Instabilities I & II

JAI Graduate Course

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Introduction

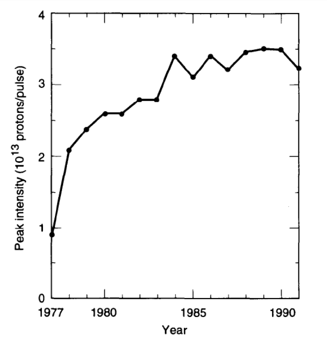
Instabilities

So you've carefully tuned the machine to produce the best possible performance at the highest intensity and you've invited the lab director to the control room to observe, for the first time, the machine working at design intensity. The intensity comes up, the pulse goes up and just when you're about to get there... there's a sudden loss of beam. You try again, and it repeats. This is characteristic of an instability.
- E. J. N. Wilson, CERN.

Ted Wilson in the SPS control room in 1977, Image ©CERN

Instabilities

Instabilities are one of the main factors that limit performance.



Peak beam intensity in the CERN PS over time, Image from [2] pp. 2 and Jacques Gareyte, 1991.

Types of Instabilties

When pushed in terms of performance, accelerators tend to reach an intensity limit. With analysis, understanding and (hopefully) mitigation, a new limit emerges. The same pattern can be seen with many high intensity and high energy accelerators.

Why does this happen?

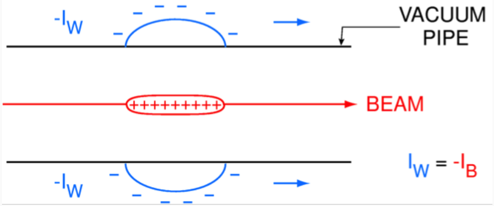
Electromagnetic interactions with the environment can affect both individual particles and the collective motion of the whole bunch.

We can have both **transverse** and **longitudinal** instabilities.
There are also both **single** and **multi-bunch** instabiltities.

Impedance of the wall

- There is a wall current I_W due to the circulating bunch
- Vacuum pipe is not smooth, so I_W sees an *impedance*.

$$Z = Z_r + iZ_i \tag{1}$$



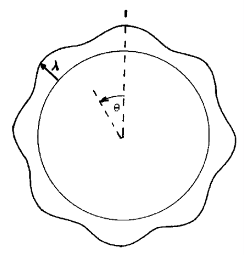
- The induced voltage is $V \approx I_W Z = -I_B Z$ which acts back on the beam.

Instabilities are intensity dependent.

Test of an Example Instability

From an initial small perturbation, we can test if the perturbation is:

- Increased, thus INSTABILITY
- Decreased, thus STABILITY



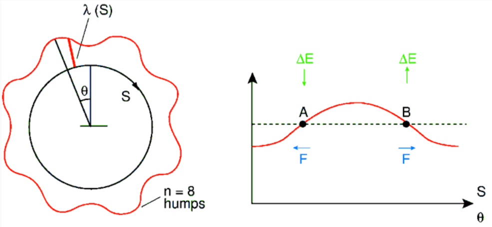
Example: perturbation in the local line density of charge around a synchrotron.

If the forces set up by a pattern of perturbation reinforce the shape, it is sure to grow exponentially.

Negative Mass Instability

No longer a problem, but helps us understand the mechanisms of instabilities.

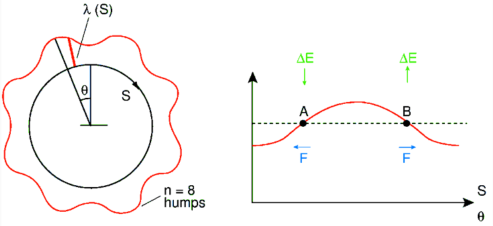
Imagine a ring with a modulation in the line density $\lambda(s)$, around the ring.



What is the result? $E = -\left[\frac{q}{4\pi\epsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$.

- Particle B finds itself with a larger charge density behind it than in front of it, pushing it forward.
- Conversely particle A will be decelerated by the mountain of charge in front of it.

Negative Mass Instability



$$E = -\left[\frac{q}{4\pi\epsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}.$$

So is this stable or unstable? Depends on γ_t.

Stability

If $\gamma < \gamma_t$: if energy is gained, revolution frequency increases, and A and B move away from the 'hump' of charge STABLE

If $\gamma > \gamma_t$: if energy is gained, revolution frequency decreases, and A and B move toward the 'hump' of charge. UNSTABLE

Impedance

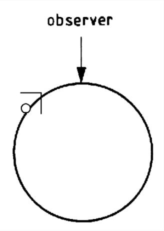
Impedance and Instabilities

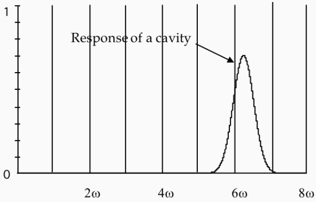
- In general, impedances are complex, and are functions of the frequency $Z(\omega) = Z(\omega)_{real} + iZ(\omega)_{imag}$
- Strong coupling between beam and vacuum chamber if the impedance and particle beam have a significant component at the same frequency
- Impedance depends on each piece of vacuum chamber including cavities, or changes in beam pipe diameter, material, shape etc...

Impedances for a particular component can be narrow band quality factor $Q >> 1$ as in an accelerating cavity

OR they can be broadband with $Q \approx 1$ due to change in vacuum chamber cross section.

Driving terms





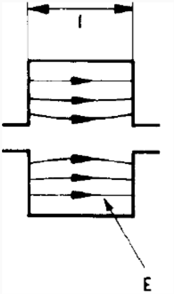
Spectrum from a bunch showing response of an r.f. cavity

Fourier analysis of a circulating delta function bunch of charge passing an observer.

$$I = \sum I_n e^{in\omega_0 t} \tag{2}$$

Produces a fundamental at the revolution frequency plus all higher harmonics in equal strength

Impedance in a cavity



The voltage experienced in local enlargement in the beam pipe (which acts like a cavity) has the form:

$$I = \hat{I}e^{-i\omega t}, V = \hat{V}e^{-i\omega t} \tag{3}$$

We can relate force on particles to the Fourier component of the beam current which excites the force.

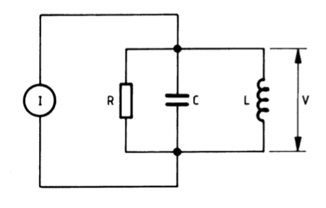
The impedance is a complex quantity. $V(\omega) = -Z(\omega)I(\omega)$

- REAL if voltage and current are in phase
- IMAGINARY if 90 degrees or i between voltage and current. (Inductive $= +$, Capacitive$= -$)
- Differs from RF wave by 90 degrees

The resistive part of the impedance can lead to a shift in the betatron oscillation frequency of the particles while the reactive or imaginary part may cause damping or anti-damping.

RLC Circuit Impedance

A cavity can be modelled as an AC resonant circuit:



$$\omega_r = \frac{1}{\sqrt{LC}} \tag{4}$$

Where the quality factor $Q = R\sqrt{C/L} = R/L\omega_r = RC\omega_r$

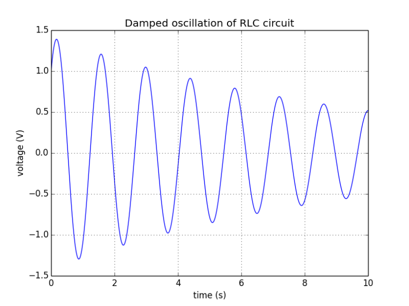
And a differential equation can be written down for voltage and current:

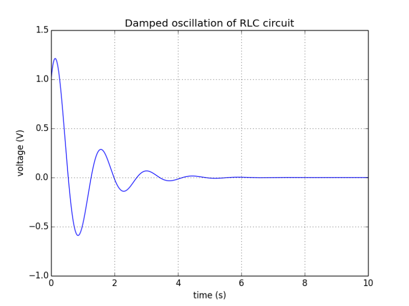
$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^2 V = \omega_r \frac{R}{Q}\dot{I} \tag{5}$$

RLC Circuit Impedance

The solution is a damped resonant circuit, damping rate $\alpha = \omega_r / 2Q$

$$V = V_0 e^{-\alpha t} \sin\left[\omega_r \sqrt{1 - \frac{1}{4Q^2}}\, t\right] + \phi \qquad (6)$$





http://www.amanogawa.com/archive/CircuitsA.html

RLC Circuit Impedance

If the current in the circuit is $I = \hat{I}e^{i\omega t}$ it can be shown [4] that the impedance seen is:

$$Z(\omega) = Z_r(\omega) + iZ_i(\omega) = R\left[\frac{1 - iQ\left(\frac{\omega^2-\omega_r^2}{\omega\omega_r}\right)}{1 + Q^2\left(\frac{\omega^2-\omega_r^2}{\omega\omega_r}\right)^2}\right] \tag{7}$$

- When ω is below resonant frequency, the reactive component is inductive or positive
- When the driving frequency is above the resonant frequency, it becomes negative and capacitive

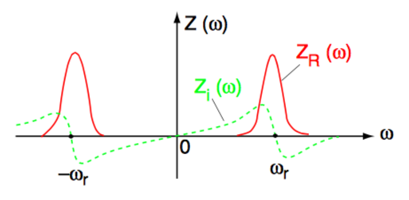
Impedance effects

- For a narrow-band impedance, high Q factor and low damping rate α. Thus signal will oscillate for many turns and produce *multi-bunch effects*
- For a broad-band cavity, Q is low, α is large, the fields collapse rapidly and don't affect subsequent bunches. May produce *single bunch effects*

RLC Circuit Impedance

For a high Q cavity (narrow band resonator) this can be simplified near the resonance frequency with $\Delta\omega = \omega - \omega_r$ to:

$$Z(\omega) \approx R_s \frac{1 - i2Q\frac{\Delta\omega}{\omega_r}}{1 + \left(2Q\frac{\Delta\omega}{\omega_r}\right)^2} \tag{8}$$



K. Schindl, "Instabilities", CERN Accelerator School

General Method for Studying Instabilities

Our Approach

- Propose a physical concept by which a perturbation to the beam might arise
- Try to determine whether this can lead to an instability
- Figure out under which conditions it is unstable

Negative Mass Instability again

We can describe the beam line density $\lambda(\theta)$ and the corresponding instantaneous current $I(\theta)$ as DC with a small AC component:

$$I = I_0 + I_1 e^{i(n\theta - \Omega t)} \tag{9}$$

Where n describes the ‘humps’ ($n = 8$ in previous slides), $\Omega = n\omega_0$ is the angular frequency.

This induces a voltage per turn due to the longitudinal impedance:

$$U_s = -I_1 e^{i(n\theta - \Omega t)} Z(\Omega) \tag{10}$$

Frequency Shift

We postulate that this produces a *complex frequency shift* $\Delta\Omega$ which modifies the pattern:

$$\Omega = n\omega_0 + \Delta\Omega \tag{11}$$

Now we take a little short-cut... recalling that the motion in an accelerating cavity with voltage V_0 and frequency hf_0 with a phase angle $\phi_s = 0$ (stationary bucket), we get an equation of motion in ϕ:

$$\left[\frac{E_0\beta^2\gamma}{2\pi\eta h f_0^2 e}\right]\ddot{\phi} + V_0\phi = 0 \tag{12}$$

And the small amplitude synchrotron oscillation frequency is:

$$\omega_s^2 = \left[\frac{e\eta h V_0 \omega_0^2}{2\pi E_0 \beta^2 \gamma}\right] \tag{13}$$

Frequency Shift

$$\omega_s^2 = \left[\frac{e\eta h V_0 \omega_0^2}{2\pi E_0 \beta^2 \gamma}\right] \tag{14}$$

From this, we replace the term describing voltage and harmonic number with: $V_0 h \rightarrow -inZI_0$

Note that the i is due to the fact that unlike the RF, the voltage induced by a resistive impedance passes zero 90 degrees after the particle passes.

Which happens to be the correct result, and gives us the frequency shift:

$$(\Delta\Omega)^2 = (\Omega - n\omega_0)^2 = -i\left[\frac{\eta\omega_0^2 nI_0}{2\pi\beta_2 E}\right](Z_r + iZ_i) \tag{15}$$

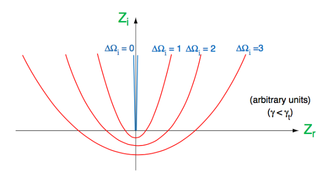
Growth, Damping and Frequency Shift

If we put the complex frequency shift back into the equation for instantaneous current, we get:

$$I(t,\theta) = I_0 + I_1 e^{\Delta\Omega_i t} e^{i(n\theta - (n\omega_0 + \Delta\Omega_r)t)} \tag{16}$$

- the $\Delta\Omega_i$ term describes the growth or damping of the mode
- the $\Delta\Omega_r$ term is the real frequency shift of the rotating pattern

Stability diagram



Take Equation 15 and lump all the beam parameters into a constant ξ, to get:

$$(\Delta\Omega)^2 = \xi(Z_r - iZ_i) = (\Delta\Omega_r + i\Delta\Omega_i)^2 \tag{17}$$

Equate the real and imaginary parts, to get parabolic contours for $\Delta\Omega_i = \mathrm{const}$:

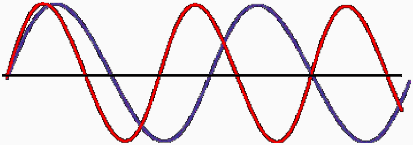
$$Z_r = 2\Delta\Omega_i\sqrt{Z_i/\xi + \Delta\Omega_i^2/\xi^2} \tag{18}$$

Relates the imaginary part of $\Delta\Omega$ the growth (or damping) rate, to the complex impedance Z as a plot of contours of constant growth rate in the Z_r, Z_i plane. The area where there is no growth is infinitely small and concentrated on the axis. So: *the negative mass instability should drive any unbunched beam mode unstable due to the resistivity of the vacuum chamber!?*

Landau Damping

Landau Damping - the idea

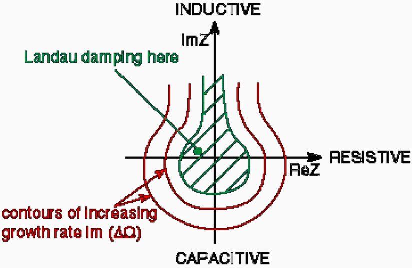
In a real machine, not all particles in the beam have the same frequency. The coherent motion from an instability therefore de-coheres over time, potentially damping the instability.



Two oscillators excited together become incoherent and give zero centre of charge motion after a number of turns comparable to the reciprocal of their frequency difference.

Landau Damping - stability diagram

Landau damping applies not just to longitudinal but also transverse, single and multi-bunch instabilities. Along with active feedback systems, it is a powerful way to overcome coherent beam instabilities



The line defining zero growth rate leads us to a handy approximation for the stability limit of unbunched beams, the 'Keil-Schnell Stability Criterion':

$$\left|\frac{Z}{n}\right| \leq \frac{F m_0 c^2 \beta^2 \gamma \eta}{I_0} \left(\frac{\Delta p}{p}\right)^2_{FWHH} \tag{19}$$

Types of Instabilities

Types of Instabilities

Table 1: A non-exhaustive list of instabilities

	Transverse	Longitudinal
Single bunch	Rigid bunch instability	Negative mass instability Head tail instability Robinson instability Longitudinal microwave instability
Multi-bunch	Coupled bunch modes Resistive wall instability	Coupled bunch modes

For some more detailed discussion on these, [1] and [3] are useful references. Useful books include Wiedemann [5] and Chao [2].

Questions?

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