Zimányi School 2017

Parton-hadron duality *(etc.)*

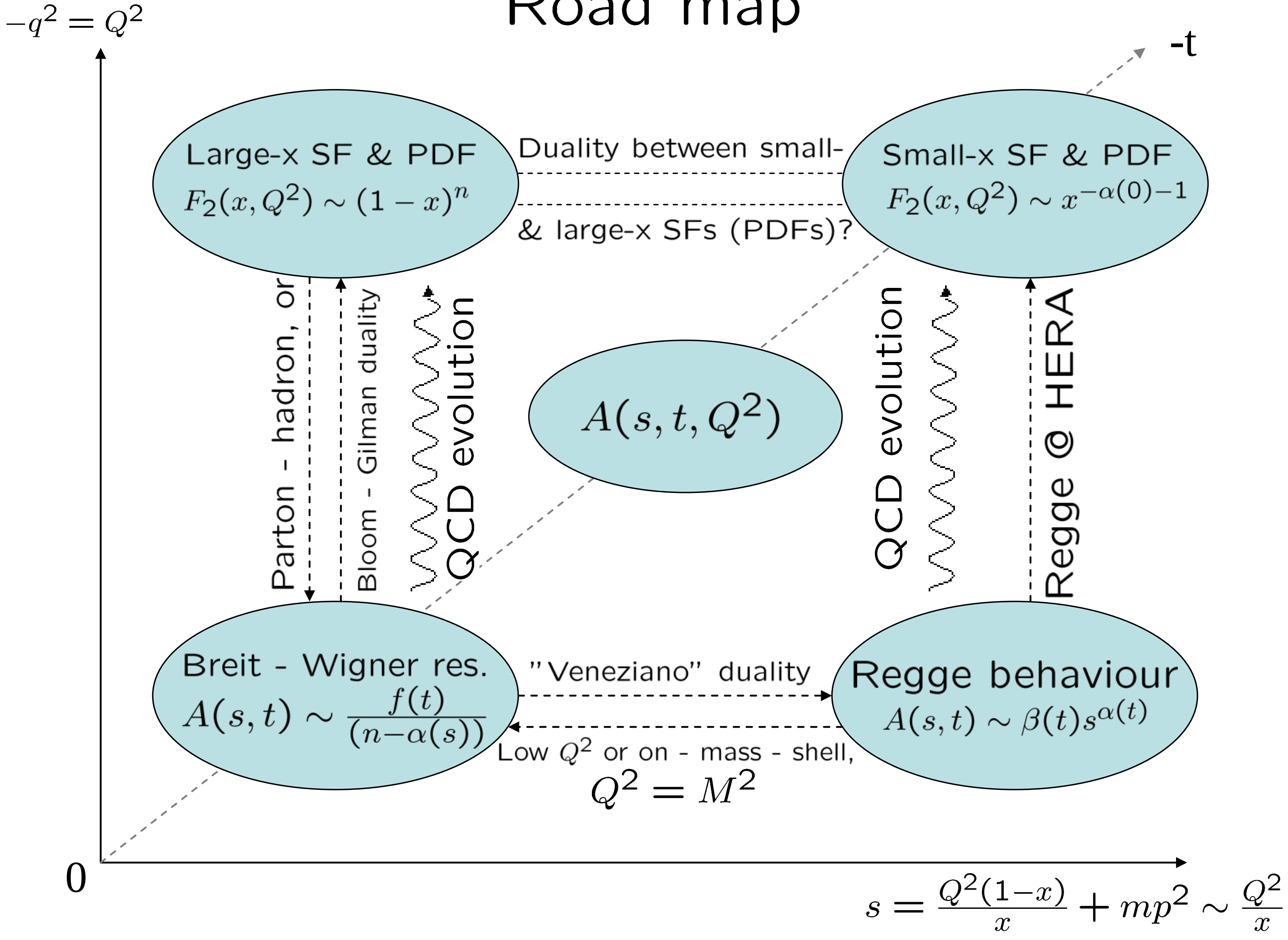
László Jenkovszky (BITP, Kiev)

jenk@bitp.kiev.ua

• *ITEMS:*

- What is duality (Veneziano and Gloom-Gilman);
- Kinematical variables, a road map (in s, t and Q^2); *NN* and *ep*;
- Basic ingredients, tools, measurables; amplitudes vs. collective properties (EOS);
- General recipe: input + unitarity; analyticity and crossing symmetry, factorization; QCD;
- Reggeons (Regge), pomeron (Pomeranchuk), odderon (no name);
- Non-linear Regge trajectories vs. strings, finite number of resonances;
- Balancing between “soft” (NN) and “hard” (DIS, VMP…) reactions;
- *Conclusions:*
- “Soft” (forward) physics – difficult, non-rewarding but indispensable;
- NN is a building block of AA.

Road map



$-q^2 = Q^2$
-t
Large-x SF & PDF
$F_2(x,Q^2) \sim (1-x)^n$
Duality between small-
& large-x SFs (PDFs)?
Small-x SF & PDF
$F_2(x,Q^2) \sim x^{-\alpha(0)-1}$
Parton - hadron, or
Bloom - Gilman duality
QCD evolution
$A(s,t,Q^2)$
QCD evolution
Regge @ HERA
Breit - Wigner res.
$A(s,t) \sim \frac{f(t)}{(n-\alpha(s))}$
"Veneziano" duality
Regge behaviour
$A(s,t) \sim \beta(t) s^{\alpha(t)}$
Low Q^2 or on - mass - shell,
$Q^2 = M^2$
0
$s = \frac{Q^2(1-x)}{x} + mp^2 \sim \frac{Q^2}{x}$

The basic object of the theory

$$A(s,t,Q^2)$$

$$\to A(s,t,Q^2=m^2) \text{ (on mass shell)}$$

$$\to \Im m A(s,t=0,Q^2) \sim F_2 \quad \text{DIS}$$

Reconstruction of the DVCS amplitude from DIS

$$F_2 \sim \Im m A(\gamma^* p \to \gamma^* p)\Big|_{t=0} \to \Im m A(\gamma^* p \to \gamma p)\Big|_{t=0}$$

$$\to A(\gamma^* p \to \gamma p)\Big|_{t=0} \to A(\gamma^* p \to \gamma p)$$

or

$$\Im m A(\gamma^* p \to \gamma^* p)\Big|_{t=0} \sim F_2(x_B,Q^2) = x_B q(x_B,Q^2)$$

$$q(x_B,Q^2) \to q(\xi,\eta,t,x_B,Q^2) \to$$

$$\to \xi q(\xi,\eta,t,x_B,Q^2) \overset{?}{=} GPD(\xi,\eta,t,x_B,Q^2)$$

$$\sigma_t(s) = \frac{4\pi}{s} Im A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2}|A(s,t)|^2; \quad n(s);$$

$$\sigma_{el} = \int_{t_{min} \approx -s/2 \approx \infty}^{t_{thr.} \approx 0} \frac{d\sigma}{dt} dt; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s,t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

$$A_{pp}^{p\bar{p}}(s,t) = P(s,t) \pm O(s,t) + f(s,t) \pm \omega(s,t) \rightarrow_{LHC} \approx P(s,t) \pm O(s,t),$$

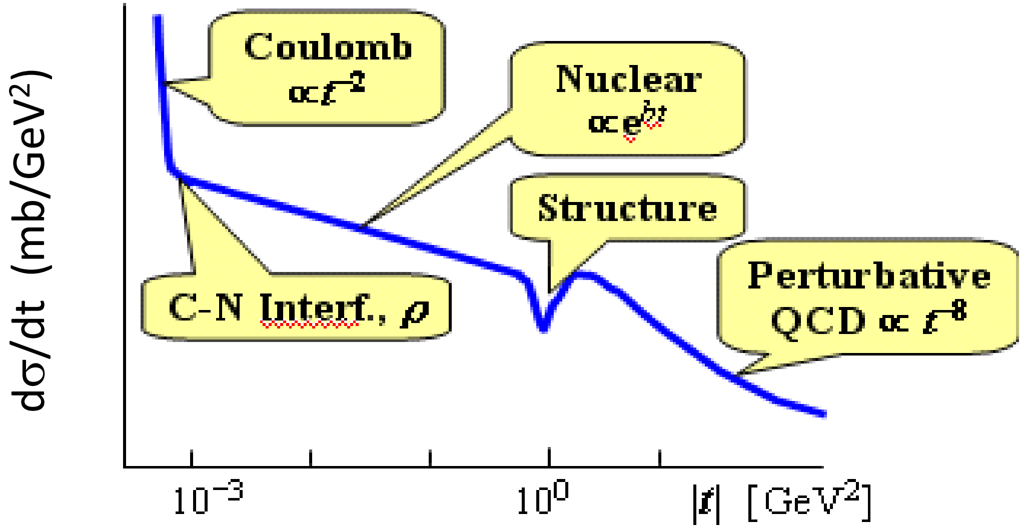
where P, O, f. ω are the Pomeron, odderon and non-leading Reggeon contributions.

α(0)\C	+	-
1	P	O
1/2	f	ω

NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!

Elastic Scattering

$\sqrt{s}$ = 14 TeV prediction of BSW model



momentum transfer $-t \sim (p\theta)^2$

θ = beam scattering angle

p = beam momentum

$$\rho = \left.\frac{\mathrm{Re}(f_{el}(t))}{\mathrm{Im}(f_{el}(t))}\right|_{t\to 0}$$

$$\left.\frac{dN}{dt}\right|_{t=CNI} = L\pi |f_C + f_N|^2 \approx L\pi \left| -\frac{2\alpha_{EM}}{|t|} + \frac{\sigma_{tot}}{4\pi}(i+\rho)e^{-\frac{b|t|}{2}} \right|^2$$

L, σ_{tot} , b, and ρ from FIT in CNI region (UA4)

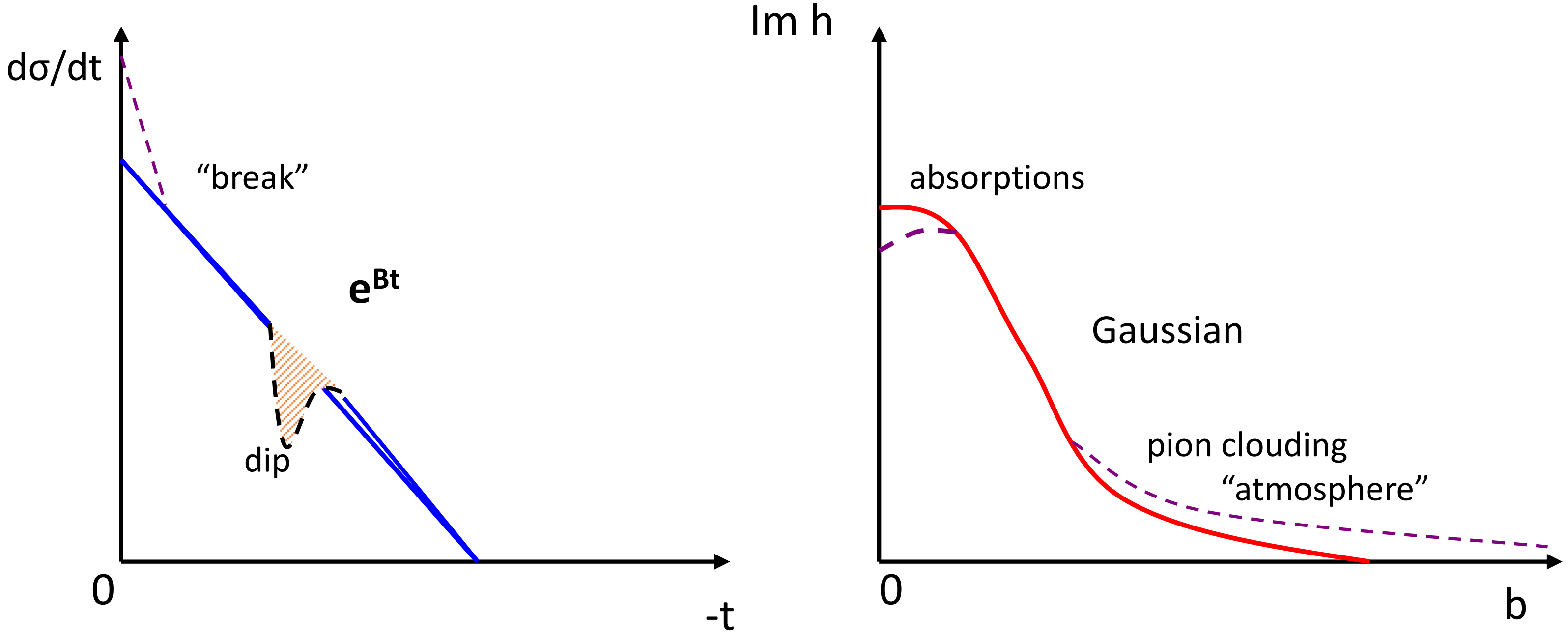
CNI region: $|f_C| \sim |f_N|$ → @ LHC: $-t \sim 6.5\ 10^{-4}$ GeV²; $\theta_{min} \sim 3.4$ μrad

($\theta_{min} \sim 120$ μrad @ SPS)

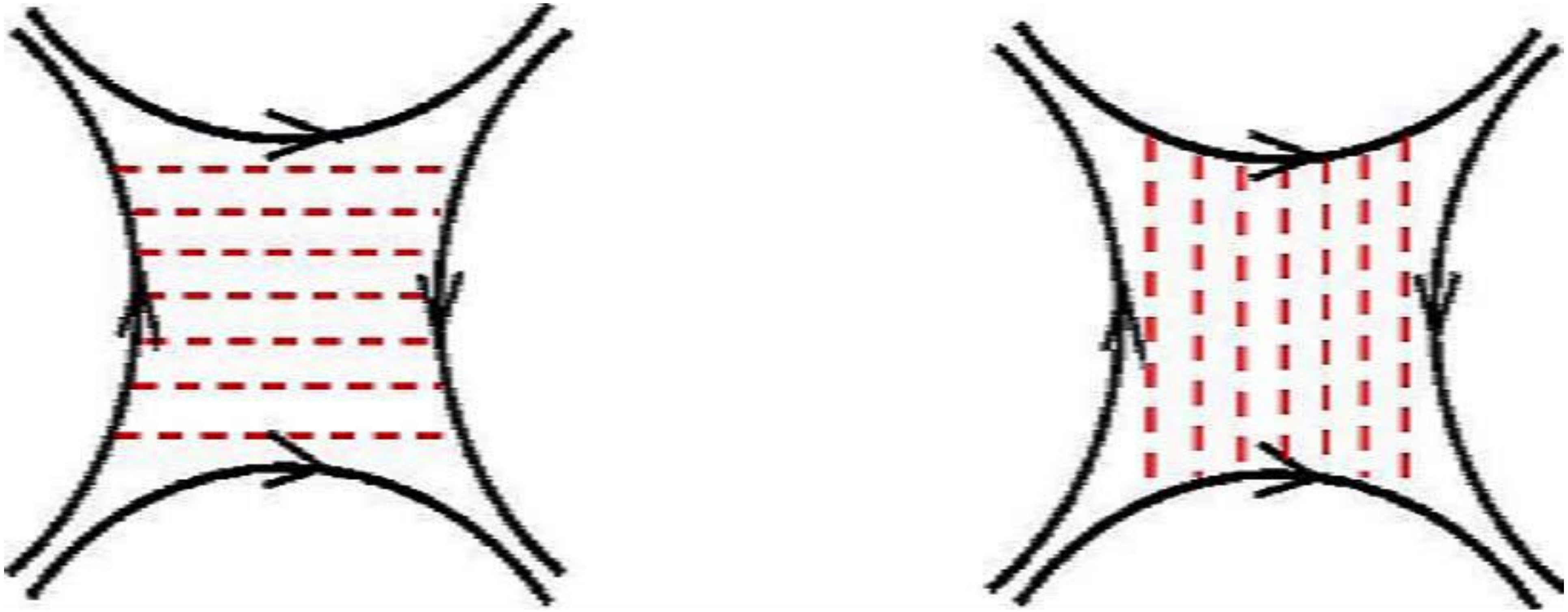
1. **On-shell (hadronic) reactions (s,t, Q^2=m^2);**

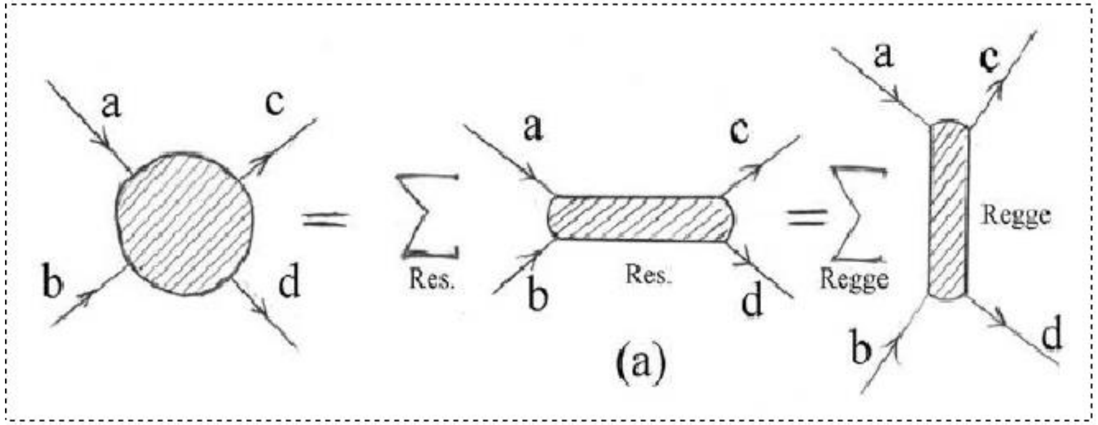
   t ←→ *b* **transformation dictionary:**

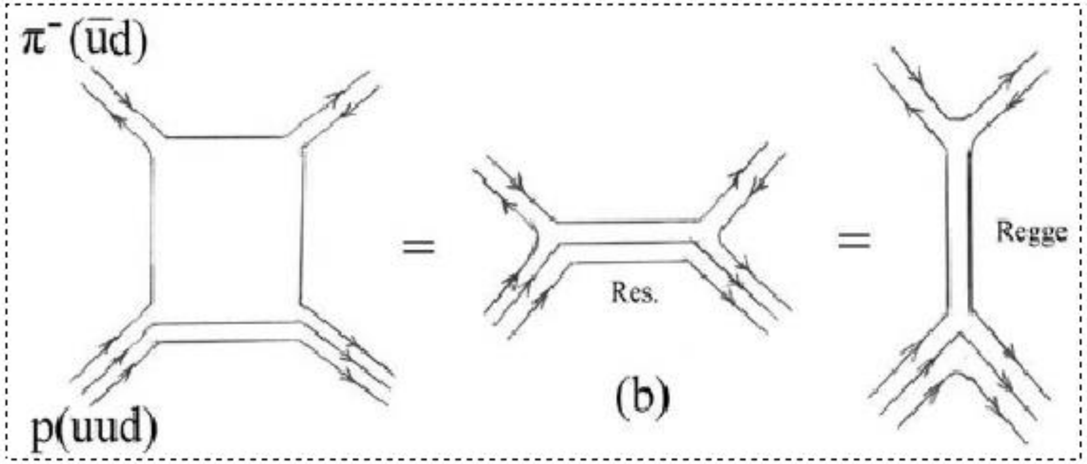
$$h(s,b) = \int_0^\infty d\sqrt{-t}\sqrt{-t}A(s,t)$$



DUALITY:




a
c
b
d
Res.
a
c
b
Res.
d
(a)
a
c
Regge
Regge
b
d


π^- ($\bar{u}$d)
p(uud)
Res.
(b)
Regge

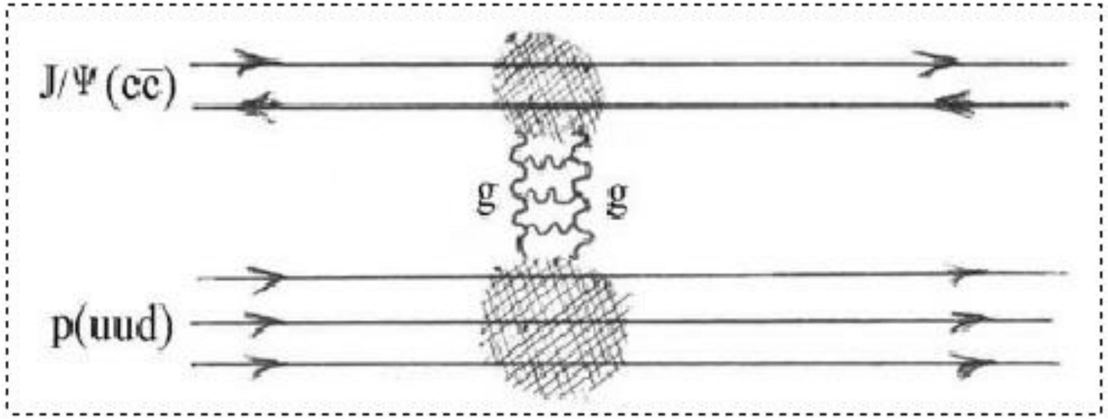

J/Ψ (c$\bar{c}$)
g
g
p(uud)

TABLE I: Two-component duality

$\mathcal{I}mA(a+b \to c+d) =$	R	Pomeron
$s-$channel	$\sum A_{Res}$	Non-resonant background
$t-$channel	$\sum A_{Regge}$	Pomeron ($I = S = B = 0;\ C = +1$)
Duality quark diagram	Fig. 1b	Fig. 2
High energy dependence	$s^{\alpha-1},\ \alpha < 1$	$s^{\alpha-1},\ \alpha \geq 1$

The (s,t) term of a dual amplitude is

$$D(s,t) = c\int_0^1 dx\Big(\frac{x}{g_1}\Big)^{-\alpha(s')-1}\Big(\frac{1-x}{g_2}\Big)^{-\alpha(t')-1},$$

where s and t are the Mandelstam variables, and g_1, g_2 are parameters, $g_1, g_2 > 1$. For simplicity, we set $g_1 = g_2 = g_0$.

1. Regge behavior, $s \to \infty$, $t = const$: $D(s,t) \sim s^{\alpha(t)-1}$;

2. Thereshold behavior, $s \to s_0$: $D(s,t) \sim \sqrt{s_0 - s}[const + \ln(1 - s_0/s)]$;

3. Direct-channel poles:

$$D(s,t) = \sum_{n=0}^{\infty} g^{n+1} \sum_{l=o}^{n} \frac{[-s\alpha'(s)]^l C_{n-l}(t)}{[n-\alpha(s)]^{l+1}}.$$

Exotic direct-channel trajectory: $\alpha(s) = \alpha(0) + \alpha_1(\sqrt{s}_0 - \sqrt{s_0 - s})$.

"GOLDEN" diffraction reaction: $J/\Psi p-$ scattering: By VMD, photoproduction is reduced to elastic hadron scattering:

$$D(\gamma p - Vp) = \sum \frac{e}{f_V} D(Vp - Vp).$$

Collective properties of the nuclear matter vs. the S matrics, or how can the EOS (equation of state) be inferred from the scattering amplitude (data)?

The answer was given in the paper *R. Dashen, S.Ma, H.J. Bernstein, Phys. Rev.* **187** *(1969) 345.*

$$\beta(\Omega-\Omega_0) = -\frac{1}{4\pi}\sum_{n=2}^{\infty} z^n \int_{nm}^{\infty} dE e^{-\beta E}(Tr_n A S^{-1}\frac{d}{dE}S),$$

where Ω is the thermodynamical potential, $z = e^{\beta\mu}, \quad \beta = 1/T$.

The S matrix can be saturated either by experimental data points or by a model for the scattering amplitude.

For the latter a direct-channel resonance model was used by *P. Fre and L. Sertorio (Nuovo Cim.* **28A** *(1975) 538;* **31A** *(1076) 365).*

At high energies, the S matrix (scattering amplitude) is Regge behaved:

$$A(s,t) = \sum_i \xi_i(t)\beta_i(t)(-is/s_0)^{\alpha_i(t)}, \quad i = P, f, ...$$

$$p(T) = p_0(T) + p_1(T) + p_2(T),$$

$$p_1(T) = \frac{T^2}{2(2\pi)^4}\int_{2m}^{\infty} dEK_2(\beta E)E^2\frac{d}{dE}[ReA(s,0)(1-\frac{4m^2}{E^2})^{1/2}],$$

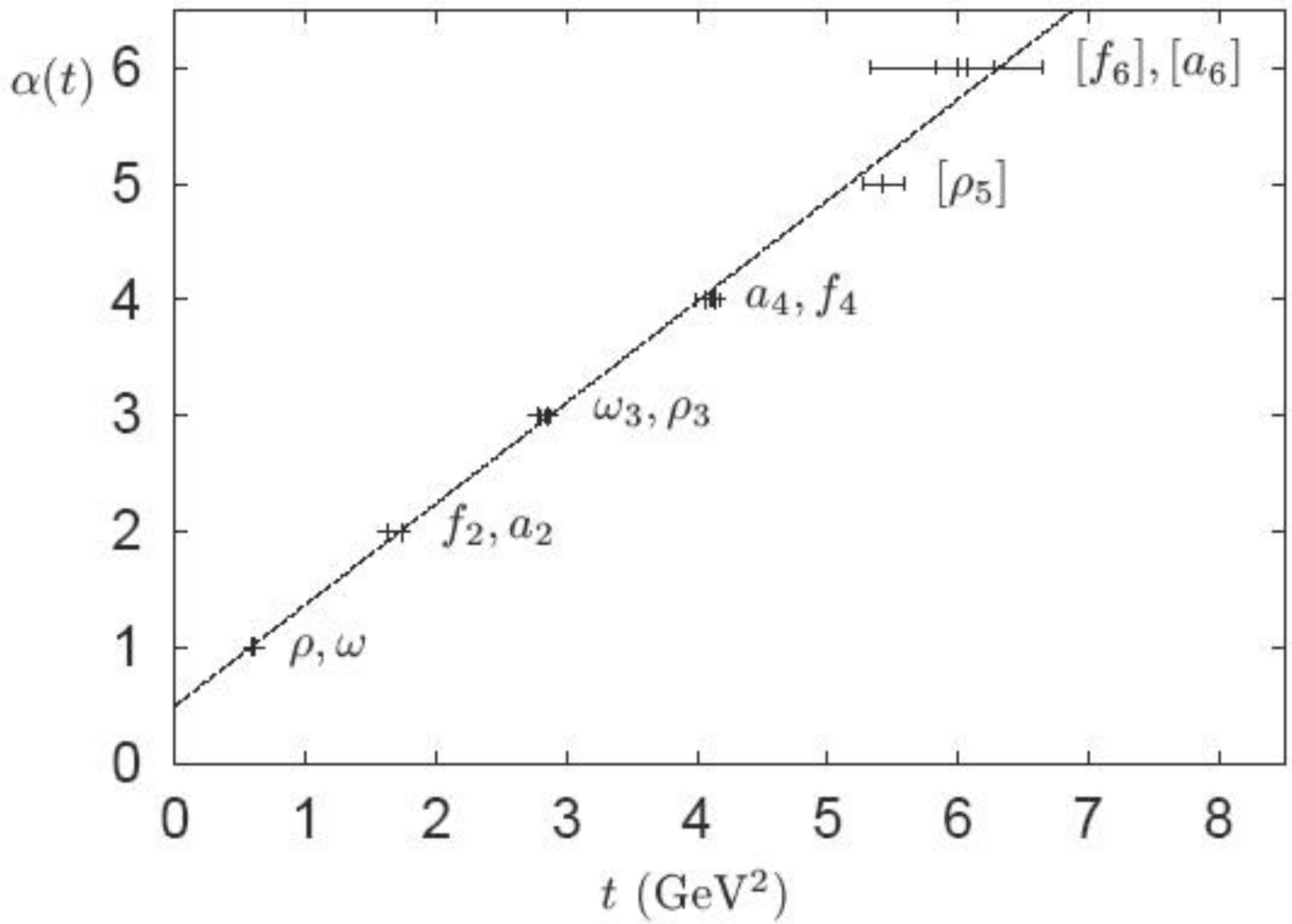
$$p_2(T) = \frac{T^2}{8(2\pi)^5}\int_{2m}^{\infty} dEK_2(\beta E)\int_{4m^2-s}^{infty}[ReA(s,t)\frac{d}{dE}ImA(s,t)],$$

where $K_2(z)$ is the Bessel funcion of imaginary argument.

L.L. Jenkovszky and A.A. Trushevsky (Nuovo Cim. **34A** *(1976) 369)* saturated the scattering amplitude with a Pomeron exchange in the t channel, resulting in:

Linear particle trajectories

Plot of spins of families of particles against their squared masses:



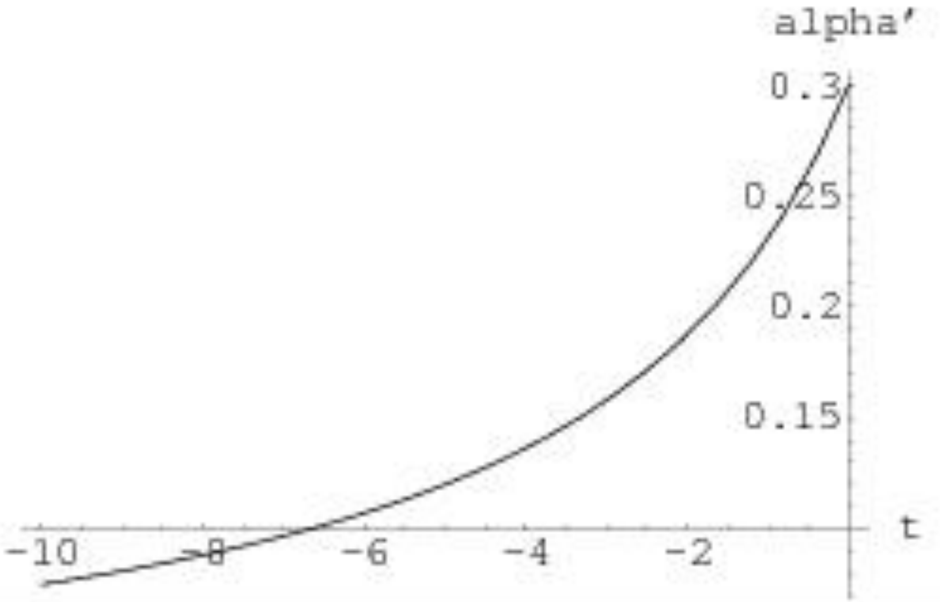
The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the $t-$channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the $t-$ channel unitarity, by which

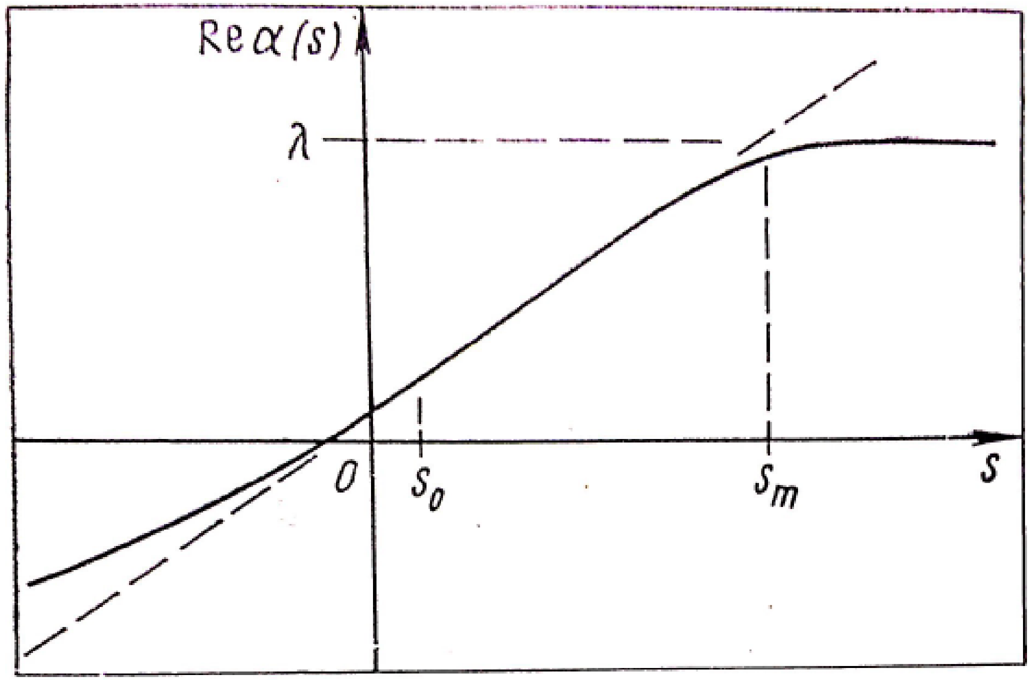
$$\Im\alpha(t) \sim (t-t_0)^{\Re\alpha(t_0)+1/2}, \quad t \rightarrow t_0,$$

where t_0 is the lightest threshold. For the Pomeron trajectory it is $t_0 = 4m_\pi^2$, and near the threshold:

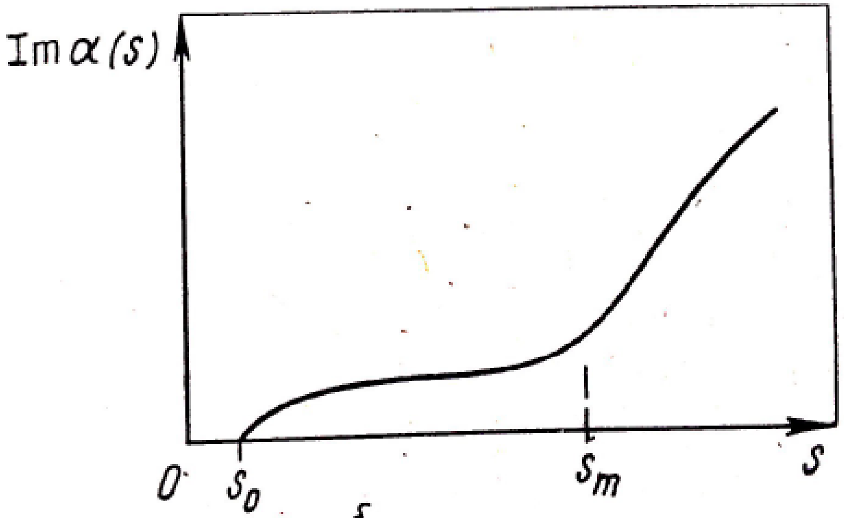
$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \qquad (1)$$



The slope of the cone for a single pole is: $B(s,t) \sim \alpha'(t) \ln s$. The Regge residue $e^{b\alpha(t)}$ with a logarithmic trajectory $\alpha(t) = \alpha(0) - \gamma \ln(1 - \beta t)$, is identical to a form factor (geometrical model).



а



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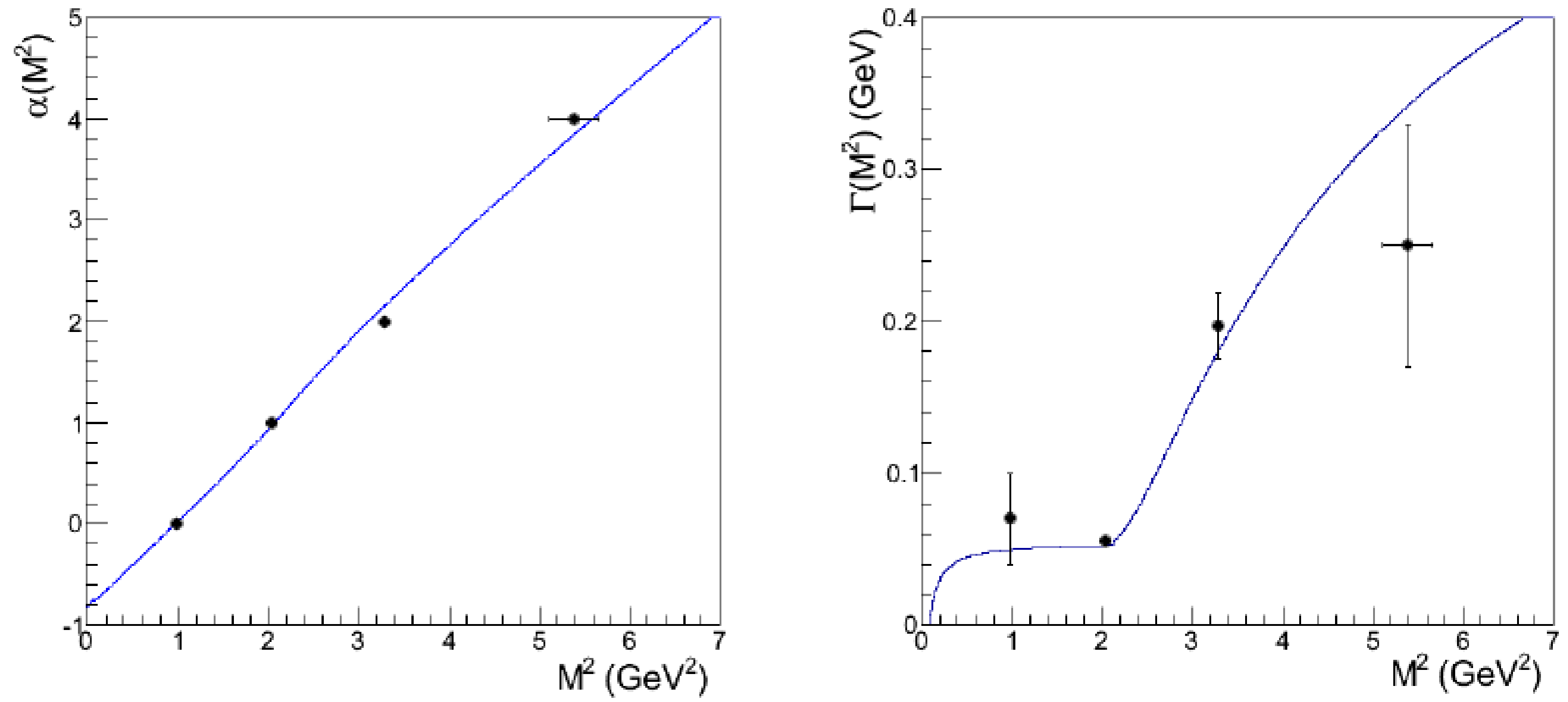
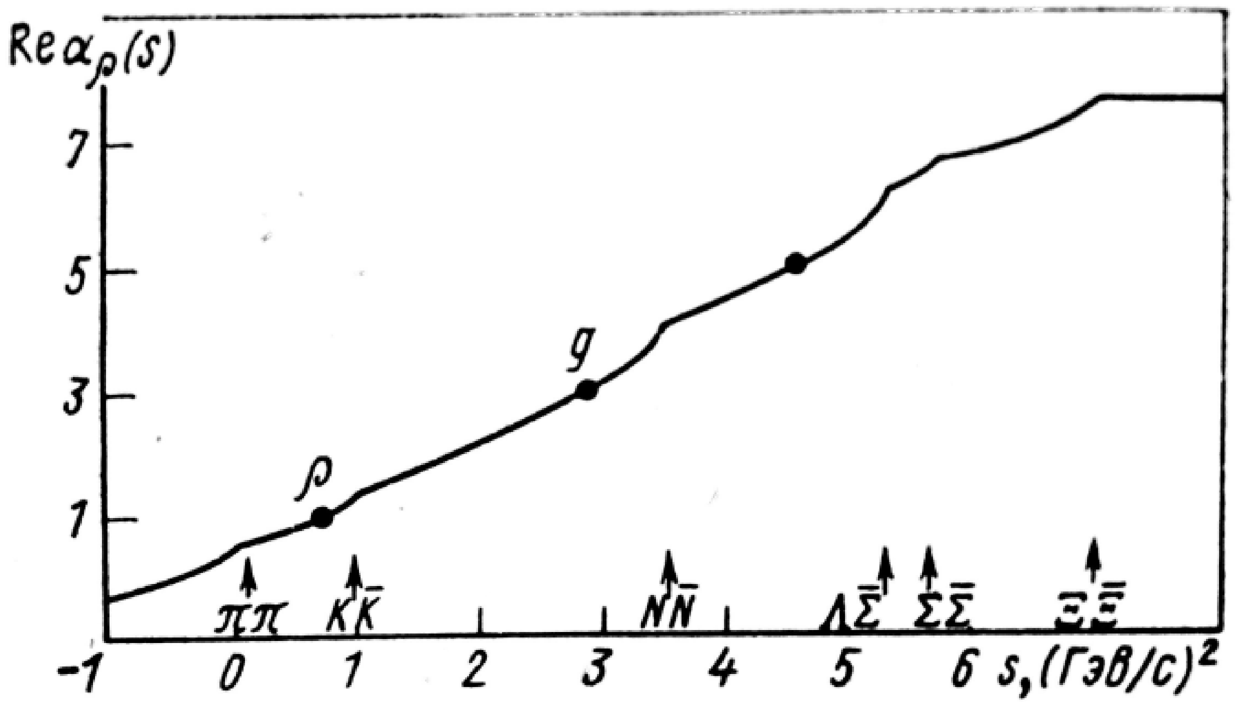


FIG. 6: Real part of f_1 trajectory on the left, width function $\Gamma(M^2)$ on the right.

Let us start with a toy model of a non-linear trajectory, Following [18, 31], we write a simple trajectory in which the (additive) thresholds are those made of stable particles allowed by quantum numbers. For the ρ trajectory these are: $\pi\pi$, $K\bar{K}$, $N\bar{N}$, $\Lambda\bar{\Sigma}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$. The relevant trajectory is:

$$\alpha_\rho(m) = 7.64 - 0.127\sqrt{m - 0.28} - 0.093\sqrt{m - 0.988} - 0.761\sqrt{m - 1.88} - \text{"}\Lambda\bar{\Sigma},\ \Sigma\bar{\Sigma},\ \Xi\bar{\Xi}\text{"}, \tag{10}$$

with the parameters of higher threshold quoterd in Ref. [18].



$$N_{theor} = \int_0^m \rho_{theor}(m')dm',$$

where

$$\rho_{theor}(m) = f(m)\exp(m/T)$$

and $f(m) \approx A/(m^2 + (500MeV)^2)^{5/4}$ (alternative choices for this slowly varying function are possible).

According to Hagedorn's conjecture, confirmed by subsequent studies, the density of hadronic resonances increases exponentially, modulus a slowly varying function of mass, $f(m)$,

$$\rho(m) = f(m)\exp(m/T) \tag{1}$$

up to about $m = 2 \div 2.5$ MeV, whereupon the exponential rise slows down

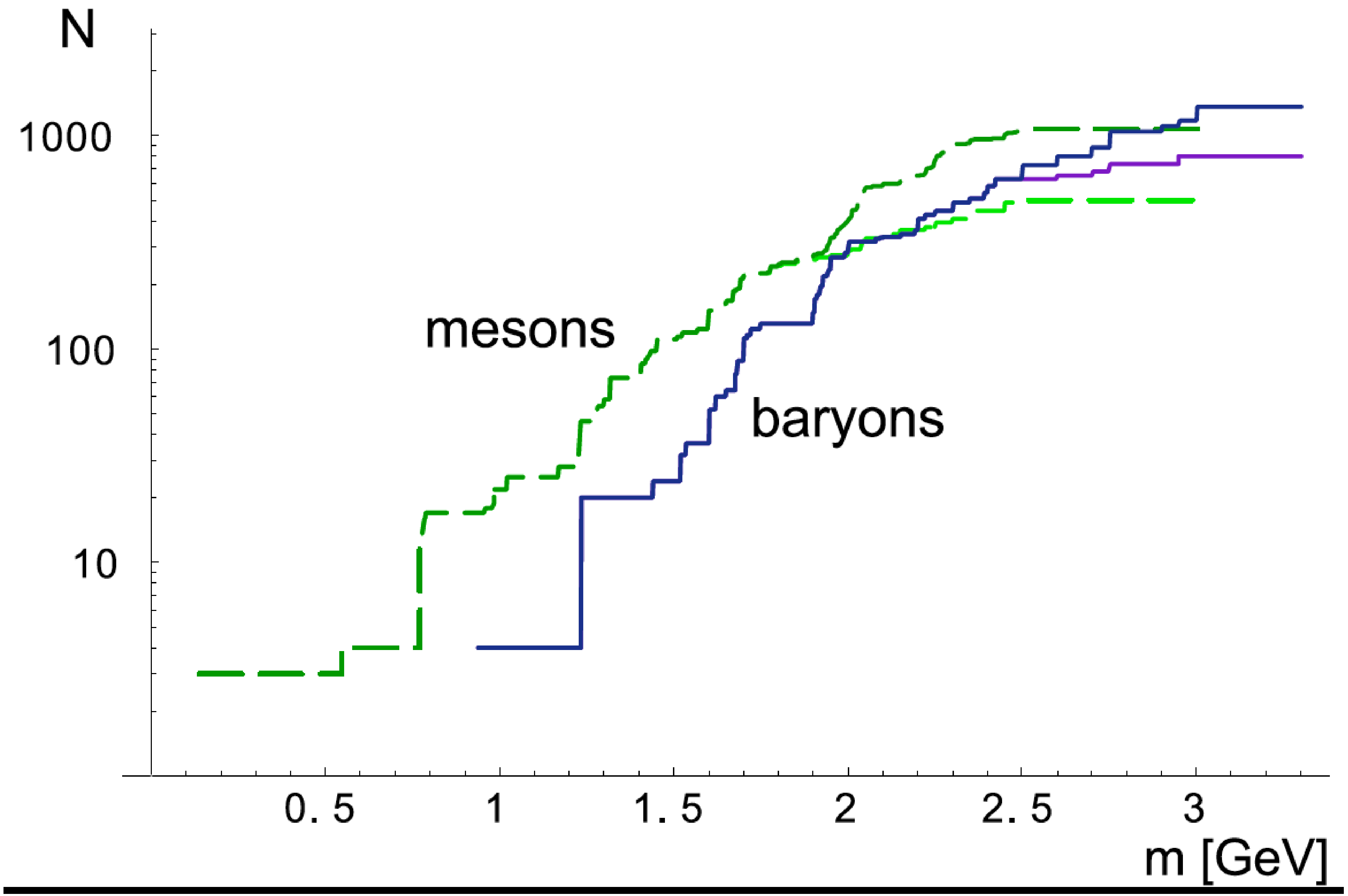
We extend the Hagedorn formula by introducing in the slope of relevant non-linear Regge trajectories. Anticipating a detailed quantitative analyses, one may observe immediately that flattening of $\Re\alpha(s = m^2)$ results in a drastic decrease of the relevant slope $\alpha'(m)$ and a corresponding change of the Hagedorn spectrum, which we parametrize as

$$\rho(m) \sim (\Re\alpha(m))' \exp(m/T). \tag{3}$$

Usually, one compares the cumulants of the spectrum, defined as the number of states with mass lower than m_i. The experimental curve is

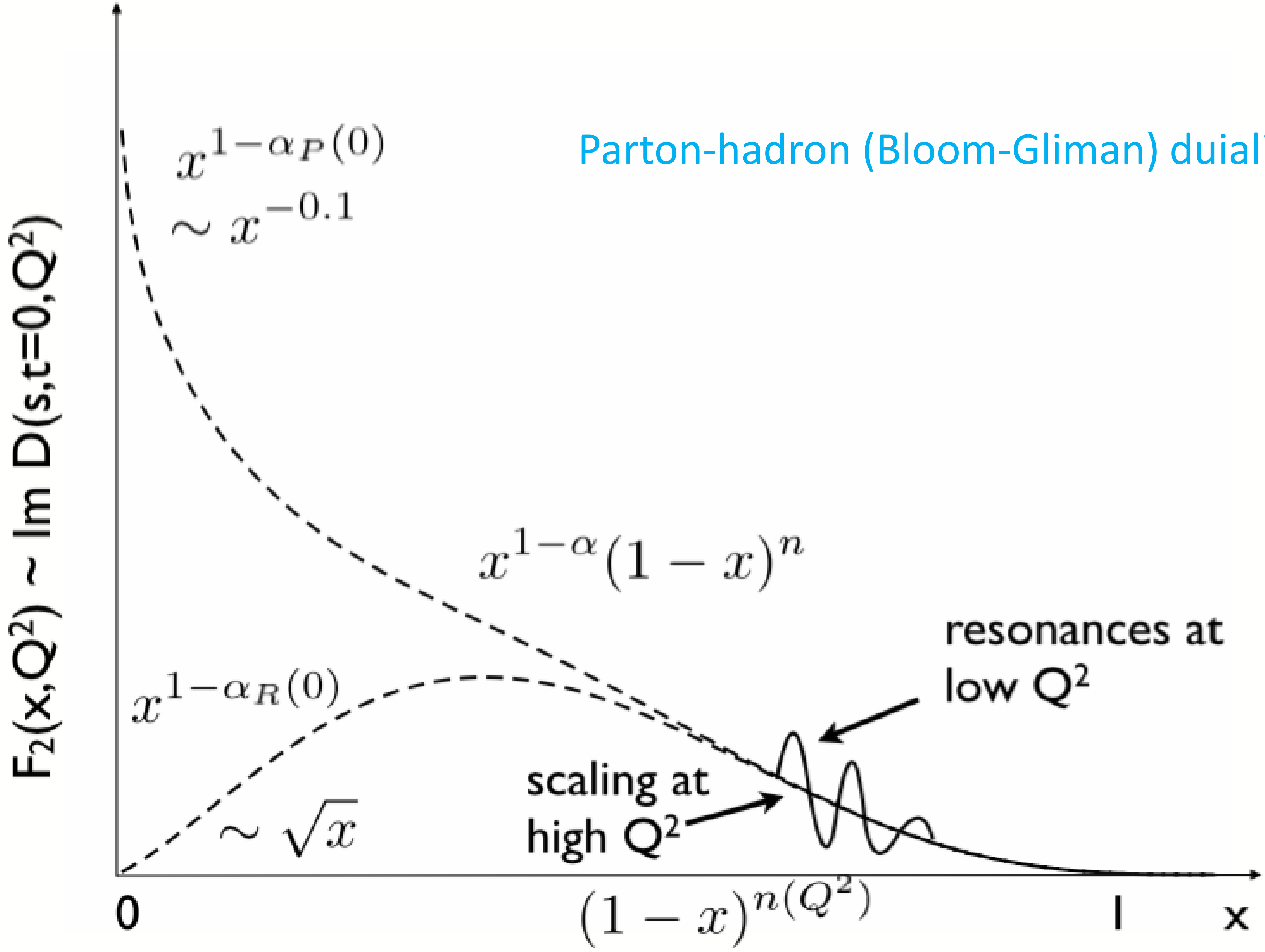
$$N_{exp}(m) = \sum_i g_i \Theta(m - m_i), \tag{4}$$

where $g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the i-th state and m_i is its mass. The theoretical curve


N
1000
100
10
mesons
baryons
0. 5
1
1. 5
2
2. 5
3
m [GeV]

[1] R. Hagedorn, Nuovo Cim. Suppl. **3** (1965) 147.
[2] Wojciech Broniowski, Wojciech Florkowski, and Leonid Glozman, hep-ph/0407290.
[3] Wojciech Broniowski and Wojciech Florkowski, hep-ph/0004104; Wojciech Broniowski, Enrique Ruiz Arriola, hep-ph/1008.2317; Enrique Ruiz Arriola and Wojciech Broniowski, hep-ph/1210.7153; Wojciech Broniowski, nucl-th/1610.0967.
[4] Wojciech Broniowski, hep-ph/0008112.
[5] K.A. Olive et al. (Particle Data Group) Chinese Physics C **38** (2014) 090001, http://pdg.lbl.gov/.
[6] Thomas D. Cohen and Vojtech Krejcirik, hep-ph/1107.2130.
[7] S.Z. Belenky and L.D. Landau, Sov. Phys. Uspekhi **56** (1955) 309.
[8] E.V. Shuryak, Sov. J. Nucl. Phys. **16** (1973) 220.
[9] L. Burakovsky, *Hadron spectroscopy in Regge Phenomenology*, hep-ph/9805286.
[10] M.M. Brisudova, L. Burakovsky, T. Goldman and A. Szczepaniak, *Nonlinear Regge trajectories and glueballs*, nucl-th/030303012.

Parton-hadron (Bloom-Gliman) duiality



troproduction will have a behavior which is corre-

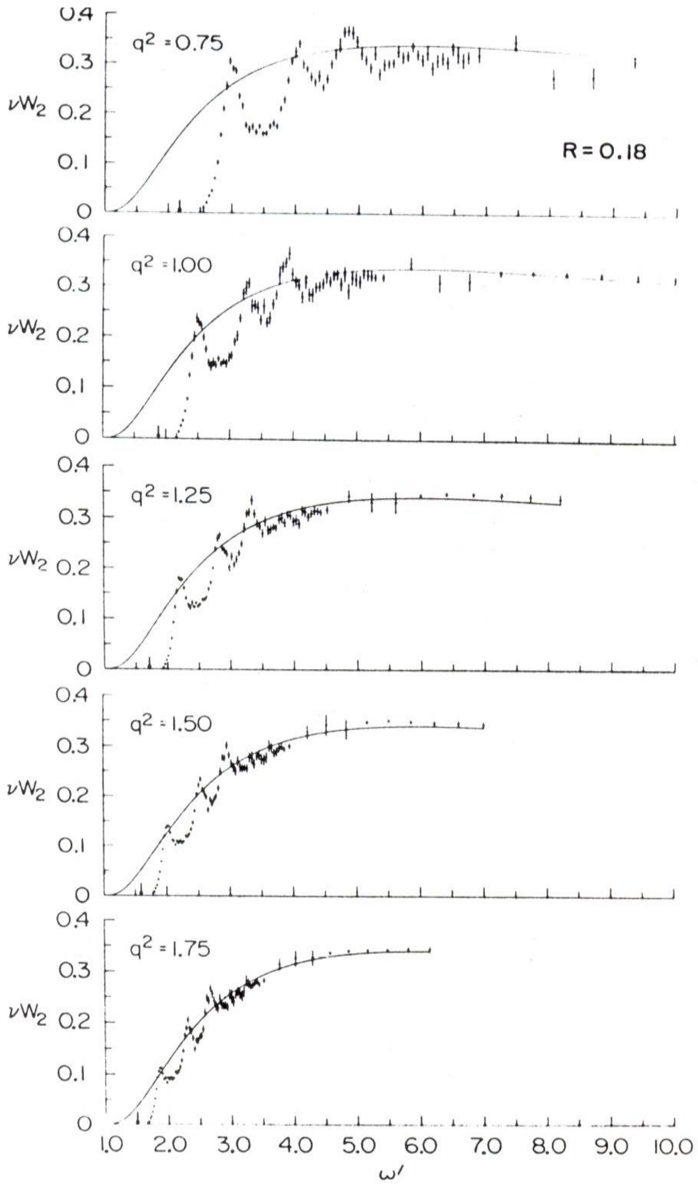


FIG. 3. The function $\nu W_2(\nu, q^2)$ plotted versus $\omega' = 1 + W^2/q^2$ from an interpolation of data to fixed q^2 values of 0.75, 1.00, 1.25, 1.50, and 1.75 GeV^2. The solid line is the scaling-limit curve, $\nu W_2(\omega')$, a smooth fit (Ref. 12) to the data in the scaling region. The arrow indicates the position of the elastic peak.

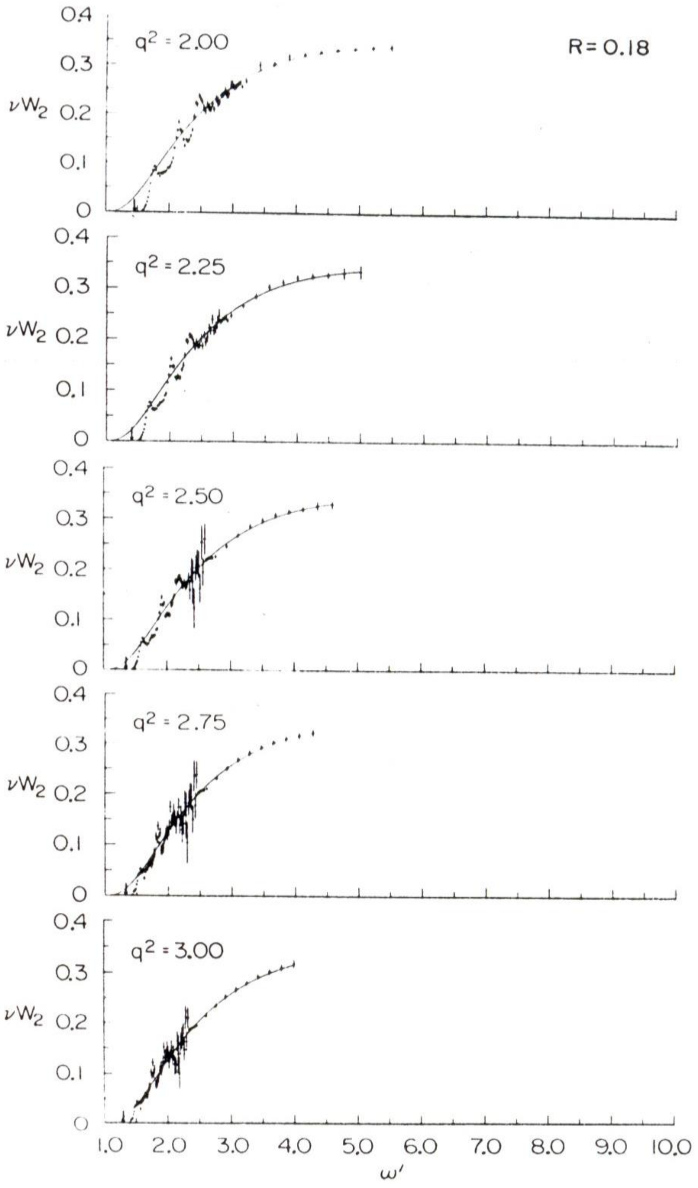
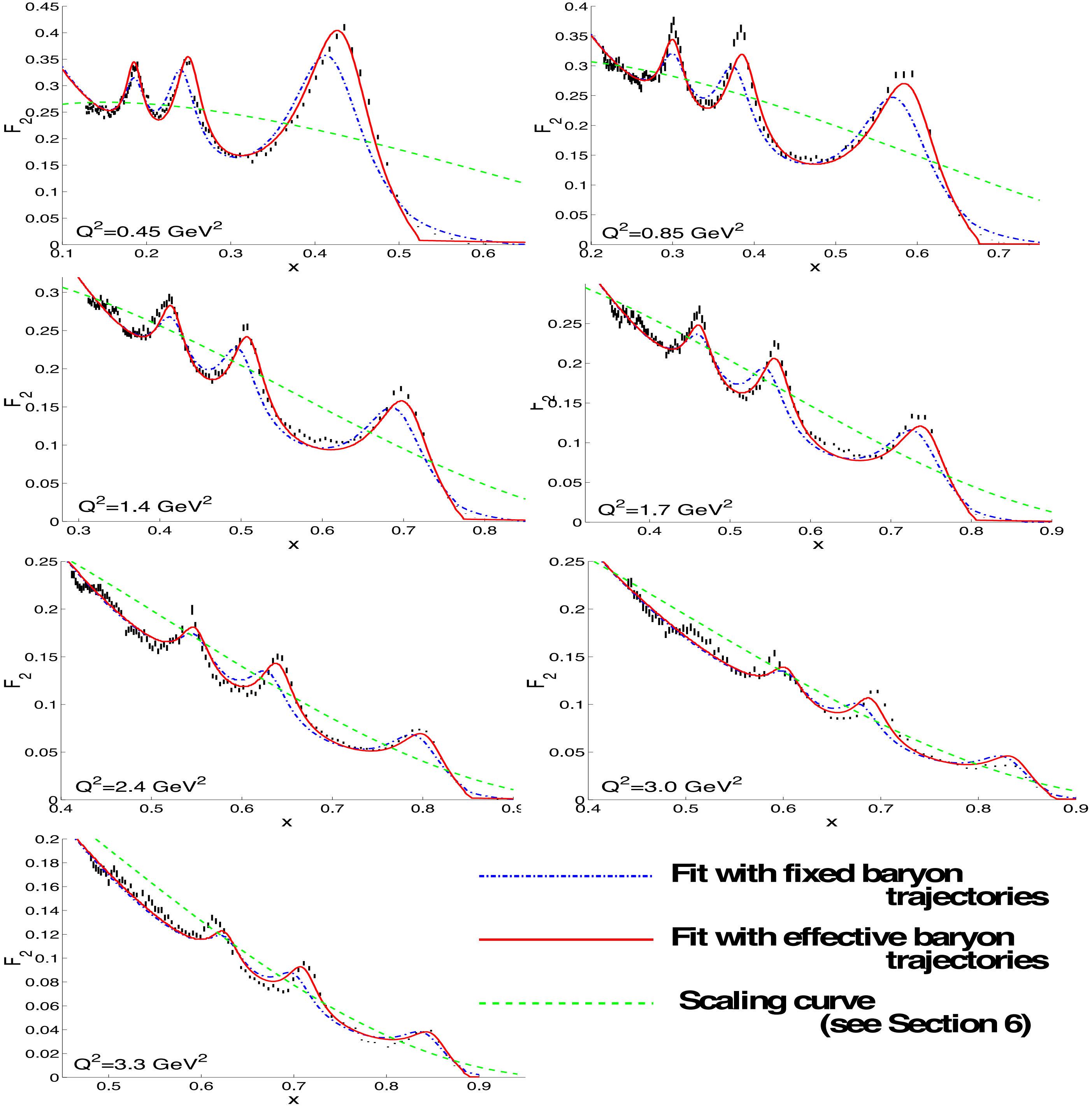
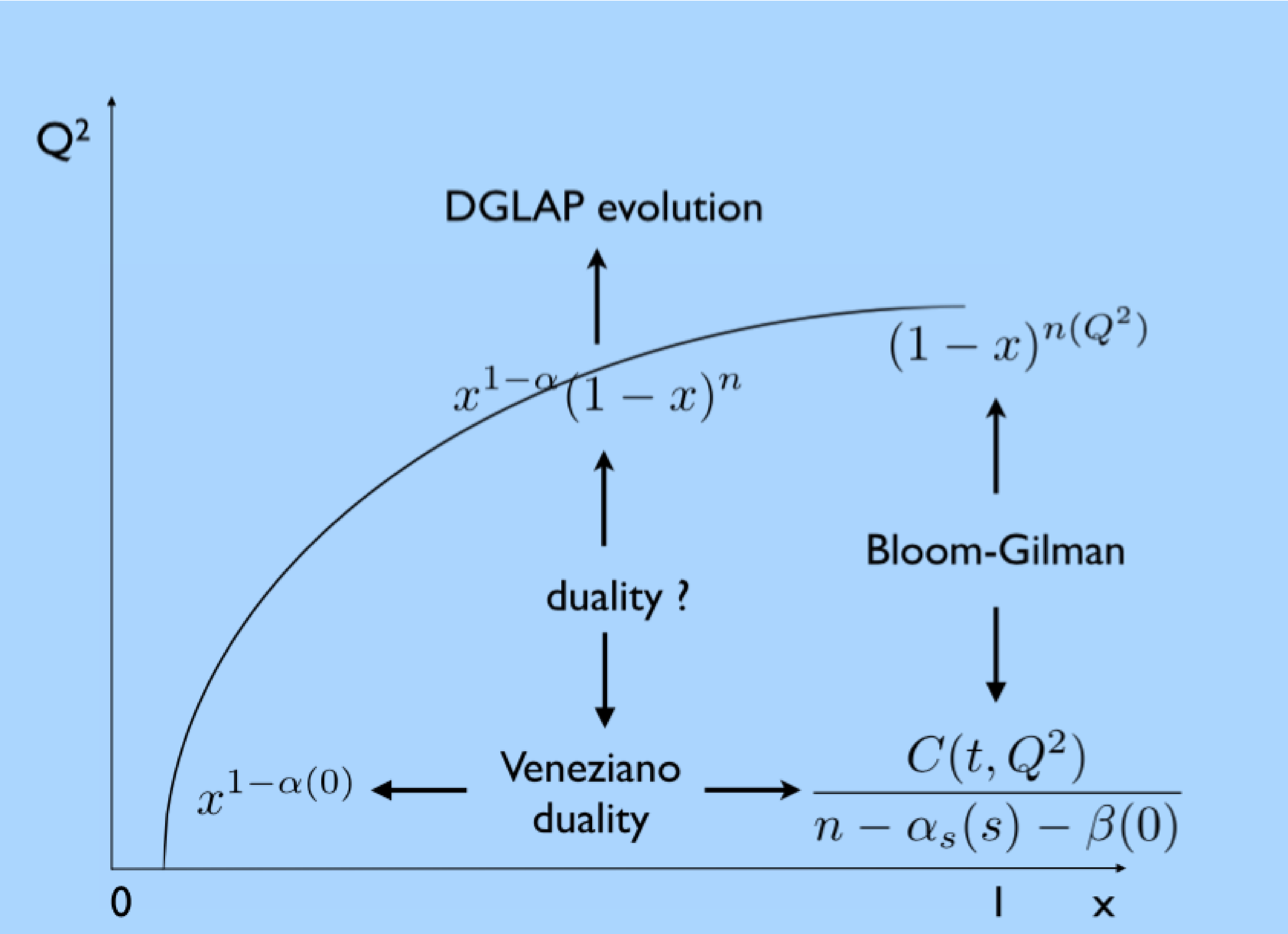


FIG. 4. Same as Fig. 3, but for $q^2 = 2.0$, 2.25, 2.50, 2.75, and 3.0 GeV^2.

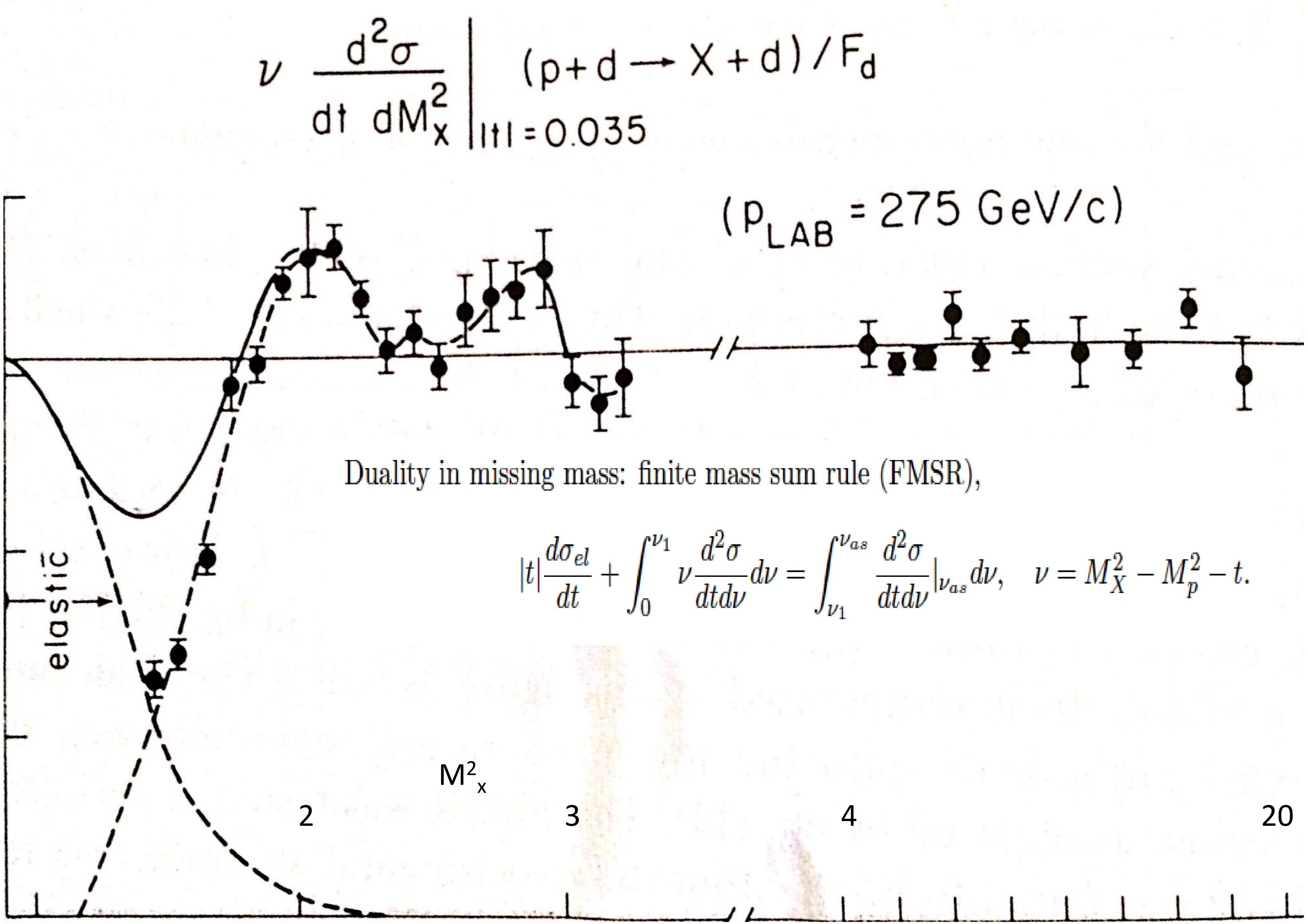
SLAC data; Bloom and Gilman



Parton-hadron(B-G) duality;
Jlab data



Q²
DGLAP evolution
$(1-x)^{n(Q^2)}$
$x^{1-\alpha}(1-x)^n$
Bloom-Gilman
duality ?
$x^{1-\alpha(0)}$
Veneziano duality
$\frac{C(t,Q^2)}{n-\alpha_s(s)-\beta(0)}$
0
1
x



Duality in missing mass: finite mass sum rule (FMSR),

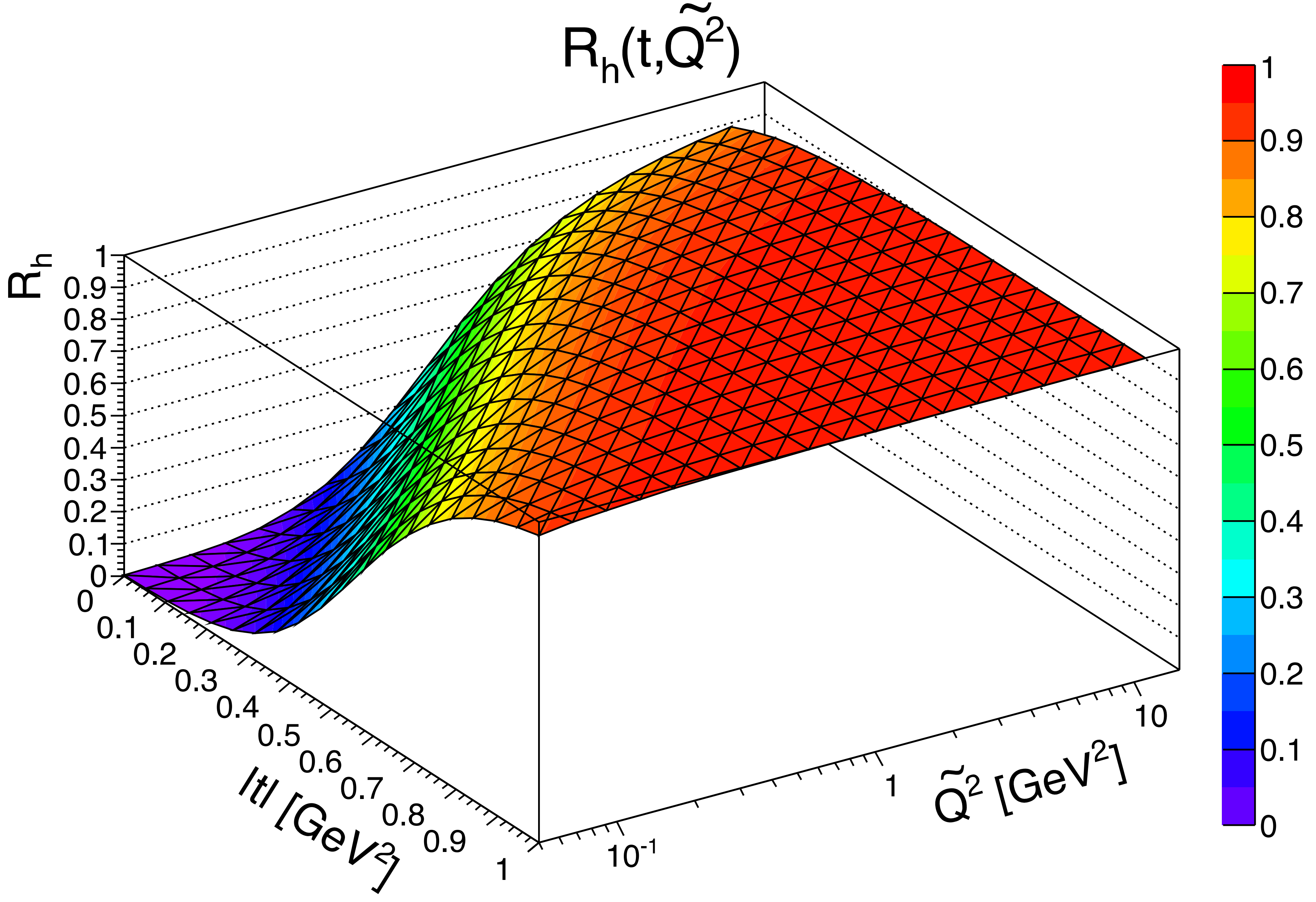
$$|t|\frac{d\sigma_{el}}{dt} + \int_0^{\nu_1} \nu \frac{d^2\sigma}{dtd\nu} d\nu = \int_{\nu_1}^{\nu_{as}} \frac{d^2\sigma}{dtd\nu}|_{\nu_{as}} d\nu, \quad \nu = M_X^2 - M_p^2 - t.$$

Unique Pomeron with two (“soft” and “hard”) components
R. Fiore et al. Phys. Rev. PR **D**90(2014)016007, arXiv 1312.5683

$$A(s,t,Q^2,{M_v}^2) = \frac{\tilde{A}_s}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q^2_s}}\right)^{n_s}} e^{-i\frac{\pi}{2}\alpha_s(t)}\left(\frac{s}{s_{0s}}\right)^{\alpha_s(t)} e^{2\left(\frac{a_s}{\widetilde{Q^2}}+\frac{b_s}{2m_p^2}\right)t}$$

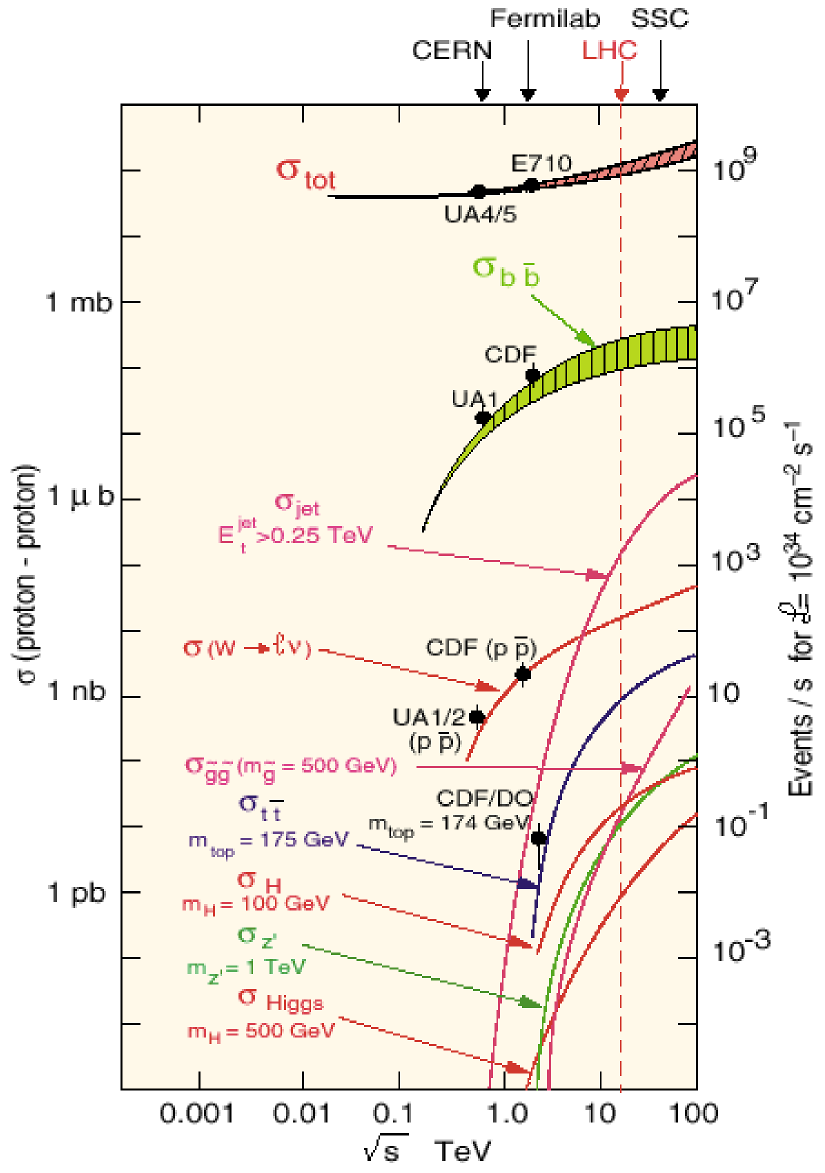
$$+\frac{\tilde{A}_h\left(\frac{\widetilde{Q^2}}{\widetilde{Q^2_h}}\right)}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q^2_h}}\right)^{n_h+1}} e^{-i\frac{\pi}{2}\alpha_h(t)}\left(\frac{s}{s_{0h}}\right)^{\alpha_h(t)} e^{2\left(\frac{a_h}{\widetilde{Q^2}}+\frac{b_h}{2m_p^2}\right)t}$$

Applicable, universally both in NN and eN scattering!


$R_h(t,\tilde{Q}^2)$
R_h
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
|t| [GeV^2]
10^{-1}
1
10
$\tilde{Q}^2$ [GeV^2]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Where we are:

- Total cross section at LHC
  $\sigma(pp \rightarrow \text{anything}) \sim 0.1$ barn
- So a 1 pb Higgs cross section corresponds to one being *produced* every 10^{11} interactions!
  (further reduced by BR × efficiency)
- Experiments have to be designed so that they can separate such a rare signal process from the background
- Rate = $L \cdot \sigma$
  where luminosity L (units $cm^{-2}s^{-1}$) is a measure of how intense the beams are
  LHC design luminosity = 10^{34} $cm^{-2}s^{-1}$



- ***Thank you!***