Neutrinos
Neutrino masses and mixing: theory
Neutrinos in cosmology

The 2018 European School on High Energy Physics

Maratea
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What will you learn from this lecture?

- Origin of neutrino masses and leptonic mixing
  - Type of neutrino masses
  - Models BSM for neutrino masses
  - See-saw mechanism
  - How to explain the observed mixing structure and Flavour symmetry models (very briefly)

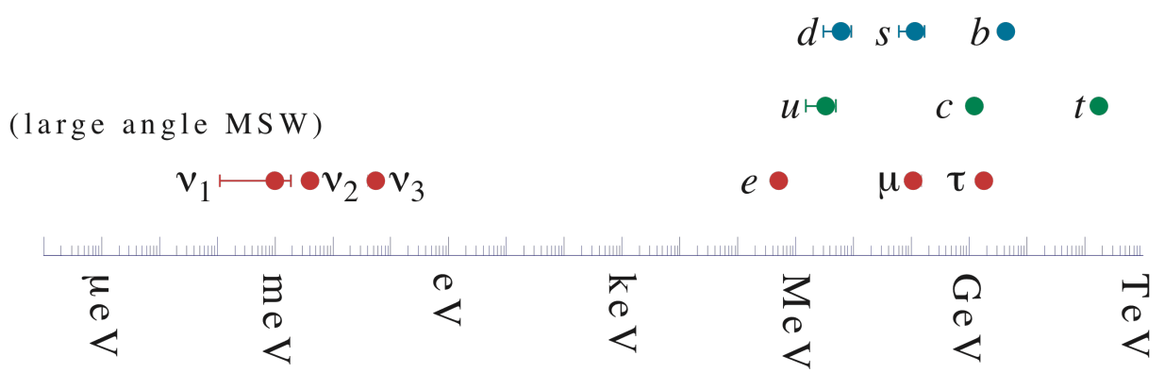
- Neutrinos in cosmology
  - neutrinos in the Early Universe
  - Leptogenesis and the baryon asymmetry
  - Sterile neutrinos as WDM (in additional material)

The ultimate goal is to understand
- where do neutrino masses come from?
- why there is leptonic mixing? and what is at the origin of the observed structure?

Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from flavour physics experiments and from colliders.

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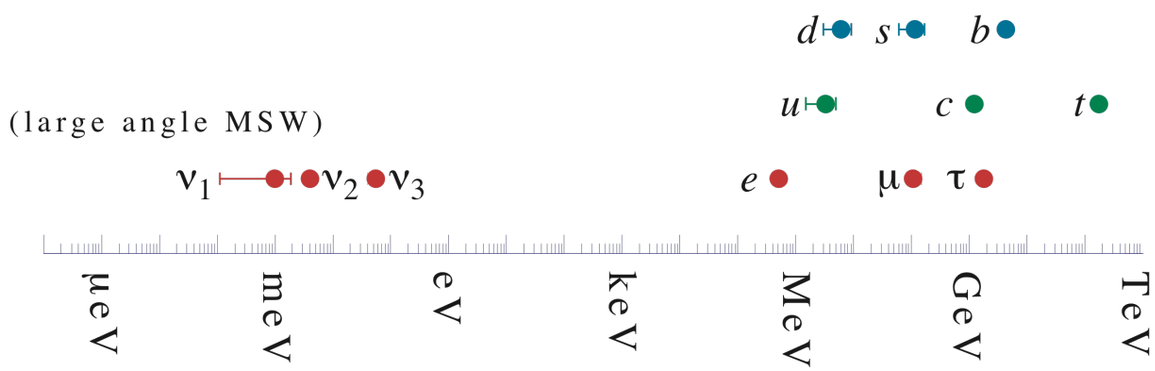

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This information is **complementary** with the one from flavour physics experiments and from colliders.

First step:
We need to augment the SM Lagrangian with terms which describe neutrino masses.

What kind of masses can neutrinos have?

Neutrino masses in the nuSM lagrangian

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R\psi_L + \text{h.c.}) = m_\psi\bar{\psi}\psi$$

Exercise
check this formula

Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: $\nu_L \quad , \quad \nu_R$
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R\nu_L + \text{h.c.})$$

Does it conserve lepton number?

$$\nu_L \to e^{i(+1)\alpha}\nu_L$$
$$\nu_R \to e^{i(?)\alpha}\nu_R$$

Neutrino masses in the nuSM lagrangian

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$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R\nu_L + \text{h.c.})$$

This conserves lepton number!

$$\nu_L \to e^{i\alpha}\nu_L$$
$$\nu_R \to e^{i\alpha}\nu_R$$

$$\mathcal{L}_{mD} \to \mathcal{L}_{mD}$$

Diagonalize a Dirac mass term

If there are several fields, there will be a Dirac mass matrix.

$$\mathcal{L}_{mD} = -\bar{\nu}_{Ra}\,(m_D)_{ab}\,\nu_{Lb} + \text{h.c.}$$

This requires two unitary mixing matrices to diagonalise it

$$m_D = V m_{\text{diag}} U^\dagger$$

and the massive states are

$$n_L = U^\dagger \nu_L \quad n_R = V^\dagger \nu_R$$

This is the mixing matrix which enters in neutrino oscillations. So the form of the mass matrix determines the mixing pattern.

Majorana masses

If we have only the left-handed field, we can still write down a mass term, called Majorana mass term. We use the fact that

$$(\psi_L)^c = (\psi^c)_R$$

then the mass term is

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

Exercise
Show that these two formulations are equivalent.

Hint:

$$\begin{aligned} \bar{\nu_L^c} \nu_L &= (C\bar{\nu}_L^T)^\dagger \gamma^0 \nu_L = \bar{\nu}_L^* C^\dagger \gamma^0 \nu_L \\ &= \nu_L^T \gamma^{0*} C^\dagger \gamma^0 \nu_L = -\nu_L^T C^{-1} \nu_L \end{aligned}$$

This breaks lepton number!

$$\nu_L \to e^{i\alpha} \nu_L \qquad \mathcal{L}_{mM} \to e^{2i\alpha} \mathcal{L}_{mM}$$

Diagonalize a Majorana mass term

If there are several fields, there will be a Majorana mass matrix. We can show that it is symmetric.

$$\boxed{M_M = M_M^T}$$

In fact:

$$\nu_L^T M_M C^{-1} \nu_L = (\nu_L^T M_M C^{-1} \nu_L)^T$$
$$= -\nu_L^T M_M^T C^{-1,T} \nu_L = \nu_L^T M_M^T C^{-1} \nu_L$$

This implies that only one unitary mixing matrix is required to diagonalise it

$$\boxed{M_M = (U^\dagger)^T m_{\rm diag} U^\dagger}$$

The massive fields are related to the flavour ones as

$$\boxed{n_L = U^\dagger \nu_L}$$

and the Lagrangian can be rewritten in terms of a Majorana field

$$\mathcal{L}_M = -\frac{1}{2}\bar{n_L^c} m_{\rm diag} n_L - \frac{1}{2}\bar{n_L} m_{\rm diag} n_L^c = -\frac{1}{2}\bar{\chi} m_{\rm diag} \chi$$

with $$\boxed{\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c}$$

A Majorana mass term (breaks L) leads to Majorana neutrinos (breaks L).

Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

What do we expect the massive neutrinos to be? Dirac, Majorana, both?

Dirac + Majorana masses

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This breaks lepton number, in both the Majorana mass terms.

The expectation is that, as lepton number is not conserved, neutrinos will be Majorana particles. Let's prove it.

We start by rewriting $\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\psi}_L^c \mathcal{M} \psi_L + \text{h.c.}$

with $\psi_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $\mathcal{M} \equiv \begin{pmatrix} M_{M,L} & m_D^T \\ m_D & M_{M,R} \end{pmatrix}$

In fact

$$\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\nu}_L^c M_{M,L} \nu_L - \frac{1}{2}\bar{\nu}_R M_{M,R} \nu_R - \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

and one can use $\bar{\nu}_L^c m_D^T \nu_R^c = \bar{\nu}_R m_D \nu_L$

Exercise
Show that these two formulations are equivalent.

Then, we need to diagonalise the full mass matrix, and we find the Majorana massive states, in analogy to what we have done for the Majorana mass case.

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

The difference is that $n_L = U_j \nu_L + U_k \nu_R^c$

Not unitary

Mixing between mass states and sterile neutrinos

Summary of neutrino mass terms

Dirac masses

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \text{h.c.})$$

This term conserves lepton number.

Majorana masses

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

This term breaks lepton number.

Dirac + Majorana masses

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2} \nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2} \nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

Lepton number is broken -> Majorana neutrinos.

Can neutrino masses arise in the SM? and if not, how can we extend the SM to generate them?

Neutrino masses in the SM and beyond

In the SM, neutrinos do not acquire mass and mixing:

- like the other fermions as there are no right-handed neutrinos.

$$m_e \bar{e}_L e_R \qquad\qquad m_\nu \bar{\nu}_L \cancel{\nu_R}$$

Solution: Introduce ν_R for Dirac masses

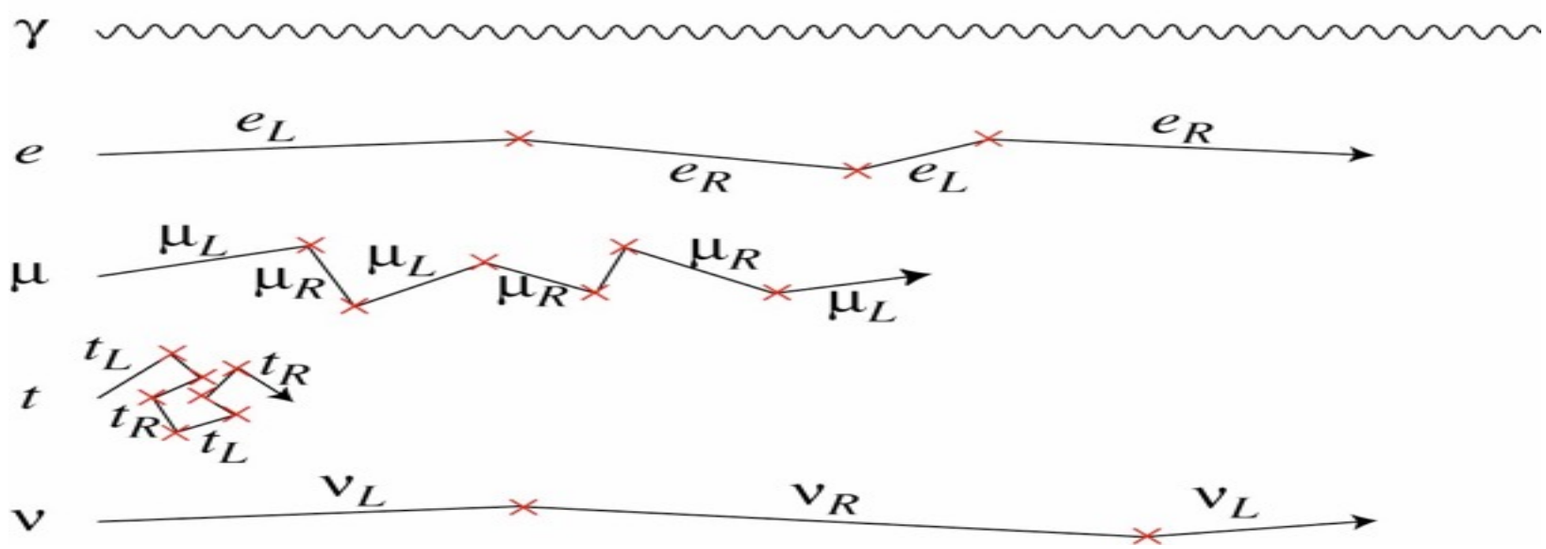
- they do not have a Majorana mass term

$$M \nu_L^T C \nu_L$$

as this term breaks the SU(2) gauge symmetry.
Solution: Introduce an SU(2) scalar triplet or gauge invariant non-renormalisable terms (D>4). This term breaks Lepton Number.

Dirac Masses

If we introduce a right-handed neutrino, then a lepton-number conserving interaction with the Higgs boson emerges.



Thanks to H. Murayama

$$\mathcal{L} = -y_\nu \bar{L} \cdot \tilde{H} \nu_R + \text{h.c.}$$

with $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and $\tilde{H} = \begin{pmatrix} H^{0,*} \\ -H^- \end{pmatrix}$

This term is
- SU(2) invariant and
- respects lepton number

When the neutral component of the Higgs field gets a vev, a Dirac mass term for neutrinos is generated.

$$\mathcal{L}_{\nu H} = -y_\nu (\bar{\nu}_L, \bar{\ell}_L) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} \nu_R + \text{h.c.}$$

$$= -y_\nu (\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-) \nu_R + \text{h.c.}$$

$$= -y_\nu \frac{v_H}{\sqrt{2}} \bar{\nu}_L \nu_R + \text{h.c.} + \ldots$$

$H^0 \to \frac{v_H}{\sqrt{2}} + h^0$

It follows that

$$y_\nu \sim \frac{\sqrt{2} m_\nu}{v_H} \sim \frac{0.2 \text{ eV}}{200 \text{ GeV}} \sim 10^{-12}$$

Tiny couplings!

Many theorists consider this explanation of neutrino masses not satisfactory. We would expect this Yukawa couplings to be similar to the ones in the quark sector:

1. why the coupling is so small????
2. why the mixings are large? (instead of small as in the quark sector)
3. why neutrino masses have at most a mild hierarchy if they are not quasi-degenerate? instead of what happens to quarks?

Dirac masses are strictly linked to lepton number conservation. But this is an accidental global symmetry. Should it be conserved at high scales?

Majorana Masses

In order to have an SU(2) invariant mass term for neutrinos, it is necessary to introduce a Dimension 5 operator (or to allow for new scalar fields, e.g. a scalar triplet):

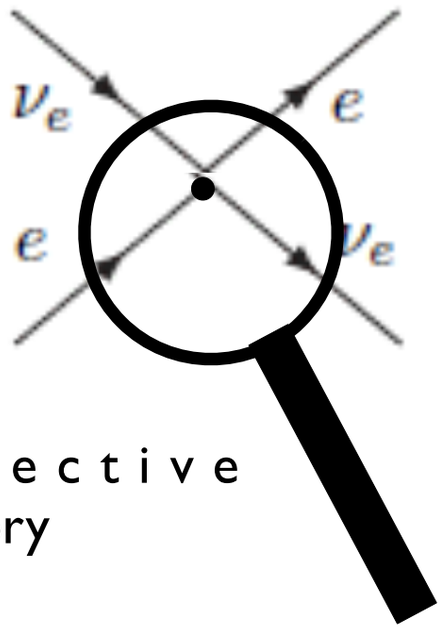
D=5 term

$$-\mathcal{L} = \lambda \frac{L \cdot H L \cdot H}{M} = \frac{\lambda v_H^2}{M} \nu_L^T C^\dagger \nu_L$$

Weinberg operator, PRL 43

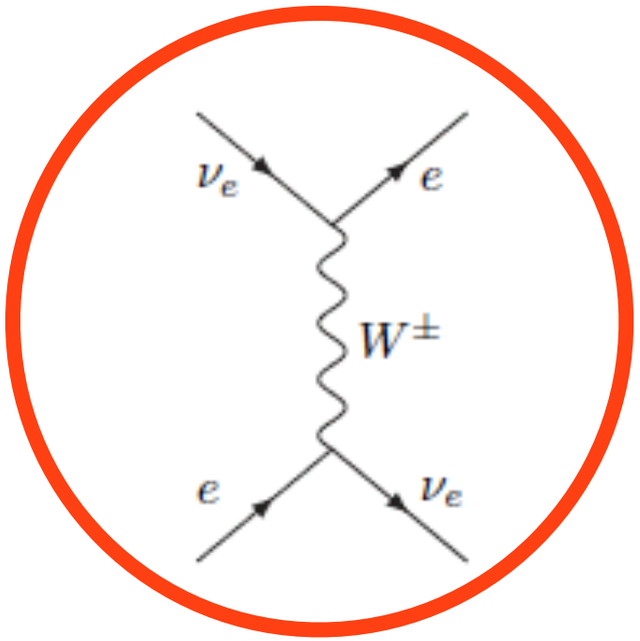
Lepton number violation!

If neutrino are Majorana particles, a **Majorana mass** can arise as the **low energy realisation of a higher energy theory (new mass scale!)**.



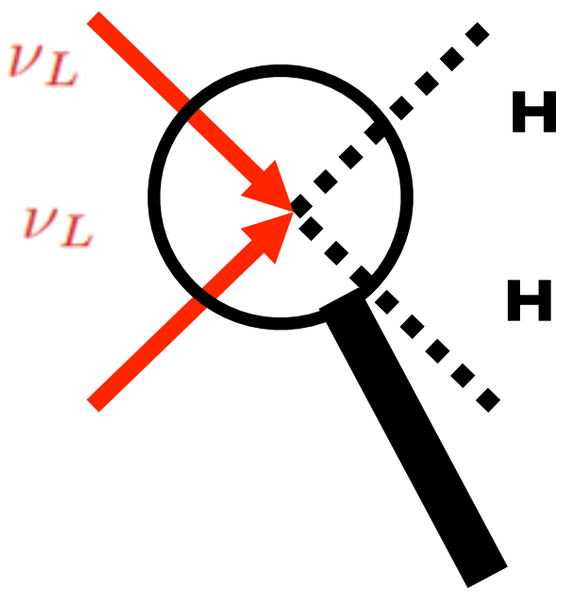
effective theory

$$\mathcal{L} \propto G_F(\bar{e}_L\gamma_\mu\nu_L)(\bar{\nu}_L\gamma^\mu e_L)$$



Standard Model: W exchange

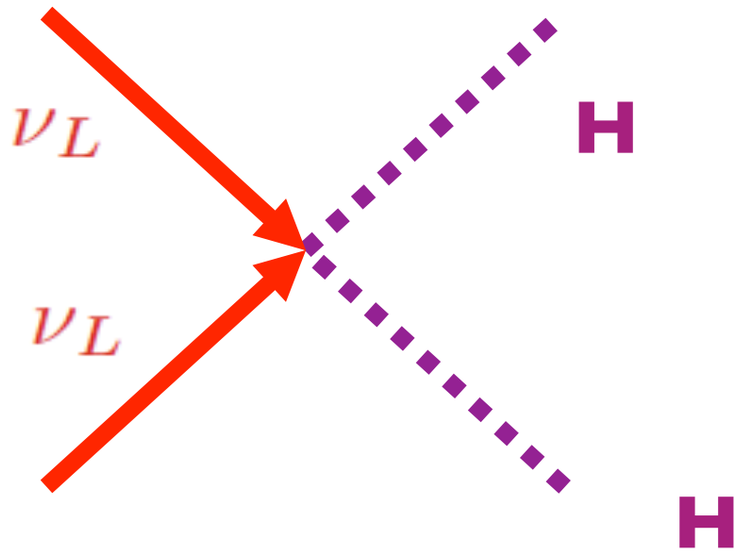
$$\mathcal{L}_{SM} \propto g\bar{\nu}_L\gamma^\mu e_L W_\mu \Rightarrow G_F \propto \frac{g^2}{m_W^2}$$



Neutrino mass

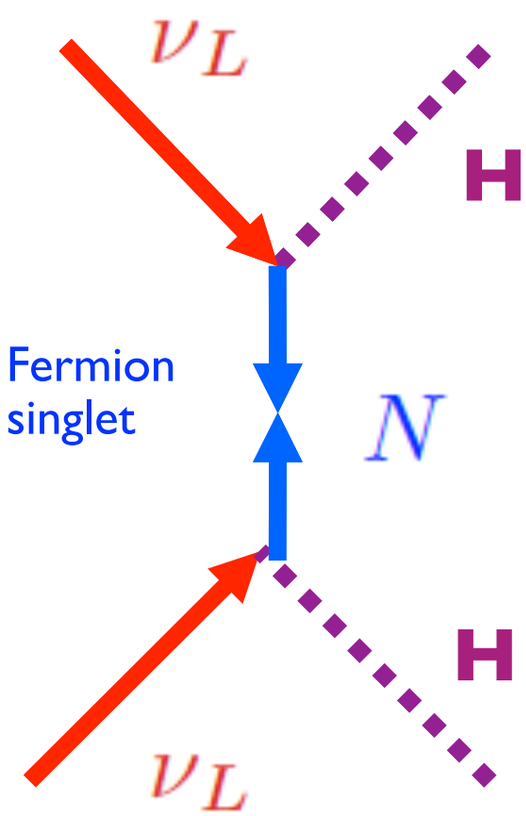
$$-\mathcal{L} = \lambda\frac{L\cdot HL\cdot H}{M}$$

New theory: new particle exchange with mass M



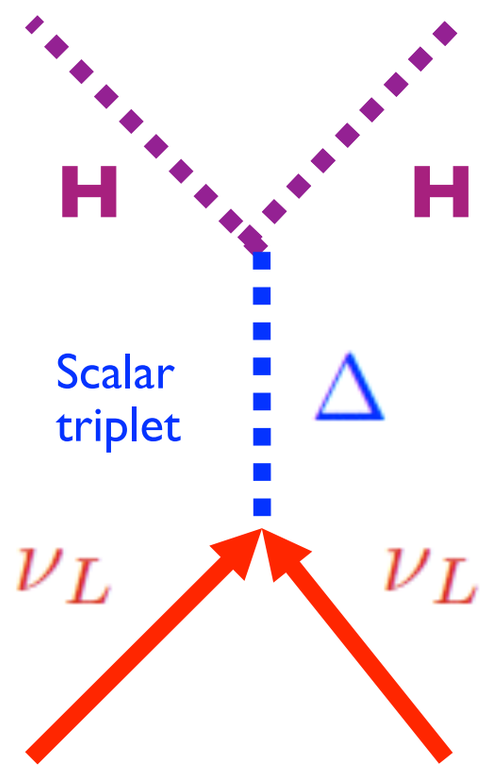
$$-\mathcal{L} = \lambda \frac{L \cdot H L \cdot H}{M}$$

See-saw Type I



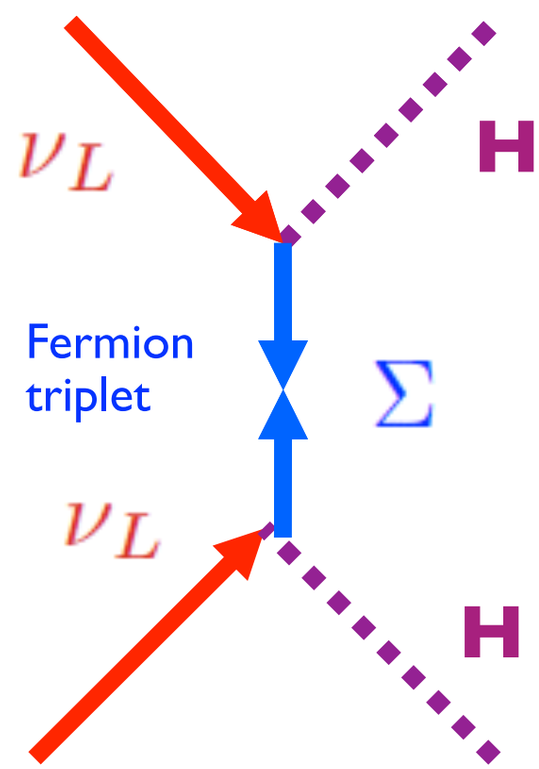
Minkowski, Yanagida, Glashow, Gell-Mann, Ramond, Slansky, Mohapatra, Senjanovic

See-saw Type II



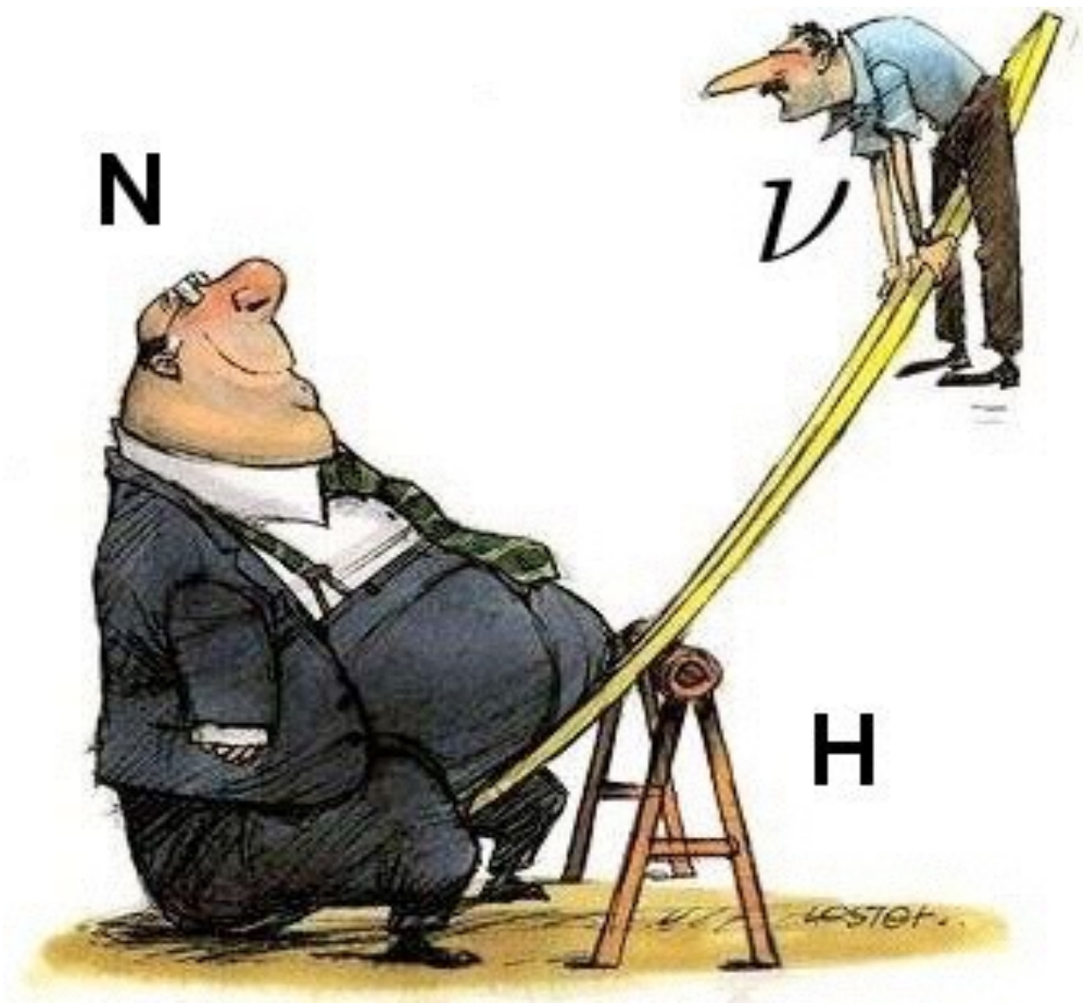
Magg, Wetterich, Lazarides, Shafi. Mohapatra, Senjanovic, Schecter, Valle

See-saw Type III



Ma, Roy, Senjanovic, Hambye

Models of neutrino masses BSM



See-saw type I

- Introduce a right handed neutrino **N** **(sterile neutrino)**
- Couple it to the Higgs and left handed neutrinos

The Lagrangian is

breaks lepton number

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

When the Higgs boson gets a vev, Dirac masses will be generated. The mass matrix will be (for one generation)

$$\mathcal{L} = \left(\nu_L^T N^T\right) \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

This is of the Dirac+Majorana type we discussed earlier. So we know that the massive states are found by diagonalising the mass matrix and the massive states will be Majorana neutrinos.

$$\begin{vmatrix} -\lambda & m_D \\ m_D & M-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - M\lambda - m_D^2 = 0$$

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \frac{M}{\frac{M-M}{2} - \frac{4m_D^2}{4M}} = -\frac{m_D^2}{M}$$

One massive state remains very heavy, the light neutrino masses acquires a tiny mass!

$$m_\nu \simeq \frac{m_D^2}{M} \sim \frac{1\,\mathrm{GeV}^2}{10^{10}\,\mathrm{GeV}} \sim 0.1\,\mathrm{eV}$$

Mixing between active neutrinos and heavy neutrinos will emerge but it will be typically very small

$$\tan 2\theta = \frac{2m_D}{M}$$

and can be related to neutrino masses $m_\nu \simeq \frac{m_D^2}{M} \simeq \sin^2\theta M$

Pros and cons of type I see-saw models

Pros:

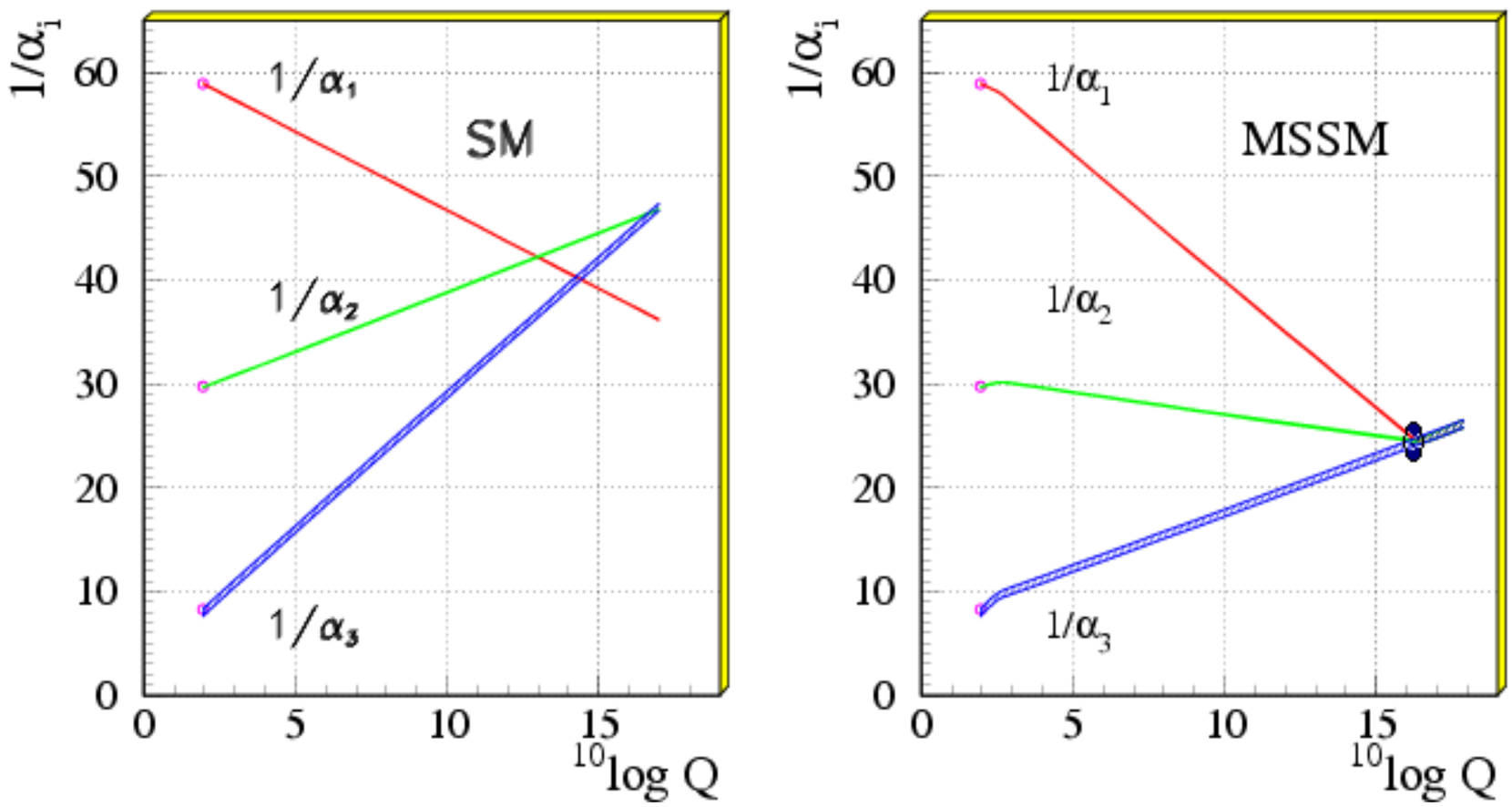
- they explain "naturally" the smallness of neutrino masses.
- can be embedded in GUT theories!
- neutrino masses are a indirect test of GUT theories
- have several phenomenological consequences (depending on the mass scale), e.g. leptogenesis, LFV

Cons:

- the new particles are typically too heavy to be produced at colliders (but TeV scale see-saws)
- the mixing with the new states are tiny
- in general: difficult to test

GUT theories and the see-saw mechanism

The SM has a very complex gauge structure (3 gauge couplings) and charge assignments for the fields. GUTs aim at providing a unified picture.

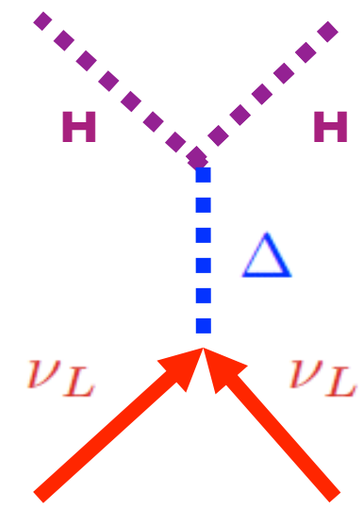


S. Rabi, PDG

Due to the renormalisation of the couplings, they "run" and unify at a very high energy scale, typically 10^{16} GeV. Proton decay is predicted and is the main signature of these models.

See-saw type II

We introduce a Higgs triplet which couples to the Higgs and left handed neutrinos. It has hypercharge 2.



$$\mathcal{L}_\Delta \propto y_\Delta L^T C^{-1} \sigma_i \Delta_i L + \text{h.c.}$$

with $$\Delta_i = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

Once the Higgs triplet gets a vev, Majorana neutrino masses arise:

$$m_\nu \sim y_\Delta v_\Delta$$

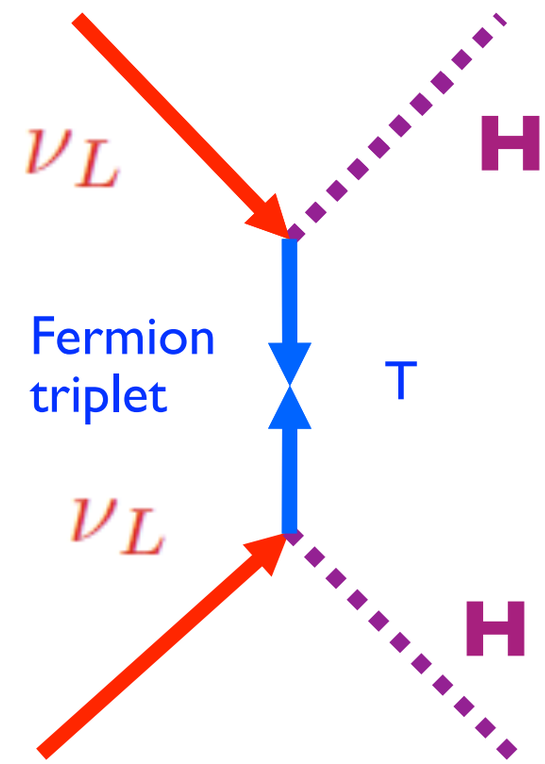
Cons: why the vev is very small?
Pros: the component of the Higgs triplet could tested directly at the LHC.

See-saw type III

We introduce a fermionic triplet which has hypercharge 0.

$$\mathcal{L}_T \propto y_T \bar{L} \sigma H \cdot T + \text{h.c.}$$

with $$T = \begin{pmatrix} T^0 & T^+ \\ T^- & -T^0 \end{pmatrix}$$



Majorana neutrino masses are generated as in see-saw type I:

$$m_\nu \simeq -y_T^T M_T^{-1} y_T v_H^2$$

Pros: the component of the fermionic triplet have gauge interactions and can be produced at the LHC
Cons: why the mass of T is very large?

Extensions of the see saw mechanism

Models in which it is possible to lower the mass scale (e.g. TeV or below), keeping large Yukawa couplings have been studied. Examples: inverse and extended see-saw.

Let's introduce two right-handed singlet neutrinos.

$$\mathcal{L} = Y\bar{L}\cdot HN_1 + Y_2\bar{L}\cdot HN_2^c + \Lambda\bar{N}_1N_2 + \mu' N_1^T C N_1 + \mu N_2^T C N_2$$

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

See e.g. Gavela et al., 0906.1461; Ibarra, Molinaro, Petcov, 1103.6217; Kang, Kim, 2007; Majee et al., 2008; Mitra, Senjanovic, Vissani, 1108.0004; Malinsky, Romao, Valle, 2005

$$m_{tree} \simeq -m_D^T M^{-1} m_D \simeq \frac{v^2}{2(\Lambda^2 - \mu'\mu)}\left(\mu Y_1^T Y_1 + \epsilon^2 \mu' Y_2^T Y_2 - \Lambda\epsilon(Y_2^T Y_1 + Y_1^T Y_2)\right)$$

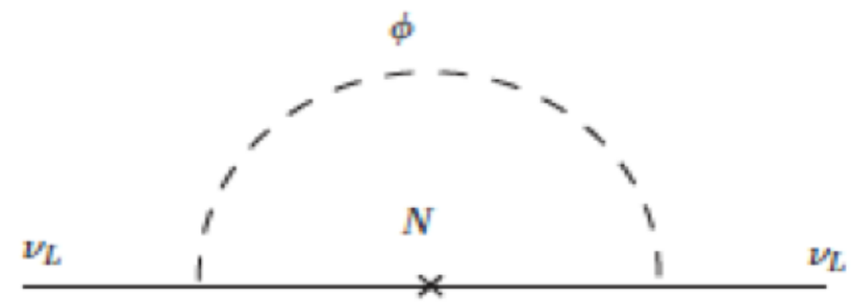
Small neutrino masses emerge due to cancellations between the contributions of the two sterile neutrinos (typically associated to small breaking of some L).

Other models of neutrino masses

Radiative masses

If neutrino masses emerge via loops, in models in which Dirac masses are forbidden, there is an additional suppression.

Some of these models have also dark matter candidates.



$$m_\nu \propto \frac{g^2}{16\pi^2} f(M, \mu_\phi^2)$$

See Ma, PRL81; also e.g. Boehm et al., PRD77; ...

R-parity violating SUSY

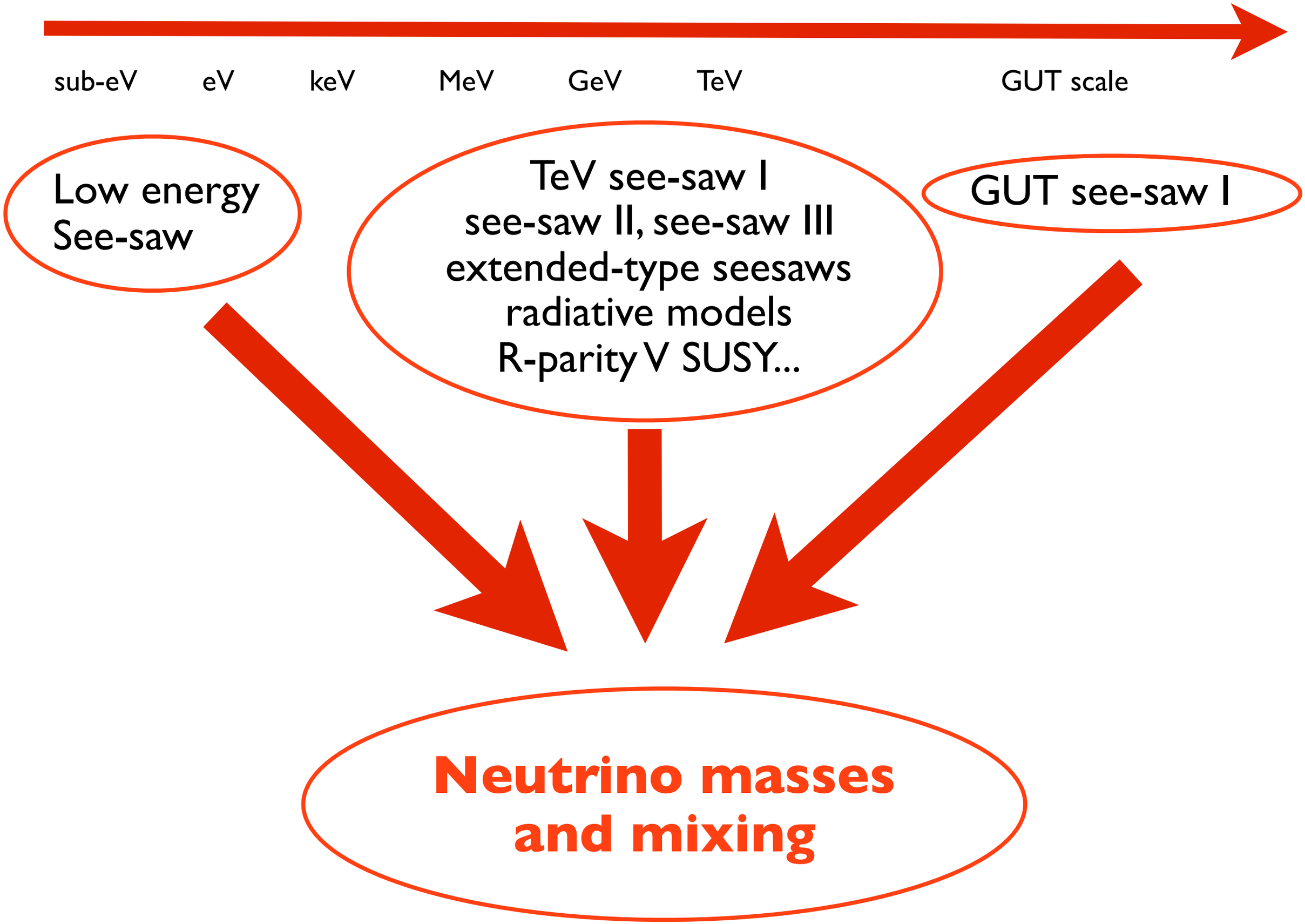
In the MSSM, there are no neutrino masses. But it is possible to introduce terms which violate R (and L).

$$V = \ldots - \mu H_1 H_2 + \epsilon_i \tilde{L}_i H_2 + \lambda'_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}_k + \ldots$$

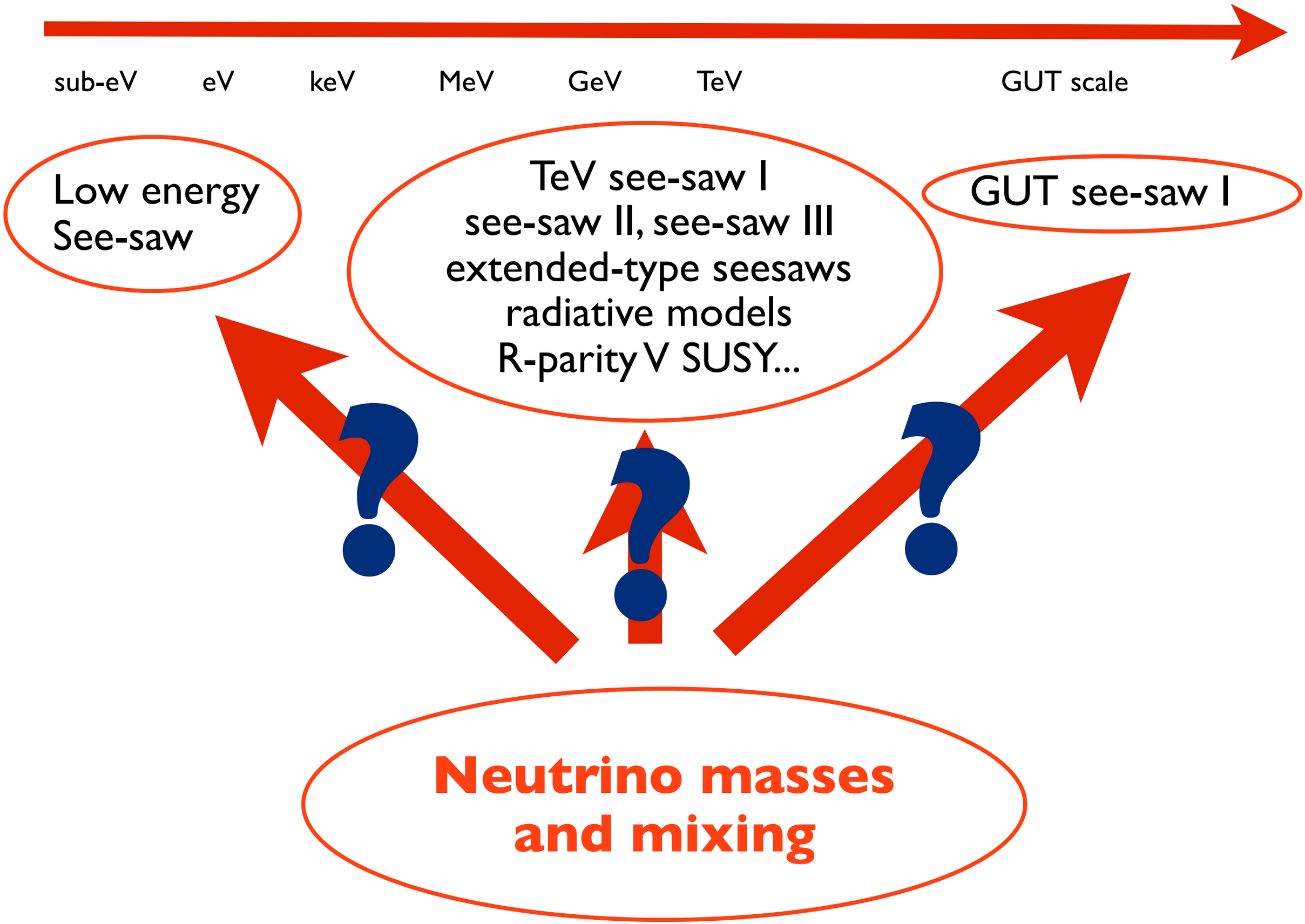
See e.g. Aulakh, Mohapatra, PLB119; Hall, Suzuki, NPB231; Ross, Valle, PLB151; Ellis et al., NPB261; Dawson, PRD57, ...

The bilinear term induces mixing between neutrinos and higgsino, the trilinear term masses at loop-level.

What is the new physics?

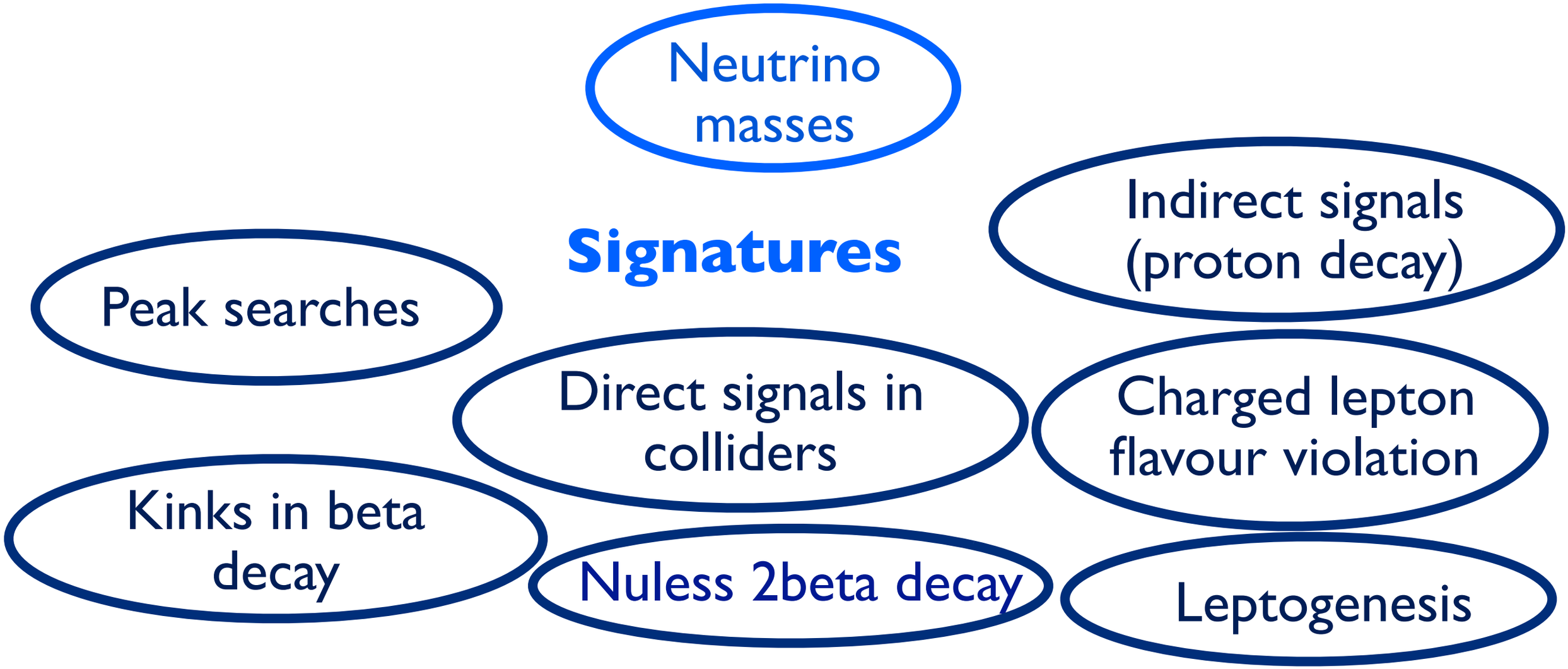


What is the new physics?



Complementarity with other searches

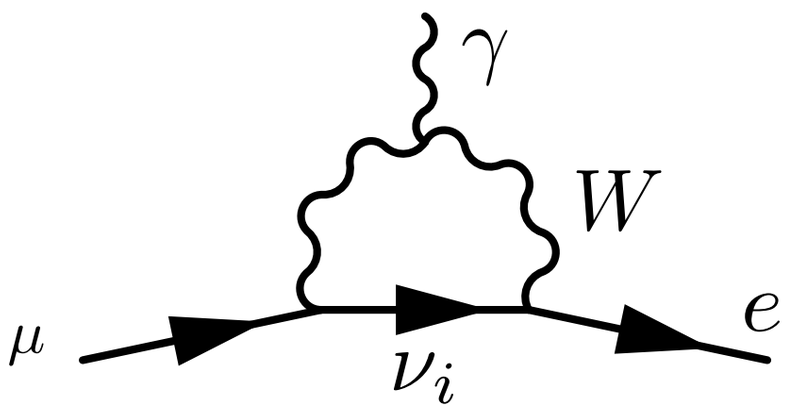
There are many (direct and indirect) signatures of these extensions of the SM.



Establishing the origin of neutrino masses requires to have as much information as possible about the masses and to combine it with other signatures of the models.

Charged lepton flavour violation

Neutrino masses induce very suppressed LFV processes.

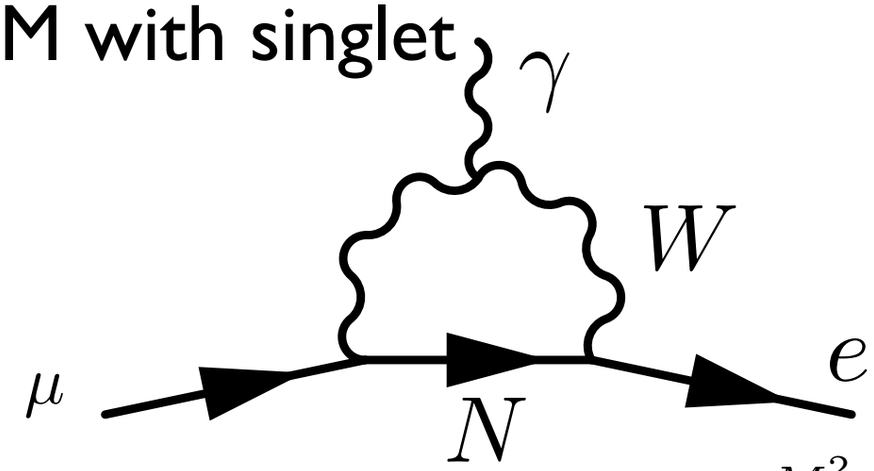


$$Br(\mu \to e\gamma)$$
$$\sim \frac{3\alpha}{32\pi}\left(\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta^2 m_{i1}}{m_W^2}\right)^2 \sim 10^{-53}$$

S. Petcov, SJNP 25 (1977)

Any observation of CLFV would show new physics BSM and provide clues on the origin of neutrino masses.

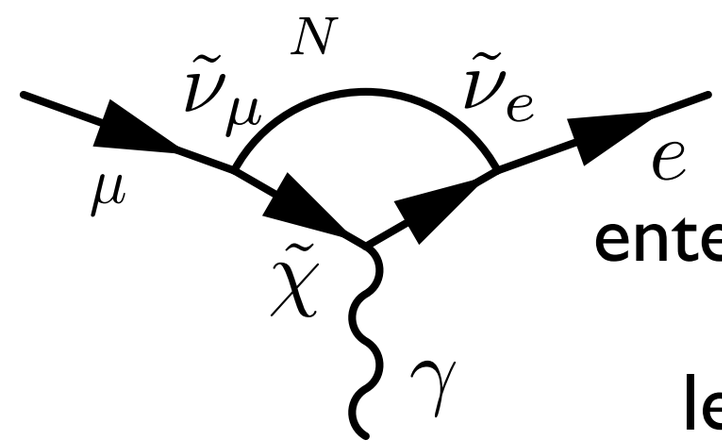
Example: extension of the SM with singlet N



$$Br(\mu \to e\gamma) \sim \frac{3\alpha}{8\pi}\left(\sum_j U^*_{\mu j} U_{ej} g\left(\frac{M_N^2}{m_W^2}\right)\right)^2$$

Example: SUSY see-saw

$$Br \propto \left|\sum_N Y^*_{N\mu} Y_{Ne} \ln(m_0/m_N)\right|^2$$

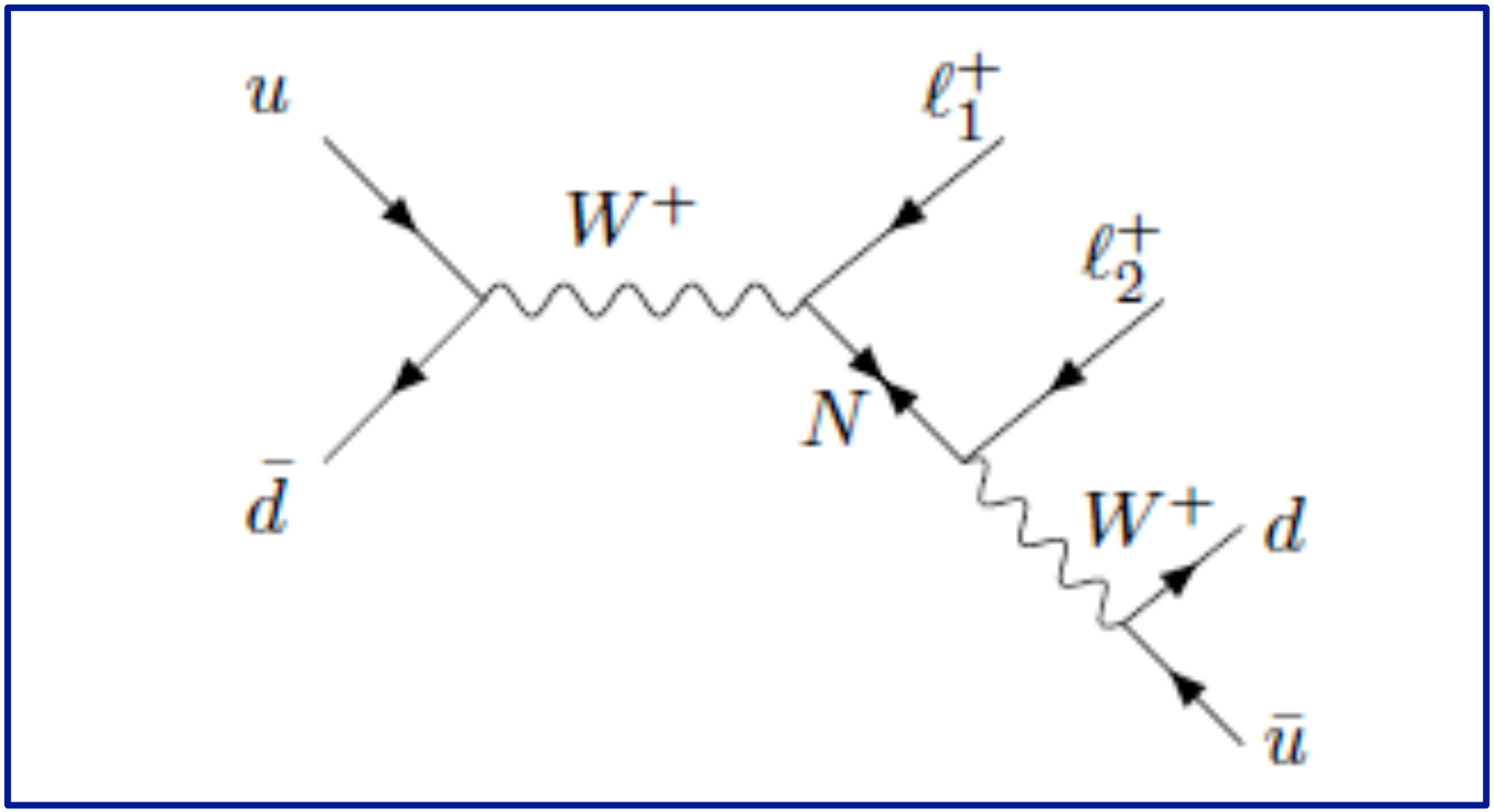


The same parameters enter in LFV, nu masses and leptogenesis.

Borzumati, Masiero, PRL 57

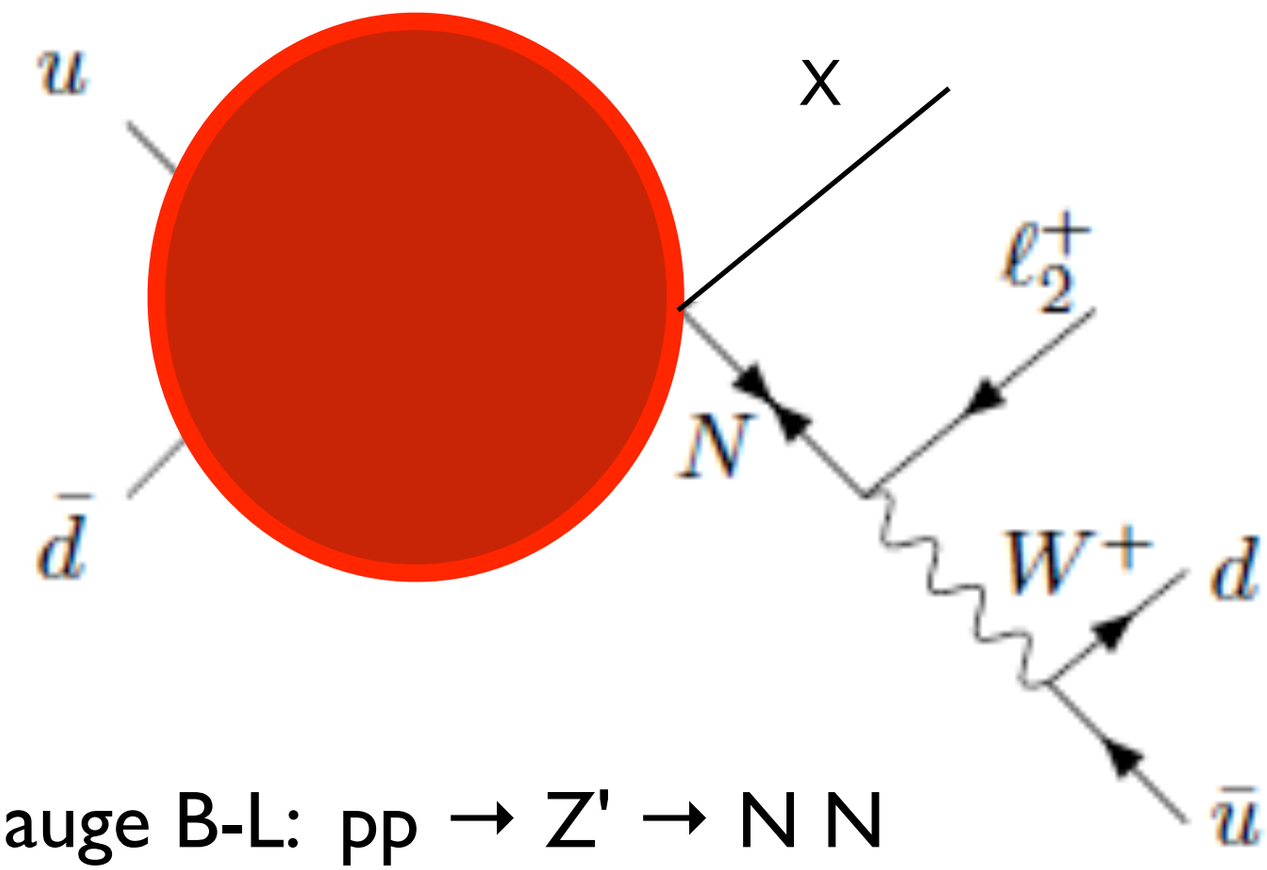
Signatures of TeV scale see-saw

Once produced, N can decay via mixing and the decay products detected.



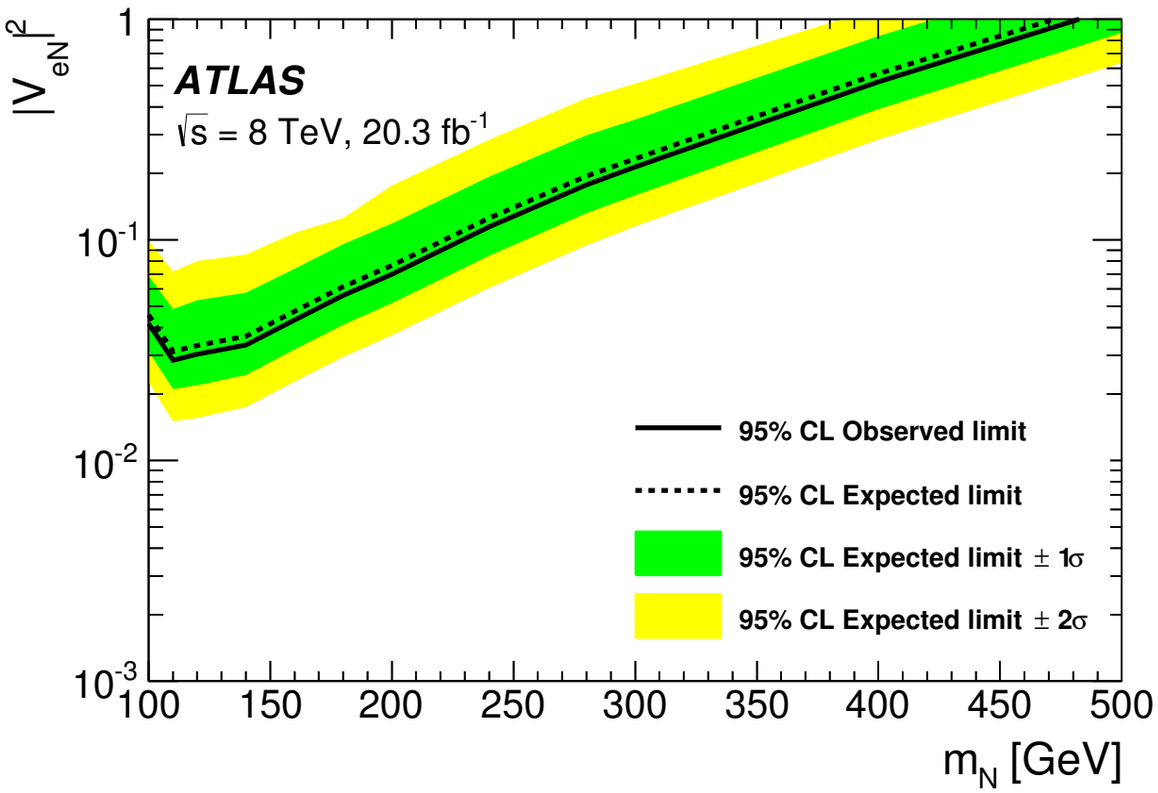
The characteristic signatures are LNV (same-sign dileptons and no missing E_T), LFV (trileptons with different flavours), displaced vertices (for low mass N).

A note: In see-saw type I, Lepton number violating signatures are typically suppressed because they are related to neutrino masses (which are tiny). But other production mechanisms are present in many BSM models:

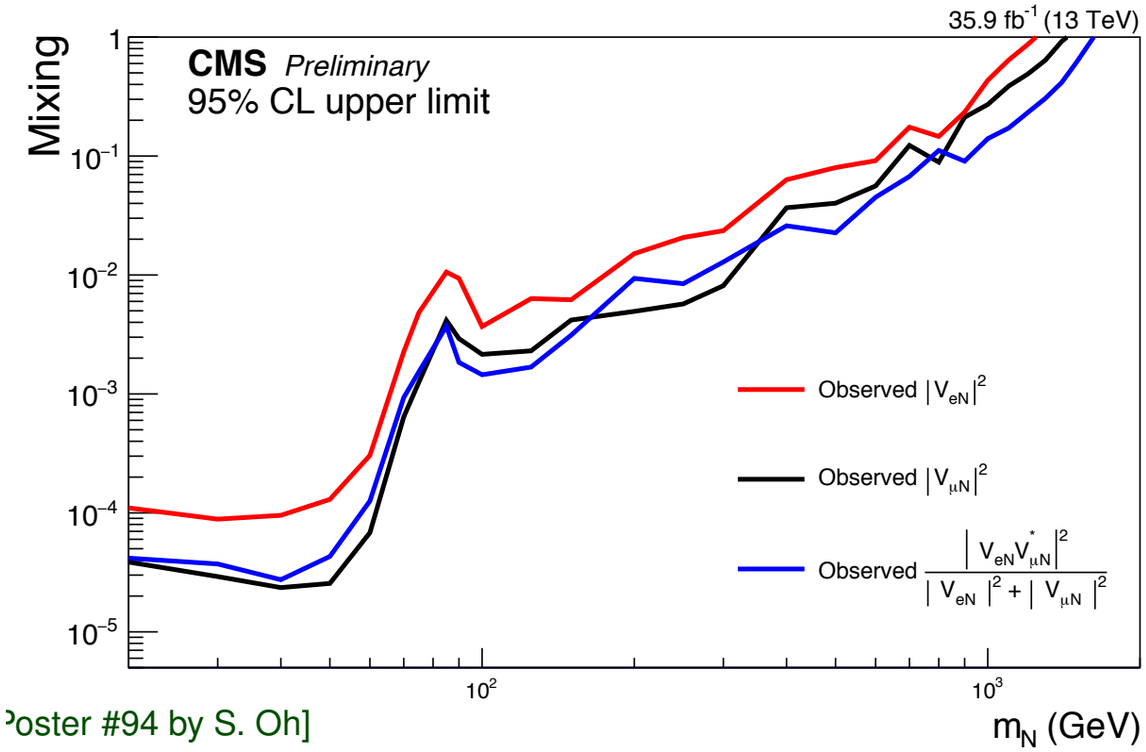


Gauge B-L: pp → Z' → N N
See-saw type II: Scalar Triplets
Triplet see-saw.
Left-Right models via WR
Inverse see-saw

Searches for LNV and LFV processes at colliders

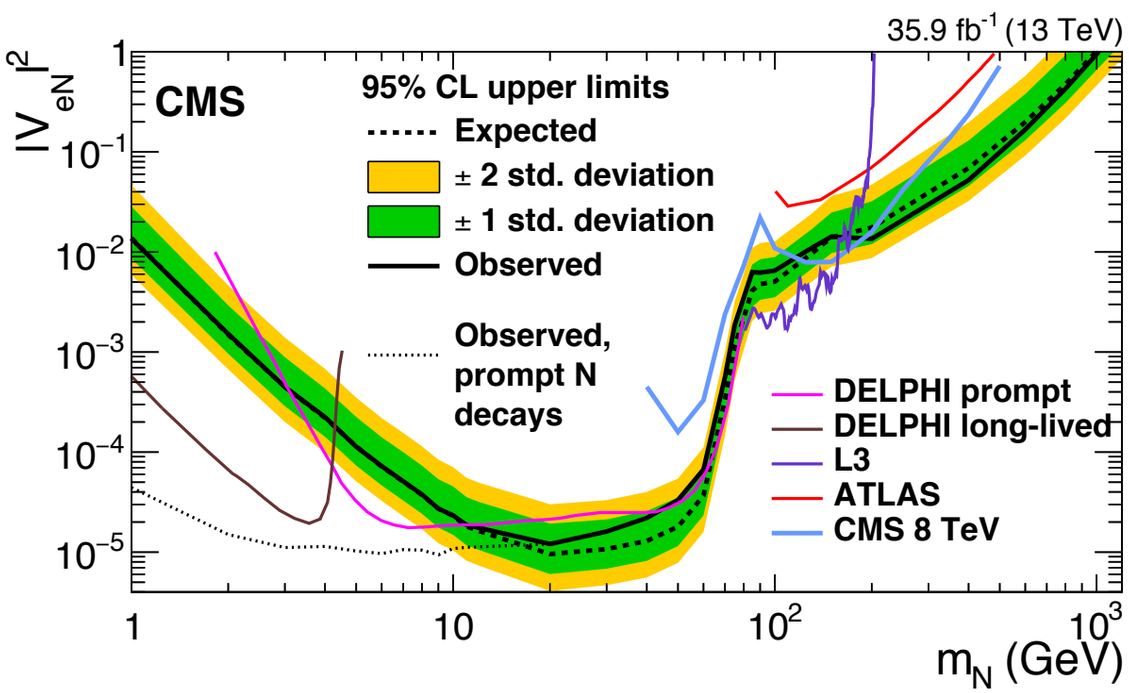


ATLAS, 1506.06020



Poster #94 by S. Oh]

CMS PAS EXO-17-028
From B. Dev talk at Neutrino 2018



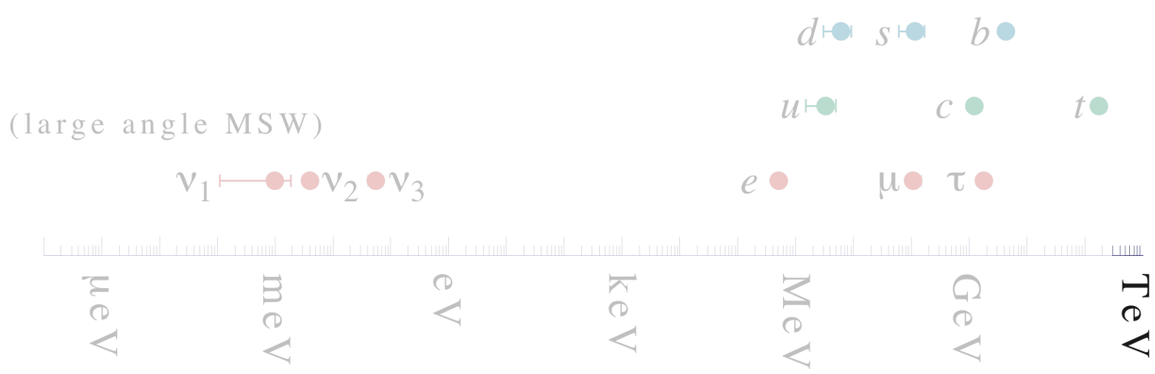
PRL 120 2018

ATLAS, CMS and LHC-b have put new bounds and with higher luminosity significantly stronger bounds can be obtained.

Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from flavour physics experiments and from colliders.

Why do we observe a specific pattern of mixing angles? Is there an underlying principle?

Masses and mixing from the mass matrix

Neutrino masses and the mixing matrix arises from the diagonalisation of the mass matrix

$$M_M = (U^\dagger)^T m_{\rm diag} U^\dagger \quad \Rightarrow \quad n_L = U^\dagger \nu_L$$

Theory

Experiments

Example. In the diagonal basis for the charged leptons

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

the angle is $\tan 2\theta = \dfrac{2b}{a-c} \gg 1 \quad \text{for} \quad a \sim c \text{ and, or } a, c \ll b$

and masses $m_{1,2} \simeq \dfrac{a + c \pm 2b}{2}$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)\mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L})\mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L})U_\nu^* U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\mathcal{M}_{\rm diag} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad (\bar{\nu}^c_{1L}, \bar{\nu}^c_{2L}, \bar{\nu}^c_{3L})\mathcal{M}_{{\rm diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

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$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\mathcal{M}_{\rm diag} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad (\bar{\nu}^c_{1L}, \bar{\nu}^c_{2L}, \bar{\nu}^c_{3L})\mathcal{M}_{{\rm diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

in the CC interactions (and oscillations):

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)\gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} W_\mu \Rightarrow \frac{g}{\sqrt{2}}(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\gamma^\mu U_{\rm osc} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W_\mu$$

$$\boxed{U_{\rm osc} = V_L^\dagger U_\nu}$$

Phenomenological approaches

Various strategies and ideas can be employed to understand the observed pattern (many many models!).

- Flavour symmetries: continous, discrete (A4, A5, S4…)

- Complementarity between quarks and leptons

$$\theta_{12} + \theta_C \simeq 45^o$$

- Anarchy (all elements of the matrix of the same order)

- Other….

What will you learn from this lecture?

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  - Type of neutrino masses
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  - neutrinos in the Early Universe
  - Leptogenesis and the baryon asymmetry
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Useful formulae

Particles in a thermal bath are described by

$$f_{\rm eq} = \frac{1}{\exp(\frac{p-\mu_\nu}{T}) \pm 1}$$

The number densities are given by

$$n_{\rm eq} \simeq g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}$$ Non relativistic

Internal d.o.f.

$$n_{\rm eq} \simeq g T^3$$ Relativistic

Entropy

$$s = \frac{2\pi^2}{45} g_* T^3$$

Relativistic d.o.f.

Freeze-out

Typically, particles were in thermal equilibrium for T above their mass, if the interactions were fast enough.

$$\phi\phi \leftrightarrow \psi\bar{\psi}$$

As the Universe expands, the T drops and interactions slow down and the particles decouple. Then their number density is redshifted and a relic remains (ex., neutrinos, DM). The condition for freezeout is

$$\boxed{\Gamma \sim H}$$

where

$\Gamma = \langle \sigma n \rangle$ interaction rate

$H = \sqrt{\frac{8\pi G_N}{3}\rho^2} \simeq \frac{T^2}{m_{\rm Pl}}$ expansion rate

For radiation domination

Hot relic

A hot relic is a particle with decouples when relativistic.

The typical example is neutrinos.

$$\sigma = G_F^2 T^2$$
$$n \sim g T^3$$
$$H \simeq \frac{T^2}{m_{\rm Pl}}$$

Exercise
Compute T more precisely.

$$\Gamma \sim H \Rightarrow T \simeq \left(\frac{1}{G_F^2 m_{\rm Pl}} \right)^{1/3} \sim 1 \text{ MeV}$$

In order to compute their contribution to the energy density of the Universe, let's consider the comoving number density (for entropy conservation)

$$Y \equiv \frac{n}{s}$$

both scale as a^{-3}

Exercise
Derive

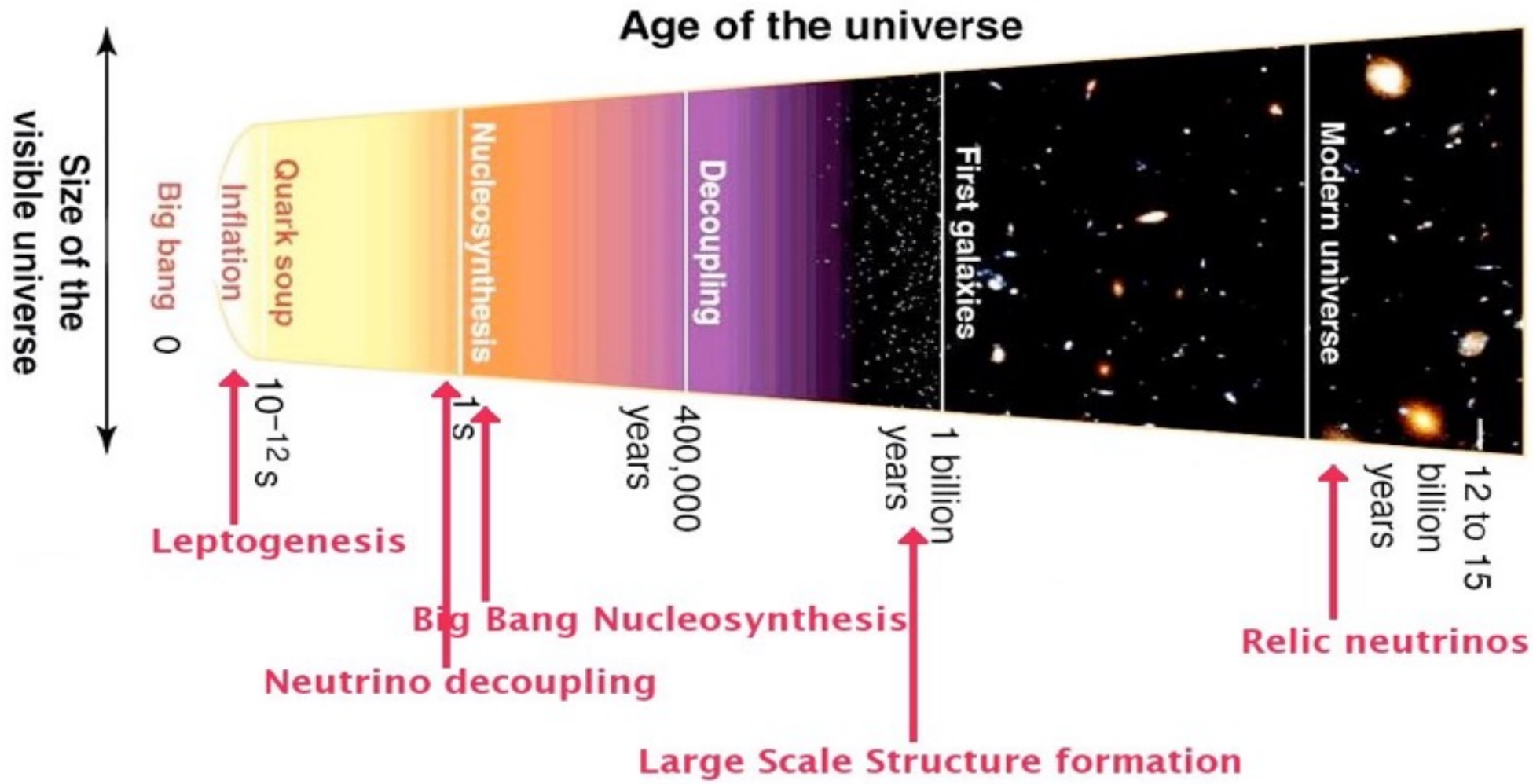
So

$$Y_{\rm today} = Y_{\rm freeze-out}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\rm cr}} h^2 = \frac{n_\nu m_\nu}{\rho_{\rm cr}} h^2 = \boxed{\frac{m_\nu}{91.5\ {\rm eV}}}$$

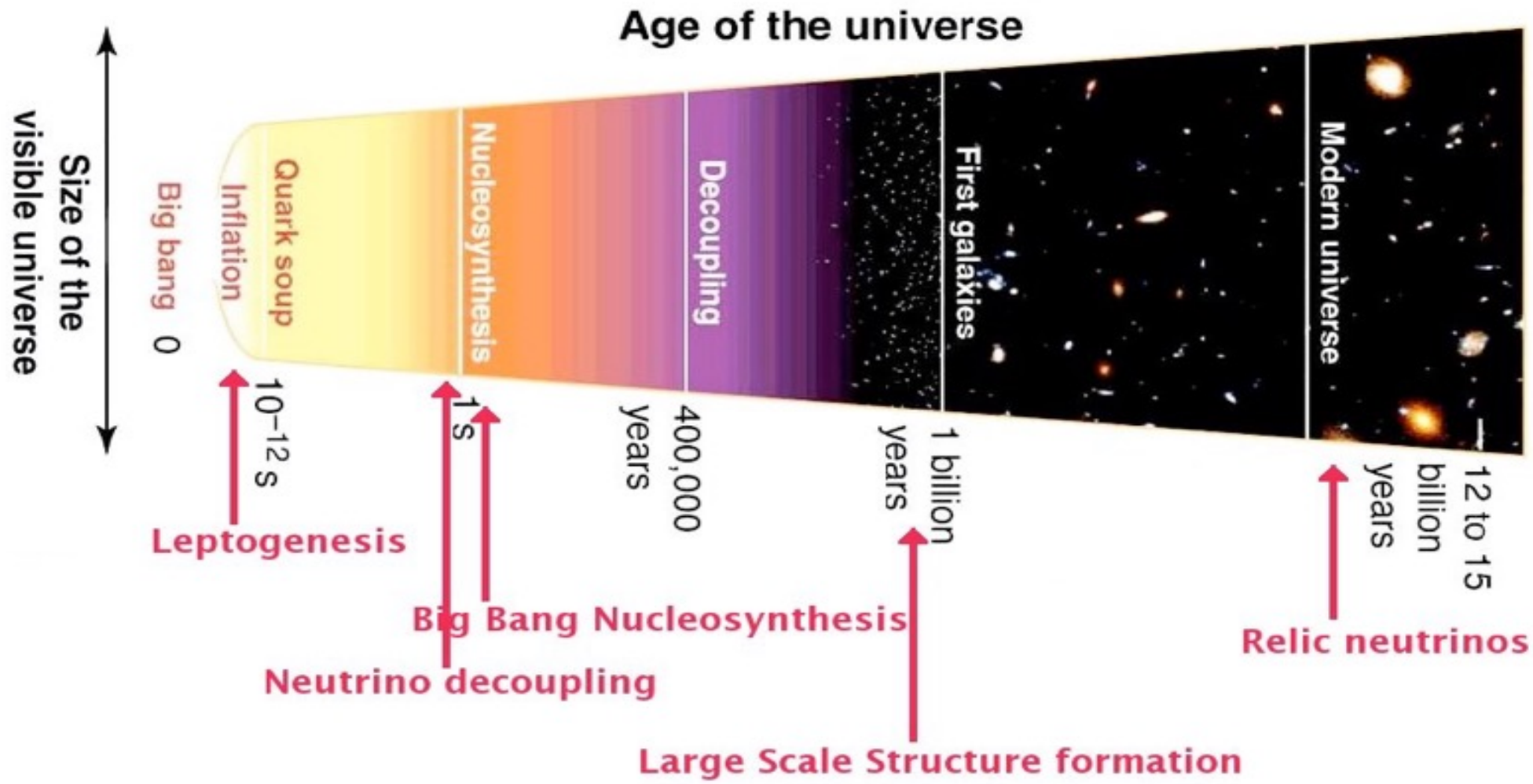
In general, the hot relic density abundance scales linearly with the mass.

Neutrinos have played an important role in shaping the Universe.



How many **relic neutrinos** are in a **cup of tea**?

Neutrinos have played an important role in shaping the Universe.



How many **relic neutrinos** are in a **cup of tea**?

5600!

New Scientist 05 March 2008: Universe submerged in a sea of chilled neutrinos

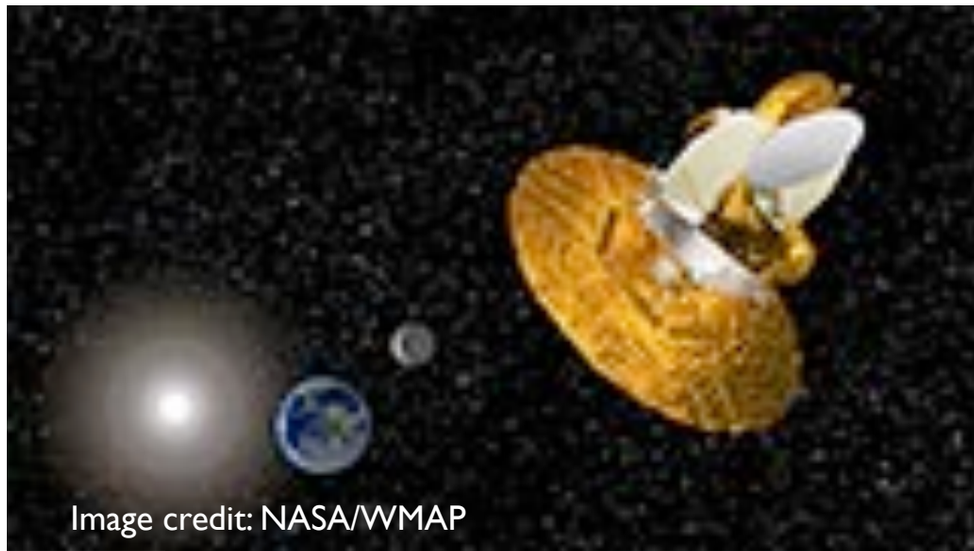


Image credit: NASA/WMAP

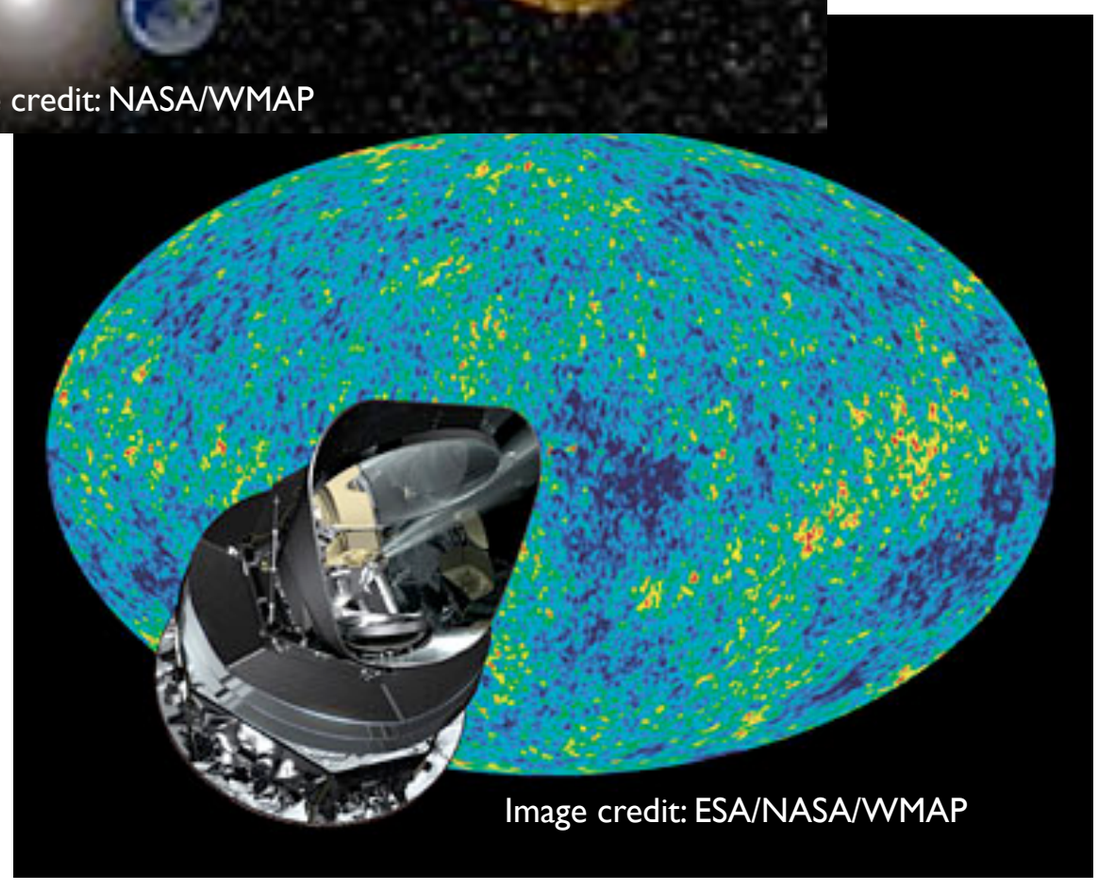
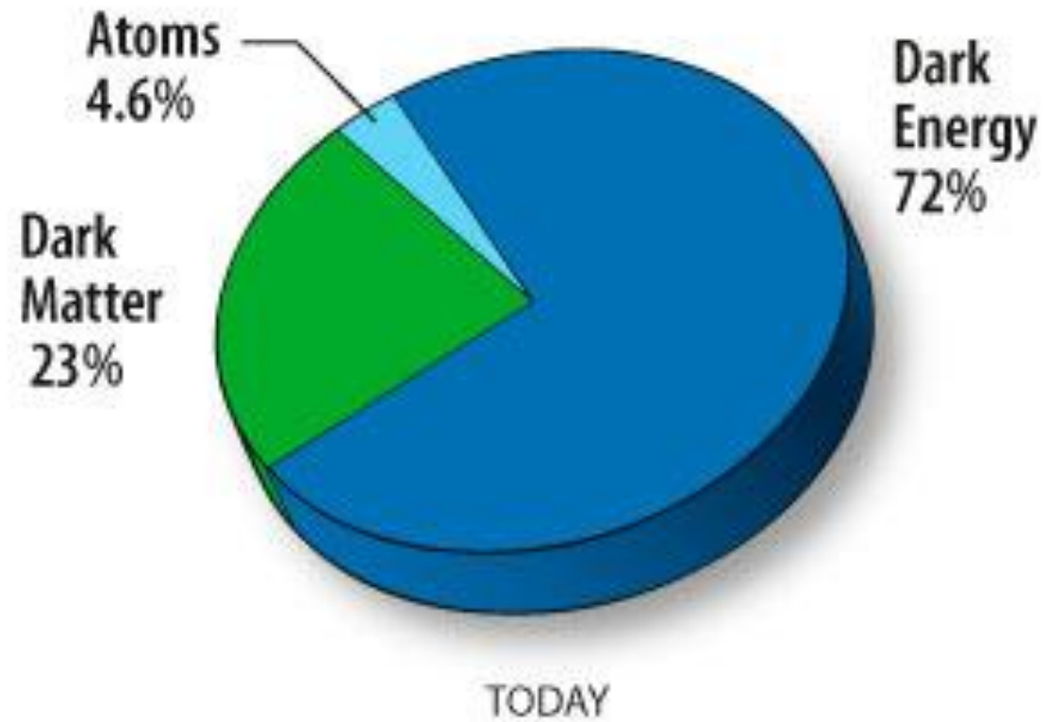
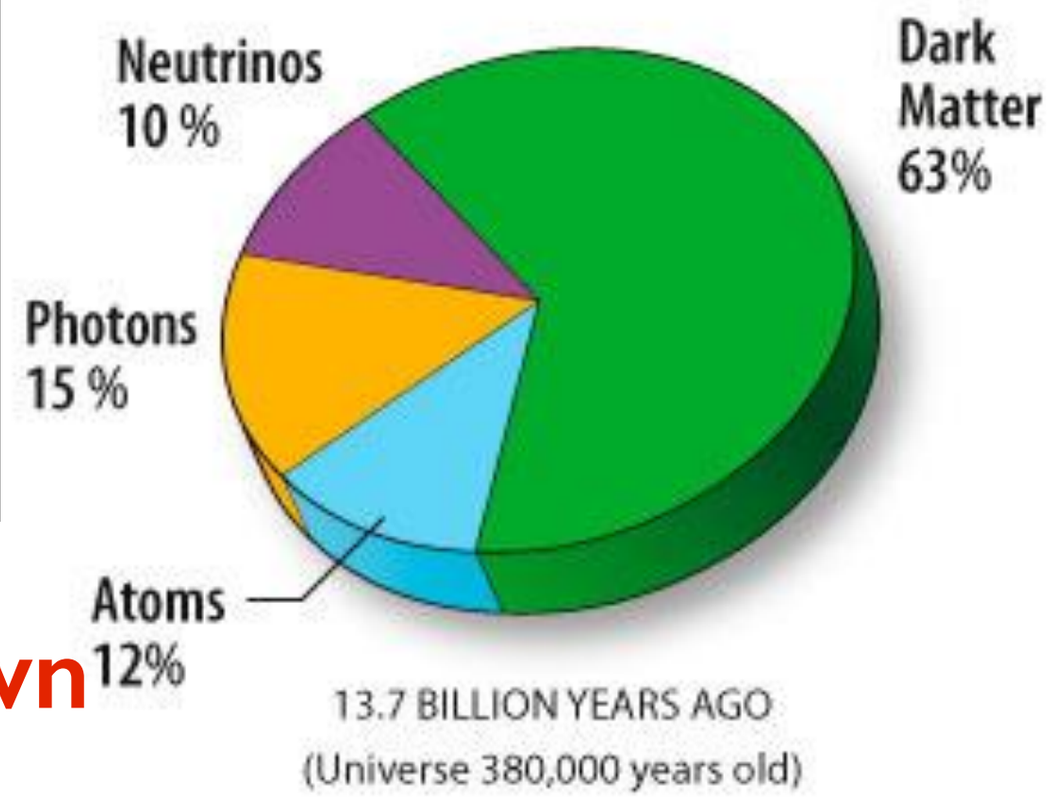


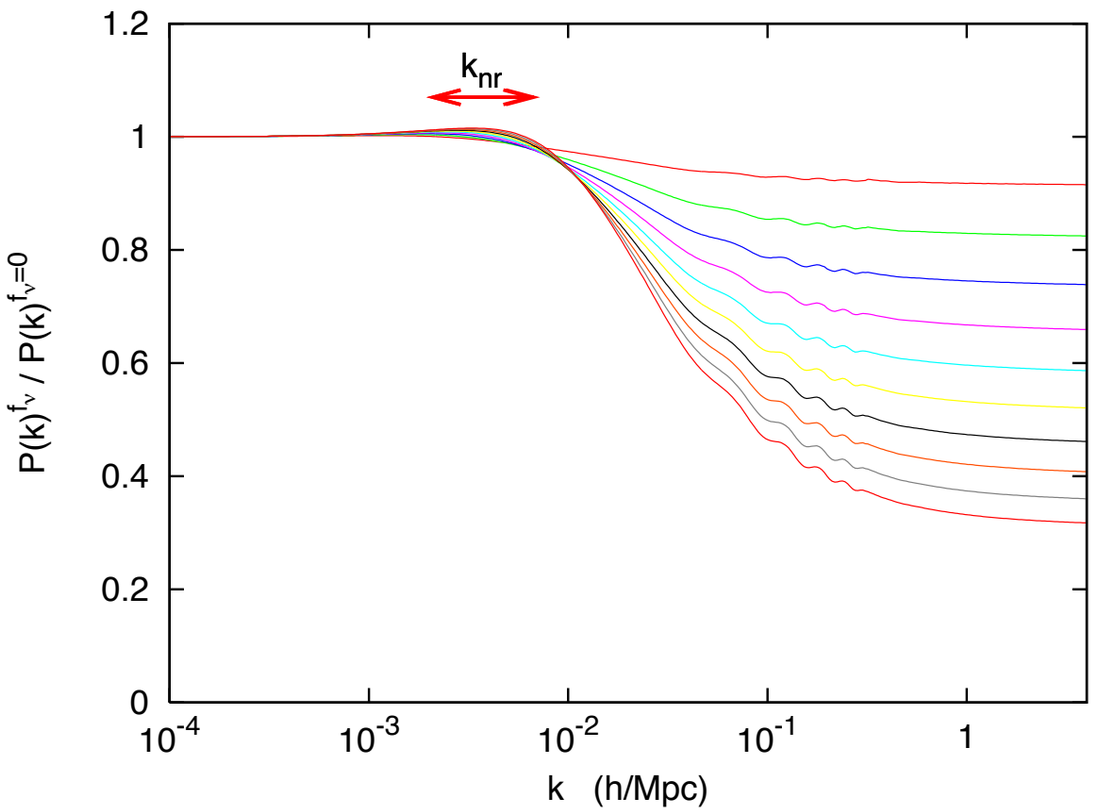
Image credit: ESA/NASA/WMAP

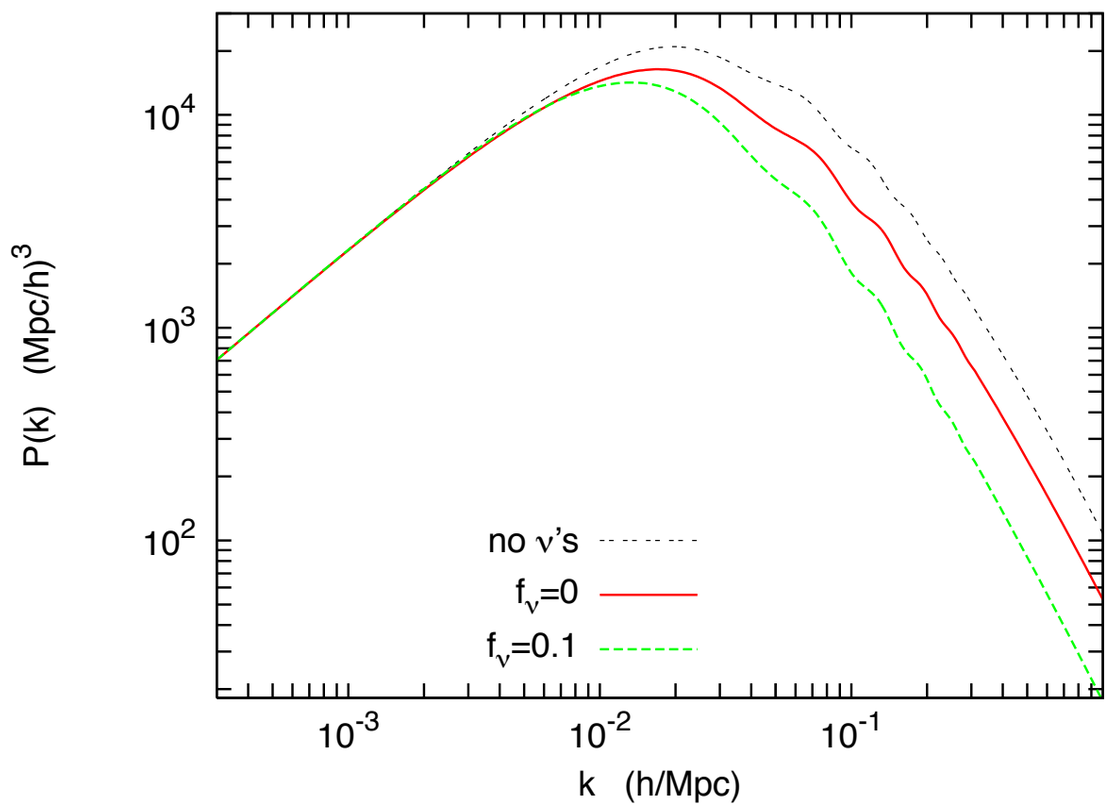




Neutrinos are the only known component of Dark Matter.

Neutrino masses suppress the matter power spectrum at small scales due to their free-streaming.





J. Lesgourgues and S. Pastor, PRept 2006

Loss of power on scales: $k_{fs} = 0.11\sqrt{\frac{\sum_i m_i}{1\ \mathrm{eV}}\frac{5}{1+z}}\ \mathrm{Mpc}^{-1}$

Way to probe the matter power spectrum:

- galaxy surveys, such as SDSS, BOSS, HETDEX...

$$\sum_i m_i < 0.1 \text{ eV} \; - \; 0.2 \text{ eV}$$

- Lyman alpha: this traces the intergalactic low density gas.

$$\sum_i m_i < 0.11 \text{ eV} \; - \; 0.17 \text{ eV}$$

- 21 cm lines: MWA, SKA and FFTT.

$$\sum_i m_i \sim 0.02 \text{ eV} \; - \; 0.003 \text{ eV}$$

- Lensing of galaxies

By using the cosmic shear, it is possible to reconstruct the matter distribution at different redshifts.

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The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- B (or L) violation;

- C, CP violation;

- departure from thermal equilibrium.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- B (or L) violation;

In the SM also L is violated at the non-perturbative level. A lepton asymmetry is converted into a baryon asymmetry by sphaleron effects.

If neutrinos are Majorana particles, L is violated.

See-saw models require L violation (typically the Majorana mass of a heavy right-handed neutrino).
In SUSY models without R-parity, L can be violated and neutrino masses generated.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- C, CP violation;

If C were conserved:

$$\Gamma(X^c \to Y^c + B^c) = \Gamma(X \to Y + B)$$

and no baryon asymmetry generated:

$$\frac{dB}{dt} \propto \Gamma(X^c \to Y^c + B^c) - \Gamma(X \to Y + B)$$

We have observed CPV in quark sector (too small) and we can search for it in the leptonic sector.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- out of equilibrium

In equilibrium

$$\Gamma(X \to Y + B) = \Gamma(Y + B \to X)$$

A generated baryon asymmetry is cancelled exactly by the antibaryon asymmetry.
When particles get out of equilibrium, this does not happen.

$$T < M_X$$

Baryogenesis

Let's consider a boson X, very heavy with BV couplings:

$$X \to lq \qquad B_1 \quad Br(1) = r$$
$$X \to q\bar{q} \qquad B_2 \quad Br(2) = 1 - r$$

The baryon number produced in the X and X decays

$$B_x = B_1 r + B_2(1-r)$$
$$B_{\bar{X}} = -B_1 \bar{r} - B_2(1-\bar{r})$$

The total lepton number produced is then

$$\boxed{\Delta B = (B_1 - B_2)(r - \bar{r})}$$

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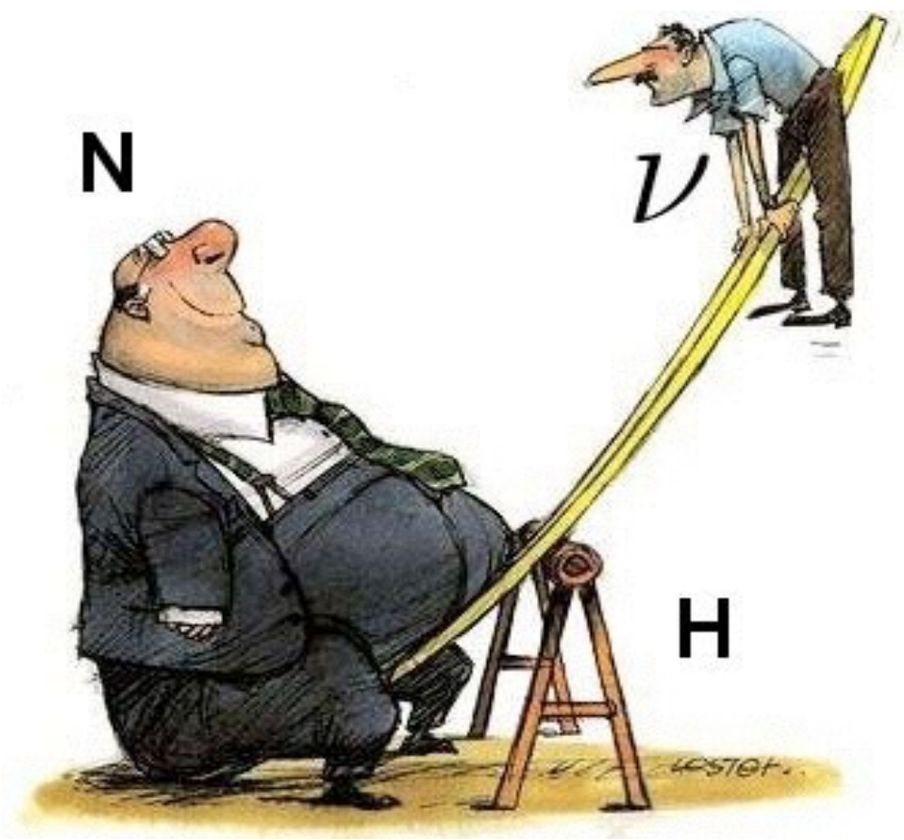
B violation

CP violation

Out of equilibrium

Leptogenesis

The excess of quarks can be explained by **Leptogenesis** (Fukugita, Yanagida): the heavy N responsible for neutrino masses generate a lepton asymmetry.



Recall: See saw mechanism type I

- Introduce a right handed neutrino N
- Couple it to the Higgs

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

- At T>M, the right-handed neutrinos N are in equilibrium thanks to the processes which produce and destroy them:

$$N \leftrightarrow \ell H$$

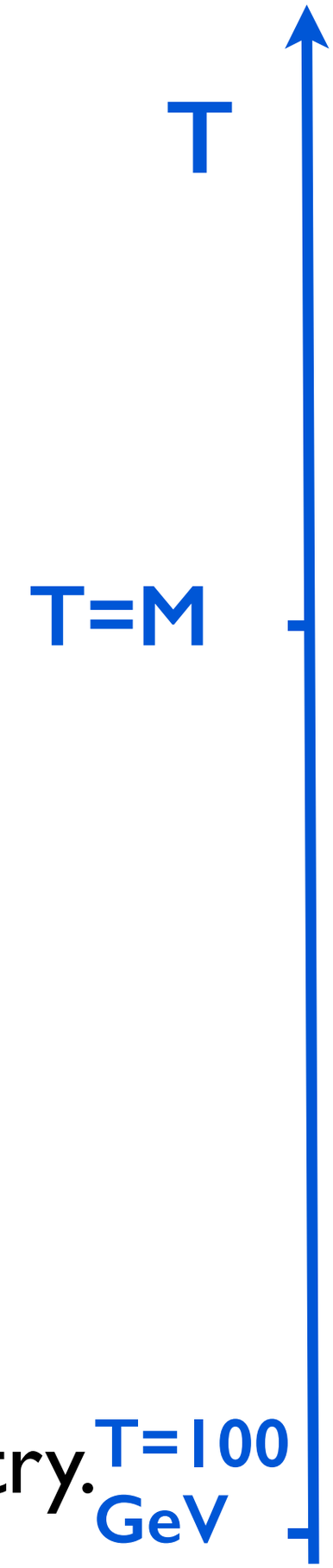
- When T<M, N drops out of equilibrium

$$N \to \ell H \qquad\qquad N \to \ell^c H^c$$

- A lepton asymmetry can be generated if

$$\Gamma(N \to \ell H) \neq \Gamma(N \to \ell^c H^c)$$

- Sphalerons convert it into a baryon asymmetry.



Fukugita, Yanagida, PLB 174; Covi, Roulet, Vissani; Buchmuller, Plumacher; Abada et al., ...

In order to compute the baryon asymmetry:

1. evaluate the CP-asymmetry:

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to lH) - \Gamma(N_1 \to \bar{l}H^c)}{\Gamma(N_1 \to lH) + \Gamma(N_1 \to \bar{l}H^c)}$$

2. solve the Boltzmann equation to take into account the wash-out of the asymmetry with a k washout factor:

$$Y_L = k\epsilon_1$$

3. convert the lepton asymmetry into baryon asymmetry.

$$Y_B = \frac{k}{g^*} c_s \epsilon_1 \sim 10^{-3} - 10^{-4} \epsilon_1$$

[Fukugita, Yanagida; Covi, Roulet, Vissani; Buchmuller, Plumacher]

Conclusions (with some personal views)

1. Neutrinos have masses and mix and a wide experimental programme will measure their parameters with precision.

2. Neutrino masses cannot be accommodated in the Standard Model: extensions can lead to Dirac or Majorana neutrinos, with the latter the most studied cases. See-saw models are particularly favoured.

3. The main question concerns the energy scale of the new physics. Neutrino masses cannot pin it down by themselves and other signatures should be studied (leptogenesis, CLFV, collider LNV, LFV for TeV scale models, ...)

4. Neutrinos were in equilibrium in the Early Universe. They decoupled around 1 MeV and since then they impacted significantly on the evolution of the Universe (BBN, CMB, LSS).

A few references

Flavour models

S. F. King and C. Luhn, Neutrino Mass and Mixing with Discrete Symmetry, Rept.Prog.Phys. 76 (2013) 056201

Neutrinos in cosmology

J. Lesgourgues and S. Pastor, Massive neutrinos and cosmology, Phys.Rept. 429 (2006) 307-379 [astro-ph/0603494]

Sterile neutrinos in cosmology

M. Drewes (Munich, Tech. U.) et al., A White Paper on keV Sterile Neutrino Dark Matter, JCAP 1701 (2017) no. 01, 025 [arXiv:1602.04816]

K. Abazajian, 1705.01837

A few references

Leptogenesis

C. S. Fong, E. Nardi, A. Riotto, Leptogenesis in the Universe, Adv.High Energy Phys. 2012 (2012) 158303 [arXiv:1301.3062]

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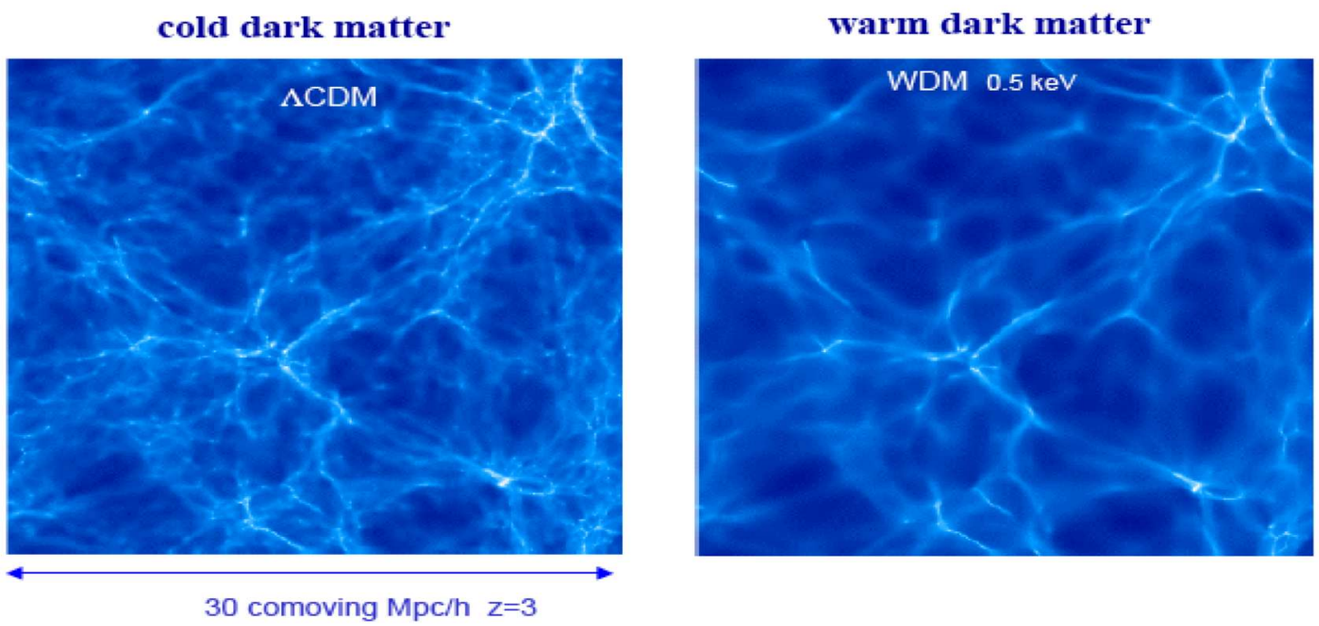
Warm Dark Matter

DM candidates with clustering properties intermediate between hot dark matter and cold dark matter is named warm dark matter. For a standard distribution, the mass is in the keV range.



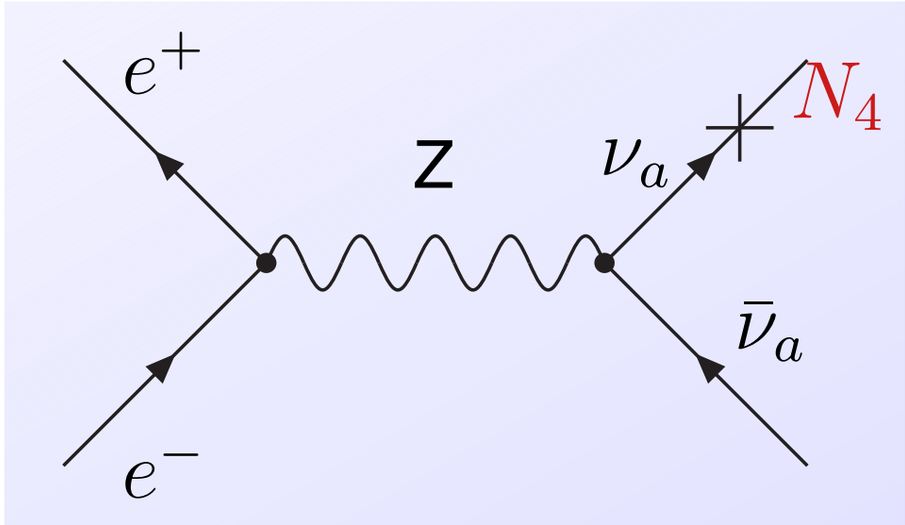
See, e.g. Haehnelt, Frenk et al., B. Moore et al.....

A prime candidate are sterile neutrinos. In the right range of masses and mixing angles, sterile neutrinos can be "stable" on the cosmic timescales.

$$\Gamma_{3\nu} \simeq \sin^2 2\theta \, G_F^2 \frac{m_4^5}{768\pi^3} \sim 10^{-30} \mathrm{s}^{-1} \frac{\sin^2 2\theta}{10^{-10}} \left(\frac{m_4}{\mathrm{keV}}\right)^5$$

@Silvia Pascoli

Their production is different from active neutrinos as they were never in equilibrium with the thermal plasma. In an interaction involving active neutrinos, a heavy neutrino would be produced via loss of coherence.



These oscillations happen in the thermal plasma, so the mixing angle will be in matter.

$$\sin^2 2\theta_m = \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + D^2 + (\Delta(p)\cos 2\theta - V_D + |V_T|)^2}$$

Analogue to matter effects in the earth and depend on the lepton asymmetry.

Genuine thermal effects. They always suppress the oscillations.

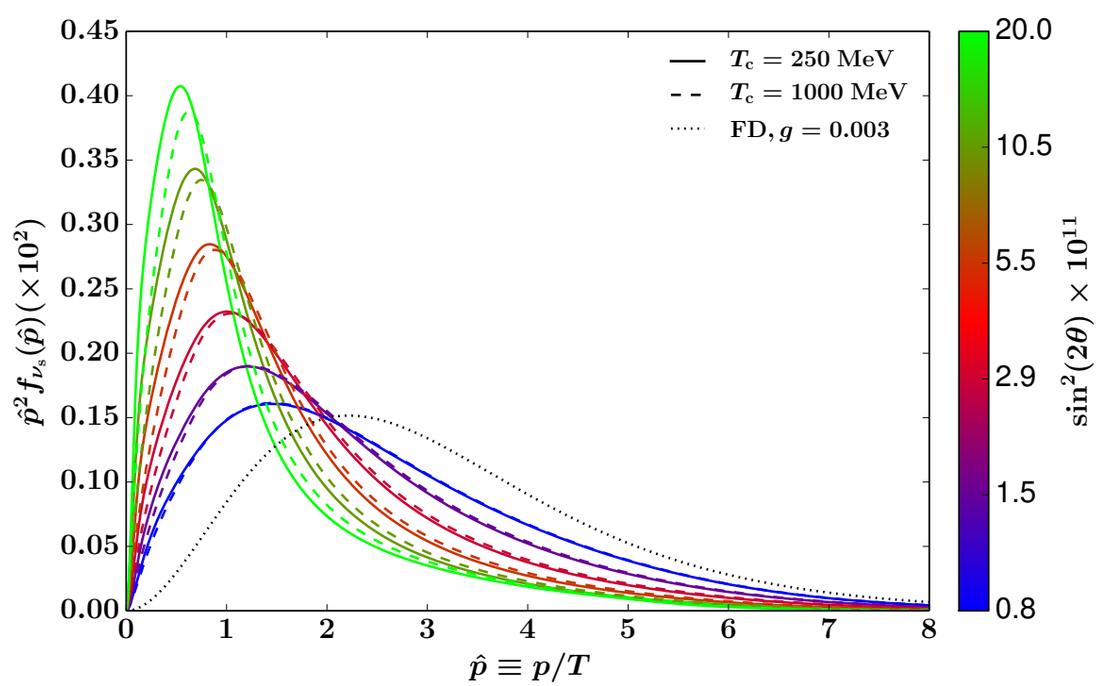
The production will depend on the mixing angle and on the interaction rate of the active neutrinos. A detailed computation requires to solve the associated Boltzmann equation for their distribution:

$$\frac{\partial}{\partial t} f_s(p,t) - Hp\frac{\partial}{\partial p} f_s(p,t) \simeq \frac{\Gamma_a}{2}\langle P(\nu_a \to \nu_s; p,t)\rangle (f_a(p,t) - f_s(p,t))$$

with $f_a(p,t) = (1+e^{E/T})^{-1}$.

Exercise
It can be solved analytically

The final abundance is $\Omega_4 h^2 \simeq 0.3 \frac{\sin^2 2\theta}{10^{-8}} \left(\frac{m_4}{10\mathrm{keV}}\right)^2$

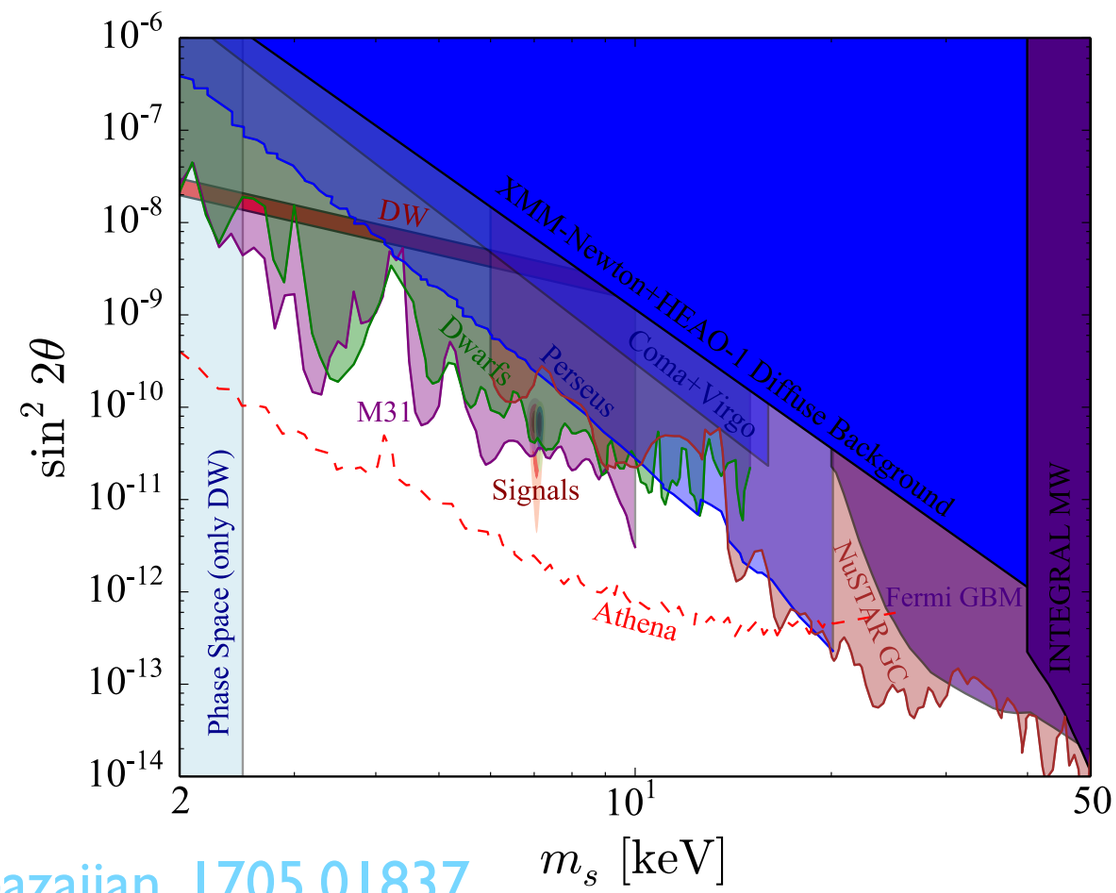


In presence of a large asymmetry, even smaller angles are required thanks to the resonant enhancement of the production.

Bounds on these DM candidates:

- Structure formation. If their mass is too low, they will behave too much as HDM erasing the structure at intermediate scales. This allows to put a bound in the several keV range.

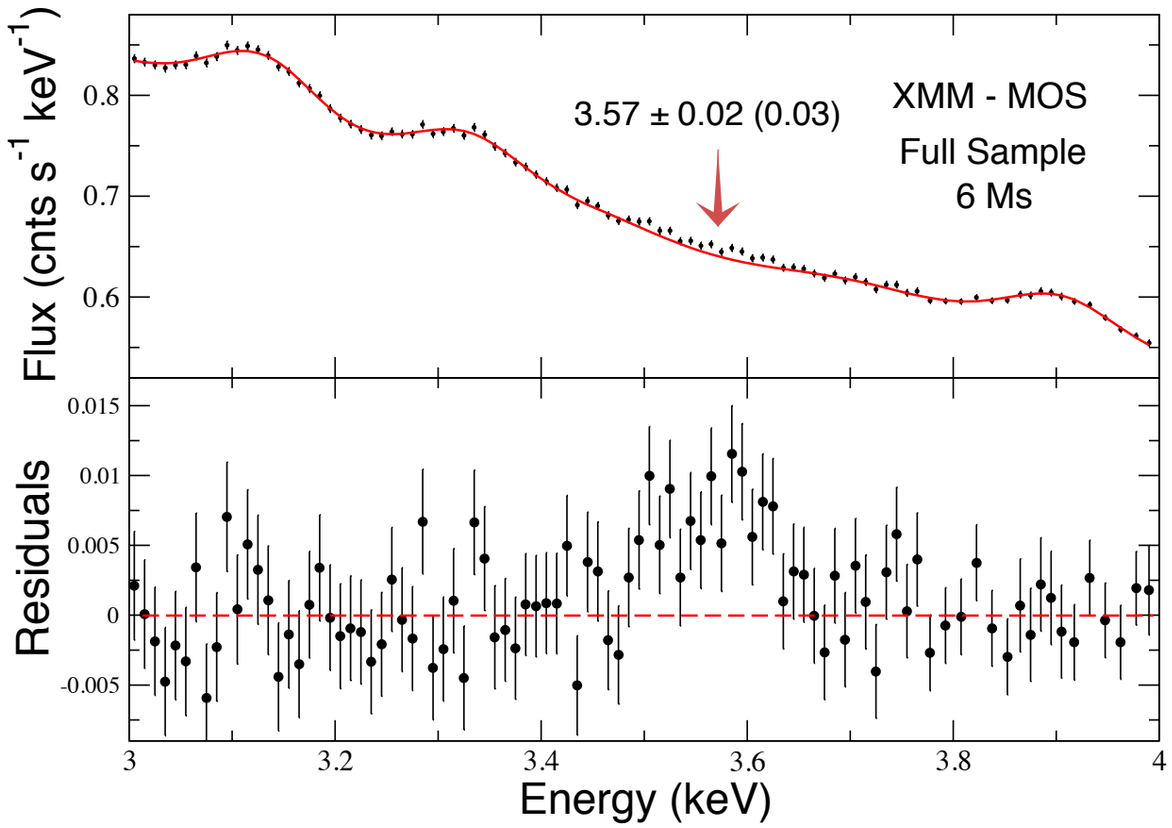
- x-ray searches. Although nearly sterile, their small mixing with active neutrinos make them decay in photons:



K. Abazajian, 1705.01837

$\nu_4 \to \nu_a \gamma$ with $E_\gamma = m_4/2$ and $Br(\nu\gamma) \sim 0.01$

In 2014 two independent groups presented indications of a line around 3.5 keV.

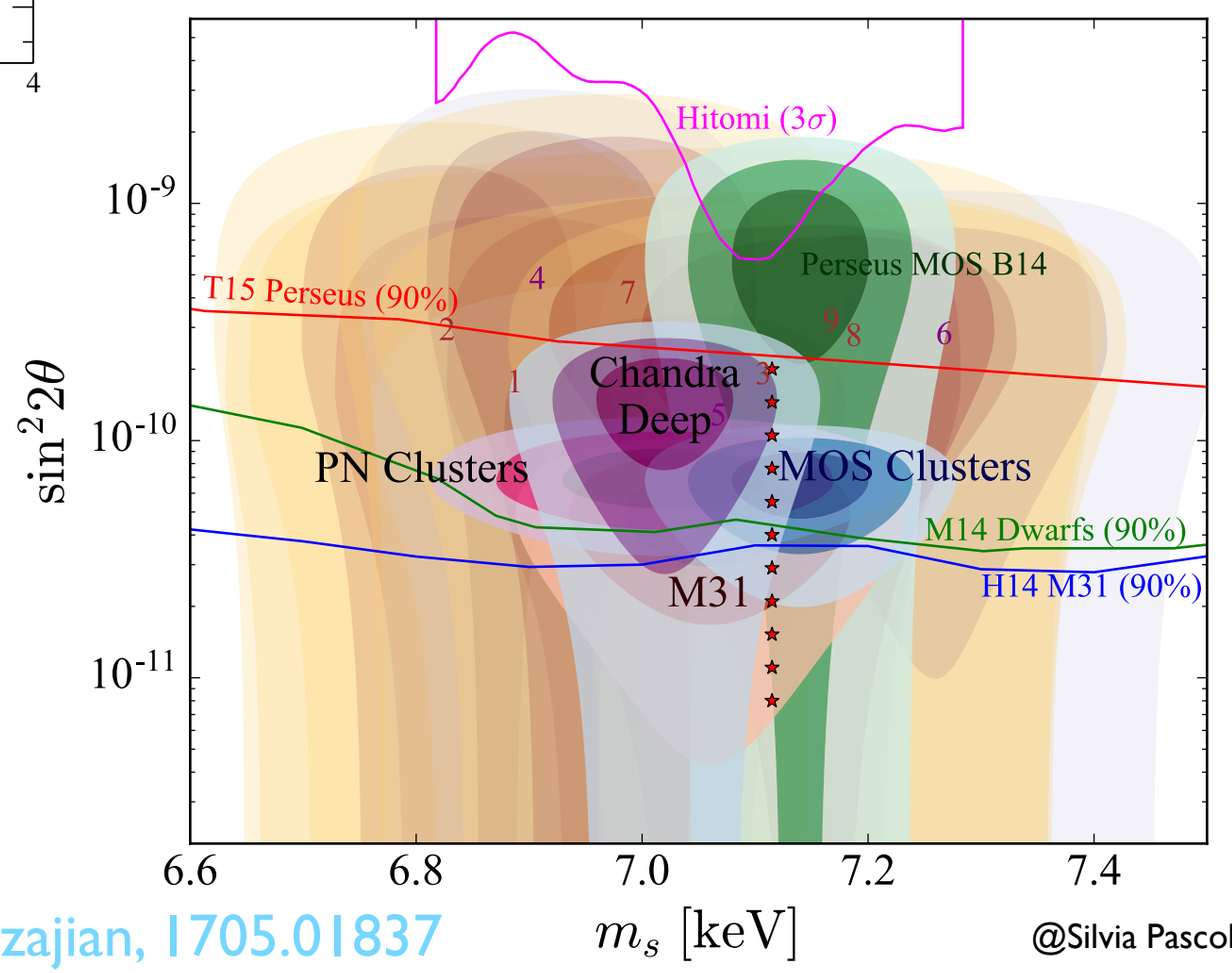


They analysed the emissions of several clusters.

E. Bulbul et al., 1402.2301. See also, A. Boyarsky et al., 1402.4119

If interpreted as sterile neutrinos, this would correspond to a 7 keV neutrino with a mixing

$$\sin^2 2\theta \simeq 7 \times 10^{-11}$$



K. Abazajian, 1705.01837

@Silvia Pascoli

Neutrinoscope App

NEUTRINOSCOPE!

Physicists know that neutrinos exist thanks to very large and complex detectors, but these particles are not rare. In fact, most natural objects emit neutrinos: every second, your own body fires off around 8000 of them!

With the **NeutrinoScope** app, we invite you to explore the sources of neutrinos around you. First, you can see simulations of some of the most important sources of neutrinos: the Sun, the atmosphere, radioactivity.

FIND BY SCANNING:

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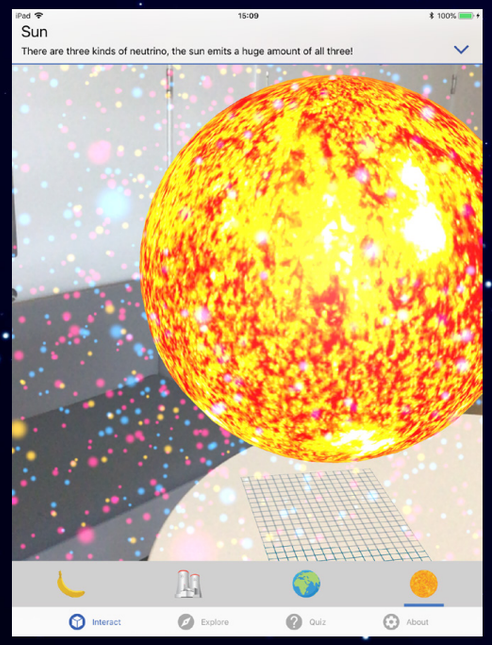
WWW.GHOSTSINTHEUNIVERSE.COM/NAME-OF-GAME



Then, turn the camera on the world around you and see what you can see! As you pan around, you will see the known sources of neutrinos in your current location. What can you find?

DOWNLOAD NEUTRINOSCOPE FOR FREE FROM THE APPLE APP STORE NOW!

NeutrinoScope was developed in partnership with Cambridge Consultants, a world leader in disruptive innovation and technology-based consulting. For more information, visit www.cambridgeconsultants.com

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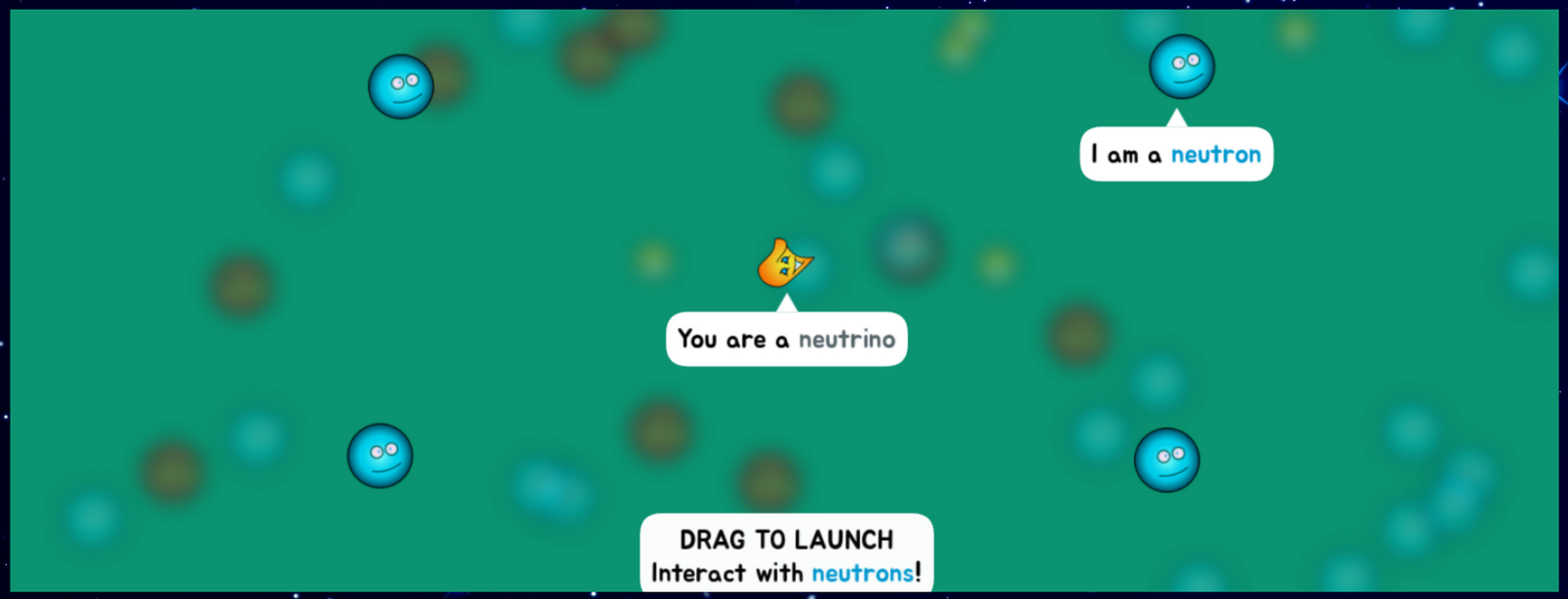
Neutrino game: NuOdyssey

NUODYSSEY

When a neutrino is produced, it travels through space close to the speed of light, nearly unstopped.
In fact, billions of neutrinos are going through you right now without you noticing them!
Nevertheless, as they travel, **once in a while**, they would interact with different types of particles in various ways.
You can explore how a neutrino interacts in this game! Would you be able to reach Super-Kamiokande and be detected?

PLAY NUODYSSEY AT: WWW.GHOSTSINTHEUNIVERSE.ORG/NUODYSSEY



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