Hadron resonance gas with repulsive interactions and baryon rich matter

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- Lattice QCD and Hadron Resonance Gas (HRG) model
- The virial expansion in the nucleon gas
- Fluctuations and correlations of conserved charges HRG with with repulsive mean-field
- EoS at non-zero density

in collaboration with P. Huovinen, arXiv:1708.00879, work in progress

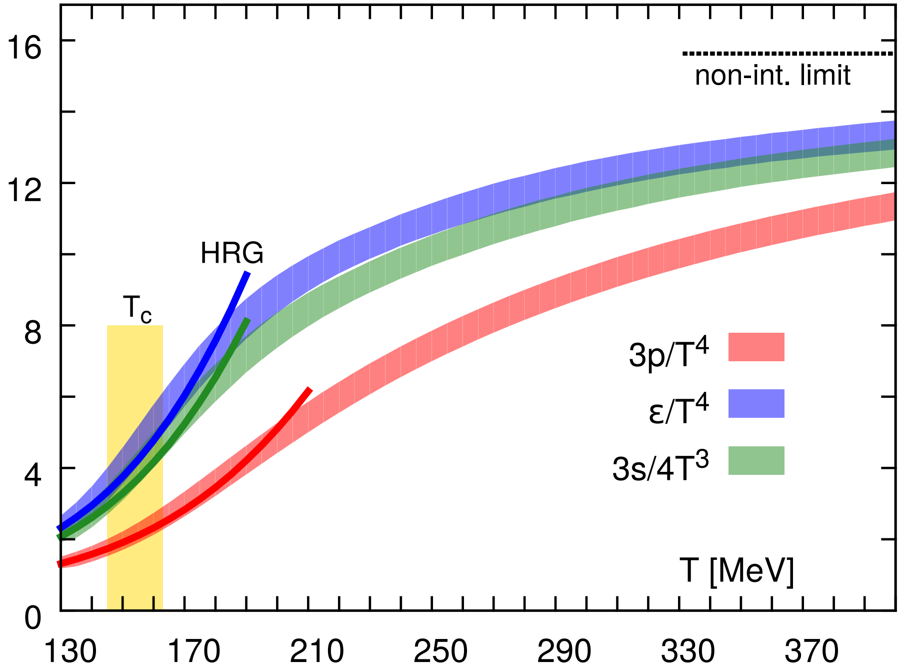
Zimányi School 2018, Budapest, December 4-8, 2017

Equation of state in the continuum limit

Equation of state has be calculated in the continuum limit up to *T=400* MeV

$p \sim \rho T, \;\; \rho \sim T^3$

in ultra-relativistic case

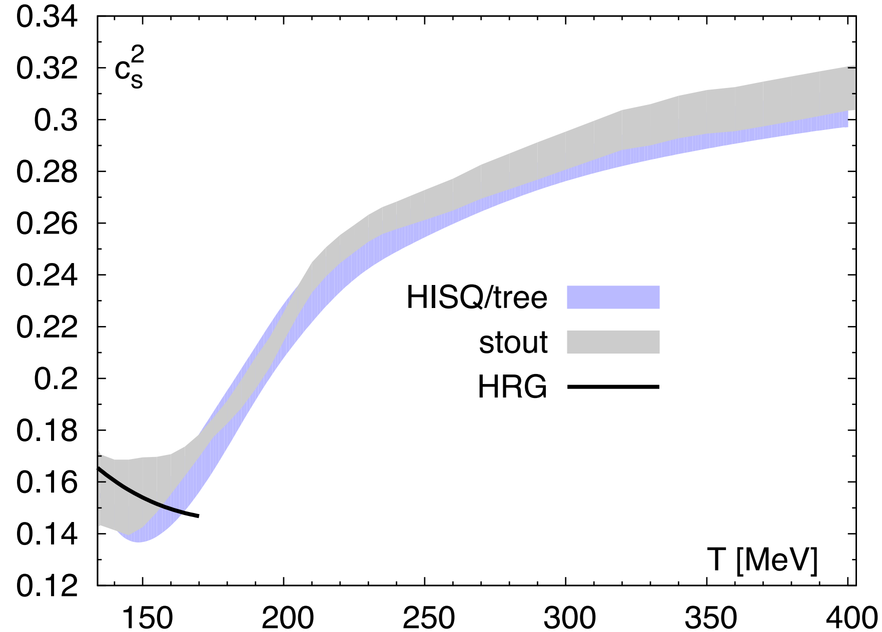


Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
(virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for *T< 145* MeV

Calculations that use two different discretization schemes agree:

Bazavov et al, PRD 90 (2014) 094503



$T_c = (154 \pm 9)\mathrm{MeV}$

$\epsilon_c \simeq 300\mathrm{MeV}/\mathrm{fm}^3$

$\epsilon_{low} \simeq 180\mathrm{MeV}/\mathrm{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\mathrm{MeV}/\mathrm{fm}^3$

$\epsilon_{high} \simeq 500\mathrm{MeV}/\mathrm{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\mathrm{MeV}/\mathrm{fm}^3$

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

⇓

information about carriers of the conserved charges (hadrons or quarks)

⇓

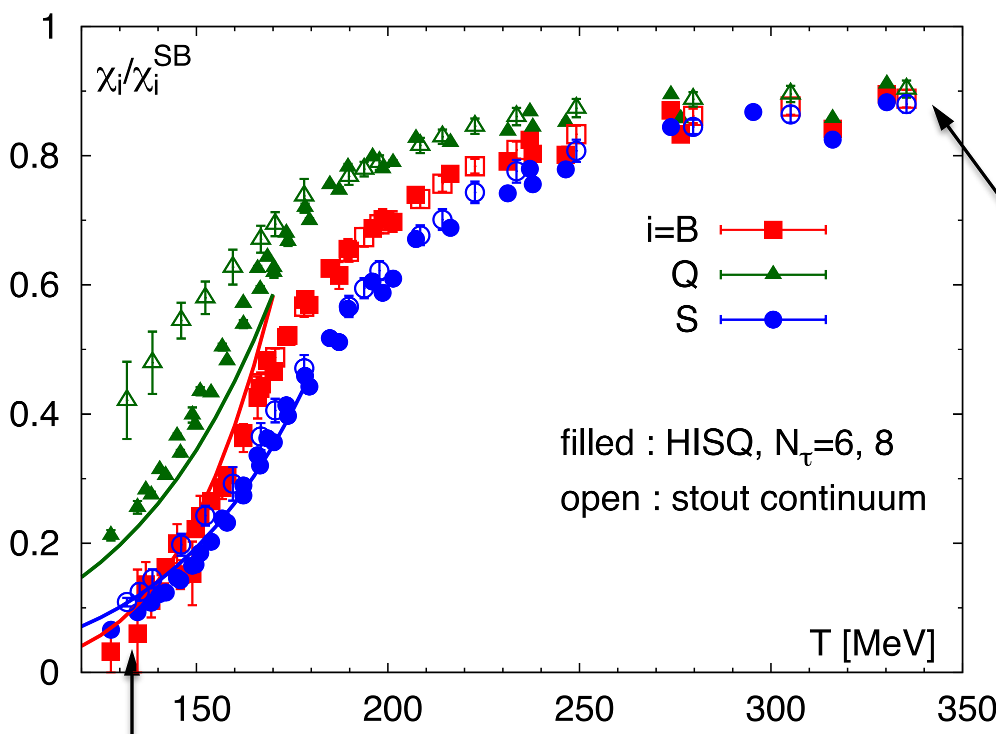
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

HotQCD: PRD86 (2012) 034509

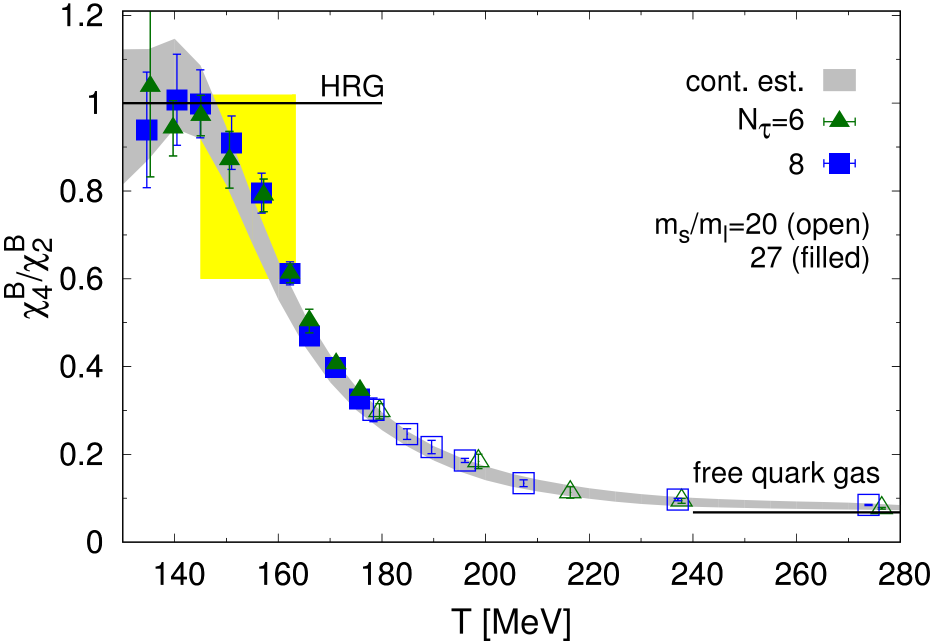
BW: JHEP 1201 (2012) 138,

Higher order fluctuations of conserved charges in T>0 QCD

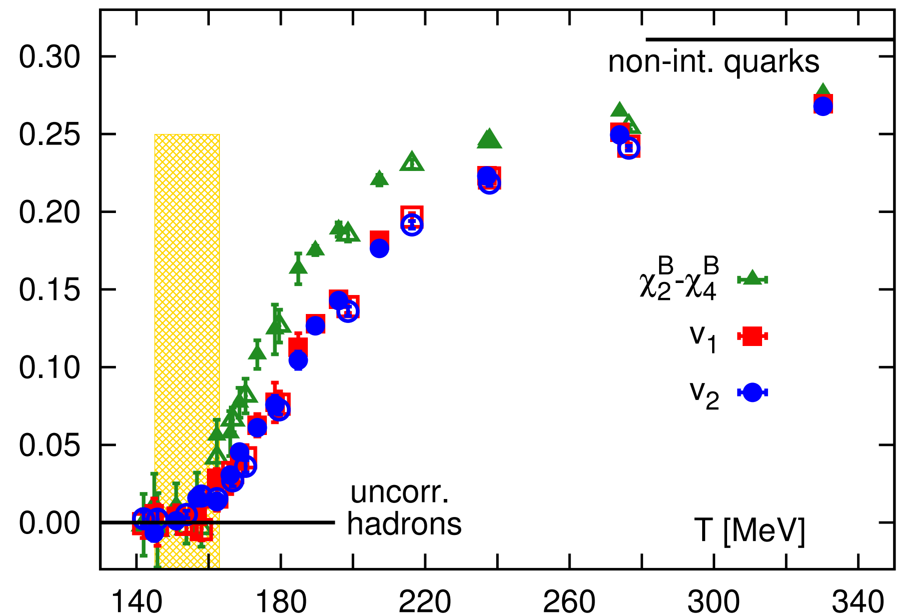
All hadrons except the pions are heavy $\Rightarrow$ use Boltzmann approximation

$p_{baryons}(T, \mu_B, \mu_S) = \sum_S p_S(T) \cdot \cosh((\mu_B + S\mu_S)/T)$

Bazavov et al, PRD 95 (2017)054504



Bazavov et al, PRL 111 (2013) 082301



$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

The above combinations should be 0 or 1 in HRG independent of details of hadron spectrum and HRG description breaks down close to the transition temperature (even below T_c)

Virial expansion in the nucleon gas

$$p = p^{ideal} + T \sum_{ij} b_2^{ij}(T) e^{\beta\mu_i} e^{\beta\mu_j}$$

b_2^{ij} can be related to the S-matrix of scattering of particles i and j

$\pi\pi$, KK, πN and NK scattering are dominated by resonances:

HRG model

$$p \rightarrow p^{ideal}_{\pi,K,N} + p^{ideal}_{resonances}$$

Dashen, Ma, Berstein, PR 187 (1969) 345
Prakash, Venugopalan, NPA 546 (1992) 718

No resonances in NN interactions

Gas of nucleons:

$$p(T,\mu) = p_0(T)\cosh(\beta\mu) + 2b_2(T) T \cosh(2\beta\mu)$$

$$p_0(T) = \frac{4M^2T^2}{\pi^2} K_2(\beta M)$$

$$b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE \left(\frac{ME}{2} + M^2\right) K_2\left(2\beta\sqrt{\frac{ME}{2}+M^2}\right) \frac{1}{4i} \mathrm{Tr}\left[S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S\right],$$

factorization in μ and T dependent part is broken

Virial expansion in the nucleon gas (cont'd)

Only the elastic part of the S-matrix is known

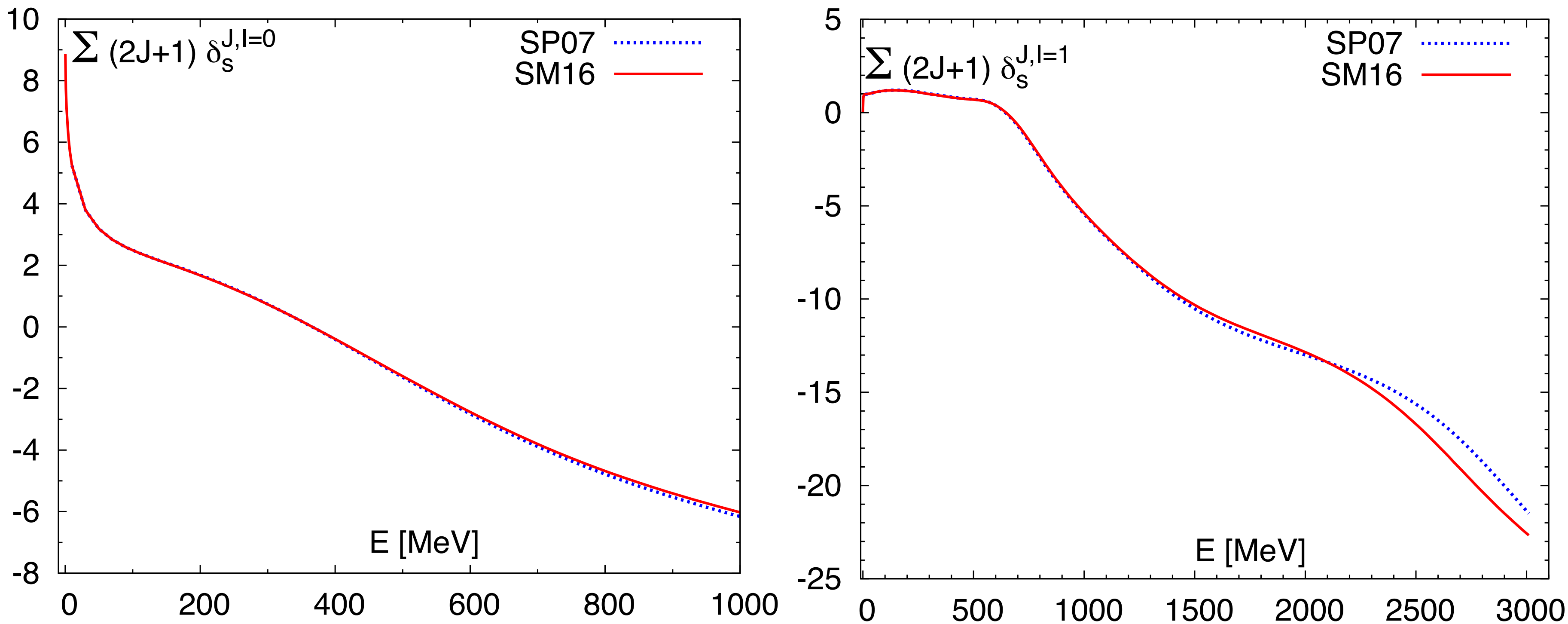
$$\frac{1}{4i}\mathrm{Tr}\left[S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE}S\right] \to \sum_{s=\pm}\sum_J (2J+1)\left(\frac{d\delta_s^{J,I=0}}{dE} + 3\frac{d\delta_s^{J,I=1}}{dE}\right)$$

Use recent partial wave analysis results for NN scattering (SM16, SP07)

Use effective range expansion for $E < 1$ MeV

Workman et al, PRC 94 (2016) 065203

Arndt et al, PRC 76 (2007) 025209



$$\frac{d}{dE}\sum_{J,s}(2J+1)\delta_s^{J,I} \Rightarrow b_2(T) < 0$$

Repulsive mean field in the nucleon gas

Assume that the repulsive interactions change the single nucleon energies by $U = Kn_b$, where n_b is the single nucleon density

$$K \sim \int d^3r V_{NN}(r) \Rightarrow K > 0$$

Nucleon and anti-nucleon densities

Olive, NPB 190 (1981) 483

$$n_b = 4\int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p-\mu+U)}, \quad \bar{n}_b = 4\int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p+\mu+\bar{U})}, \; E_p^2 = p^2 + M^2$$

$$\partial p/\partial \mu = n_b - \bar{n}_b \Rightarrow p(T,\mu) = T(n_b + \bar{n}_b) + \frac{K}{2}(n_b^2 + \bar{n}_b^2)$$

Small (zero) $\mu \Rightarrow \beta K n_b \ll 1$ and

$$n_b \simeq n_b^0(1 - \beta K n_b^0), \; \bar{n}_b \simeq \bar{n}_b^0(1 - \beta K \bar{n}_b^0) \Rightarrow$$

$$p(T,\mu) = T(n_b^0 + \bar{n}_b^0) - \frac{K}{2}\left(\left(n_b^0\right)^2 + \left(\bar{n}_b^0\right)^2\right)$$

or

$$p(T,\mu) = p_0(T)(\cosh(\beta\mu) - \frac{KM^2}{\pi^2} K_2(\beta M)\cosh(2\beta\mu))$$

Comparison of repulsive mean field and virial expansion

Repulsive mean field

$$p(T,\mu) = p_0(T) \times$$

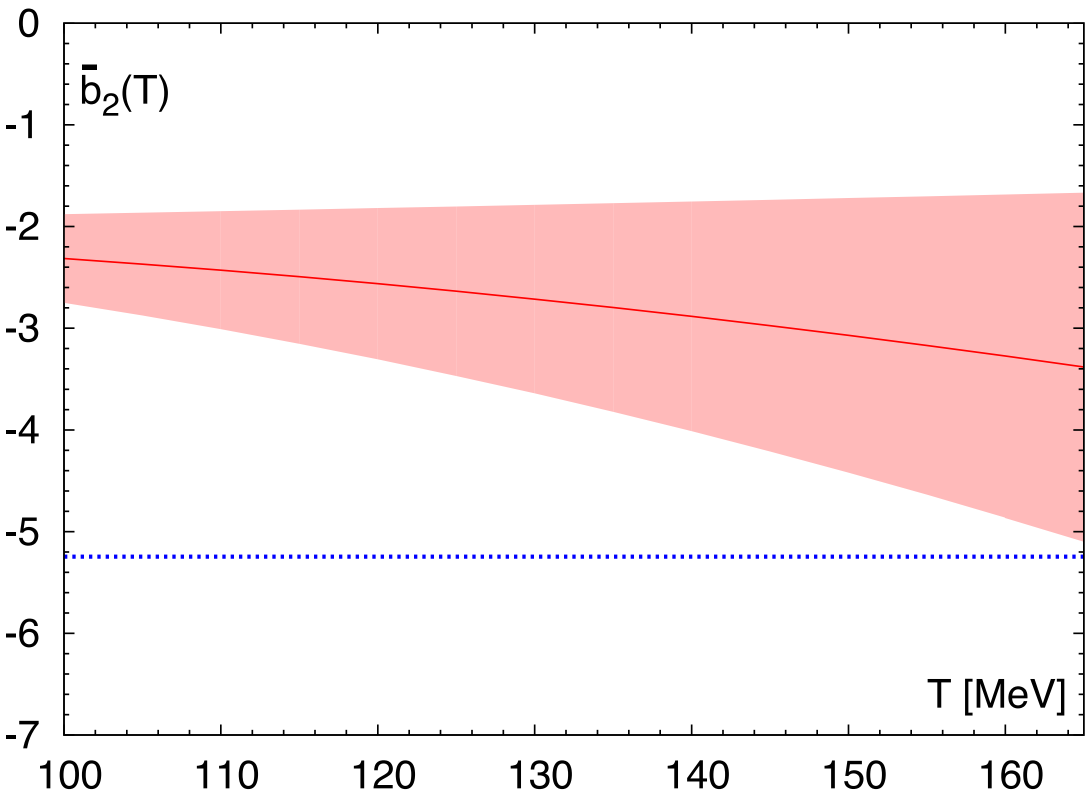
$$(\cosh(\beta\mu) - \frac{KM^2}{\pi^2} K_2(\beta M) \cosh(2\beta\mu))$$

2nd order virial expansion

$$p(T,\mu) = p_0(T) \times$$

$$(\cosh(\beta\mu) + \bar{b}_2(T) K_2(\beta M) \cosh(2\beta\mu))$$

$$\bar{b}_2(T) = \frac{2T b_2(T)}{p_0(T) K_2(\beta M)}$$



In-elastic interactions become important for $E > 400$ MeV $\Rightarrow$ use σ_{el}/σ_{tot} to estimate the uncertainties in $b_2(T)$ due to these effects

$$-\frac{KM^2}{\pi^2}$$

for typical phenomenological value $K = 450$ MeV fm^3

Sollfrank et al, PRC 55 (1997) 392

Hadron resonance gas with repulsive mean field

$$n_B(T,\mu_B,\mu_S,\mu_Q) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta\mu_{i,eff}}, \qquad \mu_{i,eff} = \sum_j q_i^j \mu_j - K n_B$$

$$\bar{n}_B(T,\mu_B,\mu_S,\mu_Q) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta\bar{\mu}_{i,eff}}, \qquad \bar{\mu}_{i,eff} = -\sum_j q_i^j \mu_j - K \bar{n}_B$$

$$(q_i^1, q_i^2, q_i^3) = (B_i, S_i, Q_i)$$

strange and non-strange baryons interact the same way

$\partial p/\partial\mu_B = n_B - \bar{n}_B$ and leading order expansion in $\beta K n_B \Rightarrow$

$$p_B(T,\mu_B,\mu_S,\mu_Q) = T(n_B^0 + \bar{n}_B^0) - \frac{K}{2}\left(\left(n_B^0\right)^2 + \left(\bar{n}_B^0\right)^2\right)$$

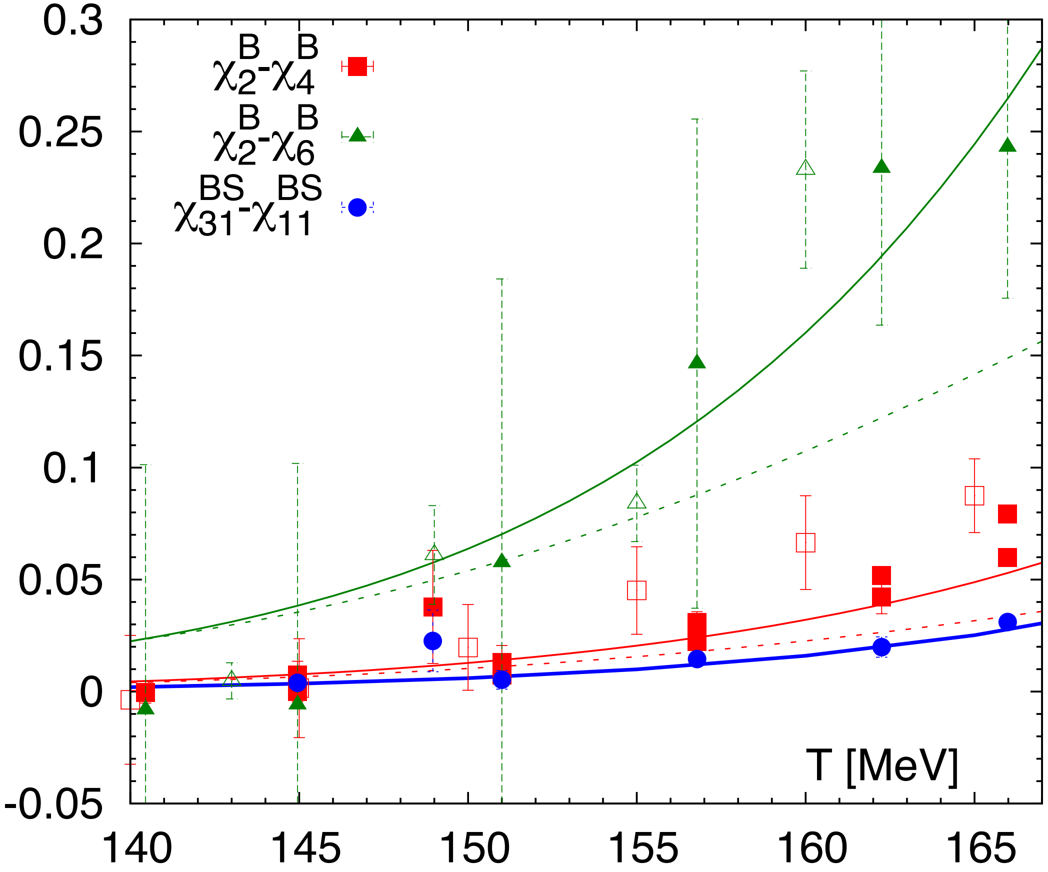
$$\Downarrow$$

$$\begin{aligned} \chi_n^B &= \chi_n^{B(0)} - 2^n \beta^4 K \left(N_B^0\right)^2, && (n \text{ even}) \\ \chi_{n1}^{BS} &= \chi_n^{BS(0)} + 2^{n+1}\beta^5 K N_B^0 (p_B^{S1} + 2p_B^{S2} + 3p_B^{S3}) && (n \text{ odd}) \end{aligned}$$

$$N_B^0(T) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i)$$

Comparison with lattice QCD results

Assume that only ground state baryons (octet + decuplet) contribute to n_B
higher resonances are treated as free particles



Filled symbols: HISQ

Bazavov et al,
PRL 111 (2013) 082301,
PRD 95 (2017) 054504

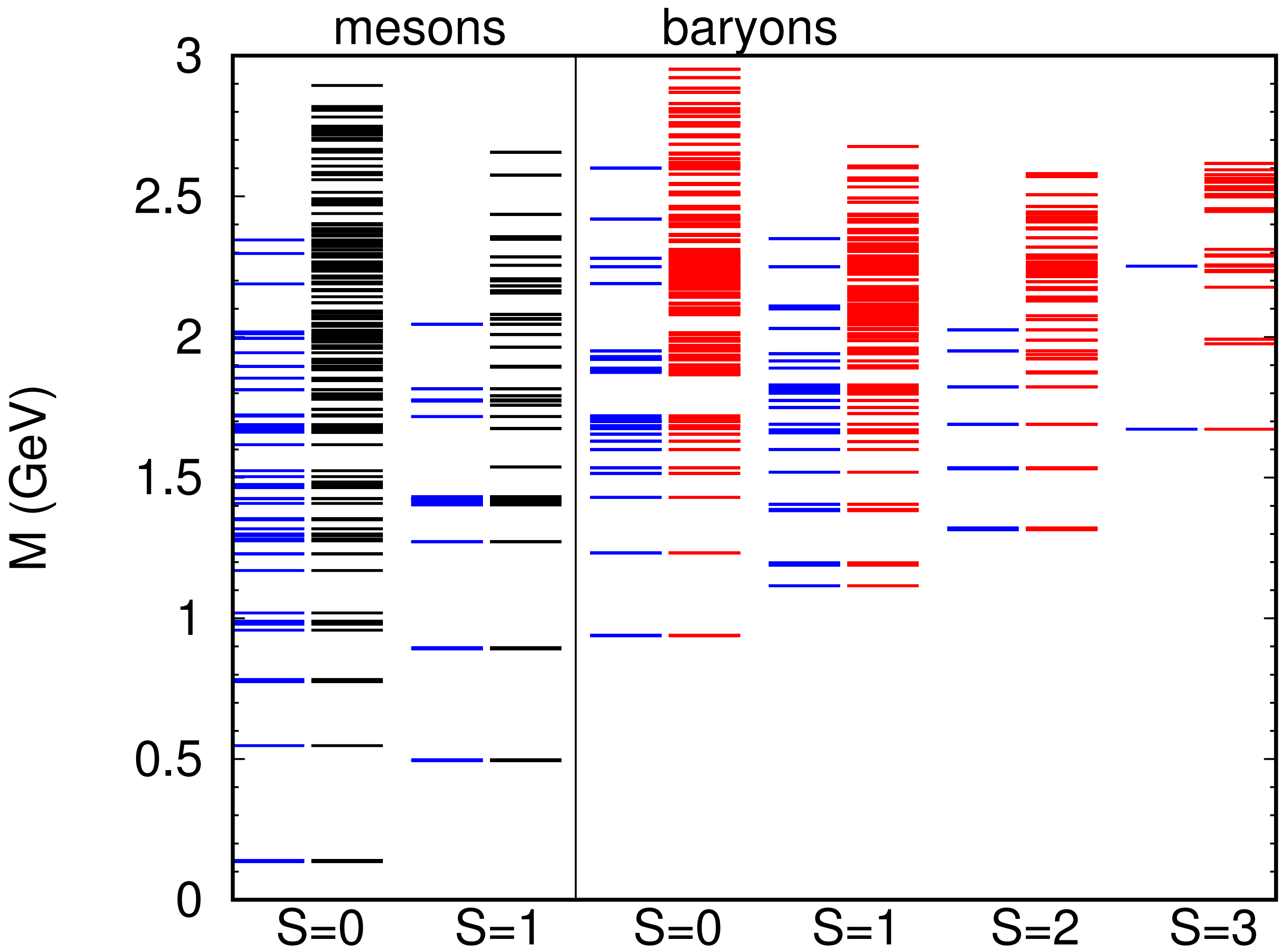
Open symbols: stout
4th order
Bellwied et al
PRD 92 (2015) 114505

6th order
D'Elia et al,
PRD 95 (2017) 094503

Repulsive mean field calculations can explain the differences between certain higher order fluctuations and correlations; v_2 is not described by this simple model

Hadron resonance gas and missing states

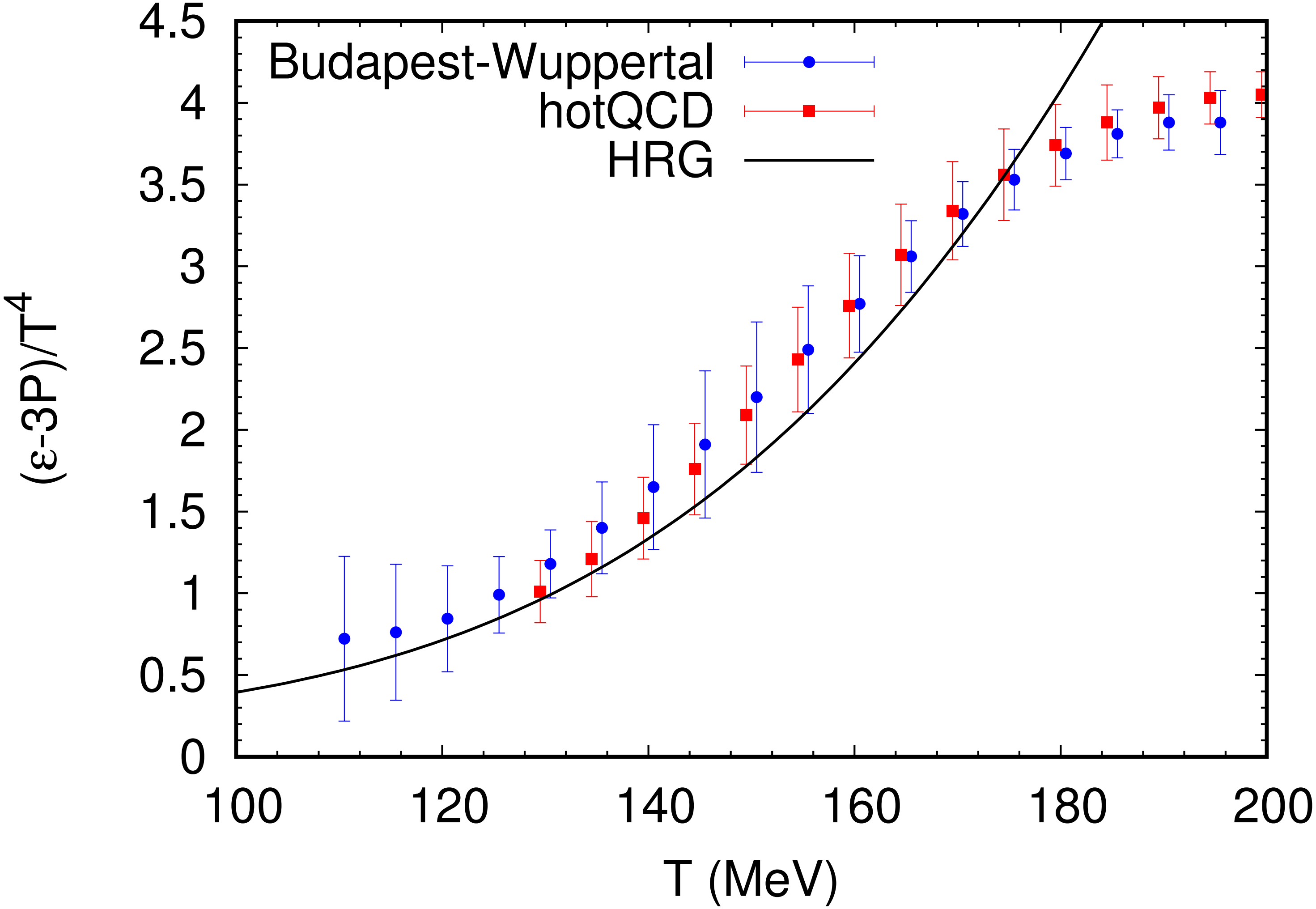
There are many more states that predicted by LQCD and quark models but are Included in PDG



Missing states are important for the fluctuations of conserved charges

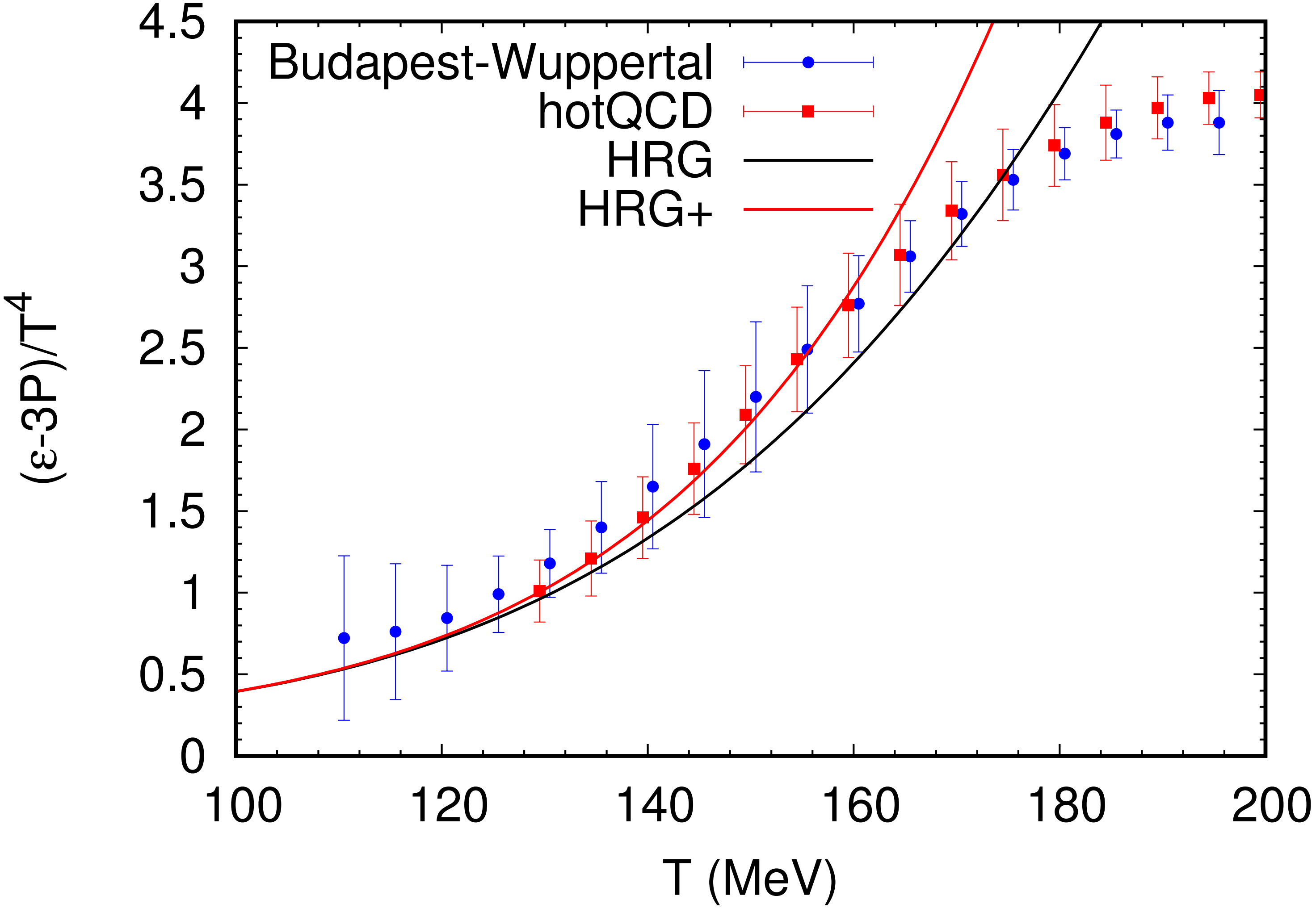
Bazavov, PRL 113 (2014) 072001

Missing states and the trace anomaly



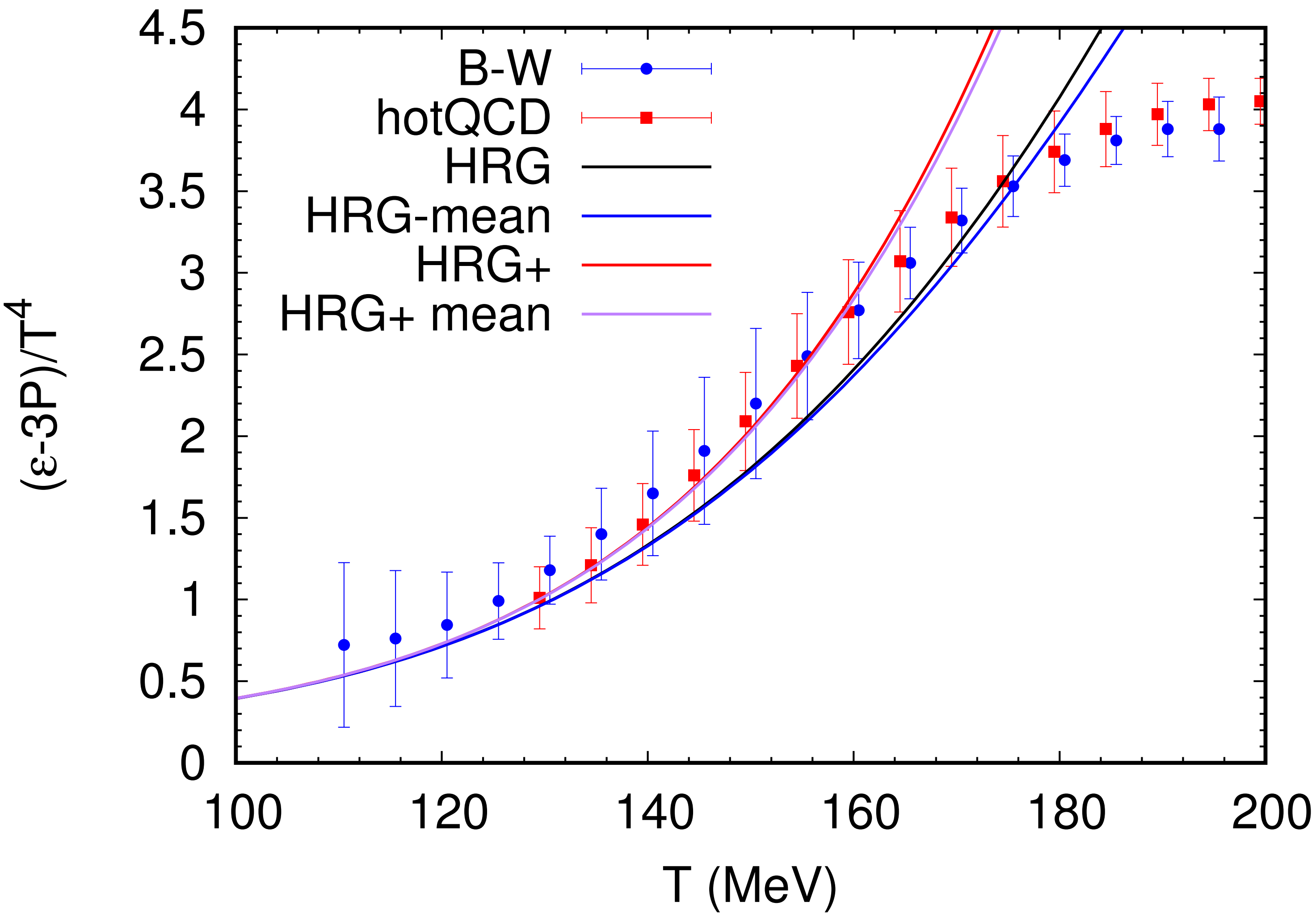
- HRG is below the lattice data

Missing states and the trace anomaly



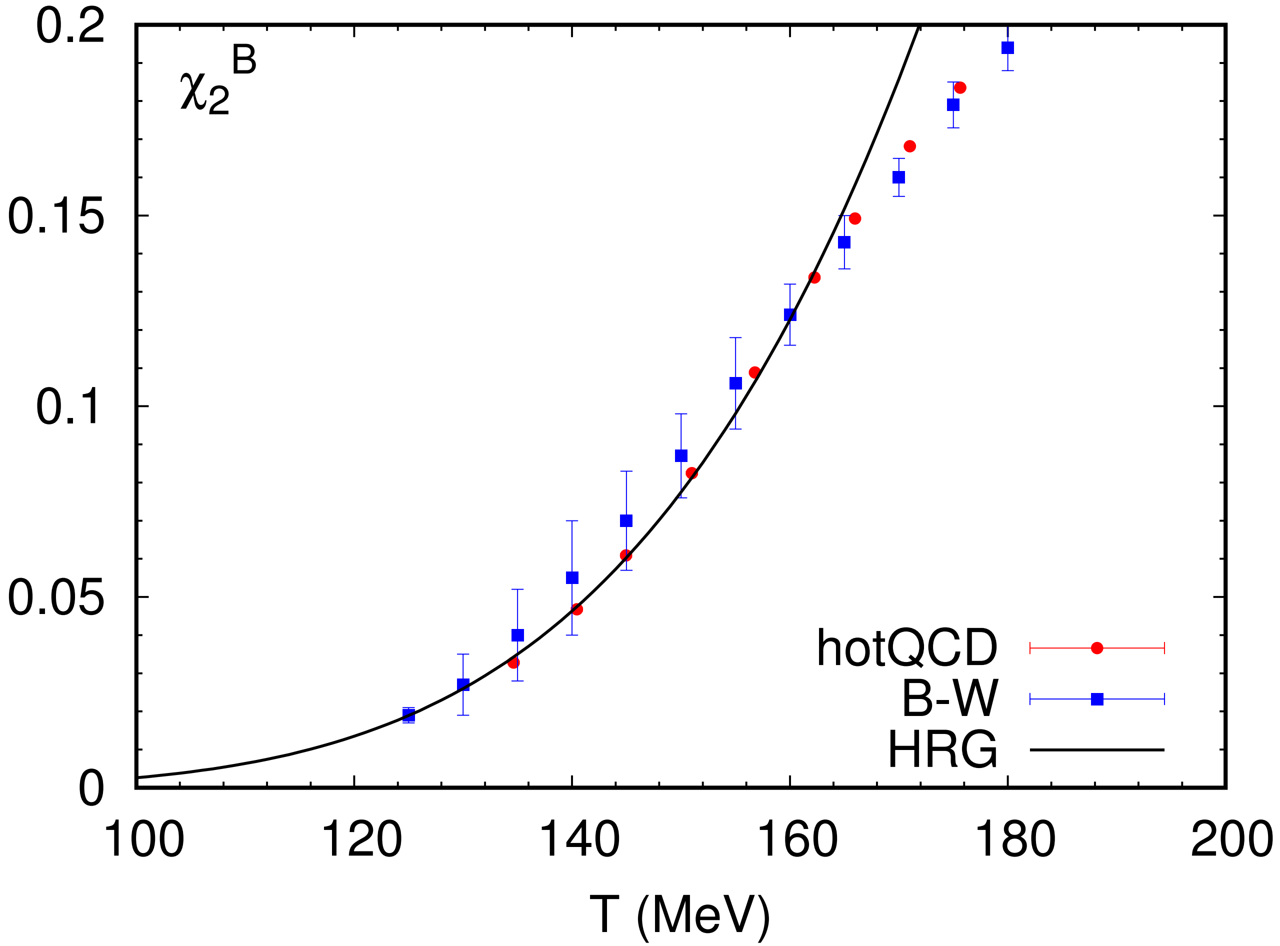
- HRG is below the lattice data
- HRG with missing states (HRG+) describes the data better

Missing states and the trace anomaly



- HRG is below the lattice data
- HRG with missing states (HRG+) describes the data better
- Repulsive mean-field slightly reduces the HRG result

Baryon number fluctuations: missing states and repulsive mean field



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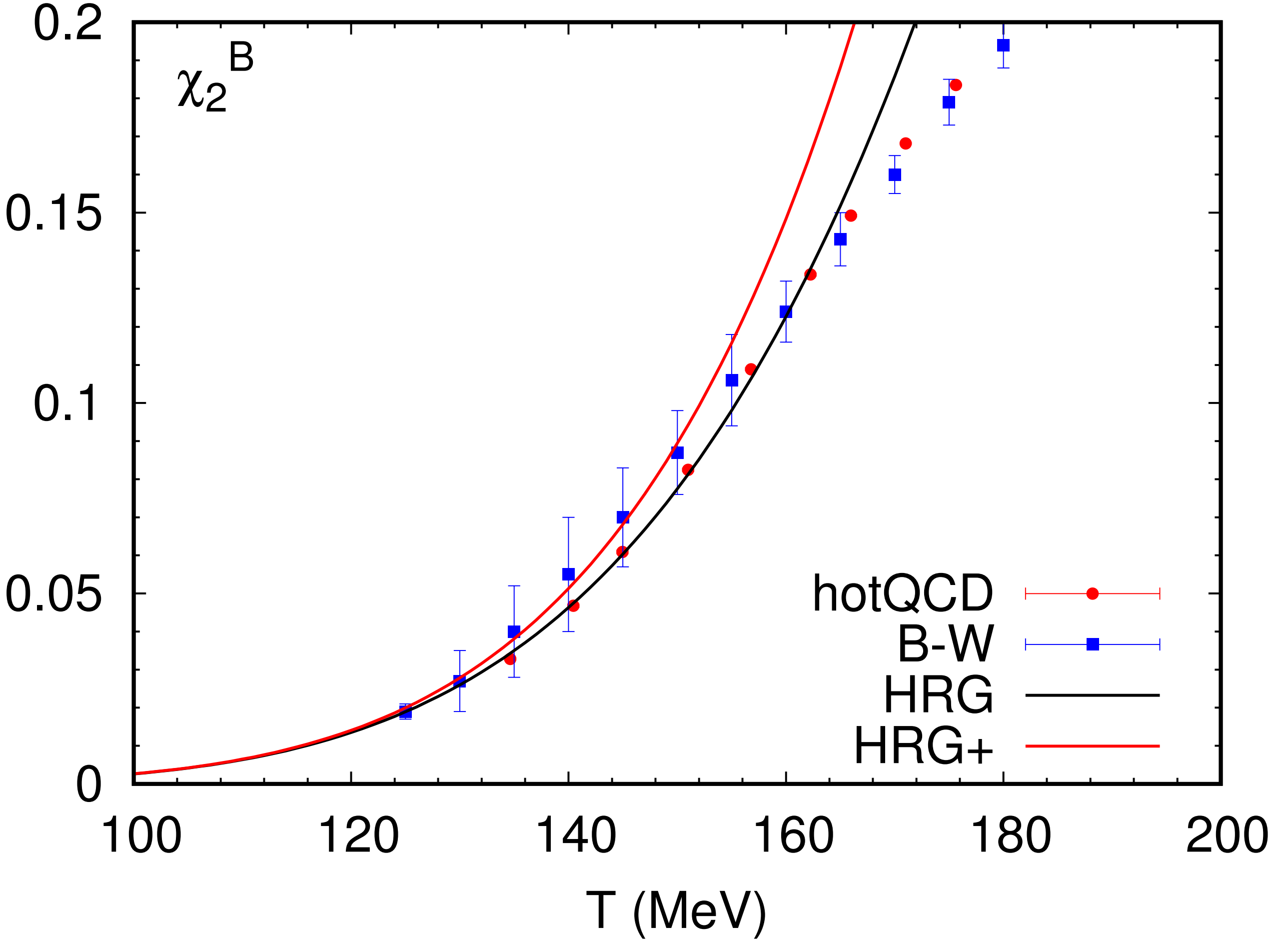

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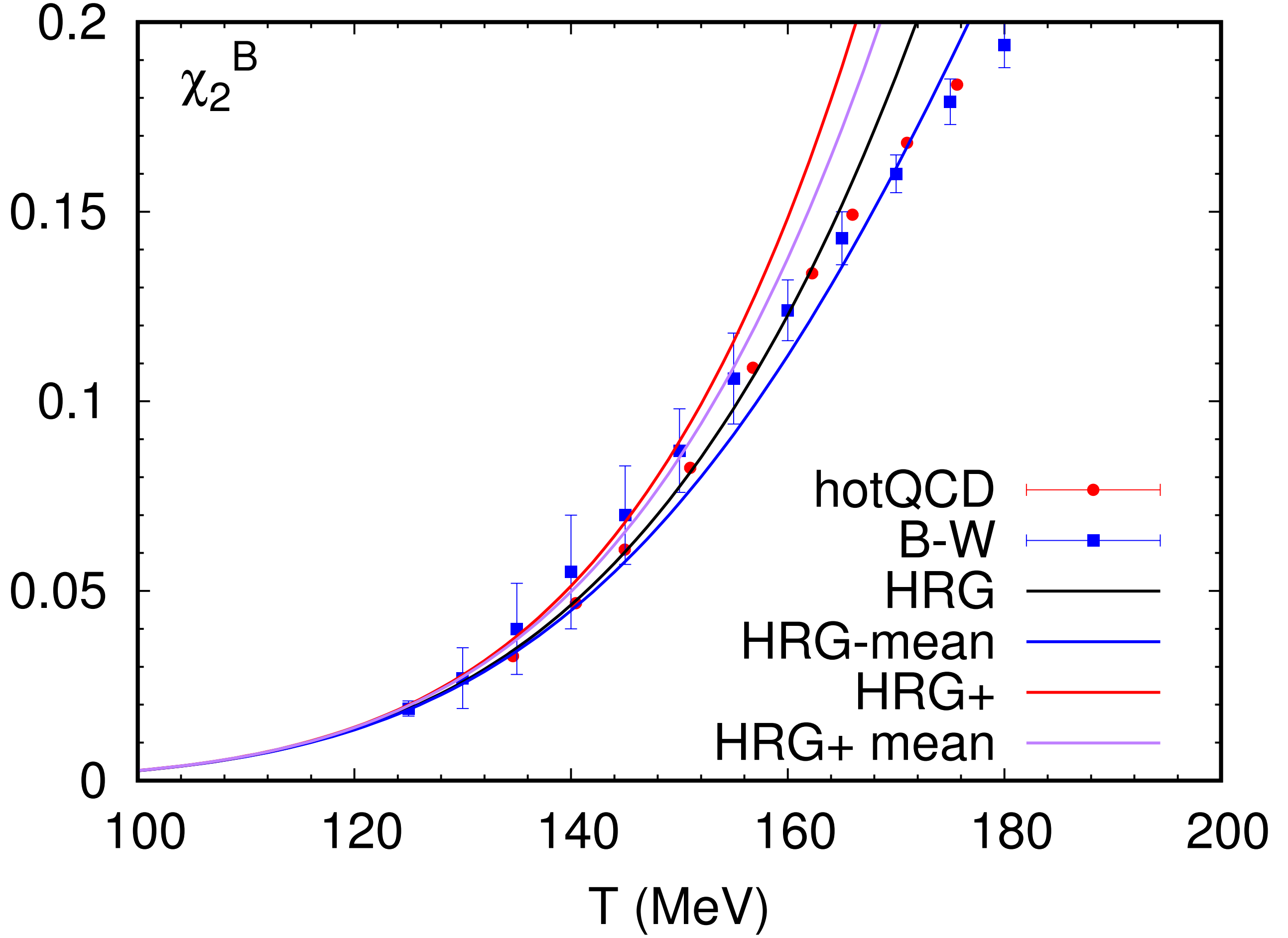

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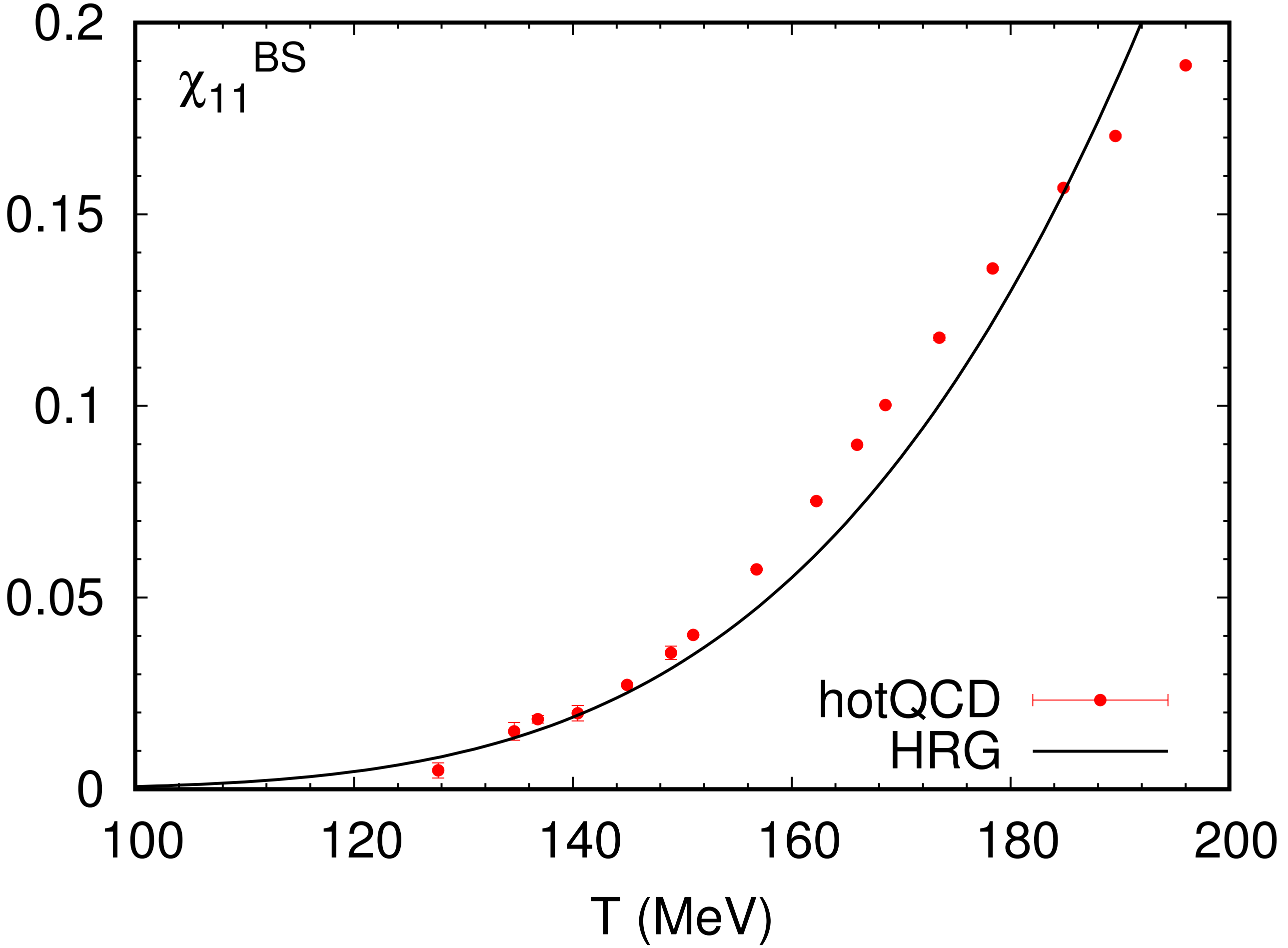
Baryon strangeness correlations:
missing states and repulsive mean field



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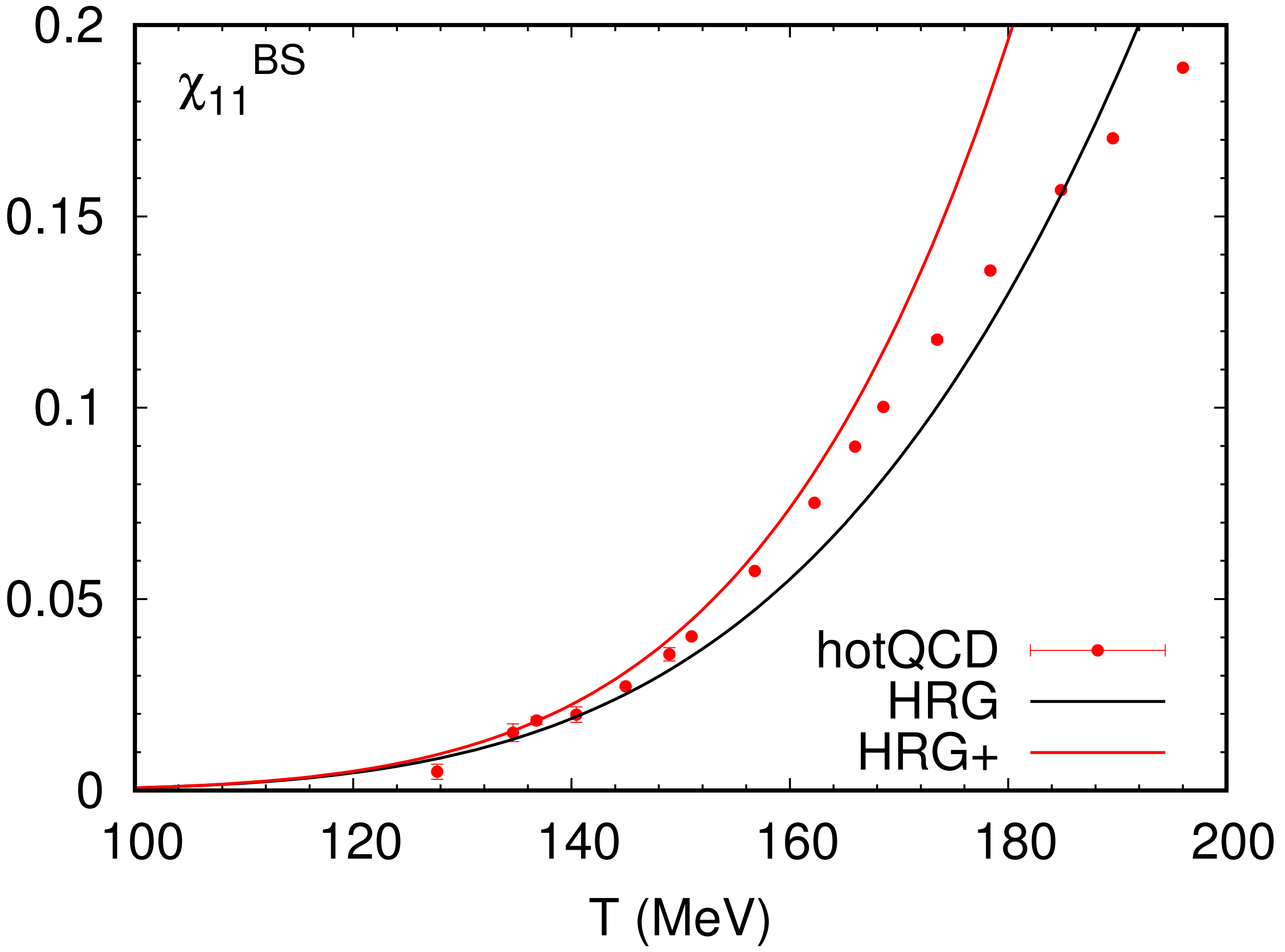

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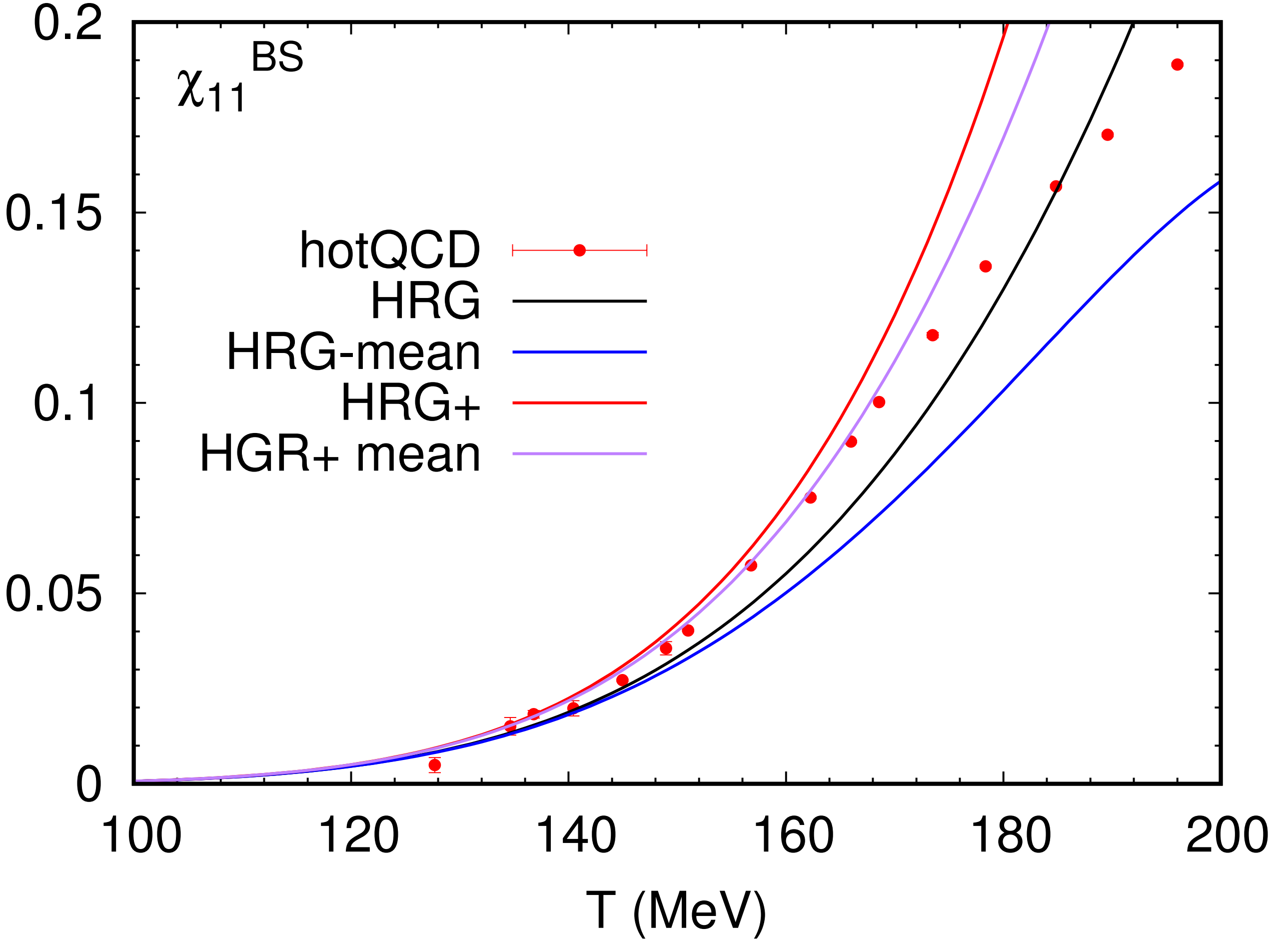
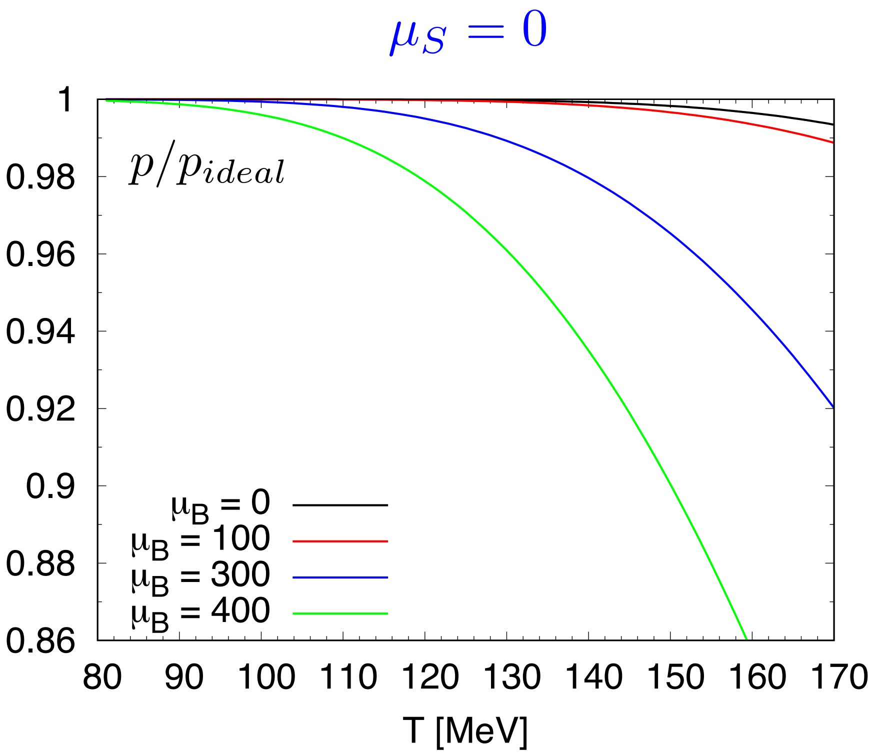
Baryon strangeness correlations: missing states and repulsive mean field

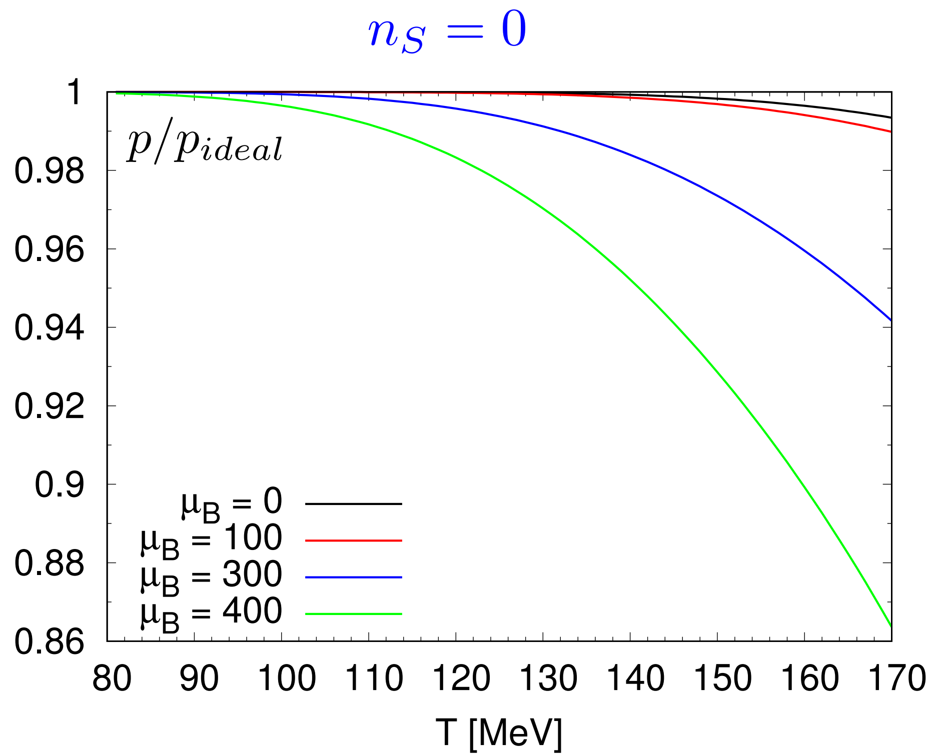


- HRG is below the lattice data
- HRG with missing states (HRG+) slightly over predicts the lattice data
- Repulsive mean-field reduces the HRG+ result

Pressure with repulsive mean field

Use expanded expressions (in $K\, n_B$) to calculate the pressure



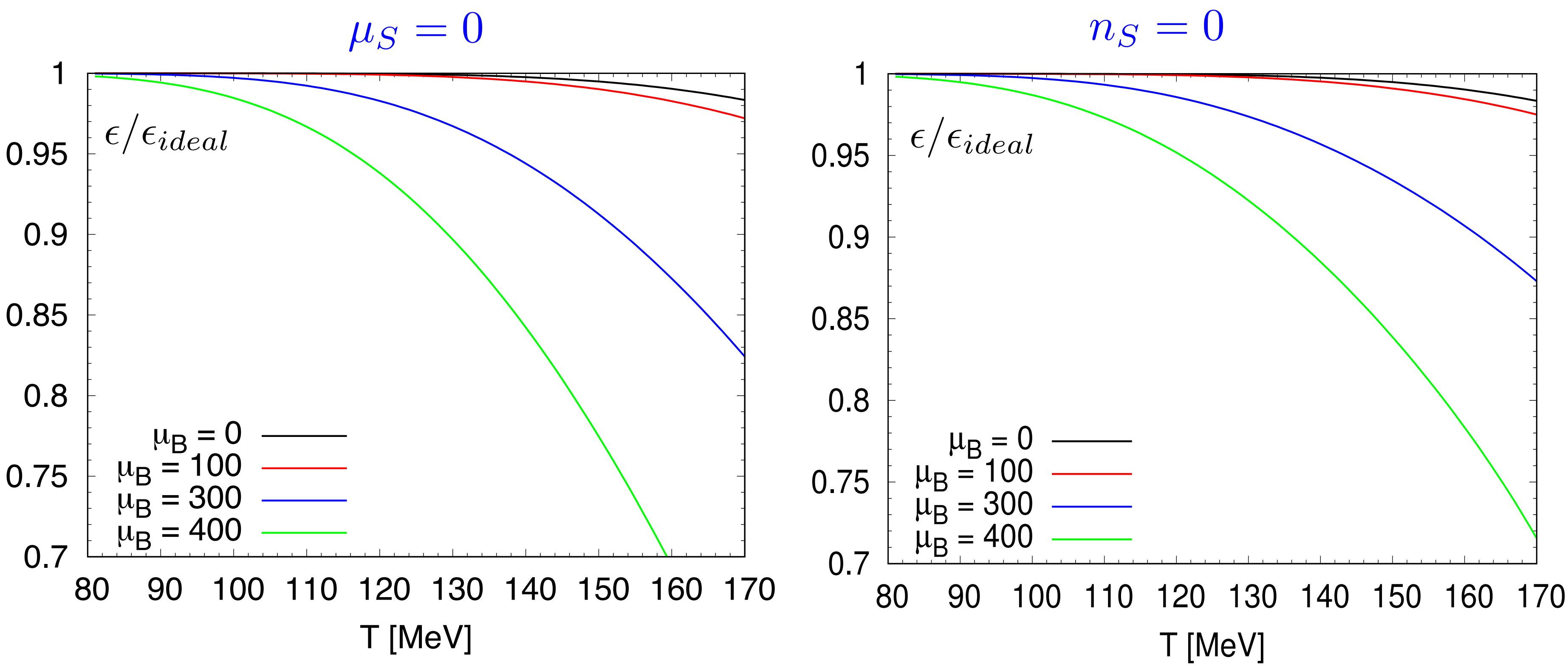


Virial expansion works only for baryon chemical potential < *400* MeV

The repulsive mean field reduces the pressure up 24%

For the strangeness neutral case the effects of the repulsive interactions are smaller.

Energy density with repulsive mean field

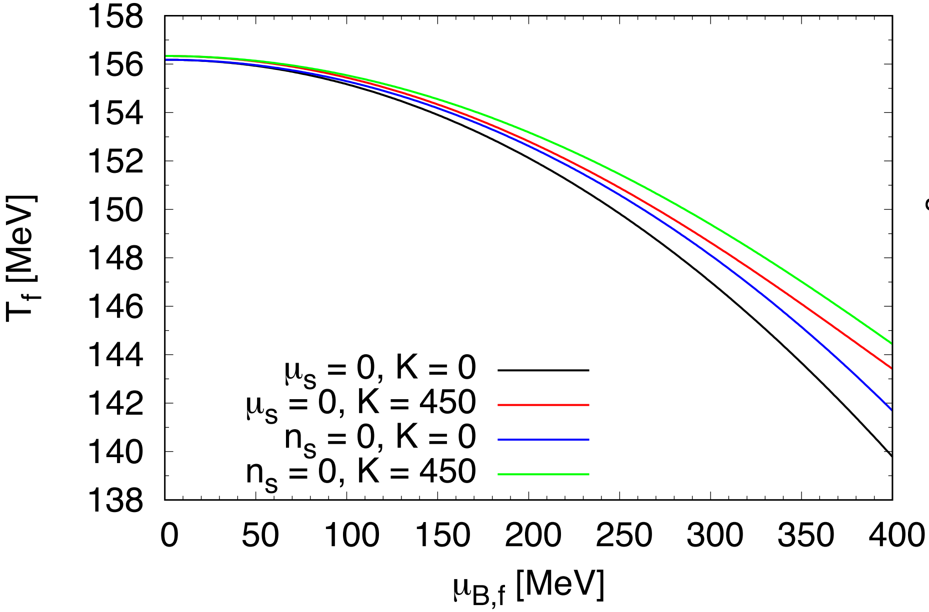


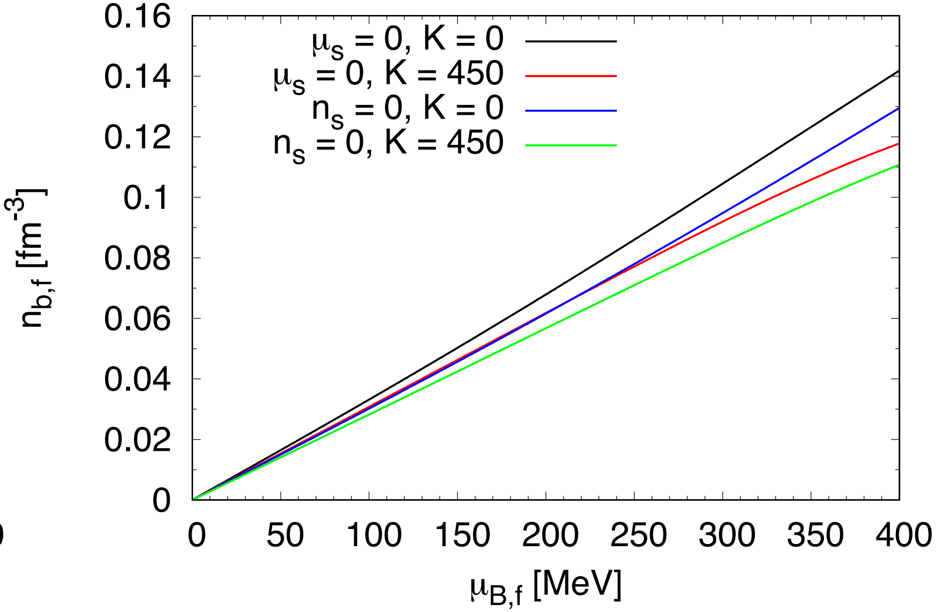
Repulsive mean field reduces the energy density up to 30%

For the strangeness neutral case the effects of the repulsive interactions are smaller.

Freezout at constant energy density

Assume that freeze-out happens at energy density of 330 MeV/fm^3

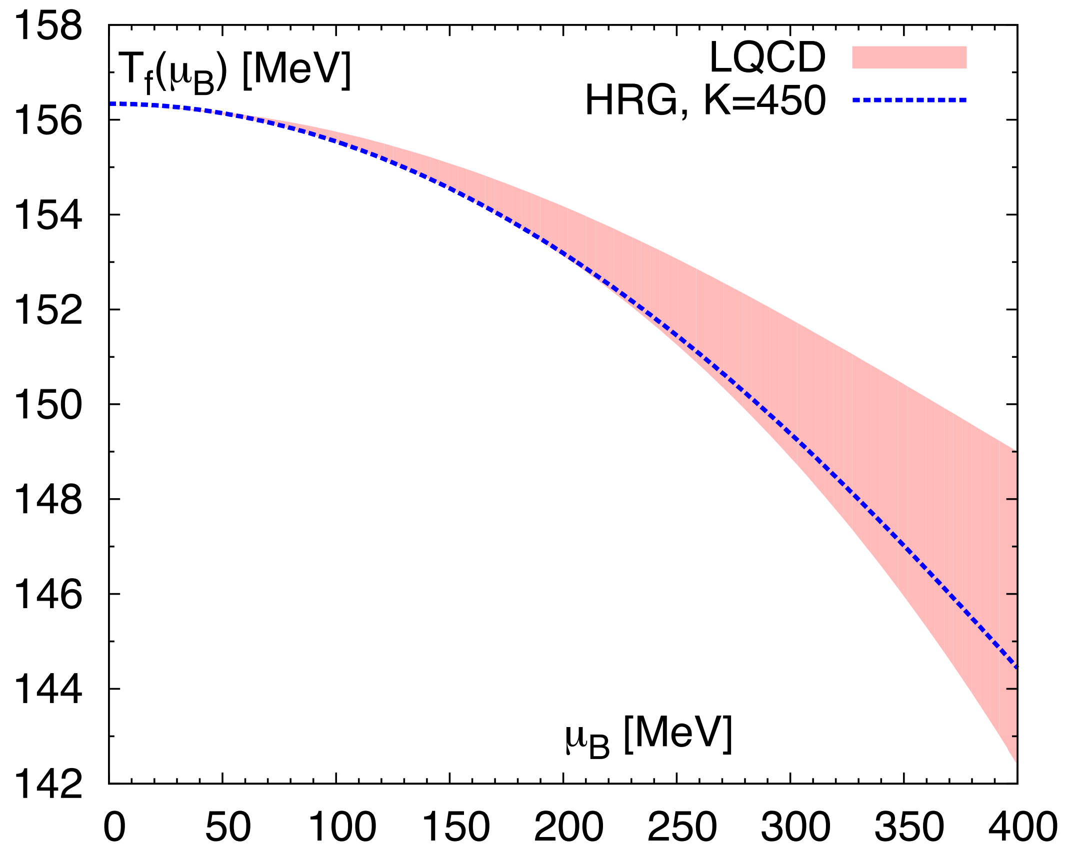




Strangeness neutrality and repulsive interaction reduce the curvature of the freeze-out temperature

Strangeness neutrality and repulsive interactions reduce the net baryon density at the freeze-out

Freeze-out line in HRG vs. lattice



The curvature of the freeze-out line corresponding to constant energy density ~330 MeV/fm^3 calculated in HRG model with repulsive interactions agrees with lattice result of Bazavov et al, PRD 95 (2017) 054504

Summary

- Repulsive baryon-baryon interactions are important for baryon number fluctuations and baryon strangeness correlations as well as for EoS at non-zero baryon density, their effect could be similar to the effect of missing states in terms of size but in opposite direction

- Mean field approach is very similar to the virial expansion in the low density regime => constraints on the mean field values

- The simplest mean field approach can describe the differences between second and higher order baryon number fluctuations as well as baryon strangeness correlations, but certain baryon-strangeness correlations cannot be described by this simple model

- The virial expansion is applicable for baryon chemical potential < *400* MeV

- Future: separate treatment of strange non-strange baryons, including the effect of repulsive interactions on resonances