# Even spin glueball masses and Pomeron Regge trajectory within twist two operator from string/gauge duality

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This talk is based on arXiv:1611.03820
Work done in collaboration with:
Diego M. Rodrigues and Henrique Boschi-Filho

**Summary of the talk:**

- motivation: Point-like scenario x String-like scenario

- Brief Review:  AdS/CFT correspondence and AdS/QCD models

- Glueballs in QCD and the Pomeron

- The Hardwall model – computing even glueballs states masses and the Regge trajecory for the Pomeron

- Results

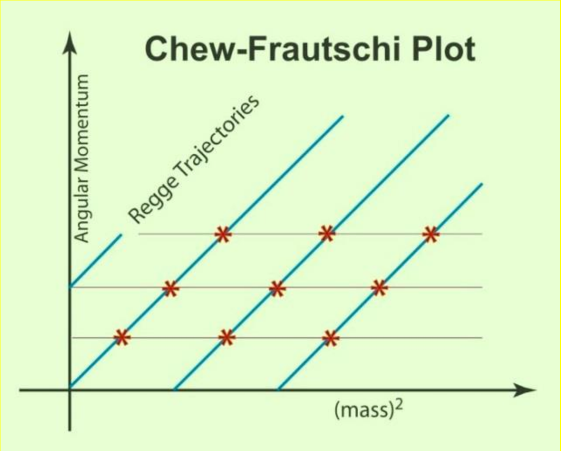
- Last comments

- **Current Thinking:** **Particles are point-like objects !**

- **60´s: hadrons that collided under high energy.**

- **Observed Hadronic Spectrum: Infinite towers of particles (ressonances) showing up in Regge trajectories.**

$$J \sim m^2$$




**REGGE BEHAVIOUR**

# But what does this have to do with strings?

- This relationship is also achieved for a classical rotating string

$$J \sim m^2$$

- **Gabriele Veneziano:**

**Particles are string-like objects !**

$$A_{\text{Ven}}(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))},$$

$\Gamma$ is the Gamma function

$$\alpha(s) = \bar{\alpha}' s + \bar{\alpha}_0 \ , \alpha(t) = \bar{\alpha}' t + \bar{\alpha}_0.$$

- For s→ ∞ with fixed t regime : $A_{Ven} \sim s^{\alpha(t)}$

**Regge behaviour**

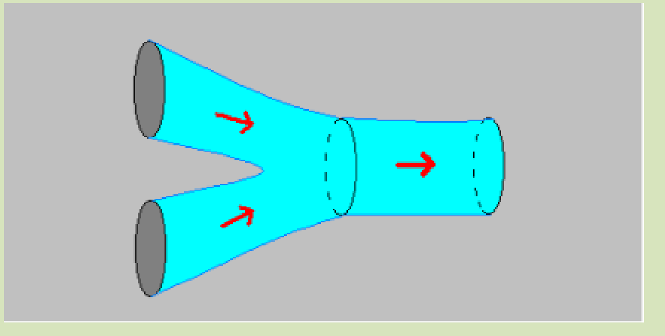
- For s→ ∞ with fixed s/t regime : $A_{Ven}$ **is not good**

# Quantum Relativistic Strings

- Masses spectra similar to the hadrons;
- Physical states corresponding to the other particles:

$$A_\mu \text{ (Photon)}, \; G_{\mu\nu} \text{ (Graviton)}, \ldots\ldots;$$

- String theory includes the gravitational interaction;
- The fundamental objects of nature are not Particles;
- The different particles appear from the vibration of strings;
- Bosonic strings: 26-D;
- SuperStrings (bosons + fermions): 10-D;
- Extra dimensions are compacted;



# Are hadrons string-like objects ?

- **This relationship is not trivial**
  - **Hadrons have some string properties, but….**

- **We cannot describe all Hadrons properties just using a string model in a flat spacetime.**

- **The complete descriptions depends on the spacetime structure.**

# Quantum Chromodynamics - QCD

✓ used as the standard theory to explain the phenomenology of strong interactions.

❑ at the low-energy limit ($g_{YM} > 1$) the QCD cannot be treated perturbatively.

❖ Regge trajectories are an example of nonpertubative behavior of strong interactions: difficult to model it using QCD.

Alternative Way

# AdS/CFT correspondence

**ANTI–DE SITER/CONFORMAL FIELD THEORY**

# AdS/CFT correspondence, J. Maldacena, 1997

(simplified version of a particular useful case)

**SUPERSTRING THEORY in the $AdS_5 \times S^5$ spacetime.**

⇔ DUAL

**YANG-MILLS THEORY**

- Supersymmetric $\mathcal{N} = 4$
- Conformal
- SU(N) symmetry, with $N \to \infty$

in a 4-dimensional Minkowski spacetime ($AdS_5 \times S^5$ boundary).

At low energies string theory is represented by an effective supergravity theory → **gravity / gauge duality**

Other versions of the Correspondence: $AdS_4 \times S^7$ or $AdS_7 \times S^4$ (M-theory in 11 dimensions)

- After breaking the conformal symmetry one can build phenomenological models that describe approximately QCD. So, AdS/QCD models (hardwall, softwall, Witten BH, etc.)
- Weak coupling theory ⇔ Strong coupling theory.

# AdS/CFT Dictionary

Isometries in the bulk ↔ Simmetries in the boundary field theory

$$
\begin{array}{ccc}
\text{Field } (\phi,\ g_{\mu\nu}\ldots.) & \leftrightarrow & \text{Operator } (TrF^2,\ T_{\mu\nu}\ldots.) \\
\text{Radial distance, } u & \leftrightarrow & \text{Energy} \\
\text{Minimal area} & \leftrightarrow & \text{Wilson loop} \\
\vdots & & \vdots \\
\text{Minimal volume} & \leftrightarrow & \text{Entanglement entropy}
\end{array}
$$

Bulk field mass ↔ boundary operator scaling dimension

$$\phi : \quad \Delta(\Delta - d) = m^2$$

$$\psi : \quad |m| = \Delta - \frac{d}{2}$$

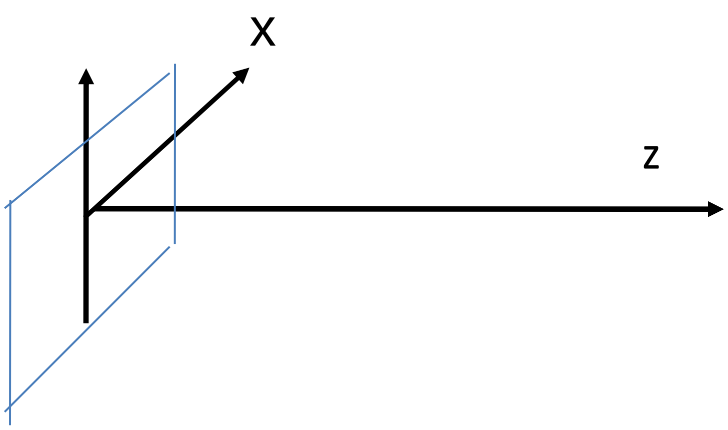
$$A_\mu : \quad m^2 = (\Delta - 1)(\Delta + 1 - d)$$

# The AdS$_5$ Spacetime

**Disregarding the S$^5$ space, the AdS$_5$ Space in Poincaré coordinates is given by:**

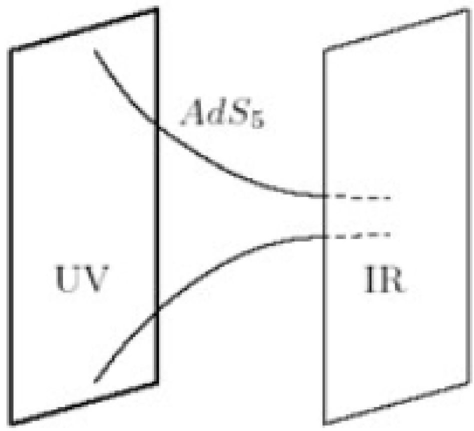
$$ds^2 = \frac{R^2}{(z)^2}(dz^2 + (d\vec{x})^2 - dt^2)$$

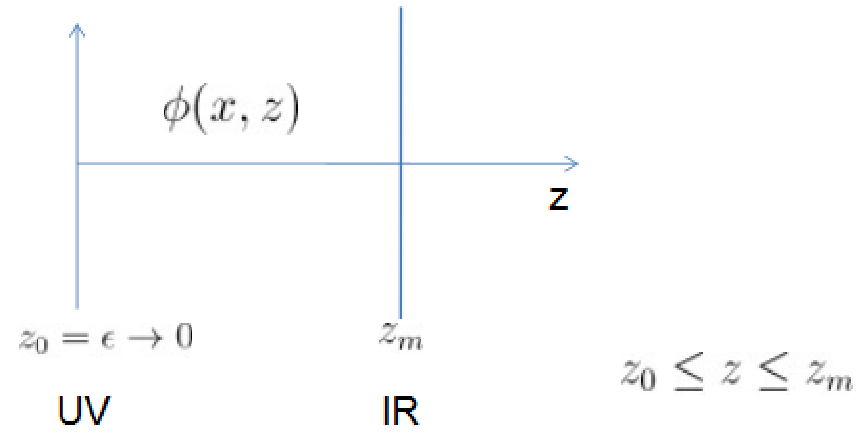
The 4-dim boundary is at z = 0




Fifth dimension z ~ 1 / E where E = Energy in 4-dim boundary

# An AdS/QCD model: Hardwall Model




Scattering of Glueballs using the AdS/CFT correspondence : P & S, 2001/2002

Finite region in AdS space $0 \leq z \leq z_{max}$ with $z_{max} = \frac{1}{\Lambda_{QCD}}$

Henrique Boschi-Filho & Nelson Braga JHEP 2003, EPJC 2004

Masses of Glueball states 0++ and its radial excited states 0++*, 0++**, 0++***, ...

Brodsky, Teramond PRL 2005, 2006; Erlich, Katz, Son, Stephanov PRL 2005.

Extension to Mesons and Baryons

# Glueballs in QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}\left(\not{D} - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \,,$$

$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_{YM} f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \,,$$

where $\mathcal{A}^a_\nu$ are the gluon fields, with $a = 1, \cdots, 8$, $f^{abc}$ are the structure constants of $SU(3)$ group and $g_{YM}$ is the coupling constant of Yang-Mills (strong) interactions.

- Gluons do not carry electric charges, but they have color charge;
- Due to this fact, they coupled to each other;
- The bound states of gluons predicted by QCD, but not detect so far, are called glueballs;
- Glueballs states are characterised by $J^{PC}$, where J is the total angular momentum,

and P and C are the P-parity (spatial inversion) and the C-parity (charge conjugation) eigenvalues, respectively.

# Regge Trajectories

**Strongly interacting particles (Hadrons) obey approximate relations between Angular Momentum (J) and quadratic masses ($m^2$)**

$$J(m^2) \approx \alpha_0 + \alpha' m^2$$

Where $\alpha_0$ and $\alpha'$ are constants

**Extended for glueball: $J^{PC}$**

# The Pomeron

For our purposes, we are interested in the reggeon (Regge pole) with intercept $\alpha_0 \approx 1$, called pomeron.

In the Chew-Frautschi plane, even spin glueball states lie on the pomeron Regge trajectory.

**Experimental Regge trajectories from proton proton scattering**

$$J(m^2) \approx 1.08 + 0.25m^2$$

Masses *m* in GeV (A. Donnachie and P. V. Landshof, Nucl. Phys. B 267, 690 (1986))

**The Pomeron is related to Glueball states** $\mathbf{2^{++}, 4^{++}, 6^{++}, 8^{++}}$

**and may be to** $\mathbf{0^{++}}$

# Using the Harwall model (1)

The starting point in our calculation is the massive symmetric second-rank tensor field action which will be related to the glueball state $2^{++}$. The $D$-dimensional action for a massive spin-2 field in a curved spacetime consistent with the flat space limit is given by

$$S[h_{\mu\nu}] = \frac{1}{2\kappa^2}\int d^Dx\sqrt{|g|}\Big(\frac{1}{4}\nabla_\mu h\nabla^\mu h - \frac{1}{4}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \frac{1}{2}\nabla^\mu h_{\mu\nu}\nabla^\nu h$$
$$+\frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} + \frac{\xi}{2D}\mathcal{R}h_{\mu\nu}h^{\mu\nu} + \frac{1-2\xi}{4D}\mathcal{R}h^2 - \frac{M^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^2)\Big),$$

where $h = g^{\mu\nu}h_{\mu\nu}$ is the trace, $\mathcal{R}$ is the Ricci scalar and $\xi$ is the only dimensionless coupling responsible for the nonminimality of interaction with the curved background. For AdS$_D$, the Ricci scalar is given by

$$\mathcal{R} = -\frac{D(D-1)}{R^2}.$$

# Using the Harwall model (2)

With these constraints:

$$\nabla^2 h_{\mu\nu} + 2\,\mathcal{R}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - M^2 h_{\mu\nu} = 0,$$
$$h_\mu^\mu = 0,$$
$$\nabla^\mu h_{\mu\nu} = 0,$$

In our particular case, we are interested in the 5-dimensional version of for AdS. Setting D = 5, ξ = 1, we have:

$$S[h_{\mu\nu}] = \frac{1}{2\kappa^2}\int_{AdS_5} d^5x\sqrt{|g|}\Big(\frac{1}{4}\nabla_\mu h \nabla^\mu h - \frac{1}{4}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \frac{1}{2}\nabla^\mu h_{\mu\nu}\nabla^\nu h$$
$$+\frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} - \frac{2}{R^2}h_{\mu\nu}h^{\mu\nu} + \frac{1}{R^2}h^2 - \frac{1}{4}M_5^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big),$$

$M_5$ is the five-dimensional mass.

➢ RECALL

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)\,, \qquad \eta_{\mu\nu} = (-,+,+,+)$$

✓EOM:

$$\left[z^3\partial_z \frac{1}{z^3}\partial_z + \Box - \frac{(M_5 R)^2}{z^2}\right]\phi(z,\vec{x},t) = 0\,,$$

# Using the Harwall model (3)

Choosing the ansatz: $\phi(z, \vec{x}, t) = e^{-iP_\mu x^\mu} \, z^2 \, f(z),$

we have the Bessel equation: $z^2 f''(z) + z f'(z) + [(m_{\nu,k}\, z)^2 - \nu^2] f(z) = 0,$

where: $\nu^2 = (M_5 R)^2 + 4.$

So, the complete solution for $\phi(z, \vec{x}, t)$ reads

$$\phi(z, \vec{x}, t) = C_{\nu,k} \, e^{-iP_\mu x^\mu} \, z^2 \, J_\nu(m_{\nu,k}\, z) + D_{\nu,k} \, e^{-iP_\mu x^\mu} \, z^2 \, N_\nu(m_{\nu,k}\, z),$$

Since we are interested in regular solutions inside AdS space, we are going to disregard the Neumann solution, then:

$$\phi(z, \vec{x}, t) = C_{\nu,k} \, e^{-iP_\mu x^\mu} \, z^2 \, J_\nu(m_{\nu,k}\, z),$$

where $m_{\upsilon,k}$ are the glueball masses and k= 1, 2, 3,... are the radial excitations.
We just consider the ground state (K=1).

# Using the Harwall model (4)

## HOW TO RAISE THE GLUEBALL STATE SPIN?

Now we are going to introduce the Twist of an operator!

$$\tau = \Delta - J$$

In this work we are considering the pomeron is a twist two object

$$\Delta - J = 2$$

To raise the spin of the glueball, we will insert symmetrised covariant derivatives in a given operator with spin S in order to raise the total angular momentum, such that, the total angular momentum after the insertion is now S + J.

# Using the Harwall model (5)

Now, we consider a spin $J$ operator in the four-dimensional space, with conformal dimension $\Delta$, denoted by $\mathcal{O}_\Delta$ and constructed in the following way

$$\mathcal{O}_\Delta \sim \mathrm{SymTr}(F_{\beta\alpha_1} D_{\alpha_2} \ldots D_{\alpha_{J-1}} F^{\beta}_{\alpha_J}),$$

➢ **RECALL** $(M_5 R)^2 = \Delta(\Delta - 4) = J^2 - 4,$ ⟹ $\Delta = J + 2$

➢ **RECALL, AGAIN** $\nu^2 = (M_5 R)^2 + 4.$ ⟹ $\nu = J$

Imposing boundary conditions:

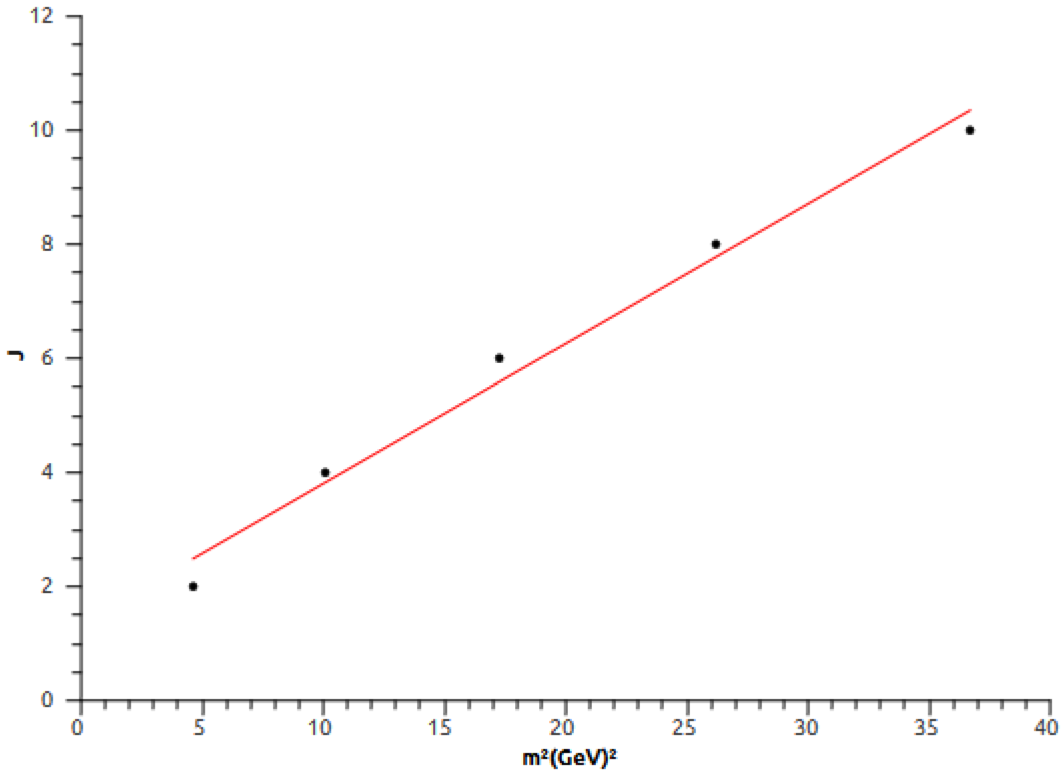
- **Dirichlet b.c:** $m_\nu = \frac{\lambda_\nu}{z_{max}}; \quad J_\nu(\lambda_\nu) = 0,$
- **Neumann b.c:** $(2-\nu) J_\nu(\gamma_\nu) + \gamma_\nu J_{\nu-1}(\gamma_\nu) = 0, \quad m_\nu = \frac{\gamma_\nu}{z_{max}}.$

In order to set $Z_{max}$:
(from lattice)

$z^D_{max} = 2.389 \text{ GeV}^{-1},$ $z^N_{max} = 1.782 \text{ GeV}^{-1}.$

# Results Achieved!

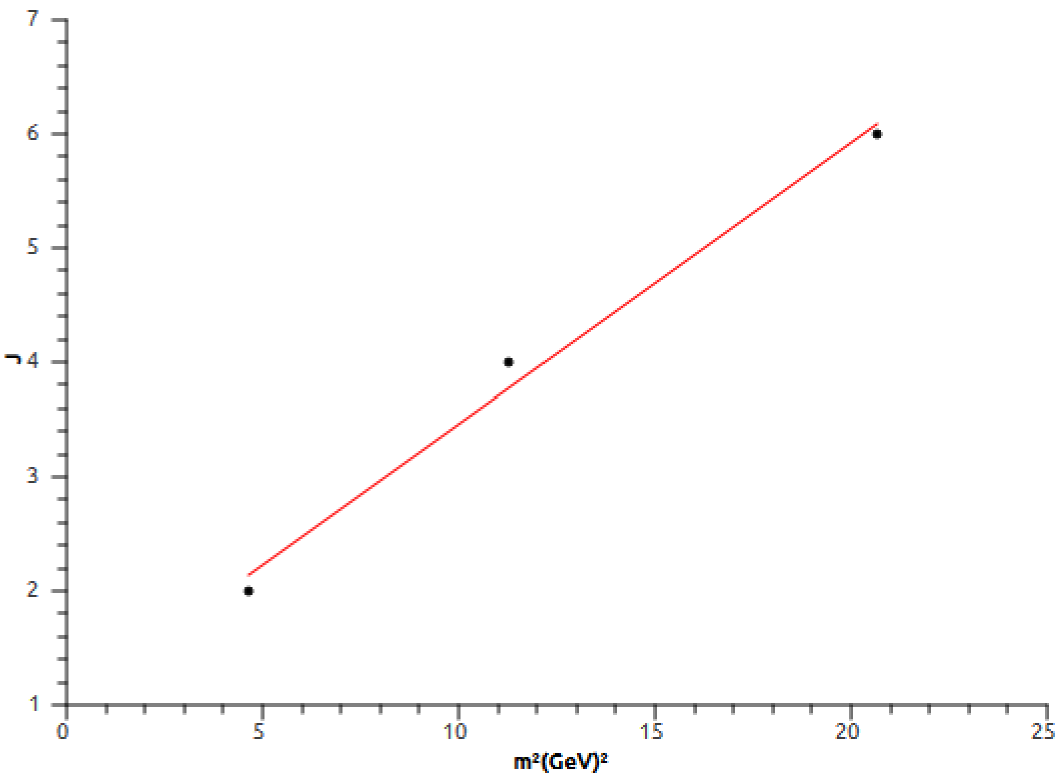
# Even Glueball states in the Hardwall with Dirichlet Boundary condition




| $J^{PC}$ | Mass (GeV) |
|---|---|
| $2^{++}$ | 2.150 |
| $4^{++}$ | 3.176 |
| $6^{++}$ | 4.159 |
| $8^{++}$ | 5.117 |
| $10^{++}$ | 6.059 |

$$\boxed{J(m^2) = (1.34 \pm 0.39) + (0.25 \pm 0.01)m^2.}$$

# Even Glueball states in the Hard-wall with Neumann Boundary condition




| $J^{PC}$ | Mass (GeV) |
|---|---|
| $2^{++}$ | 2.150 |
| $4^{++}$ | 3.356 |
| $6^{++}$ | 4.546 |
| $8^{++}$ | 5.725 |
| $10^{++}$ | 6.899 |

$$\boxed{J(m^2) = (0.99 \pm 0.34) + (0.25 \pm 0.02)m^2.}$$

# Last Comments

*** The AdS/CFT correspondence an excellent tool to tackle QCD out of perturbative regime;**

*** Here, the values for the glueball masses are in agreement with those found in the literature;**

*** Here, the values for the Regge trajectories related to the pomeron are in agreement with those found in the literature;**

*** The complete work can be seen in Phys.Rev. D95 (2017) no.7, 076011, arXiv:1611.03820; (including all details and references).**


# Nagyon szépen köszönöm!!