

Polyakov loop fluctuations in the presence of external fields

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Polyakov loop \rightarrow probes the free energy of a static quark

$$|L| = \exp(-F_Q/T)$$

Pure gauge: deconfinement \rightarrow 1st order

$$T < 270 \text{ MeV} \Rightarrow L = 0, F_Q = \infty$$

$$T > 270 \text{ MeV} \Rightarrow L > 0, F_Q \text{ finite}$$

QCD: deconfinement \rightarrow crossover

- ▶ Polyakov loop \rightarrow approximate order parameter

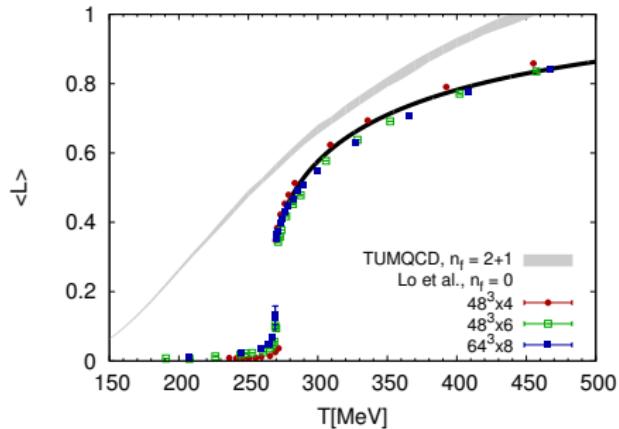


Figure: Polyakov loop in pure gauge²(colored points+black line) and in 2+1 QCD³(gray band)

¹P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

²A. Bazavov, N. Brambilla, H.-T Ding, P. Petreczky, H. -P. Schadler, A. Vairo and J. H. Weber, Phys. Rev. D **93**, 114502 (2016).

Polyakov loop susceptibility → peak and width

$$\chi_A = V \langle |L|^2 \rangle_c = V (\langle |L|^2 \rangle - \langle |L| \rangle^2)$$

In addition

- ▶ Longitudinal (real) susceptibility

$$\chi_L = V \langle (L_L)^2 \rangle_c = V (\langle (L_L)^2 \rangle - \langle L_L \rangle^2)$$

- ▶ Transverse (imaginary) susceptibility

$$\chi_T = V \langle (L_T)^2 \rangle_c = V (\langle (L_T)^2 \rangle - \langle L_T \rangle^2)$$

Ratios of susceptibilities → alternative probes of deconfinement¹

$$R_A = \frac{\chi_A}{\chi_L} \qquad R_T = \frac{\chi_T}{\chi_L}$$

¹ P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

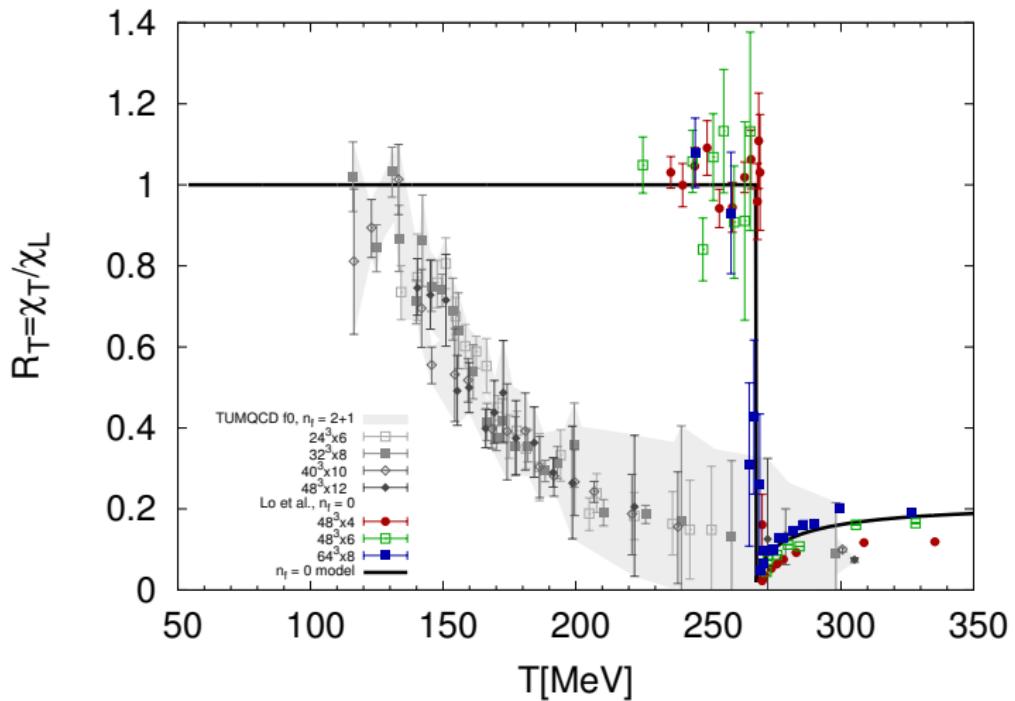


Figure: Lattice data on the R_T ratio in pure gauge¹ (colored points+black line) and 2+1 QCD² (gray band).

¹P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

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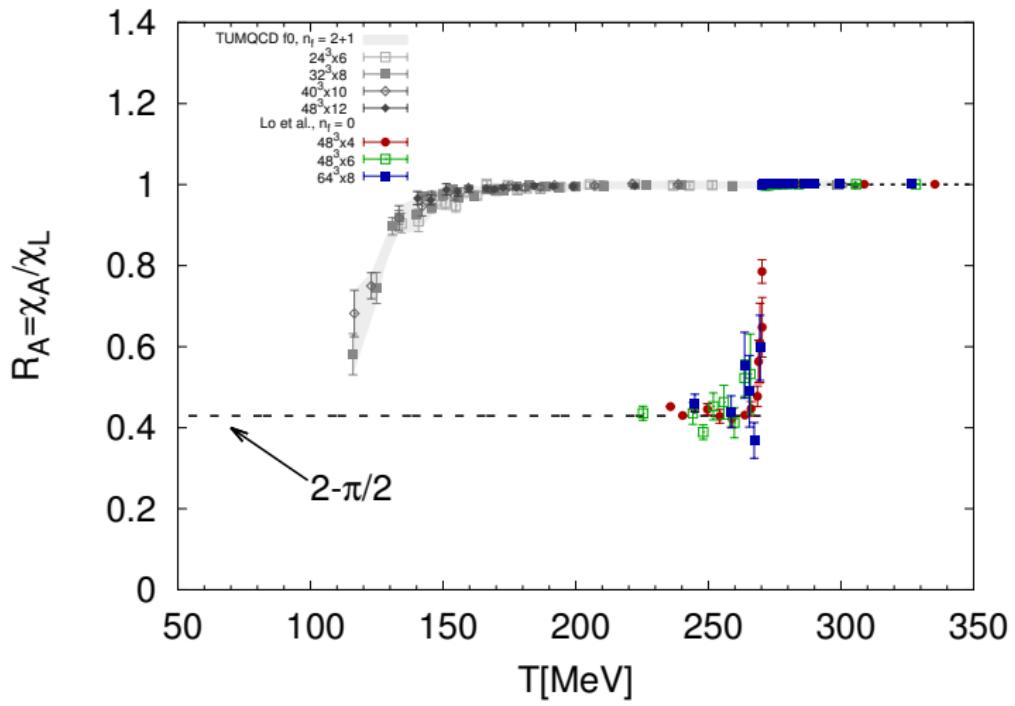


Figure: Lattice data on the R_A ratio in pure gauge¹ (colored points) and 2+1 QCD² (gray band).

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Pure SU(3): $T < 270 \text{ MeV} \rightarrow R_A \approx 0.43$

Gaussian approximation

$$Z = \int dL_L dL_T e^{-\textcolor{blue}{V}\textcolor{red}{U}/T}, \quad \textcolor{red}{U} = \alpha T^4 (L_L^2 + L_T^2),$$

$$\langle O \rangle = \frac{1}{Z} \int dL_L dL_T O(L_L, L_T) e^{-\textcolor{blue}{V}\textcolor{red}{U}/T}$$

Susceptibilities

$$\langle |L|^2 \rangle_c = \left(2 - \frac{\pi}{2}\right) \frac{1}{2\alpha \textcolor{blue}{V} T^3}, \quad \langle (L_{L,T})^2 \rangle_c = \frac{1}{2\alpha \textcolor{blue}{V} T^3}$$

Ratios

$$R_T = 1, \quad R_A = 2 - \frac{\pi}{2} \approx 0.43$$

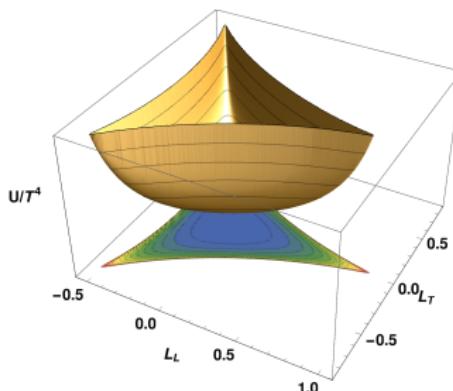
Generalization: to capture details of phase transition

$$\mathcal{U} \rightarrow \mathcal{U} = \mathcal{U}_{Gluon} + \mathcal{U}_{Int}$$

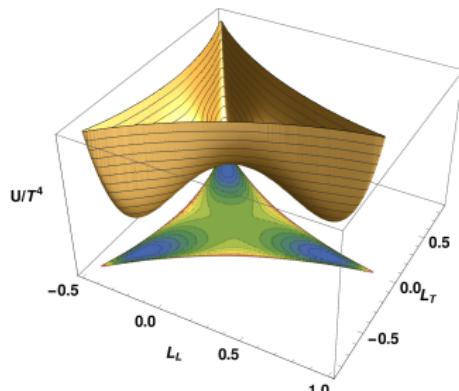
\mathcal{U}_{Gluon} : Pure gauge potential

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)L\bar{L} + b(T)\ln M_H(L, \bar{L}) + \frac{1}{2}c(T)(L^3 + \bar{L}^3) + d(T)(L\bar{L})^2$$

- Reproduces pure gauge P , L , χ_L and χ_T



$T < 270$ MeV



$T > 270$ MeV

Generalization: to capture details of phase transition

$$\mathcal{U} \rightarrow \mathcal{U} = \mathcal{U}_{Gluon} + \mathcal{U}_{Int}$$

\mathcal{U}_{Gluon} : Pure gauge potential

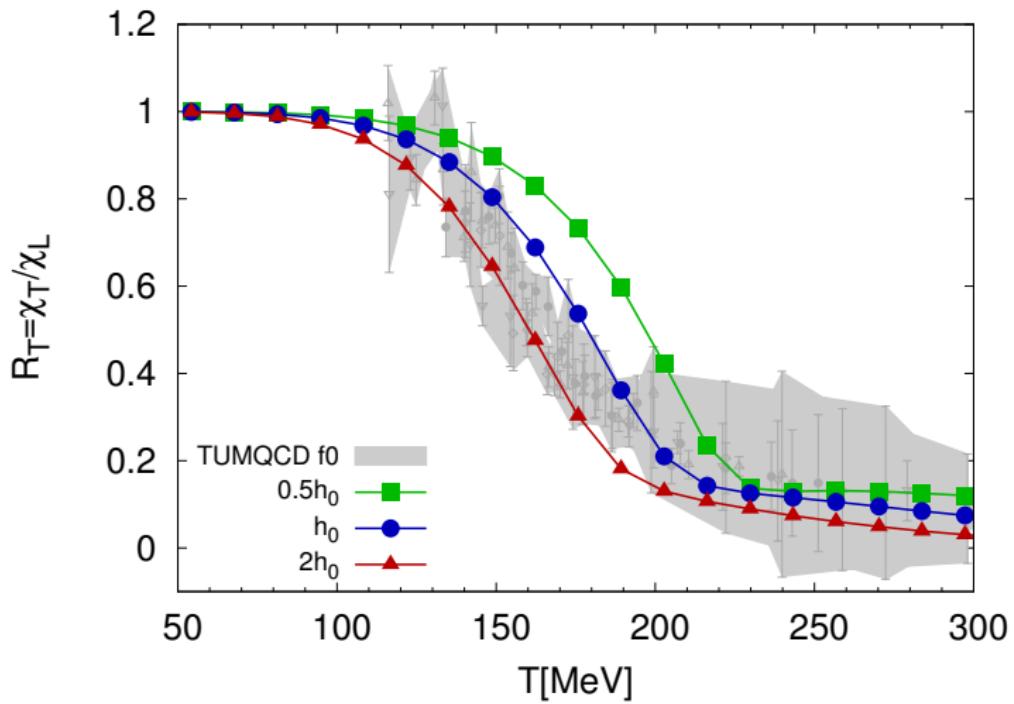
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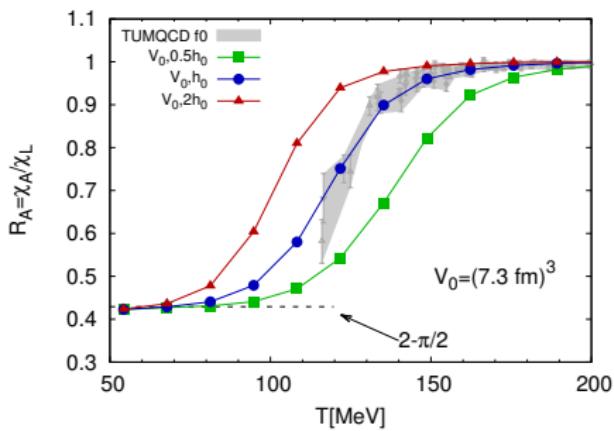
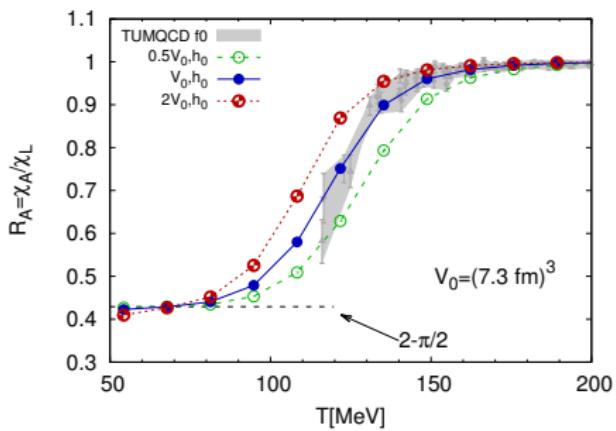
- ▶ Reproduces pure gauge P , L , χ_L and χ_T

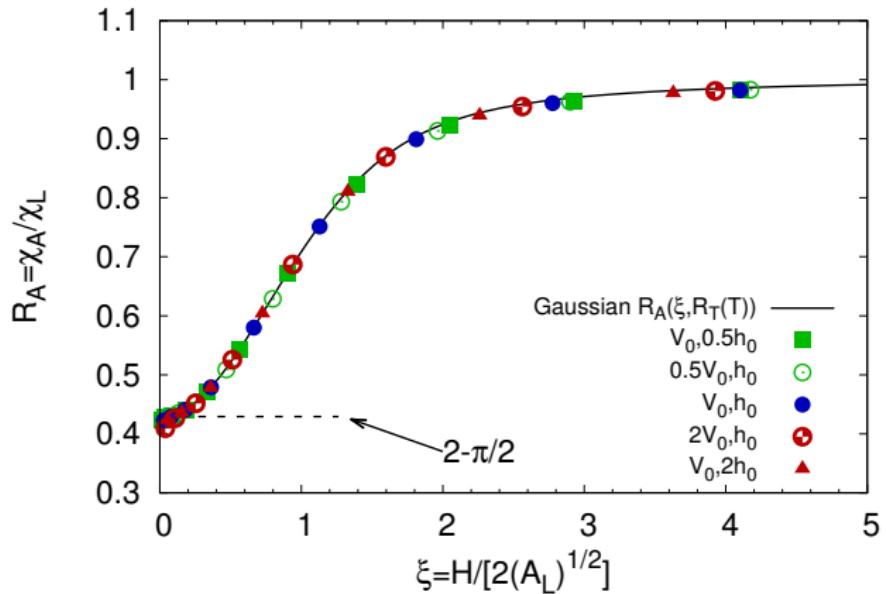
\mathcal{U}_{Int} : Quark-gluon interaction:

$$\frac{\mathcal{U}_{Int}}{T^4} = -\underbrace{[2h(T, m_l) + h(T, m_s)]}_{h_0} L_L, \quad h(T, m_q) = \frac{6m_q^2}{(\pi T)^2} K_2\left(\frac{m_q}{T}\right).$$

- ▶ $m_l = 6$ MeV, $m_s = 20m_l = 120$ MeV







$$\mathcal{A}_L = \frac{VT^3}{2\chi_L(T, h=0)}, \quad \mathcal{H} = VT^3 \left[h + \frac{\langle L \rangle(T, h=0)}{\chi_L(T, h=0)} \right]^{-1}$$

Conclusions

- ▶ Ratios of Polyakov loop are sensitive to deconfinement and strength of explicit symmetry breaking
 - ▶ Useful for constraining effective models
- ▶ The model discussed here captures lattice trends for R_A and R_T ratios
- ▶ We reveal scaling properties of R_A ratio
 - ▶ Possible lattice test?

Appendix

Effective gluon potential \mathcal{U}_G

Parametrization of the effective potential:

- ▶ $a(T)$, $c(T)$ and $d(T)$ of the form:

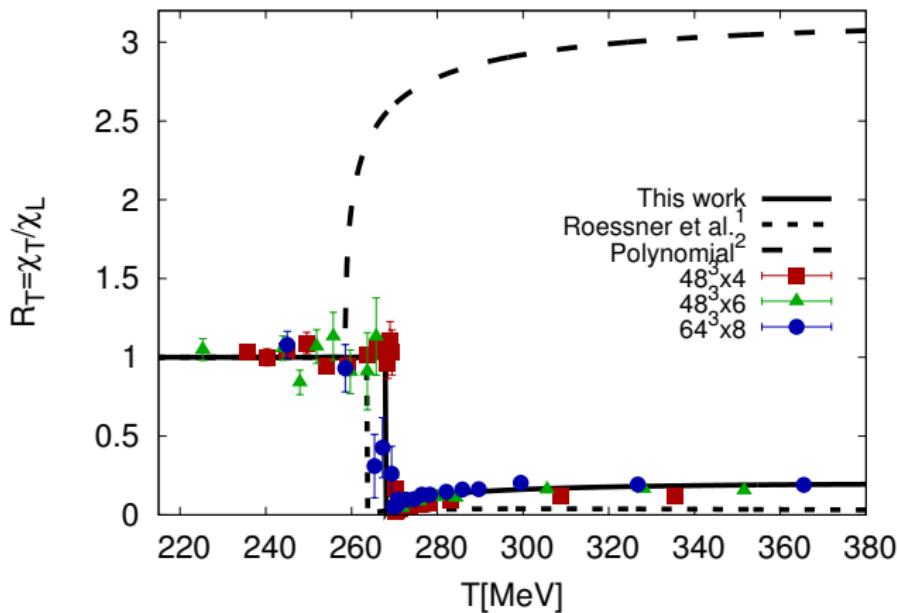
$$x(t \equiv T/T_c) = \frac{x_1 + x_2/t + x_3/t^2}{1 + x_4/t + x_5/t^2}$$

- ▶ $b(T)$ of the form:

$$b(t \equiv T/T_c) = x_1 t^{-x_4} (1 - e^{x_2/t^{x_3}})$$

Parameters x_n can be found in: *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, 074502 (2013)

R_T for different potentials

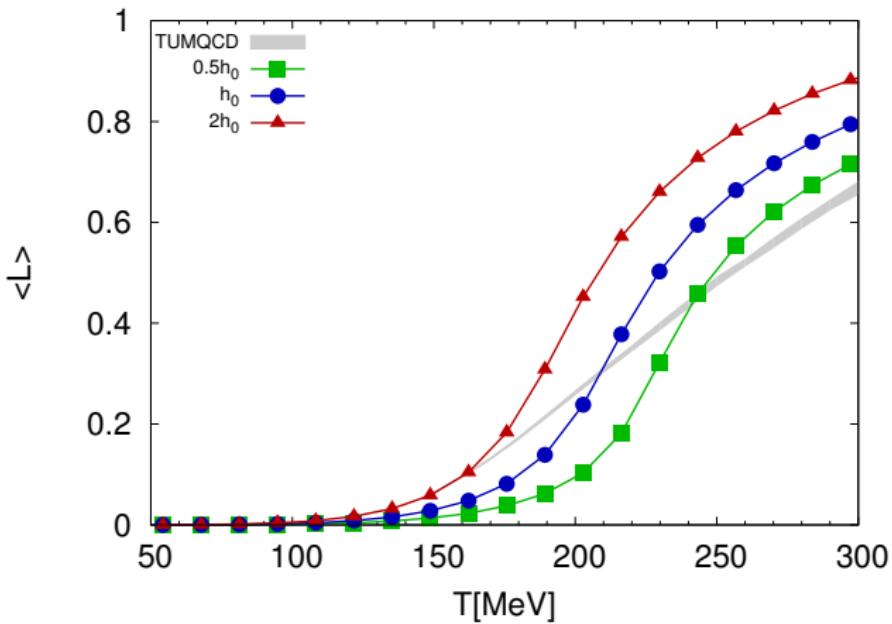


¹ S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75**, 034007 (2007)

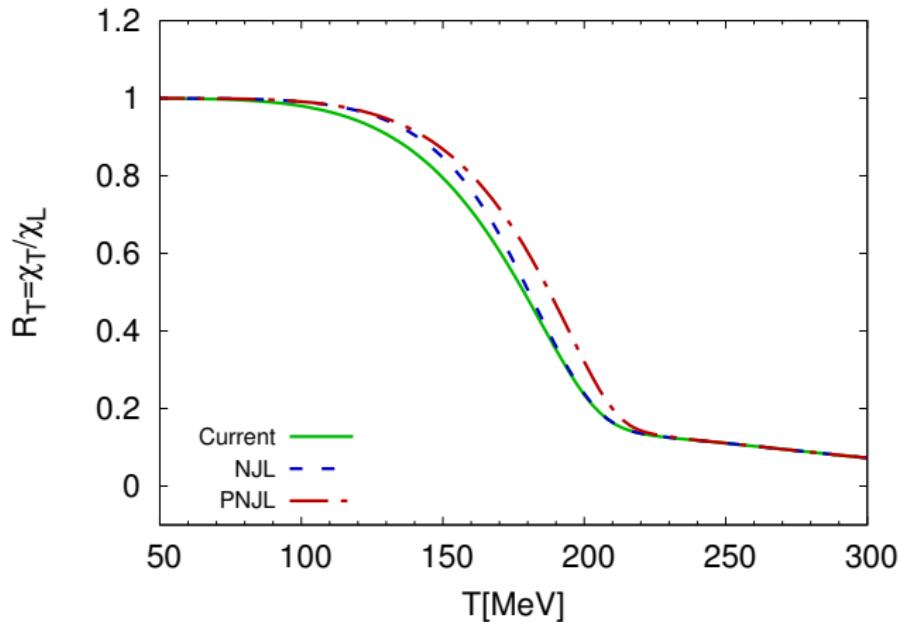
² Potential: C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73**, 014019 (2006),

parametrization: A. V. Friesen, Y. L. Kalinovsky and V. D. Toneev, Int. J. Mod. Phys. A **27**, 1250013 (2012)

Model Polyakov loop



Model R_T for different quark mass profiles



Matching to Gaussian model

The general form of Gaussian distribution function:

$$Z = \int dL_L dL_T \exp \left[-\mathcal{A}_1 L_L^2 - \mathcal{A}_2 L_T^2 + \mathcal{H} L_L \right]$$

- ▶ In this case $R_A(\mathcal{A}_1, \mathcal{A}_2, \mathcal{H}) = R_A(\xi, R_T)$, with

$$\xi = \mathcal{H}/(2\sqrt{\mathcal{A}_1}), \quad R_T = \mathcal{A}_1/\mathcal{A}_2.$$

- ▶ The R_A ratio \rightarrow exact:

$$R_A(\xi, R_T) = 1 + R_T + 2\xi^2 - \frac{2}{\pi^2} R_T e^{-2\xi^2} [\mathcal{I}(\xi, R_T)]^2,$$

$$\mathcal{I}(\xi, R_T) = \int_{-\infty}^{\infty} dx e^{-x^2+2\xi x} \frac{x^2}{2R_T} \left[K_0\left(\frac{x^2}{2R_T}\right) + K_1\left(\frac{x^2}{2R_T}\right) \right]$$

The effective scaling variable $\xi \rightarrow$ Gaussian matching:

$$\mathcal{A}_{1,2} = VT^3 [2\chi_{L,T}(T, h=0)]^{-1}, \quad \mathcal{H} = VT^3 \left[h + \frac{\langle L \rangle(T, h=0)}{\chi_L(T, h=0)} \right]^{-1}$$