Chiral Magnetic Effect in the Dirac-Heisenberg-Wigner formalism

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Table of Contents

Table of Contents

Chiral Magnetic Effect

What is the Chiral Magnetic Effect?

We see a normally unexpected electric current in a non-Abelien system evolving under a strong magnetic field.

Chiral Magnetic Effect

What is the Chiral Magnetic Effect?

- Given a background EM magnetic field, and the QCD gauge fields.
- An initially vanishing chiral imbalance could obtain non-zero value due to the interaction with the gauge fields with non-zero Q_w winding number.

$$Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z} \tag{1}$$

Axial charge:

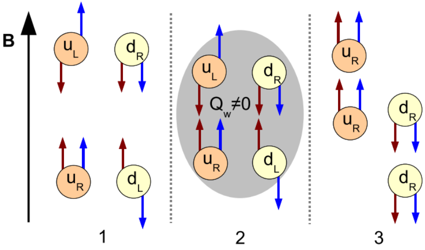
$$(N_L - N_R)_{t=\infty} = 2 N_f Q_w \tag{2}$$

Axial current (on the background field):

$$j^5_\mu = \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle_A \tag{3}$$

D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008).

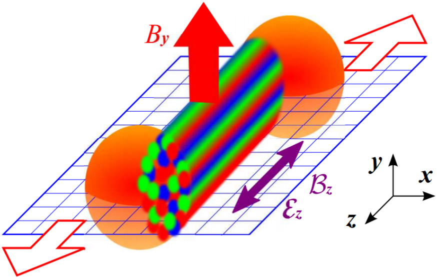
Chiral Magnetic Effect



1. Chirally neutral mixture in very strong B field: particles constrained to Lowest Landau Level.
2. Gauge interaction with non-zero Q_w fields change chirality.
3. Chirality separation leads to charge separation, that leads to current.

Chiral Magnetic Effect

Possible realisation in heavy-ion collisions:



- Background: very strong B field due to highly charged nuclei passing near each other.
- Gauge: QCD gluons

Chiral Magnetic Effect

- Transition between different topologies can happen via tunneling.
- The simplest configuration is a flux-tube, where the gauge fields are $E||B$.
- This can be described by the Schwinger effect $\rightarrow$ connection to pair production.
- Already investigated for constant fields

  Kenji Fukushima, Dmitri E. Kharzeev, and Harmen J. Warringa Phys. Rev. Lett. 104, 212001 2010.

- Main idea: color diagonalisation leads to QED description with E_z, B_z from chromoelectric/magnetic fields and with B_y from EM.

Chiral Magnetic Effect

Main characteristics of the CME (electric) current j_μ:

- $E_z = B_z = B_y = 0$, nothing happens :)
- $E_z = 0, B_z \neq 0, B_y \neq 0$, nothing happens.
- $E_z \neq 0, B_z = 0, B_y = 0$, nothing happens.
- $E_z \neq 0, B_z \neq 0, B_y = 0$, still nothing...
- $E_z \neq 0, B_z = 0, B_y \neq 0$, still nothing...
- Only in the case, when none of the three is zero, is there a CME current!

Chiral Magnetic Effect

- Q: How can we investigate the time dependence of this process?

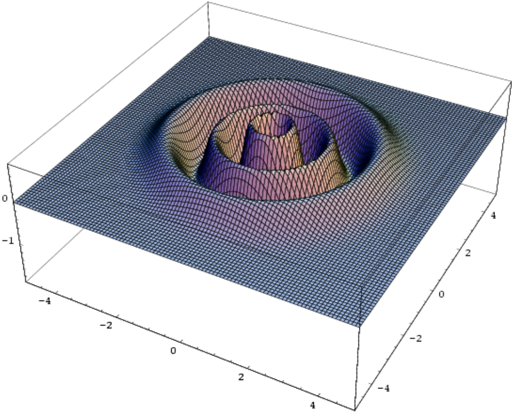
- A: Generalizing the Schwinger description as usual:
  Wigner functions in the real time formalism.

Table of Contents

Wigner function

Tool of description: the Wigner function

- Quantum analogue of the classical phase space distribution.



Wigner function of an n=5 Fock state.

E. Wigner

Wigner function

How it is defined?

- Constructed from the wave function:

$$\hat{\rho}(\vec{x},\vec{s},t) = e^{-ig\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \qquad (4)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle\Omega|\hat{\rho}(\vec{x},\vec{s},t)|\Omega\rangle\mathrm{d}^3s \qquad (5)$$

The Wigner function is usually decomposed by using the Dirac matrix basis:

$$W(\vec{x},\vec{p},t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \qquad (6)$$

Equations of motion of the Wigner function

A system of 16 unknown real functions:

$$
\begin{aligned}
&D_t\mathbb{s} & & & &- &&2\vec{P}\cdot\vec{\mathbb{t}}_{\mathbb{1}} &&=0 &&(7)\\
&D_t\mathbb{p} & & & &+ &&2\vec{P}\cdot\vec{\mathbb{t}}_{\mathbb{2}} &&=2m\mathbb{a}_0 &&(8)\\
&D_t\mathbb{v}_0 &&+ &&\vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & & &=0 &&(9)\\
&D_t\mathbb{a}_0 &&+ &&\vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & & &=2m\mathbb{p} &&(10)\\
&D_t\vec{\mathbb{v}} &&+ &&\vec{D}_{\vec{x}}\mathbb{v}_0 &&+ &&2\vec{P}\times\vec{\mathbb{a}} &&=-2m\vec{\mathbb{t}}_{\mathbb{1}} &&(11)\\
&D_t\vec{\mathbb{a}} &&+ &&\vec{D}_{\vec{x}}\mathbb{a}_0 &&+ &&2\vec{P}\times\vec{\mathbb{v}} &&=0 &&(12)\\
&D_t\vec{\mathbb{t}}_{\mathbb{1}} &&+ &&\vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_{\mathbb{2}} &&+ &&2\vec{P}\mathbb{s} &&=2m\vec{\mathbb{v}} &&(13)\\
&D_t\vec{\mathbb{t}}_{\mathbb{2}} &&- &&\vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_{\mathbb{1}} &&- &&2\vec{P}\mathbb{p} &&=0 &&(14)
\end{aligned}
$$

Simplification

For the CME calculation we are first interested in light quarks...
Let's simplify things: $m = 0$.

Only the vector current / charge and the axial current / charge remains in the equations...

Equations of motion for the $m = 0$ spin-1/2 Wigner function

A system for 8 unknown real functions remains:

$$D_t \mathbb{v}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{v}} = 0 \tag{15}$$

$$D_t \mathbb{a}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{a}} = 0 \tag{16}$$

$$D_t \vec{\mathbb{v}} + \vec{D}_{\vec{x}} \mathbb{v}_0 + 2\vec{P} \times \vec{\mathbb{a}} = 0 \tag{17}$$

$$D_t \vec{\mathbb{a}} + \vec{D}_{\vec{x}} \mathbb{a}_0 + 2\vec{P} \times \vec{\mathbb{v}} = 0 \tag{18}$$

Wigner function non-local operators

Also, to reduce dimensions, we consider homogeneous external fields. In this case, the non-local operators are given exactly (without gradient expansion) as:

$$D_t = \partial_t + g\vec{\mathcal{E}}(t)\vec{\nabla}_{\vec{p}} \tag{19}$$

$$\vec{D}_{\vec{x}} = g\vec{\mathcal{B}}(t) \times \vec{\nabla}_{\vec{p}} \tag{20}$$

$$\vec{P} = \vec{p} \tag{21}$$

Observables

For the CME calculation we record the time evolution of the phase-space integrals of the currents and charges:

$$\mathbb{v}^{\mu}(t) = \frac{1}{(2\pi)^3} \int\limits_{-\infty}^{\infty} \mathrm{d}p^3 \mathbb{v}^{\mu}(t, \vec{p}) \ , \tag{22}$$

$$\mathbb{a}^{\mu}(t) = \frac{1}{(2\pi)^3} \int\limits_{-\infty}^{\infty} \mathrm{d}p^3 \mathbb{a}^{\mu}(t, \vec{p}) \ . \tag{23}$$

Numerical Solution

Ingredients of the 3+1 D solver:

- Pseudospectral collocation
- Rational Chebyshev polynomial basis $\mathrm{TB}_n(x) = \cos\left(n \cdot \mathrm{acot}(x/L)\right)$
- 4th order Runge-Kutta
- GPU Acceleration

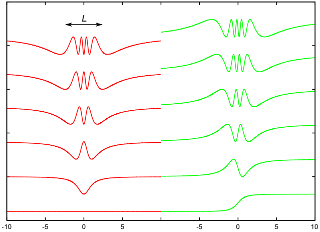
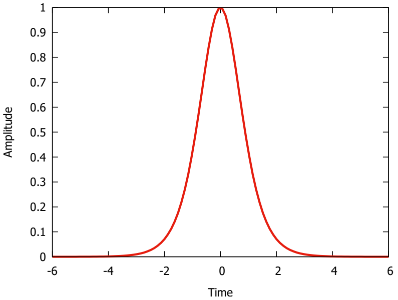


Table of Contents

Sauter field

Start with a simple model field that we know very well:

$$S(t) = \cosh^{-2}(t/\tau) \tag{24}$$



Sauter field

$$S(t) = \cosh^{-2}(t/\tau) \tag{25}$$

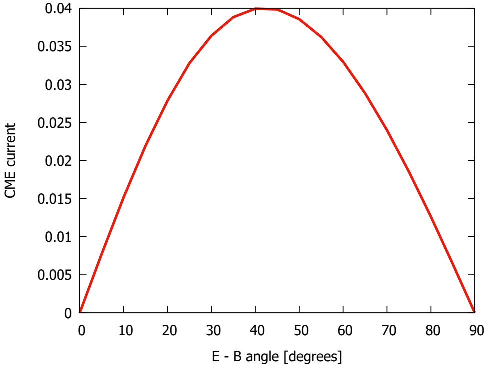
Let the fields be:

$E_z = A \cdot S(t),$

$B_z = A \cdot \cos(\alpha) S(t),$

$B_y = A \cdot \sin(\alpha) S(t)$

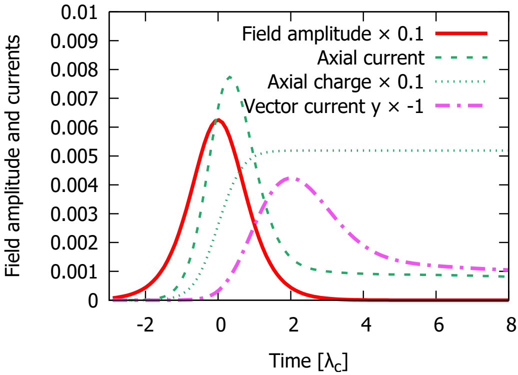
Sauter field



Only in the case, when none of the three is zero, is there a CME current! This is what we expect from a CME model!

Sauter field

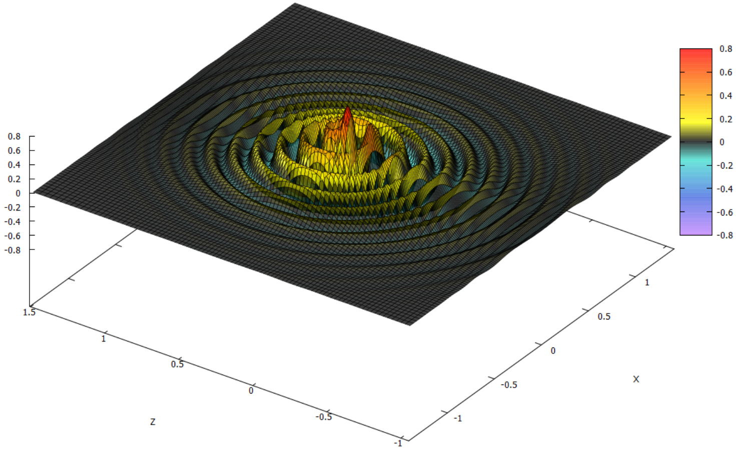
CME formation during the interaction:



External fields comes first, then axial current and then electric current!

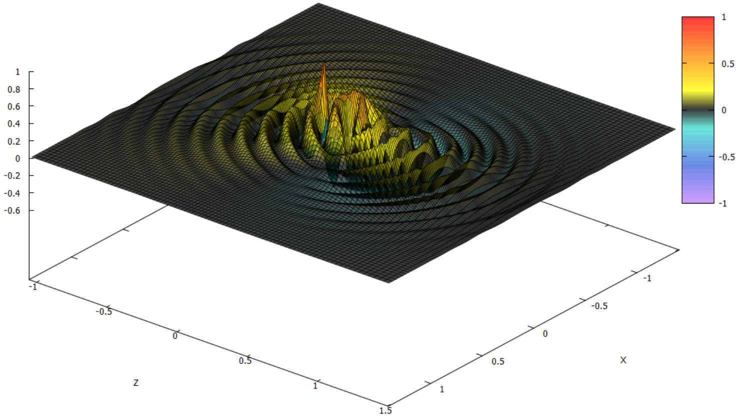
Results

The Anomalous component of the electric current:



Results

The Axial current in X direction:



Model field for high energy collisions

Assume the following fields for a high energy collision:
Consider the following time dependent function:

$$\Phi(t,\tau,A,\kappa) = A \cdot \begin{cases} \cosh^{-2}(10\mathrm{t}/\tau) & t < 0, \\ (1+t/\tau)^{-\kappa} & t \geq 0, \end{cases} \tag{26}$$

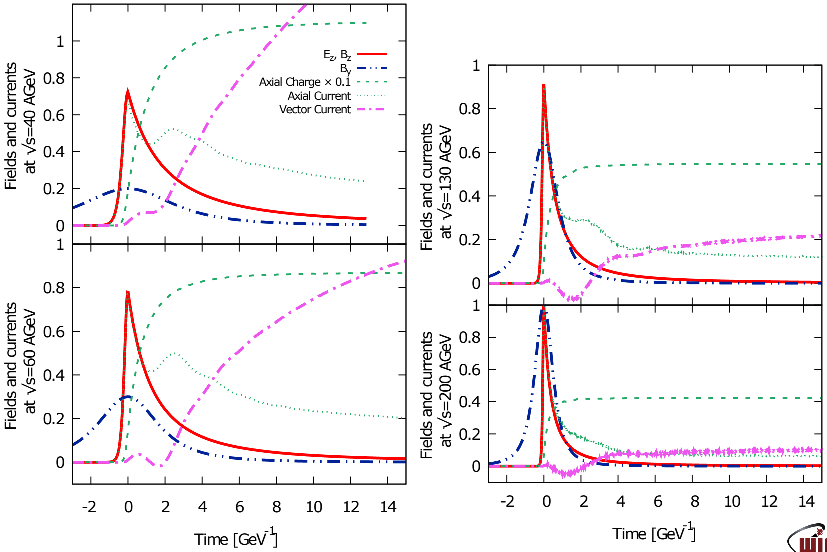
where we set $\kappa = 2$ and model the external fields as:

$$e\vec{E}(t) = \{0, 0, \qquad \Phi(t,\tau,A_{Ez},\kappa)\}\ , \tag{27}$$

$$e\vec{B}(t) = \{0, A_{By}\left(1+\frac{t^2}{\tau^2}\right)^{-3/2}, \Phi(t,\tau,A_{Bz},\kappa)\}\ . \tag{28}$$

with some phenomenological parameters.

Model field for high energy collisions



Summary

- The DHW equations can predict electric and axial currents consistent with what would be expected in the Chiral Magnetic Effect.
- The fields are connected to the electromagnetic B, and the non-Abelian chromoelectromagnetic $\mathcal{E}, \mathcal{B}$ fields.
- The DHW description can act as a real time microscopic source for the effect in non-central heavy-ion collisions
- A phenomenologically parametrized model field shows the disappearance of the effect at high energies and a sign change at intermediate energies.

arxiv:1707.03621

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Wigner GPU Lab