Neutrinos
Lecture I: theory and phenomenology of neutrino oscillations

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What will you learn from these lectures?

- The basics of neutrinos: a bit of history and the basic concepts
- Neutrino oscillations: in vacuum, in matter, experiments
- Nature of neutrinos, neutrino less double beta decay
- Neutrino masses and mixing BSM
- Neutrinos in cosmology
Today, we look at

- A bit of history: from the initial idea of the *neutrino* to the solar and atmospheric neutrino anomalies
- The basic picture of neutrino oscillations (mixing of states and coherence)
- The formal details: how to derive the probabilities
- Neutrino oscillations both in vacuum and in matter
- Their relevance in present and future experiments (with some additional slides about experiments to illustrate the latest results. Not discussed in detail.)
Useful references

- C. Giunti, C.W. Kim, Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press, USA (May 17, 2007)

- M. Fukugita, T. Yanagida, Physics of Neutrinos and applications to astrophysics, Springer 2003


- Talks at the Neutrino 2018 conference (10 days ago)
A brief history of neutrinos

- The proposal of the “neutrino” was put forward by W. Pauli in 1930. [Pauli Letter Collection, CERN]

Dear radioactive ladies and gentlemen,

...I have hit upon a desperate remedy to save the ... energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 ... The mass of the neutron must be ... not larger than 0.01 proton mass. ... in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.

- Since the neutron was discovered two years later by J. Chadwick, Fermi, following the proposal by E. Amaldi, used the name “neutrino” (little neutron) in 1932 and later proposed the Fermi theory of beta decay.
• Reines and Cowan discovered the neutrino in 1956 using inverse beta decay. [Science 124, 3212:103]

• Madame Wu in 1956 demonstrated that P is violated in weak interactions.

• Muon neutrinos were discovered in 1962 by L. Lederman, M. Schwartz and J. Steinberger.

The Nobel Prize in Physics 1988
• The first idea of neutrino oscillations was considered by B. Pontecorvo in 1957.

• Mixing was introduced at the beginning of the ‘60 by Z. Maki, M. Nakagawa, S. Sakata,

• First indications of ν oscillations came from solar ν.

• R. Davis built the Homestake experiment to detect solar ν, based on an experimental technique by Pontecorvo.
• Compared with the predicted solar neutrino fluxes (J. Bahcall et al.), a significant deficit was found. First results were announced [R. Davis, Phys. Rev. Lett. 12 (1964)302 and R. Davis et al., Phys. Rev. Lett. 20 (1968) 1205].

• This anomaly received further confirmation (SAGE, GALLEX, SuperKamiokande, SNO...) and was finally interpreted as neutrino oscillations.

![Diagram of SNO's CC, NC and ES measurements from the D2O Phase. The -axies are inferred fluxes of electron neutrinos and muon plus tau neutrinos. Since the NC and ES measurements are sensitive to both $\nu_e$ and $\nu_\mu/\nu_\tau$, the ES and NC bands have definite slopes. The CC measurement is sensitive to $\nu_e$ only, so has an infinite slope. The widths of the bands represent the uncertainties of the measurements. The intersection of the three bands gives the best estimate of $\phi_\mu/\phi_\tau$ and $\phi_e$.

The flux of neutrinos predicted by the SSM is indicated by $\phi_{SSM}$.]

5.3. SNO's night-day flux asymmetry measurement in $D_2O$

In addition to measuring the time integrated fluxes, the difference between the solar neutrino fluxes at night and day has also been studied [12]. If the mixing of solar neutrino flavours is due to interactions with matter (the MSW effect) [13, 14], then $\nu_e$ might regenerate while passing through the Earth at night time. For more details on the MSW effect, see Boris Kayser's lectures in these proceedings [8]. The probability to regenerate depends on the neutrino mixing parameters, $\Delta m^2_{12}$ (the difference of the squared neutrino masses) and $\theta_{12}$ (the solar neutrino mixing angle), the path length of the neutrinos through the Earth, and the local electron density that the neutrinos encounter. SNO has determined the night-day asymmetry $A = 2 (\phi_{\text{night}} - \phi_{\text{day}}) / (\phi_{\text{night}} + \phi_{\text{day}})$ for the flux of $\nu_e$ under two different assumptions. The first assumption is that $A_{\text{NC}}$ may be non-zero (possible if there is matter enhanced mixing with sterile neutrinos). The asymmetry of the NC rate was allowed to float in a fit to the data that simultaneously determined the asymmetries of the CC and NC rates. The result of the fit was $A_{\text{CC}} = (1.4 \pm 0.6)\%$, $A_{\text{NC}} = (-20.4 \pm 16.9)\%$.

The second assumption is that there is no mixing with sterile neutrinos. When $A_{\text{NC}}$ is fixed at zero, SNO measures $A_{\text{NC}} = (7.0 \pm 4.9)\%$, $A_{\text{NC}} = 0\%$. Both of these results are consistent with no night-day asymmetry.

The Nobel Prize in Physics 2015

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An anomaly was also found in atmospheric neutrinos.

- Atmospheric neutrinos had been observed by various experiments but the first relevant indication of an anomaly was presented in 1988 \([\text{Kamiokande Coll., Phys. Lett. B205 (1988) 416}]\), subsequently confirmed by MACRO.


The Nobel Prize in Physics 2015
Neutrinos in the SM

- Neutrinos come in 3 flavours, corresponding to the charged lepton.

- They belong to SU(2) doublets:
  
  \[
  \begin{pmatrix}
  \nu_e \\
  e
  \end{pmatrix}
  \quad
  \begin{pmatrix}
  \nu_\mu \\
  \mu
  \end{pmatrix}
  \quad
  \begin{pmatrix}
  \nu_\tau \\
  \tau
  \end{pmatrix}
  \]

  electron
  
  electron antineutrino
Neutrino mixing

Mixing is described by the Pontecorvo-Maki-Nakagawa-Sakata matrix:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} (U_{\alpha k}^* \bar{\nu}_k L \gamma^\rho l_{\alpha L} W_\rho + h.c.)$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.
• 2-neutrino mixing matrix depends on 1 angle only. The phases get absorbed in a redefinition of the leptonic fields (a part from 1 Majorana phase).

\[
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

• 3-neutrino mixing matrix has 3 angles and 1(+2) CPV phases.

\[
\begin{pmatrix}
\bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3
\end{pmatrix}
\begin{pmatrix}
e^{i\phi_1} & 0 & 0 \\
0 & e^{i\phi_2} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\text{CKM-type}
\end{pmatrix}
\begin{pmatrix}
e^{i\rho_e} & 0 & 0 \\
0 & e^{i\rho_\mu} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e \\
\mu \\
\tau
\end{pmatrix}
\]

Rephasing
\[
\begin{align*}
e & \rightarrow \quad e^{-i(\rho_e+\psi)}e \\
\mu & \rightarrow \quad e^{-i(\rho_\mu+\psi)}\mu \\
\tau & \rightarrow \quad e^{-i\psi}\tau
\end{align*}
\]
the kinetic, NC and mass terms are not modified: these phases are unphysical.
For Dirac neutrinos, the same rephasing can be done. For Majorana neutrinos, the Majorana condition forbids such rephasing: 2 physical CP-violating phases.

$$U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \begin{pmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{13}
\end{pmatrix} \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha_{21}/2} & 0 \\
0 & 0 & e^{i\alpha_{31}/2}
\end{pmatrix}$$

For antineutrinos, $$U \rightarrow U^*$$

**CP-conservation requires** $$U$$ is real $$\Rightarrow \delta = 0, \pi$$
Contrary to what expected in the SM, neutrinos oscillate: after being produced, they can change their flavour.

Neutrino oscillations imply that neutrinos have mass and they mix. First evidence of physics beyond the SM.
Neutrino oscillations and Quantum Mechanics analogs

Neutrino oscillations are analogous to many other systems in QM, in which the initial state is a **coherent superposition of eigenstates of the Hamiltonian**:

- **NH3 molecule**: produced in a superposition of “up” and “down” states

- **Spin states**: for example a state with spin up in the z-direction in a magnetic field aligned in the x-direction $B=(B,0,0)$. This gives rise to spin-precession, i.e., the state changes the spin orientation with a typical oscillatory behaviour.
Neutrino oscillations: the picture

Production

Flavour states

Propagation

Massive states (eigenstates of the Hamiltonian)

Detection

Flavour states

At production, coherent superposition of massive states:

\[ |\nu_{\mu}\rangle = U_{\mu 1} |\nu_1\rangle + U_{\mu 2} |\nu_2\rangle + U_{\mu 3} |\nu_3\rangle \]
Production

\[ |\nu_\mu\rangle = \sum_i U_{\mu i} |\nu_i\rangle \]

Propagation

\[ \nu_1 : \quad e^{-iE_1t} \]
\[ \nu_2 : \quad e^{-iE_2t} \]
\[ \nu_3 : \quad e^{-iE_3t} \]

Detection: projection over \[ \langle \nu_e | \]

As the propagation phases are different, the state evolves with time and can change to other flavours.
Neutrinos oscillations in vacuum: the theory

Let’s assume that at \( t=0 \) a muon neutrino is produced

\[
|\nu, t = 0\rangle = |\nu_\mu\rangle = \sum_i U_{\mu i} |\nu_i\rangle
\]

The time-evolution is given by the solution of the Schroedinger equation with free Hamiltonian:

\[
|\nu, t\rangle = \sum_i U_{\mu i} e^{-iE_i t} |\nu_i\rangle
\]

In the same-momentum approximation:

\[
E_1 = \sqrt{p^2 + m_1^2} \quad E_2 = \sqrt{p^2 + m_2^2} \quad E_3 = \sqrt{p^2 + m_3^2}
\]

Note: other derivations are also valid (same E formalism, etc).
At detection one projects over the flavour state as these are the states which are involved in the interactions. The **probability of oscillation** is

\[
P(\nu_\mu \rightarrow \nu_\tau) = |\langle \nu_\tau | \nu, t \rangle|^2
\]

\[
= \left| \sum_{ij} U_{\mu i} U^*_{\tau j} e^{-iE_it} \langle \nu_j | \nu_i \rangle \right|^2
\]

\[
= \left| \sum_i U_{\mu i} U^*_{\tau i} e^{-iE_it} \right|^2
\]

Typically, neutrinos are very relativistic: \( E_i \simeq p + \frac{m_i^2}{2p} \)

\[
= \left| \sum_i U_{\mu i} U^*_{\tau i} e^{-i \frac{m_i^2}{2E_i} t} \right|^2
\]

\[
= \left| \sum_i U_{\mu i} U^*_{\tau i} e^{-i \frac{m_i^2 - m_1^2}{2E_i} t} \right|^2
\]

\[\Delta m_{i1}^2\]
Implications of the existence of neutrino oscillations

The oscillation probability implies that

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i U_{\alpha_1}U_{\beta_1}^* e^{-i \frac{\Delta m_{i1}^2}{2E} L} \right|^2 \]

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

- **neutrinos mix** (as \( U \) needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)
General properties of neutrino oscillations

• Neutrino oscillations conserve the total lepton number: a neutrino is produced and evolves with times.

• They violate the flavour lepton number as expected due to mixing.

• Neutrino oscillations do not depend on the overall mass scale and on the Majorana phases.

• CPT invariance: $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$

• CP-violation: $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ requires $U \neq U^*(\delta \neq 0, \pi)$
2-neutrino case

Let’s recall that the mixing is

\[
\begin{pmatrix}
\nu_\alpha \\
\nu_\beta
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

We compute the probability of oscillation

\[
P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2
\]

\[
= \left| \cos \theta \sin \theta - \cos \theta \sin \theta e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2
\]

\[
= \sin^2(2\theta) \sin^2\left( \frac{\Delta m^2_{21}}{4E} L \right)
\]

\[
\frac{\Delta m^2_{21}}{4E} L = 1.27 \frac{\Delta m^2_{21}}{4} \frac{[eV^2]}{E[GeV]} L[\text{km}]
\]

Exercise
Derive
$P(\nu_\alpha \rightarrow \nu_\beta) \simeq 0$

$P(\nu_\alpha \rightarrow \nu_\beta) \simeq \frac{1}{2} \sin^2(2\theta)$

First oscillation maximum
Properties of 2-neutrino oscillations

- **Appearance probability:**

  \[ P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2_{21}}{4E} L\right) \]

- **Disappearance probability:**

  \[ P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2_{21}}{4E} L\right) \]

- **No CP-violation as there is no Dirac phase in the mixing matrix**

  \[ P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \]

- **Consequently, no T-violation (using CPT):**

  \[ P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha) \]
3-neutrino oscillations

They depend on two mass squared-differences

\[ \Delta m_{21}^2 \ll \Delta m_{31}^2 \]

In general the formula is quite complex

\[
P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2}{2E} L} + U_{\alpha 3} U_{\beta 3}^* e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|^2
\]

**Interesting 2-neutrino limits**

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations.

\[
\frac{\Delta m_{21}^2}{4E} L \ll 1
\]

- applies to atmospheric, reactor (Daya Bay...), current accelerator neutrino experiments...
The oscillation probability reduces to a 2-neutrino limit:

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* + U_{\alpha 3} U_{\beta 3}^* e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|^2
\]

We use the fact that

\[
U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* + U_{\alpha 3} U_{\beta 3}^* = \delta_{\alpha \beta}
\]

\[
= \left| -U_{\alpha 3} U_{\beta 3}^* + U_{\alpha 3} U_{\beta 3}^* e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|^2
\]

\[
= \left| U_{\alpha 3} U_{\beta 3}^* \right|^2 \left| -1 + e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|^2
\]

The same we have encountered in the 2-neutrino case

\[
= 2 \left| U_{\alpha 3} U_{\beta 3} \right|^2 \sin^2 \left( \frac{\Delta m_{31}^2}{4E} L \right)
\]

Exercise

Derive
\[ \frac{\Delta m_{31}^2}{4E} L \gg 1 \] : for reactor neutrinos (KamLAND).

The oscillations due to the atmospheric mass squared differences get averaged out.

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) \approx c_{13}^4 \left( 1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m_{21}^2 L}{4E} \right) + s_{13}^4
\]
CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let’s consider the CP-asymmetry:

\[
P(\nu_\alpha \rightarrow \nu_\beta; t) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; t) =
\]

\[
= \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} + U_{\alpha 3} U_{\beta 3}^* e^{-i \frac{\Delta m_{31}^2 L}{2E}} \right|^2 - (U \rightarrow U^*)
\]

\[
= U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2} U_{\beta 2} e^{i \frac{\Delta m_{21}^2 L}{2E}} + U_{\alpha 1}^* U_{\beta 1} U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} - (U \rightarrow U^*) + \cdots
\]

\[
= 4s_{12}c_{12}s_{13}c_{13} s_{23} c_{23} \sin \delta \left[ \sin\left( \frac{\Delta m_{21}^2 L}{2E} \right) + \left( \frac{\Delta m_{23}^2 L}{2E} \right) + \left( \frac{\Delta m_{31}^2 L}{2E} \right) \right]
\]

- CP-violation requires all angles to be nonzero.
- It is proportional to the sine of the delta phase.
- If one can neglect \( \Delta m_{21}^2 \), the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.
Neutrinos oscillations in matter

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.

- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.

- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.
Effective potentials


Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.

We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

\[ \langle \bar{e}\gamma_0 e \rangle = N_e \quad \langle \bar{e}\gamma e \rangle = \langle \bar{\nu}_e \rangle \quad \langle \bar{e}\gamma_0\gamma_5 e \rangle = \left\langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e} \right\rangle \quad \langle \bar{e}\gamma\gamma_5 e \rangle = \langle \vec{\sigma}_e \rangle \]

For an unpolarised at rest background, the only term is the first one. \( N_e \) is the electron density.

The neutrino dispersion relation can be found by solving the Dirac eq with plane waves, in the ultrarelativistic limit

\[ E \simeq p \pm \sqrt{2} G_F N_e \]

<table>
<thead>
<tr>
<th>medium ( e, \bar{e} )</th>
<th>( A_{CC} ) for ( \nu_e, \bar{\nu}_e ) only</th>
<th>( A_{NC} ) for ( \nu_{e,\mu,\tau}, \bar{\nu}_{e,\mu,\tau} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, \bar{p} )</td>
<td>0 ( \pm \sqrt{2} G_F (N_e - N_{\bar{e}}) )</td>
<td>( \pm \sqrt{2} G_F (N_e - N_{\bar{e}})(1 - 4s_W^2)/2 )</td>
</tr>
<tr>
<td>( n, \bar{n} )</td>
<td>0 ( \pm \sqrt{2} G_F (N_p - N_{\bar{p}}) )</td>
<td>( \pm \sqrt{2} G_F (N_n - N_{\bar{n}})(1 - 4s_W^2)/2 )</td>
</tr>
<tr>
<td>ordinary matter</td>
<td>( \pm \sqrt{2} G_F N_e )</td>
<td>( \pm \sqrt{2} G_F N_n/2 )</td>
</tr>
</tbody>
</table>
**The Hamiltonian**

Let’s start with the vacuum Hamiltonian for 2-neutrinos

\[
i \frac{d}{dt} \left( \begin{array}{c} |\nu_1\rangle \\ |\nu_2\rangle \end{array} \right) = \left( \begin{array}{cc} E_1 & 0 \\ 0 & E_2 \end{array} \right) \left( \begin{array}{c} |\nu_1\rangle \\ |\nu_2\rangle \end{array} \right)
\]

Recalling that \(|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle\), one can go into the flavour basis

\[
i \frac{d}{dt} \left( \begin{array}{c} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{array} \right) = \left( \begin{array}{cc} E_1 & 0 \\ 0 & E_2 \end{array} \right) \left( \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right) \left( \begin{array}{c} |\nu_1\rangle \\ |\nu_2\rangle \end{array} \right)
\]

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.
The **full Hamiltonian in matter** can then be obtained by adding the potential terms, diagonal in the flavour basis. For electron and muon neutrinos

\[
i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
\]

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semi-analytical results can be obtained.
2-neutrino case in constant density

\[ i \frac{d}{dt} \left( \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right) = \left( \begin{array}{cc} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & -\frac{\Delta m^2}{4E} \cos 2\theta \end{array} \right) \left( \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right) \]

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.
Effective Hamiltonian

$$\begin{pmatrix} M \end{pmatrix}.$$ 

Mixing angle

vacuum

$$\tan 2\theta \sim \frac{2}{M}.$$ 

matter suppression (Sun, SN)

$$\tan 2\theta^M \sim \frac{2}{M} + \frac{2}{-M} \ll \tan 2\theta.$$ 

MSW resonance (Sun, SN)

$$\tan 2\theta^M \sim \frac{2}{M} - \frac{2}{-M} \sim \infty.$$
2-neutrino case in constant density

\[ i \frac{d}{dt} \left( \begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array} \right) = \left( \begin{array}{cc} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}GFN_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{array} \right) \left( \begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array} \right) \]

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- **Eigenvalues:**

\[ E_A - E_B = \sqrt{\left( \frac{\Delta m^2}{2E} \cos(2\theta) - \sqrt{2}GFN_e \right)^2 + \left( \frac{\Delta m^2}{2E} \sin(2\theta) \right)^2} \]

- **The diagonal basis and the flavour basis are related by a unitary matrix with angle in matter**

\[ \tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E} \sin(2\theta)}{\frac{\Delta m^2}{2E} \cos(2\theta) - \sqrt{2}GFN_e} \]
• If $\sqrt{2} G_F N_e \ll \frac{\Delta m^2}{2E} \cos 2\theta$, we recover the vacuum case and
  $\theta_m \approx \theta$

• If $\sqrt{2} G_F N_e \gg \frac{\Delta m^2}{2E} \cos(2\theta)$, matter effects dominate and oscillations are suppressed.

• If $\sqrt{2} G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta$: resonance and maximal mixing
  $\theta_m = \pi/4$

• The resonance condition can be satisfied for
  - neutrinos if $\Delta m^2 > 0$
  - antineutrinos if $\Delta m^2 < 0$

$$P(\nu_e \to \nu_\mu; t) = \sin^2(2\theta_m) \sin^2 \left( \frac{E_A - E_B}{2} \frac{L}{E} \right)$$
In long baseline experiments

\[
\frac{\Delta m^2}{2E} \cos(2\theta) \nu + \sqrt{2}G_F N_e \quad \bar{\nu} - \sqrt{2}G_F N_e
\]

For neutrinos

\[
\Delta m^2 > 0 \quad \text{enhancement} \quad \tan 2\theta^M \sim \frac{2}{- + +}
\]

For antineutrinos

\[
\Delta m^2 > 0 \quad \text{suppression} \quad \tan 2\theta^M \sim \frac{2}{- + -}
\]
Matter effects modify the oscillation probability in LBL experiments.

\[ P_{\nu_\mu \rightarrow \nu_e} = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{13} L}{2} \]

The probability enhancement happens for

- neutrinos if \( \Delta m^2 > 0 \)
- antineutrinos if \( \Delta m^2 < 0 \)

The impact of matter effects is stronger at higher energies and at longer baselines.
2-neutrino oscillations with varying density

Let’s consider the case in which $N_e$ depends on time. This happens, e.g., if a beam of neutrinos is produced and then propagates through a medium of varying density (e.g. Sun, supernovae).

\[
\begin{align*}
\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} &= \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e(t) & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
\end{align*}
\]

At a given instant of time $t$, the Hamiltonian can be diagonalised by a unitary transformation as before. We find the instantaneous matter basis and the instantaneous values of the energy. The expressions are exactly as before but with the angle which depends on time, $\theta(t)$. 

We have
\[ |\nu_\alpha\rangle = U(t)|\nu_I\rangle, \quad U^\dagger(t)H_{m,f}U(t) = \text{diag}(E_A(t), E_B(t)) \]

Starting from the Schroedinger equation, we can express it in the instantaneous basis

\[
\begin{align*}
    i \frac{d}{dt} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} &= 
    \begin{pmatrix}
        -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_FN_e(t) & \frac{\Delta m^2}{4E} \sin 2\theta \\
        \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta
    \end{pmatrix} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} \\
    \frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} &= 
    \begin{pmatrix}
        E_A(t) & -i\dot{\theta}(t) \\
        i\dot{\theta}(t) & E_B(t)
    \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}
\end{align*}
\]

The evolution of $\nu_A$ and $\nu_B$ are not decoupled. In general, it is very difficult to find an analytical solution to this problem.
**Adiabatic case**

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can “jump” from one to the other.

If the evolution is sufficiently slow (adiabatic case):

\[ |\dot{\theta}(t)| \ll |E_A - E_B| \]

we can follow the evolution of each component independently.

**Adiabaticity condition**

\[
\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta) \frac{\Delta m^2}{2E}}{|E_A - E_B|^3} |\dot{V}_{CC}| \ll 1
\]

In the Sun, typically we have

\[
\gamma \sim \frac{\Delta m^2}{10^{-9} \text{eV}^2} \frac{\text{MeV}}{E_\nu}
\]
Solar neutrinos: MSW effect

The oscillations in matter were first discussed by L. Wolfenstein, S. P. Mikheyev, A. Yu Smirnov.

- Production in the centre of the Sun: matter effects dominate at high energy, negligible at low energy.

The probability of $\nu_e$ to be

- $\nu_A$ is $\cos^2 \theta_m$
- $\nu_B$ is $\sin^2 \theta_m$

If matter effects dominate,

- $\sin^2 \theta_m \simeq 1$

- $P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2(2\theta)$ (averaged vacuum oscillations), when matter effects are negligible (low energies)

- $P(\nu_e \rightarrow \nu_e) = \sin^2 \theta$ (dominant matter effects and adiabaticity) (high energies)
In CC (NC) SU(2) interactions, the $W$ boson ($Z$ boson) will be exchanged leading to the production of neutrinos.

Neutrino production

Beta decay.

Pion decay

Decay into electrons is suppressed.
Neutrino detection proceeds via CC (and NC) SU(2) interactions. Example:

\[ m_e = 0.5 \text{ MeV} < m_{\mu} = 105 \text{ MeV} < m_{\tau} = 1700 \text{ MeV} \]

A certain lepton will be produced in a CC only if the neutrino has sufficient energy.
We are interested mainly in produced **charged particles** as these can emit light and/or leave tracks in segmented detectors (magnetisation -> charge reconstruction).

Super-Kamiokande detector

T2K experiment

NOvA detector

MINOS experiment
Neutrino sources

J. Formaggio and S. Zeller, 1305.7513
**Solar neutrinos**

Electron neutrinos are copiously produced in the Sun.

- **Energies:** 0.1 - 10 MeV.
- One can observed CC $\nu_e$ and NC: measuring the oscillation disappearance and the overall flux.

O. Smirnov, for Borexino, Neutrino 2018

http://www.sns.ias.edu/~jnb/
BOREXINO (in operation)

- 18m of liquid organic scintillator PC + PPO (1.5 g/l)
- $(\nu, e)$-scattering with low threshold (~200 keV)
- Outer muon detector

Expected contributions to the observed spectrum (MC)

- Solar neutrino $\rightarrow$ electron recoil spectra
- Irreducible $^{14}$C and other internal radioactive contaminants: $^{210}$Po, $^{210}$Bi (both not in secular equilibrium), $^{85}$Kr, $^{11}$C
- External $\gamma$ (high energy)

Results arXiv: 1707.09279

Systematics Backgrounds

- $^{210}$Bi, E-scale, response $R(^{85}$Kr)$ < 7.5 @ 95%$

LS mass

- Data-set: Dec 14th 2011 - May 21st 2016
- Total exposure: 1291.51 days x 71.3 tons
- Fit range: (0.19 - 2.93) MeV

Results arXiv: 1707.09279

O. Smirnov, for Borexino, Neutrino 2018

Elastic neutrino scattering in WC: directional information

M. Ikeda, for Super-Kamiokande, Neutrino 2018
Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance. MSW effect at high energies, vacuum oscillations at low energy.
Strumia and Vissani

SAGE, GALLEX, SNO, Borexino, SuperKamiokande

MSW/LMA: electron neutrino survival probabilities

High metallicity SSM

Low metallicity SSM

MSW errors (1σ) are shown by rose band.

Total error on $P_{\text{ee}}$:

- For pp and pep neutrinos, contribution of experimental errors dominates (easy to predict, difficult to measure)
- For $^7$Be and $^8$B theoretical predictions of the Solar model are worse than measurements

O. Smirnov, for Borexino, Neutrino 2018

M. Ikeda, for Super-Kamiokande, Neutrino 2018
Solar experiments best constrain the “solar mixing” angle of $\theta_{12}$ to be large (but non-maximal). The mass squared difference is around $7 \times 10^{-5} \text{ eV}^2$. 

- Super-K data best constrains $\Delta m^2_{21}$
- SNO data best constrains $\sin^2 \theta_{12}$
- Complementarity makes combined fit beneficial
- Correlation via $^8\text{B}$ flux further tightens constraints
**Atmospheric neutrinos**

Cosmic rays hit the atmosphere and produce pions (and kaons) which decay producing lots of muon and electron (anti-) neutrinos.

- Typical energies: 100 MeV - 100 GeV
- Typical distances: 100-10000 km.

SuperKamiokande Coll.

I started in Apr. 1, 1996.

SK-IV finished on May 31, 2018.

Photo coverage 40%

20%

40%

40%

2002

2006

2008

1996

SK-II

SK-III

SK-IV

Accident

Full reconstruction

Replace electronics & DAQ system

Preparation for SK-Gd

SuperKamiokande Coll.
SK reported the first evidence of neutrino oscillations in 1998 with atmospheric neutrinos (\(\nu_{\mu}\rightarrow\nu_{\tau}\)).

T. Kajita's talk at Neutrino 1998

SK and MINOS went on to measure the atmospheric mixing angle to be large (mainly maximal) and the atm mass squared different at \(\sim 2.5 \times 10^{-3} \text{ eV}^2\).

# of tau events
\[338.1 \pm 72.7 \text{ (stat.+ sys.) events}\]
Reject no-tau-appearance @ 4.6\(\sigma\).
( Exp. significance is 3.3\(\sigma\))
**Reactor neutrinos**

Copious amounts of electron antineutrinos are produced from reactors.

- Typical energy: 1-3 MeV;
- Typical distances: 1 km (Daya Bay, DCHOOZ, RENO) —100 km (KamLAND).

- At these energies inverse beta decay interactions dominate.

\[ \overline{\nu}_e + p \rightarrow e^+ + n \]

These detectors use liquid scintillator.

J. P. Ochoa-Ricoux, for Daya Bay, at Neutrino 2018
At ~few km, the disappearance probability is

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) = 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}
\]

**Sensitivity to \( \theta_{13} \).** Reactors played an important role in the discovery of \( \theta_{13} \) and in its precise measurement.

---

**Plot Description:**

- **X-axis:** Length (km) [at E~3MeV]
- **Y-axis:** Probability \( \bar{\nu}_e \rightarrow \bar{\nu}_e \)

---

**Graphical Elements:**

- **Bars:** Short baseline experiments, \( \theta_{13} \) experiments: Double Chooz, Daya Bay, RENO, KamLAND: "Solar" Parameters, JUNO

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**Notes:**

- C. Buck, for DoubleCHOOZ Coll., at Neutrino 2018
- J. P. Ochoa-Ricoux, for Daya Bay, at Neutrino 2018
In 2012, previous hints (DoubleCHOOZ, T2K, MINOS) for a nonzero third mixing angle were confirmed by Daya Bay and RENO: important discovery.

This discovery has very important implications for the future neutrino programme and understanding of the origin of mixing.
At ~100 km, the disappearance probability is

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) \simeq c_{13}^4 \left( 1 - \sin^2(2\theta_{12}) \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right) \right) + s_{13}^4 \]

Sensitivity to Delta \( m_{12}^2 \).
Accelerator neutrinos

Conventional beams: muon neutrinos from pion decays

- **Typical energies:**
  - MINOS: $E \sim 4$ GeV;
  - T2K: $E \sim 700$ MeV;
  - NOvA: $E \sim 2$ GeV.
  - OPERA and ICARUS: $E \sim 20$ GeV.

- **Typical distances:** 100 km - 2000 km.
  - MINOS: $L = 735$ km;
  - T2K: $L = 295$ km;
  - NOvA: $L = 810$ km.
  - OPERA and ICARUS: $L = 700$ km.
At these energies, one can detect electron, muon (and tau) ν via CC interactions.

**MINOS, T2K, NOvA:**

\[
P(\nu_\mu \rightarrow \nu_\mu; t) = 1 - 4s_{23}^2c_{13}^2(1 - s_{23}^2c_{13}^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E}
\]

**T2K, NOvA:**

\[
P(\nu_\mu \rightarrow \nu_e; t) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E} + ...
\]

**OPERA (and ICARUS):**

\[
P(\nu_\mu \rightarrow \nu_\tau; t) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}
\]

Sensitivity to \(\Delta m_{31}^2, \theta_{23}, \theta_{13}\)
At these energies, one can detect electron, muon (and tau) ν via CC interactions.

**MINOS, T2K, NOvA:**

\[
P(\nu_\mu \rightarrow \nu_\mu; t) = 1 - 4s_{23}^2c_{13}^2(1 - s_{23}^2c_{13}^2)\sin^2 \frac{\Delta m_{31}^2 L}{4E}
\]

**T2K, NOvA:**

\[
P(\nu_\mu \rightarrow \nu_e; t) = s_{23}^2 \sin^2 (2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E} + \ldots
\]

**OPERA (and ICARUS):**

\[
P(\nu_\mu \rightarrow \nu_\tau; t) = c_{13}^4 \sin^2 (2\theta_{23}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}
\]

**Sensitivity to** $\Delta m_{31}^2$, $\theta_{23}$, $\theta_{13}$

$\delta, \text{sgn}(\Delta m_{31}^2)$
MINOS and MINOS+ were designed to study neutrino oscillations over long baselines using two detectors that are vonwscintillator tracking calorimeters to contain muons – functionally identical for systematic uncertainty reduction – Magnetized for sign selection and energy estimation.

Detectors are on-axis for NuMI neutrino beam.

Far Detector
- Underground in Soudan mine
- 735 km from target
- 5.4 kton mass

Near Detector
- At Fermilab
- 1 km from target
- 1 kton mass

Best fit
\[ \Delta m_{32}^2 = 2.42 \times 10^{-3} \text{ eV}^2 \]
\[ \sin^2 \theta_{23} = 0.42 \]

A. Aurisano, for MINOS and MINOS+, at Neutrino 2018
M. Sanchez, NOvA Coll., at Neutrino 2018

• Mayly Sanchez - ISU

1.49e21 POT

T2K Run 1-9c Preliminary

νμ / νμ at Neutrino 2018

M. Wasko, T2K Coll., at Neutrino 2018

- Fieldman-Cousins
- Reconstr. Energy (GeV)
- No Feldman-Cousins
- Reconstructed Neutrino Energy (GeV)

Neutrino beam

NOvA Preliminary

- FD Data
- Prediction
- 1-σ syst. range
- Wrong Sign νμ CC
- Total bkg.
- Cosmic bkg.

Events / 0.1 GeV

Reconstructed Neutrino Energy (GeV)

Morgan O.

NOvA Preliminary

Δm^2 (10^-3 eV^2)

No Feldman-Cousins

- NOvA NH 90% CL
  - νμ + νe, 2018
  - νμ, 2017

Best fit

sin^2θ_{23}
The accelerator muon neutrino disappearance channel has allowed to measure with good precision $|\Delta m^2_{31}|$ and $\theta_{23}$. 

A. Aurisano, for MINOS and MINOS+, at Neutrino 2018
Measurement of oscillation parameters

Atmospheric neutrinos
SK (MINOS, IceCube)

Reactor neutrinos:
JUNO

LBL exp numu disapp.:
MINOS, MINOS+, T2K, NOvA

LBL exp nue app.:
T2K, NOvA

Also: Tests of standard neutrino paradigm
Current status of neutrino parameters

3 sizable mixing angles
2 mass squared differences

NuFit 3.0: M. C. Gonzalez-Garcia et al., 1611.01514, Pre-Neutrino 2018
See also F. Capozzi et al., 1703.04471

- neutrinos have mass
- neutrinos mix (Misaligned flavour and massive states)

First evidence of physics beyond the Standard Model.
• 20 years ago, Neutrino oscillations were discovered. They play a critical role in the study of neutrino properties: their discovery implies that neutrinos have mass and mix.

• Three mixing angles, two mass squared differences have been measured with good precision. The data are also giving the first hints in favour of MO and LCPV.

• A wide experimental program has taken place and new experiments are underway. Stay tuned!
Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let’s consider for simplicity a 2-body decay: \( \pi \to \mu \bar{\nu}_\mu \).

Energy-momentum conservation seems to require:

\[
E_\pi = E_\mu + E_1 \quad \text{with} \quad E_1 = \sqrt{p^2 + m_1^2}
\]
\[
E_\pi = E_\mu + E_2 \quad \text{with} \quad E_2 = \sqrt{p^2 + m_2^2}
\]

How can the picture be consistent?
Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let’s consider for simplicity a 2-body decay: \( \pi \rightarrow \mu \bar{\nu}_\mu \).

Energy-momentum conservation seems to require:

\[
E_\pi = E_\mu + E_1 \quad \text{with} \quad E_1 = \sqrt{p^2 + m_1^2}
\]

\[
E_\pi = E_\mu + E_2 \quad \text{with} \quad E_2 = \sqrt{p^2 + m_2^2}
\]

These two requirements seems to be incompatible. Intrinsic quantum uncertainty, localisation of the initial pion lead to an uncertainty in the energy-momentum and allow coherence of the initial neutrino state.
The need for wavepackets

- In deriving the oscillation formulas we have implicitly assumed that neutrinos can be described by plane-waves, with definite momentum.

- However, production and detection are well localised and very distant from each other. This leads to a momentum spread which can be described by a wave-packet formalism.

Typical sizes:
- E.g. production in decay: the relevant timescale is the pion lifetime (or the time travelled in the decay pipe),

\[ \Delta t \sim \tau_\pi \Rightarrow \Delta E \Rightarrow \Delta p \quad \Delta x \]

For details see, Akhmedov, Smirnov, 1008.2077; Giunti and Kim, Neutrino Physics and Astrophysics.
Decoherence and the size of a wave-packet

- The different components of the wavepacket, $\nu_1$, $\nu_2$ and $\nu_3$, travel with slightly different velocities (as their mass is different).

- If the neutrinos travel extremely long distances, these components stop to overlap, destroying coherence and oscillations.

- In terrestrial experimental situation this is not relevant. But this can happen for example for supernovae neutrinos.