Dynamical Thermalization in the Quark-Meson Model

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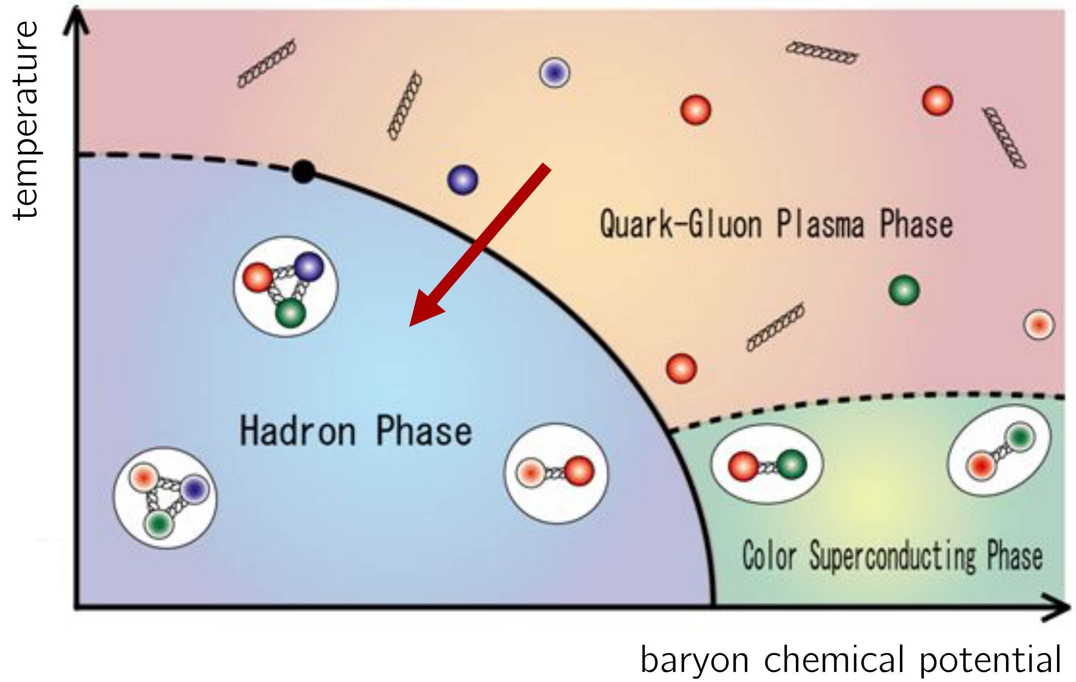
with J. Berges, J. Pawlowski, A. Rothkopf

From heavy ion collisions towards the QCD phase diagram: an equilibration process



Figures from http://wl33.web.rice.edu/images/HI-cartoon.png, http://images.slideplayer.com/25/7893277/slides/slide_3.jpg

- QCD phase diagram: an equilibrium concept
- deconfinement + chiral phase transition

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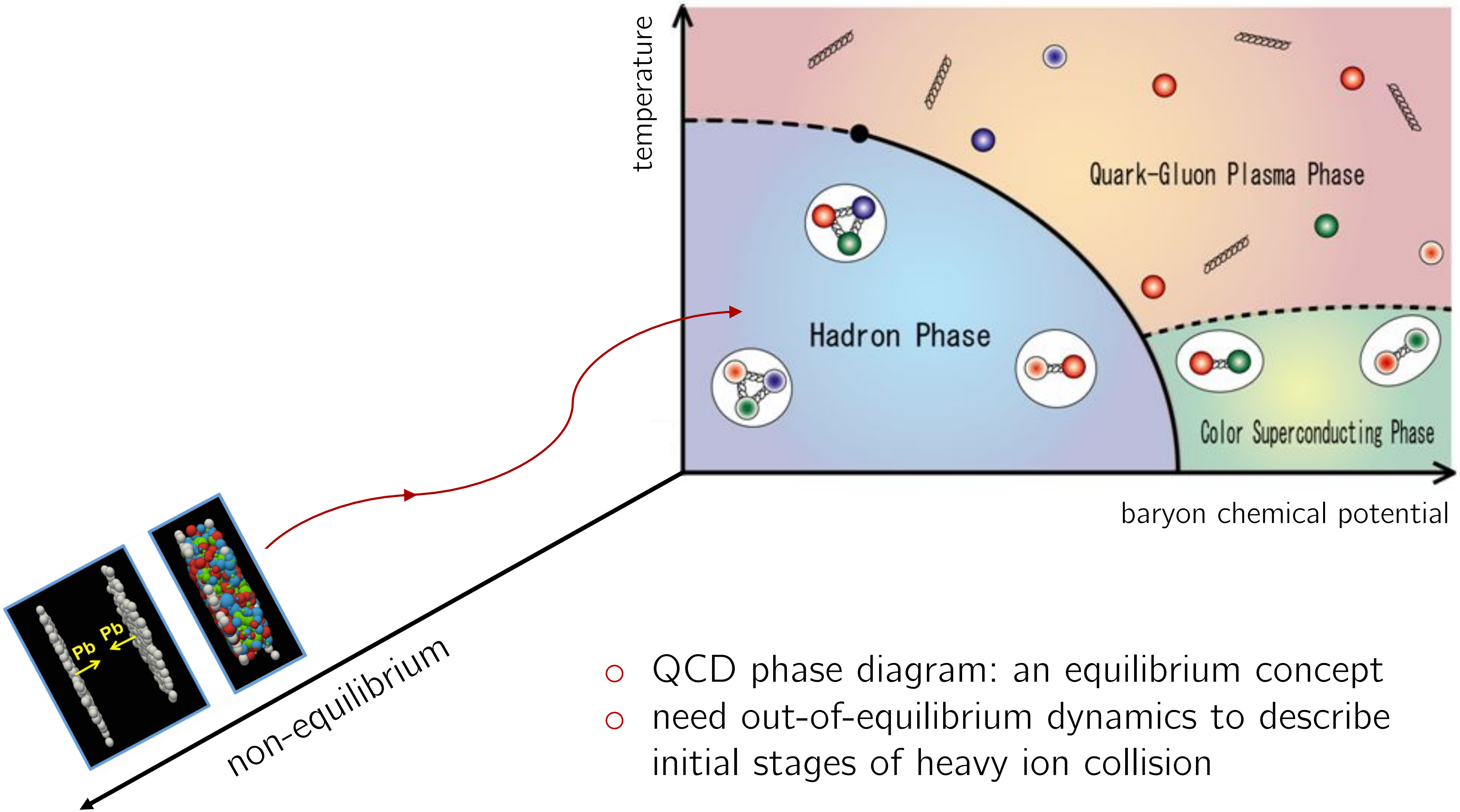

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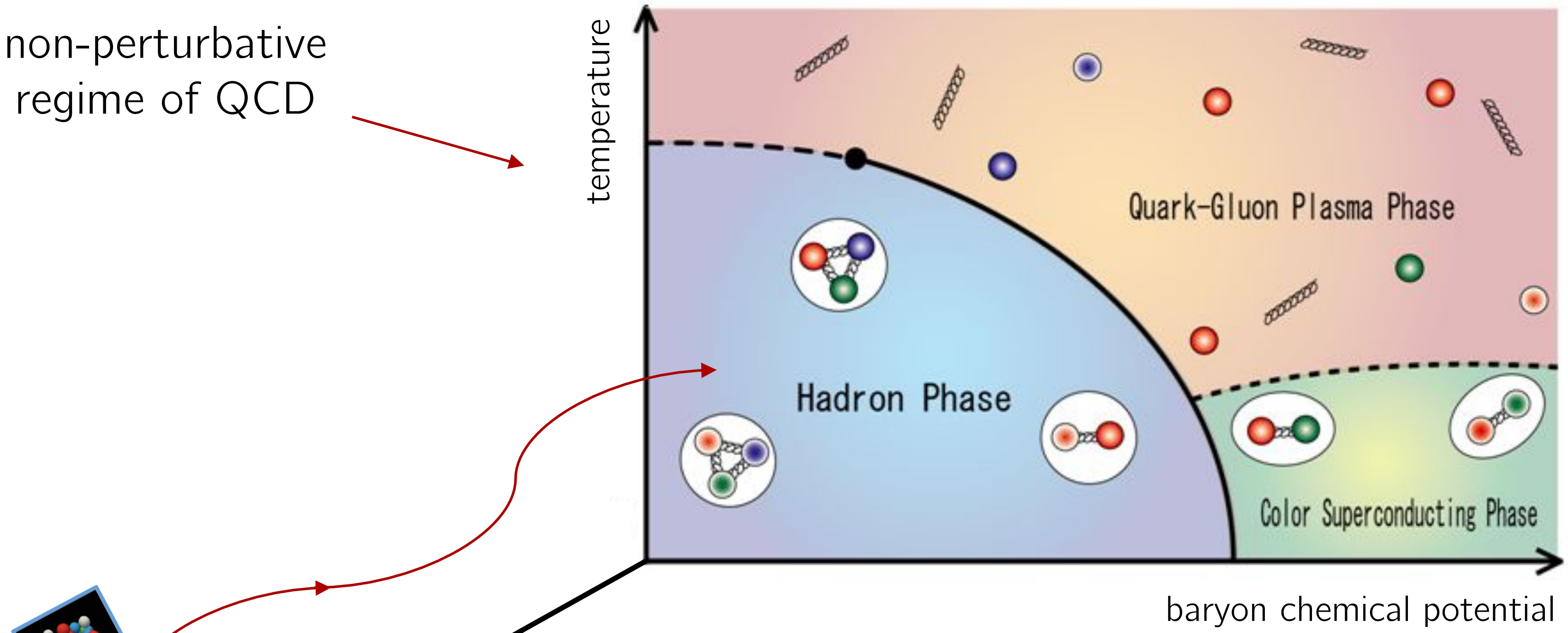

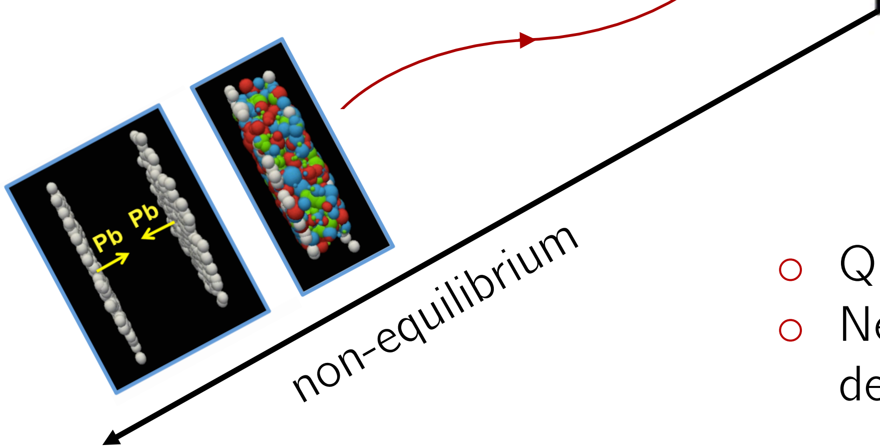

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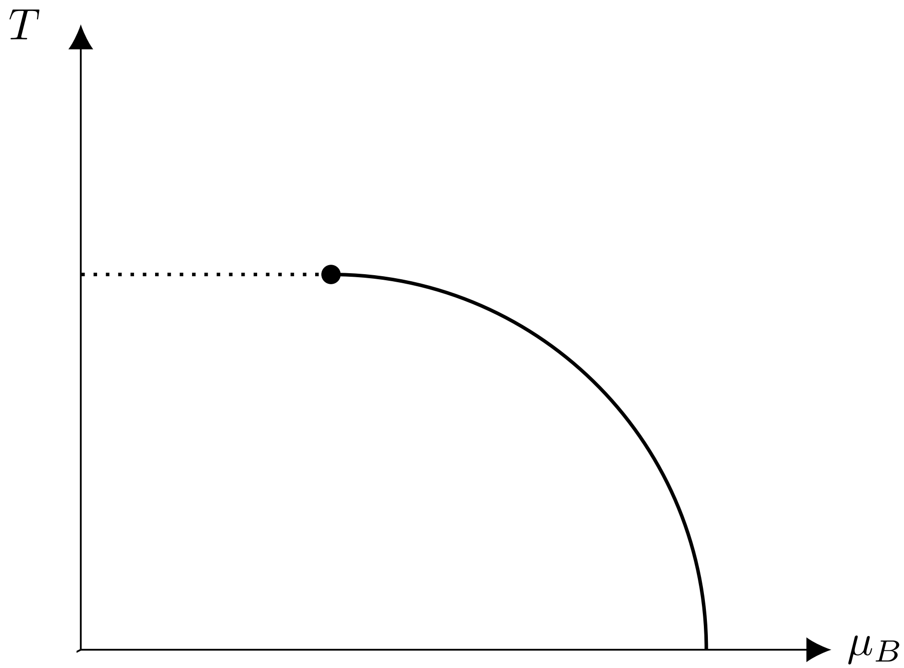
We can investigate this equilibration using effective field theories.

- low-energy QCD



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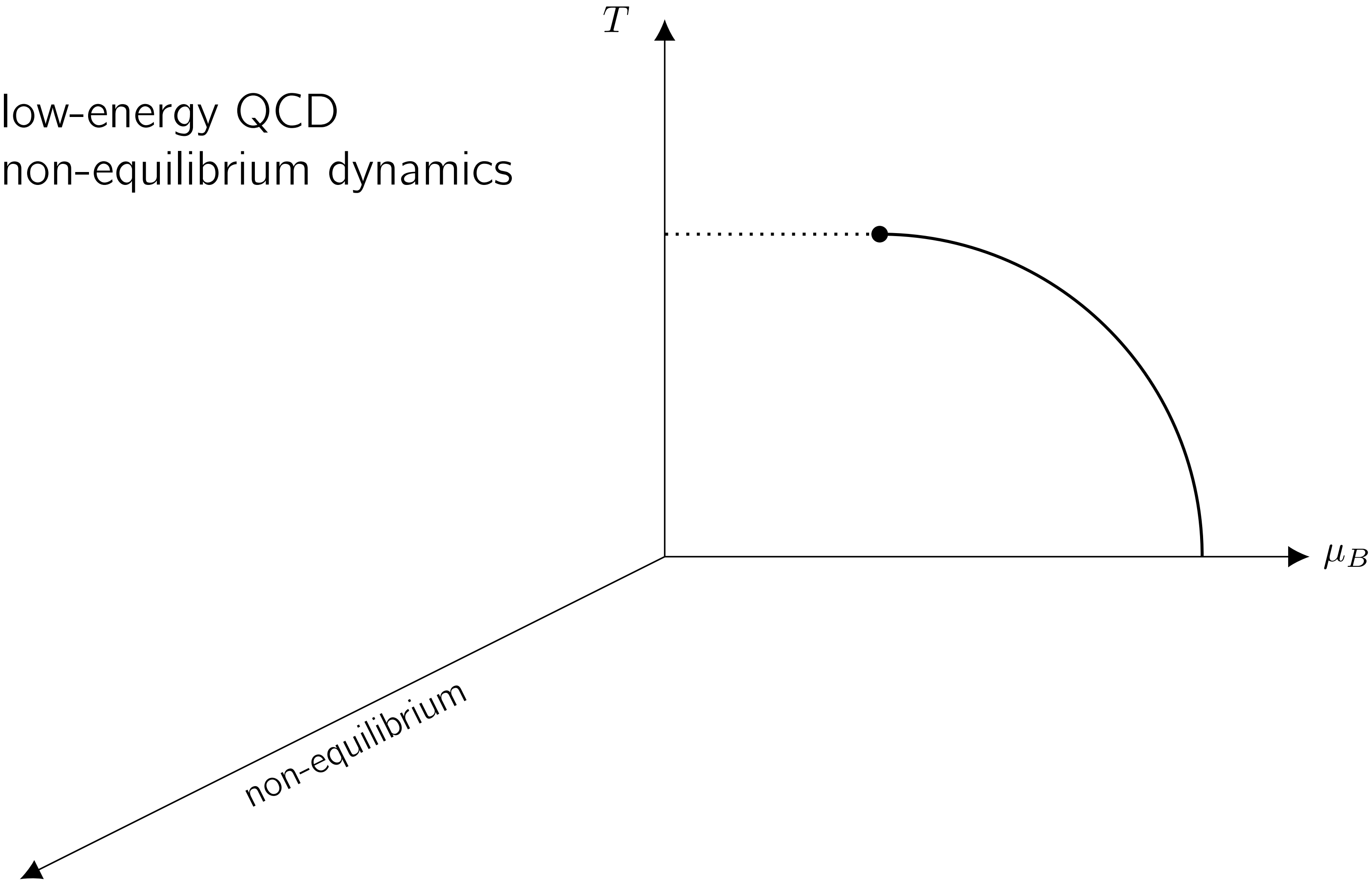

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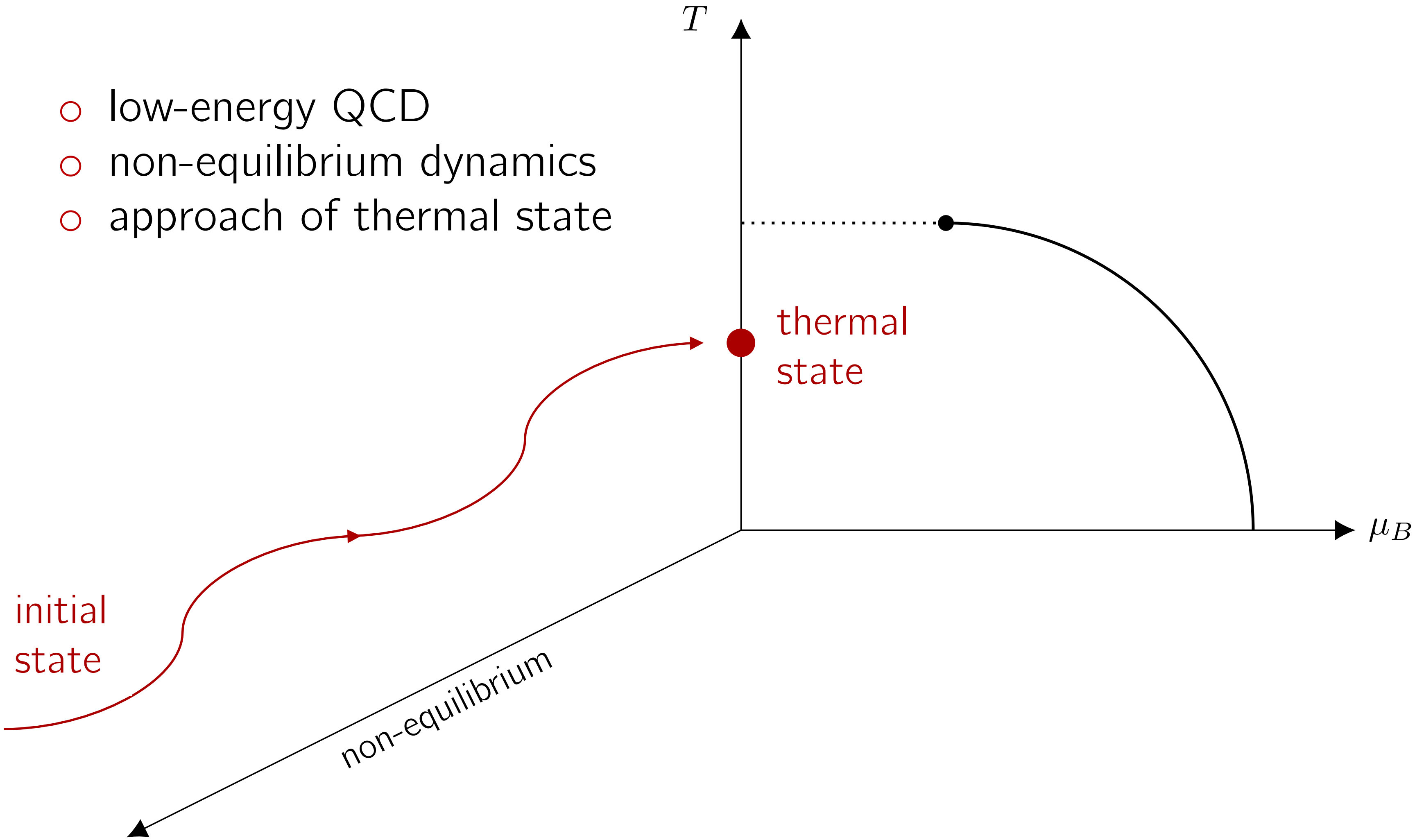

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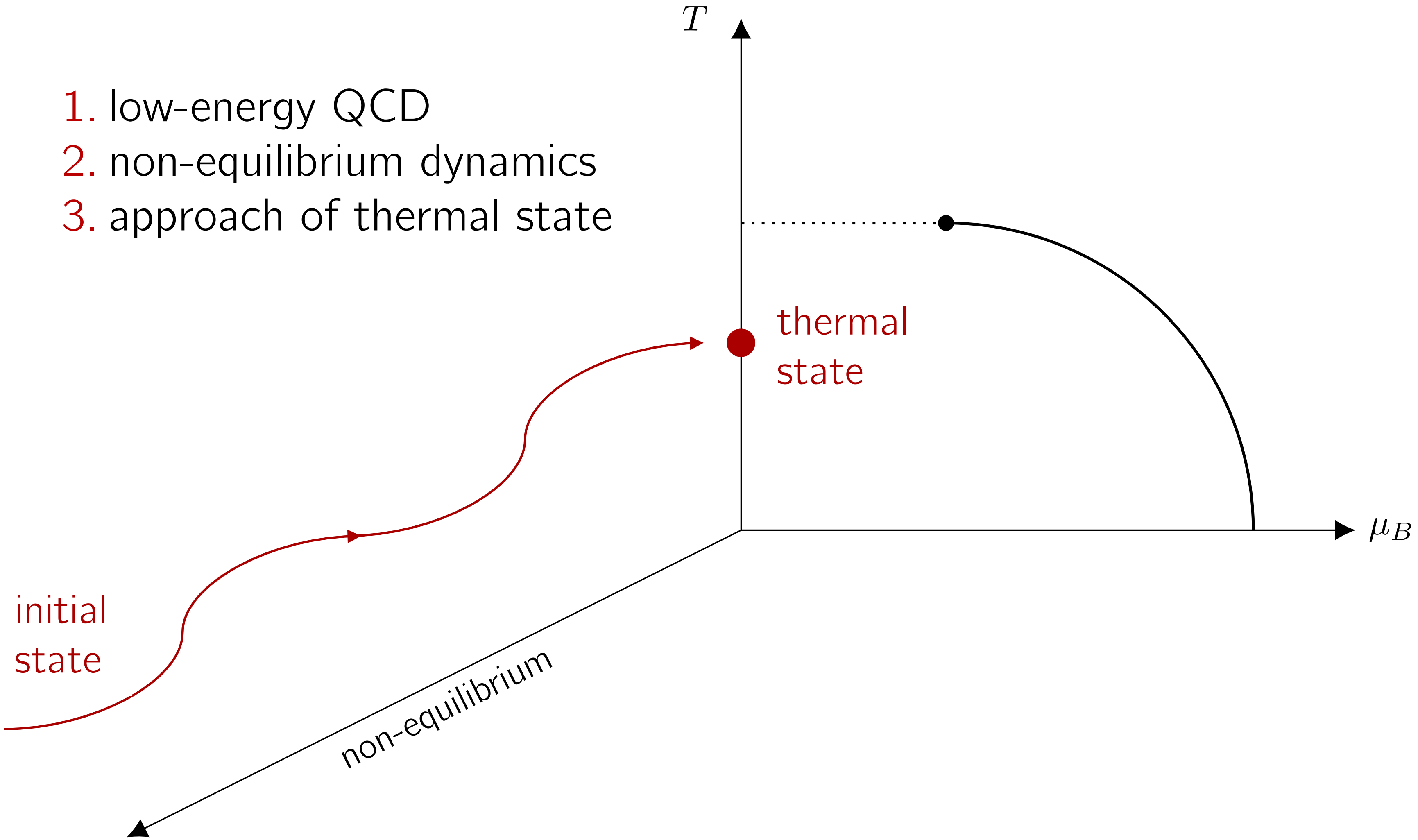
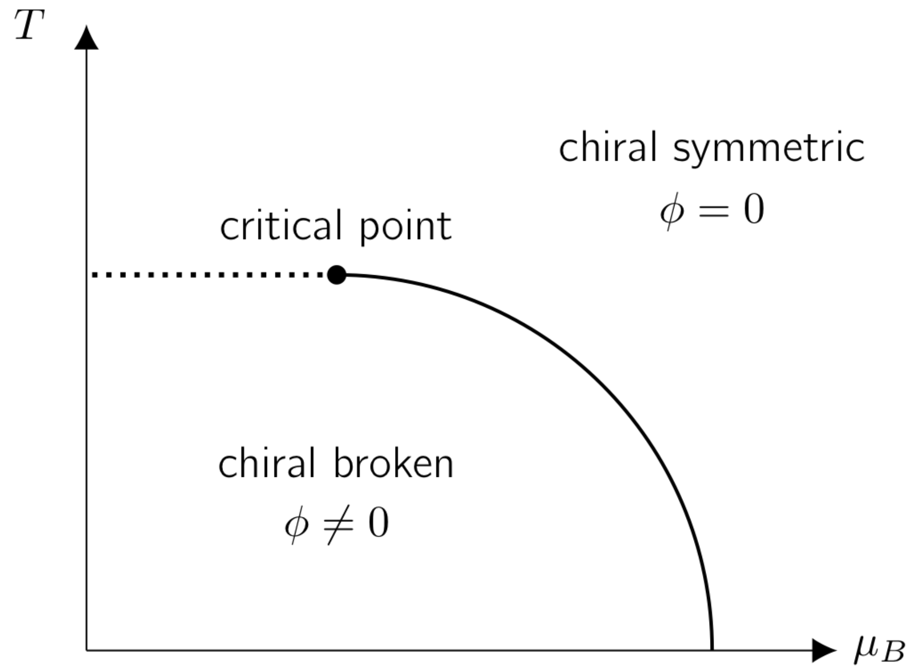


The quark-meson model provides a successful formulation of QCD below scales ~1 GeV.

- d.o.f.: light quarks and mesons
- chiral symmetry breaking
- phase diagram with 1st order & 2nd order/crossover transition



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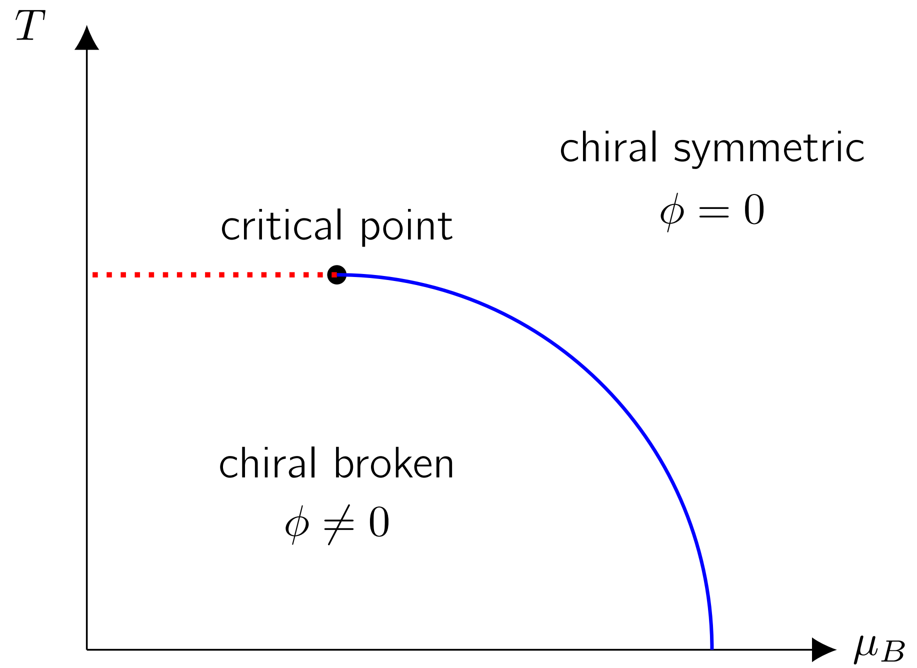

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sigma meson & pions

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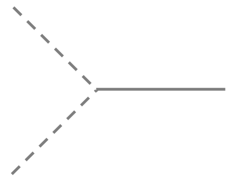

u & d quark

Yukawa coupling

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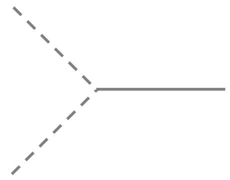

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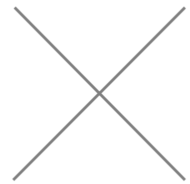
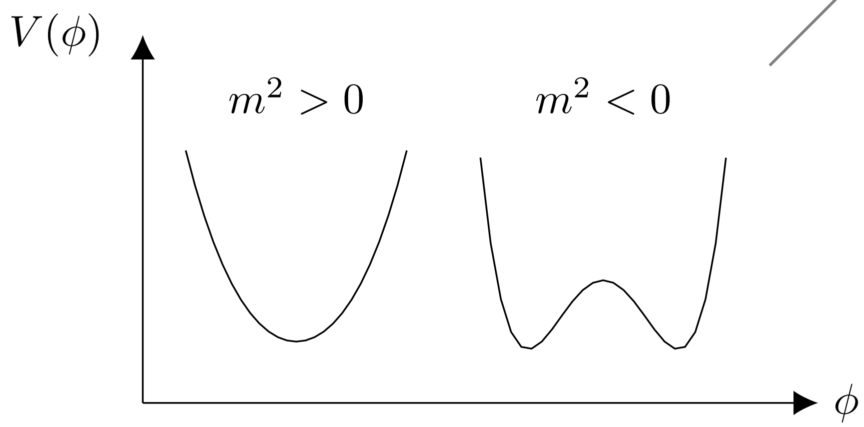
sigma meson & pions   scalar potential





The 2PI effective action is a practical tool to study thermalization.

Berges. AIP Conference Proc. (2004)
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classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

$\downarrow$

2PI effective action
$\Gamma[\phi, G]$

$\downarrow$

evolution equations
for ϕ and G

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non-equilibrium | 2PI effective action $\Gamma[\phi, G]$ | ✓ non-perturbative approximations

$\downarrow$

non-equilibrium | evolution equations for ϕ and G | ✓ non-secular and universal

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double Legendre transf.

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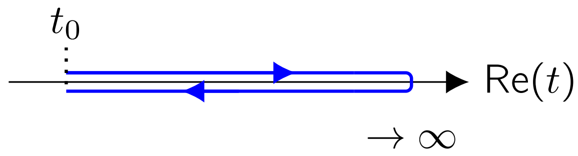
non-equilibrium generating functional for $\rho^{\text{Gauss}}(t_0)$

$$Z[J,R] = e^{iW[J,R]} = \int \mathcal{D}\varphi \; e^{iS[\varphi] + iJ\cdot\varphi + \frac{i}{2}\varphi\cdot R\cdot\varphi}$$

with



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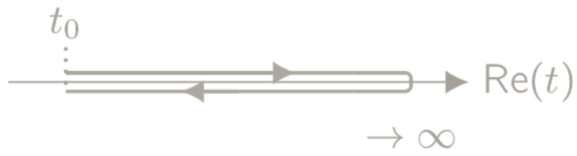

$= S[\phi] +$ 1-loop quantum corrections $+$ 2PI diagrams

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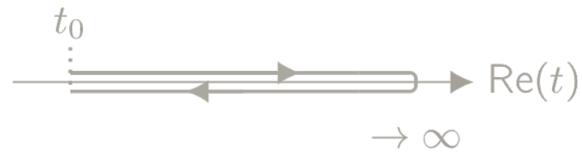

2PI effective action
$\Gamma[\phi, G]$

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large N expansion

$\underbrace{\text{LO diagrams}}_{\sim N^1} + \underbrace{\text{NLO diagrams}}_{\sim N^0} + \; \ldots \; +$ fermion-boson-loop

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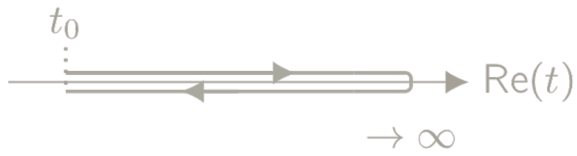

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$$\left[\partial_t^2 + M^2(x;\phi)\right]\phi(t) = \text{fermion backreaction + 2PI corrections}$$

$$\left[\Box_x + M^2(x;\phi)\right] G(x,y) = -i \int_z \left[\Sigma(x,z;\phi,G)\right] G(z,y) - i\delta(x-y)$$

effective mass   self-energy

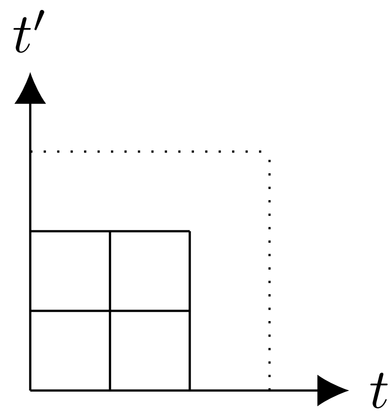
Numerical solution of the equations of motion

- symmetries: spatial homogeneity & isotropy

- propagator decomposition:

$$G(x,y) = \underbrace{F(x,y)}_{\text{statistical function}} + \frac{i}{2}\underbrace{\rho(x,y)}_{\text{spectral function}}\,\mathrm{sgn}(x^0 - y^0)$$

- iterative real-time evolution



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statistical function   spectral function

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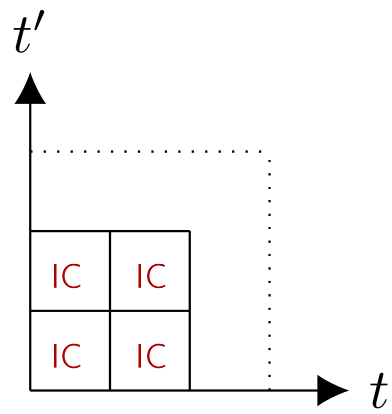

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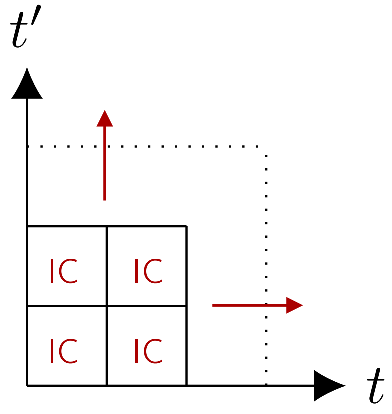
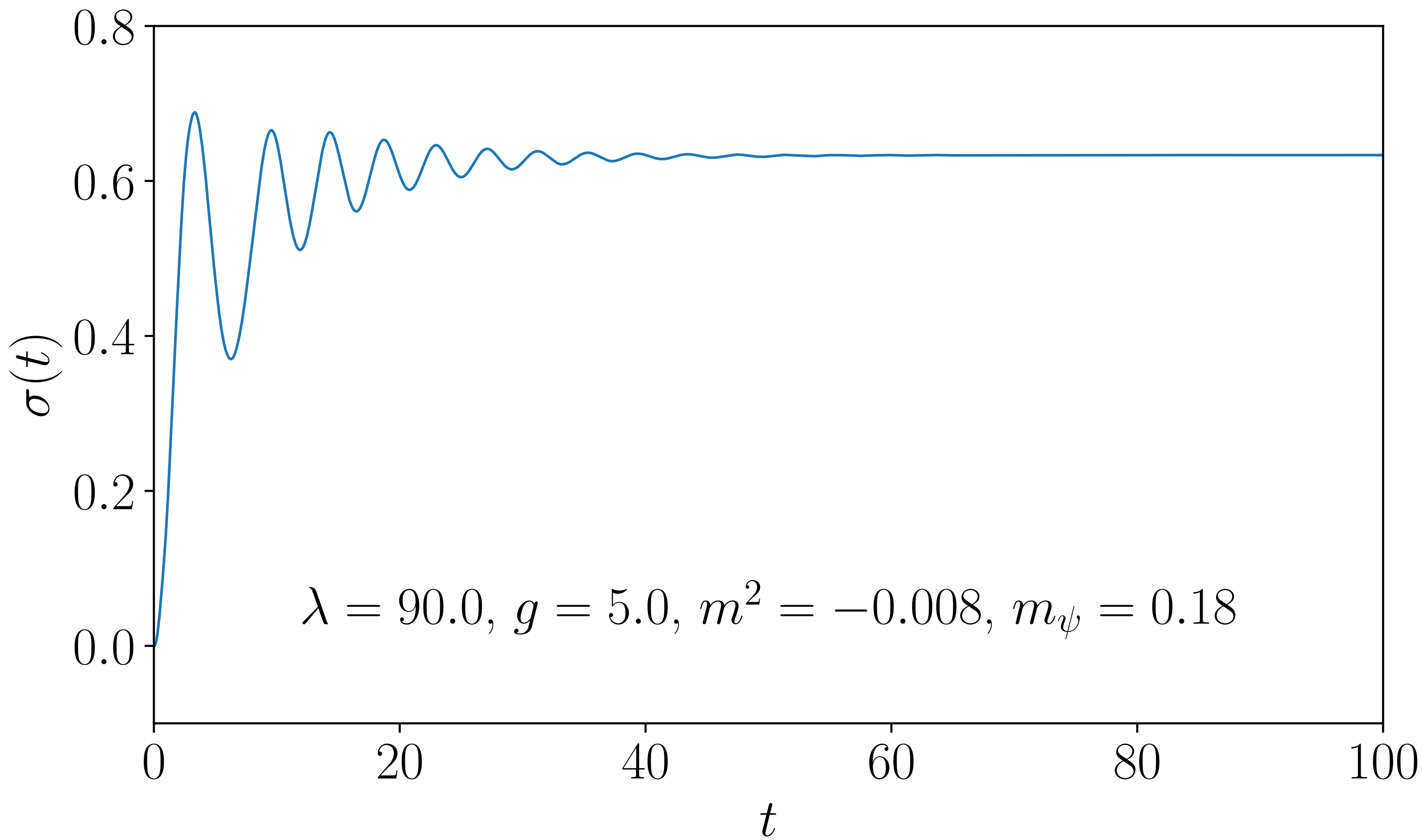


Real-time evolution of the macroscopic field



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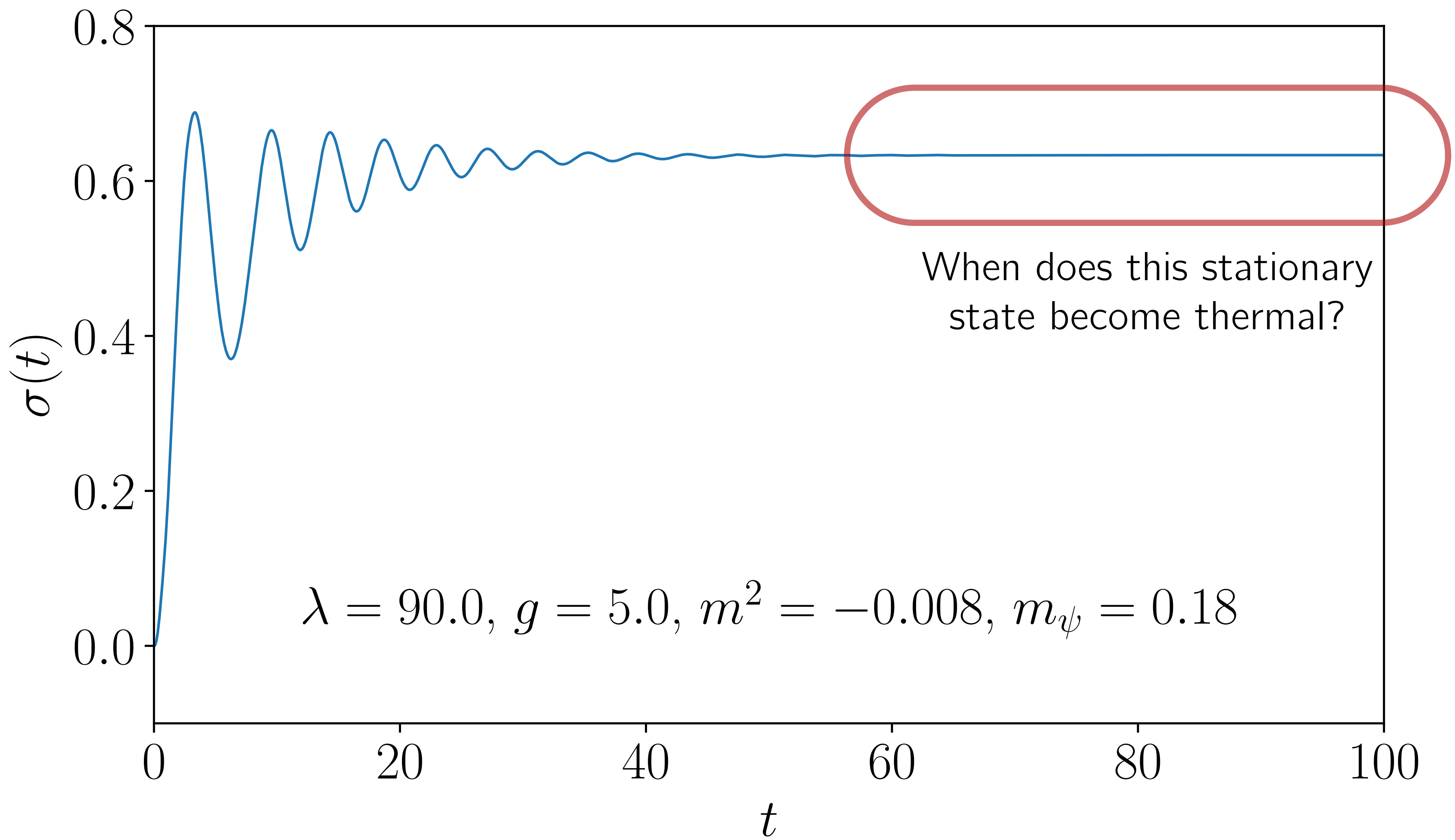


(1) Thermal equilibrium is a time-translation invariant state.

- time-translation invariance implies

$$G(t, t', |\mathbf{p}|) \rightsquigarrow G(\omega, |\mathbf{p}|)$$

in general depending on $t+t'$ and $t-t'$

independent of $t+t'$

here is something

- temporal Wigner transformation:

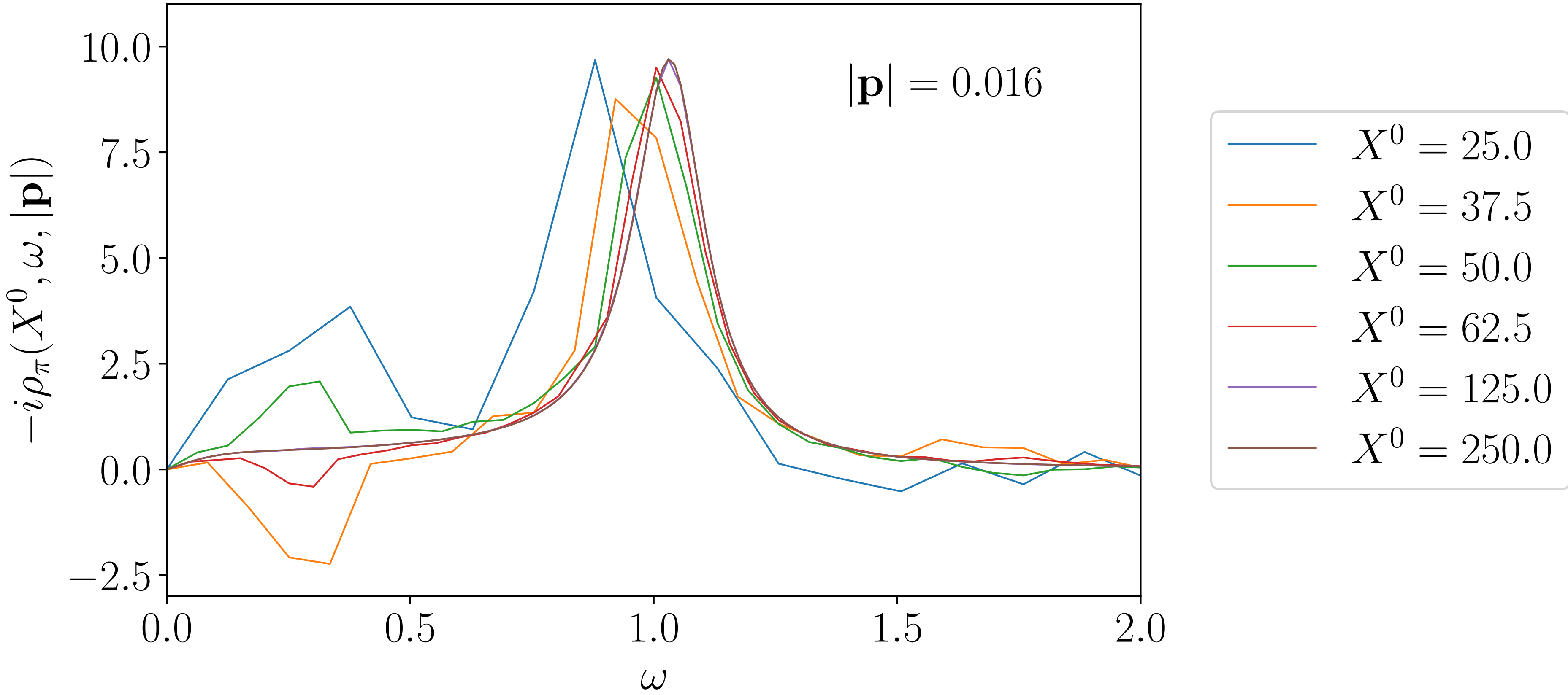
$$\rho(t, t', |\mathbf{p}|) \;\to\; \rho(X^0, \omega, |\mathbf{p}|)$$

center-of-mass time $X^0 = \frac{t+t'}{2}$

frequency

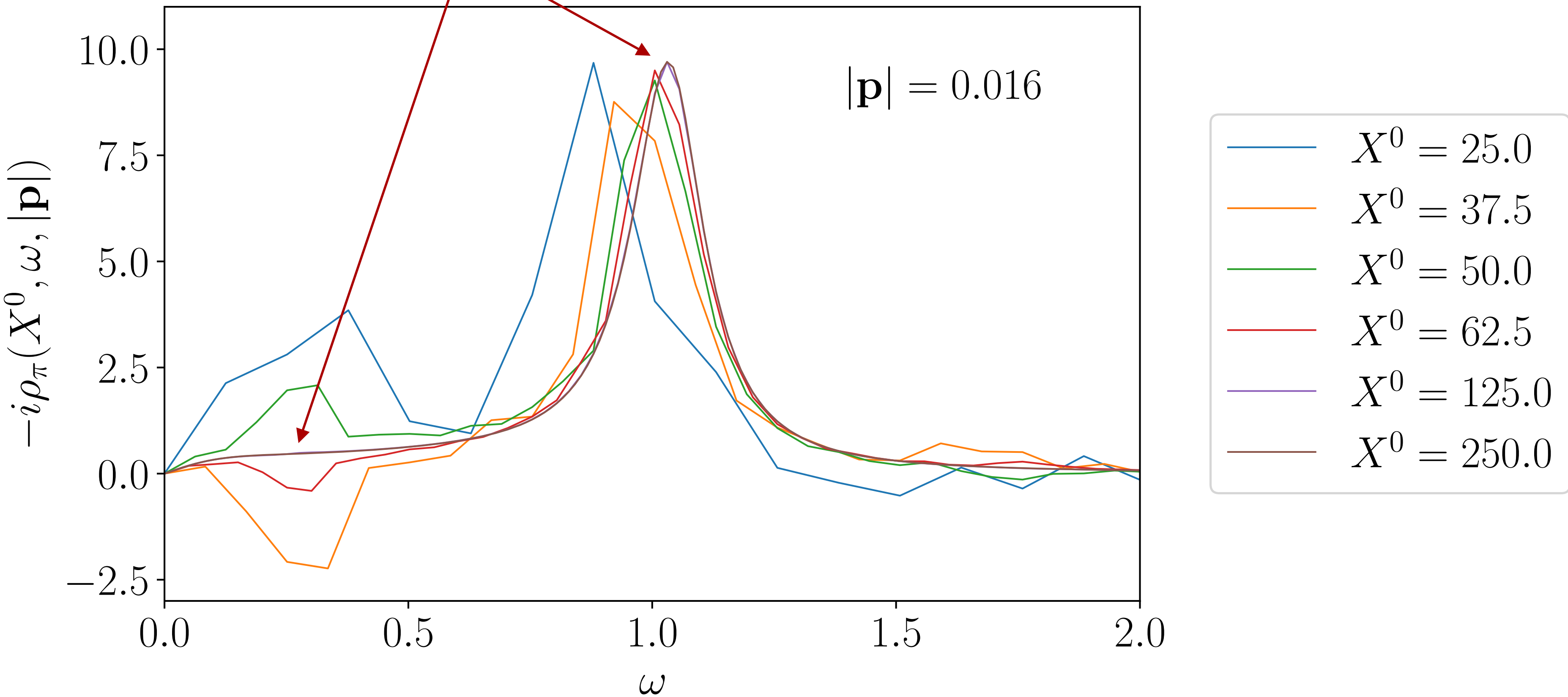
$X^0 = \frac{t+t'}{2}$

The two-point functions become time-translation invariant.



The two-point functions become time-translation invariant.

constant in $X^0 \sim t + t'$ for $X^0 \gtrsim 150$



(2) Thermal eq. as state with thermal particle distributions.

- thermal initial density matrix implies fluctuation-dissipation relation:

$$F_{\mathrm{eq}}(\omega, |\mathbf{p}|) = -i \left(\frac{1}{2} + n_{\mathrm{th}}(\omega) \right) \rho_{\mathrm{eq}}(\omega, |\mathbf{p}|)$$

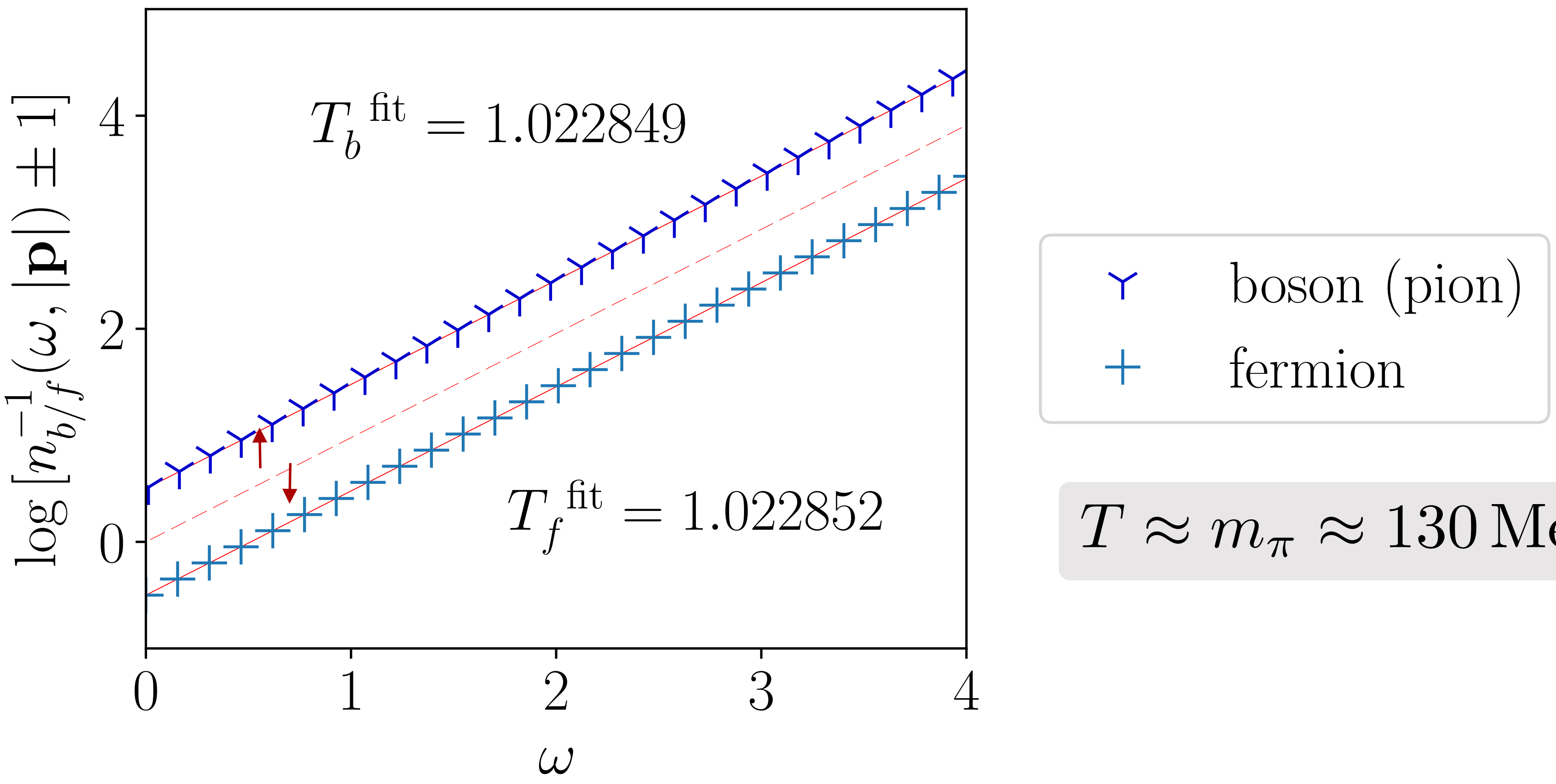
- effective particle number:

$$n(\omega, |\mathbf{p}|) = i \frac{F(\omega, |\mathbf{p}|)}{\rho(\omega, |\mathbf{p}|)} - \frac{1}{2}$$

- in thermal equilibrium:

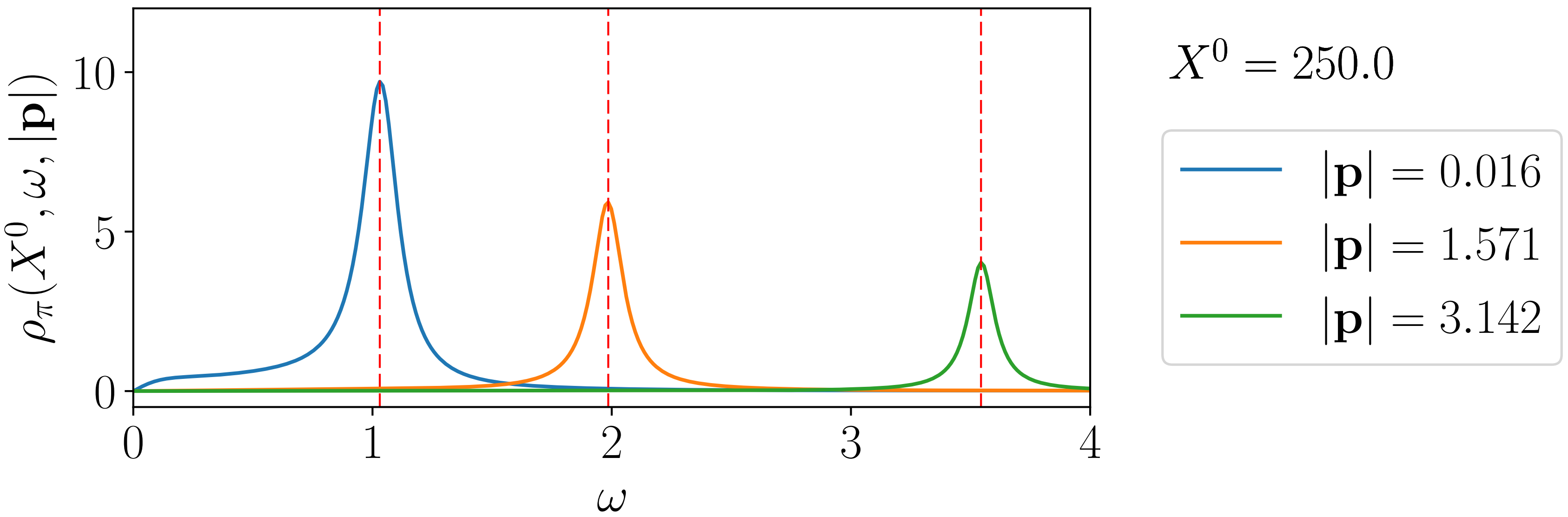
$$n(\omega, |\mathbf{p}|) \to n_{\mathrm{BE/FD}}(\omega) = \frac{1}{e^{\beta\omega} \mp 1} \quad \text{with } \beta = 1/T$$

Determination of the thermalization temperature using the Bose-Einstein and Fermi-Dirac distribution



$$T \approx m_\pi \approx 130\,\text{MeV}$$

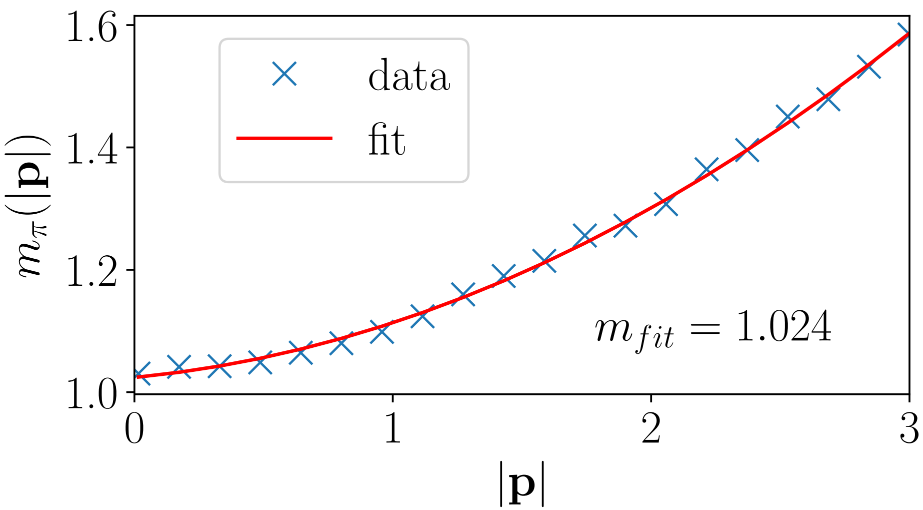
Particle masses from spectral functions by dispersion relation



- particle mass from peak position:

$$m(|\mathbf{p}|) = \sqrt{\omega^2_{\text{peak}}(|\mathbf{p}|^2) - |\mathbf{p}|^2}$$

- physical mass at zero momentum



A step forward
in describing the thermalizing of the QGP in a heavy ion collision

We were able to

- include non-equilibrium dynamics
- observe the approach of thermal equilibrium
- determine the physical mass spectrum

Next steps:

- non-zero baryon-chemical potential
- expanding box size
- scaling behavior around critical point