CGC scattering at NLO accuracy

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Outline

Outline of this talk

- Color fields in initial stages of heavy ion collision
- Dilute-dense control processes, QCD perturbation theory
- Recent progress in dilute-dense processes at NLO

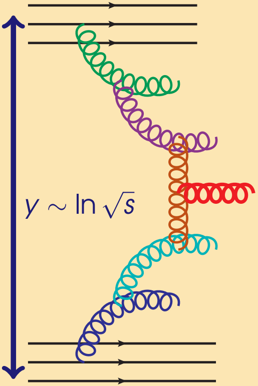
Trinity of dilute-dense CGC calculations

- Evolution equation (BK)
- Total DIS cross section
- Single inclusive hadron production in pA-collisions

+ essential question: doing the three consistently.

Initial stage of heavy ion collision: color fields

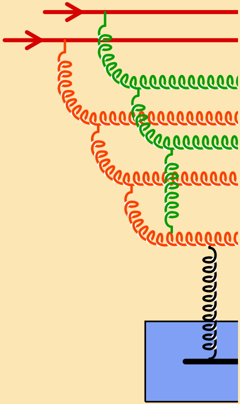
Initial state is small x



- Gluons ending in central rapidity region: multiple splittings from valence quarks
- Emission probability $\alpha_s \, dx/x$
  $\Longrightarrow$ rapidity plateau for $\Delta y \ll /\alpha_s$
- Many gluons, in fact

$$N \sim \sum_n \frac{1}{n!} (\alpha_s \ln \sqrt{s})^n \sim \sqrt{s}^{\alpha_s}$$

Small x and saturation



- Eventually gluons in cascade overlap
- YM covariant derivative
  $-iD_\mu = -i\partial_\mu + gA_\mu = p_\mu + gA_\mu$
- Nonlinearities when $p_T \sim gA_\perp \sim Q_s$
  $\Longrightarrow A_\perp \sim Q_s/g$
  (LC gauge gluons have $\mu \to \perp$)

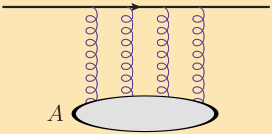
Weak coupling, but nonperturbative

- α_s small
- $f(k) \sim A_\mu A_\mu \sim 1/\alpha_s$ large

True when $Q_s \gg \Lambda_{QCD}$

$\Longrightarrow$ gets better at large $\sqrt{s}$

Eikonal scattering off target of glue



How to measure small-x glue?

- Dilute probe through target color field
- At high energy interaction is eikonal

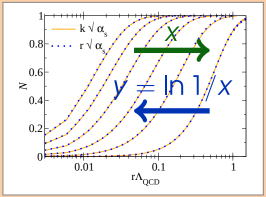
Eikonal scattering amplitude: Wilson line V

$$V = \mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in SU(N_c)$$

- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x} - \mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \operatorname{Tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

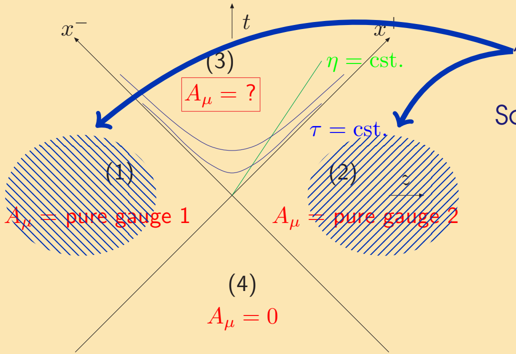
from color transparency to saturation



- $1/Q_s$ is Wilson line **correlation length**

Classical Yang-Mills initial state

Classical **Y**ang-**M**ills



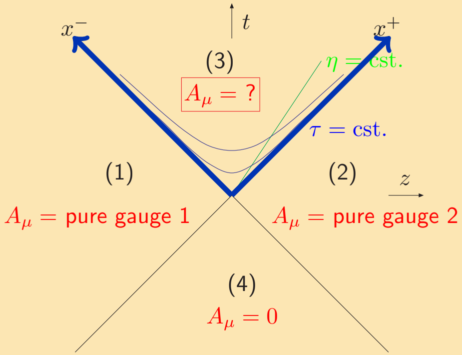
Change to LC gauge:

$$A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}) \partial_i V^\dagger_{(1,2)}(\mathbf{x})$$

Same Wilson line $V(\mathbf{x})$

Classical Yang-Mills initial state

Classical Yang-Mills



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Same Wilson line $V(\mathbf{x})$

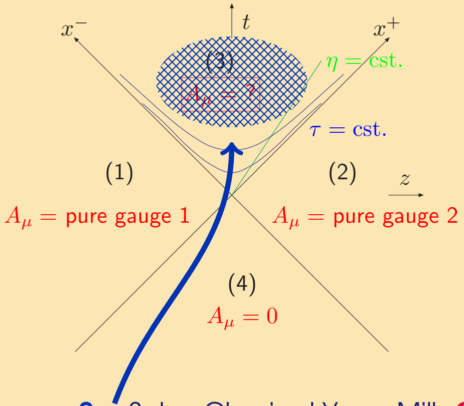
At $\boldsymbol{\tau = 0}$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

Classical Yang-Mills initial state

Classical Yang-Mills



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$$A^\eta\Big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

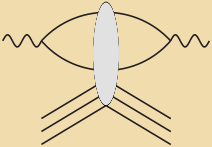
$\boldsymbol{\tau > 0}$ Solve Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over $V(\mathbf{x})$.
In dilute limit reduces to k_T-factorization.

Dilute-dense processes: probing the color fields

Dilute-dense process at LO

Physical picture at small x

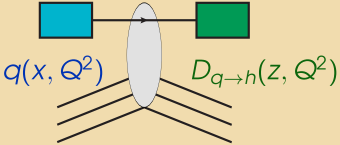
DIS



- $\gamma^* \to q\bar{q}$ dipole interacts with target color field
- Total cross section is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991
Fits to HERA data:
e.g. Golec-Biernat, Wüsthoff 1998

Forward hadrons



- q/g from probe: collinear pdf
- $|\text{amplitude}|^2 \sim$ dipole
- Indep. fragmentation

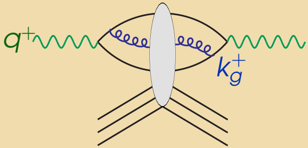
"Hybrid formalism";
Dumitru, Jalilian-Marian 2002

Both involve same dipole amplitude $\mathcal{N} = 1 - S$

Dilute-dense process at LL

Add one **soft** gluon: large logarithm of energy/x

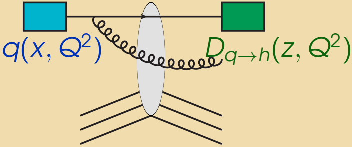
DIS



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Forward hadrons



- Soft gluon $k^+ \to 0$: same large log
- Collinear gluon $k_T \to 0$: DGLAP evolution of pdf, FF

Dumitru et al 2005

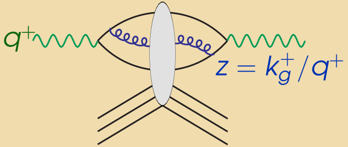
Absorb large log into renormalization of target:

BK equation Balitsky 1995, Kovchegov 1999

Dilute-dense process at NLO

Add one gluon, but **not** necessarily soft

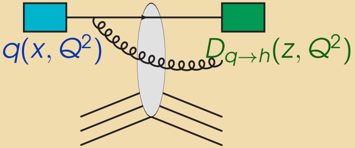
DIS



- DIS impact factor

  Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons



- NLO single inclusive

  Chirilli et al 2011

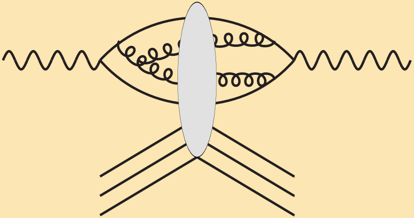
- Leading small-k^+ gluon already in BK-evolved target
- Need to **subtract** leading log from cross section:

$$\sigma_{NLO} = \int \mathrm{d}z \Big[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}} \Big] \quad z = \frac{k_g^+}{P^+_{\text{tot}}}$$

NLO to NLL

NLO evolution equation:

- Consider NNLO DIS
- Extract leading soft logarithm
- Lengthy calculation:
  Balitsky & Chirilli 2007
- But additional resummations needed for practical phenomenology



(+ many diagrams at same order)

- $\alpha_s^2 \ln^2(1/x)$: two iterations of LO BK
- $\alpha_s^2 \ln 1/x$: NLO BK
- α_s^2: part of NNLO impact factor (not calculated)

Summary: power counting

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \underbrace{\mathcal{O}(\alpha_s \ln 1/x)}_{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \underbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}_{\text{NLL}}$$

- Current phenomenology LL
- Theory recently becoming understood at NLO & NLL
- Moving to phenomenology, numerical implementations:
  - Fit to DIS data with (approx) NLL evolution (but not NLO) : Albacete 2015, Iancu et al 2015
  - Single inclusive hadrons at NLO (but not NLL) : Stasto et al 2013, Ducloué et al 2015
  - Full NLL evolution (Not yet NLO) Mäntysaari 2015
  - NLO DIS cross section (Not yet NLL) Ducloué et al 2017

} next

Recent progress in NLO dilute-dense calculations

The NLO BK equation

as derived by Balitsky and Chirilli, 2007

Equation: $y = \ln 1/x$**-dependence from**

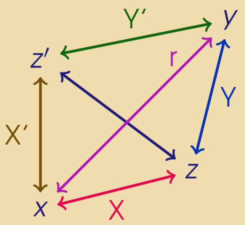
$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

Notations & approximations

- $S(x-y) \equiv (1/N_c) \langle \mathrm{Tr}\, V^\dagger(x) V(y) \rangle$
- $\otimes = \int d^2 z$ or $\int d^2 z\, d^2 z'$
- Here large N_c & mean field:

$$\langle \mathrm{Tr}\, V^\dagger V\, \mathrm{Tr}\, V^\dagger V \rangle \to \langle \mathrm{Tr}\, V^\dagger V \rangle \langle \mathrm{Tr}\, V^\dagger V \rangle$$

Coordinates



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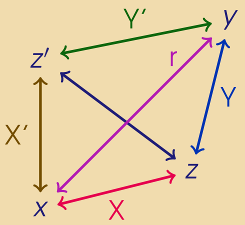


NLO BK equation: Resummations

Discussion here following Iancu et al 2015

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z - z')S(Y') - S(X)S(Y)]$$

Resum:

- β-function terms in K_1 into running coupling: K_{Bal}
- Double transverse logarithms in K_1 into $K_{\mathrm{DLA}} \sim J_1(\ln r^2)/\ln r^2$.
- Single transverse logs in K_2 into $K_{\mathrm{STL}} \sim r^{\alpha_s A_1}$ with DGLAP anomalous dimension A_1
- Subtract double counting K_{sub}, include rest of NLO K_1^{fin}

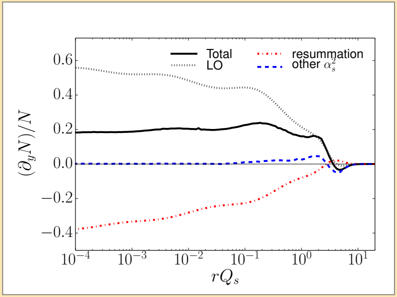
$\Longrightarrow$ Solve equation Mäntysaari, T.L. 2016 :

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \left[K_{DLA} K_{STL} K_{Bal} - K_{\mathrm{sub}} + K_1^{\mathrm{fin}} \right] \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z - z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

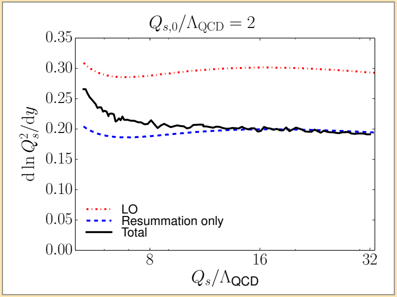
NLL evolution with resummation

Mäntysaari, T.L. 2016

Evolution speed vs r



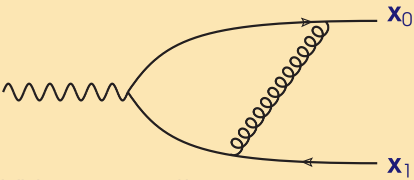
Evolution speed of Q_s



- Resummations essential to get stable results
  $\Longrightarrow$ good HERA fit with "resummation only" Iancu et al 2015
- Importance of non-resummed terms can be tuned
  (choice of 'constant under log' in resummation)
- Here simple rapidity-local resummation Iancu et al 2015
  Alternative: impose cumbersome but better defined kinematical constraint Beuf 2014, implementation Albacete 2015

DIS at NLO: impact factor

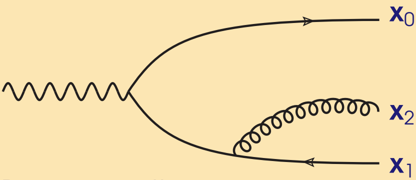
Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017



Virtual corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

+ UV divergence in loop



Real corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$$

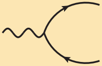
UV divergence in $\mathbf{x}_2$-integral

UV-divergence cancels because

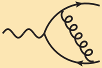
$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_1) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

DIS at NLO: subtraction of BK

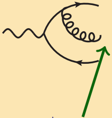
Evaluate cross section as $\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{LO}}_{L,T} + \sigma^{\mathrm{dip}}_{L,T} + \sigma^{qg}_{L,T,\mathrm{sub.}}$



$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



- LL

$k_g^+ \sim z_2$

$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Bigg[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Bigg].$$

* UV-divergence

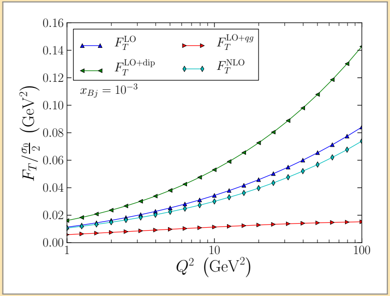
- LL: subtract leading log, already in BK-evolved $\mathcal{N}$

$\Longrightarrow$ good ways and bad ways to do this . . .

Numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017

- Major cancellation between different NLO terms



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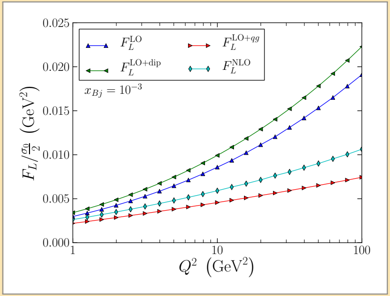
Ducloué, Hänninen, T.L., Zhu 2017

- Major cancellation between different NLO terms (similar for F_L)



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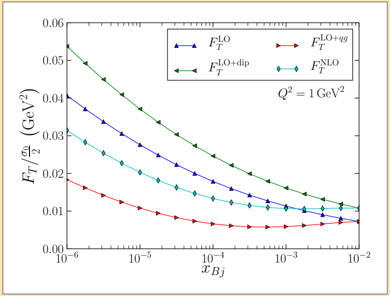
- *qg*-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect



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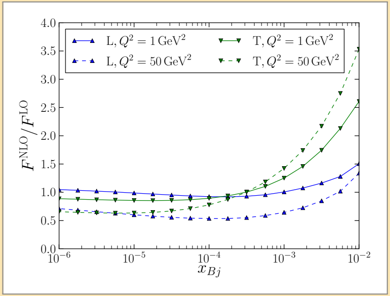
- Major cancellation between different NLO terms (similar for F_L)
- qg-term explicitly zero at $x_{Bj} = x_0 \Longrightarrow$ transient effect
- Running coupling (parent dipole)
  - Transient effect larger



NLO/LO ratio

Numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017

- Major cancellation between different NLO terms (similar for F_L)
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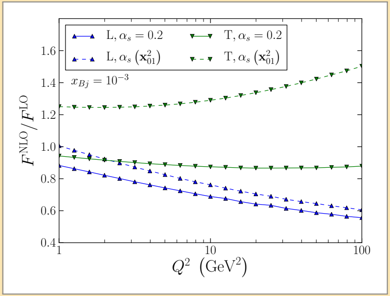
- Running coupling (parent dipole)
  - Transient effect larger
  - But Q^2-dependence stable



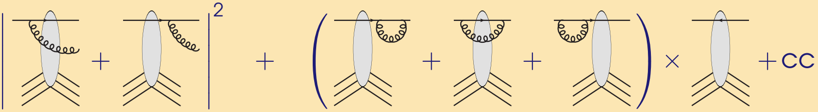
NLO/LO ratio

Relative NLO corrections of the magnitude one would expect

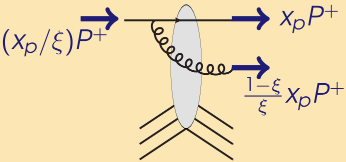
Forward single inclusive: CXY calculation

Chirilli, Xiao, Yuan 2012

$1-\xi$=longitudinal momentum fraction of gluon



- Kinematical limit: $\xi < 1 - x_g$
- Limit: all target P_A^- to qg state!



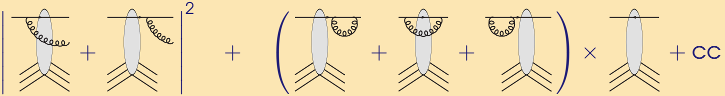
After some color algebra

$$\frac{\mathrm{d}N^{pA\to qX}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}_0(k_T)}{(2\pi)^2}$$

$$+\frac{\alpha_s}{2\pi^2}\int_{x_p}^{\xi_{\max}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\left\{C_F\mathcal{I}(k_T,\xi) + \frac{N_c}{2}\mathcal{J}(k_T,\xi)\right\}$$

$$-\frac{\alpha_s}{2\pi^2}\int_{0}^{\xi_{\max}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} x_p q(x_p)\left\{C_F\mathcal{I}_v(k_T,\xi) + \frac{N_c}{2}\mathcal{J}_v(k_T,\xi)\right\}$$

Collinear divergence



Terms with $C_F = \frac{N_c{}^2 - 1}{2N_c}$:

$$\mathcal{I}(k_T, \xi) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\frac{\mathbf{k} - \mathbf{q}}{(\mathbf{k} - \mathbf{q})^2} - \frac{\mathbf{k} - \xi\mathbf{q}}{(\mathbf{k} - \xi\mathbf{q})^2} \right]^2 \mathcal{S}(q_T)$$

$$\mathcal{I}_V(k_T, \xi) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\frac{\mathbf{k} - \mathbf{q}}{(\mathbf{k} - \mathbf{q})^2} - \frac{\xi\mathbf{k} - \mathbf{q}}{(\xi\mathbf{k} - \mathbf{q})^2} \right]^2 \mathcal{S}(k_T)$$

- Linear in $S(k_T)$, like the LO cross section, vanish at $\xi = 1$
- Logarithmic collinear divergence
- CXY: Calculate in $d_\perp = 2 - 2\varepsilon$ dimensions
- Absorb $1/\varepsilon$ into DGLAP for pdf and frag. fun.

Rapidity divergence

Terms with coefficient N_c

$$\mathcal{J}(k_T,\xi) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_T) - \int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{l}}{(2\pi)^2} \frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_T)\mathcal{S}(l_T)\,,$$

$$\mathcal{J}_v(k_T,\xi) = \Bigg[\int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2} - \int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{l}}{(2\pi)^2} \frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(l_T)\Bigg]\mathcal{S}(k_T).$$

- Collinear finite, approach finite value for $\xi \to 1$
- With $\int^{1-x_g} \frac{d\xi}{1-\xi}$ get large log; "rapidity divergence" for $x_g = 0$ that needs to be resummed with BK equation.
- Negative cross sections first obtained by Stasto, Xiao, Zaslavsky 2013
  $\Longrightarrow$ Solved by proper treatment of this subtraction

Ducloué, T.L., Zhu 2016,2017 ; Iancu, Mueller, Triantafyllopoulos 2016

Before conclusions: to do

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \overbrace{\mathcal{O}(\alpha_s \ln 1/x)}^{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \overbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}^{\text{NLL}}$$

- Next: fit to HERA data with NLO impact factor
  (with LL or NLL evolution)
- Needs implementation (both DIS and single inclusive) :
  match NLL evolution with NLO cross section:
  - Evolution variable k^+ vs k^-
  - Kinematical constraint vs
    rapidity local resummation of double logs
  - Corresponding different subtractions from cross sections
- Needs loop calculation: quark masses
- Other:
  - Exclusive processes
  - Dihadron correlations

Conclusions

- Initial stage in heavy ion collision:
  nonperturbatively strong color field
- CGC effective theory:
  systematical weak coupling description of this color field
- The same color field:
  probed in many dilute-dense processes: DIS, fwd pA, . . .
- Description of dilute-dense processes:
  moving to NLO accuracy in QCD coupling

Backups: details

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2 Y^2 + Y'^2 X^2 - r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)}\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

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- Leading order

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

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- Leading order
- Running coupling (Terms with β function coefficient)

Kernels

$$K_1 = \frac{r^2}{X^2Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2-Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

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- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)

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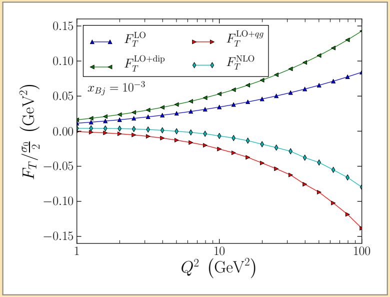
- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)
- Nonconformal double log $\Longrightarrow$ blows up for $r = 0$

DIS: Numerical implementation

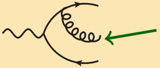
Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz_2}{z_2}\left[\mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2))\right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable
    (like single inclusive)



$X(z_2) = x_{Bj}$



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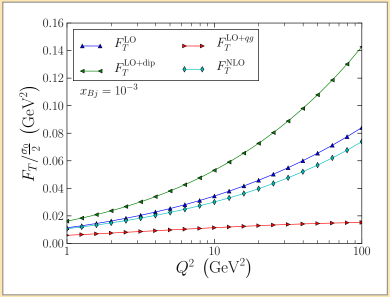
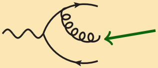
  - $X(z_2) = x_{Bj}/z_2$ OK



$X(z_2) = x_{Bj}/z_2$



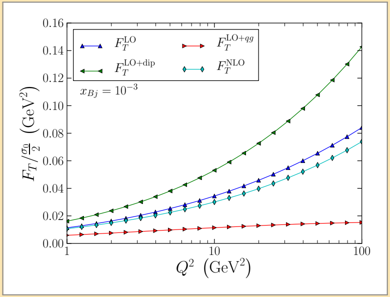
$k_g^+ \sim z_2$

DIS: Numerical implementation

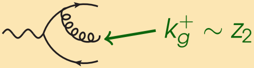
Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz_2}{z_2} \left[\mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2)) \right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK
- Lower limit of z_2
  - $z_2 > \frac{x_{Bj}}{x_0}$ from target k^- (assuming $k_T{}^2 \sim Q^2$)
  - Strict k^+ factorization: $z_2 > \frac{x_{Bj}}{x_0} \frac{M_p^2}{Q^2}$ $\Longrightarrow$ would require kinematical constraint
  - For "dipole" term integrate to $z_2 = 0$

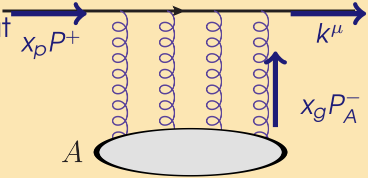


$X(z_2) = x_{Bj}/z_2$



Single inclusive: hybrid formalism at LO

- Quark/gluon from collinear pdf at $x_p \sim 1$
  Here: just consider q channel
- LO: deflected by target field
  - Transverse momentum $k_T \sim Q_s$
  - Longitudinal momentum $x_g = k_T e^{-y}/\sqrt{s} \ll 1$



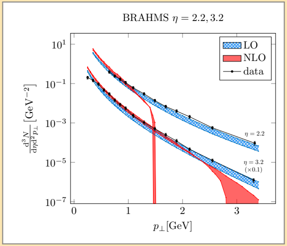
Result:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}} = \frac{1}{(2\pi)^2} xq(x, Q^2)\mathcal{S}(\mathbf{k})$$

$$\mathcal{S}(\mathbf{k}) = \int \mathrm{d}^2\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} \mathcal{S}(\mathbf{r}) \quad ; \quad \mathcal{S}(\mathbf{x}-\mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

(For quark production. leaving out fragmentation function)

Negative cross sections

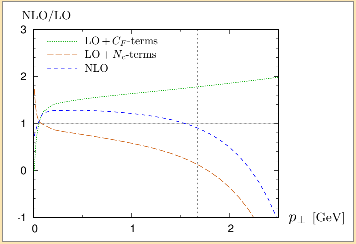


- Analytical calculation
  Chirilli, Xiao, Yuan 2012
- Numerics: Stasto, Xiao, Zaslavsky 2013
  $\Longrightarrow$ cross section negative
  (large N_C; mix C_F and N_C terms)
- Kinematics? Large k_T logs?? Beuf et al 2014, Watanabe, Xiao & Zaslavsky 2015

Ducloué, T.L., Zhu 2016: q channel at finite N_C

also Kang et al 2014

- Problem is in the rapidity divergence
- Most easily identified by color factor



Unsubtracted cross section, N_c-term

Discussion here following Iancu et al 2016 leave out C_F/DGLAP-terms

$$\frac{dN^{\mathrm{LO}+N_c}}{d^2\mathbf{k}\,dy} \sim \mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\mathcal{K}(k_T, \xi, X(\xi))$$

- Dipole operator $\mathcal{S}_0$ is "bare"
- Rapidity at which dipoles are evaluated $X(\xi)$
- x_g: the target momentum fraction for LO kinematics
- Multi-Regge-kinematics: $X(\xi) = x_g/(1-\xi)$
- Only target $X(\xi) < x_0 \implies$ phase sp. limit $\xi < 1 - x_g/x_0$:

$$\text{BK:} \quad \mathcal{S}(k_T, x_g) = \mathcal{S}(k_T, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\mathcal{K}(k_T, 1, X(\xi))$$

Combine these, taking $\mathcal{S}(k_T, x_0) \equiv \mathcal{S}_0(k_T) \ldots$

Subtracted form for cross section

Unsubtracted form

$$\mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \mathcal{K}(k_T, \xi, X(\xi))$$

$$= \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \left[\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))\right]$$

subtracted form

(Recall: dipoles evaluated at rapidity $X(\xi)$)

- These are strictly equivalent, perfectly positive at all k_T
- Subtracted form is a true perturbative series
  unsubtracted has $\alpha_s \ln 1/x$ and α_s together

Origin of negativity in CXY

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\left[\mathcal{K}(k_T,\xi,X(\xi)) - \mathcal{K}(k_T,1,X(\xi))\right]$$

How do CXY get a negative cross section?

- $\mathcal{K}(k_T,\xi,X(\xi)) - \mathcal{K}(k_T,1,X(\xi))$ dominated by $\xi \ll 1$
- Replace $X(\xi) \to X(\xi = 0) = x_g$
- Change ξ integration limit to 1 (+ distribution!)

This gives CXY subtraction scheme

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^1 \frac{\mathrm{d}\xi}{1-\xi}\Big[\overbrace{\mathcal{K}(k_T,\xi,x_g)}^{\sim \xi/k_T{}^4 \text{ for } k_T \gg Q_s} - \mathcal{K}(k_T,1,x_g)\Big]$$

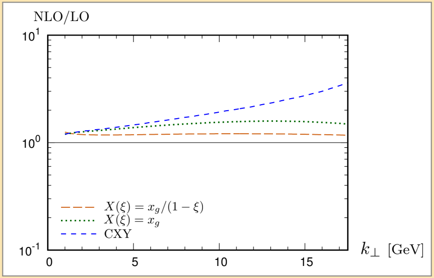
- Formally ok in α_s expansion
- Nice factorized form: only dipoles at x_g, like LO
- But subtraction no longer integral form of BK

Comparing subtraction procedures

First:
must also make choice for $X(\xi)$ in the C_F-term:
scheme dependence

▼

Take same $X(\xi)$ & limits as N_c-term



Comparing subtraction procedures

First:
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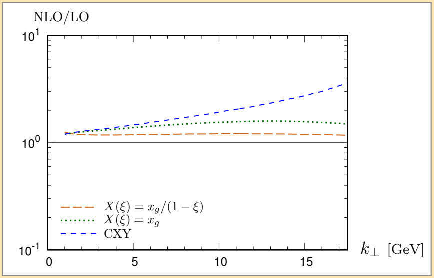
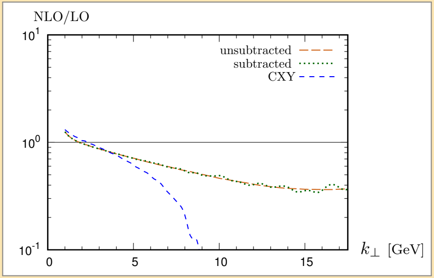
Take same $X(\xi)$ & limits as N_c-term





Two forms for NLO cross section

- Explicitly equivalent
- Positive, although $\ll$ LO