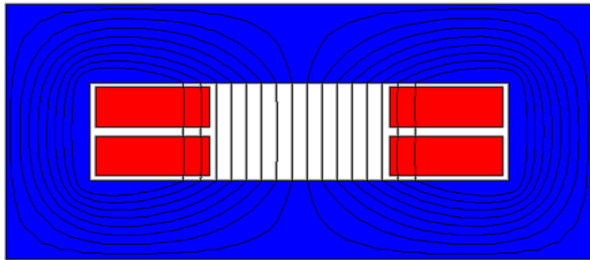
These are the most common types of resistive dipoles



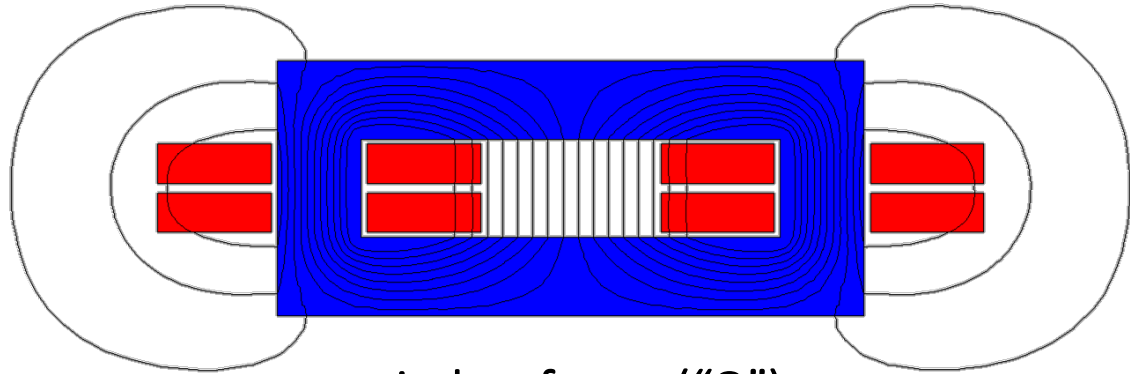
"C"



"H"

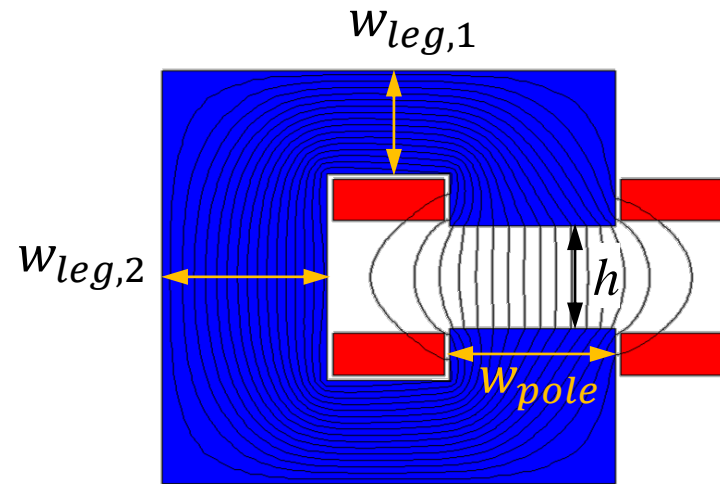


window frame
("O")



window frame ("O")
with windings on both backlegs

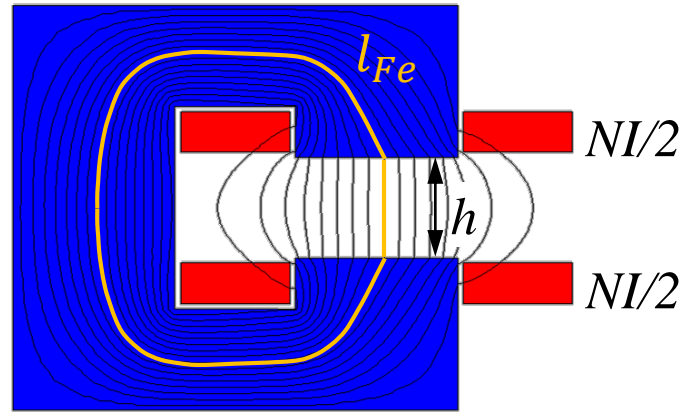
The magnetic circuit is dimensioned so that the pole is wide enough for field quality, and there is enough room for the flux in the return legs



$$w_{pole} \cong w_{GFR} + 2.5h$$

$$B_{leg} \cong B_{gap} \frac{w_{pole} + 1.2h}{w_{leg}}$$

The Ampere-turns are a linear function of the gap and of the B field (at least up to saturation)



$$NI = \oint \vec{H} \cdot d\vec{l} = \frac{B_{Fe}}{\mu_0 \mu_r} \cdot l_{Fe} + \frac{B_{gap}}{\mu_0} \cdot h \cong \frac{B_{gap} h}{\mu_0}$$

$$NI = \frac{Bh}{\eta \mu_0} \quad \eta = \frac{1}{1 + \frac{1}{\mu_r} \frac{l_{Fe}}{h}}$$

The same can be solved using magnetic reluctances and Hopkinson's law, which is a parallel of Ohm's law



$$\mathcal{R} = \frac{NI}{\Phi}$$

$$R = \frac{V}{I}$$

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A}$$

$$R = \frac{l}{\sigma S}$$

$$\eta = \frac{1}{1 + \frac{\mathcal{R}_{Fe}}{\mathcal{R}_{gap}}}$$

Example of computation of Ampere-turns and current

central field $B = 1.5 \text{ T}$
total gap 80 mm

$$\eta \cong 0.97$$

$$NI = \frac{Bh}{\eta\mu_0}$$



$$NI = (1.5 \cdot 0.080) / (0.97 \cdot 4 \cdot \pi \cdot 10^{-7}) = 98446 \text{ A total}$$

low inductance option

$$64 \text{ turns, } I \cong 98500 / 64 = 1540 \text{ A}$$

$$L = 62.9 \text{ mH, } R = 15.0 \text{ m}\Omega$$

low current option

$$204 \text{ turns, } I \cong 98500 / 204 = 483 \text{ A}$$

$$L = 639 \text{ mH, } R = 160 \text{ m}\Omega$$

Besides the number of turns, the overall size of the coil depends on the current density j , which drives the resistive power consumption (linearly)



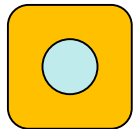
ex. $NI = 50000 \text{ A (rms)}$



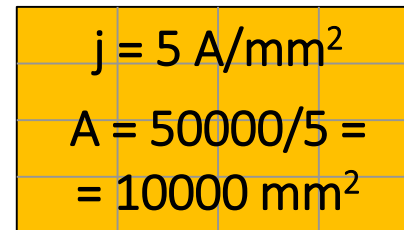
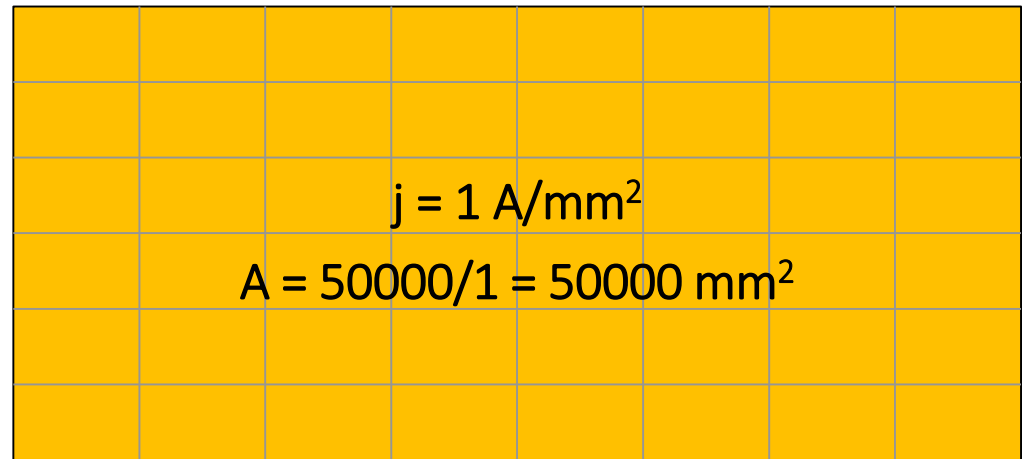
air cooled
(by conduction on
external surface)

----- $1 - 1.5 \text{ A/mm}^2$ -----
(for Cu)

water cooled
(hollow conductor)



j
(rms)



These are common formulae for the main electric parameters of a resistive dipole (1/2)



Ampere-turns (total) $NI = \frac{Bh}{\eta\mu_0}$

current $I = \frac{(NI)}{N}$

resistance (total) $R = \frac{\rho N L_{turn}}{A_{cond}}$

inductance $L \cong \eta\mu_0 N^2 A / h$

$$A \cong (w_{pole} + 1.2h)(l_{Fe} + h)$$

These are common formulae for the main electric parameters of a resistive dipole (2/2)



voltage

$$V = RI + L \frac{dI}{dt}$$

resistive power (rms)

$$\begin{aligned} P_{rms} &= RI_{rms}^2 \\ &= \rho j_{rms}^2 V_{cond} \\ &= \frac{\rho L_{turn} B_{rms} h}{\eta \mu_0} j_{rms} \end{aligned}$$

magnetic stored energy

$$E_m = \frac{1}{2} LI^2$$

These are useful formulae for the main cooling parameters of a water cooled dipole



cooling flow $Q_{tot} \cong 14.3 \frac{P}{\Delta T} \quad Q_{tot} \cong N_{hydr} Q$

water velocity $v = \frac{1000}{15\pi d^2} Q$

Reynolds number $Re \cong 1400 d v$

pressure drop $\Delta p = 60 L_{hydr} \frac{Q^{1.75}}{d^{4.75}}$

The table describes the field quality – in terms of allowed multipoles – for the different layouts of these examples

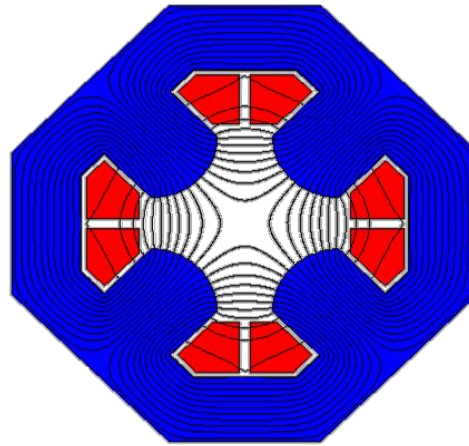


	C-shaped	H-shaped	O-shaped
b_2	1.4	0	0
b_3	-88.2	-87.0	0.2
b_4	0.7	0	0
b_5	-31.6	-31.4	-0.1
b_6	0.1	0	0
b_7	-3.8	-3.8	-0.1
b_8	0.0	0	0
b_9	0.0	0.0	0.0

b_n multipoles in units of 10^{-4} at $R = 17$ mm

$NI = 20$ kA, $h = 50$ mm, $w_{\text{pole}} = 80$ mm

These are the most common types of resistive quadrupoles



standard
quadrupole

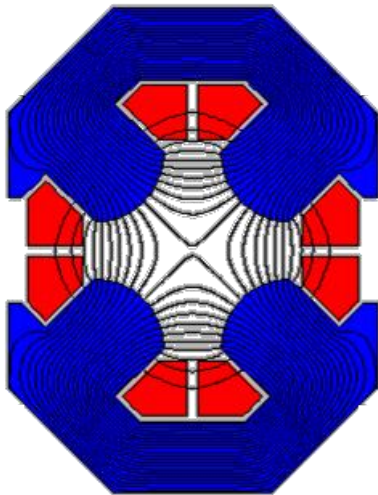
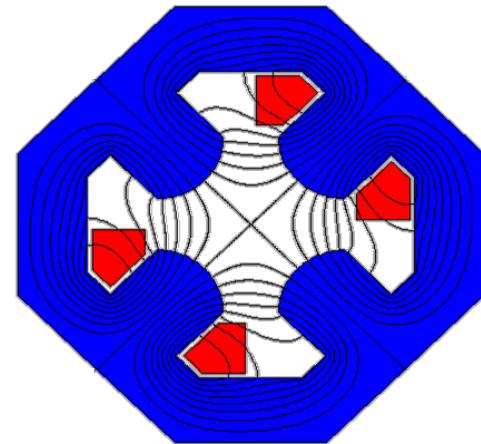


figure-of-8
(useful because narrow)



quadrupole
with half the coils
(maybe not so common)

These are useful formulae for standard resistive quadrupoles



Pole tip field

$$B_{pole} = Gr$$

Ampere-turns (per pole)

$$NI = \frac{Gr^2}{2\eta\mu_0}$$

current

$$I = \frac{(NI)}{N}$$

resistance (total)

$$R = 4 \frac{\rho N L_{turn}}{A_{cond}}$$

The *ideal* poles for dipole, quadrupole, sextupole, etc. are lines of constant scalar potential

dipole

$$\rho \sin(\theta) = \pm h/2$$

$$y = \pm h/2$$

straight line

quadrupole

$$\rho^2 \sin(2\theta) = \pm r^2$$

$$2xy = \pm r^2$$

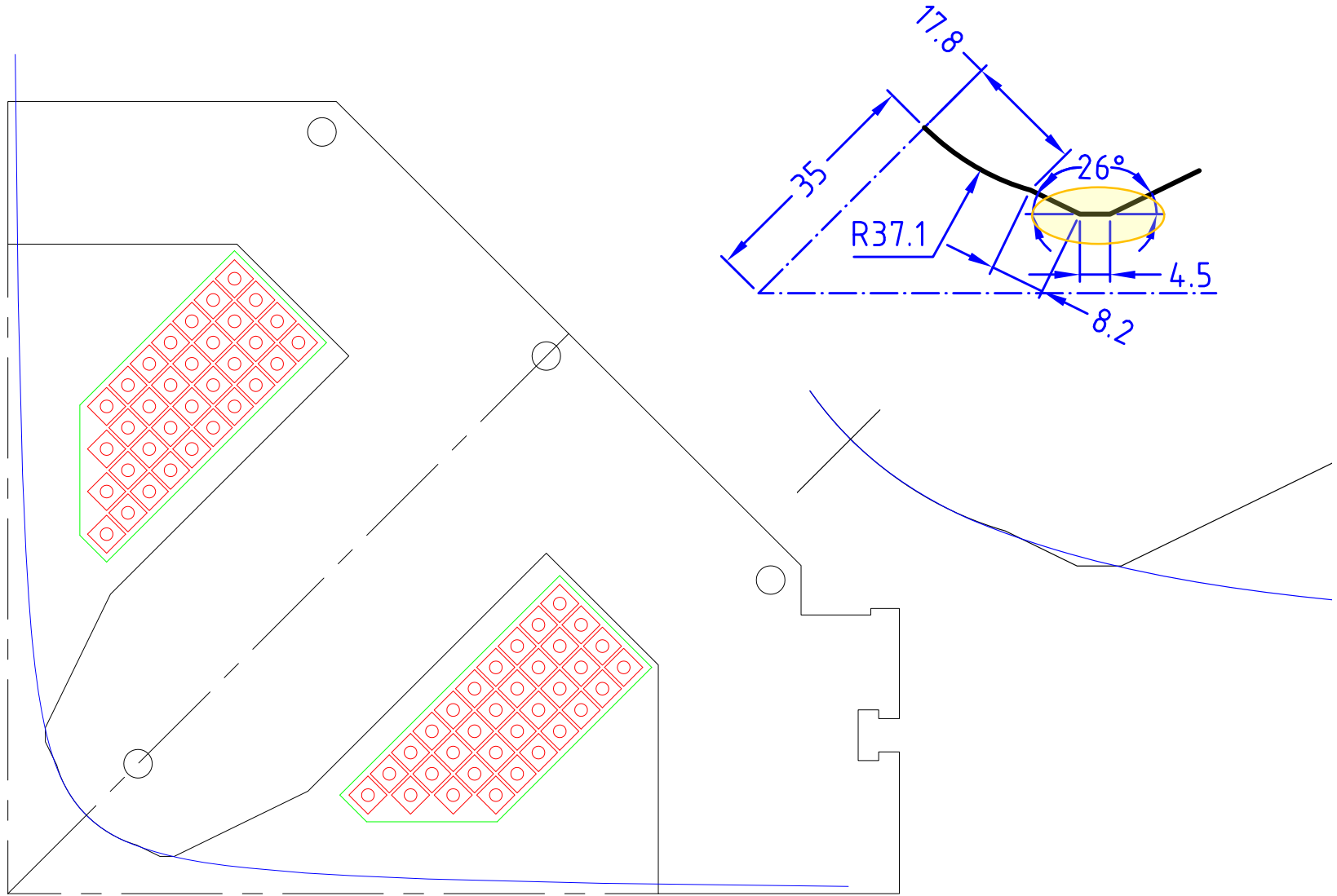
hyperbola

sextupole

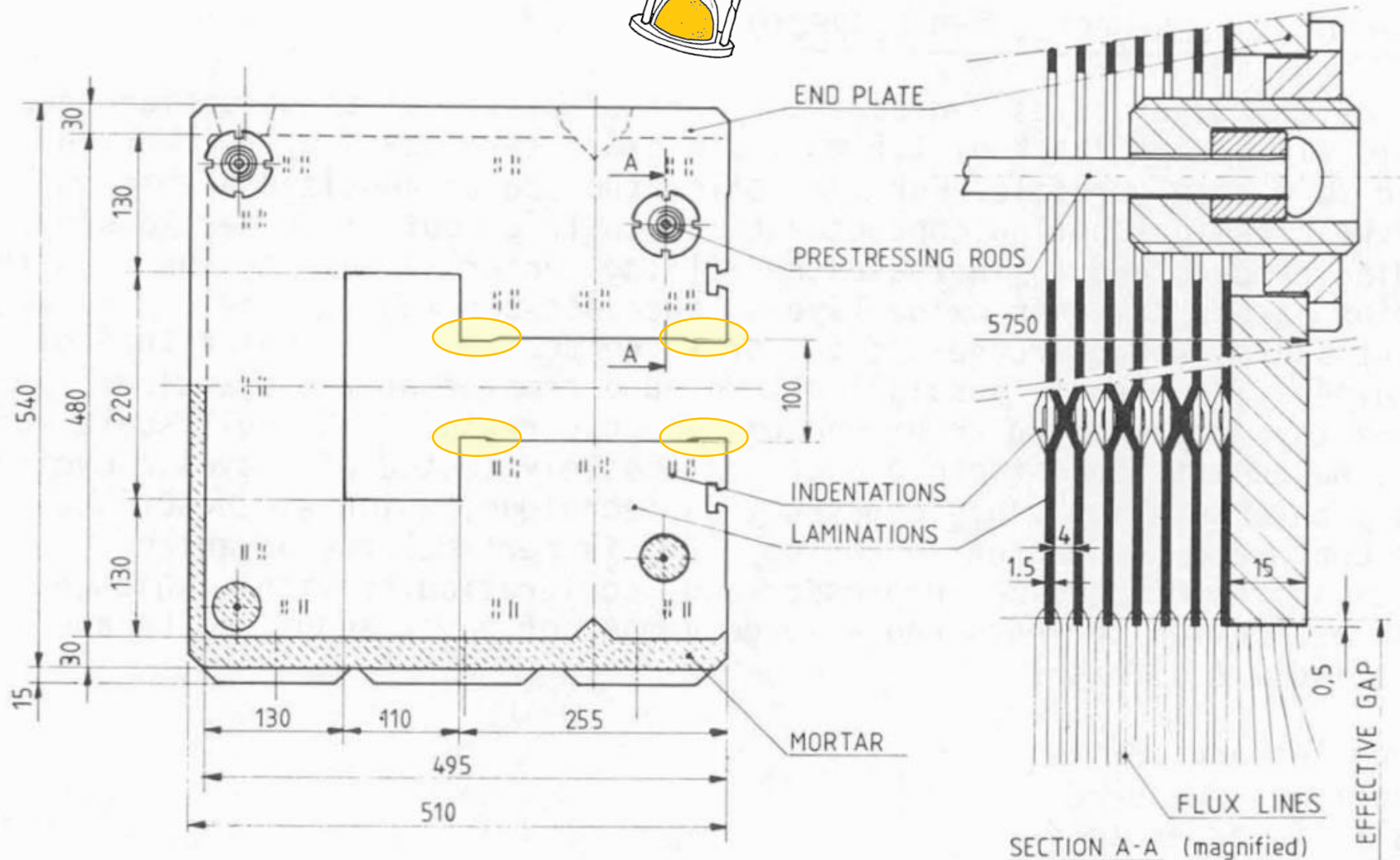
$$\rho^3 \sin(3\theta) = \pm r^3$$

$$3x^2y - y^3 = \pm r^3$$

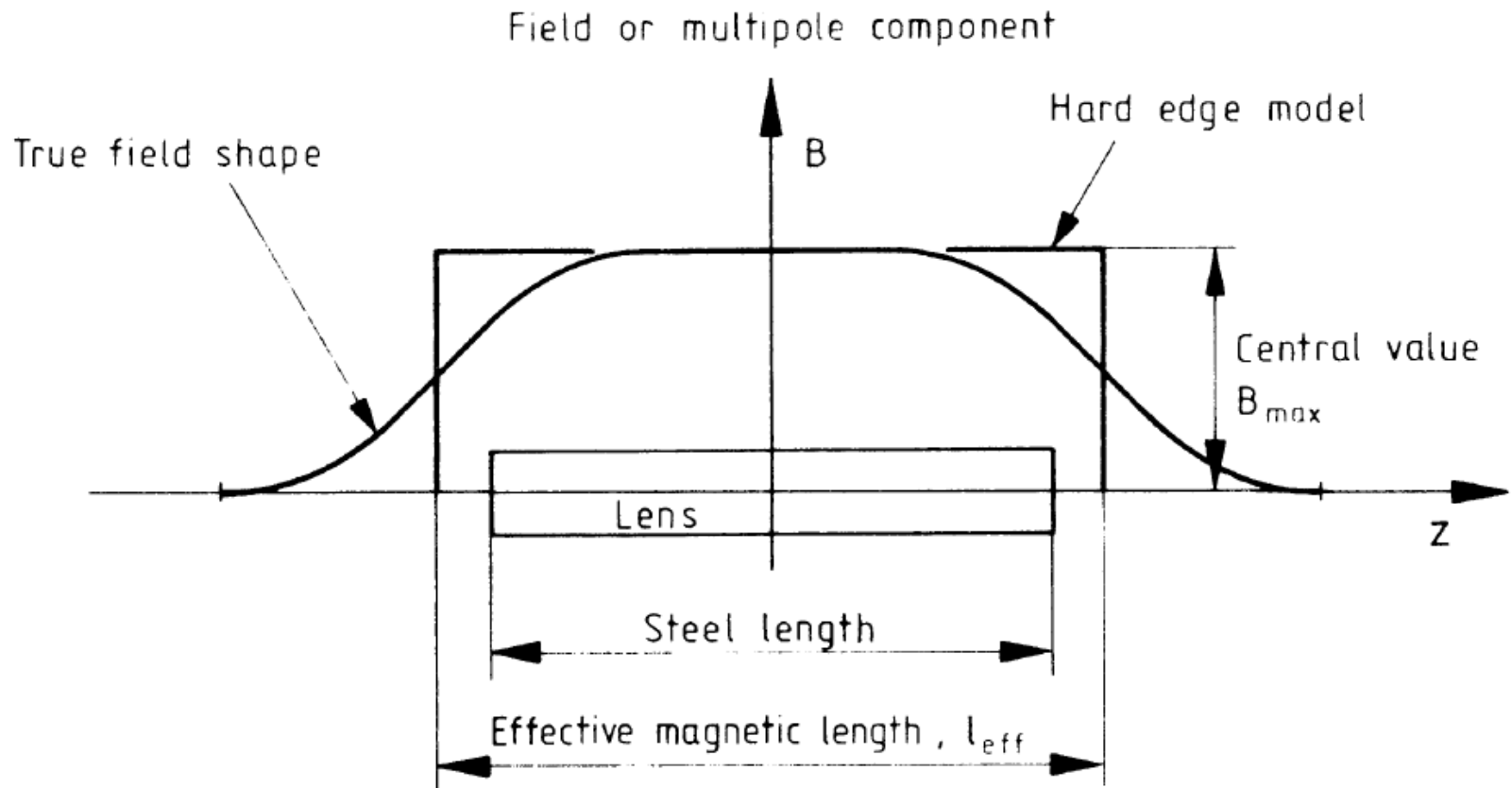
As an example, this is the pole tip used in the SESAME quadrupoles vs. the theoretical hyperbola



This is the lamination of the LEP main bending magnets, with the pole shims well visible



In 3D, the longitudinal dimension of the magnet is described by a magnetic length



$$l_m B_0 = \int_{-\infty}^{\infty} B(z) dz$$

The magnetic length can be estimated at first order with simple formulae

$$l_m > l_{Fe}$$



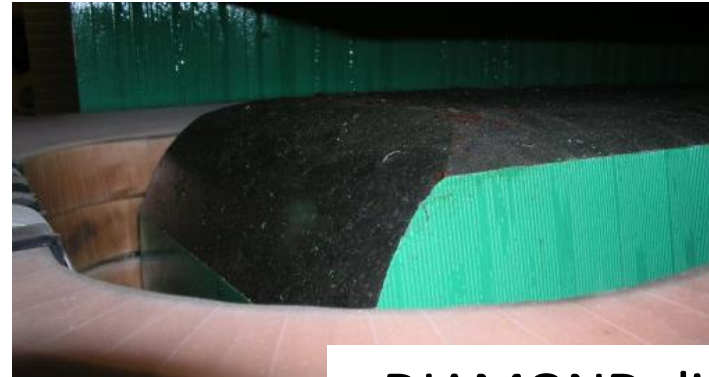
dipole

$$l_m \cong l_{Fe} + h$$

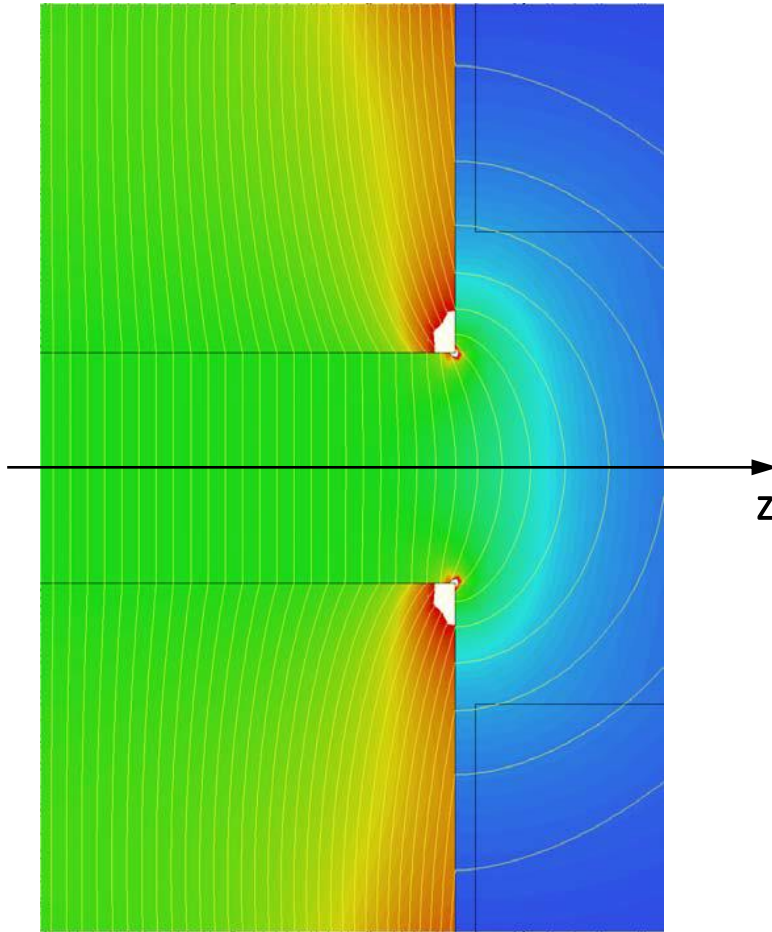
quadrupole

$$l_m \cong l_{Fe} + 0.80r$$

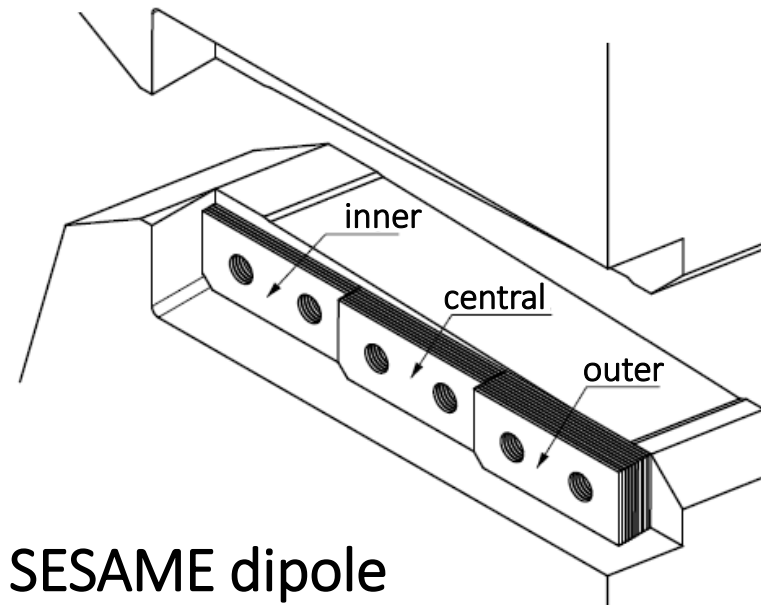
There are many different options to terminate the pole ends, depending on the type of magnet, its field level, etc.



DIAMOND dipole



abrupt

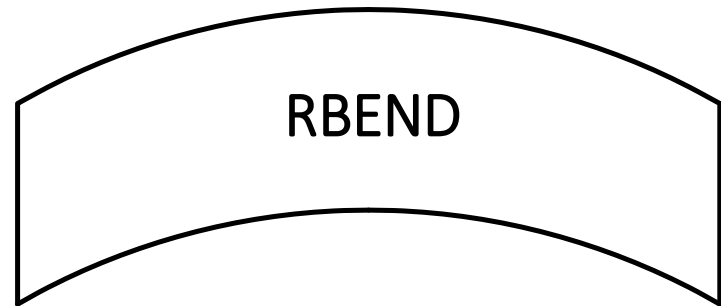
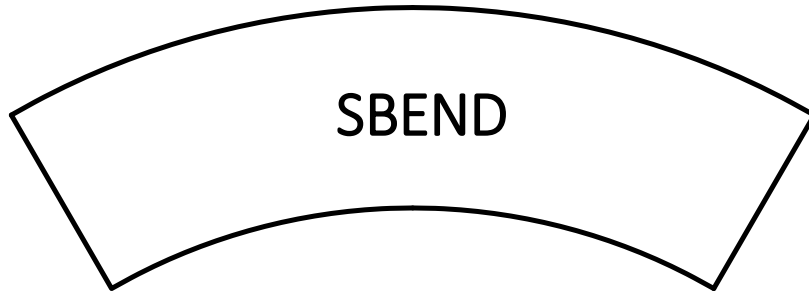


SESAME dipole

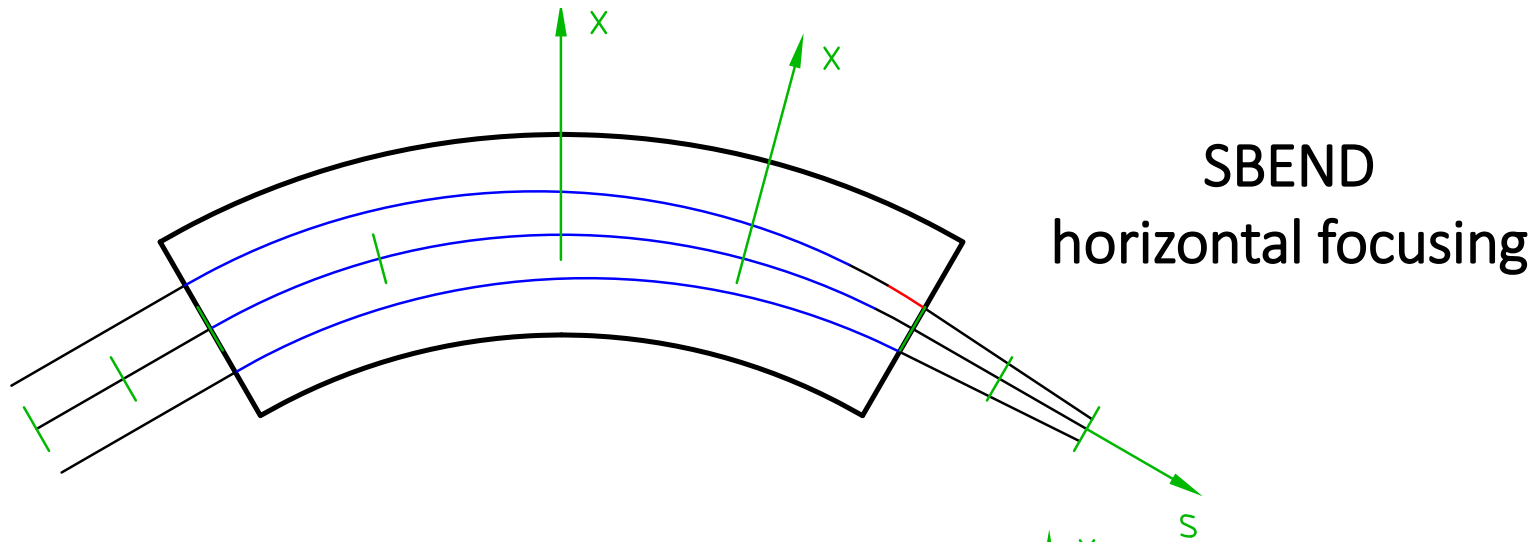
Usually two dipole elements are found in lattice codes: the sector dipole (SBEND) and the parallel faces dipole (RBEND)



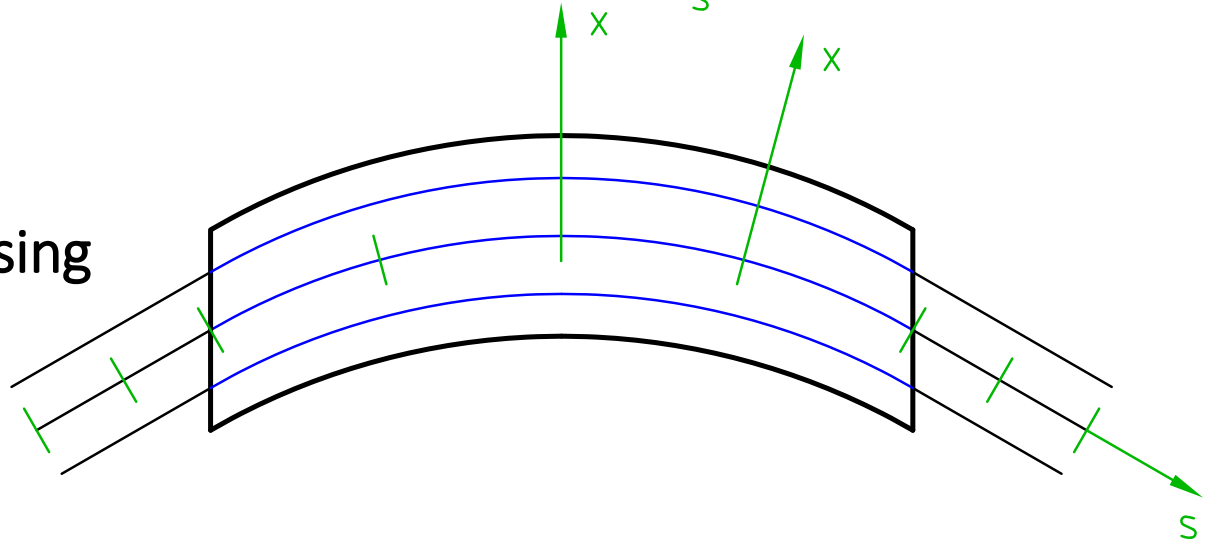
top views



The two types of dipoles are slightly different in terms of focusing, for a geometric effect



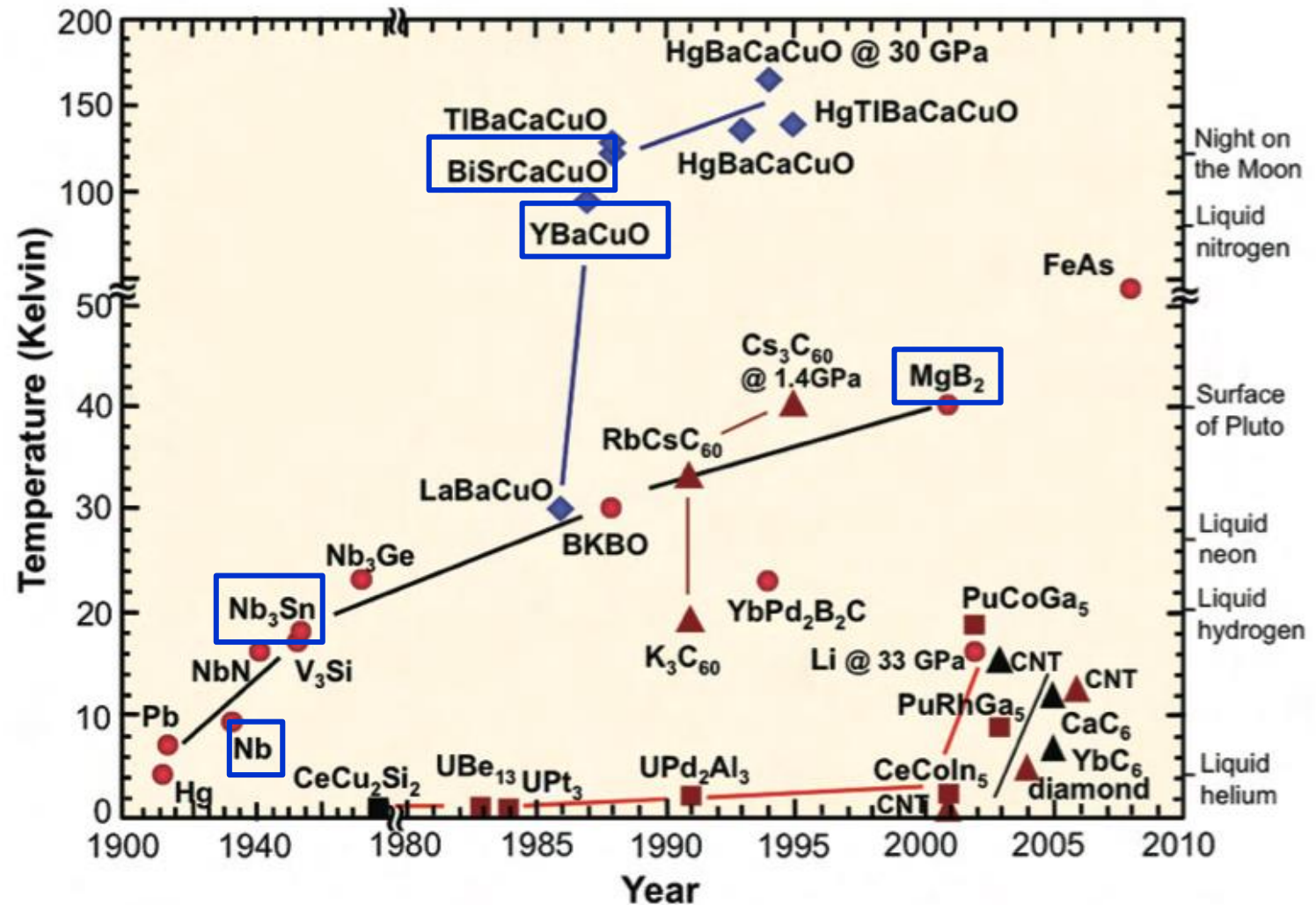
RBEND
vertical edge focusing



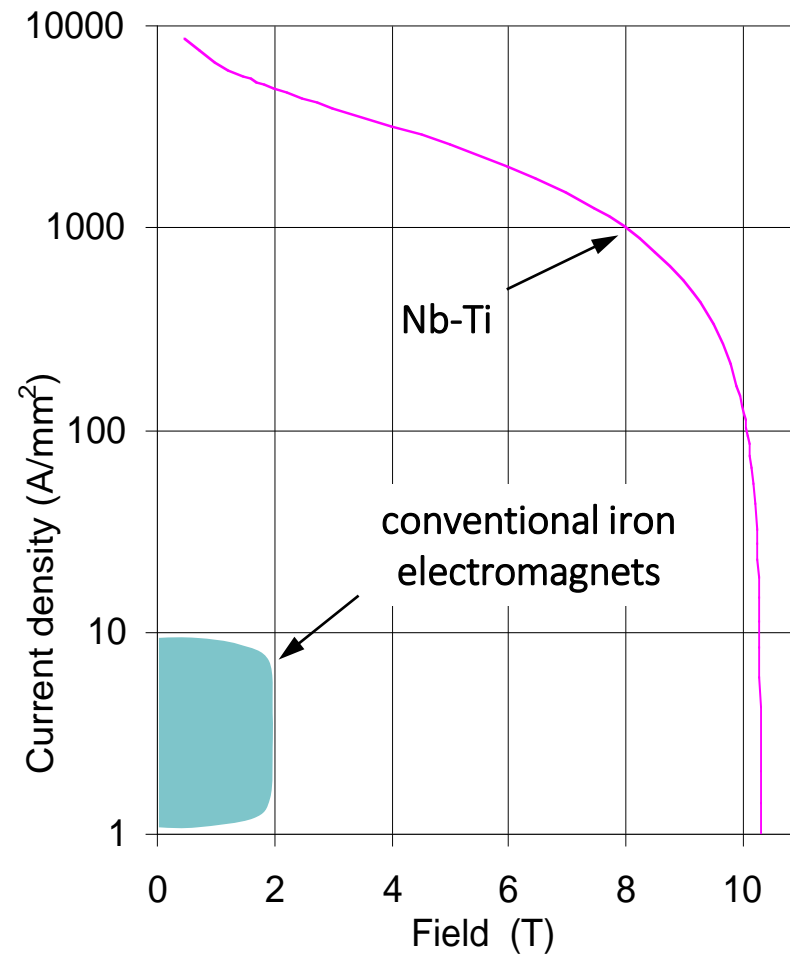
- and anything in between (playing with the edge angles) -

1. Introduction, jargon, general concepts and formulae
2. Resistive magnets
3. Superconducting magnets (thanks to Luca Bottura for the material of many slides)
4. Tutorial with OPERA-2D

This is a history chart of superconductors, starting with Hg all the way to HTS (High Temperature Superconductors)



Superconductivity makes possible large accelerators with fields well above 2 T



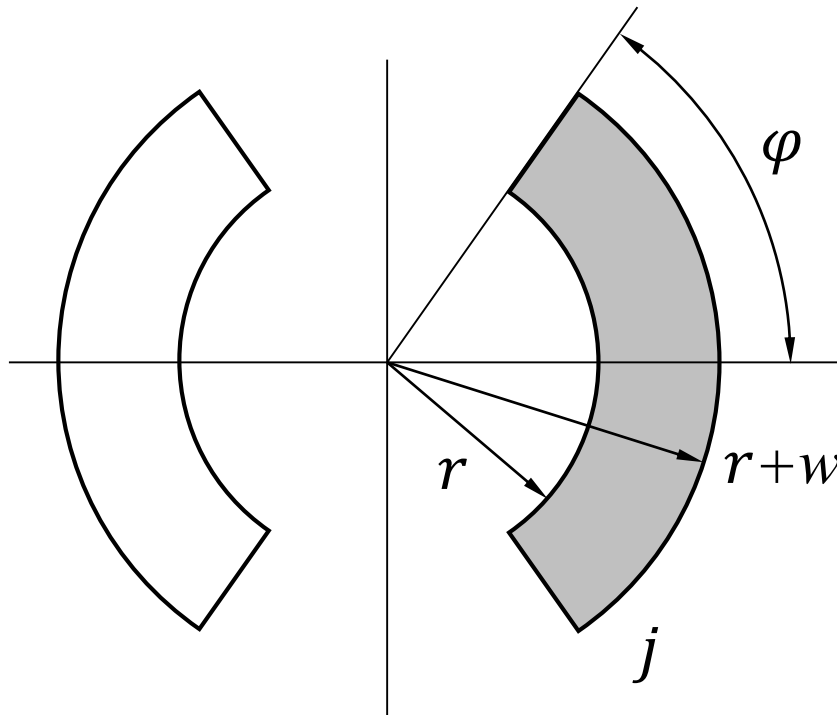
This is a summary of (somehow) practical superconductors

	LTS			HTS	
material	Nb-Ti	Nb ₃ Sn	MgB ₂	YBCO	BSCCO
year of discovery	1961	1954	2001	1987	1988
T _c [K]	9.2	18.2	39	≈93	95 / 108
B _c [T]	14.5	≈30	36...74	120...250	≈200

The field in the aperture of a superconducting dipole can be derived using Biot-Savart law (in 2D)

$$B_{\theta} = \frac{\mu_0 I}{2\pi\rho}$$

Biot-Savart law for an infinite wire



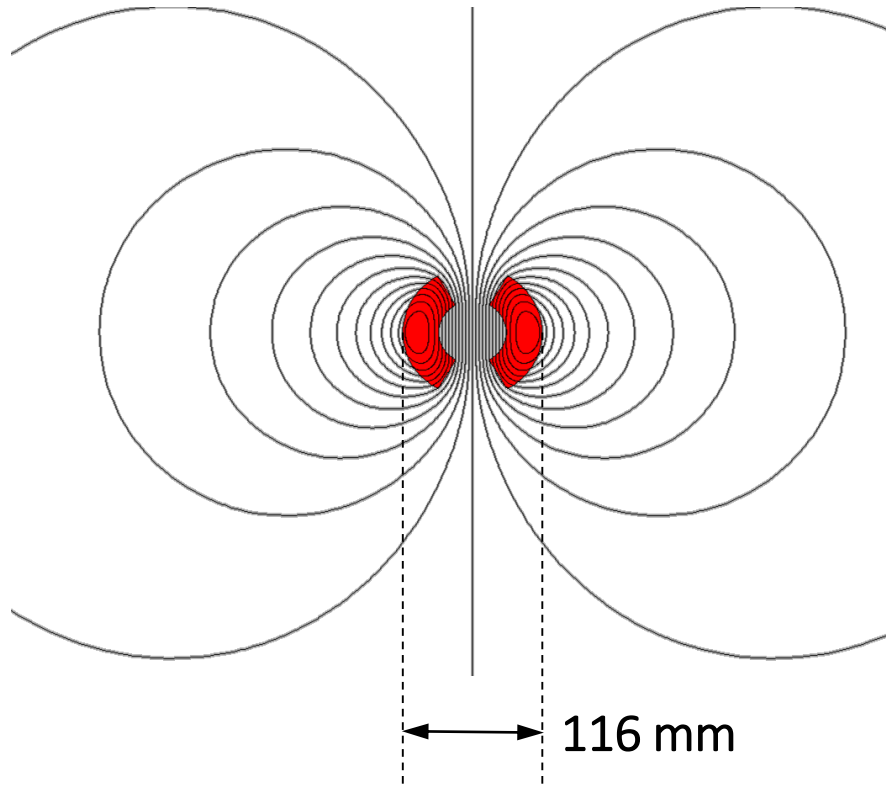
$$B = \frac{2\mu_0 \sin \varphi}{\pi} jw$$

for a sector coil

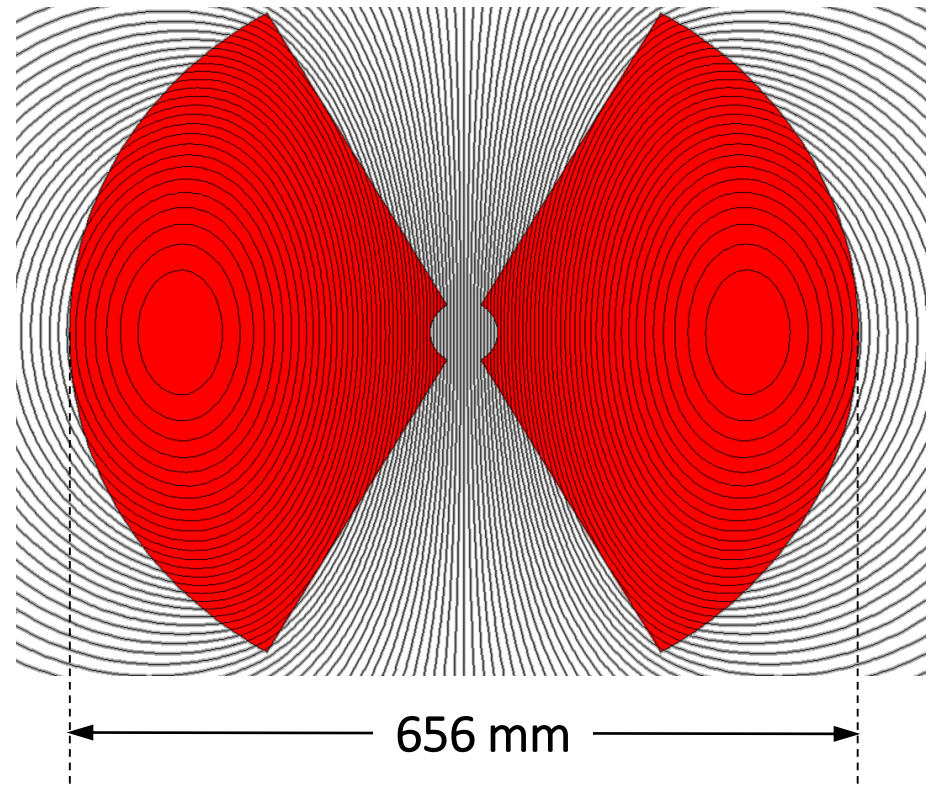
$$B = \frac{\sqrt{3}\mu_0}{\pi} jw$$

for a 60 deg sector coil

This is how it would look like one aperture of the LHC dipoles at 8.3 T, with two different current densities (without iron)

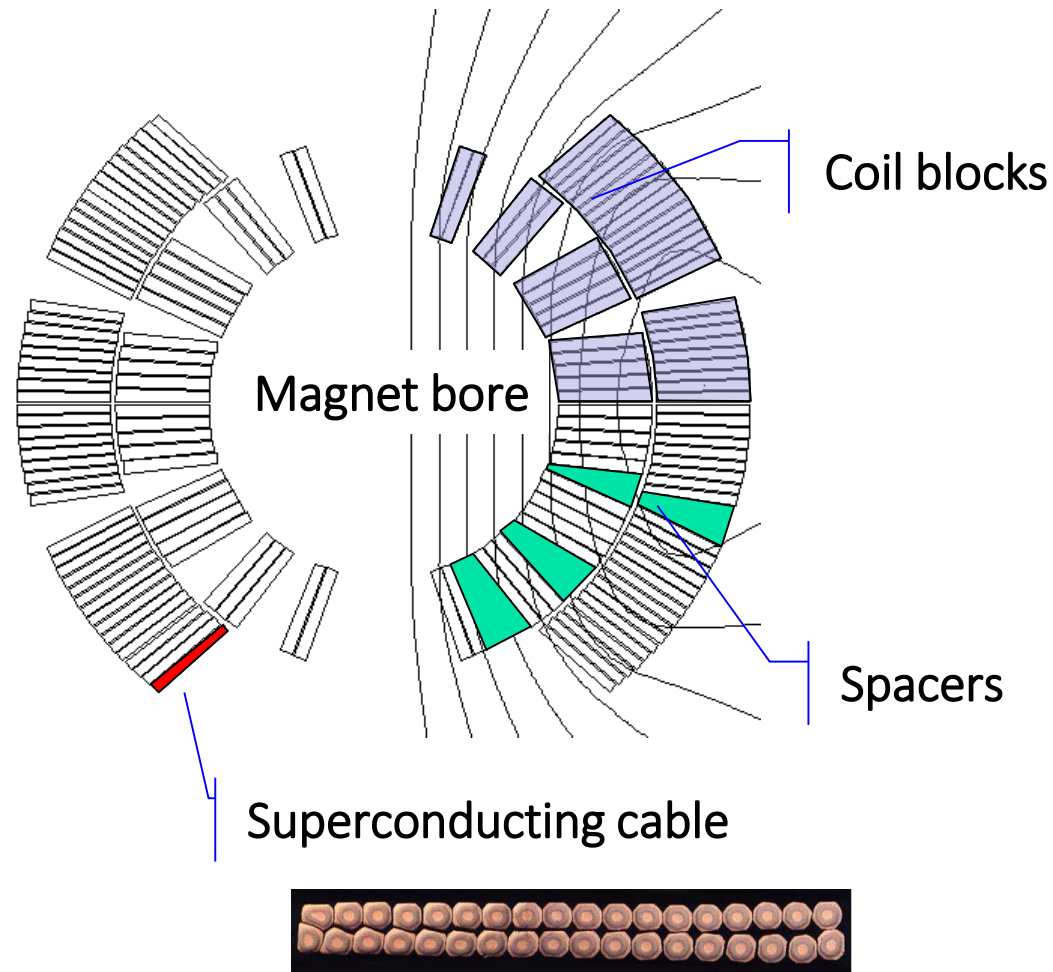


$j = 400 \text{ A/mm}^2$
 $w = 30 \text{ mm}$
 $NI = 1.2 \text{ MA}$
 $P = 14.9 \text{ MW/m}$ (if Cu at room temp.)

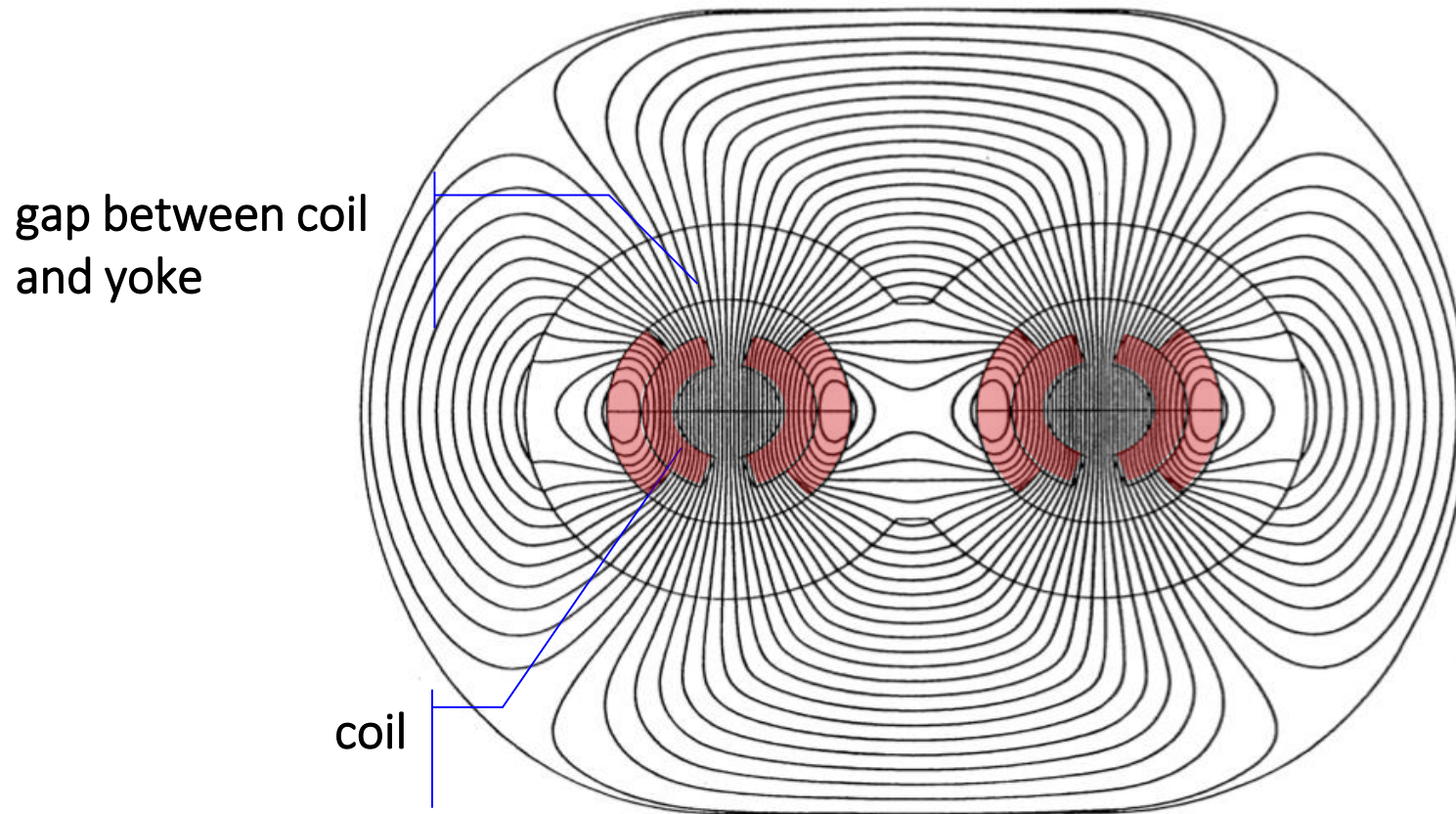


$j = 40 \text{ A/mm}^2$
 $w = 300 \text{ mm}$
 $NI = 4.5 \text{ MA}$
 $P = 6.2 \text{ MW/m}$ (if Cu at room temp.)

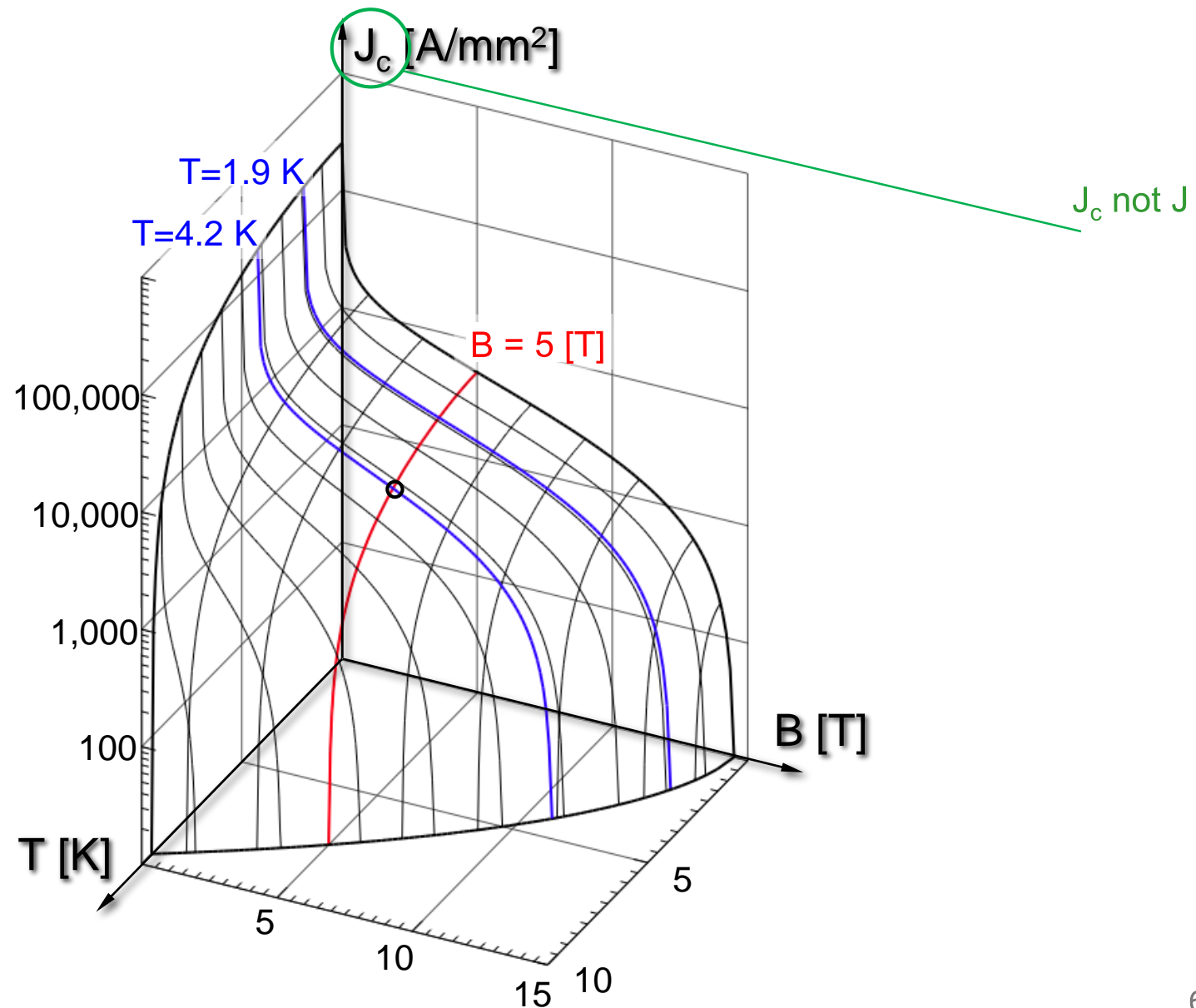
This is the actual coil of the LHC main dipoles (one aperture), showing the position of the superconducting cables



Around the coils, iron is used to close the magnetic circuit

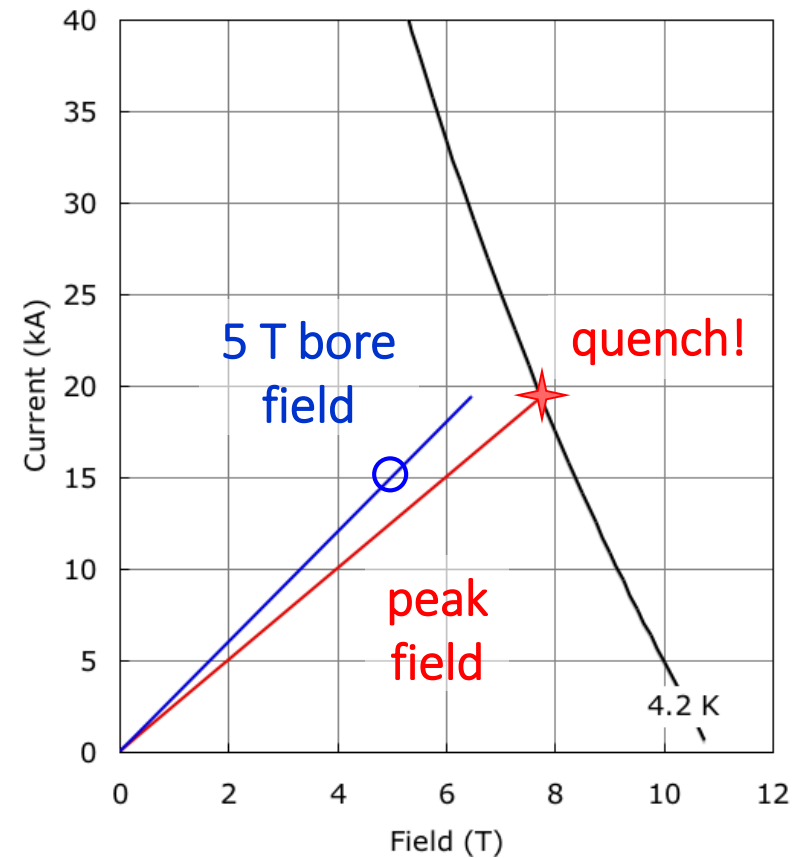
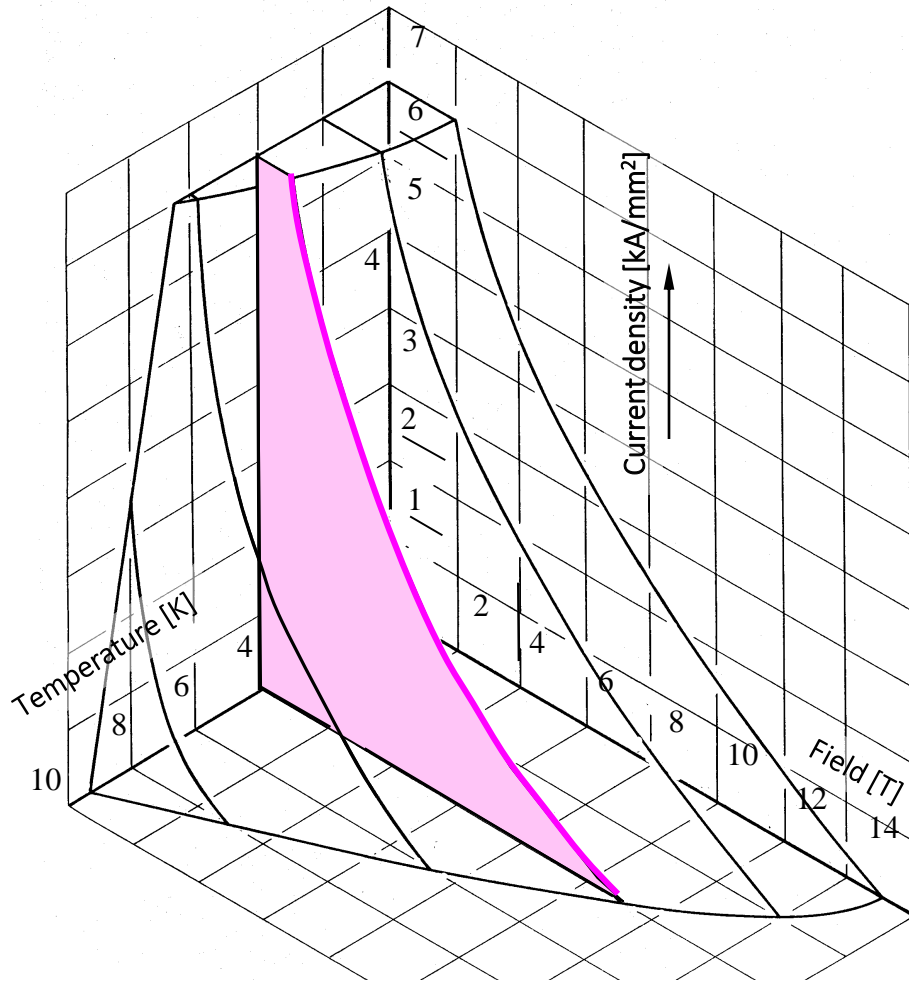


The allowable current density is high – though finite – and it depends on the temperature and the field

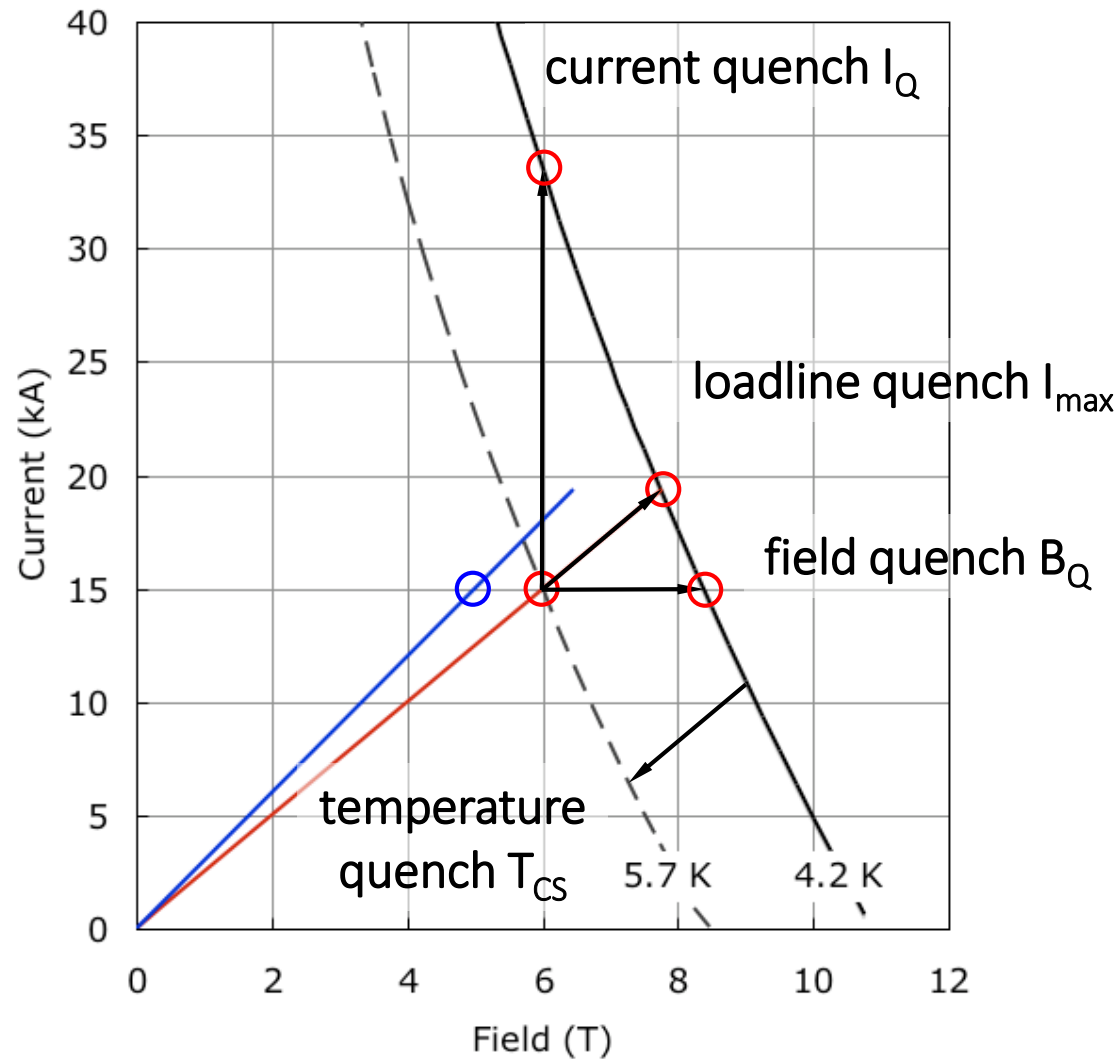


The maximum achievable field (on paper) depends on the amount of conductor and on the superconductor's critical line

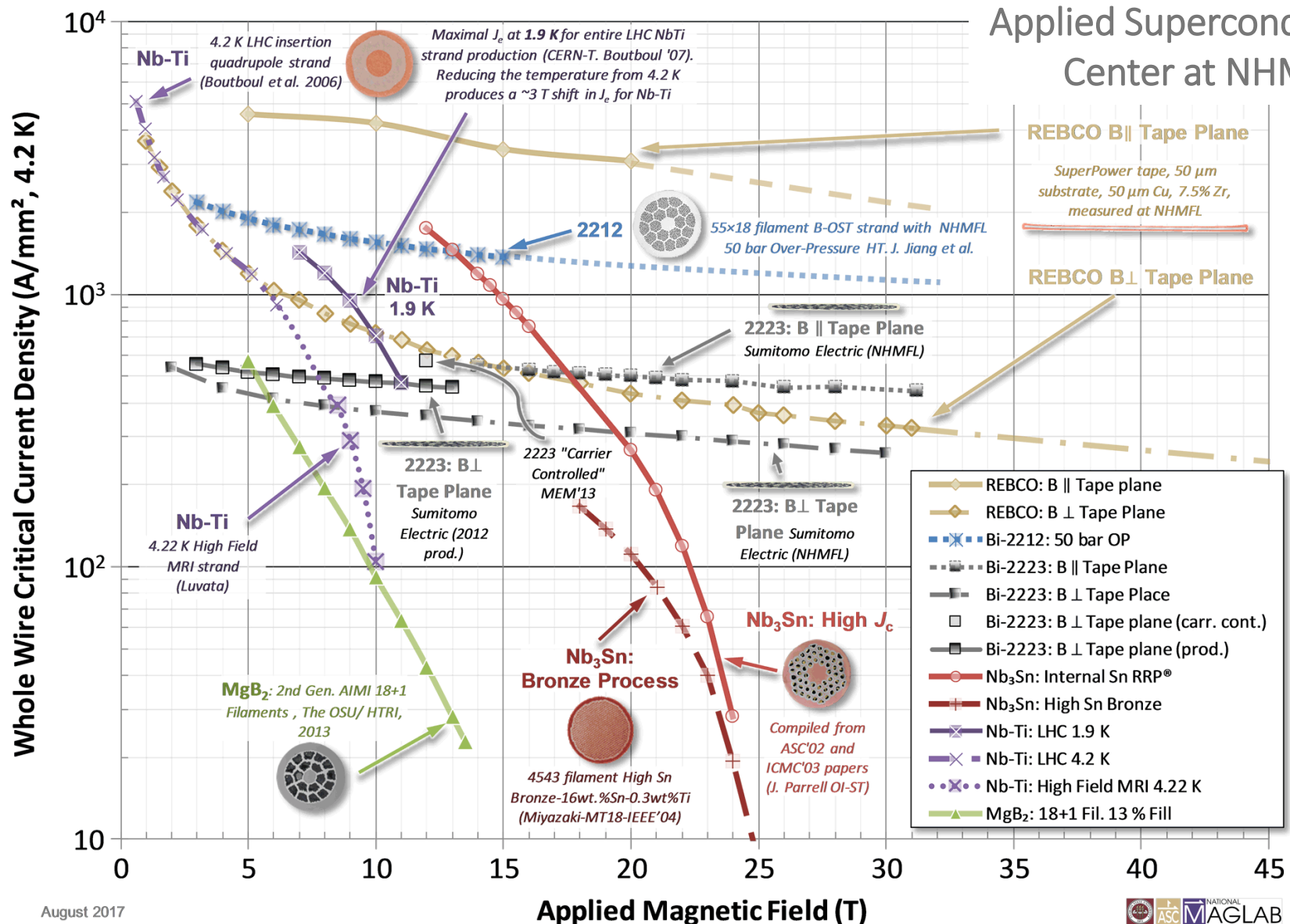
Nb-Ti critical surface $\longrightarrow I_C = J_C \times A_{SC} \longrightarrow$ Nb-Ti critical current I_C (B)



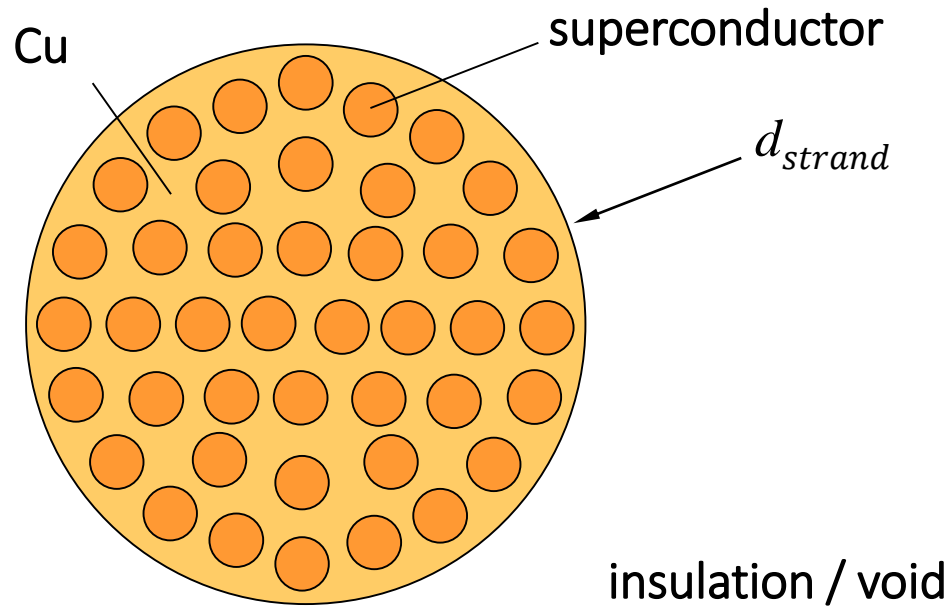
In practical operation, margins are needed with respect to this short sample limit



This is the best (Aug. 2017) critical current for several superconductors



The overall current density is lower than the current density on the superconductor

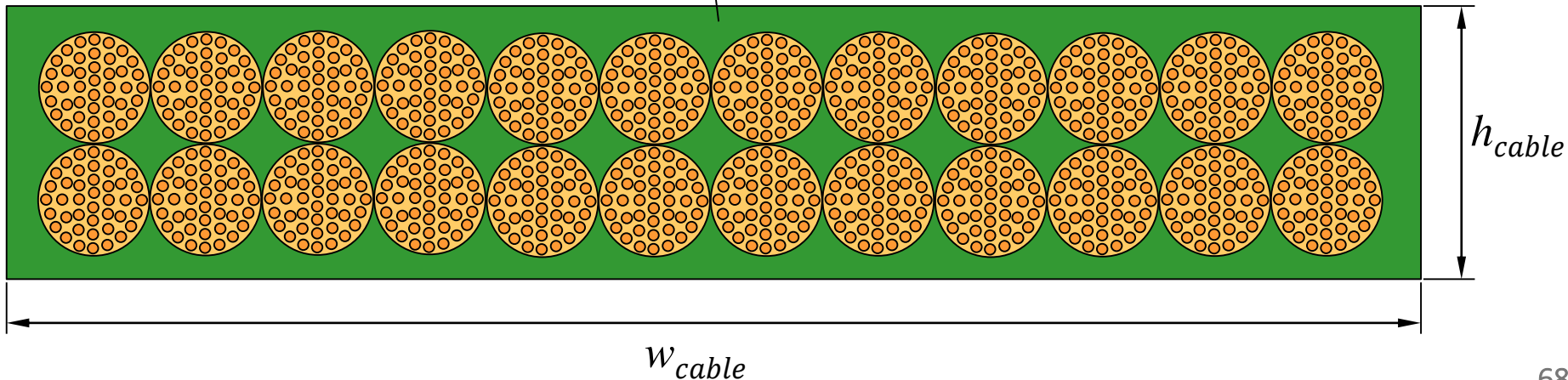


$$j_{overall} = \frac{I}{w_{cable} t_{cable}}$$

$$j_{cond} = \frac{I}{N_{strand} \frac{\pi d_{strand}^2}{4}}$$

$$j_{sc} = (1 + v_{Cu-sc}) j_{cond}$$

$$v_{Cu-sc} = \frac{A_{Cu}}{A_{sc}}$$



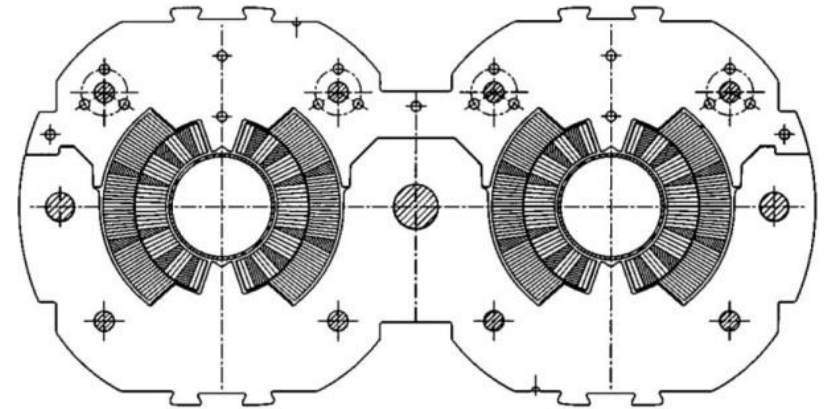
The forces can be very large, so the mechanical design is important

Nb-Ti LHC MB @ 8.3 T

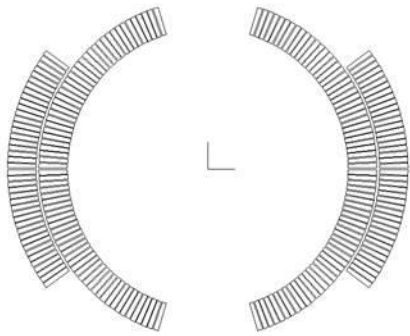
$F_x \approx 350 \text{ t per meter}$

precision of coil positioning: 20-50 μm

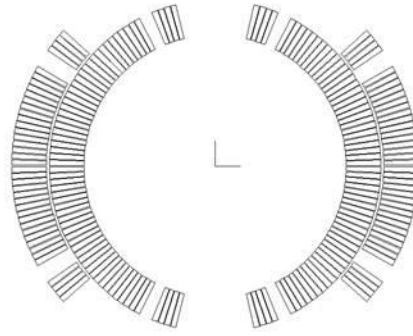
$F_z \approx 40 \text{ t}$



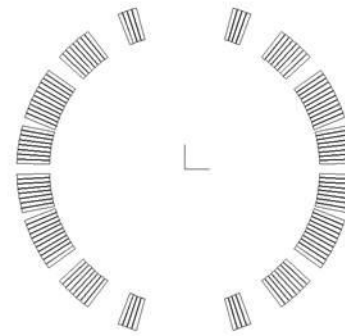
The coil cross sections of several superconducting dipoles show a certain evolution; all were (are) based on Nb-Ti



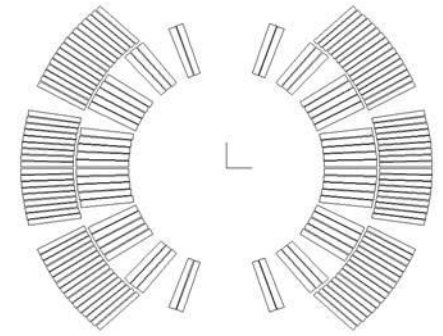
Tevatron



HERA

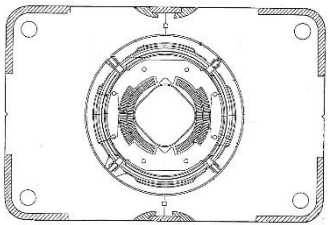


RHIC



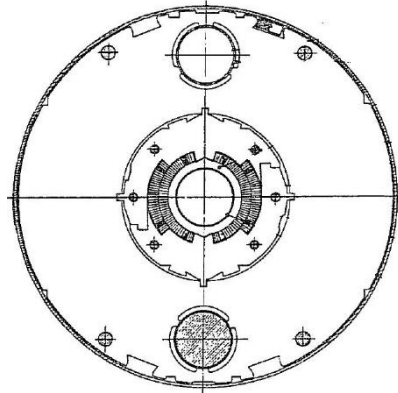
LHC
(one aperture)

Also the iron, the mechanical structure and the operating temperature can be quite diverse



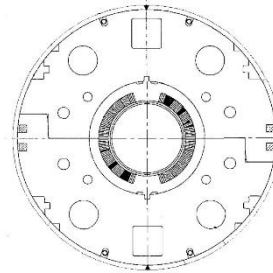
Tevatron

76 mm bore
 $B = 4.3 \text{ T}$
 $T = 4.2 \text{ K}$
first beam 1983



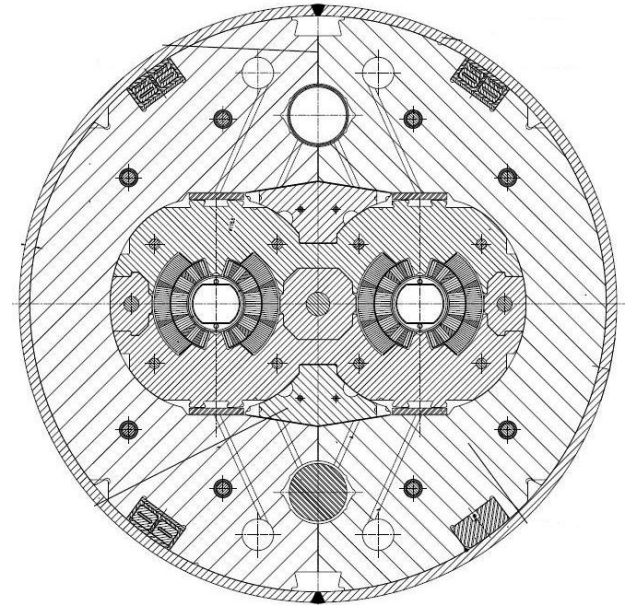
HERA

75 mm bore
 $B = 5.0 \text{ T}$
 $T = 4.5 \text{ K}$
first beam 1991



RHIC

80 mm bore
 $B = 3.5 \text{ T}$
 $T = 4.3\text{-}4.6 \text{ K}$
first beam 2000



LHC

56 mm bore
 $B = 8.3 \text{ T}$
 $T = 1.9 \text{ K}$
first beam 2008

This is how they look in their machines



1. Introduction, jargon, general concepts and formulae
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as an example, we will do a simplified 2D model of
FRESCA2, a large aperture (100 mm) high field (13 T)
Nb₃Sn dipole
(thanks to **Paolo Ferracin** and **Etienne Rochepault**)

There are different programs used for magnetic simulations

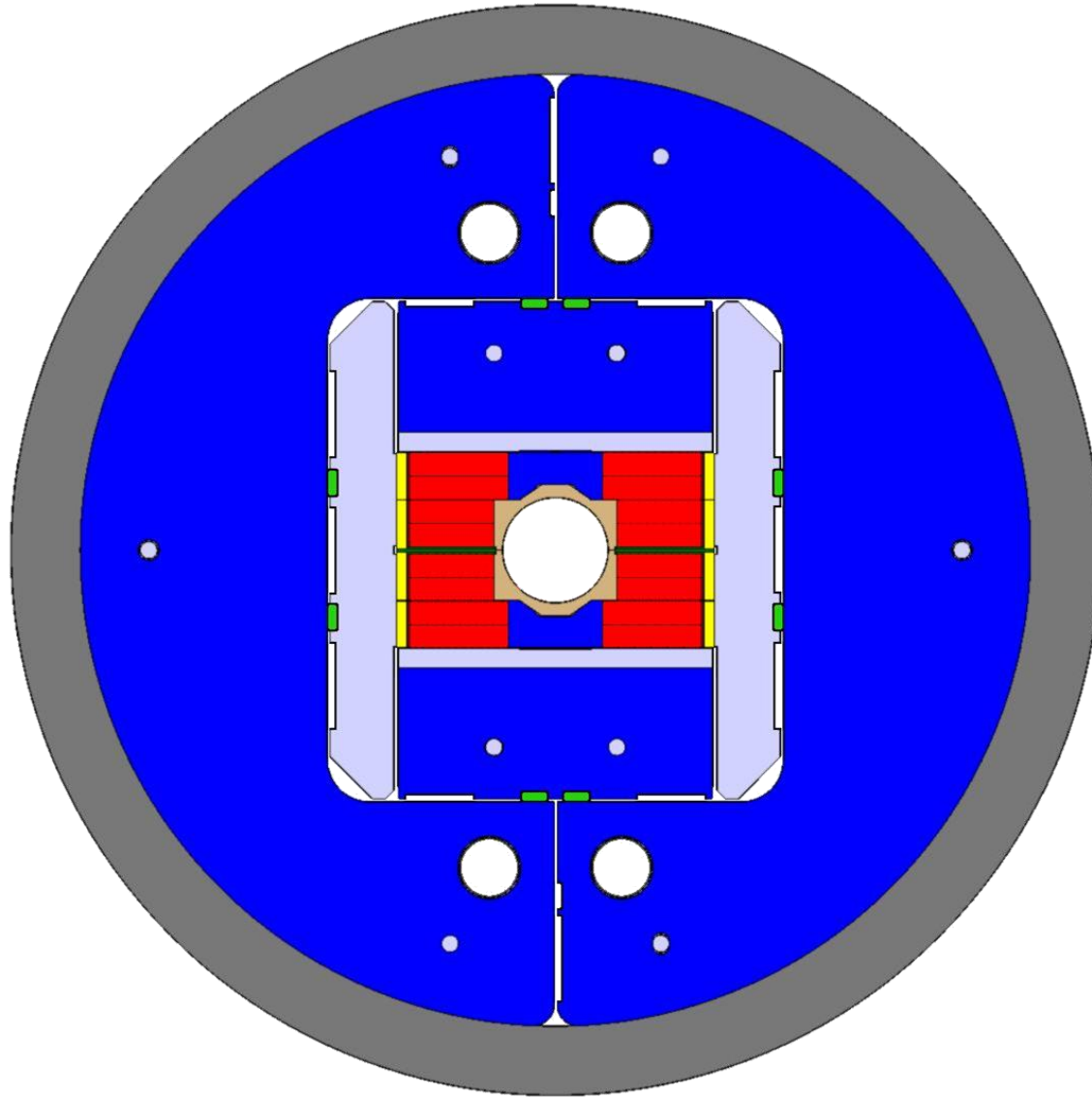


1. OPERA-2D and OPERA-3D, by COBHAM
2. ROXIE, by CERN
3. POISSON, by Los Alamos
4. FEMM
5. RADIA, by ESRF
6. ANSYS
7. Mermaid, by BINP
8. COMSOL

Here are a few references for FRESCA2

1. A. Milanese *et al.*, Design of the EuCARD high field model dipole magnet FRESCA2, MT22 conference, 2011
2. P. Ferracin *et al.*, Development of the EuCARD Nb₃Sn dipole magnet FRESCA2, ASC conference, 2012
3. F. Rondeaux *et al.*, “Block type” coils fabrication procedure for the Nb₃Sn dipole magnet FRESCA2, MT24 conference, 2015
4. E. Rochepault *et al.*, Fabrication and assembly of the Nb₃Sn dipole magnet FRESCA2, ASC conference, 2016
5. G. Willering *et al.*, Cold powering tests and protection studies of the FRESCA2 100 mm bore Nb₃Sn block coil magnet, MT25 conference, 2017

FRESCA2 is a high field, large aperture, dipole for a cable test facility at CERN

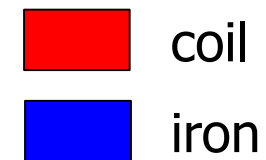


bore diameter 100 mm
outer diameter 1030 mm

nominal field 13.0 T

shell length 1.6 m
mass 10 t

material Nb_3Sn



These are the four superconducting coils before assembly

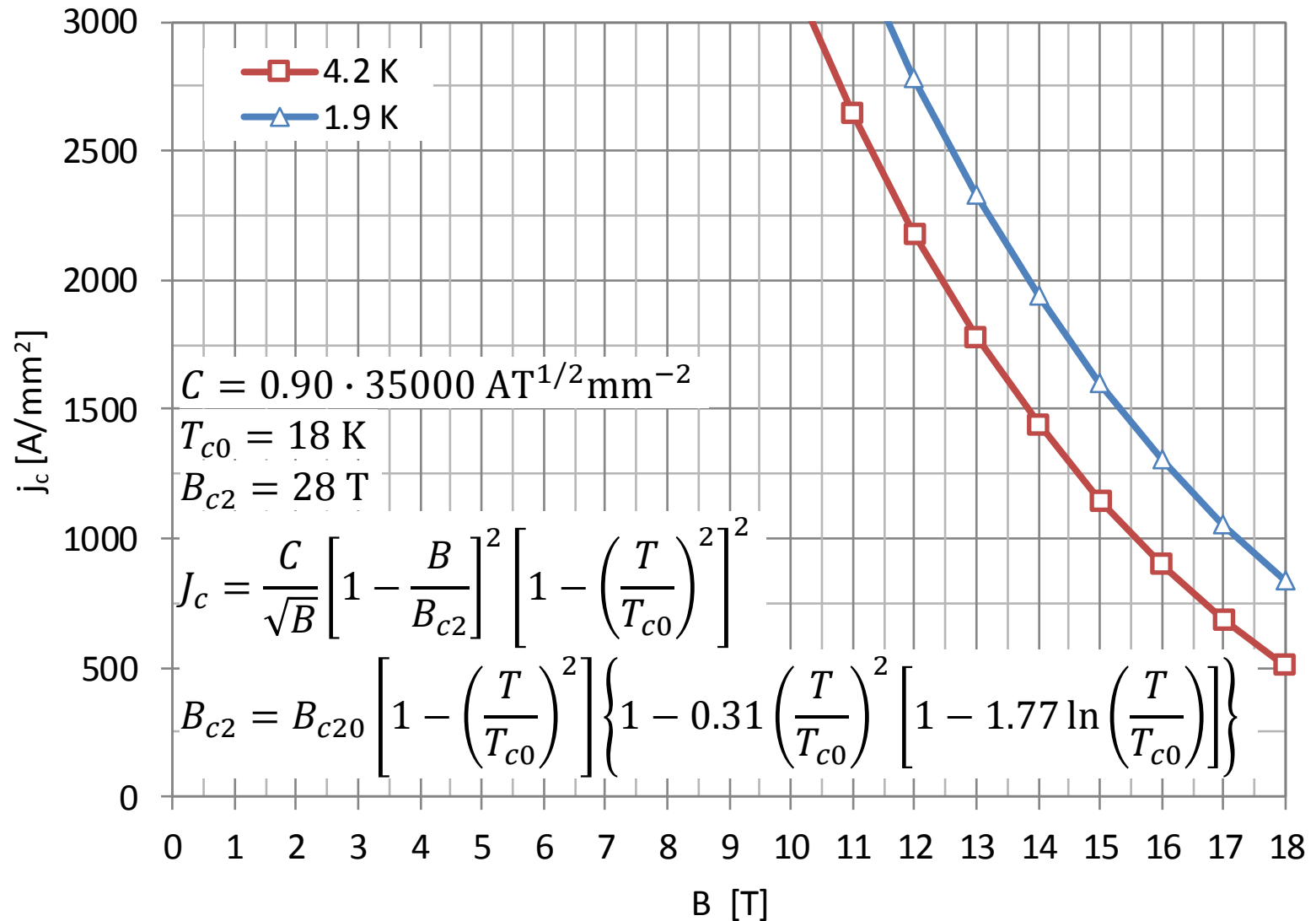


This is the FRESCA2 magnet ready to be tested in a vertical cryostat

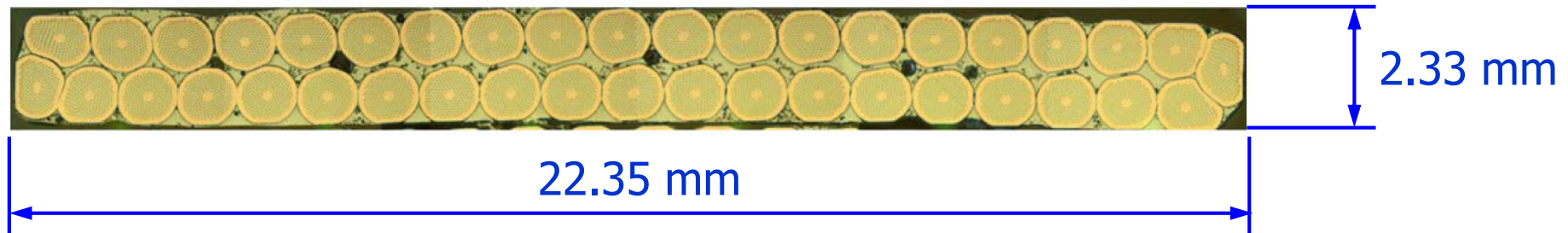


1.6 m

For our exercise, we assume the following critical curve for the Nb₃Sn conductor of FRESCA2



With the geometry of the cable and the nominal current, we can then compute the current densities for FRESCA2



$$N_{str} = 40$$

$$d_{str} = 1 \text{ mm}$$

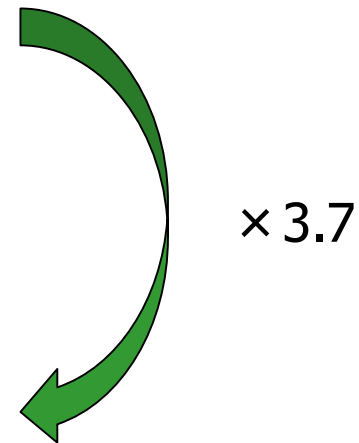
$$\nu_{\text{Cu-sc}} = 1.25$$

$$I = 11100 \text{ A}$$

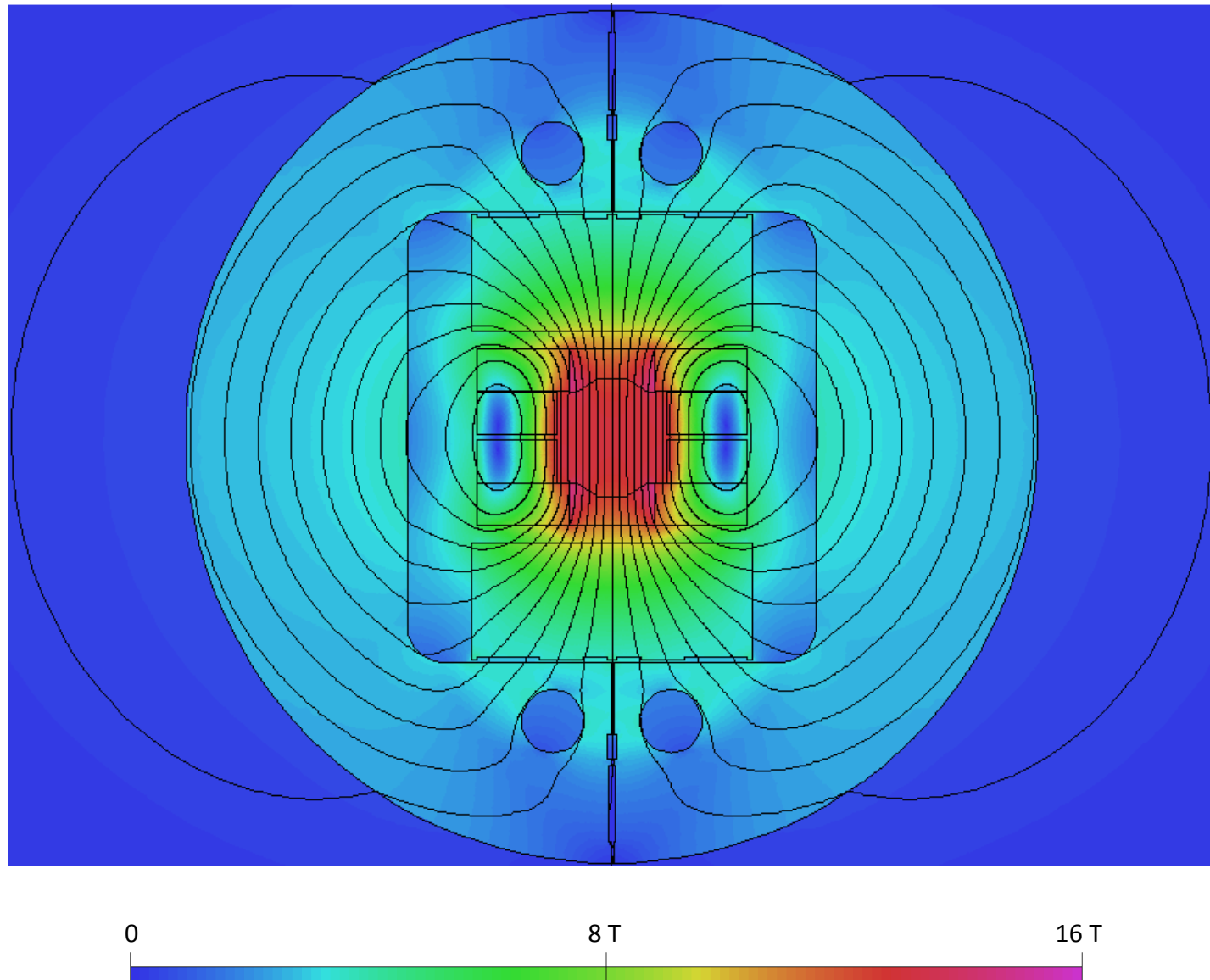
$$J_{\text{ovr}} = \frac{I}{w_{\text{cable}} t_{\text{cable}}} = 213.2 \text{ A/mm}^2$$

$$J_{\text{cond}} = \frac{I}{N_{str} \frac{\pi d_{str}^2}{4}} = 353.3 \text{ A/mm}^2$$

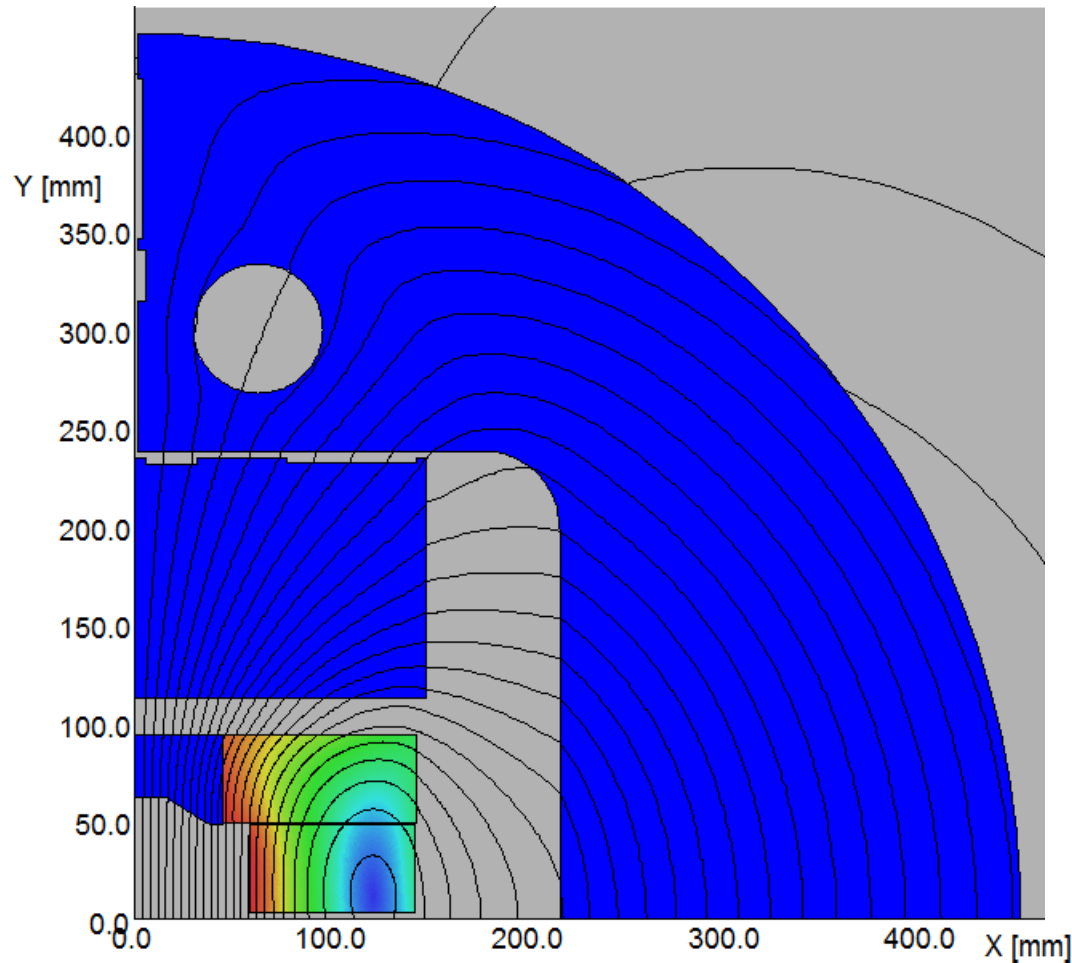
$$J_{\text{sc}} = (1 + \nu_{\text{Cu-sc}}) J_{\text{cond}} = 795.0 \text{ A/mm}^2$$



Here are the field and flux lines as computed in 2D with our OPERA model, for a central field of 13 T



Considering the symmetries, only one quarter of the dipole can be modeled; here we plot in particular the field in the coils



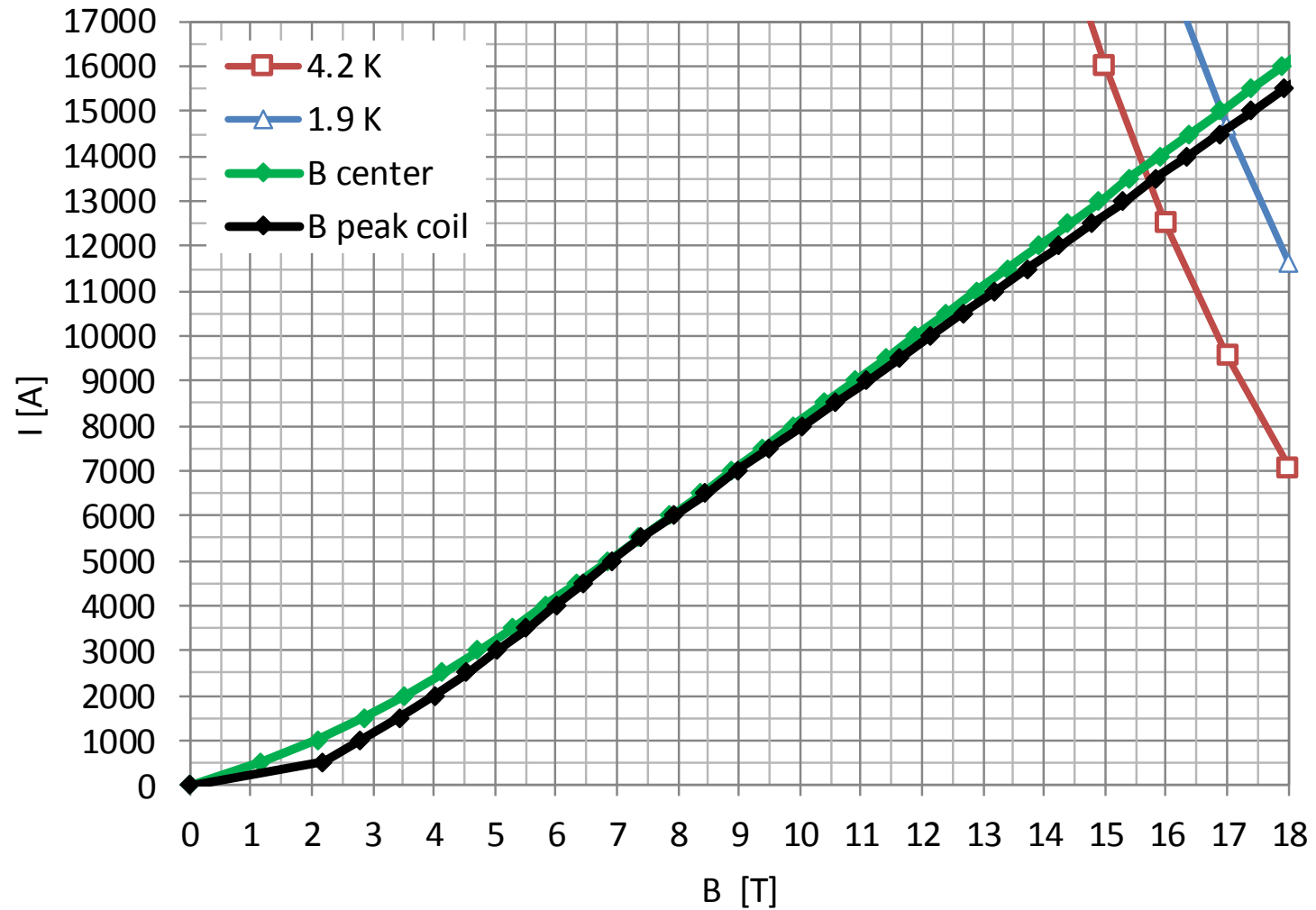
$$B_{\text{center}} = 13.00 \text{ T}$$

$$B_{\text{peak}} = 13.31 \text{ T}$$

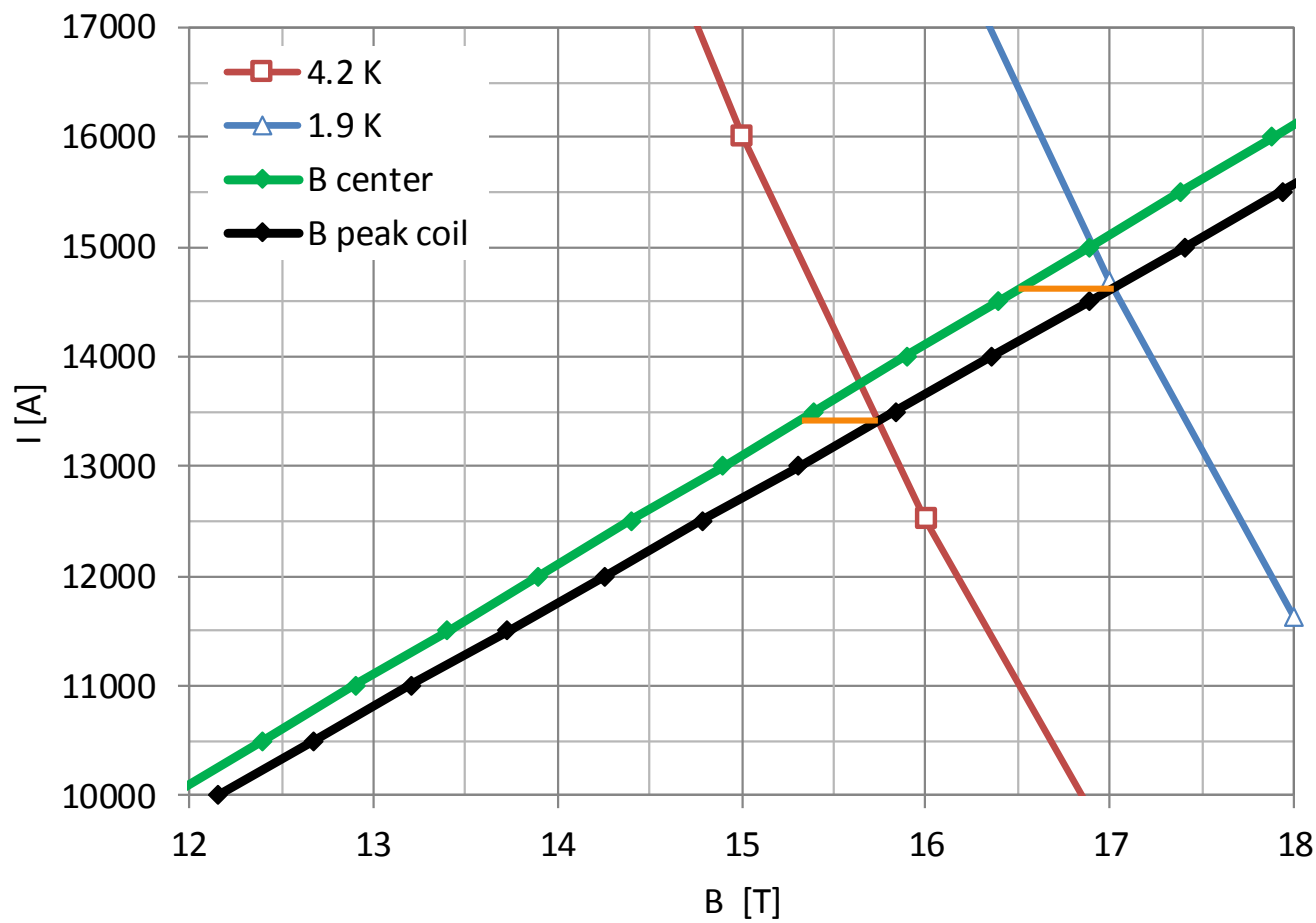
$$I_{\text{nom}} = 11100 \text{ A}$$



This is the “load line” of FRESKA2 using our 2D model



And this is a zoom of it, to show the “short sample” values

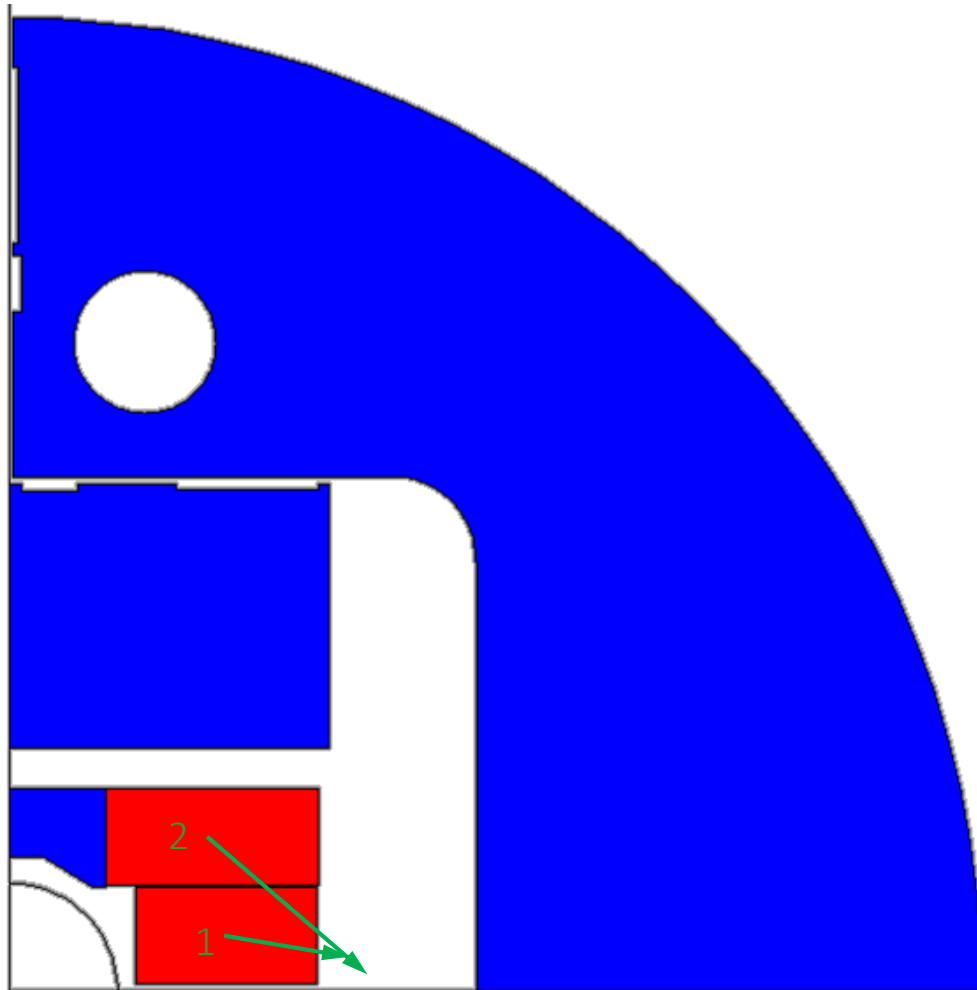


$I_{\text{nom}} = 11100 \text{ A}$
(13 T)

$I_{\text{ss}, 4.2 \text{ K}} = 13410 \text{ A}$
 $11100/13410 = 83\%$
 $B_{\text{ss}, 4.2 \text{ K}} = 15.3 \text{ T}$

$I_{\text{ss}, 1.9 \text{ K}} = 14630 \text{ A}$
 $11100/14630 = 76\%$
 $B_{\text{ss}, 1.9 \text{ K}} = 16.5 \text{ T}$

The Lorentz forces can be quite impressive, as they scale as B^2



at 13 T

coil 1

$$F_x = 3.33 \text{ MN/m}$$

$$F_y = -0.56 \text{ MN/m}$$

coil 2

$$F_x = 4.23 \text{ MN/m}$$

$$F_y = -3.60 \text{ MN/m}$$

in total

$$F_x = 1540 \text{ t/m}$$

To complete the 2D analysis, these are the allowed multipoles, computed with our model

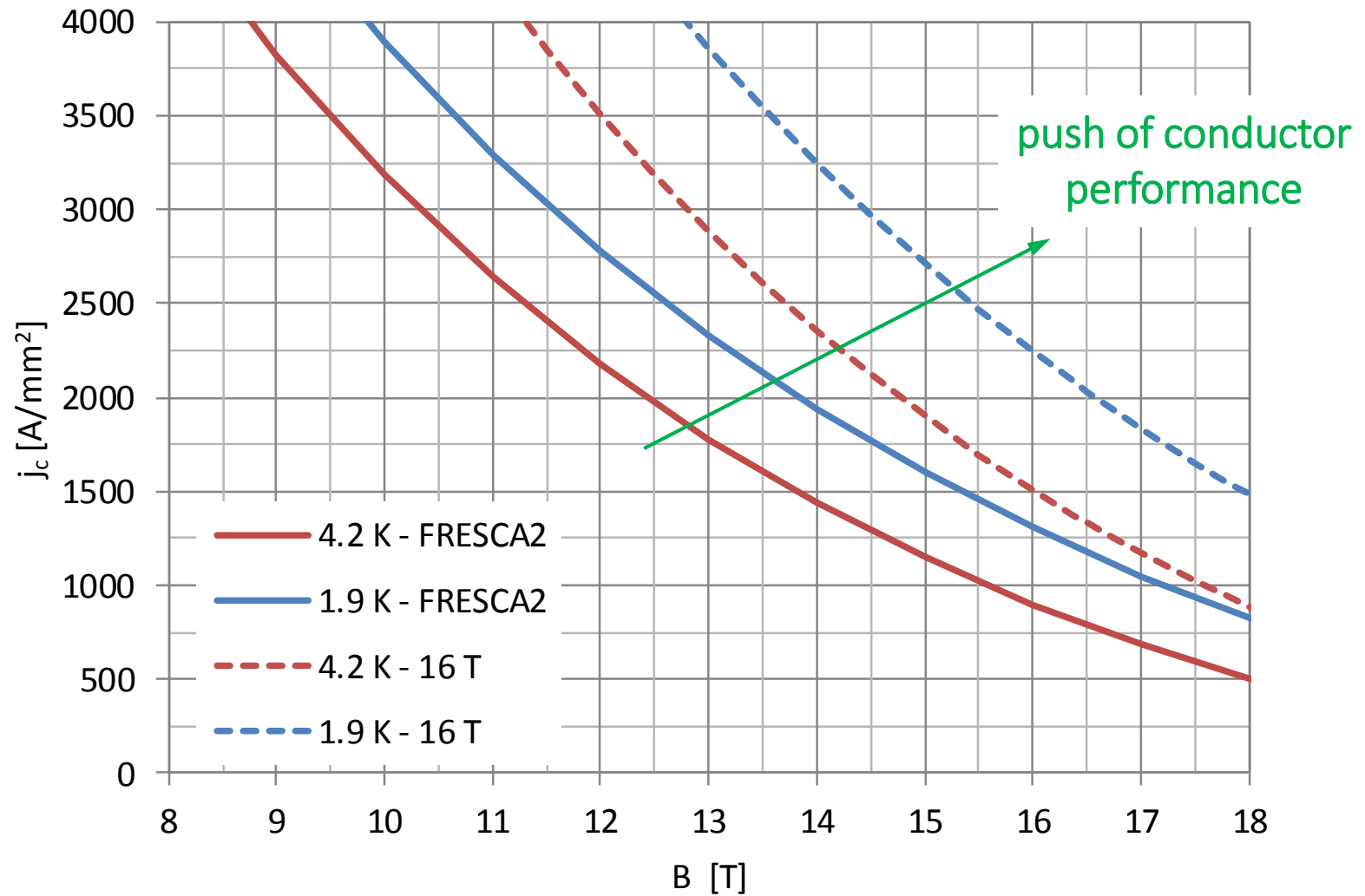
		I = 500 A	I = 5000 A	I = 11100 A
B_1	[T]	1.17	6.86	13.00
b_1	[1e-4]	10000	10000	10000
b_3	[1e-4]	44.7	12.7	79.5
b_5	[1e-4]	-221.3	-36.8	-28.0
b_7	[1e-4]	27.5	5.0	3.1
b_9	[1e-4]	8.0	0.8	0.1
b_{11}	[1e-4]	-2.6	-0.7	-0.5

$$R_{\text{ref}} = 33.33 \text{ mm}$$

Here are a few references for your project – 16 T dipoles for HE-LHC

1. Proceedings of the Malta Workshop “The High-Energy Large Hadron Collider”, Oct. 2010, CERN-2011-003
2. E. Todesco *et al.*, Dipoles for High-Energy LHC, MT23 conference, 2013
3. D. Tommasini *et al.*, Baseline specifications and assumptions for accelerator magnet, EuroCirCol-P1-WP5-M5.2, Apr. 2016
4. Various contributions in the FCC week 2017
 - 4.1 D. Tommasini, Baseline parameters of the 16 T dipoles for FCC
 - 4.2 C. Lorin and M. Durante, EuroCirCol Block Electromagnetic Design
 - 4.3 J. Munilla and F. Toral, Common Coil Configuration, Electromagnetic Computations
 - 4.4 V. Marinozzi *et al.*, EuroCirCol Cosine Theta Electromagnetic Design
5. D. Tommasini *et al.*, Status of the 16 T Dipole Development Program for a Future Hadron Collider, presented at MT25, Aug. 2017

These are the J_c curves of Nb₃Sn – at 4.2 K at 1.9 K – that can be used in your design work



Proposed steps for your 16 T dipole work

1. take the time to do a (limited) **literature review**; you could read [5] and some of the papers cited in there; also the presentations in [4] can be useful to see the various designs being explored
2. draft a **functional specification**, based on the input from the other groups (ex. optics) to define for ex. field and aperture; you can then also list the assumptions about the superconducting material (like J_c fit, operating temperature, amount of stabilizer, load line margin, cable size), following [3]
3. you can then sketch a **cross-section**, starting with a **single aperture** dipole, setting up a 2D magnetic model (one quarter), to decide on the number of turns, their position (for field quality), the size of the return yoke, etc.; this is an iterative process, where you might want to change the cable dimensions or other parameters set in 2.; you can adapt the scripts we used for FRESCA2
4. once you find a satisfying cross-section for a single aperture magnet, you can then move on to a **double aperture** one, again in an iterative way
5. at the end, you can write up your **report**, compiling in particular a table with basic properties of your design (like you find for the other layouts)