# Coulomb effects on HBT correlations in Fourier space

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In preparation...

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*Máté Csanád, András László, Daniel Kincses, Tamás Csörgő...*

# Outline

*Experimentally motivated ,,theoretical" work; quest for exact formulas...*

- Introduction
  - HBT correlations, Coulomb effect, usual treatments
  - Basic formulas
  - Need for refinement of calculations: non-Gaussian sources, precision measurements, etc.
  - Numerical & methodological motivation
- Calculating the Coulomb kernel
  - Some details (hope to enrich the knowledge on the Coulomb problem)
    $\rightarrow$ Take over & generalize & reinterpret (old) formulas (*Bremsstrahlung*)
  - The spherically symmetric Coulomb kernel
  - Examples of application
- Outlook

  *Work in progress...*

# Introduction

- HBT correlations of like particles (eg. $\pi^+\pi^+$, $\pi^-\pi^-$, $K^+K^+$): measure space-time extent of source $S(x,p)$
- Some definitions:

single part. distr.: $N_1(\mathbf{p}) = \int \mathrm{d}x\, S(x,p)$

pair wave function: $\psi(x_1,x_2)$

pair mom. distr.: $N_2(\mathbf{p}_1,\mathbf{p}_2) = \int \mathrm{d}x_1 dx_2\, S(x_1,p_1) S(x_2,p_2) |\psi(x_1,x_2)|^2$

corr. function: $C(\mathbf{p}_1,\mathbf{p}_2) = \frac{N_2(\mathbf{p}_1,\mathbf{p}_2)}{N_1(\mathbf{p}_1)N_1(\mathbf{p}_2)}$

pair source: $D(\mathbf{r},\mathbf{K}) = \int \mathrm{d}^4\rho\, S\left(\rho + r/2, \mathbf{K}\right) S\left(\rho - r/2, \mathbf{K}\right)$

- Approximately thus

$$C(\mathbf{k},\mathbf{K}) = \frac{\int D(\mathbf{r},\mathbf{K}) |\psi_{\mathbf{k}}(\mathbf{r})|^2 \mathrm{d}\mathbf{r}}{\int D(\mathbf{r},\mathbf{K}) \mathrm{d}\mathbf{r}}, \qquad \mathbf{K} := \frac{\mathbf{p}_1 + \mathbf{p}_2}{2}, \quad \mathbf{k} := \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}.$$

# Introduction

- Core-halo model intercept parameter $\lambda$: $S = \sqrt{\lambda} S_c + (1 - \sqrt{\lambda}) S_h$

$$S_h \text{ ,,large''}: \quad \Rightarrow \quad C(\mathbf{k},\mathbf{K}) = 1 - \lambda + \lambda \frac{\int D_c(\mathbf{r},\mathbf{K}) |\psi_{\mathbf{k}}(\mathbf{r})|^2 \mathrm{d}\mathbf{r}}{\int D_c(\mathbf{r},\mathbf{K}) \mathrm{d}\mathbf{r}}.$$

- No **f**inal **s**tate **i**nteractions: $C(\mathbf{k}) \equiv C^{(0)}(\mathbf{k})$, Fourier transform of source

$$\left|\psi^{(0)}_{\mathbf{k}}(\mathbf{r})\right|^2 = 1 + \cos(2\mathbf{kr}) \quad \Rightarrow \quad C^{(0)}(\mathbf{k}) = 1 + \lambda \frac{\int D_c(\mathbf{r},\mathbf{K}) \cos(2\mathbf{kr}) \mathrm{d}\mathbf{r}}{\int D_c(\mathbf{r},\mathbf{K}) \mathrm{d}\mathbf{r}}.$$

- Final state Coulomb interaction: $\psi^{(0)}$ replaced by solution of two-body Coulomb Schr. eq.; NR case: well known formulas (see below)

$$C^{(0)}(\mathbf{k}) = \frac{C(\mathbf{k})}{K(\mathbf{k})}, \quad K(\mathbf{k}) \equiv \frac{\int D_c(\mathbf{r}) |\psi_{\mathbf{k}}(\mathbf{r})|^2 \mathrm{d}\mathbf{r}}{\int D_c(\mathbf{r}) |\psi^{(0)}_{\mathbf{k}}(\mathbf{r})|^2 \mathrm{d}\mathbf{r}} \quad \text{Coulomb correction}$$

- Final state strong interaction: small (?) for $\pi\pi$, $KK$
  Usual treatment: only $s$-wave (1 parameter: strong scattering length $f_0$)

# Coulomb interaction

- Non-relativistic treatment: well known formulas.
- $\mathbf{p}=\hbar\mathbf{k}$: relative momentum, $E=\frac{p^2}{2m}$, $m$: reduced mass $(=M/2)$.
- Coulomb parameter $\eta$: ratio of classical closest distance $r_0$ to wave-length $\lambda$

$$\eta\equiv\alpha\frac{mc}{\hbar k}=\frac{\pi r_0}{\lambda},\quad \text{with}\quad \alpha\equiv\frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{\hbar c}\approx\frac{1}{137},\quad r_0\equiv\frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{E},\quad \lambda=\frac{2\pi\hbar}{p}.$$

- Two-particle wave function (symmetrized):

$$\psi^{(C)}=e^{i\mathbf{KR}}\times\frac{\mathcal{N}}{\sqrt{2}}\left\{e^{i\mathbf{kr}}F\big(-i\eta,1,i(kr-\mathbf{kr})\big)\;+\;(\mathbf{k}\leftrightarrow-\mathbf{k})\right\}.$$

*Remark:* $\psi^{(+)}$ is usual, would take $\psi^{(-)}$ instead; results identical...

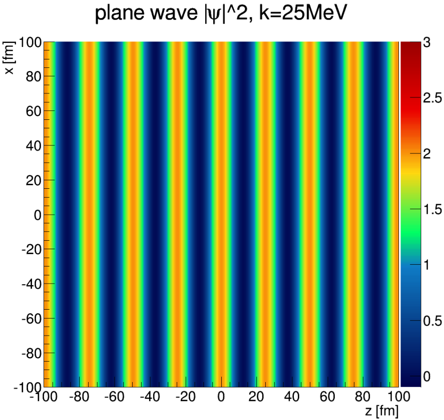
See e.g. Landau & Lifshitz, Quantum Mechanics, §134 (Pergamon Press, 1965)

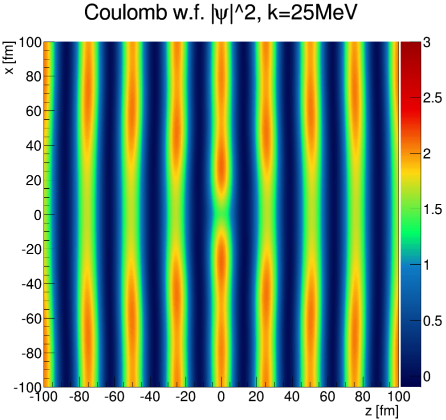
- Normalization ($\mathcal{N}$) and Gamow penetration factor ($|\mathcal{N}|^2$):

$$\mathcal{N}=e^{-\pi\eta/2}\Gamma(1+i\eta),\quad |\mathcal{N}|^2=e^{-\pi\eta}|\Gamma(1+i\eta)|^2=\frac{2\pi\eta}{e^{2\pi\eta}-1}.$$

# Coulomb interaction

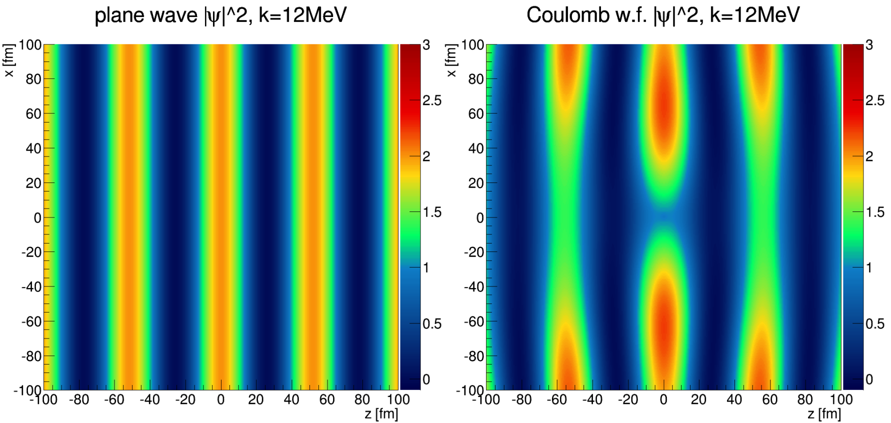
$$\left|\psi_{\mathbf{k}}^{(0)}(\mathbf{r})\right|^2 = 1+\cos(2\mathbf{kr}) \quad \text{vs.} \quad \left|\psi_{\mathbf{k}}^{(C)}(\mathbf{r})\right|^2:$$



# Coulomb interaction

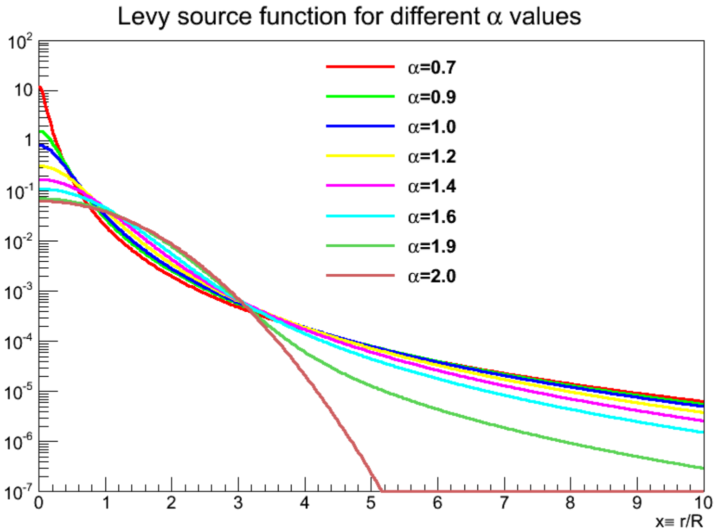
$$\left|\psi_{\mathbf{k}}^{(0)}(\mathbf{r})\right|^2 = 1+\cos(2\mathbf{kr}) \quad \text{vs.} \quad \left|\psi_{\mathbf{k}}^{(C)}(\mathbf{r})\right|^2:$$



# Source types

- Gaussian: the usual choice. $D_c(\mathbf{r}) \propto \exp\left(-r_k r_l R_{kl}^{-1}\right)$.
  - Fit parameters: $R_{kl}(\mathbf{K})$ and $\lambda(\mathbf{K})$
  - A generalization: Edgeworth expansion *of* $C(k)$ $\Rightarrow$ source: FT of $C^{(0)}(k)$
    see eg. T. Csörgő, S. Hegyi, Phys. Lett. B **489**, 15 (2000)
- Levy sources: ,,generalized Gaussian", $\alpha$: Levy parameter
  - Expressed as a Fourier transform: $D(\mathbf{r}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\mathbf{q}\, e^{i\mathbf{qr}} \exp\left(-|\mathbf{q}R|^\alpha\right)$
    $\Rightarrow$ $C^{(0)}(\mathbf{k})$ is easy, $D(r)$ itself not.
  - For $\alpha \neq 2$, slowly decreasing $(\propto r^{-3-\alpha})$
  - At PHENIX: prelim. data strongly favor $\alpha < 2$
  - Generalization: Levy polynomials (cf. Edgeworth for Gaussians)
    T. Novák, *et al.*, Acta Phys. Polon. Supp. **9**, 289 (2016)
- Cauchy sources $\Leftrightarrow$ exponential $C(k)$: special case of Levy ($\alpha = 1$)
  employed at CMS for HBT in p+p collisions. . .

# Illustration



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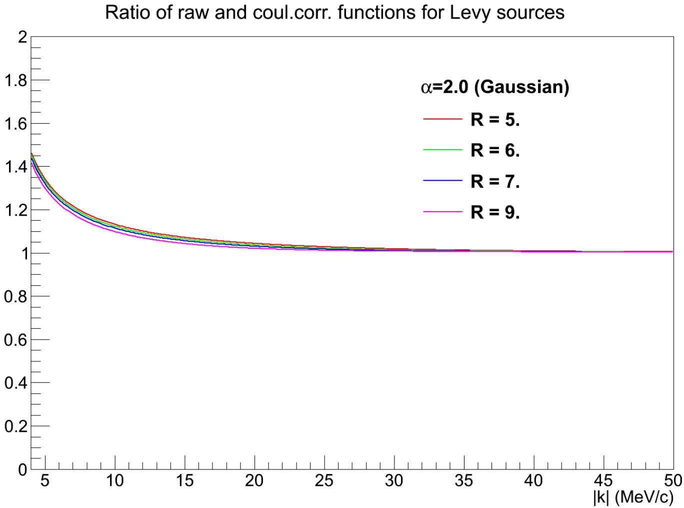

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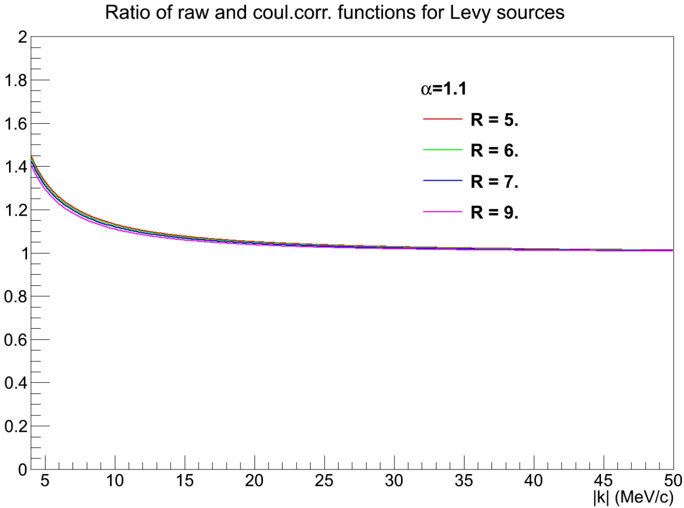

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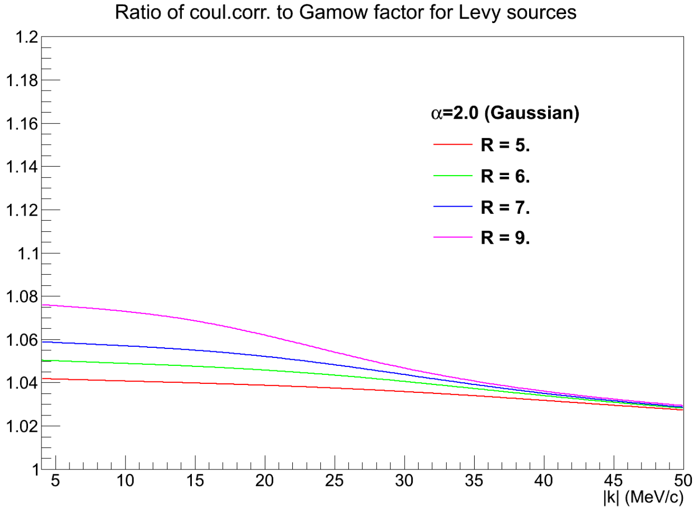

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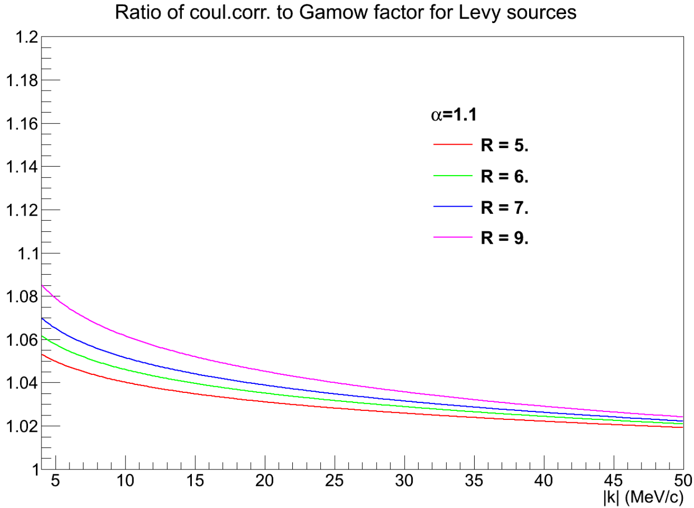

# Motivation (finally ☺)

- Simple methods of Coulomb correction need upgrade (especially for $\lambda$)
  - Gamow factor, Sinyukov approximation, pre-calculated $K(k)$: good, but may need better!
  - Need to simultaneously fit & re-calculate Coulomb correction!
- Method of *calculating* the Coulomb correction:
  - Calculate $S(r)$ (not unusually by FT)
  - Numerically integrate $S(r)$ with $|\psi^{(C)}_{\mathbf{k}}(\mathbf{r})|^2$ (cumbersome if $S(r)$ is not rapidly decreasing)
  - Frequently: *a Fourier transform + an ,,almost inverse Fourier transform"*
- One would prefer: directly from $C^{(0)}(\mathbf{q}')$ to $C(\mathbf{q})$.
  - Need to calculate: $\mathcal{K}(\mathbf{k},\mathbf{q})\equiv\int \mathrm{d}^3\mathbf{r}\, e^{i\mathbf{qr}}|\psi^{(C)}_{\mathbf{k}}(\mathbf{r})|^2$, once and for all
  - Asymptotics of $\psi^{(C)}_{\mathbf{k}}(\mathbf{r})$: distorted plane wave (logarithmic)
    $\Rightarrow$ *distribution theory*...
  - $\mathcal{K}(\mathbf{k},\mathbf{q})$ kernel: a distribution (irregular), $C^{(0)}(\mathbf{q})$: ,,test function".
  - **Task:** calculate & specify the action of $\mathcal{K}(\mathbf{k},\mathbf{q})$.
- At this stage: only **spherically symmetric** case investigated.

# Calculating the Coulomb kernel

- Some definitions:

$$D(\mathbf{r})=\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} f(\mathbf{q}) e^{i\mathbf{qr}} \quad \Rightarrow \quad C^{(0)}(\mathbf{k})=1+f(2\mathbf{k})$$

$$C(\mathbf{k})=\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} f(\mathbf{q}) \left(\mathcal{K}^{(1)}(\mathbf{k},\mathbf{q})+\mathcal{K}^{(2)}(\mathbf{k},\mathbf{q})\right),$$

- Kernel: sum of ,,direct" and ,,symmetrized" parts:

$$\mathcal{K}^{(1)}(\mathbf{k},\mathbf{q})=|\mathcal{N}_\mathbf{k}|^2 \int \mathrm{d}^3\mathbf{r}\, e^{i\mathbf{qr}} F\big(1+i\eta,1,-i(kr-\mathbf{kr})\big) F\big(1-i\eta,1,i(kr-\mathbf{kr})\big),$$

$$\mathcal{K}^{(2)}(\mathbf{k},\mathbf{q})=|\mathcal{N}_\mathbf{k}|^2 \int \mathrm{d}^3\mathbf{r}\, e^{i\mathbf{qr}} F\big(1+i\eta,1,-i(kr-\mathbf{kr})\big) F\big(1-i\eta,1,i(kr+\mathbf{kr})\big).$$

- Historic connection: similar integrals known in 1950s (in the context of Bremsstrahlung)!

  A. Nordsieck, Phys. Rev. **93**, 785 (1954).

# Spherically symmetric case

- If $f(\mathbf{q})\equiv f(q)$:

$$D(r)=\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}q}{4\pi^2 r}q\sin(qr)f(q),\qquad C(k)=\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}qf(q)\left\{\mathcal{K}_k^{(1)}(q)+\mathcal{K}_k^{(2)}(q)\right\}$$

$$\mathcal{K}_k^{(1,2)}(q)=\int_0^\infty \mathrm{d}r\, r\sin(qr)\tilde{\mathcal{K}}_k^{(1,2)}(r),\quad \text{where}$$

$$\tilde{\mathcal{K}}_k^{(1)}(r)=|\mathcal{N}_k|^2\int_{-1}^{1}\mathrm{d}y\, F\big(1+i\eta,1,-ikr(1-y)\big)\big)F\big(1-i\eta,1,ikr(1-y)\big),$$

$$\tilde{\mathcal{K}}_k^{(2)}(r)=|\mathcal{N}_k|^2\int_{-1}^{1}\mathrm{d}y\, F\big(1+i\eta,1,-ikr(1-y)\big)\big)F\big(1-i\eta,1,ikr(1+y)\big).$$

# Spherically symmetric case

- A new expression:

$$\tilde{\mathcal{K}}_k^{(1)}(r)=e^{-\pi\eta}\int_{-1}^{1}\mathrm{d}\tau\, e^{2i\arctan\tau}e^{-2i\tau kr}I_0(2kr\sqrt{1-\tau^2}),$$
$$\tilde{\mathcal{K}}_k^{(2)}(r)=e^{-\pi\eta}\int_{-1}^{1}\mathrm{d}\tau\, e^{2i\arctan\tau}e^{-2i\tau kr}.$$

- Another method (in the footsteps of Nordsieck):

$$F(a,1,z)=\oint^{(0+,1+)}\frac{\mathrm{d}t}{2\pi i}\frac{e^{tz}}{t}\left(1-\frac{1}{t}\right)^{-a}\quad\Rightarrow\quad\ldots$$

# Spherically symmetric case

- Result:

$$\mathcal{K}_k^{(1)}(q)=\lim_{\lambda\to 0}\frac{2}{e^{\pi\eta}}\mathrm{Im}\left\{\frac{(\lambda-i(q+q_0))^{i\eta-1}}{(\lambda-i(q-q_0))^{i\eta+1}}\times\right.$$
$$\left.\times{}_2F_1\left(1+i\eta,1-i\eta,1,\frac{q_0^2}{(\lambda-iq)^2+q_0^2}\right)\right\},$$
$$\mathcal{K}_k^{(2)}(q)=\lim_{\lambda\to 0}\frac{2}{e^{\pi\eta}}\mathrm{Im}\left\{\frac{(\lambda-i(q+q_0))^{i\eta-1}}{(\lambda-i(q-q_0))^{i\eta+1}}\right\},$$

- Well-defined limits (in terms of non-regular distributions); $\mathcal{K}^{(2)}$ simpler.

# Spherically symmetric case

- Example for $\mathcal{K}_k^{(2)}(q)$:

$$C_\eta^{(2)}(k)=\int_0^\infty \frac{\mathrm{d}x}{e^{\pi\eta}}\frac{\cos(\eta x)}{2\mathrm{ch}^2\frac{x}{2}}\frac{d}{dq'}\left(q'f\left(q'\right)\right)\Big|_{q'=q_0\mathrm{th}\frac{x}{2}}-\int_0^\infty \frac{\mathrm{d}x}{e^{\pi\eta}}\frac{\sin(\eta x)}{\mathrm{ch}^2\frac{x}{2}}\frac{q_0\mathrm{th}\frac{x}{2}}{\pi}$$

$$\int_0^\infty \mathrm{d}q\frac{\frac{d}{dq}\left(qf(q)\right)-\frac{d}{dq'}\left(q'f\left(q'\right)\right)\Big|_{q'=q_0\mathrm{th}\frac{x}{2}}}{q^2-q_0^2\mathrm{th}^2\frac{x}{2}}.$$

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# Summary and outlook

- A (possibly) new and efficient method for calculating Coulomb interacting HBT correlation functions presented
  - Calculations directly in momentum (Fourier) space
  - Distribution theory invoked; however, numerically OK
  - Some examples presented: Levy, Cauchy sources. . .
  - Restricted to spherically symmetry (as of now)
- Prospective generalization I: short-range strong interactions

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New exact analytic formulas for QM Coulomb problem! ☺

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*Much work to be done. . .*

- Outlook — need to:
  - Go beyond spherical symmetry (not hopeless, but. . . )
  - Properly (?) treat relativistic kinematics. . .

# *Thank you for your attention!*

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