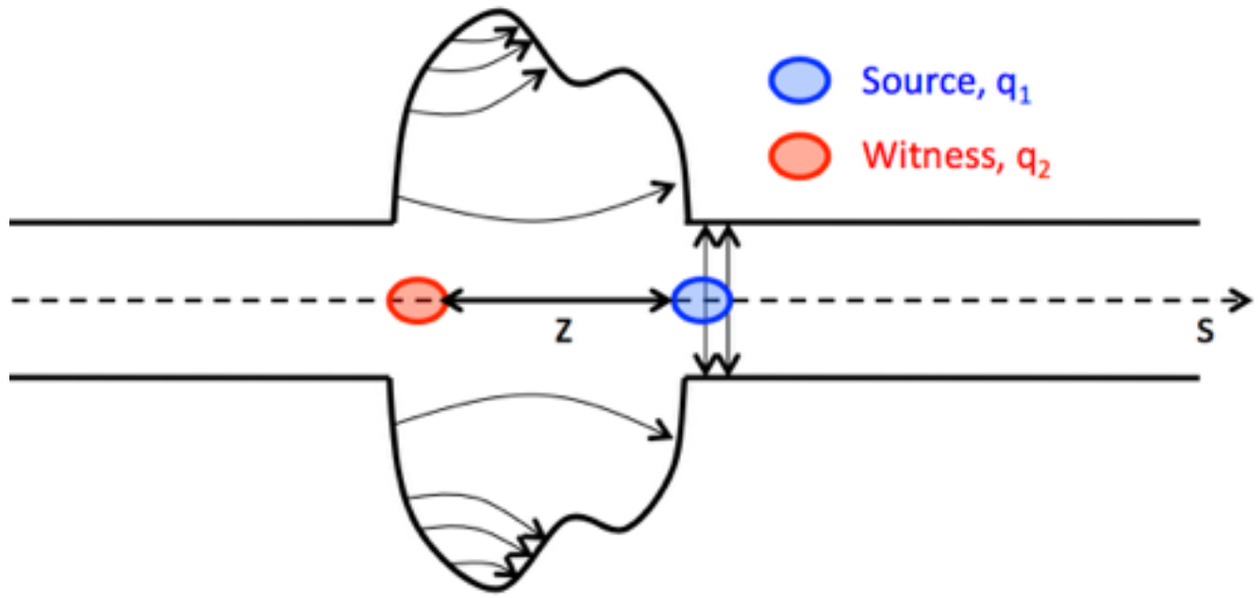
Robinson Instability



- Ignoring head-tail effects of beam on itself or multiple bunches, you get longitudinal instability caused by multi-turn wakefield seen by particle bunch across entire ring.

- However, wakefield response strongly peaked in rf cavity structure as $h\omega \sim \omega_r$.

[1] Rumolo, *Beam Instabilities*, arXiv:1601.05201

Robinson Instability

- Regular longitudinal equation of motion has additional driving term due to wakefields

longitudinal displacement relative to synchronous particle

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \omega_s^2 z = \frac{N_b \eta e^2}{m_0 \gamma C} \sum_{k=0}^{\infty} W_{||} \left[z(t) - z(t - kT_0) - kC \right].$$

synchrotron frequency

wakefield function

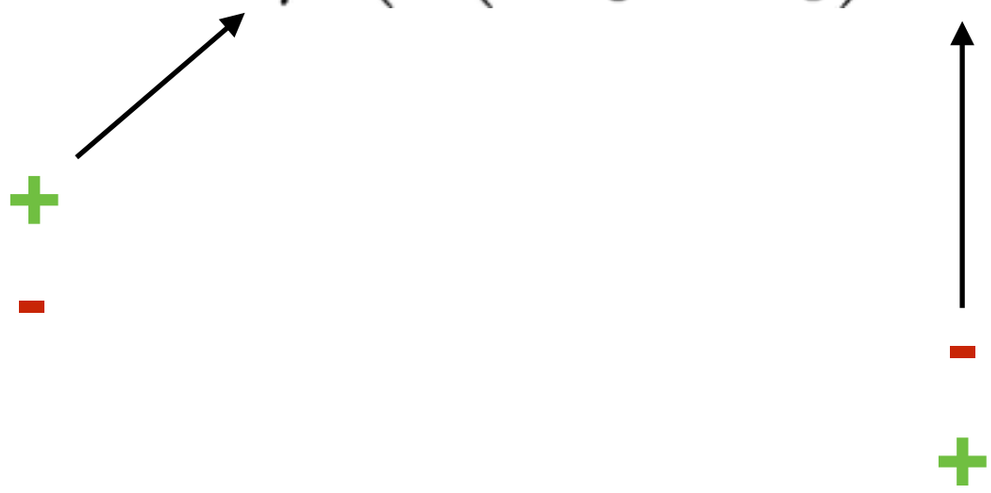
$$\frac{d\omega}{\omega_0} = \eta \frac{dp}{p}$$

[1] Rumolo, *Beam Instabilities*, arXiv:1601.05201

Robinson Instability

- Can taylor expand wakefield function about phase at previous turn (as long as wakefield function has no abrupt changes over bucket length)
- In frequency space, damping/anti-damping response on bunch is dependent on

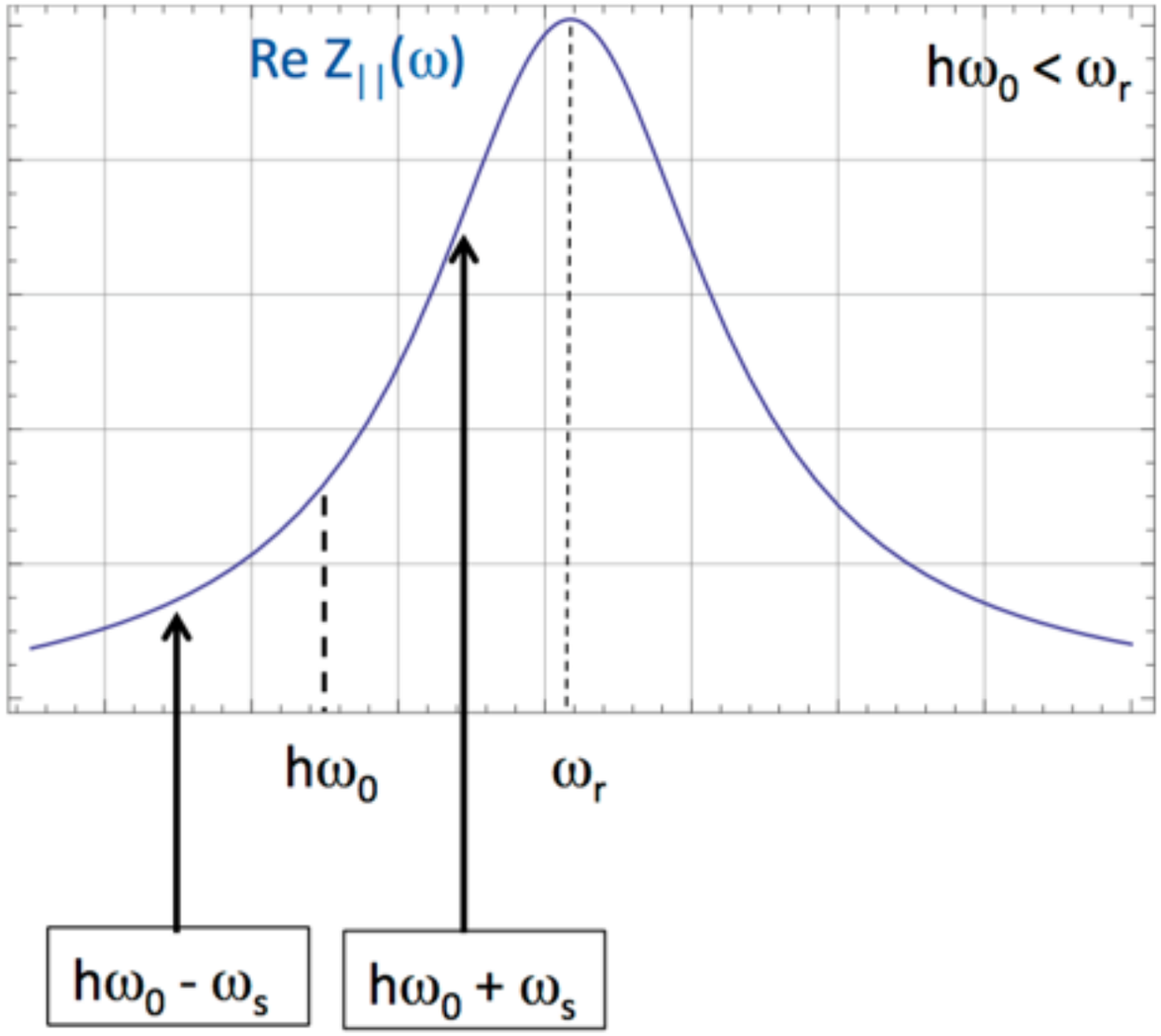
$$\tau \propto \eta \cdot (Z(h\omega_0 + \omega_s) - Z(h\omega_0 - \omega_s))$$

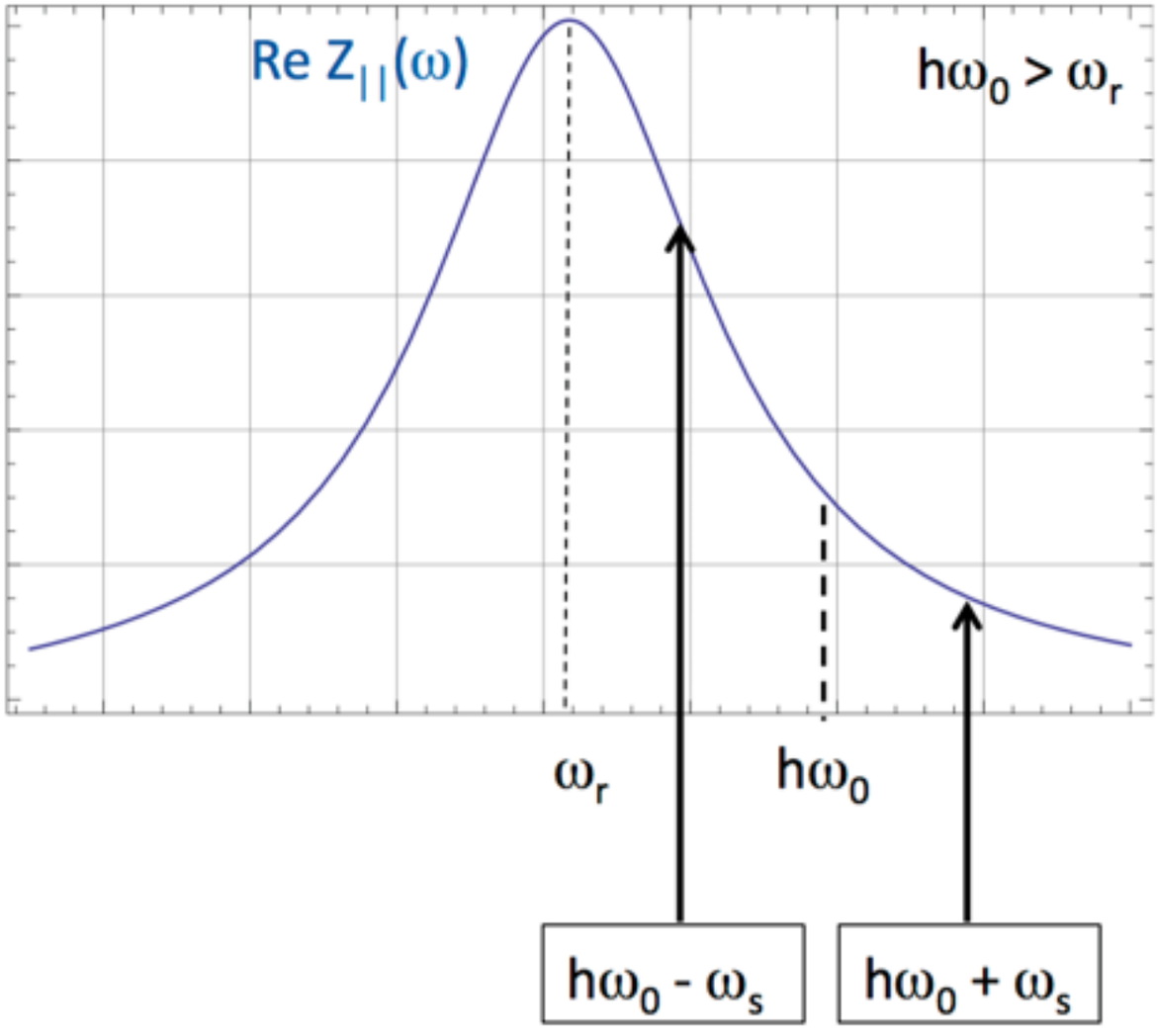


[1] Rumolo, *Beam Instabilities*, arXiv:1601.05201

Robinson Instability

- How to satisfy the two regions of stability:





[1] Rumolo, *Beam Instabilities*, arXiv:1601.05201