Properties of geometrical clusters and deconfinement transition in SU(2) gluodynamics

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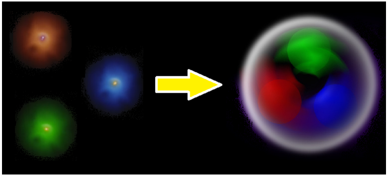
Zimanyi Winter School, 2017

Introduction

- QCD operates with quarks and gluons,

  Only hadrons are directly observed

  If partons clusterize to hadrons, then how to define these clusters?



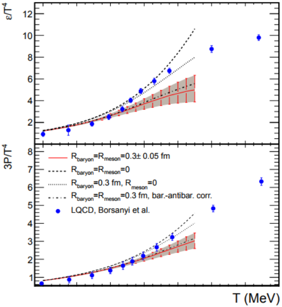
- Effective degrees of freedom $\Rightarrow$ simplified description

  QCD with quarks and gluons– **difficult**

  Hadron Resonance Gas – **simple**

  Hadrons are effective degrees of freedom at low temperatures

  How to define effective degrees of freedom at high temperatures?



A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, PLB 718, 1, 80 (2012)

Motivation

- Non abelian nature of QCD $\Rightarrow$ confinement/deconfinement transition
  SU(2) gluodynamics – simplest gauge theory (2 types of color charge, no quarks)
- **Svetitsky -Jaffe conjecture**:
  deconfinement transitions in (d+1)dimensional SU(N) gluodynamics is equivalent to magnetic transition in the d-dimensional Z(N) spin system

  L. G. Yaffe and B. Svetitsky, PRD, 26, 963 (1982)

  SU(2) gluodynamics $\Leftrightarrow$ Ising spin model

  Polyakov loop - gauge invariant analog of continuous spin
- Deconfinement in QCD (probably) is a phase transition of the liquid-gas type
  - hadron matter at low densities and temperatures is a gas
  - QGP (probably) is the most perfect liquid
    (hydrodynamic expansion, small η/s, typical value of $\frac{U_{potential}}{T}$)

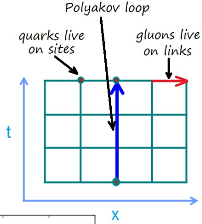
    E. Shuryak, Prog. Part. Nucl. Phys. 62, 48-101 (2009)

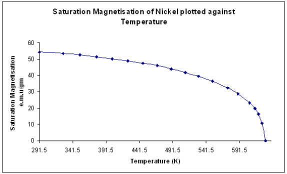
Is phase transition in gluodynamics related to the liquid-gas one?

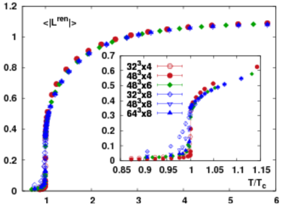
- Polyakov loop L can be defined for any configuration of gluon field A_μ

$$SU(N_c): \quad L(\vec{x}) = \frac{1}{N_c} Tr \exp\left(\int dt \; A_4(\vec{x}, t) \right)$$

- Polyakov loop at discrete lattice of spatial-temporal extent $N_\sigma^3 \times N_\tau$
  - gluon field $A_\mu \Rightarrow$ gauge link $U_\mu = \exp(iA_\mu)$
  - Polyakov loop $L(\vec{x}) = \frac{1}{N_c} Tr \prod_{t=0}^{N_\tau - 1} U_4(\vec{x}, t)$
- SU(2) gauge group (2 types of color) $\Rightarrow L \in [-1, 1]$ – real
- Polyakov loop – order parameter of deconfinement







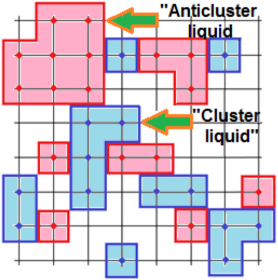
M. P. Weiss and R. Forrer, Ann. Phys. Paris, 5, 153 (1926)

Pok Man Lo et al., Phys.Rev. D 88, 074502 (2013)

Identification of geometrical clusters

- Definition of (anti)clusters:
  - $|L(\vec{x})| < L_{cut} \Rightarrow$ auxiliary vacuum
  - $|L(\vec{x})| \geq L_{cut} \Rightarrow$ (anti)clusters
  - L_{cut} - vacuum cut-off parameter,
    minimal value of Polyakov loop inside (anti)cluster

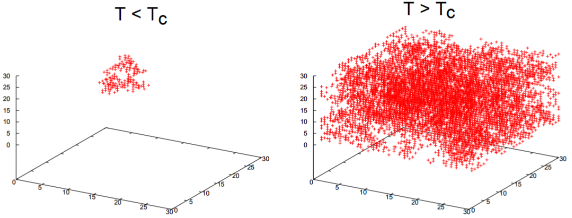
C. Gattringer, PLB, 690, 179 (2010)
C. Gattringer, A. Schmidt, JHEP 1101, 051, 2011



- Two signs of Polyakov loop in SU(2) gluodynamics $\Rightarrow$ (anti)clusters can be either "spin up"or "spin down"ones
  - Largest fragment - "anticluster liquid droplet"
  - Next to the largest fragment - "cluster liquid droplet"
  - Gas of (anti)clusters has the same Polykov loop sign as their "liquids"

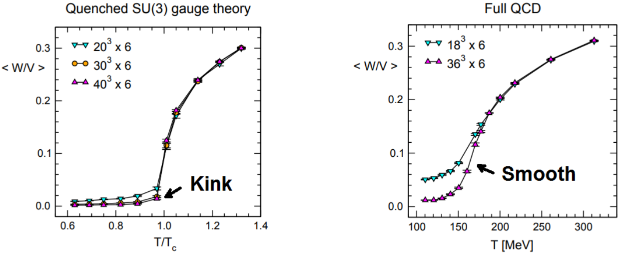
Percolation in gluodynamics

- Properties of the largest cluster are studied in SU(2), SU(3) and SU(4) GD



S. Borsanyi, J. Danzer, Z. Fodor, C. Gattringer, A. Schmidt, J. Phys. Conf. Ser. 312, 012005, 2011

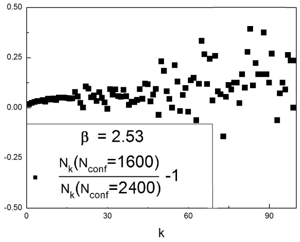
- Statistical weight of the largest cluster is smooth in presence of quarks



How to recognize phase transition? Can small clusters help?

Numerical simulation of distributions

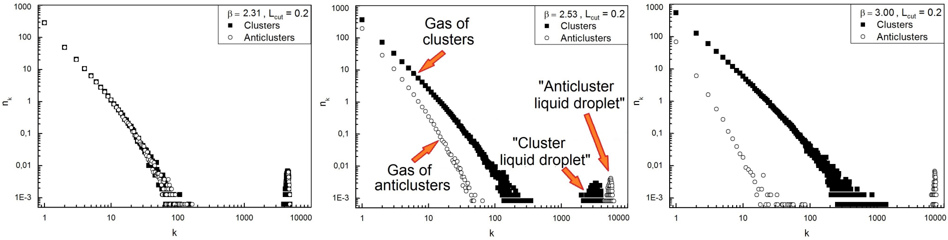
- $3+1$ dimensional lattice of size $N_\sigma = 24$, $N_\tau = 8$
- 13 values of inverse coupling $\beta \in [2.31, 3]$ $\Rightarrow$ 13 values of physical temperature
- vacuum cut-off parameter $L_{cut} = 0.1$ and 0.2
- Average over 800, 1600 and 2400 independent configurations for all β and L_{cut}
  - n_k^{conf} – number of k-mers in given configuration
  - $n_k = \frac{1}{N_{conf}} \sum_{conf} n_k^{conf}$ – average number of k-mers



Saturation of cluster size distributions

$N_{conf} = 2400$ **is taken as the high statistics limit**

Size distributions of (anti)clusters



Distributions at low $\beta \leq \beta_c \simeq 2.52$ (phase of restored global Z(2) symmetry)

- symmetry between (anti)cluster distributions
- gas and “liquid"domains are well separated

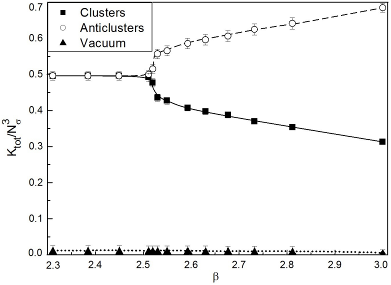
Distributions at high $\beta > \beta_c \simeq 2.52$ (phase of broken global Z(2) symmetry)

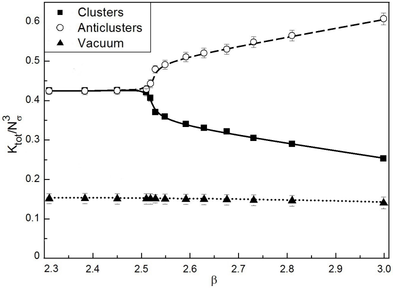
- no symmetry between (anti)cluster distributions
- “cluster liquid"evaporates to cluster gas
- anticluster gas condensates to “anticluster liquid"

Deconfinement phase transition (at least in SU(2) GD)
= special kind of the liquid-gas phase transition?

Volume fraction

$$K_{tot} = \begin{cases} \sum_k k n_k^{(a)Cl} \Big/ N_\sigma^3, & (anti)clusters \\ 1 - K_{tot}^{aCl} - K_{tot}^{Cl}, & auxiliary\ \ vacuum \end{cases}$$





Volume fraction of vacuum is independent on β and/ore temperature

Incompressible auxiliary vacuum?

Liquid droplet approach

- Deconfinement transition in SU(2) gluodynamics is a special kind of the liquid-gas phase transition with two liquids and two gases

Polyakov loop clusters ⇔ droplets?

- Liquid droplet formula for average number of (anti)clusters of size k

first introduced in M.E. Fisher, Physics 3, 255 (1967)

$$n_{k \geq k_{min}} = C \exp\left(\nu k - \sigma k^{2/3} - \tau \ln k\right)$$

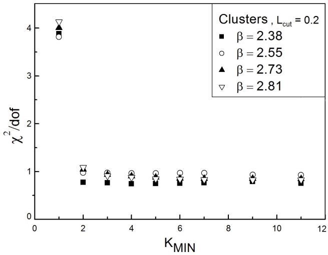
  - C - normalization factor (absolute amount)
  - ν - reduced chemical potential (liquid-gas phase transition)
  - σ - reduced surface tension coefficient (appearance of critical point)
  - τ - Fisher topological exponent
    (size distribution at critical point and critical exponents)
  - k_{min} - size of the minimal (anti)cluster described by the liquid droplet formula

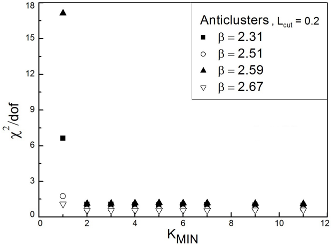
Applicability of the Liquid Droplet formula

- Too small (anti)clusters can not be treated as droplets

  What is minimal size of (anti)clusters which are described by the LDF?

- Overall χ^2/dof calculated for distributions of clusters with size $k \geq k_{min}$





- LDF describes size distributions with almost the same quality for all $k_{min} \geq 2$

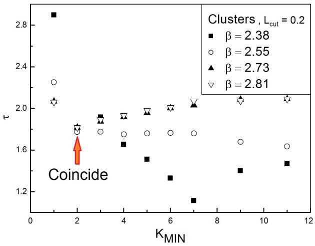
How to define k_{min}?

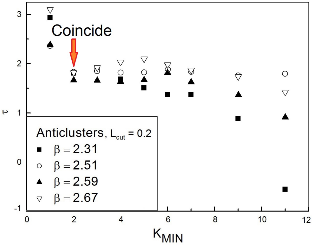
Determination of k_{min} and τ

- Fisher topological exponent τ is supposed to be constant

  M.E. Fisher, Physics 3, 255 (1967)

- Fit of distributions of clusters with size $k \geq k_{min}$





- $\tau = 1.806 \pm 0.008$ is independent on temperature at $k_{min} = 2$
  Found value of τ agrees with exactly solvable statistical models for:
  - nuclear matter

    P. T. Reuter, K. A. Bugaev, Phys. Let. B 517, 233 (2001); Ukr. J. Phys. 52, 489 (2007).
  - tricritcal endpoint of QCD

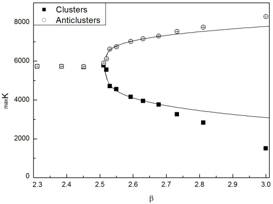
    OI and and K. A. Bugaev, Ukr. J. Phys. 57, (2012) 964

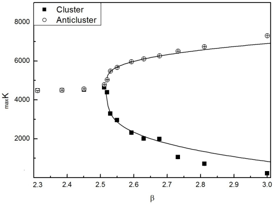
Average maximal (anti)cluster

- **Average Polyakov loop** is SU(2) gluodynamics order parameter, not observable
- **Largest (anti)cluster** occupies almost all lattice $\Rightarrow |L| \sim \max K_{aCl} - \max K_{Cl}$

$$\max K = \sum_{\vec{x}} k^{1+\tau} n_k \Big/ \sum_{\vec{x}} k^{\tau} n_k$$

$$\beta > \beta_c : \ \max K(\beta) - \max K(\beta_c) = a \cdot (\beta_c - \beta)^b$$



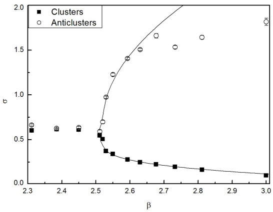


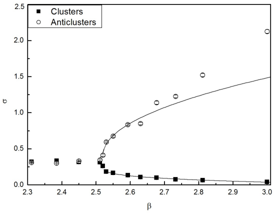
L_{cut}	type	a	b	χ^2/dof
0.1	Cl	-3056 ± 246	0.2964 ± 0.0284	$16.32/4 \simeq 4.08$
0.1	aCl	2129 ± 160	0.3315 ± 0.0269	$8.94/4 \simeq 2.235$
0.2	Cl	-4953 ± 443	0.3359 ± 0.0289	$12.3/3 \simeq 4.01$
0.2	aCl	2462 ± 87.7	0.3750 ± 0.0129	$2.068/4 \simeq 0.517$

Exponent b coinside with b_{Ising} of the Ising model - Svetitsky-Jaffe conjecture

Reduced surface tension coefficient

$$\beta > \beta_c : \ \sigma(\beta) - \sigma(\beta_c) = d \cdot (\beta_c - \beta)^B$$





L_{cut}	type	d	B	χ^2/dof
0.1	Cl	-0.485 ± 0.014	0.2920 ± 0.0012	$1.43/4 \simeq 0.36$
0.1	aCl	2.059 ± 0.028	0.4129 ± 0.0077	$1.68/4 \simeq 0.48$
0.2	Cl	-0.2796 ± 0.0118	0.2891 ± 0.0016	$1.11/4 \simeq 0.28$
0.2	aCl	1.344 ± 0.033	0.4483 ± 0.0021	$0.66/2 \simeq 0.33$

$$|L| \sim \max K_{aCl} - \max K_{Cl} \sim (\sigma_c - \sigma_{Cl})^{b/B_{Cl}} - (\sigma_{aCl} - \sigma_c)^{b/B_{aCl}}$$

Reduced surface tension coefficient - order parameter

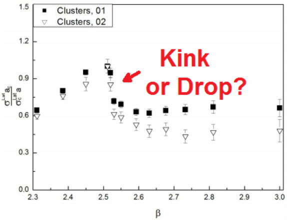
Physical surface tension of (anti)clusters

- Reduced surface tension σ enters LDF as part of the Boltzmann exponential

$$n_k \sim e^{\cdots - \sigma k^{2/3} - \cdots}, \qquad \sigma k^{2/3} = \frac{\sigma_{phys} (a^3 k)^{2/3}}{T}$$

a – T-dependent lattice spacing

σ_{phys} – physical surface tension

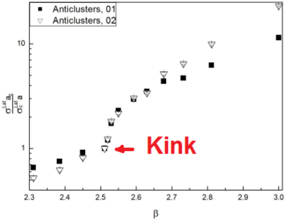


- Physical surface tension of gluonic tube is related to tension of color string $\Rightarrow$ σ_{str} also has a kink
  K. Bugaev, A.I. Ivanytskyi et al., Yad. Fys. 75, 6, 757 -759 (2012)

- Kink of σ_{str} is observed in QC_2D with quarks
  See talk of V. Braguta on "LFT and QCD", Dubna 2017

Irregularity of surface tension is not washed out in presence of quarks?

New order parameter?



- Kink of the surface tension coefficient is responsible for generation of CEP (not 3CEP!) in exactly solvable cluster model
  K.A.Bugaev, V. K. Petrov, G. M. Zinovjev, Yad. Fiz. 76, 2, (2013)

Conclusion

- The approach to study the properties of geometrical clusters is developed
- Novel interpretation of the deconfinement phase transition as the condensation/evaporation of large anticluster/cluster "liquid droplet"is proposed
- It is shown that the reduced surface tension of (anti)clusters is able to distinguish the phases of restored and broken Z(2) global symmetry and can serve as an order parameter

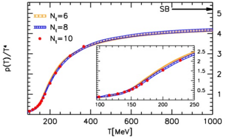
Thank you for your attention

QGP is the most perfect fluid

- **QGP is not asymptotically free gas**:

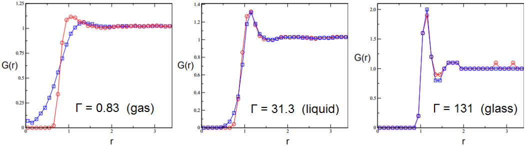
  There is no Stefan-Boltzmann limit
  of noninteracting massless particles
  at hight temperatures

  interaction is strong $\Rightarrow$ no gas regime



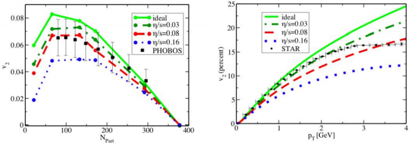
Sz. Borsanyi et al., JHEP 1011 (2010) 077

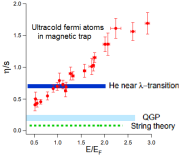
- **Plasma parameter of QGP**: $\Gamma_{QGP} \equiv \frac{\langle E_{potential} \rangle}{\langle E_{kinetic} \rangle} \simeq 10 - 100$ – liquid



B. A. Gelman, E. V. Shuryak and I. Zahed, Phys. Rev. C 74, 044908 (2006)

- **Shear viscosity η to entropy density s** (experimental data prefer $\frac{\eta}{s} \simeq 0.03$)



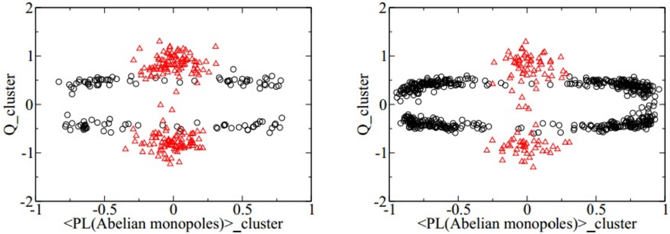


P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007)

E. Shuryak, Prog. Part. Nucl. Phys. 62, 48-101 (2009)

Topology and SU(2) Polyakov loop

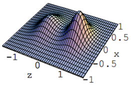
- Semiclassical structure of finite temperature gauge fields for at low temperatures is dominated by calorons with non-trivial holonomy

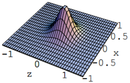


P. Gerhold et al., AIP Conf. Proc. 892, 213-216 (2007)

- KvBLL caloron splits into two abelian dyons L and M

T. C. Kraan and P. van Baal, Phys. Lett. B 428, 268, 268 (1998)
K. Lee and C. Lu, Phys. Rev. D 58, 025011 (1998)





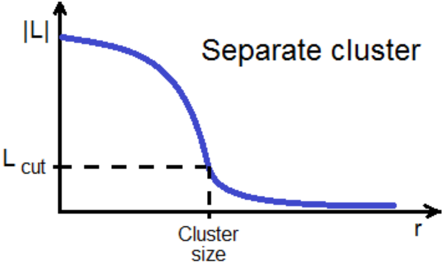
L-dyon: $$L^L = -\cos\left[\bar{\nu}\coth(2\bar{\nu}Tr) - \frac{1}{2rT}\right]$$

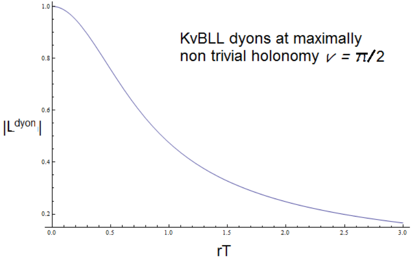
M-dyon: $$L^M = \cos\left[\nu\coth(2\nu Tr) - \frac{1}{2rT}\right]$$

$\bar{\nu} = 1 - \nu$ and $\nu \in [0, \pi]$ – holonomy parameter

Caloron inspired model of the SU(2) Polyakov clusters?

- Polyakon loop has non zero value inside separate cluster ($|L_0| > 0$) while at spatial infinity it approaches zero ($L_\infty = 0$)





Is it possible to represent the Polyakov cluster by ensemble of dyons with $\nu = \frac{\pi}{2}$