Anomalous transport phenomena at finite temperature

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Zimányi Winter School on Heavy Ion Physics
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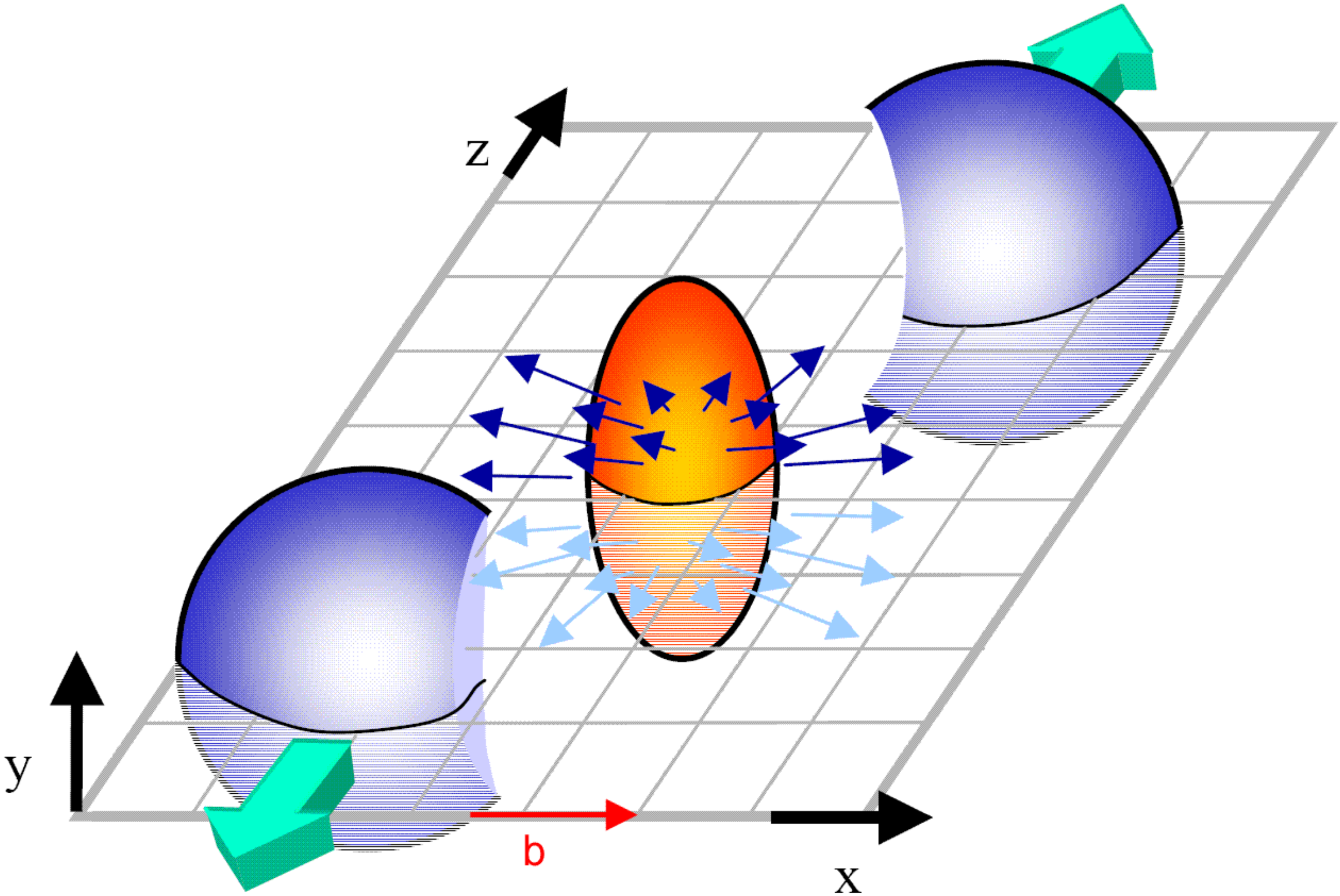
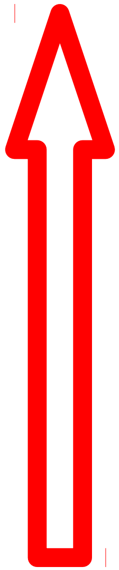
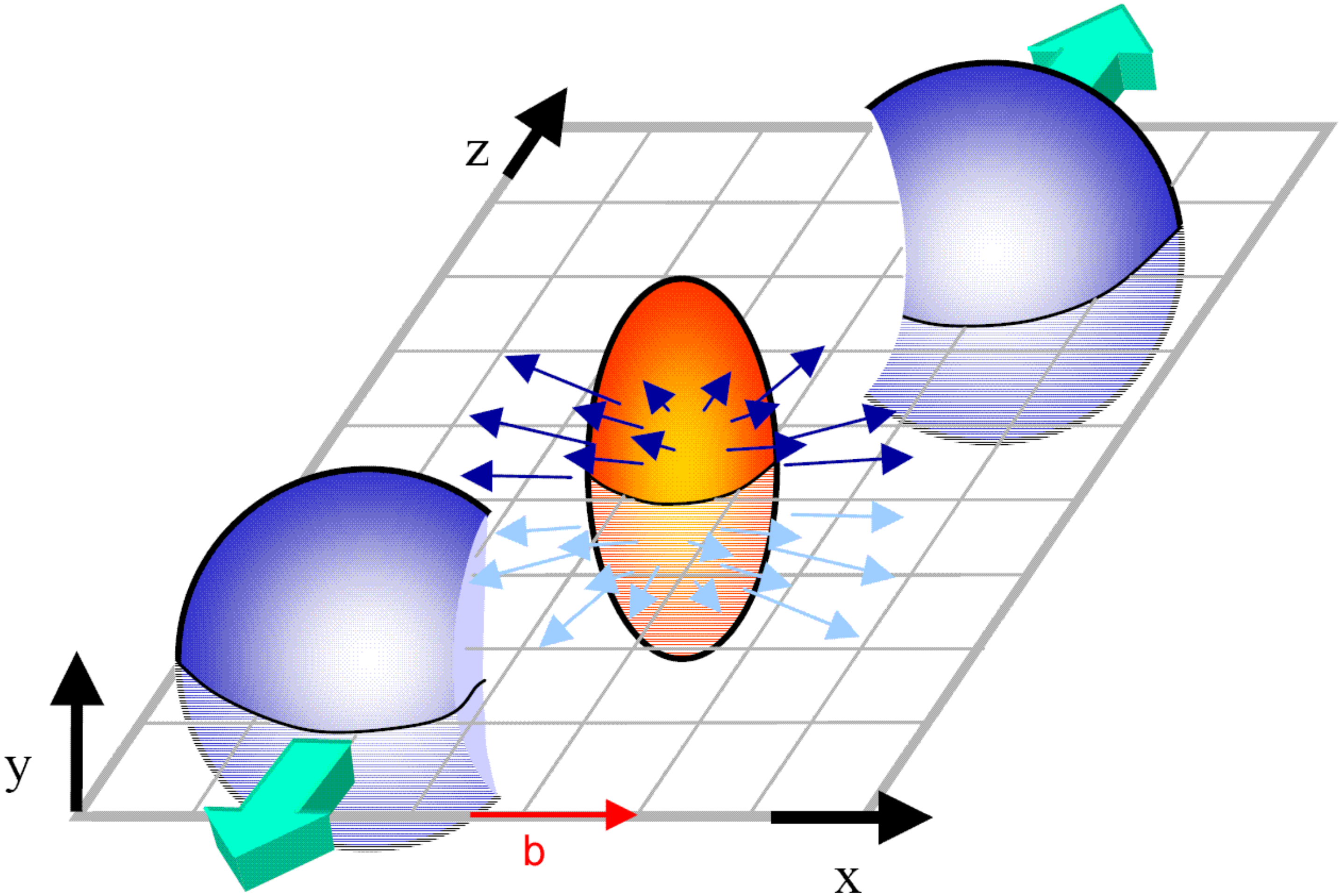
Characteristic direction in HIC



Characteristic direction in HIC

- *magnetic field*
- *net angular momentum*





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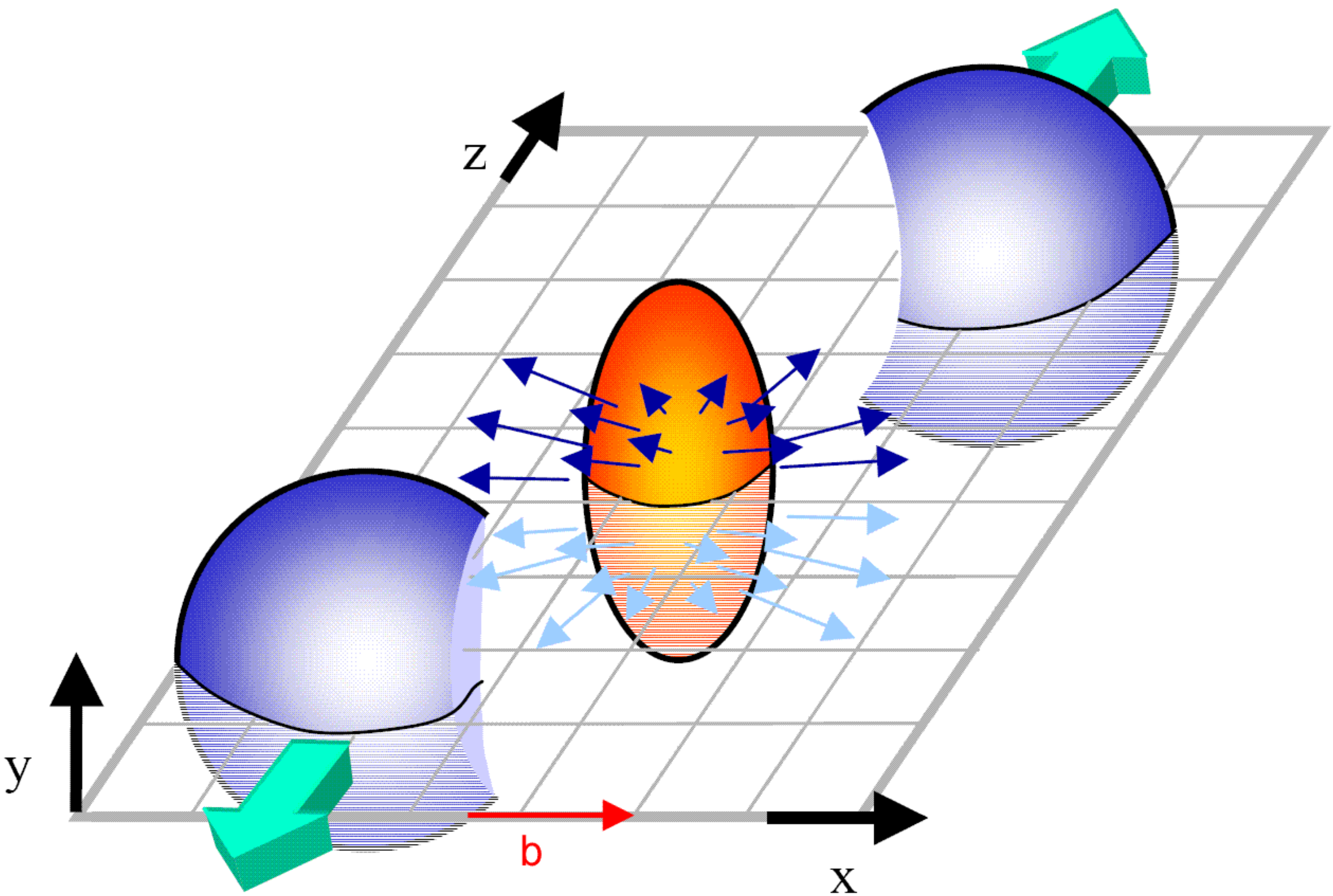
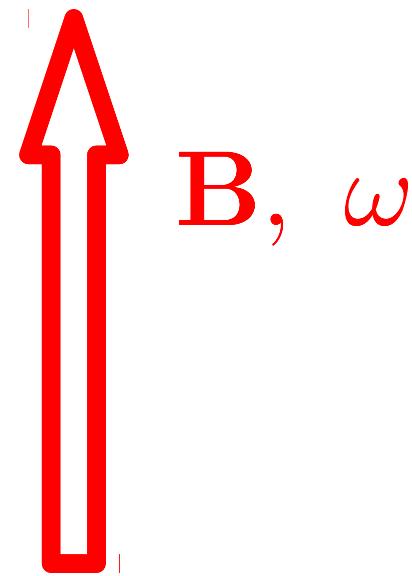
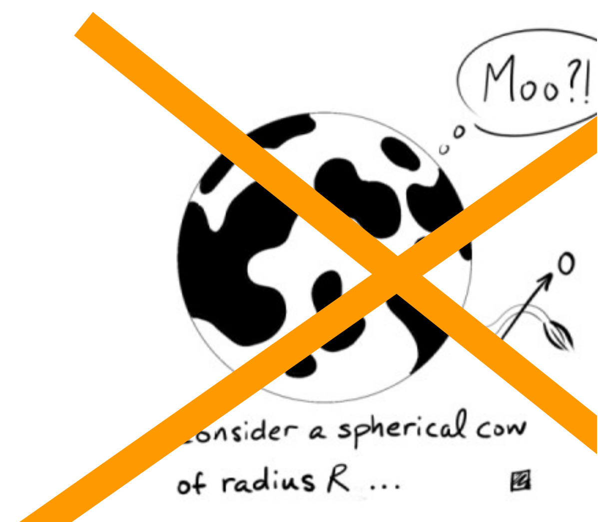




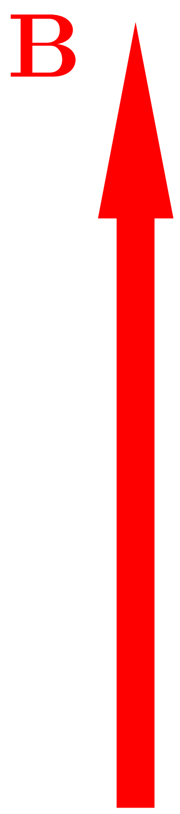


Outline

- **Phenomenology**
  - currents in the QGP
  - CME, CSE, CESE, CVE…
  - possible ways to measure
- **What’s anomalous?**
  - origin of chiral imbalance
  - universality of the static CME conductivity
  - hydrodynamic descr. of chiral medium
- **What theory predicts**
  - linear response computation
  - non-static setup for the CME conductivity

Transport in chiral medium: CME

see: arXiv 1511.04050



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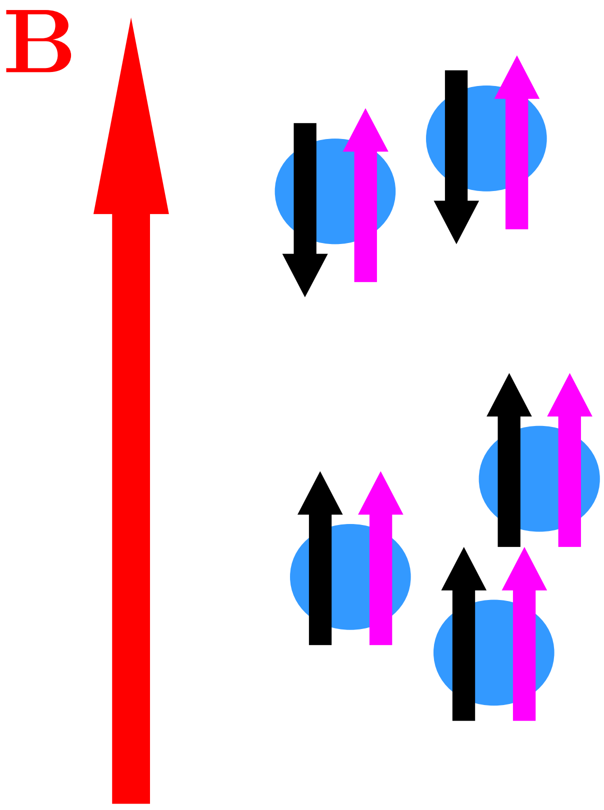

(+ opposite charge)

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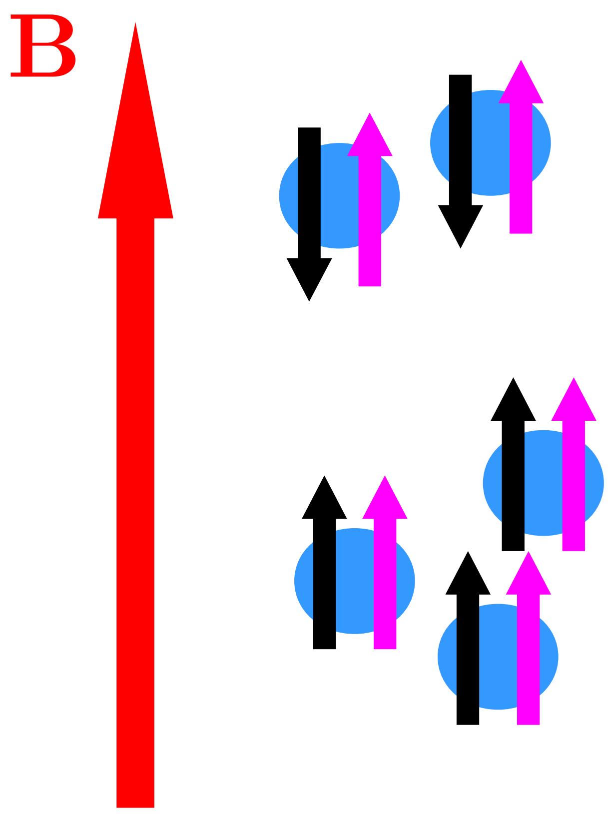

$\mu_5 \neq 0$

$n_R \neq n_L$

$\langle \mathbf{s} \rangle \sim \mathbf{B}$

$\langle \mathbf{p} \rangle \sim \mu_5 \langle \mathbf{s} \rangle$

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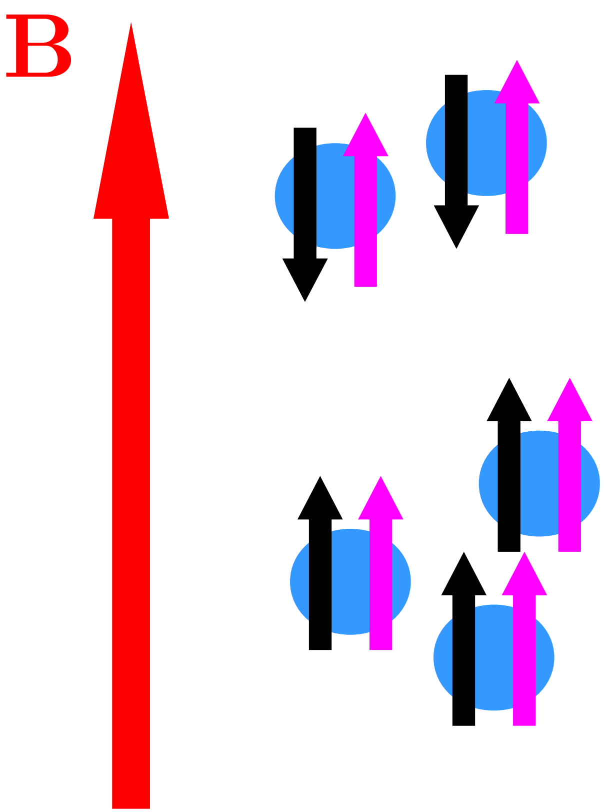

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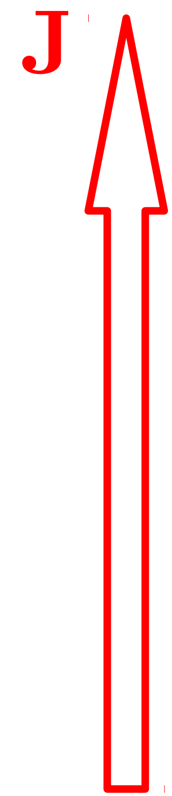

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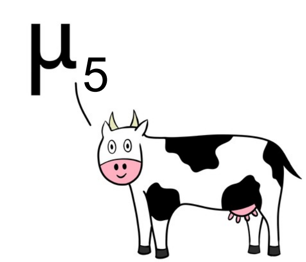


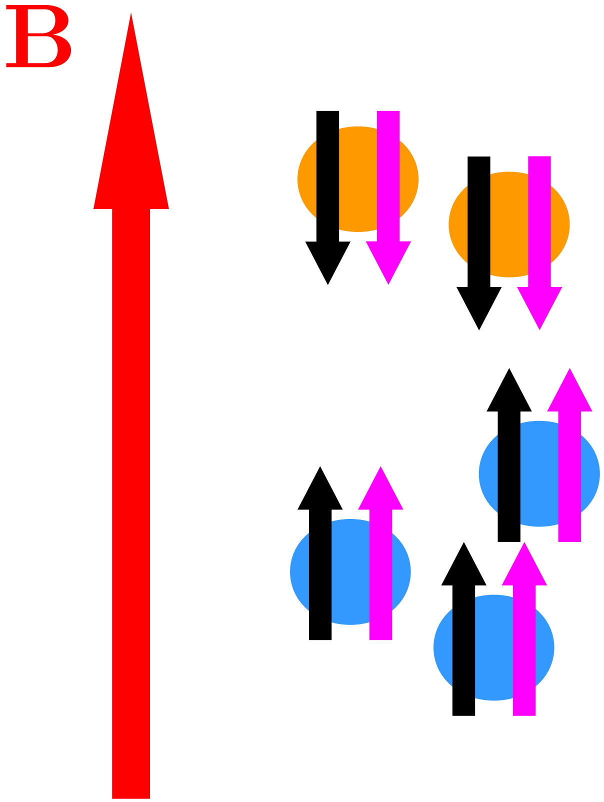
Transport in chiral medium

$$\mathbf{J} = \sigma\mathbf{E} + C_A\mu_5\mathbf{B}$$

chiral imbalance!

Transport in chiral medium: CSE

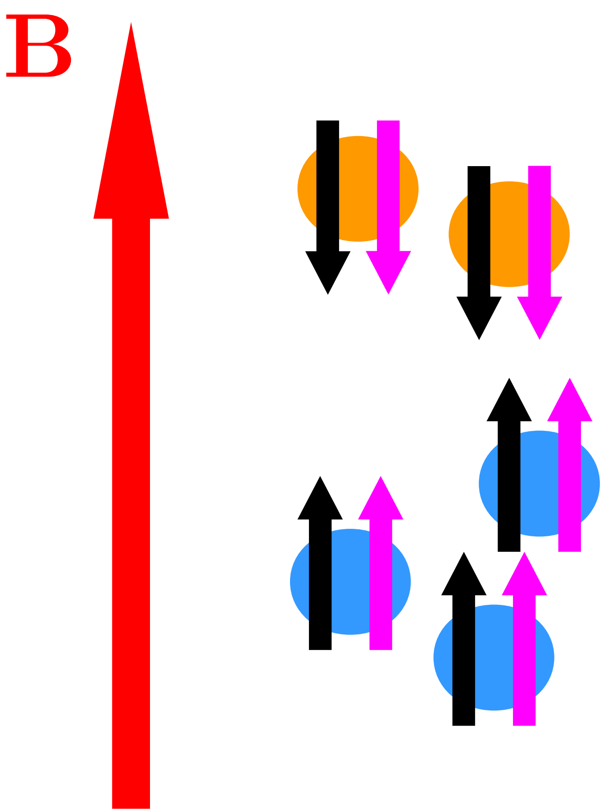
see: arXiv 1511.04050



(+ opposite chirality)

Transport in chiral medium: CSE

see: arXiv 1511.04050

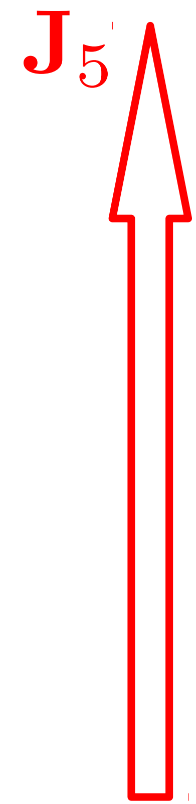


$\mu \neq 0$

$n_q \neq n_{\overline{q}}$

$\langle \mathbf{s} \rangle \sim e\mathbf{B}$

$\langle \mathbf{J}_{L/R} \rangle \sim e\mu\mathbf{B}$



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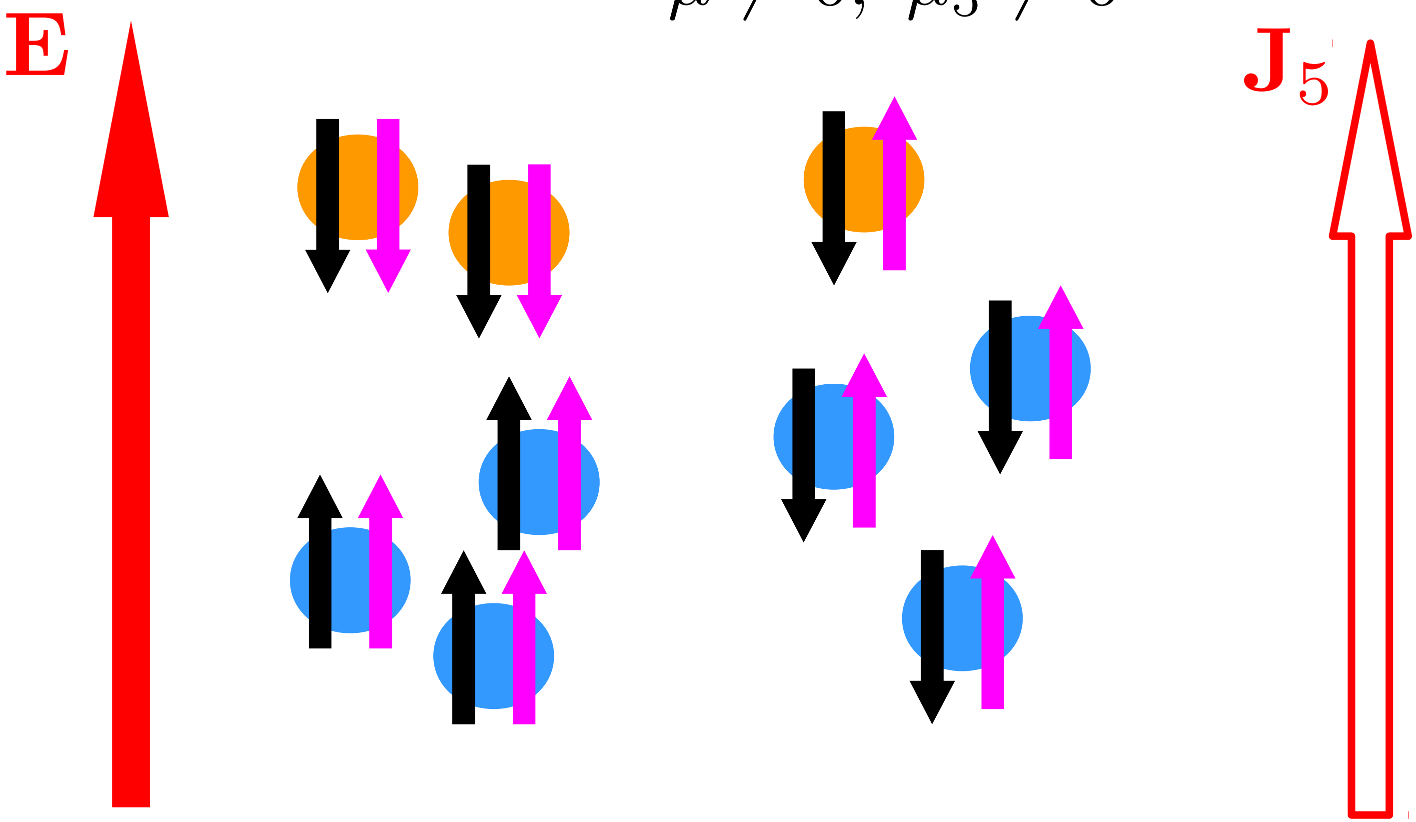
Transport in chiral medium: CESE

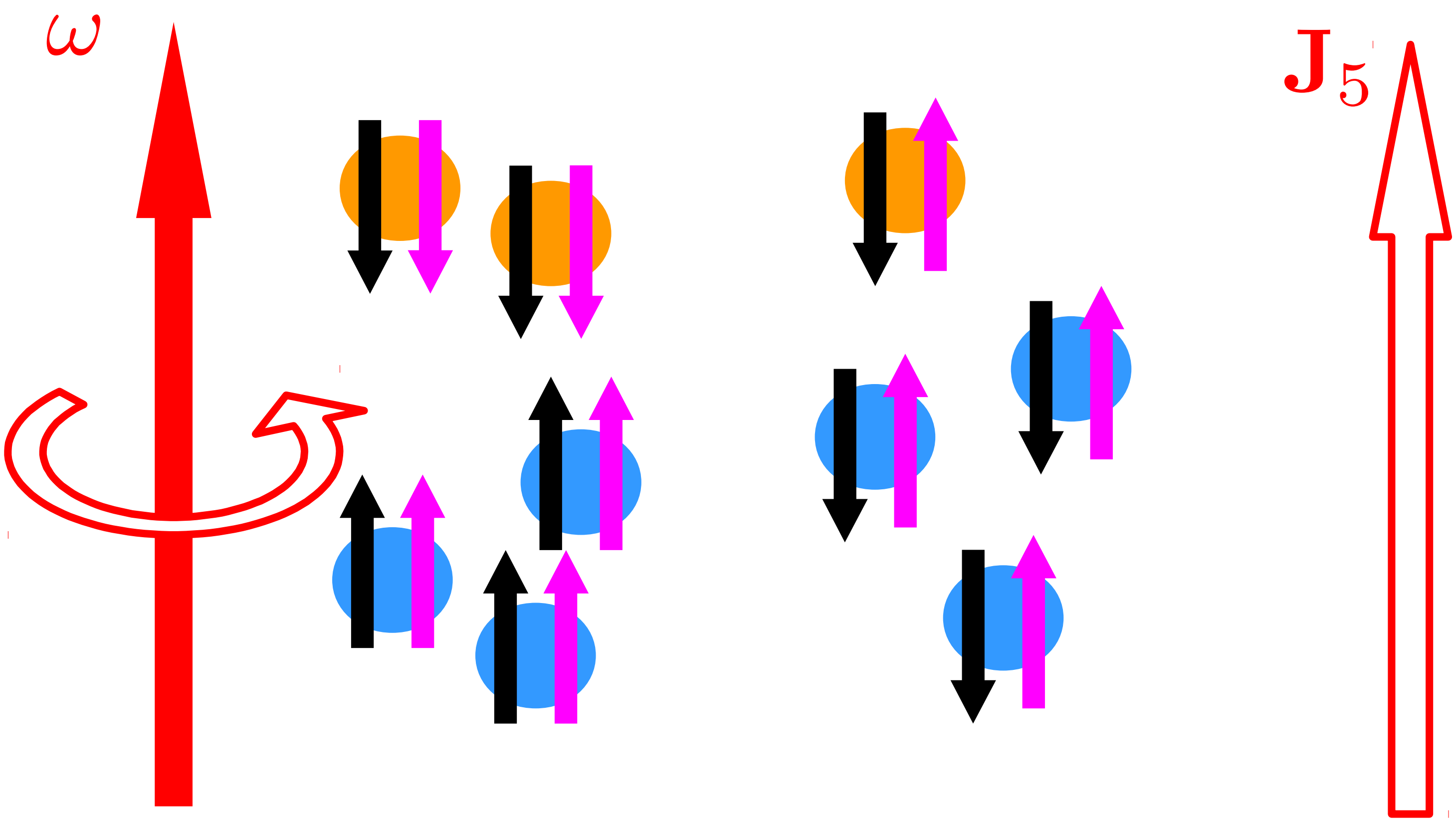
see: arXiv 1511.04050

$\mu \neq 0, \ \mu_5 \neq 0$



Transport in chiral medium: CVE

see: arXiv 1511.04050



$\langle \mathbf{s} \rangle \sim \omega$ **BUT charge-blind!**

Discrete symmetries of currents

	$\mathbf{J}$	$=$ $\sigma\mathbf{E}$	$+$ $\sigma_5\mathbf{B}$
$\mathcal{P}$	odd	$even \times odd$	$odd \times even$
$\mathcal{T}$	odd	$odd \times even$	$even \times odd$

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Discrete symmetries of currents

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$\mathcal{P}$	$even$	$odd \times odd$	$even \times even$
$\mathcal{T}$	$even$	$even \times even$	$odd \times odd$

Transport in chiral medium

$$\mathbf{J} = \sigma \mathbf{E} + C_A \mu_5 \mathbf{B}$$

$$\mathbf{J}_5 = \# \mu \mu_5 \mathbf{E} + C_A \mu \mathbf{B}$$

Transport in chiral medium

$$\mathbf{J} = \sigma \mathbf{E} + \sigma_5 \mathbf{B}$$

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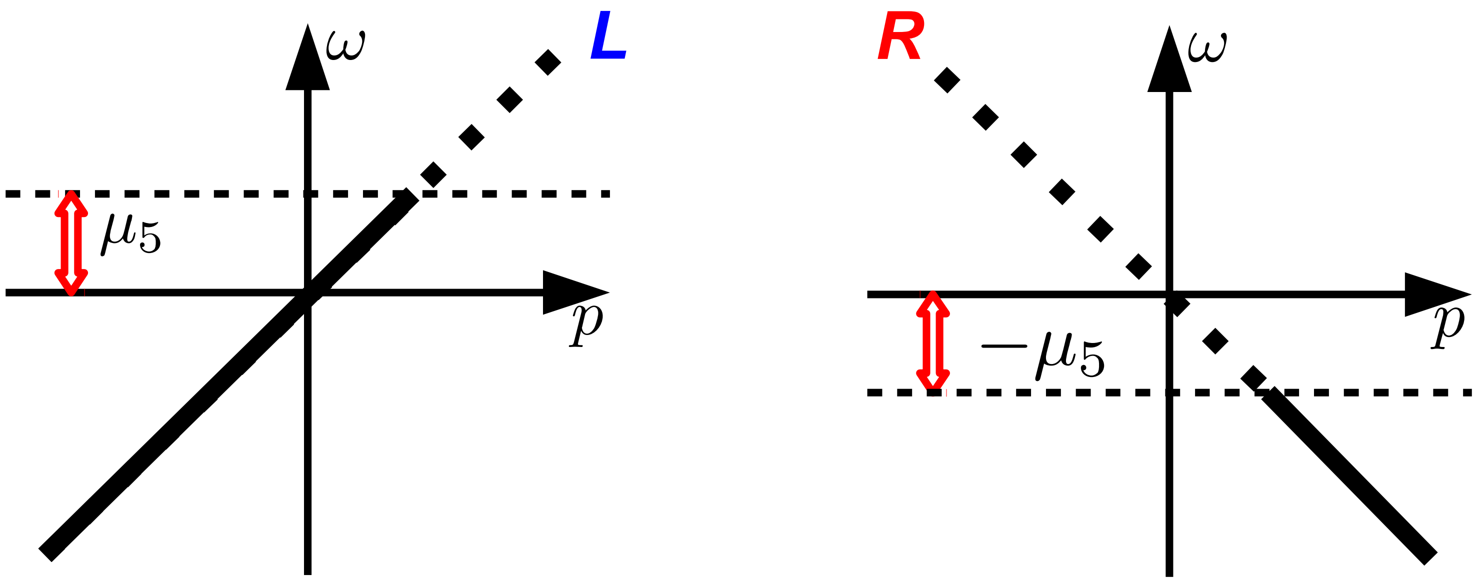
conductivity is fixed by the chiral anomaly! $\partial_\mu J_5^\mu = \frac{N_f e^2}{8\pi^2} \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$

$$P = \mu_5 \frac{\mathrm{d}Q_5}{\mathrm{d}t} = \mu_5 C_A \int_V \mathbf{E} \cdot \mathbf{B} \stackrel{!}{=} \int_V \mathbf{J} \cdot \mathbf{E} = \sigma_5 \int_V \mathbf{E} \cdot \mathbf{B} = P$$

Simple way to anomaly

chiral fermions in 1+1D, affected by homog. E field



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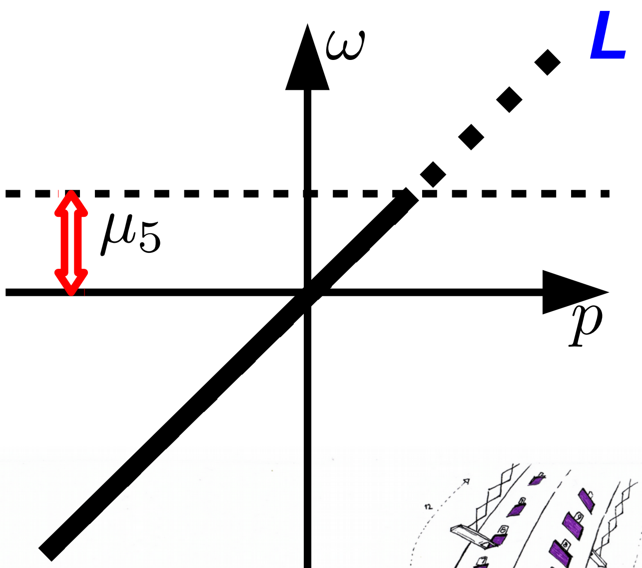

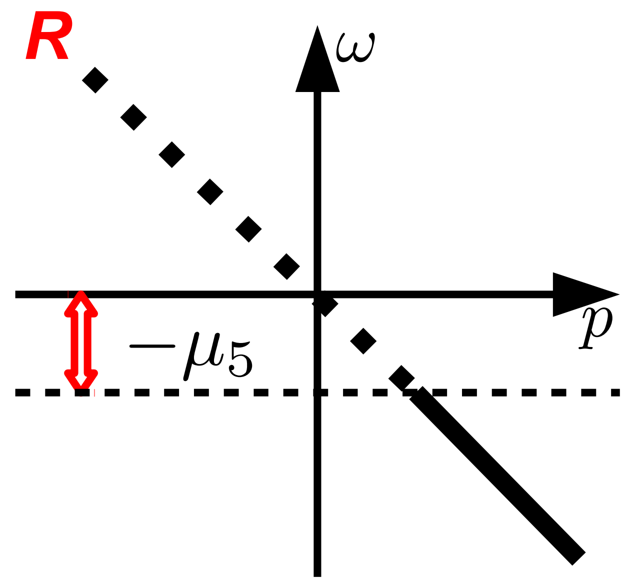

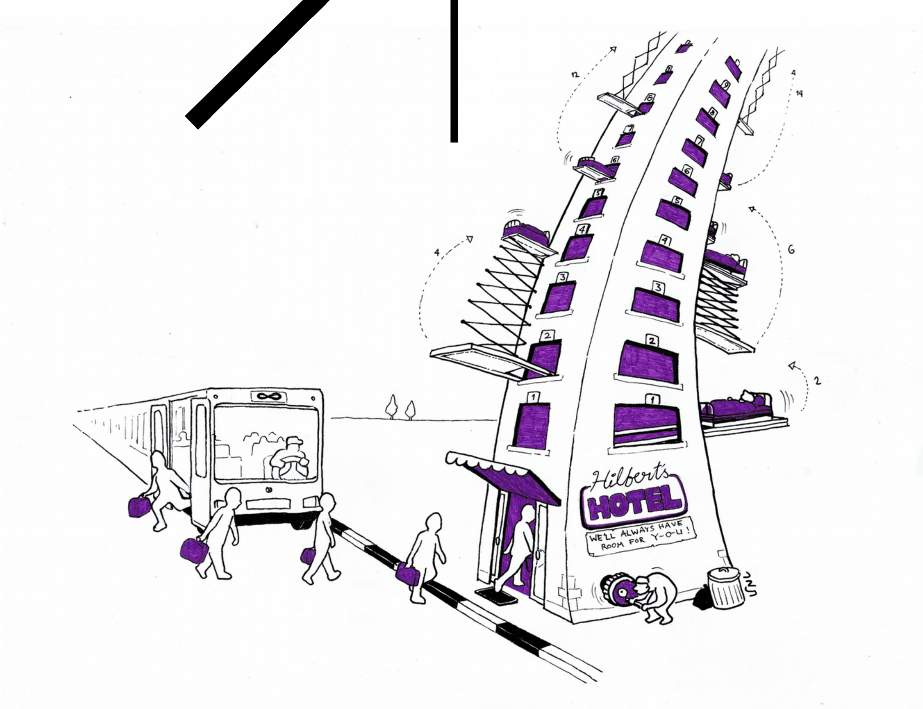

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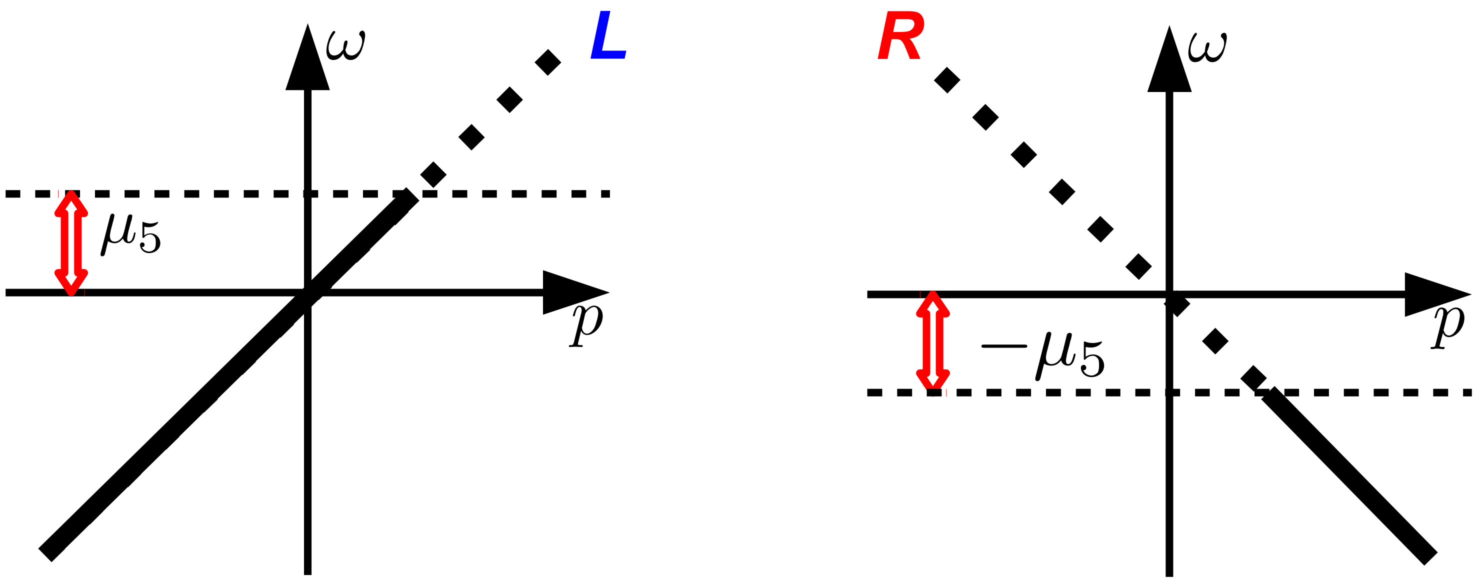

$$\omega = \pm p$$

$$\dot{p} = E$$

$$\dot{n}_{L/R} = \pm \frac{e}{2\pi} E$$

$$\frac{dQ_5^{1+1}}{dt} = \frac{d}{dt}(n_R - n_L) = \frac{e}{\pi} E$$

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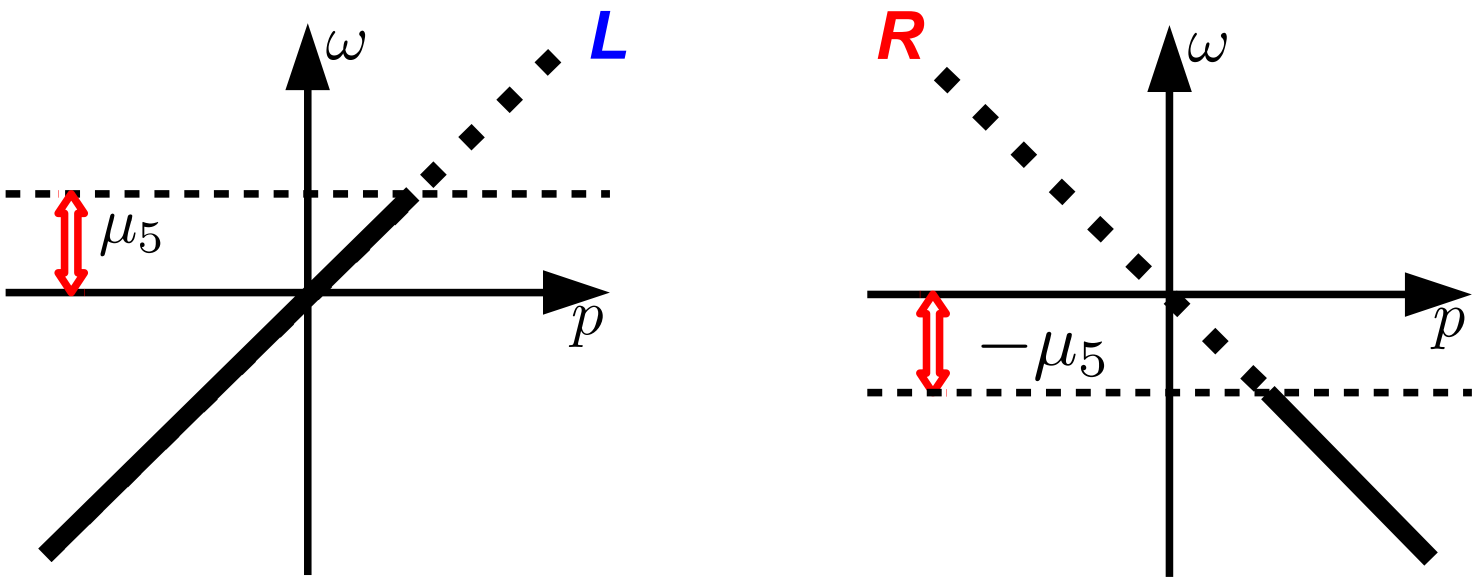

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3+1D: Landau levels!

$$\frac{dQ_5}{dt} = (1{+}1\text{D res.}) \times \frac{eBL^2}{2\pi}$$

$$\frac{1}{V}\frac{dQ_5}{dt} = \frac{e^2}{2\pi^2} EB$$

How to measure CME in HIC?

expectation: charge-dipole $\perp$ to the reaction plane

$$\frac{dN^{\pm}}{d\phi} \sim 1 + 2a_1^{\pm}\sin(\phi - \Psi_{\mathrm{RP}}) + 2v_2\cos(2\phi - 2\Psi_{\mathrm{RP}}) + \ldots$$

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BUT: $\langle a_1^{\pm} \rangle = 0$

SO:

$$\gamma_{\alpha\beta} := \langle \cos(\phi_i + \phi_j - 2\Psi_{\mathrm{RP}}) \rangle_{\alpha\beta}$$

$$\delta_{\alpha\beta} := \langle \cos(\phi_i - \phi_j) \rangle_{\alpha\beta}$$

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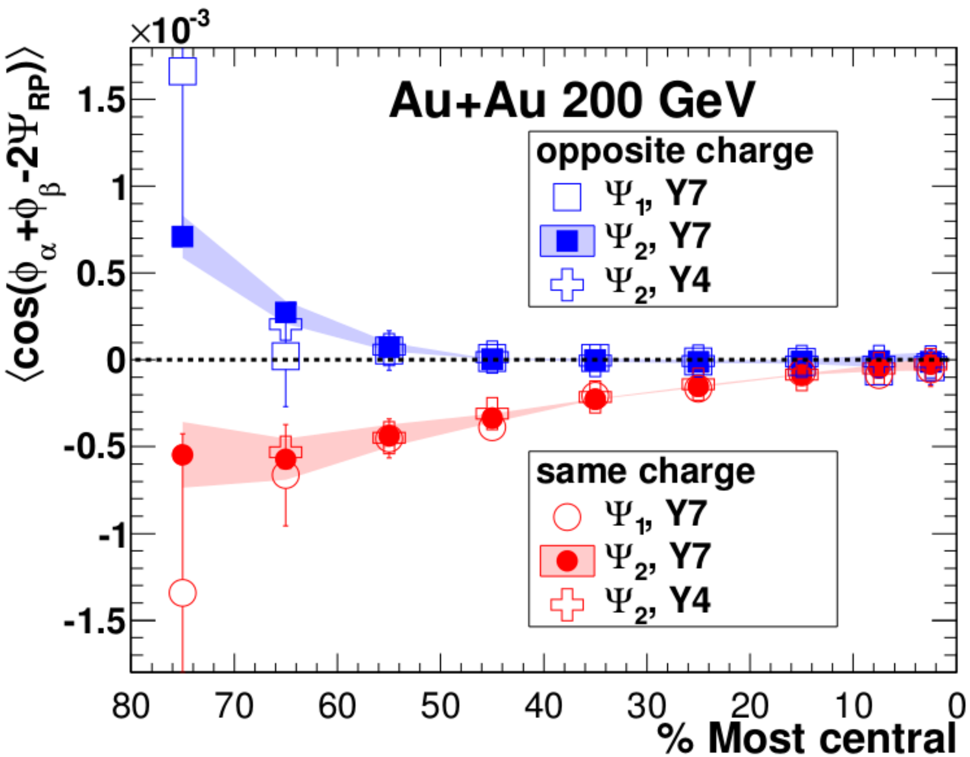
ALSO background subtraction needed!

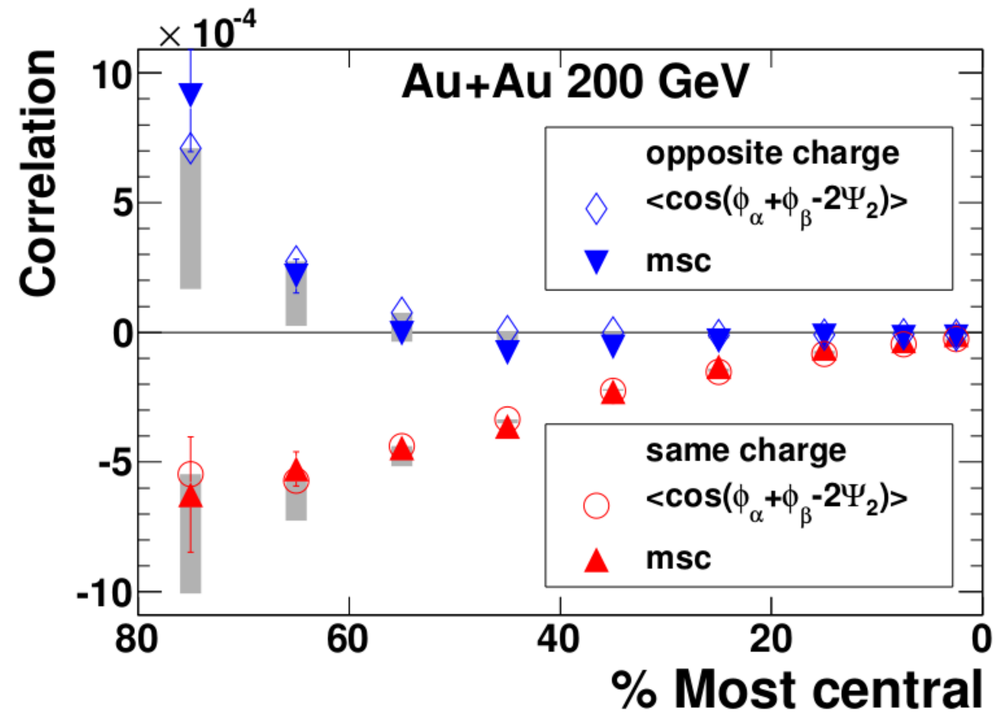
$$\delta^{\mathrm{bckg.}} = F \qquad \delta = F + H$$

$$\gamma^{\mathrm{bckg.}} = \kappa v_2 F \qquad \gamma = \kappa v_2 F - H$$

How to measure CME in HIC?

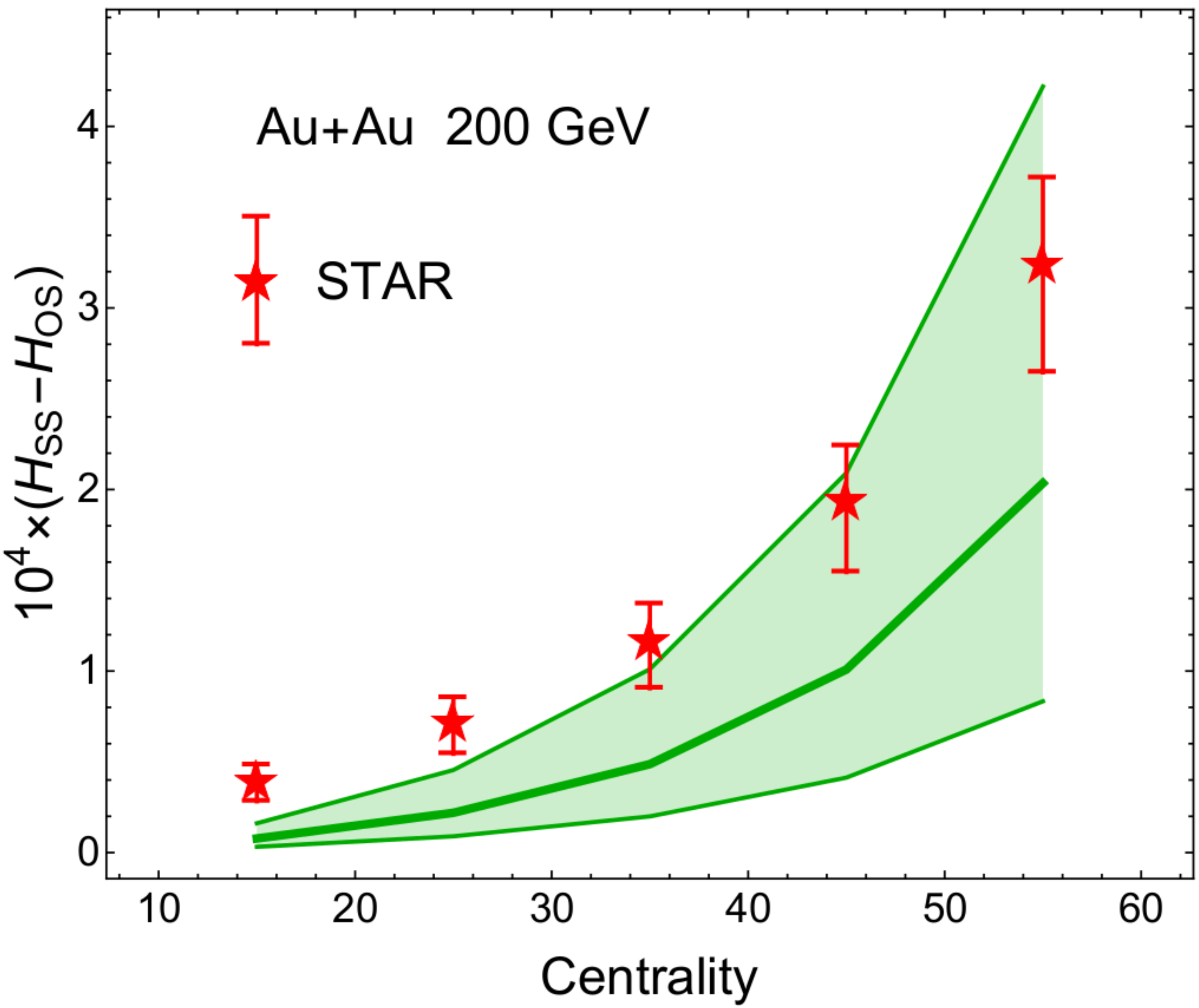
Kharzeev *etal.*
arXiv: 1511.04050





How to measure CME in HIC?

Liao *etal.*
arXiv: 1611.04586



Anomalous hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id.}} + \tau^{\mu\nu}$$

$$J^{\mu} = n u^{\mu} + \nu^{\mu}$$

$$\partial_\mu T^{\mu\nu} = F_{\nu\lambda} J^{\lambda}$$

$$\partial_\mu J^{\mu} = 0$$

$$T^{\mu\nu}_{\mathrm{id.}} = (\epsilon + P) u^{\mu} u^{\nu} + P \eta^{\mu\nu}$$

$$s^{\mu}_{\mathrm{id.}} := s u^{\mu} = \frac{\epsilon + P - \mu n}{T} u^{\mu}$$

Anomalous hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id.}} + \tau^{\mu\nu} \qquad \partial_\mu T^{\mu\nu} = F_{\nu\lambda} J^\lambda$$

$$J^\mu = n u^\mu + \nu^\mu \qquad \partial_\mu J^\mu = 0$$

$$T^{\mu\nu}_{\mathrm{id.}} = (\epsilon + P) u^\mu u^\nu + P \eta^{\mu\nu} \qquad s^\mu_{\mathrm{id.}} := s u^\mu = \frac{\epsilon + P - \mu n}{T} u^\mu$$

$$\left.\begin{aligned} \nu^\mu u_\mu &= 0 \\ \tau^{\mu\nu} u_\mu = \tau^{\mu\nu} u_\nu &= 0 \end{aligned}\right\}$$

transverse conditions:
no energy flow in the rest frame! (Landau)

Ensure positive entropy production:

$$\partial_\mu \underbrace{\left(s u^\mu - \frac{\mu}{T} \nu^\mu \right)}_{=: s^\mu} = -\nu^\mu \partial \left(\frac{\mu}{T} \right) + \frac{1}{T} \tau^{\mu\nu} \partial_\mu u_\nu \geq 0$$

$$\nu^\mu \overset{!}{\sim} \partial^\mu \left(\frac{\mu}{T} \right)$$

$$\tau^{\mu\nu} \overset{!}{\sim} \partial^\mu u^\mu + \partial^\nu u^\mu$$

$\Longrightarrow$ ***introduction of diffusion const. & viscosities!***

Anomalous hydrodynamics

Son: PRL ***103,*** *191601 (2009)*

$$\partial_\mu J^\mu_{R/L} = \pm C E^\rho B_\rho$$

$$\nu^\mu = -\sigma T P^{\mu\nu}\partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu$$

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + \underbrace{D\omega^\mu + D_B B^\mu}_{= s^\mu_A}$$

doubling density:
L & **R** chirality

additional terms to currents:
coupling to ***B*** and $\boldsymbol{\omega}$

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doubling density:
L & **R** chirality

additional terms to currents:
coupling to ***B*** and $\boldsymbol{\omega}$

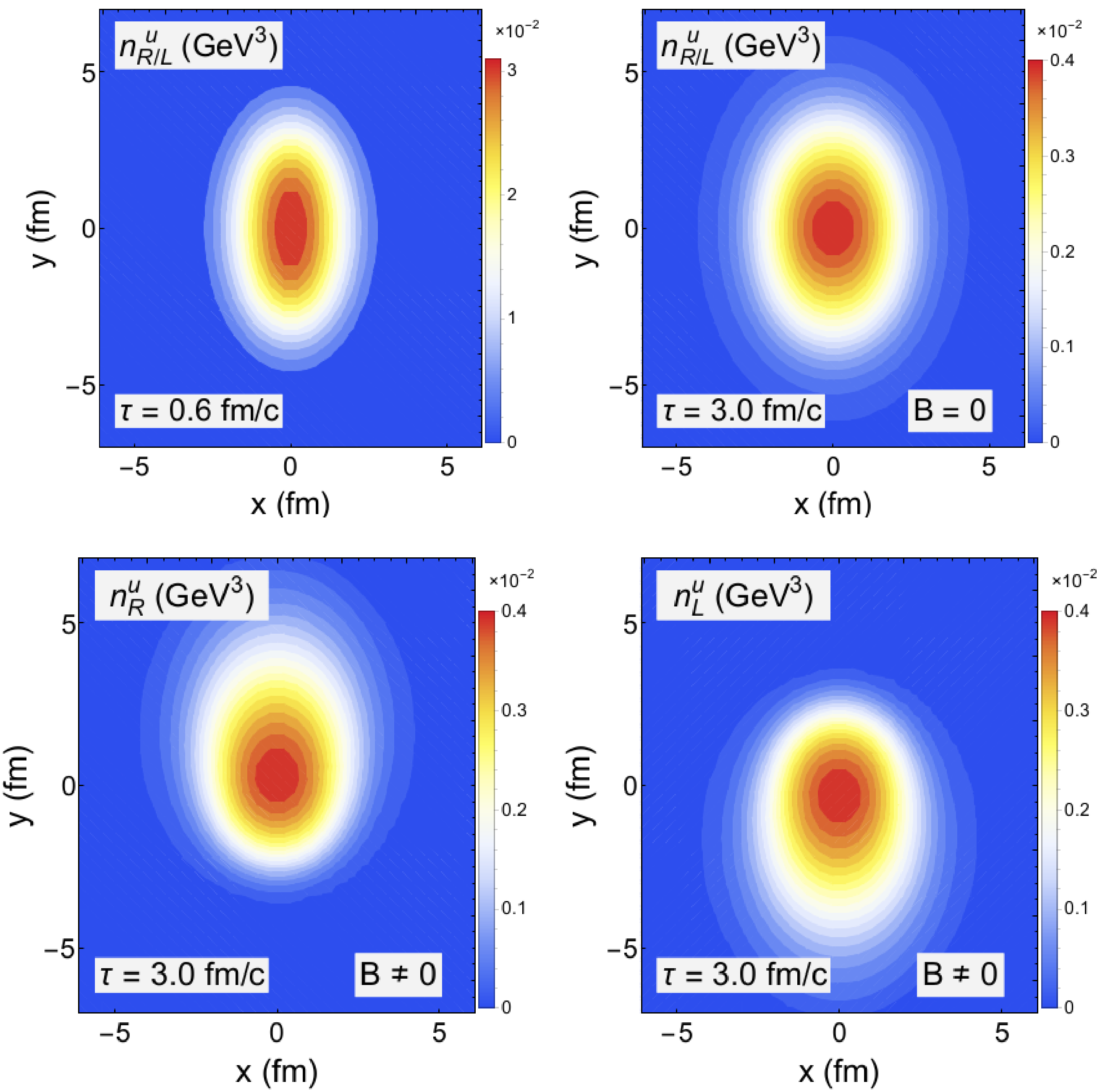
No additional entropy generation!
(for ***P***-odd and ***T***-even transport coefficients)

$$\partial_\mu s^\mu_A = 0$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right) \qquad \xi_B = C\left(\mu - \frac{1}{3}\frac{n\mu^2}{\epsilon + P}\right)$$

Anomalous hydrodynamics

Liao *etal.*
arXiv: 1611.04586



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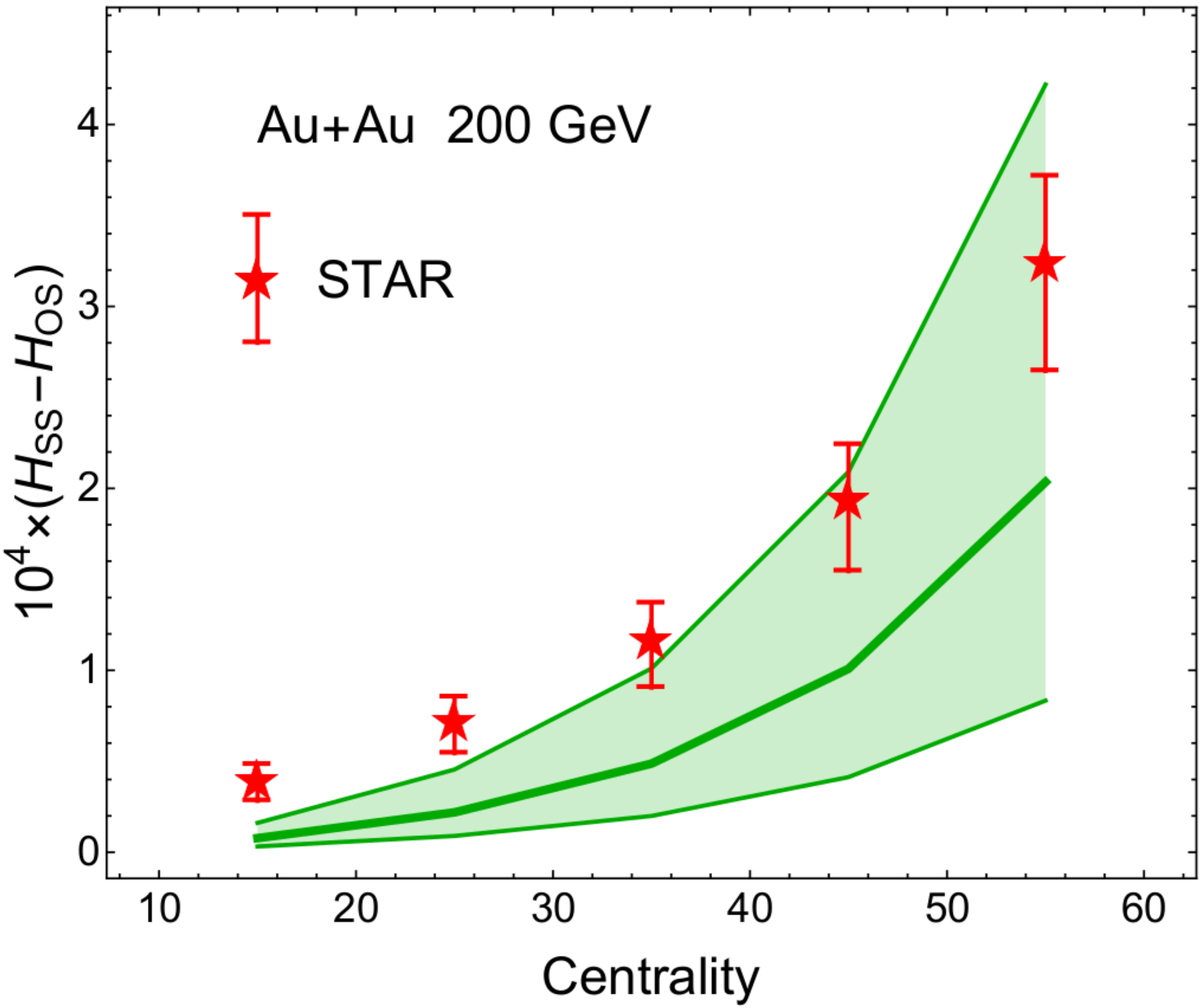

Anomaly in QFT

U(1) vector current: $J^\mu = \overline{\Psi}\gamma^\mu\Psi$

U(1) axialvector current: $J_5^\mu = \overline{\Psi}\gamma^\mu\gamma^5\Psi$

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U(1) axialvector current: $J_5^\mu = \overline{\Psi}\gamma^\mu\gamma^5\Psi$

$$\partial_\mu J^\mu = 0 \qquad\qquad \partial_\mu J_5^\mu = {\color{red}0}$$

Anomaly in QFT

U(1) vector current: $$J^\mu = \overline{\Psi}\gamma^\mu\Psi$$

U(1) axialvector current: $$J_5^\mu = \overline{\Psi}\gamma^\mu\gamma^5\Psi$$

$$\partial_\mu J^\mu = 0 \qquad\qquad \partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$$

fermions coupled to gauge fields:

- ✔ maintaining gauge invariance
  - $\rightarrow$ *costs the anomalous divergence of the axial current*
- ✔ the anomaly comes from the UV behaviour of the fermionic propagator

Linear response in QFT

See Kovtun: J. Phys. A **45,** 473001 (2012)
arXiv: 1205.5040

perturbation in ***(axial)vector current***

$$\delta H(t) = \int_{-\infty}^{t} \mathrm{d}\tau \int \mathrm{d}^3\mathbf{x} J^{\mu}_{(5)}(\tau,\mathbf{x}) A^{(5)}_{\mu}(\tau,\mathbf{x})$$

change of avr. ***current*** to linear order in the strength $A^{(5)}_{\nu}$

$$\delta\langle J^{\mu}\rangle \sim A_{\nu}(\omega,\mathbf{k}) \int_{-\infty}^{\infty} \mathrm{d}\widetilde{\omega} \frac{\rho^{\mu\nu}_{JJ}(\widetilde{\omega},\mathbf{k})}{(\widetilde{\omega}-\omega+i\epsilon)^2}\Bigg|_{\epsilon\to 0} \to A_{\nu}(k=0) \lim_{\omega\to 0} \frac{\rho^{\mu\nu}_{JJ}(\omega,\mathbf{k}=0)}{\omega}$$

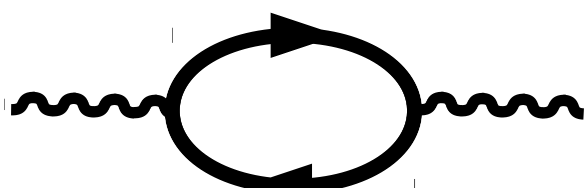
$\boxed{\sigma}$

$$\rho^{\mu\nu}_{JJ} = \langle [J^{\mu}(\omega,\mathbf{k}), J^{\nu}(x=0)]\rangle$$

Anomalous currents

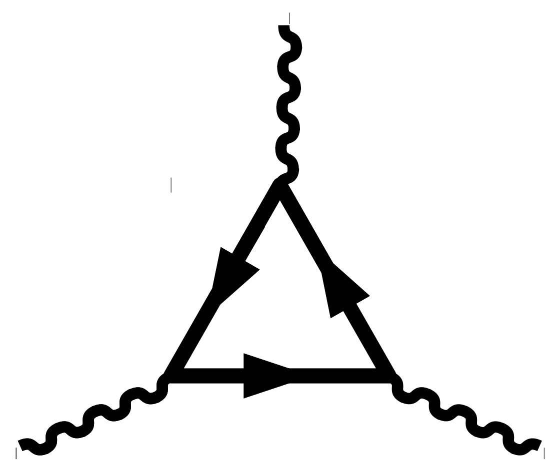
$$\delta\langle J^i\rangle \sim \langle J^i J^j\rangle A_j \sim \langle\overline{\Psi}\gamma^i\Psi\overline{\Psi}\gamma^j\Psi\rangle \sim F_1^{ij}E_i + \mu_5 F_2^{ij}B_j + \mathcal{O}(\mu^2,\mu_5^2)$$

$$\delta\langle J_5^i\rangle \sim \langle J_5^i J^j\rangle A_j \sim \langle\overline{\Psi}\gamma^i\gamma^5\Psi\overline{\Psi}\gamma^j\Psi\rangle \sim \mu\mu_5 G_1^{ij}E_i + \mu G_2^{ij}B_j + \mathcal{O}(\mu^2,\mu_5^2)$$



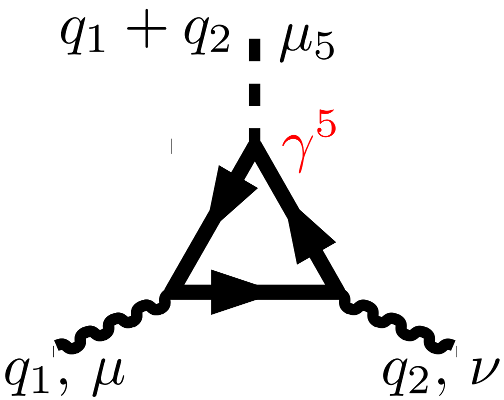
$$\langle\overline{\Psi}\gamma^\mu\Psi\overline{\Psi}\gamma^\nu\Psi\rangle$$

$$\xRightarrow{\dfrac{\partial}{\partial\mu_{(5)}}}$$



$$\langle\overline{\Psi}\gamma^\mu\Psi\overline{\Psi}\gamma^\rho(\gamma^5)\Psi\overline{\Psi}\gamma^\nu\Psi\rangle\Big|_{\mu,\mu_5=0}$$

Anomalous currents – formulas: Work in progress!!!



Useful for:

- *time dependent & inhomogenous external fields (the anomaly does not fix the conductivity uniquely)*
- *finite temperature effects in non-static setup*

$$\Delta^{\mu\nu}(q_1,q_2) = \frac{e^2}{2}\int_p \mathrm{tr}\left\{\gamma^\mu \rho(p+q_1)\gamma^0\gamma^5\rho(p-q_2)\gamma^\mu\rho(p)\right\}(n(p_0+q_{10})-n(p_0-q_{20}))+$$

$$-\frac{e^2}{2}\int_P \mathrm{tr}\left\{\gamma^\nu\left(iG^R(P)+iG^A(P)\right)\gamma^0\gamma^5\rho(P-q_1-q_2)\gamma^\mu\rho(P-q_1)(n(P_0-q_{10}-q_{20})-n(P_0))+\right.$$

$$-\frac{e^2}{2}\int_P \mathrm{tr}\left\{\gamma^\nu\rho(P+q_1+q_2)\gamma^0\gamma^5\left(iG^R(P)+iG^A(P)\right)\gamma^\mu\rho(P+q_2)(n(P_0+q_{10}+q_{20})-n(P_0))+\right.$$

$$+\{\mu\leftrightarrow\nu,\ q_1\leftrightarrow q_2\}$$

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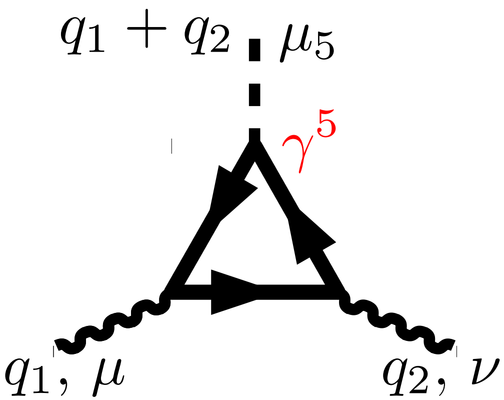

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- ✔ *finite temperature effects in non-static setup*

$$\Delta^{\mu\nu}(q_1,q_2) = \boxed{\frac{e^2}{2}\int_p \mathrm{tr}\left\{\gamma^\mu \rho(p+q_1)\gamma^0\gamma^5\rho(p-q_2)\gamma^\mu\rho(p)\right\}\left(n(p_0+q_{10}) - n(p_0-q_{20})\right)+}$$

$$\boxed{\begin{aligned} &-\frac{e^2}{2}\int_P \mathrm{tr}\left\{\gamma^\nu\left(iG^R(P)+iG^A(P)\right)\gamma^0\gamma^5\rho(P-q_1-q_2)\gamma^\mu\rho(P-q_1)(n(P_0-q_{10}-q_{20})-n(P_0))+\right. \\ &-\frac{e^2}{2}\int_P \mathrm{tr}\left\{\gamma^\nu\rho(P+q_1+q_2)\gamma^0\gamma^5\left(iG^R(P)+iG^A(P)\right)\gamma^\mu\rho(P+q_2)(n(P_0+q_{10}+q_{20})-n(P_0))+\right. \end{aligned}}$$

$$+\{\mu \leftrightarrow \nu,\ q_1 \leftrightarrow q_2\}$$

Some properties:

these terms are subleading for small q_1 & q_2

for CME, only **this term** survives in the homogenous limit

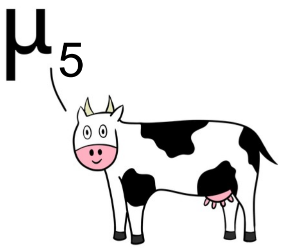
Take-aways

- **We have learned about:**
  - **a peculiar sign of chiral symmetry restoration**
  - **possible signals to identify anomalous chir. eff.**
  - **out-of-equilibrium description still poses challenge**
  - life isn’t that easy for theorists either

- **Needed:**
  - **realistic simulations based on microscopic prop.**
  - (meaning further work, *in progress!)*
  - **better observables to measure**

Thank you for listening!

Questions? Comments?



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