Transport coefficients of the Quark-Gluon Plasma in perturbative QCD

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Outline

- Transport in heavy ion collisions
- A modern approach to an effective kinetic theory for transport (and jets, thermalization, ...)
- Incorporating NLO (O(g)) and non-perturbative effects: testing the stability of these perturbative results

Pedagogical review in JG Teaney **1502.03730** (in QGP5)
Gritty details for jets in JG Moore Teaney **JHEP1603** (2016)
NLO transport JG Moore Teaney, in preparation

Overview

Flow: a bulk property

- Initial asymmetries in position space are converted by collective, macroscopic (many body) processes into final state momentum space asymmetries

- Quantitatively: azimuthal Fourier decomposition of the final state particle spectra

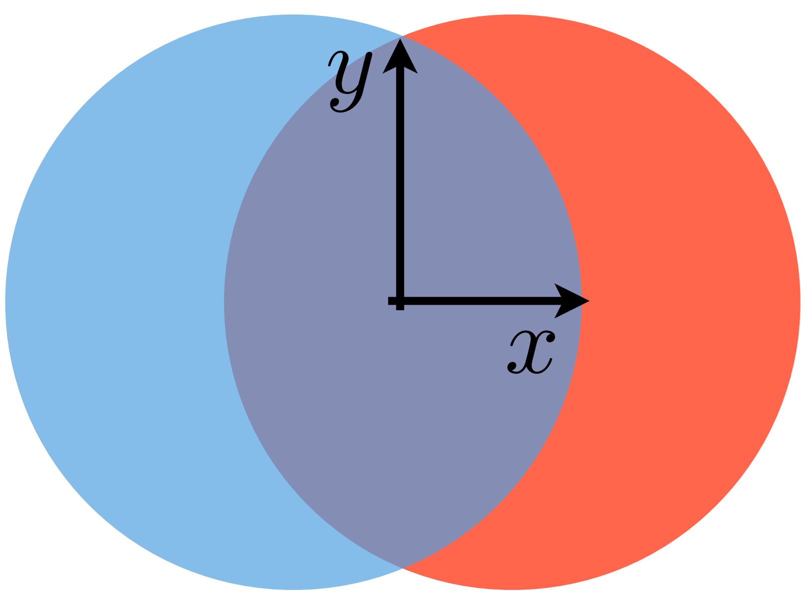
$$\frac{dN_i}{dy\, d^2p_T} = \frac{dN_i}{2\pi p_T dP_T dy}\left(1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y)\cos(n\phi)\right)$$

vzero amplitude + v_n coefficients

- 2D analogue of the multipole expansion of the CMB

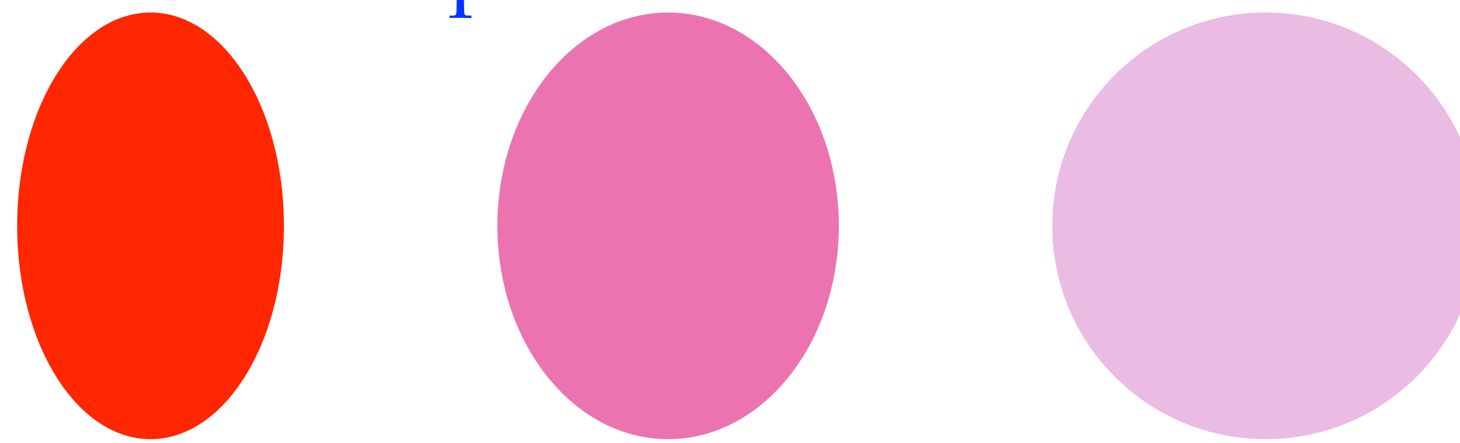
A famous example:elliptic flow

Initial asymmetry



Beam along z

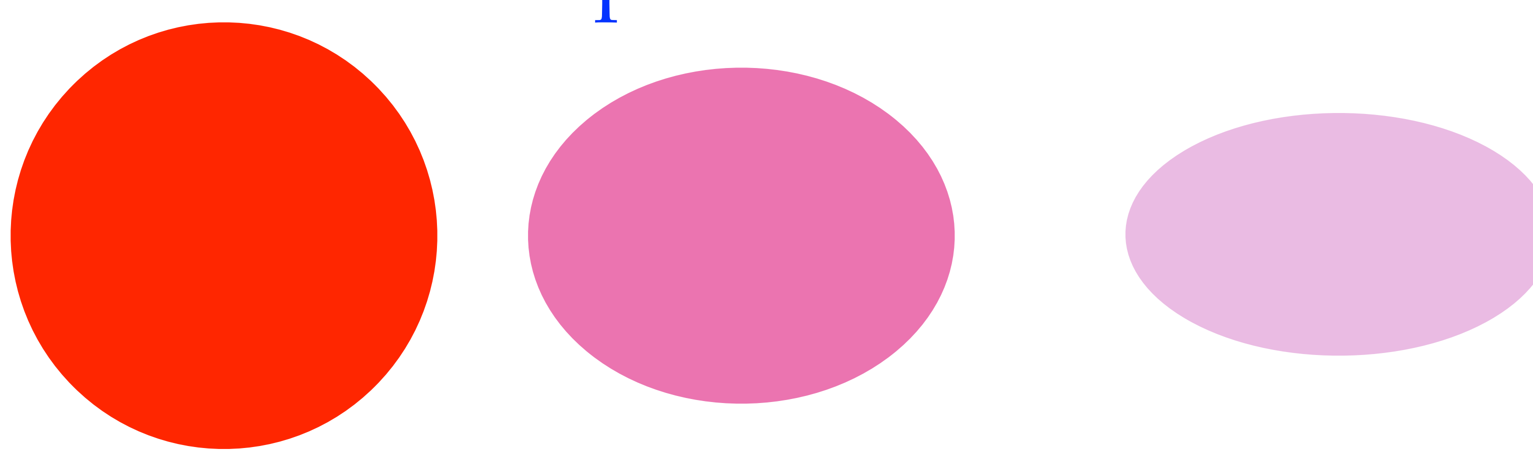
Position space



Large pressure gradients

No more flow

Momentum space



From initial symmetry

to final fixed anisotropy

- Hydrodynamics describes the buildup of flow. The shear viscosity parametrizes the efficiency of the conversion

Hydrodynamics

- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity

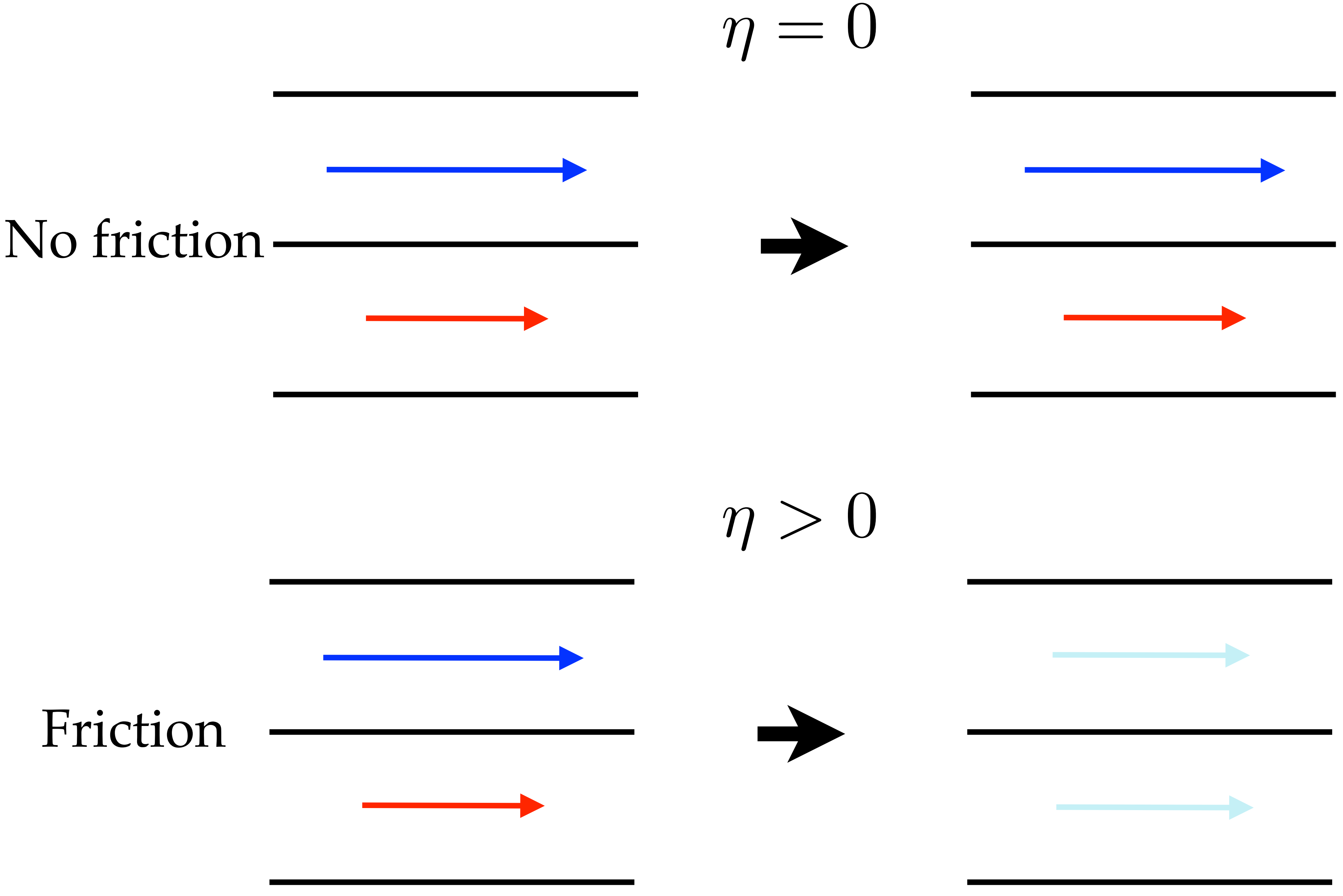
- For hydro fluctuations with local flow velocity $\mathbf{v}$ around an equilibrium state (with temp. T), at first order in the gradients and in $\mathbf{v}$

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

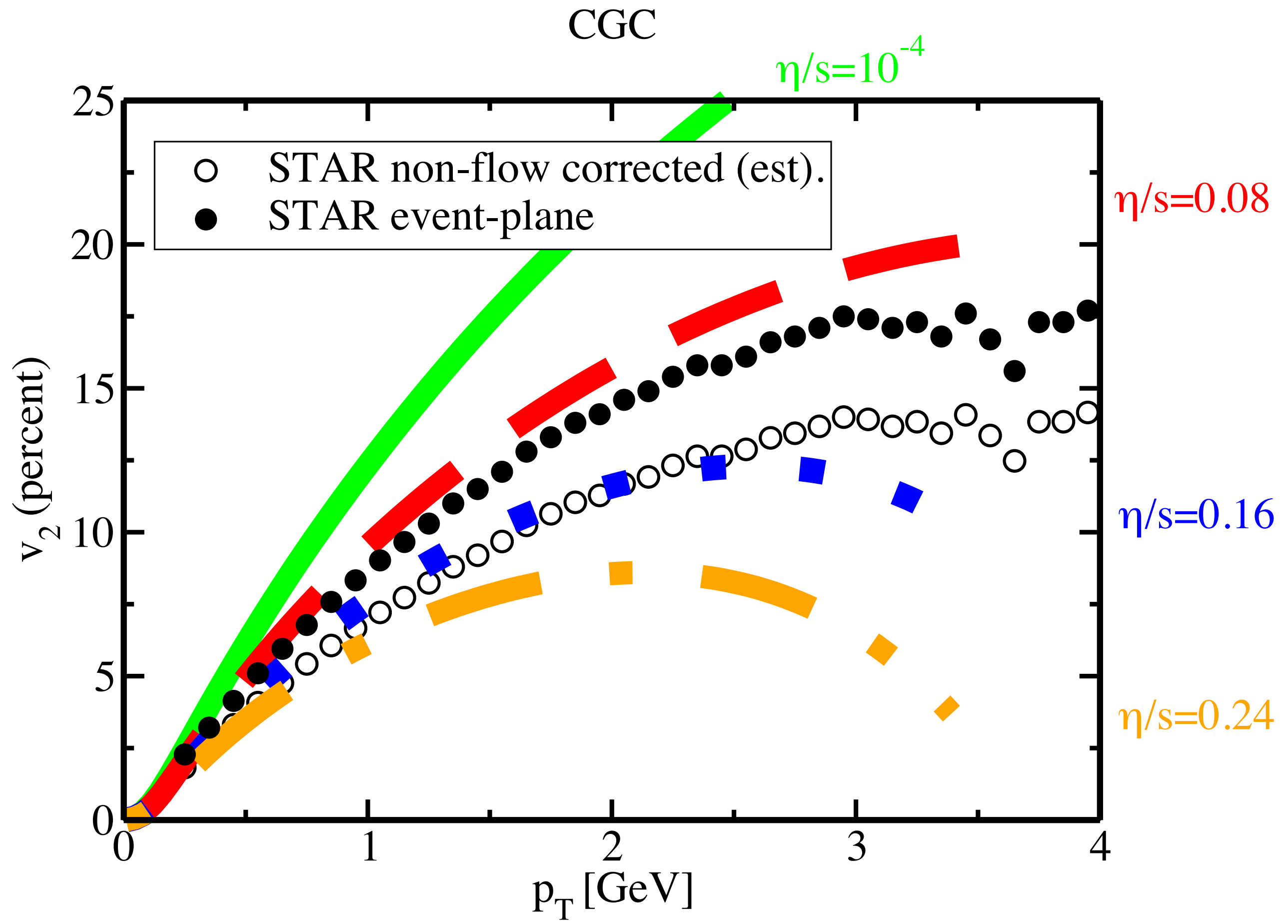
Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity

The shear viscosity



- Finite shear viscosity smears out flow differences (diffusion)

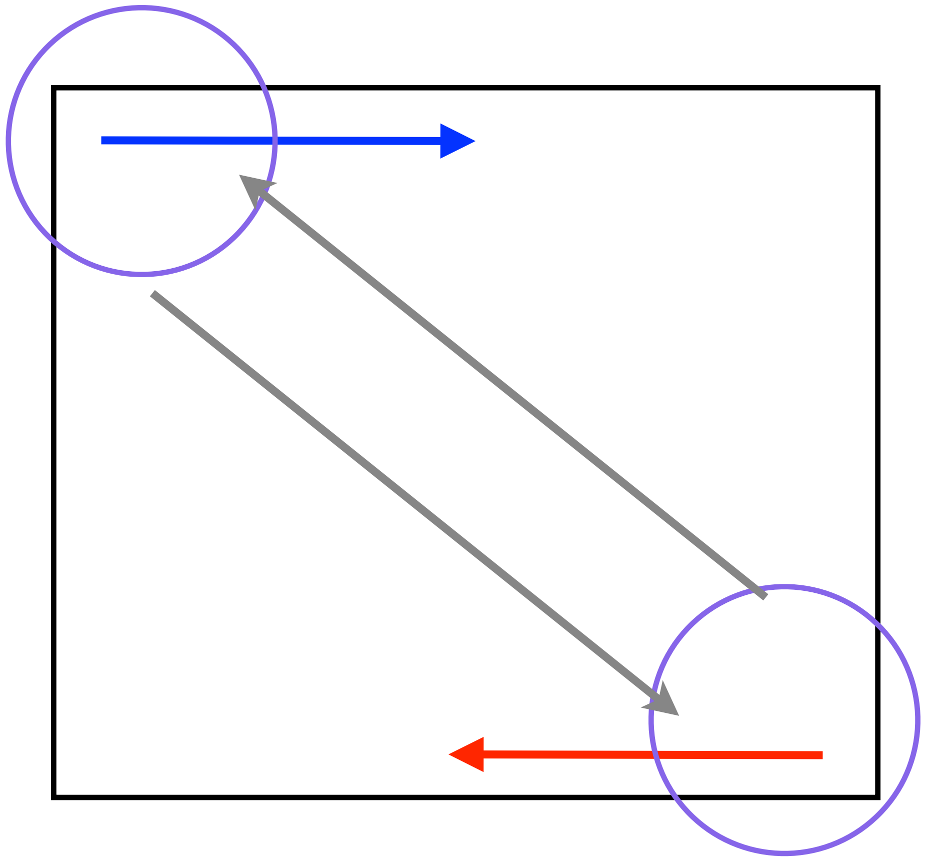
Hydro meets data

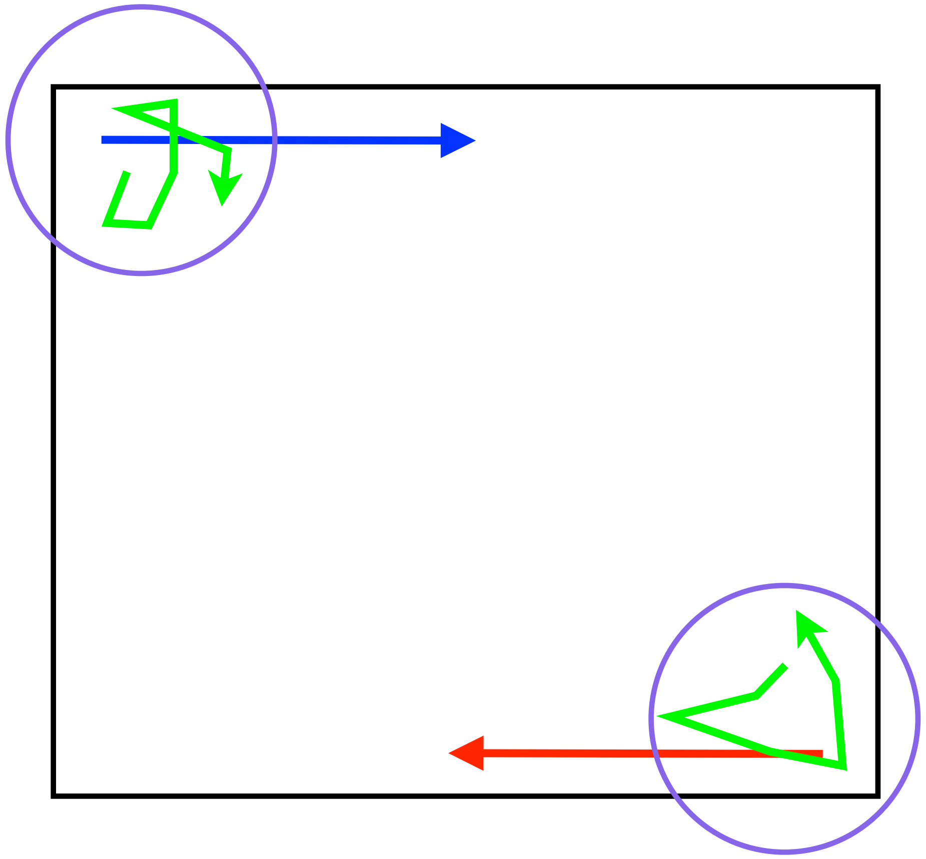


- The shear viscosity, being dissipative, smears out flow differences and makes the position→momentum conversion less efficient

  Plot from Luzum Romatschke **PRC78** (2008)

Estimating η: counterintuitive?





- Weak coupling: long distances between collisions, easy diffusion. **Large** η
- Strong coupling: short distances between collisions, little diffusion. **Small** η

Estimating η
(or why is η/s natural)

- u flow velocity, v_x microscopical velocity of particles

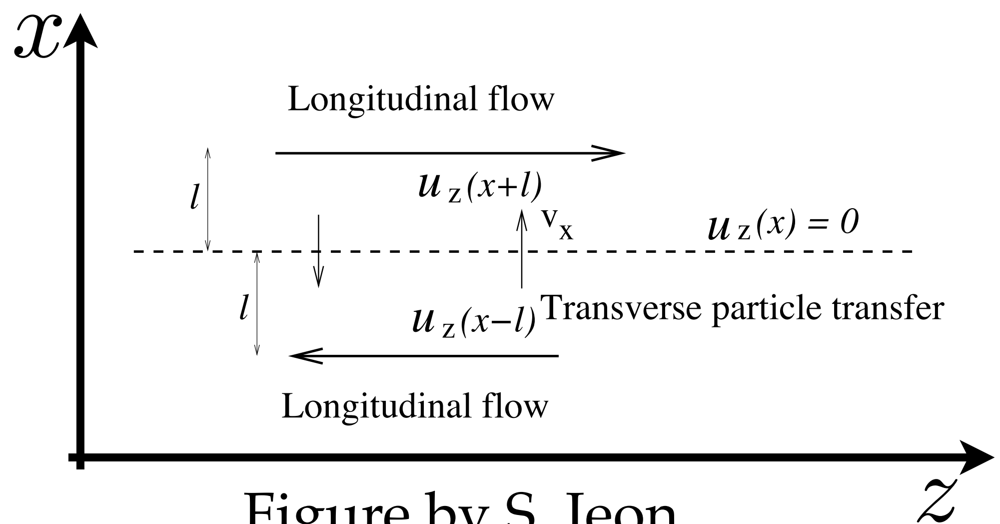


Figure by S. Jeon

- $T^{0z}=(e+P)u^0u^z$ diffuses along x with $v^x=u^x/u^0$. Net change

$$(e+p)v^x u^0 (u^z(x-l_{\rm mfp}) - u^z(x+l_{\rm mfp}) \approx -2(e+p)v^x u^0 l_{\rm mfp}\partial_x u^z(x) \sim -\eta u^0 \partial_x u^z(x)$$

- Using $e+p = sT$ and in the high-T limit ($v^x\sim 1$)

$$\boxed{\frac{\eta}{s} \sim T l_{\rm mfp}}$$

Estimating η
(or why is η/s natural)

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

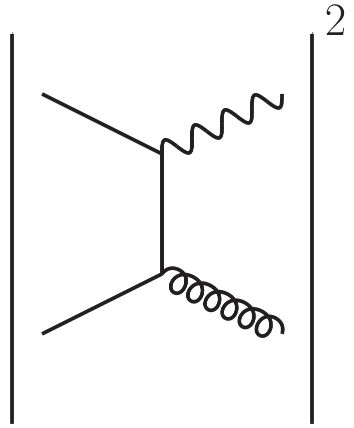
* Coulomb divergences and screening scales (m_D~gT) in gauge theories

$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

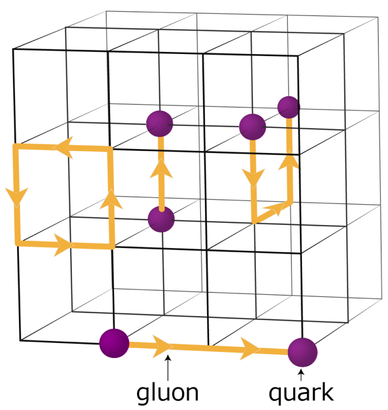
- From holography one instead has η/s=1/(4π) (for $\mathcal{N}=4$ SYM) and a conjectured lower limit
  Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004)

The effective kinetic theory

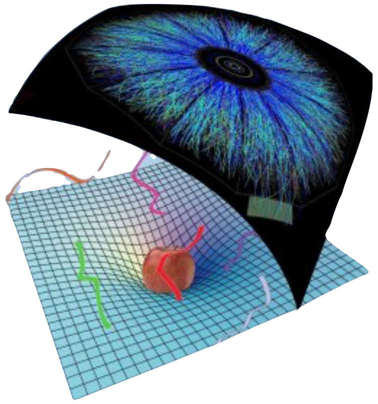
Theory approaches to transport coefficients



pQCD: QCD action (and EFTs thereof). Can be done both in and out of equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$



lattice QCD: Euclidean QCD action, equilibrium only. Real world: analytically continue to Minkowskian domain



AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

The weak-coupling picture

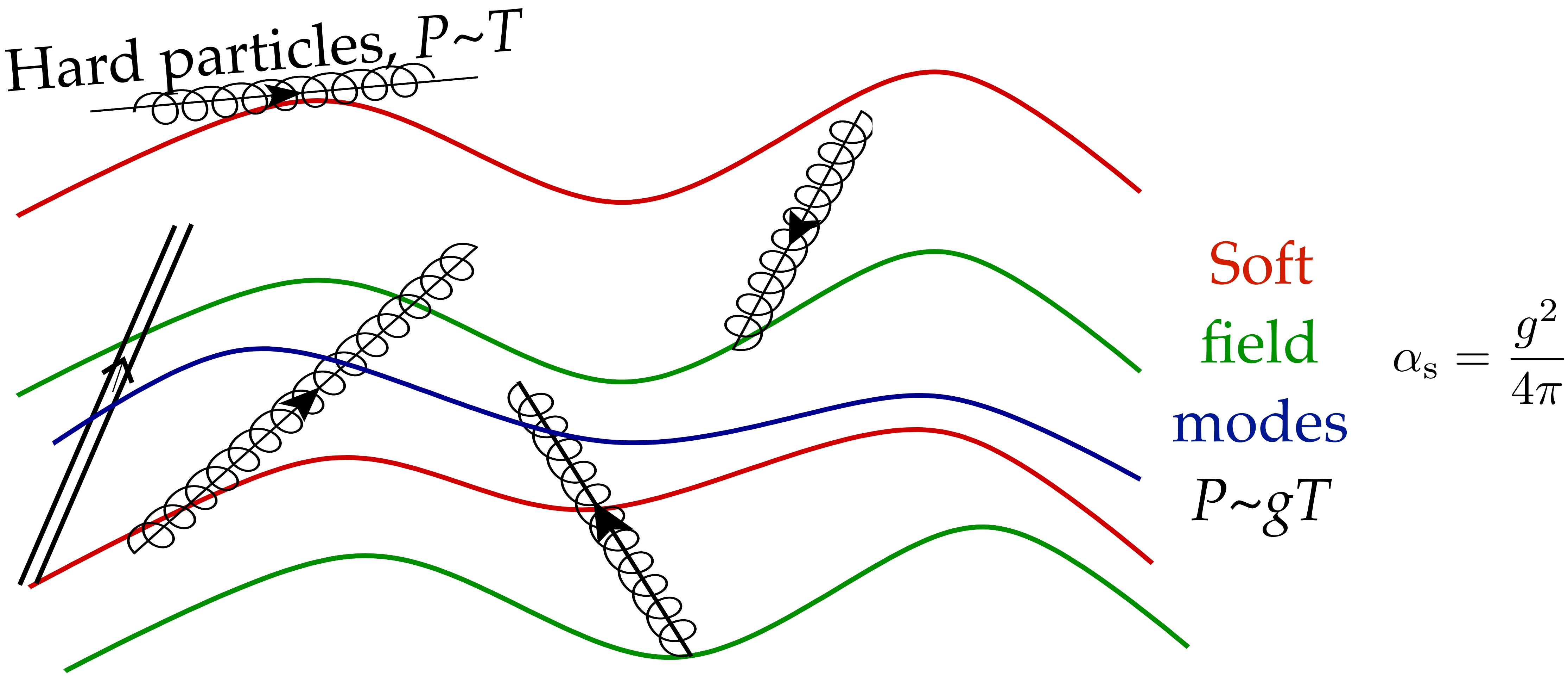


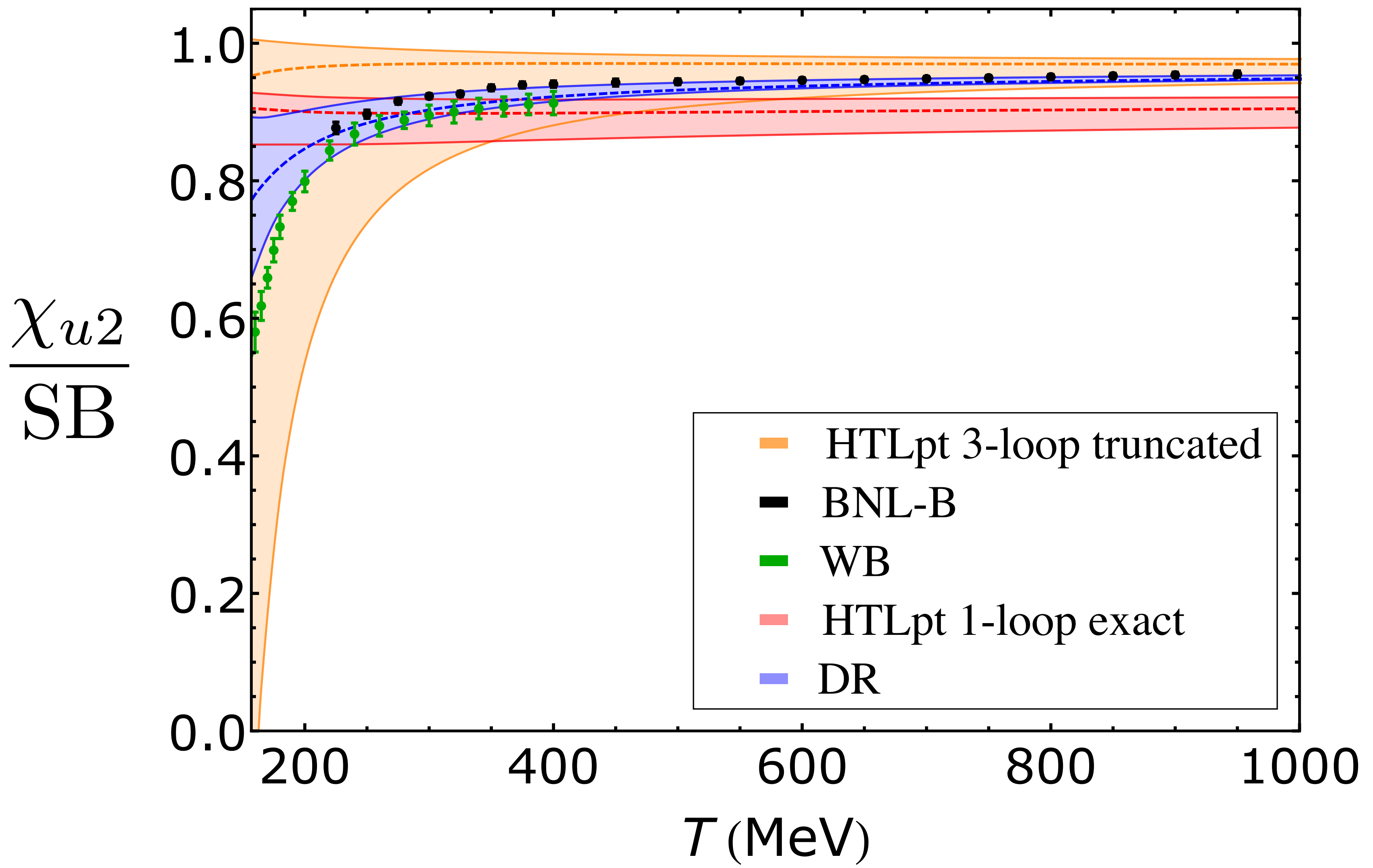
Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T} - 1} \overset{\omega \sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics

$$\chi_{u2} = \frac{\partial^2 p(T,\mu)}{\partial \mu_u^2}$$



Mogliacci Andersen Strickland Su Vuorinen **JHEP1312** (2013)

- Successful for static (thermodynamical) quantities. Possibility of solving the soft sector non-perturbatively (dimensionally-reduced theory on the lattice)

The effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

- Justified at weak coupling, but can be extended to factor in non-perturbative contributions (in progress, more later)
- The effective theory is obtained by integrating out (off-shell) quantum fluctuations (for instance from Kadanoff-Baym equations). Appropriate for describing the dynamics of excitations on scales large compared to $1/T$, which is the size of the typical de Broglie wavelength of an excitation.
- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator

$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

The effective kinetic theory

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- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator $$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

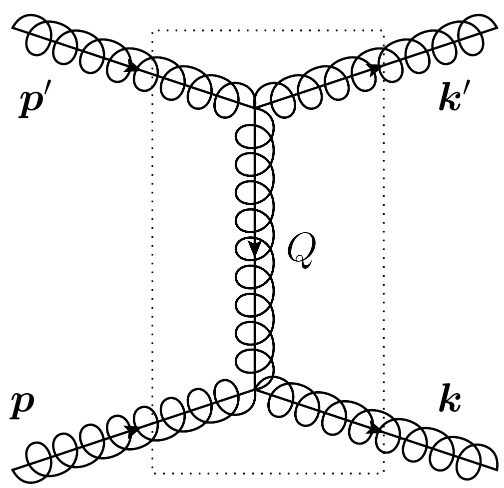
- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a *mean free time* $(1/g^4T)$ *large compared to the actual duration of an individual collision* $(1/T)$

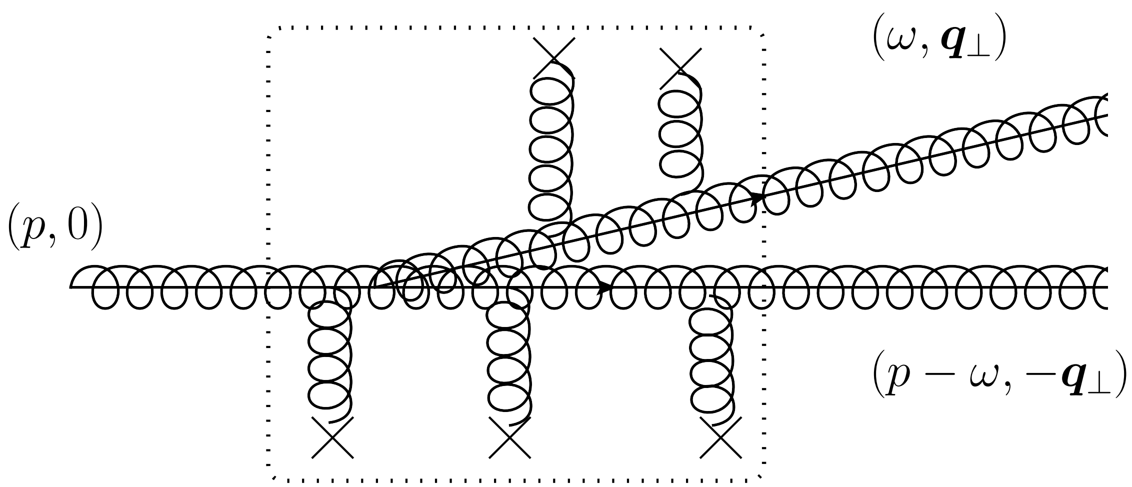
The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, AMY, all that) AMY (2003)





Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\rm EQ}(\mathbf{p}) + \sum_\ell \delta f_\ell(\mathbf{p}) \qquad \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\rm EQ}(\mathbf{p},u,\beta,\mu) = C_{\rm lin}[\delta f_\ell]$$

- Driving term equates linearized collision operator. Since $\langle T^{i\neq j}\rangle \propto \eta$, $\langle \mathbf{J}_q\rangle = -D_q \boldsymbol{\nabla}\langle n_q\rangle$ η requires ℓ=2, D_q ℓ=1

- Transport coefficients obtained by the kinetic thy definitions of T, J once δf_ℓ has been obtained. Solution easier in **quadratic form** (variational). LO η,D~$1/g^4$

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\rm EQ}(\mathbf{p},u,\beta,\mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p})\, C_{\rm lin}[\delta f_\ell]$$

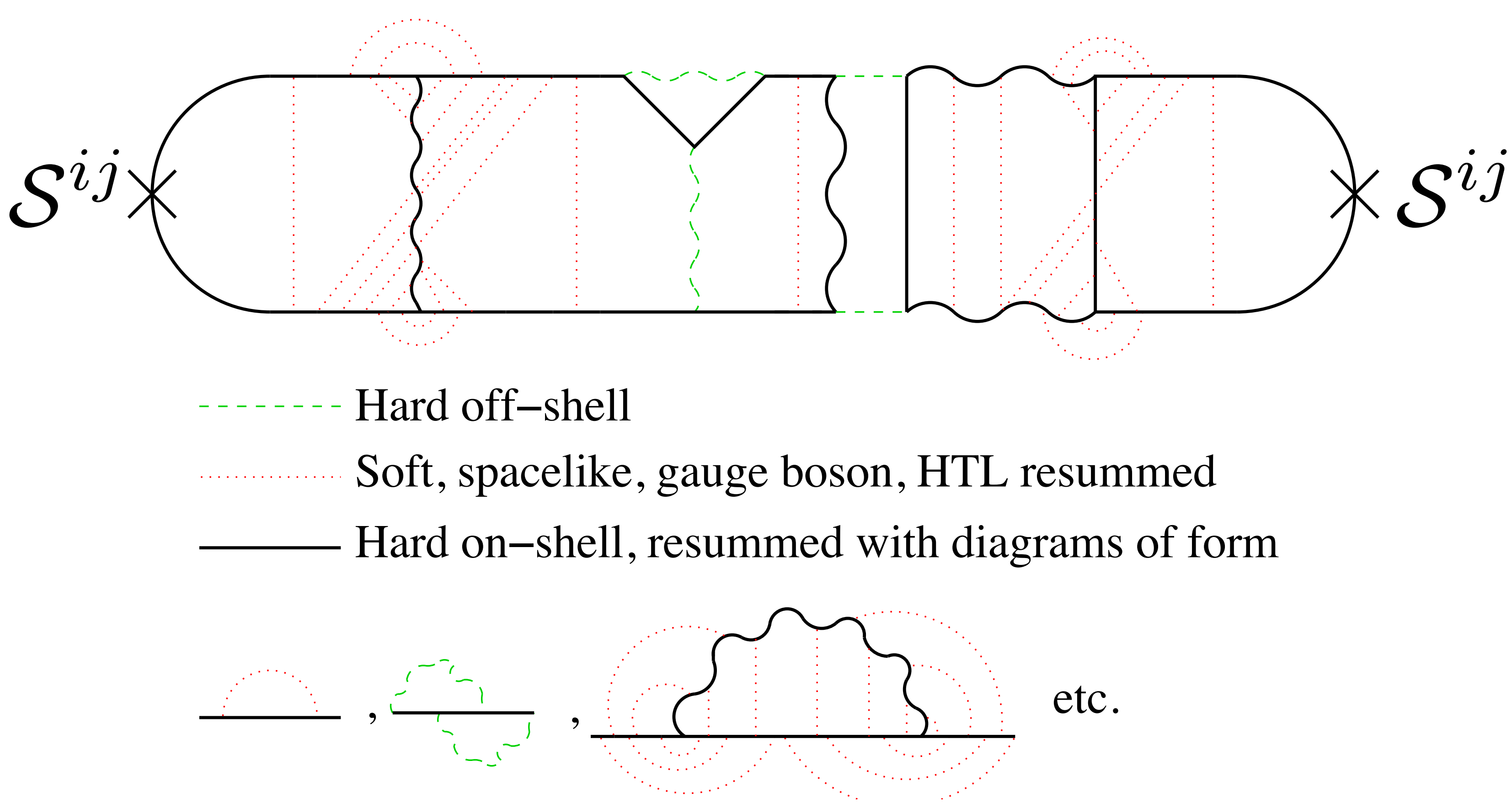
Arnold Moore Yaffe (2003)

The EKT and transport

- Linearized EKT equivalent to Kubo formula (S TT part of T)

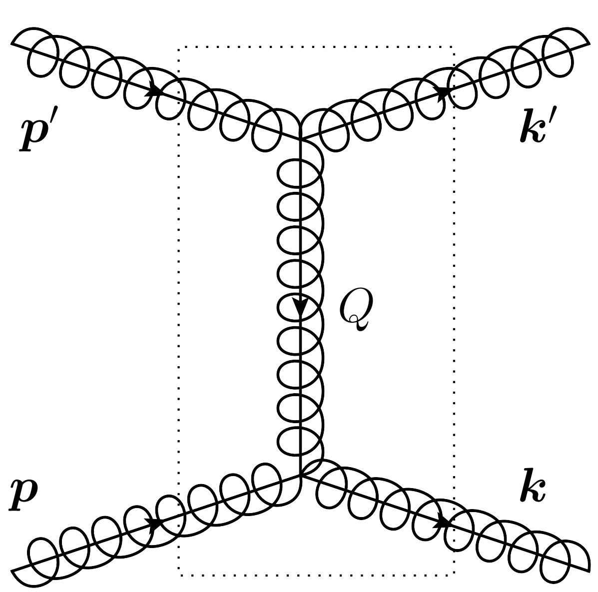
$$\eta = \frac{1}{20} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x\, e^{i\omega t} \left\langle \left[\mathcal{S}^{ij}(t,\mathbf{x}),\, \mathcal{S}^{ij}(0,\mathbf{0}) \right] \right\rangle \theta(t)$$

- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)



Reorganization

- The NLO corrections come from regions sensitive to soft gluons (no quarks in this illustration)
- Before we get there, let's have a reorganized perspective on these regions at LO
- Look at 2↔2 scattering

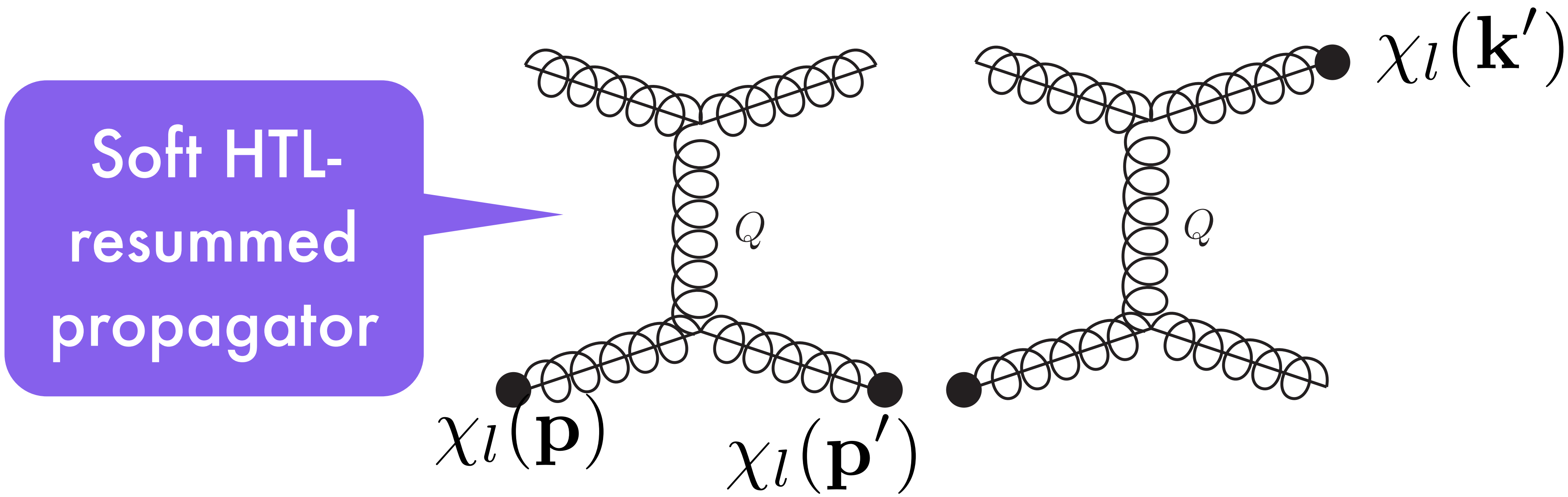


$$\int_{\mathbf{pkp'k'}} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}',\mathbf{k}')\right|^2 (2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\, f_{\rm EQ}(p)\, f_{\rm EQ}(k)\,[1+f_{\rm EQ}(p')]\,[1+f_{\rm EQ}(k')]$$
$$\times \Big[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\Big]^2$$

$$\delta f_l(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,\chi_l(\mathbf{p})$$

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- Left: **diffusion terms**, **p** and **p′** strongly correlated

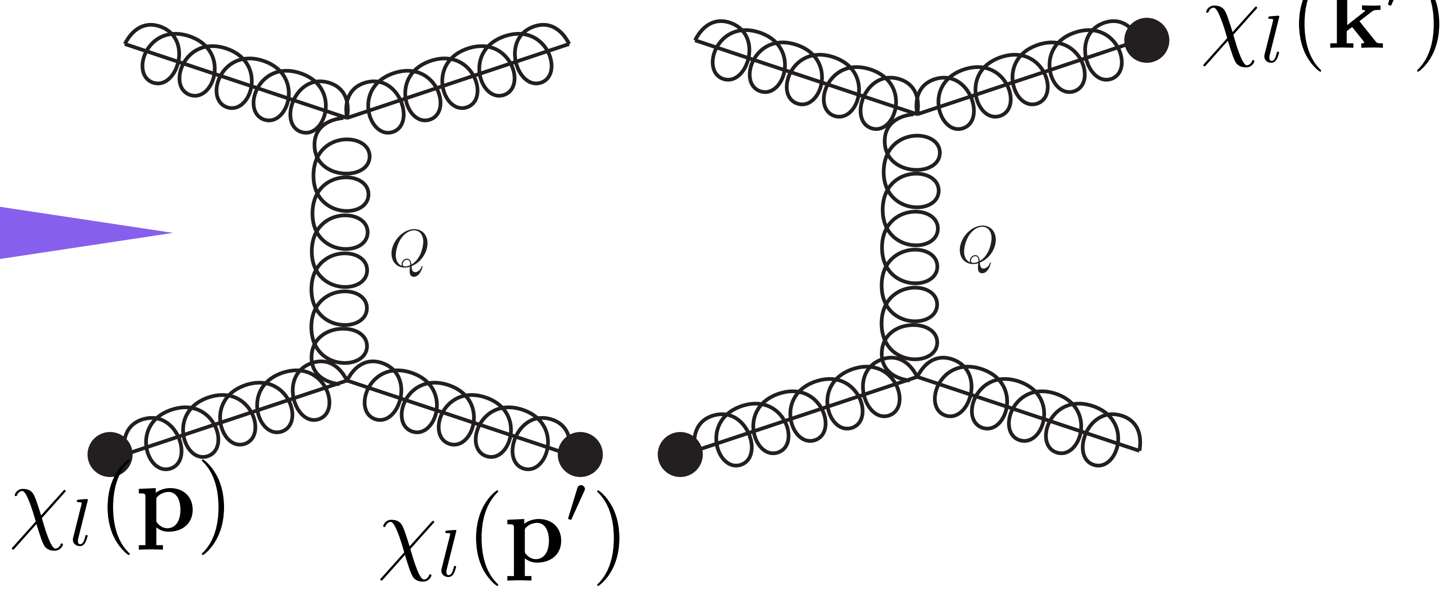
$$\left(\chi_\ell(\mathbf{p})-\chi_\ell(\mathbf{p}')\right)^2=(\hat{\mathbf{p}}\cdot\mathbf{q})^2[\chi'(p)]^2+\frac{\ell(\ell+1)}{2}\,\frac{q^2-(\hat{\mathbf{p}}\cdot\mathbf{q})^2}{p^2}[\chi(p)]^2$$

identify a longitudinal and a transverse momentum broadening contribution, $\hat{q}_L$ and $\hat{q}$

LO soft gluon scattering

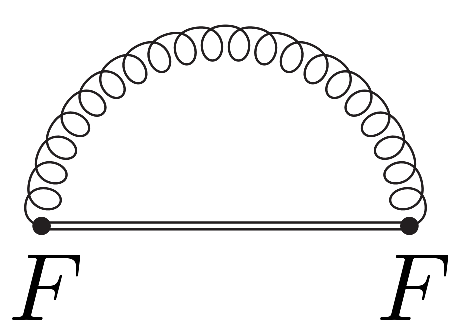
- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

Soft HTL-resummed propagator



- **Diffusion terms:** transverse becomes Euclidean

$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^-{}_\perp \rangle_{q^-=0}$$
$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_D^2}{q_\perp^2 + m_D^2} = \frac{g^2 C_A T m_D^2}{2\pi} \ln \frac{\mu_\perp}{m_D}$$

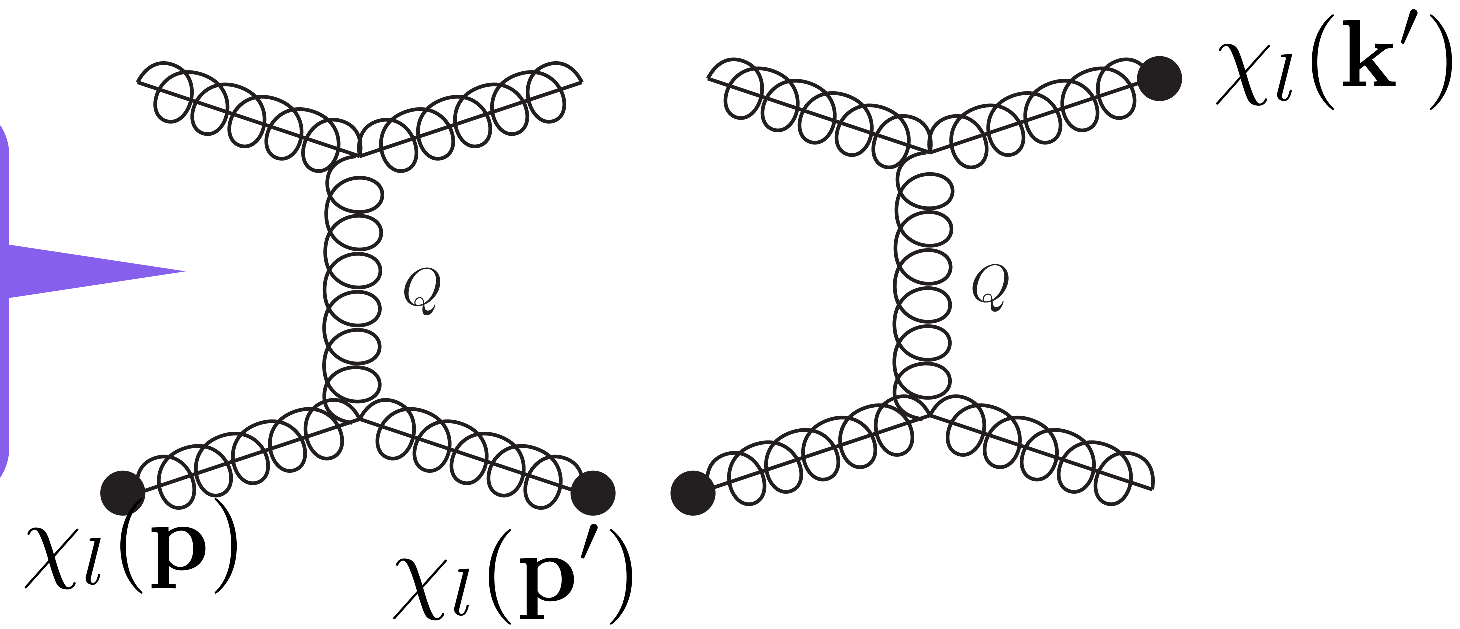


Aurenche Gelis Zaraket **JHEP0205** (2002), Caron-Huot **PRD79** (2009)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

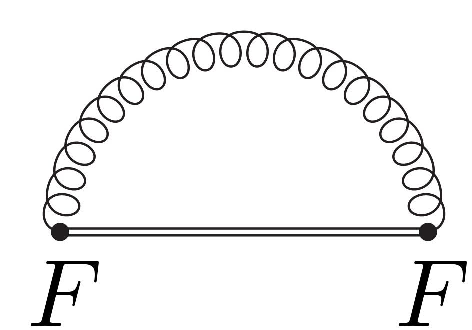
Soft HTL-resummed propagator



- **Diffusion terms:** longitudinal with lightcone sum rule

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-z}(Q) F^{-z} \rangle_{q^-=0}$$

$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2} = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln \frac{\mu_\perp}{m_\infty}$$

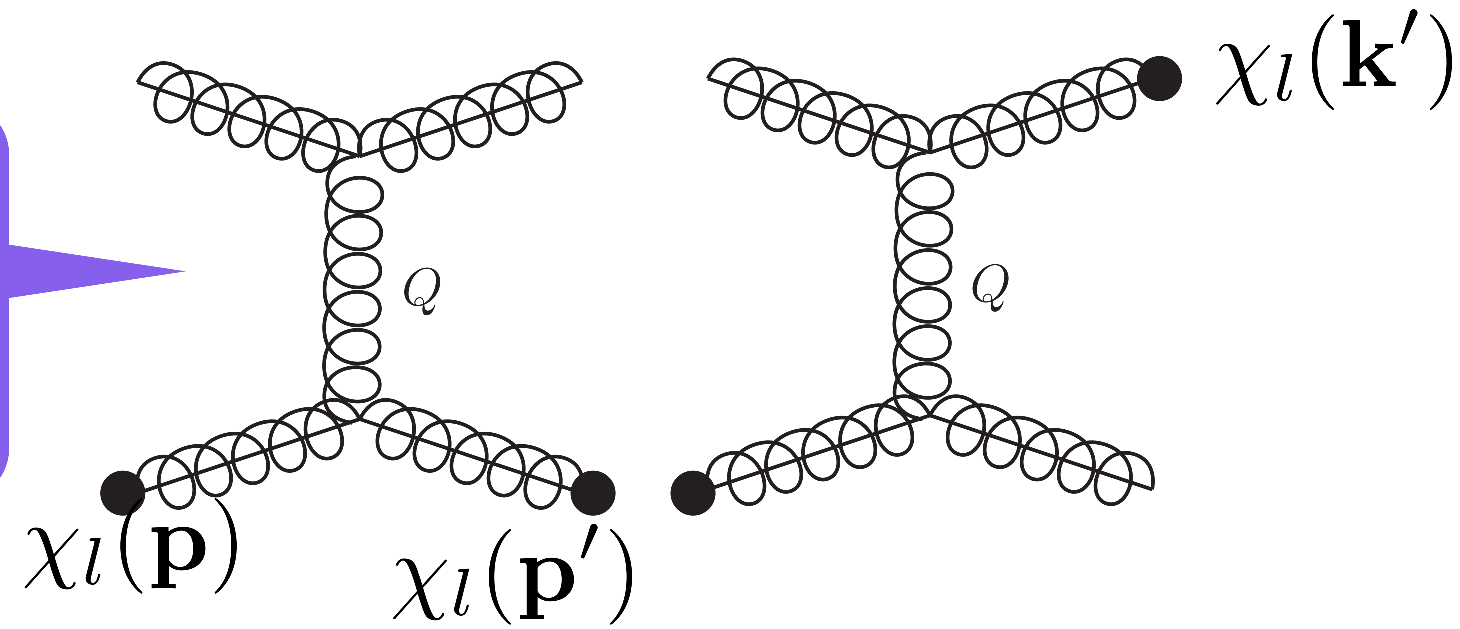


JG Moore Teaney (2015)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$

Soft HTL-resummed propagator



- **Diffusion terms:** easy with light-cone techniques*

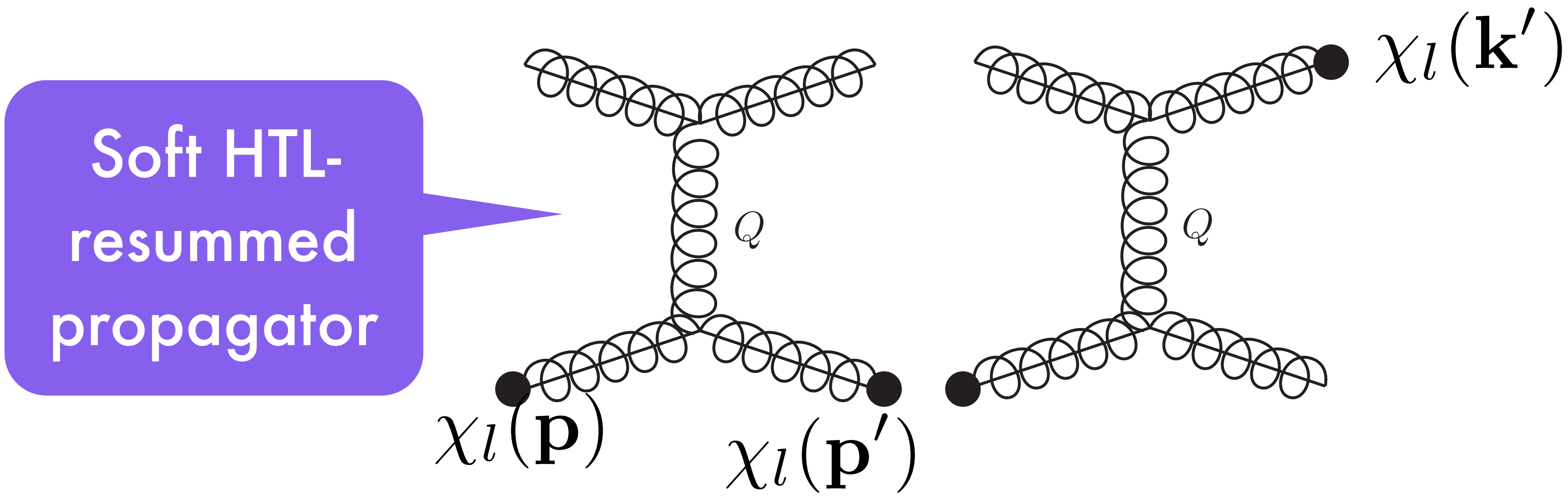
$$\hat{q}_L^a\Big|_{\mathrm{soft}}=\frac{g^2C_{R_a}Tm_D^2}{4\pi}\ln\frac{\sqrt{2}\mu_\perp}{m_D}\qquad \hat{q}^a\Big|_{\mathrm{soft}}=\frac{g^2C_{R_a}Tm_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

give rise to the leading log contribution

*Caron-Huot PRD82 (2008) JG Moore Teaney (2015)

LO soft gluon scattering

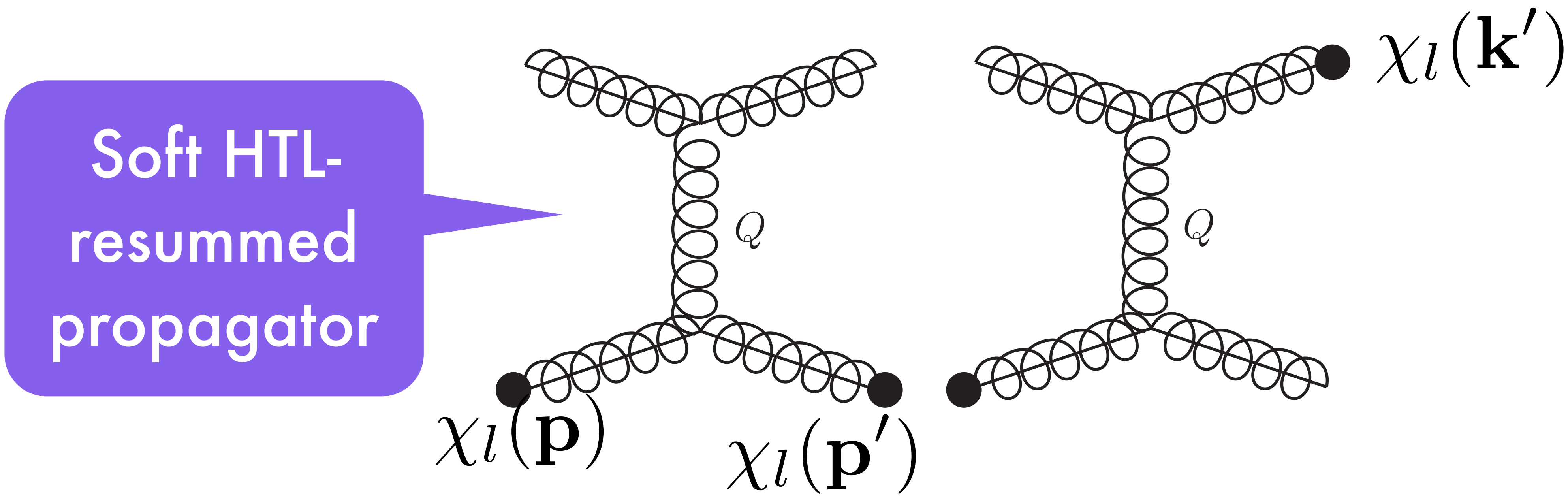
- When $Q{=}P'{-}P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\qquad (\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Two-point function of two uncorrelated deviations from equilibrium
(diffusion was the response of an off-eq leg to the equilibrium bath)

LO soft gluon scattering

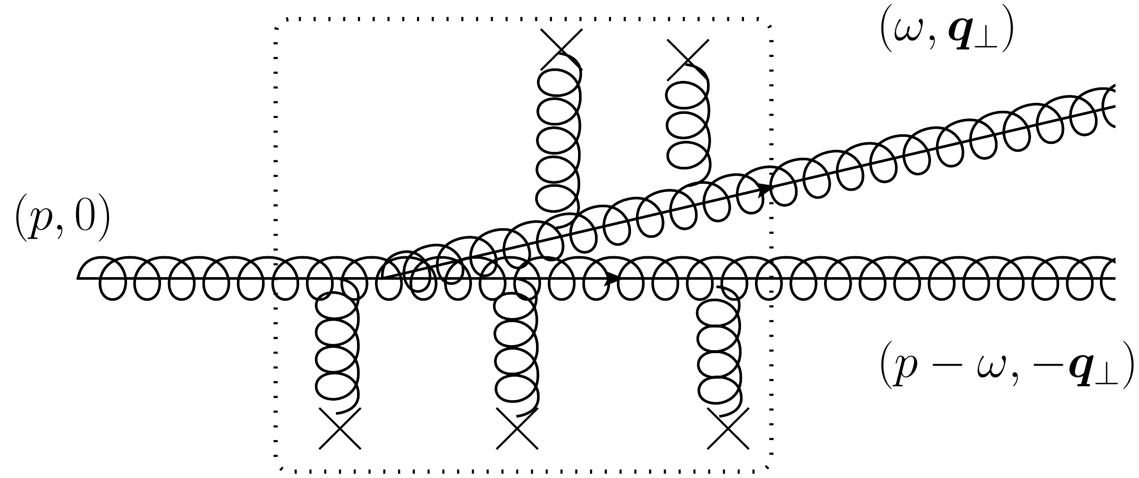
- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Light-cone techniques not applicable, have to use numerical integration. Easy at LO, where they are finite (no leading log contribution)

Reorganization

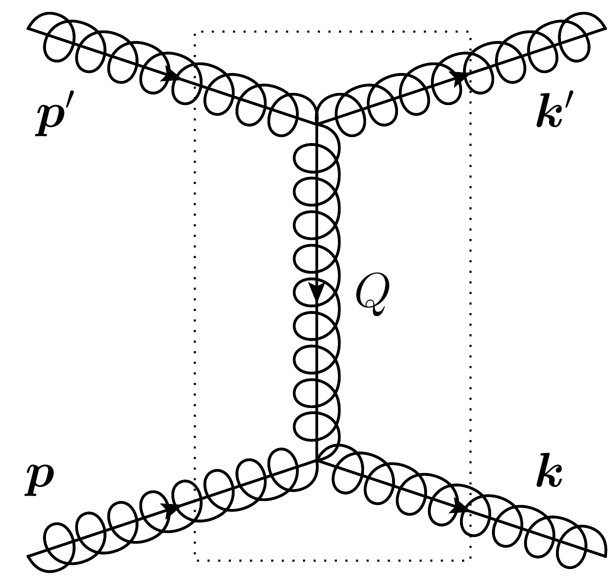
- 1↔2 processes: strictly collinear kinematics, unaffected by reorganization



- Reorganization of the LO collision operator

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left[C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{cross}} + C^{\mathrm{coll}} \right]$$

- Final ingredient: 2↔2 large angle scatterings, IR-regulated to avoid the soft region



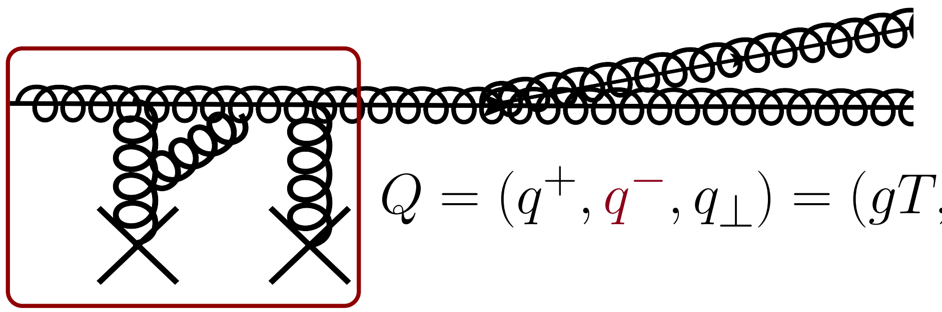
Going to NLO

- The diffusion, cross and collinear terms receive $O(g)$ corrections
- There is a new semi-collinear region

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO $C(q_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

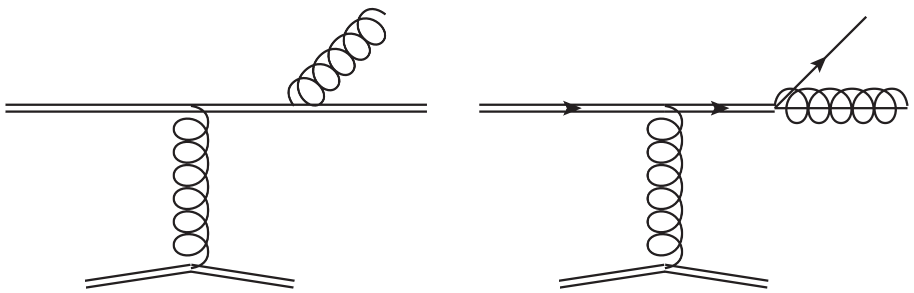
$$\mathcal{C}_{\mathrm{LO}}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$



$$Q = (q^+, q^-, q_\perp) = (gT,\, g^2T,\, gT)$$

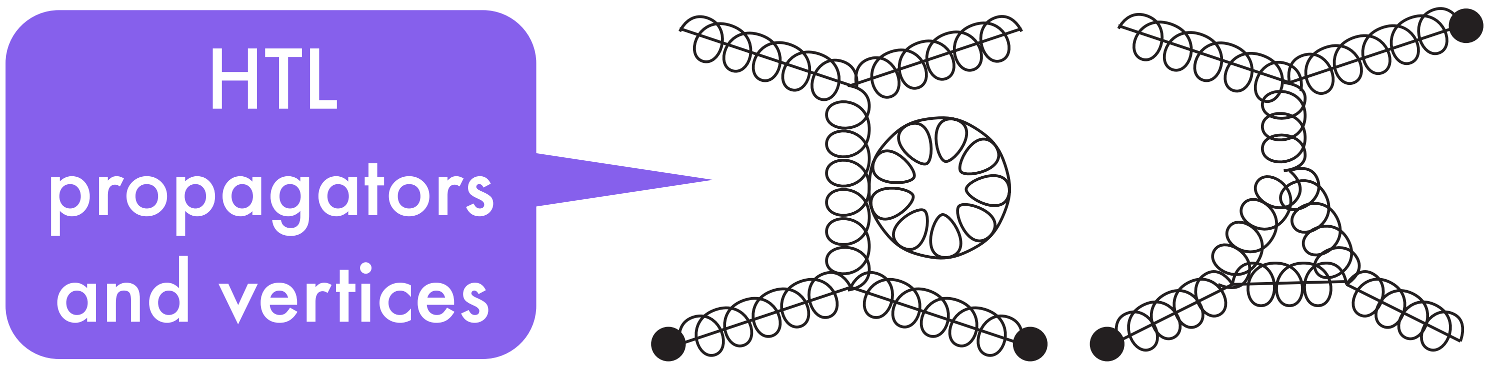
$\mathcal{C}_{\mathrm{NLO}}(q_\perp)$ complicated but analytical (Euclidean tech)
Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013)

- Regions of overlap with the diffusion and semi-collinear regions need to be subtracted



NLO diffusion and cross

- At NLO one has these types of diagrams



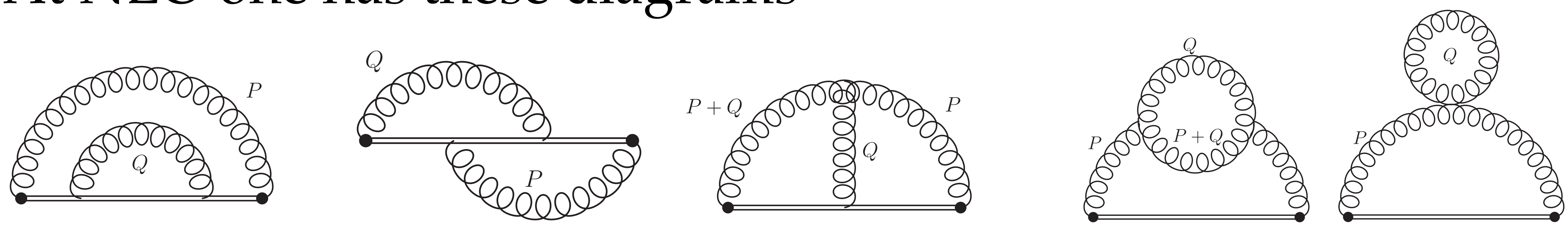
- For diffusion (left): application of light-cone techniques still possible, huge simplification and closed-form results
Transverse (NLO $\hat{q}$) is finite Caron-Huot (2008)
Longitudinal (NLO $\hat{q}_L$) is UV log-divergent JG Moore Teaney (2015)

$$\hat{q}_{\mathrm{NLO}} = \hat{q}_{\mathrm{LO}} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

$$\hat{q}_L(\mu_\perp)_{\mathrm{NLO}} = g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \frac{q_\perp^2\, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}\right]$$

Diffusion corrections

- At NLO one has these diagrams



- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\mathrm{NLO}} = \hat{q}_{\mathrm{LO}} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

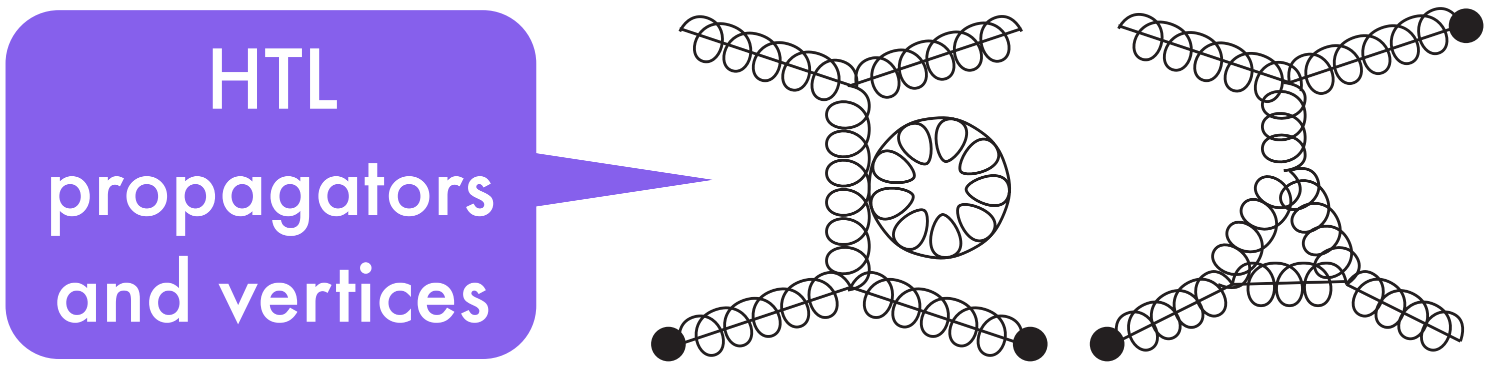
$$\hat{q}_L(\mu_\perp)_{\mathrm{LO}} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\mathrm{NLO}} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \boxed{\frac{q_\perp^2\, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}}\right]$$

after collinear subtraction light-cone sum rule still sees only dispersion relation ($O(g)$ correction). NLO still UV-log sensitive

NLO diffusion and cross

- At NLO one has these types of diagrams

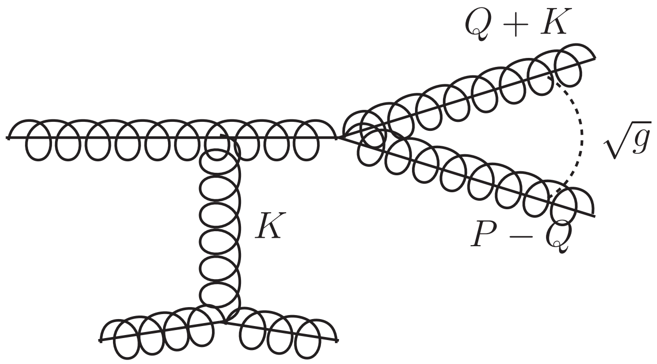


- For cross (right): no diffusion picture = no "easy" light-cone sum rules, only way would be bruteforce HTL. **Missing**, but silver lining: they're finite, so just estimate the number and vary it
  NLO test ansatz: LO cross x $m_D/T(\sim g)$ x arbitrary constant that we vary

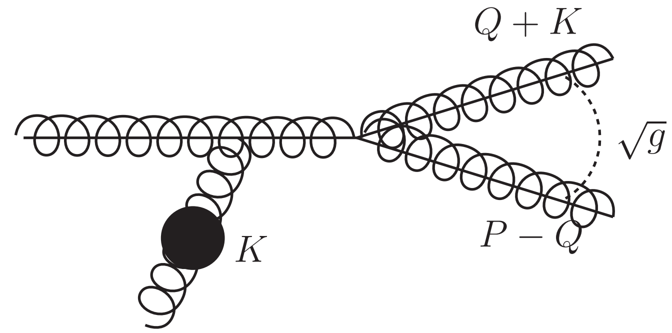
$$C_{\mathrm{NLO}}^{\mathrm{cross}} = C_{\mathrm{LO}}^{\mathrm{cross}} \times \frac{m_D}{T} \times c_{\mathrm{cross}}$$

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike



K soft plasmon, timelike

- Evaluation: introduce "*modified* $\hat{q}$" tracking the changes in the small light-cone component p^- of the gluons. Can be evaluated in EQCD

"standard" $$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$

"modified" $$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=\delta E}$$

- Rate ∝ "*modified*$\hat{q}$ " x DGLAP splitting. IR log divergence makes collision operator finite at NLO

Semi-collinear processes

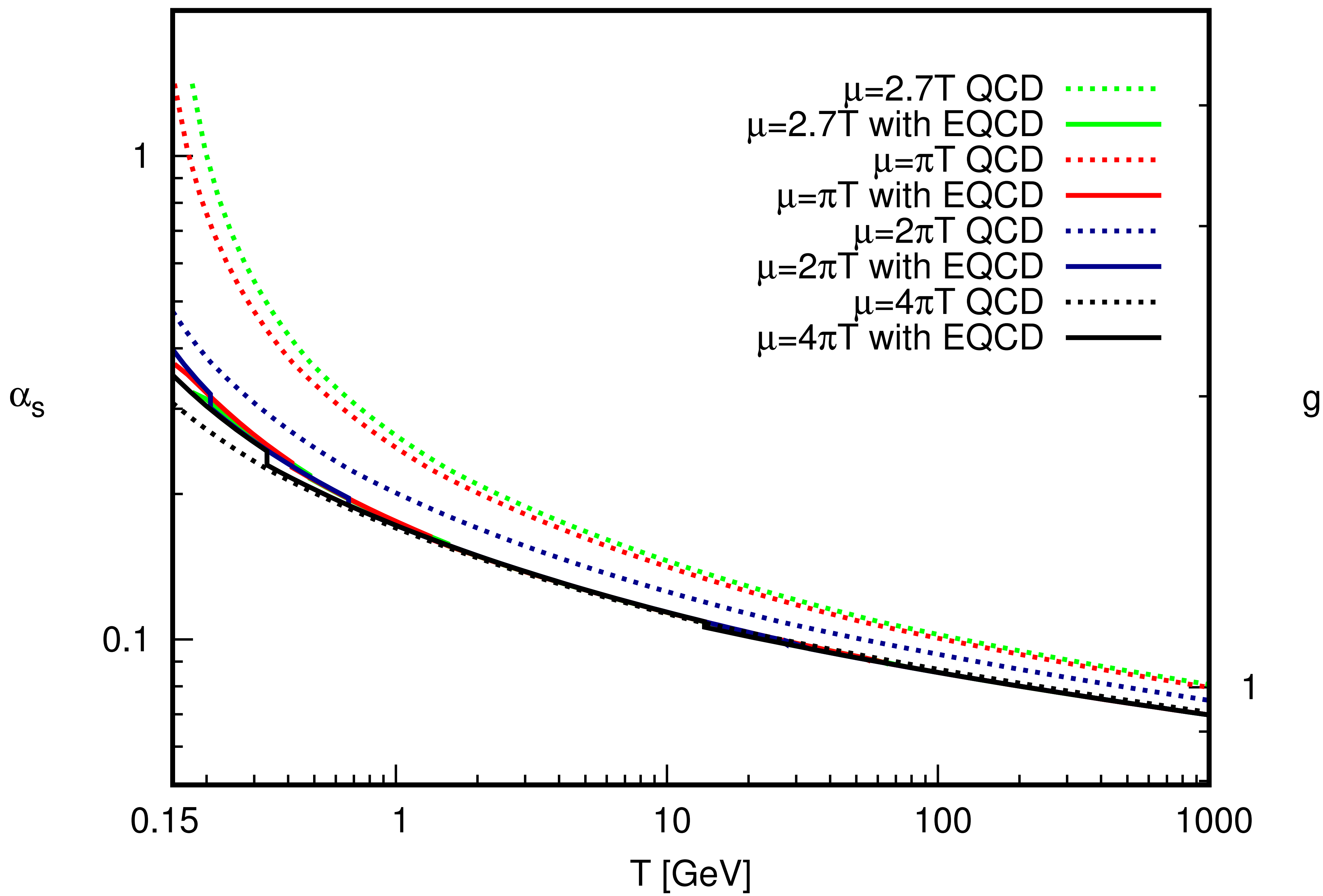
- Important technical detail: subtractions (no, I am not talking about first grade algebra)
- Pure *O(g)* semicollinear rate actually involves subtraction of collinear and hard limits ,i.e. $\hat{q}(\delta E) - \hat{q}(0) - \hat{q}(\delta E, m_D \to 0)$
- This makes it mostly negative: when extrapolating to larger *g* we risk a negative collision operator
- We devised a new implementation that, while equivalent at *O(g)*, is better behaved when extrapolating due to resummations
- In a nutshell, make $C(q_\perp)$ δE-dependent in the first-order of the LPM ladder resummation.

Results

Results

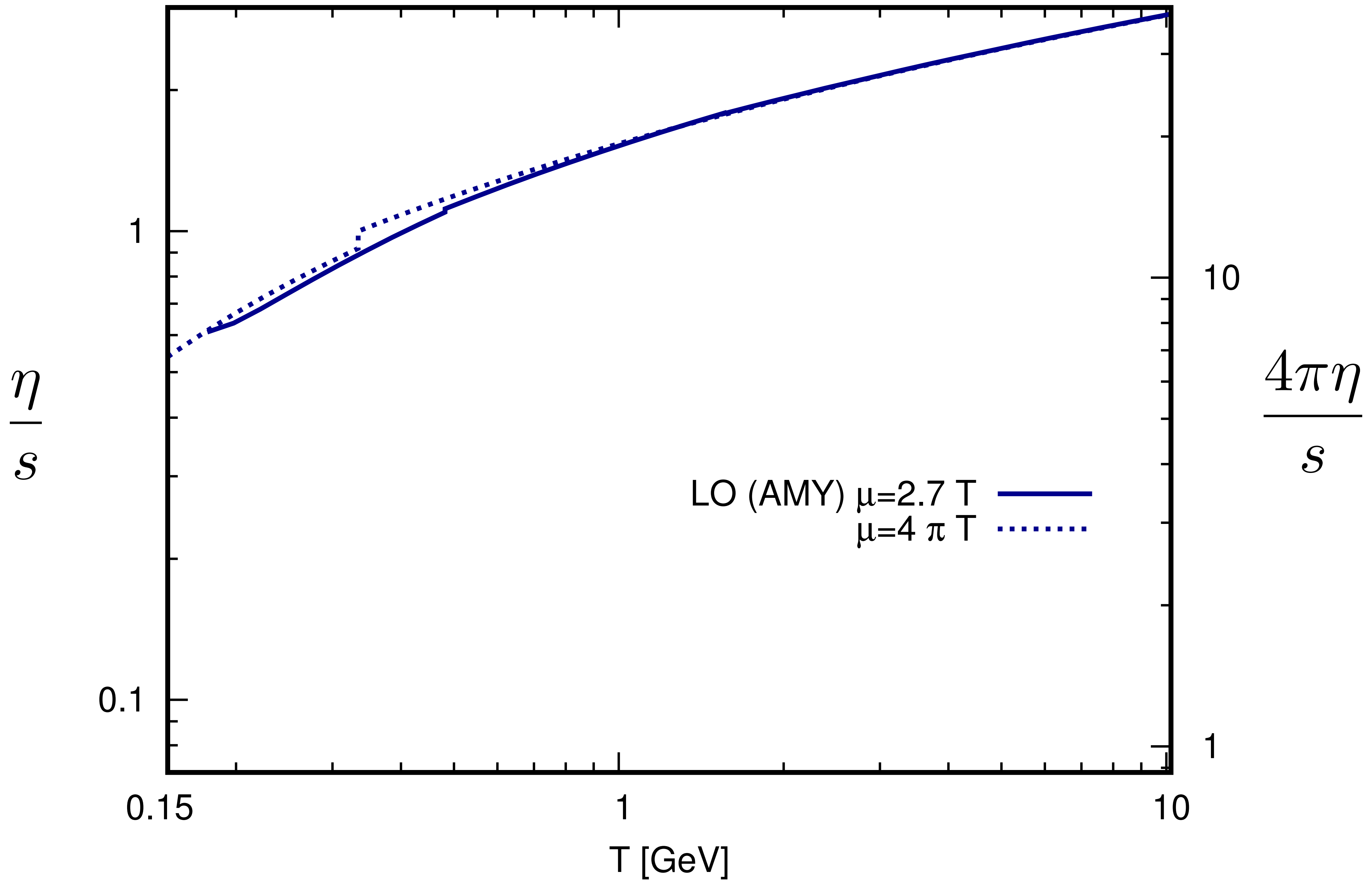
- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion
- Kinetic theory with massless quarks still conformal to NLO
- Relate parameter $m_D/T \sim g$ to temperature through
  - Two-loop EQCD $g(T)$ as in Laine Schröder **JHEP0503** (2005)
  - Simple two-loop MSbar with various μ/T
- Degree of arbitrariness in the choice of quark mass thresholds, test several values of μ/T

Results



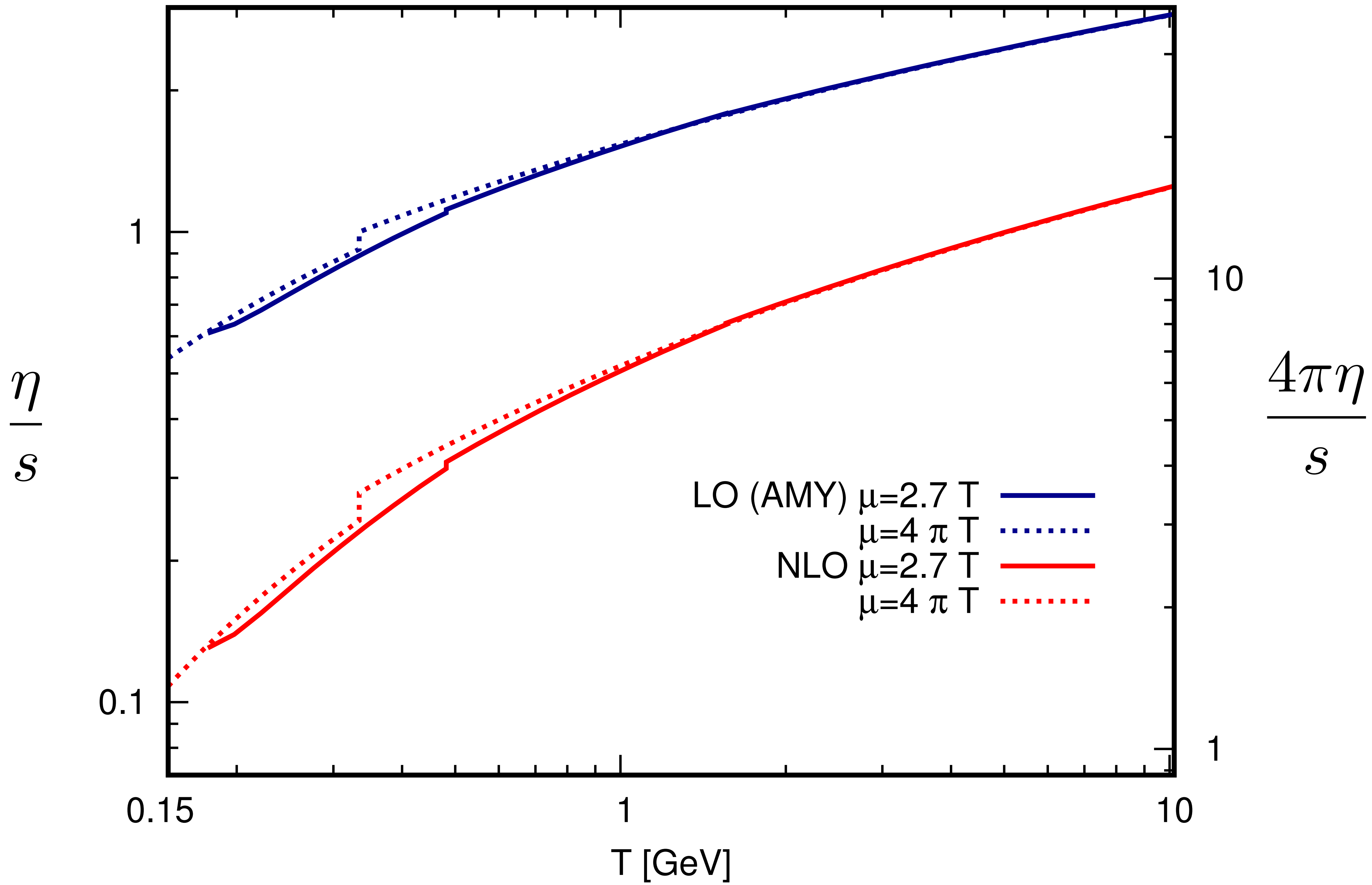
μ=2.7T QCD
μ=2.7T with EQCD
μ=πT QCD
μ=πT with EQCD
μ=2πT QCD
μ=2πT with EQCD
μ=4πT QCD
μ=4πT with EQCD
α_s
g
1
0.1
1
0.15
1
10
100
1000
T [GeV]

η/s(T) of QCD



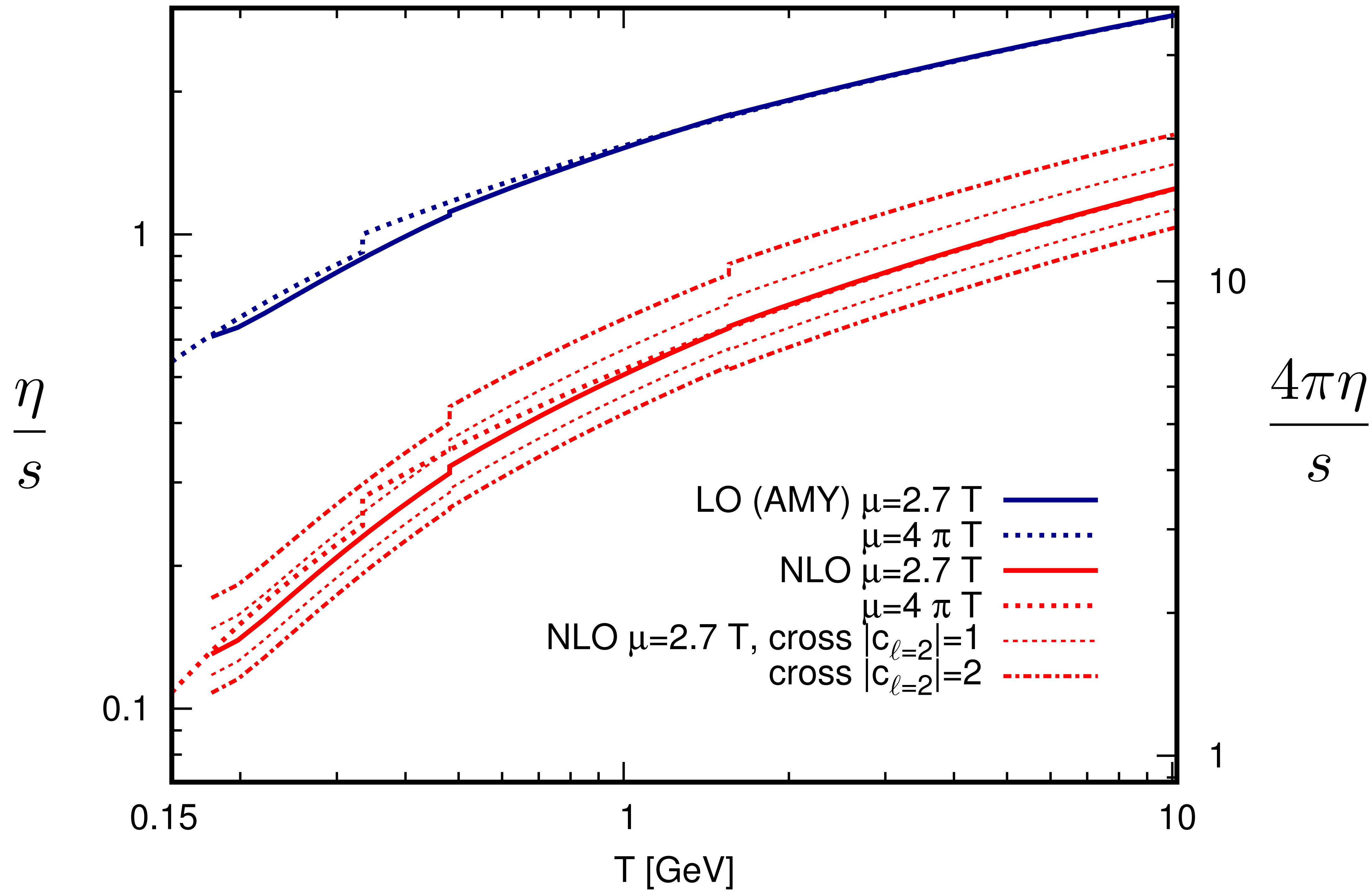
- LO results from AMY (2003)

η/s(T) of QCD



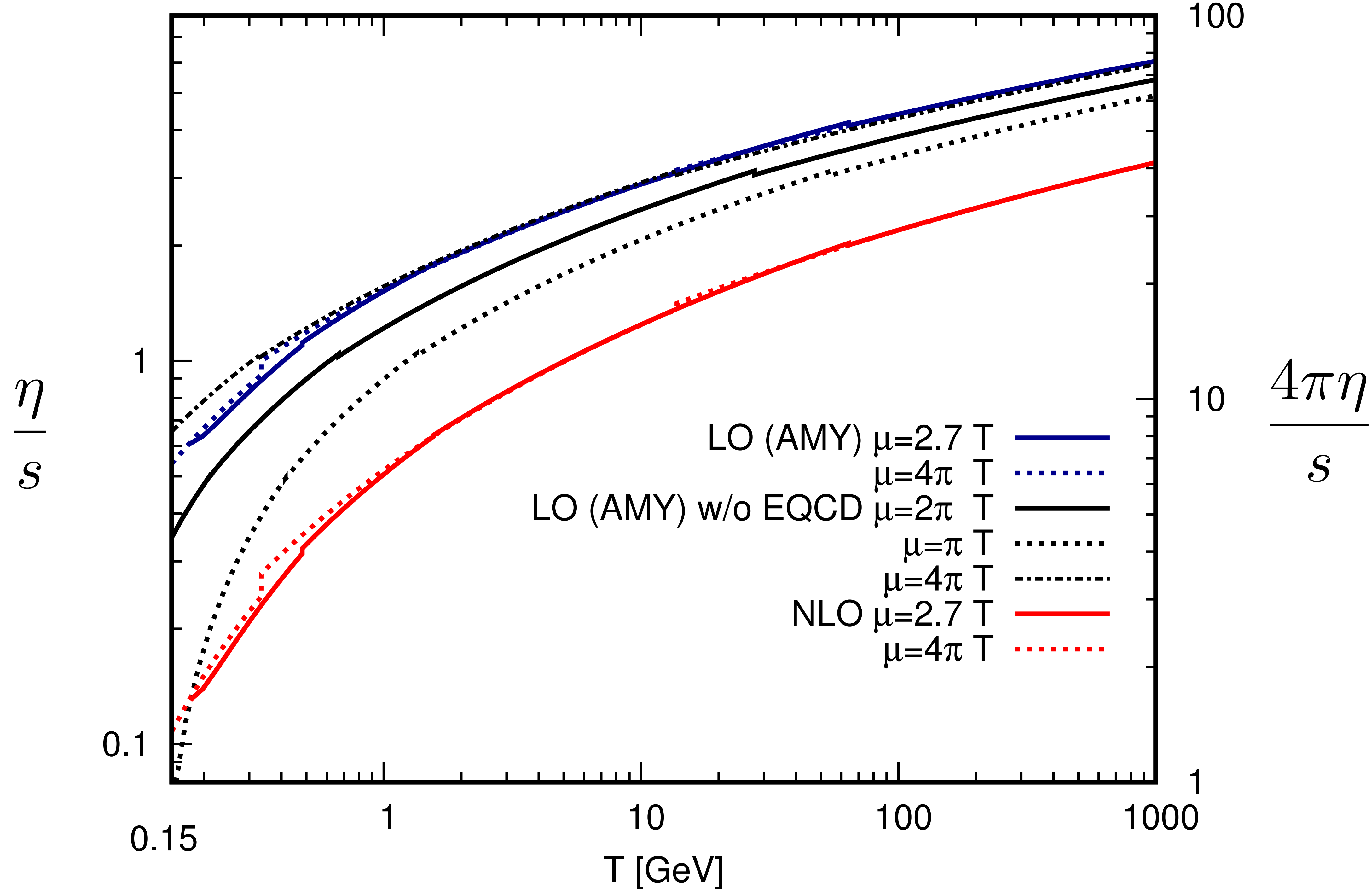
- All known NLO terms, no cross ansatz yet

η/s(T) of QCD



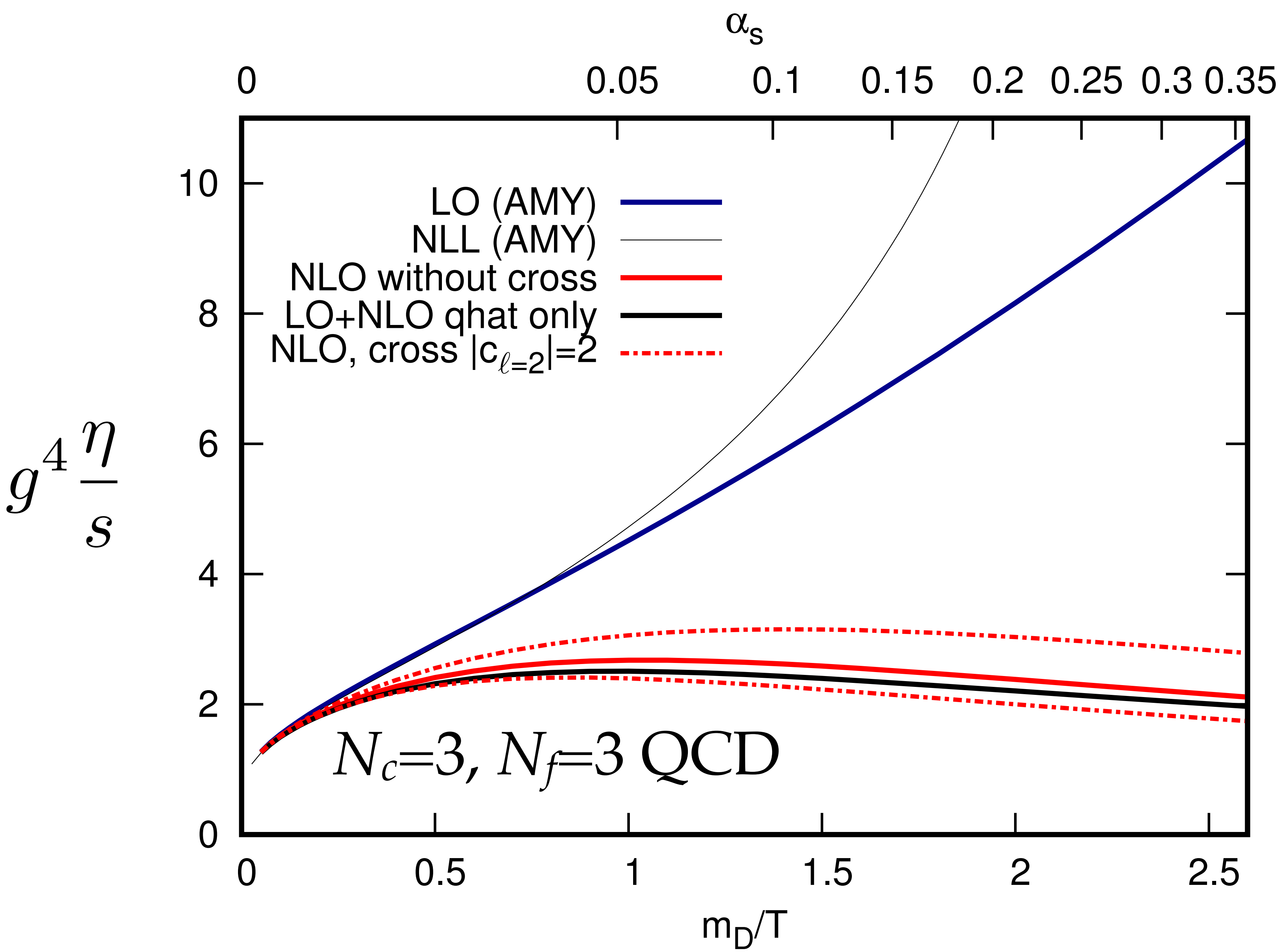
- Cross ansatz introduces $O(\pm 30\%)$ uncertainty

η/s(T) of QCD



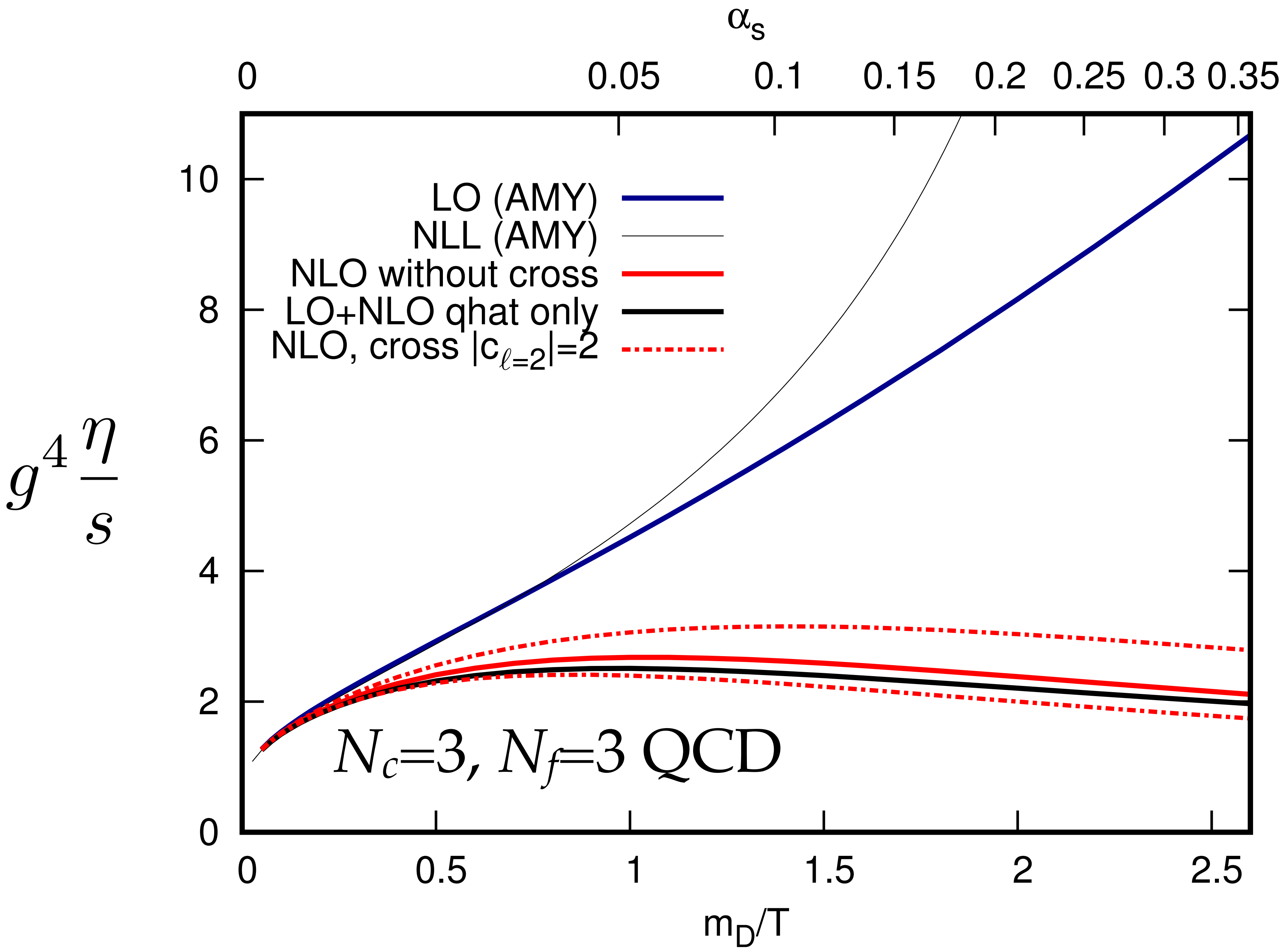
- Pure QCD running uncertainty band at LO (NNLO) smaller than NLO deviation from LO

η/s convergence



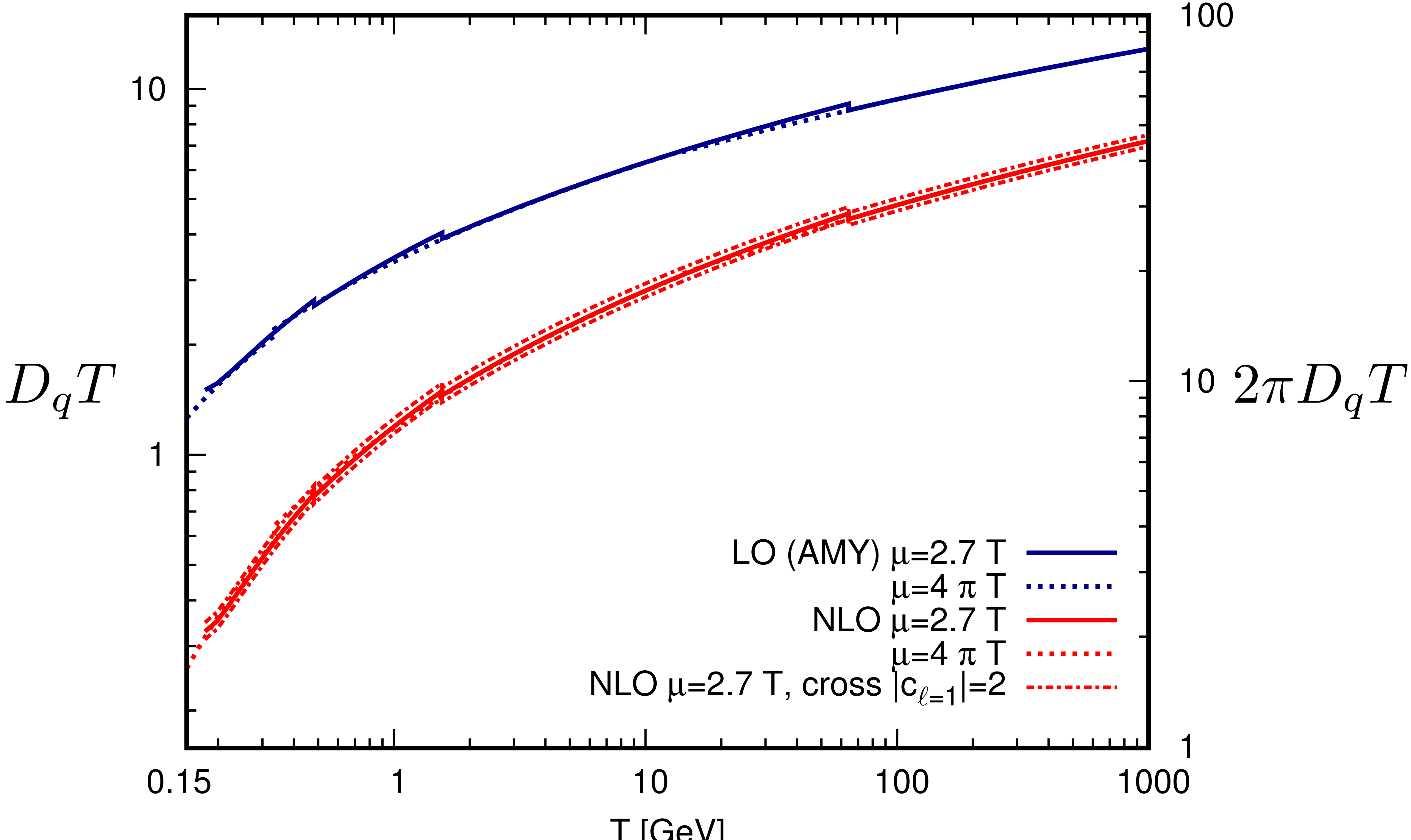
- Convergence realized at m_D~$0.5T$

η/s convergence



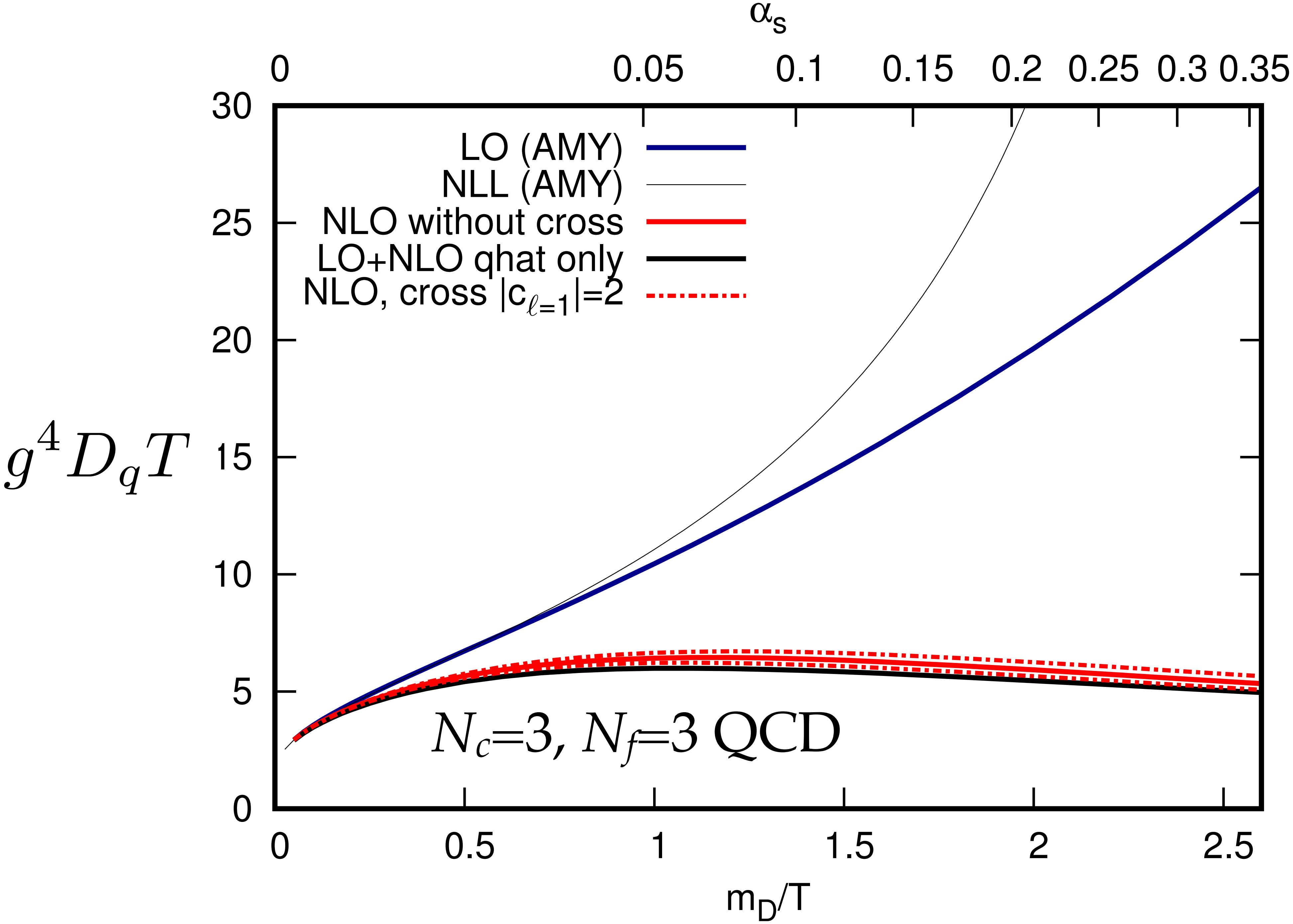
- The ~entirety of the downward shift comes from NLO O(g) corrections to $\hat{q}$

$D_q T(T)$ of QCD



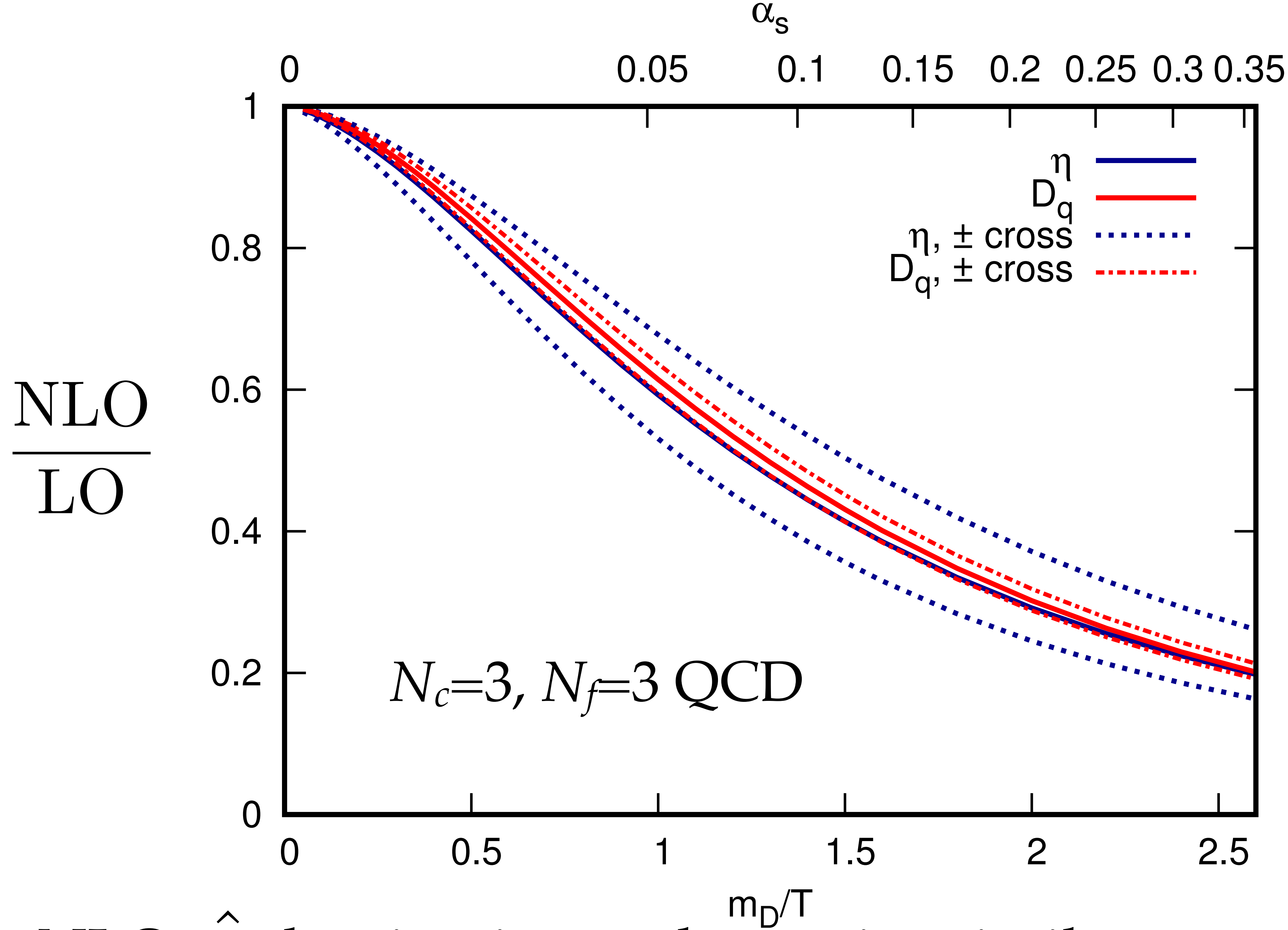
- Cross ansatz uncertainty much smaller (soft quarks here)

D_qT convergence



- Convergence realized again at m_D~$0.5T$

Ratios



- NLO $\hat{q}$ domination makes ratios similar

Conclusions

- We have computed all contributions to the NLO linearized collision operator but one (for each ℓ)
- NLO corrections are #large, η and D down by a factor of ~5 in the phenomenological region
- Convergence below m_D~$0.5T$
- Second-order τ_Π will be available in the papers
- Corrections dominated by NLO $\hat{q}$. Could it be that observables directly sensitive to transverse momentum broadening show bad convergence and those who are not show good convergence? Why? #statisticswithsmallnumbers

Backup

Euclideanization of light-cone soft physics

- For t/x_z=0: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z|<1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)}\left(\frac{1}{2}+n_{\rm B}(p^0)\right)(G_R(P)-G_A(P))$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)}\left(\frac{1}{2}+n_{\rm B}(p^0)\right)(G_R(p^0,\mathbf{p}_\perp,\tilde{p}^z+(t/x^z)p^0)-G_A)$$

- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

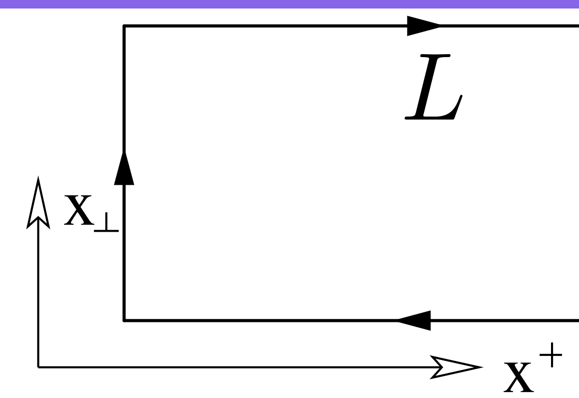
$$G_{rr}(t,\mathbf{x}) = T\sum_n \int d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n,\mathbf{p}_\perp,\tilde{p}^z + i\omega_n t/x^z)$$

$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}} G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

LPM resummation



$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

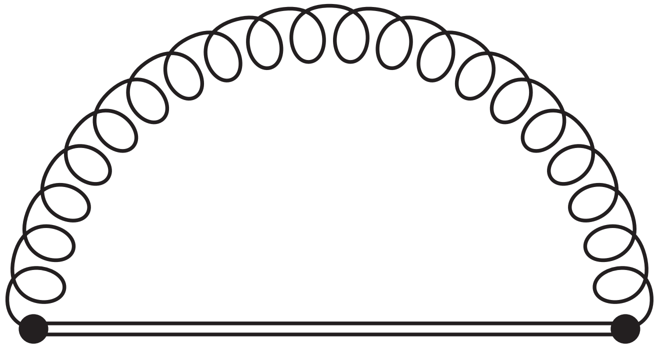
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R} \int_{-\infty}^{+\infty} dx^+ \mathrm{Tr} \left\langle U(-\infty, x^+) F^{+-}(x^+) U(x^+, 0) F^{+-}(0) U(0, -\infty) \right\rangle$$

$F^{+-} = E^z$, longitudinal Lorentz force correlator

- At leading order

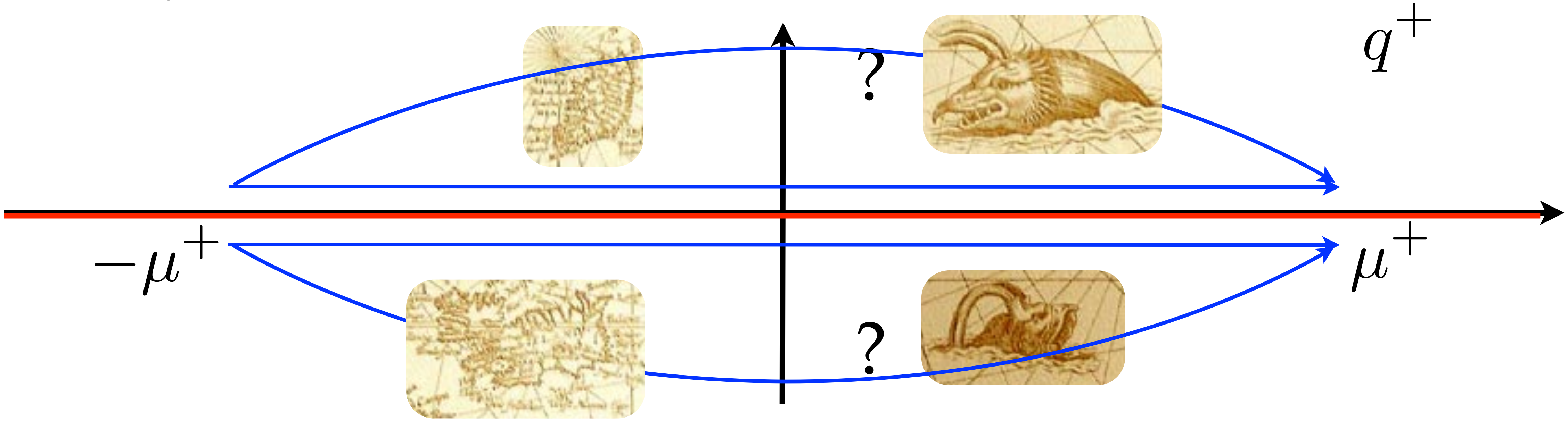


$$\hat{q}_L \propto \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} (q^+)^2 G^>_{++}(q^+, q_\perp, 0)$$

$$= \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+, q_\perp, 0) - G^A)$$

Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



- Use analyticity to deform the contour away from the real axis and keep *1/q⁺* behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$