Critical point of nuclear matter

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Outline

1. VDW - GCE
2. VDW with quantum statistics (QVDW)
3. Nuclear matter with the VDW equation of state
    - Phase diagram
    - Fluctuations in the region of the critical point
    - Comparison with Walecka model

Van der Waals model in GCE

Van der Waals EoS (CE)

$$P(T,V,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2}, \qquad a>0\ ,\ b>0\ .$$

Transition to GCE[1]:

$$\left(\frac{\partial F}{\partial V}\right)_{T,N} = -p(T,V,N) \Rightarrow F(T,V,N)\ ;$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} \Rightarrow \mu = \mu(T,V,N) \Rightarrow N = N(T,V,\mu)\ ;$$

Equations for $n(T,\mu)$

$$\frac{N}{V} \equiv n(T,\mu) = \frac{n_{\rm id}(T,\mu^*)}{1 + b\, n_{\rm id}(T,\mu^*)}\ , \qquad \mu^* = \mu - b\,\frac{n\,T}{1-b\,n} + 2a\,n\ .$$

when $a=b=0$: $n(T,\mu) = n_{\rm id}(T,\mu)$.

[1]Vovchenko, Anchishkin, Gorenstein, J. Phys. A, (2015).

Quntum van der Waals model (QVDW)

Requirements for quantum version of EoS

1) Should reduce to ideal gas Eos as $a = 0$ and $b = 0$
2) Should reduce to classical VDW EoS when quantum statistics can be neglected
3) $s \geq 0$ and $s \to 0$ as $T \to 0$

Pressure VDW-GCE

$$p(T,\mu) = p^{\rm id}(T,\mu^*) - an^2, \quad \mu^* = \mu - b\,p - a\,b\,n^2 + 2an \ .$$

$$p^{\rm id}(T,\mu) \;=\; \frac{d}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{m^2+\boldsymbol{k}^2}}\left[\exp\left(\frac{\sqrt{m^2+\boldsymbol{k}^2}-\mu}{T}\right)+\eta\right]^{-1}$$

Density

$$n(T,\mu) = \left(\frac{\partial p}{\partial \mu}\right)_T = \frac{n^{\rm id}(T,\mu^*)}{1 + b\,n^{\rm id}(T,\mu^*)}$$

Vovchenko, Anchishkin, Gorenstein, Phys. Rev. C, (2015).

Nuclear matter with van der Waals EoS

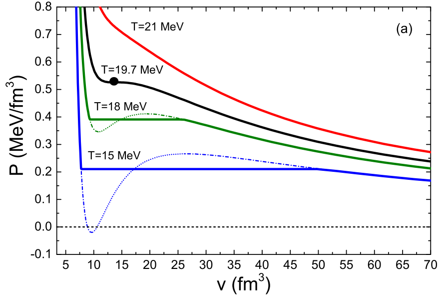
parameters values

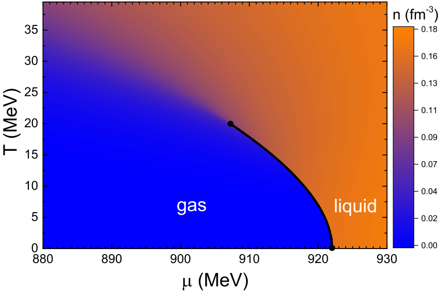
$$\mu \geq 0 \;, \qquad \eta = +1 \;, \qquad g = 4 \;, \qquad m = 938 \text{ MeV} \;.$$

ground state properties, $T = 0$

$$p \;=\; 0 \;, \qquad \varepsilon/n \;\cong\; m + E_B \;\cong\; 922 \text{ MeV} \;, \qquad n \;=\; n_0 \;\cong\; 0.16 \text{ fm}^{-3} \;.$$

$$\Rightarrow \qquad b \;\cong 3.42 \text{ fm}^3 \;, \qquad a \;\cong\; 329 \text{ MeV fm}^3 \;.$$



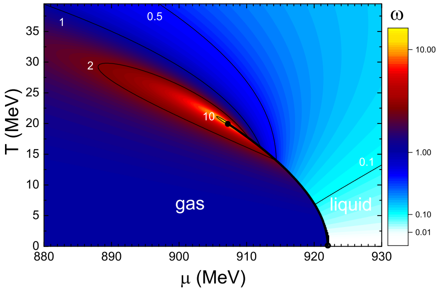


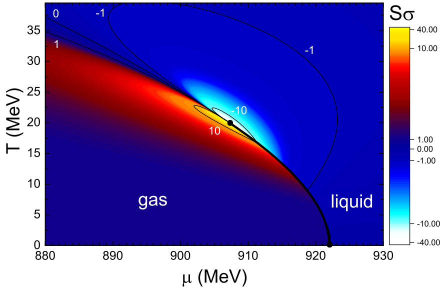
Nucleon number fluctuations

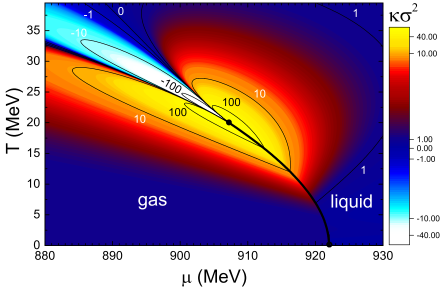
$$\omega[N] = \omega_{\rm id}\left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\,\omega_{\rm id}\right]^{-1}$$

$$S\sigma = \frac{k_3}{k_2} = \omega[N] + \frac{T}{\omega[N]}\left(\frac{\partial\omega[N]}{\partial\mu}\right)_T$$

$$\kappa\sigma^2 = \frac{k_4}{k_2} = (S\sigma)^2 + T\left(\frac{\partial[S\sigma]}{\partial\mu}\right)_T$$







Nuclear matter: QVDW vs Walecka

QVDW:

$$p(T,\mu) = p^{\mathrm{id}}(T,\mu^*) - a\, n^2(T,\mu)$$

$$\mu^* = \mu \;-\; b\, p(T,\mu) - a\, b\, n^2(T,\mu) + 2\, a\, n(T,\mu)$$

$$n(T,\mu) = \frac{n^{\mathrm{id}}(T,\mu^*)}{1 + b\, n^{\mathrm{id}}(T,\mu^*)}$$

Walecka model:

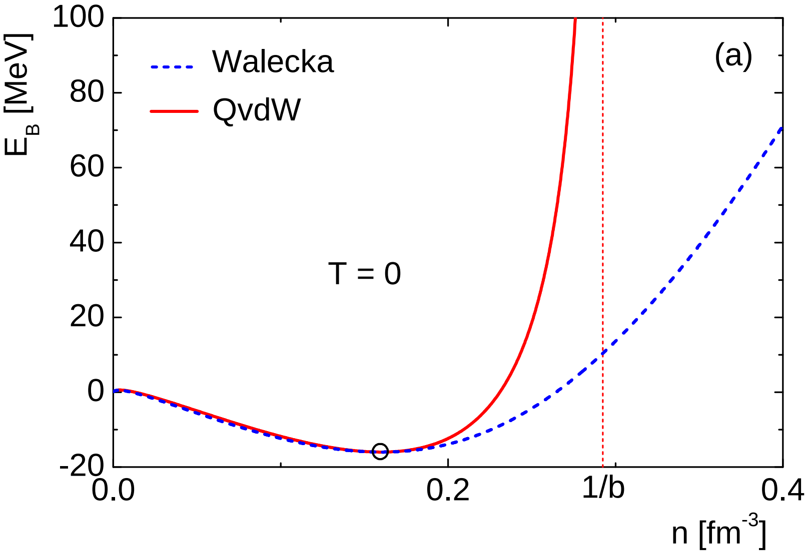
$$p(T,\mu) \;=\; p^{\mathrm{id}}(T,\mu^*;m^*) \;+\; \frac{c_{\mathrm{v}}^2}{2}\, n(T,\mu) \;-\; \frac{(m-m^*)^2}{2c_{\mathrm{s}}^2}$$

$$\mu^* \;=\; \mu \;-\; c_{\mathrm{v}}^2\; n(T,\mu) \qquad \frac{m}{m^*} \;=\; 1 \;+\; c_{\mathrm{s}}^2\, \frac{g_N}{2\pi^2} \int_0^\infty k^2 dk\; \frac{f_{\mathrm{k}}(T,\mu^*;m^*)}{\sqrt{m^{*2}+k^2}}$$

$$n(T,\mu) \;=\; n^{\mathrm{id}}(T,\mu^*;m^*)$$

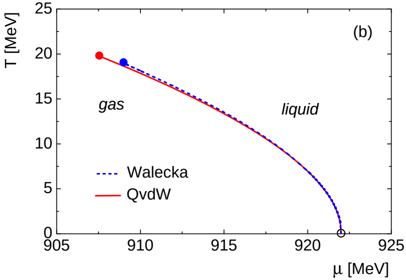
from the same ground state properties

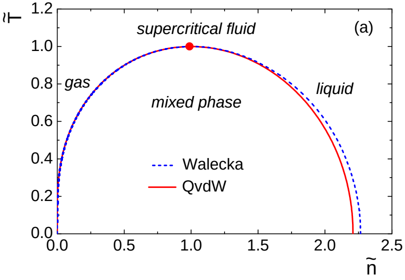
$$c_{\mathrm{v}}^2 \;\cong\; 11.0~\mathrm{fm}^2\;, \qquad c_{\mathrm{s}}^2 \;\cong\; 14.6~\mathrm{fm}^2\;.$$


100
80
60
40
20
0
-20
0.0
0.2
1/b
0.4
E_B [MeV]
n [fm^{-3}]
Walecka
QvdW
(a)
T = 0

Critical point

	Walecka	Hybrid	QvdW	Experiment[1]
T_c [MeV]	18.9	19.2	19.7	17.9 ± 0.4
n_c [fm^{-3}]	0.070	0.071	0.072	0.06 ± 0.01
p_c [MeV fm^{-3}]	0.48	0.50	0.52	0.31 ± 0.07
K_0 [MeV]	553	674	763	250 - 315

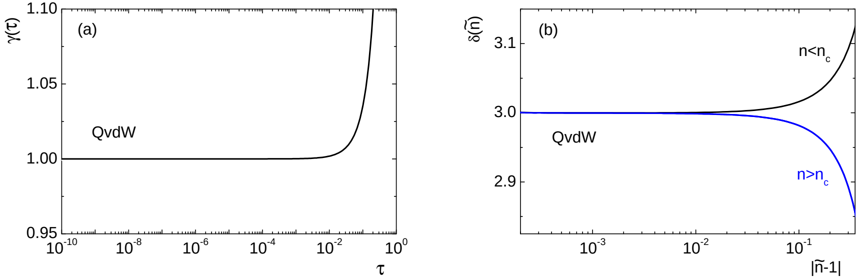




[1] Elliott, Lake, Moretto, Phair, Phys. Rev. C (2013).

Critical exponents

	1	2	3	4
exponent	scaling	path	mean-field theory	empirical
α	$c_{\rm V} = A\,\tau^{-\alpha}$	$\tau \to 0^+\,,\ \widetilde{n} = 1$	0	0.11
β	$\frac{\widetilde{n}_{\rm l} - \widetilde{n}_{\rm g}}{2} = B\,(-\tau)^{\beta}$	$\tau \to 0^-$	$\frac{1}{2}$	0.33
γ	$\kappa_T = p_c^{-1}\,G\,\tau^{-\gamma}$	$\tau \to 0^+\,,\ \widetilde{n} = 1$	1	1.24
δ	$\widetilde{p} - 1 = D\,\lvert\widetilde{n} - 1\rvert^{\delta}\,{\rm sgn}(\widetilde{n} - 1)$	$n \to 1^{\pm}\,,\ \tau = 0$	3	4.79



Critical amplitudes: $G \simeq 0.257$, $D \simeq 1.4$.

Summary

1. Van der Waals equation can be generalized to include quantum statistics effects.
2. Applied to the description of nuclear matter the model predicts rich structures of fluctuations in the region of critical point.
3. In the region of critical point model predictions are very similar to the corresponding predictions of Walecka model.

Thank you for attention!

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