OUTLINE LECTURE 1

• in lecture 1:
  • flavor structure of the standard model
  • testing the Kobayashi-Maskawa mechanism
USEFUL REFERENCES

• some excellent introductions to flavor physics
  • Kamenik, 1708.00771
  • Nir, 0708.1872, 1605.00433
  • Grossman, Tanedo, 1711.03624
  • Gedalia, Perez, 1005.3106
  • Blanke, 1704.03753
  • Ligeti, 1502.01372
• why such hierarchical structure of SM fermions?

• Standard Model flavor puzzle
• what lies above the electroweak scale?
  • flavor physics a way to probe well above EW scale
LOW ENERGY PRECISION BOUNDS

• an impressive progress on flavor bounds in last 10 years
  \( |c\bar{u}| \rightarrow |\bar{b}\bar{s}| \)
• in \( D, B_s \) mixing
• also from \( \varepsilon_K \rightarrow \bar{d}s \)

\( \frac{1}{\Lambda^2} \left( \bar{b}_L \gamma_\mu d_L \right) \left( \bar{b}_L \gamma_\mu d_L \right) \)
FLAVOR STRUCTURE OF THE STANDARD MODEL

- in the SM flavor refers to the type/generation of fermion
- below electroweak scale the unbroken SM gauge group is $SU(3)_c \times U(1)_{em}$
  - three generations of fermions

| 3_{2/3} : | up type quarks; | $u, c, t$ |
| 3_{-1/3} : | down type quarks; | $d, s, b$ |
| 1_{-1} :   | charged leptons;  | $e, \mu, \tau$ |
| 1_{0} :    | neutrinos;        | $\nu_e, \nu_\mu, \nu_\tau$ |
THE NAME

• origin of the name "flavor"

The term *flavor* was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice-cream has both color and flavor so do quarks (Fritzsch, 2008).
THE NEXT FEW SLIDES...

• what defines a quantum field theory model
• how to count physical parameters
WHAT DEFINES A QFT MODEL?

- gauge group
- field content
- then write down the most general renormalizable Lagrangian
STANDARD MODEL

• SM defined by
  • the gauge group
  \[ G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y. \]
  • the field content (we set neutrino masses to zero!)
  \[ H \sim (1, 2)_{1/2}, \]
  \[ Q_{Li} \sim (3, 2)_{+1/6}, \quad u_{Ri} \sim (3, 1)_{+2/3}, \quad d_{Ri} \sim (3, 1)_{-1/3}, \]
  \[ L_{Li} \sim (1, 2)_{-1/2}, \quad \ell_{Ri} \sim (1, 1)_{-1}, \]

• then write down the most general renormalizable Lagrangian
  \[ \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}. \]
FLAVOR IN THE SM

- the kinetic terms completely determined by covariant derivatives

\[ D_\mu = \partial_\mu + i g_s G^a_\mu t^a + i g W^i_\mu \tau^i + i g' B_\mu Y. \]

- flavor blind - all three generations in the same gauge representations

\[ \mathcal{L}_{\text{kin}}|_{Q_L} = i \tilde{Q}_L^i (\partial_\mu + i g_s G^a_\mu \frac{1}{2} \lambda^a + i g W^i_\mu \frac{1}{2} \sigma^i + i \frac{1}{6} g' B_\mu) \delta^{ij} Q^j_L, \]

\[ \mathcal{L}_{\text{kin}}|_{u_R} = i \tilde{u}_R^i (\partial_\mu + i g_s G^a_\mu \frac{1}{2} \lambda^a + i \frac{2}{3} g' B_\mu) \delta^{ij} u^j_R, \]

- can rotate each of the fields by global $SU(3) \times U(1)$
  - SM large global flavor symmetry

\[ \mathcal{G}_{\text{flavor}} = U(3)_q^3 \times U(3)_{\text{lep}}^2, \]
FLAVOR IN THE SM

• kinetic terms global flavor symmetry

\[ G_{\text{flavor}} = U(3)_q^3 \times U(3)_{\text{lep}}^2, \]

\[ U(3)_q^3 = U(3)_Q \times U(3)_u \times U(3)_d, \]

\[ U(3)_{\text{lep}}^2 = U(3)_L \times U(3)_{\ell}, \]

• but broken by the Yukawa terms

\[ \mathcal{L}_{\text{Yukawa}} = -Y_d^{ij} \bar{Q}^i_L H d^j_R - Y_u^{ij} \bar{Q}^i_L H^c u^j_R - Y_\ell^{ij} \bar{L}^i_L H \ell^j_R + \text{h.c..} \]

\[ G_{\text{flavor}} \rightarrow U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau, \]

• is the source of quark and charge lepton masses

• after Higgs obtains a vev

\[ \langle H \rangle = (0, v/\sqrt{2}), \; v = 246 \text{ GeV} \]
INTERMEZZO

• a Standard Model

vs.

the Standard Model
a (the) Standard Model

• a Standard Model
  • gauge group+field content \Rightarrow a\ renormalizable Lagrangian
  • has accidental symmetries: $U(1)_B \times U(1)_I^3$
    - this for any values of parameters in the Lagrangian
    - can be broken by non-renormalizable terms

• the Standard Model
  • with the actual values of the parameters
  • there can be approximate symmetries
ISOSPIN

• isospin is an approximate symmetry
• in QCD interactions can replace $u \leftrightarrow d$
  • because $m_u, m_d$ small

\[
\frac{|m_u - m_d|}{\Lambda_{\text{QCD}}} \ll 1
\]
HOW DO WE COUNT PHYSICAL PARAMETERS

• SM has 18 parameters*
  ● 3 gauge couplings
  ● 3 lepton masses
  ● 6 quark masses
  ● 4 parameters in the CKM matrix
  ● 2 params in the Higgs sector

* and the strong CP parameter $\theta$
PHYSICAL PARAMETERS

• what are physical parameters?
  • parameters that cannot be rotated away
  • for instance: charged lepton masses
DIAGONALIZING LEPTON YUKAWA

- lepton Yukawa can be made diagonal, real, positive

\[ \mathcal{L}_{\text{Yukawa}} \supset -Y_{\ell}^{ij} \bar{L}_{L}^{i} H \ell_{R}^{j} + \text{h.c.} \]

\[ L_{L} \rightarrow V_{L} L_{L}, \quad \ell_{R} \rightarrow V_{\ell} \ell_{R}, \]

- how many physical parameters?
  - \( Y_{\ell} \): 9 real + 9 imaginary #'s
  - \( V_{L}, V_{\ell} \): 2x(3 real+6 im.) #'s
  - when rotate \( L_{L}^{i} \) and \( l_{R}^{i} \) by the same phase no change in \( y_{i} \)
    - 3 phases (im. #'s) no effect
  - 9-2x3=3 real, 9-(2x6-3)=0 im. physical parameters

\[ Y_{\ell} \rightarrow V_{L}^{\dagger} Y_{\ell} V_{\ell} = \text{diag}(y_{e}, y_{\mu}, y_{\tau}). \]
HOW DO WE COUNT PHYSICAL PARAMETERS

• the general rule

# physical parameters = # parameters - # broken symmetry generators
AN EXAMPLE

• an example: spin in a magnetic field
  • if no magnetic field: SO(3) symmetry (3 generators)
  • two degenerate eigen-states
• if magnetic field: Zeeman effect, the states are split
  • the splitting depends on strength of magnetic field \( B \): 1 physical param.
• but \( B \) in general has three components
  • 3 parameters \( \mathbf{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \).
• use rotat. around \( x \) and \( y \) axis to align \( B \) along \( z \) axis (set \( B_x = B_y = 0 \))
  • 2 broken generators

\[
\text{# physical parameters} = \text{# parameters} - \text{# broken symmetry generators}
\]

\( 1 = 3 - 2 \)
Diagonalizing Quark Yukawas

- Use unitary transformations

\[ \mathcal{L}_{\text{Yukawa}} \supset -Y_d^{ij} \bar{Q}^i_L H d_R^j - Y_u^{ij} \bar{Q}^i_L H^c u_R^j + \text{h.c.} \]

\[ Q_L \rightarrow V_Q Q_L, \quad u_R \rightarrow V_u u_R, \quad d_R \rightarrow V_d d_R, \]

- Can bring the \( Y_u, Y_d \) Yukawas to the form

\[ Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V^\dagger \text{diag}(y_u, y_c, y_t) \]

- How many physical parameters?
  - \( Y_d, Y_u \): 2x(9 real + 9 im.) #'s
  - \( V_Q, V_u, V_d \): 3x(3 real + 6 im.) #'s
  - One global phase no effect
  - 2x9-3x3=9 real, 2x9-(3x6-1)=1 im. physical parameters
  - 6 quark masses, 3 mixing angles, one phase
Diagonalizing quark Yukawas

\[ \mathcal{L}_{\text{Yukawa}} \supset -Y_d^{ij} \bar{Q}_L^i H d_R^j - Y_u^{ij} \bar{Q}_L^i H^c u_R^j + \text{h.c.} \]

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Unitary CKM matrix
ONALIZING QUARK YUKAWAS

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\[ V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ s_{ij} = \sin \theta_{ij}, \quad c_{ij} = \cos \theta_{ij}, \]

- can bring the \( Y_u, Y_d \) Yukawas to the form

\[ Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V^\dagger \text{diag}(y_u, y_c, y_t) \]

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FLAVOR IN THE SM

- kinetic terms global flavor symmetry

\[ G_{\text{flavor}} = U(3)_q^3 \times U(3)_{\text{lep}}^2, \]

\[ U(3)_q^3 = U(3)_Q \times U(3)_u \times U(3)_d, \]

\[ U(3)_{\text{lep}}^2 = U(3)_L \times U(3)_{\ell}, \]

- broken by the Yukawa terms

\[ \mathcal{L}_{\text{Yukawa}} = -Y_d^{ij} \, \overline{Q}_L^i \, H d_R^j - Y_u^{ij} \, \overline{Q}_L^i \, H^c u_R^j - Y_{\ell}^{ij} \, \overline{L}_L^i \, H \ell_R^j + h.c.. \]

- since \( Y_{\ell} \neq 1 \): \( U(3)_L \times U(3)_{\ell} \rightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau \), i.e., family lepton number,

- since \( Y_u \neq 1 \): \( U(3)_Q \times U(3)_u \rightarrow U(1)_u \times U(1)_c \times U(1)_t \), i.e., up-quark family number,

- since \( Y_d \neq 1 \): \( U(3)_Q \times U(3)_d \rightarrow U(1)_d \times U(1)_s \times U(1)_b \), down-quark family number,

- since \([Y_d, Y_u] \neq 0\): \( U(1)_q^6 \rightarrow U(1)_B \), i.e., the above quark \( U(1)s \) further break to a global baryon number.
FLAVORS IN THE SM

- kinetic terms generate flavor symmetry
  \[ G_{\text{flavor}} = U(3)^3_q \times U(3)^2_{\text{lep}}, \]

- broken by the Yukawa terms
  \[ L_{\text{Yukawa}} = -Y^{ij}_d \bar{Q}^i_L H d^j_R - Y^{ij}_u \bar{Q}^i_L H^c u^j_R - Y^{ij}_\ell \bar{L}^i_L H \ell^j_R + \text{h.c.}. \]

- since \( Y_\ell \neq 1 \): \( U(3)_L \times U(3)_\ell \to U(1)_e \times U(1)_\mu \times U(1)_\tau \), i.e., family lepton number,
- since \( Y_u \neq 1 \): \( U(3)_Q \times U(3)_u \to U(1)_u \times U(1)_c \times U(1)_t \), i.e., up-quark family number,
- since \( Y_d \neq 1 \): \( U(3)_Q \times U(3)_d \to U(1)_d \times U(1)_s \times U(1)_b \), down-quark family number,
- since \([Y_d, Y_u] \neq 0\): \( U(1)^6_q \to U(1)_B \), i.e., the above quark \( U(1)\)'s further break to a global baryon number.
FLAVOR IN THE SM

- the main message:
  - in the SM the flavor structure resides in the Yukawa interactions

\[ \mathcal{L}_{\text{Yukawa}} \supset -Y_d^{ij} \bar{Q}_L^i H d_R^j - Y_u^{ij} \bar{Q}_L^i H^c u_R^j + \text{h.c.} \]

\[ Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V^\dagger \text{diag}(y_u, y_c, y_t) \]

- can move flavor changing interactions to kinetic term by field redefinition

\[ \mathcal{M}_q = Y_q \frac{(\nu + h)}{\sqrt{2}}. \]

\[ Q_L \rightarrow \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix}, \]

- in the so-called mass basis

\[ \mathcal{L}_{\text{SM}} \supset (\bar{q}_i D_{\text{NC}} q_i) + \frac{g}{\sqrt{2}} \bar{u}_L^i W^+ V_{\text{CKM}}^{ij} d_L^j + m_u \bar{u}_L^i u_R^i (1 + \frac{h}{\nu}) + m_d \bar{d}_L^i d_R^i (1 + \frac{h}{\nu}) + \text{h.c.}, \]
**Flavor in the SM**

- neutral currents are flavor conserving (at tree level)
  - photon, gluon, Z: have *flavor (generation) universal* interactions

- Higgs has *flavor diagonal* interactions
  - proportional to quark mass

- charged currents are *flavor changing*
- W couplings are flavor changing
FLAVOR IN THE SM

- neutral currents are flavor conserving (at tree level)
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- charged currents are flavor changing
  - W couplings are flavor changing
• neutral currents are flavor conserving (at tree level)
  • photon, gluon, $Z$: have flavor (generation) universal interactions
  • Higgs has flavor diagonal interactions
    • proportional to quark mass
  • charged currents are flavor changing
  • $W$ couplings are flavor changing
• neutral currents are flavor conserving (at tree level)
• photon, gluon, Z: have flavor (generation) universal interactions
• Higgs has flavor diagonal interactions
  • proportional to quark mass
• charged currents are flavor changing
• $W$ couplings are flavor changing
### Charged Currents vs. Neutral Currents

<table>
<thead>
<tr>
<th>Charged Currents</th>
<th>Neutral Currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s \rightarrow u\mu^-\nu$</td>
<td>$s \rightarrow d\mu^+\mu^-$</td>
</tr>
<tr>
<td>$\text{Br}(K^+_u \rightarrow \mu^+\nu) = 64%$</td>
<td>$\text{Br}(K^0_{sd,d\bar{s}} \rightarrow \mu^+\mu^-) = 7 \times 10^{-9}$.</td>
</tr>
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<td>$b \rightarrow c\ell\nu$</td>
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</tr>
<tr>
<td>$\text{Br}(B^-_{b\bar{u}} \rightarrow D^0 \ell^+\ell^-) = 2.3%$</td>
<td>$\text{Br}(B^-_{b\bar{u}} \rightarrow K^{*-}\ell^+\ell^-) = 5 \times 10^{-7}$.</td>
</tr>
<tr>
<td>$c \rightarrow s\mu^-\nu$</td>
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</tr>
<tr>
<td>$\text{Br}(D^\pm_{c\bar{d}} \rightarrow K^0 \mu^+\mu^-) = 9%$</td>
<td>$\text{Br}(D^0_{c\bar{u}} \rightarrow \pi^0 \mu^+\mu^-) &lt; 1.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
CHARGED CURRENTS VS. NEUTRAL CURRENTS

• no tree level Flavor Changing Neutral Currents (FCNCs) in the SM

**charged currents**

\[
\text{Br}(K^{+}_{ud} \to \mu^{+}\nu) = 64\%
\]

\[
\text{Br}(B^{-}_{bu} \to D^{0}_{cu}\ell\bar{\nu}) = 2.3\%
\]

\[
\text{Br}(D^{\pm}_{cd} \to K^{0}_{sd, d\bar{s}}\mu^{\pm}\nu) = 9\%
\]

**neutral currents**

\[
\text{Br}(K_L^{+} \to \mu^{+}\mu^{-}) = 7 \times 10^{-9}
\]

\[
\text{Br}(B^{-}_{bu} \to K^{*-}_{s\bar{u}}\ell^{+}\ell^{-}) = 5 \times 10^{-7}
\]

\[
\text{Br}(D^{0}_{cu} \to \pi^{0}_{u\bar{u}-dd}\mu^{+}\mu^{-}) < 1.8 \times 10^{-4}
\]
**CKM MATRIX**

- 3x3 matrix, is hierarchical

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\sim \begin{pmatrix}
1 & 0.2 & 0.004 \\
0.2 & 1 & 0.04 \\
0.008 & 0.04 & 1
\end{pmatrix},
\]

- is unitary

\[
V_{CKM}^\dagger V_{CKM} = V_{CKM} V_{CKM}^\dagger = 1.
\]
$V_{\text{CKM}} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix},$

- is unitary

$V_{\text{CKM}}^\dagger V_{\text{CKM}} = V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1.$
CKM MATRIX

• hierarchical structure + unitarity
  • encoded in Wolfenstein parametrization

\[ V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4) \]

\[ \lambda \equiv |V_{us}| \sim 0.22 \]

• CKM matrix depends on 3 real params, 1 phase
  • 3 mixing angles, 1 phase
  • in Wolfenstein param. trade for
    • 3 real params: \( \lambda, A, \rho, \)
    • 1 imag. param: \( \eta \)
CKM MATRIX

- hierarchical structure + unitarity
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\[ V_{\text{CKM}} = \begin{pmatrix}
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- CKM matrix depends on 3 real params, 1 phase
  - 3 mixing angles, 1 phase
  - in Wolfenstein param. trade for
    - 3 real params: \(\lambda, A, \rho,\)
    - 1 imag. param: \(\eta\)
CP VIOLATION IN THE STANDARD MODEL

- CP violation in the SM
  - all terms invariant apart from Yukawa terms

\[
Y_{ij} \bar{\psi}_L^i H \psi_R^j + Y_{ij}^* \bar{\psi}_R^j H^\dagger \psi_L^i \xrightarrow{\text{CP}} Y_{ij} \bar{\psi}_R^j H^\dagger \psi_L^i + Y_{ij}^* \bar{\psi}_L^i H \psi_R^j
\]

- CP conserved if Yukawas real
  \[Y_{ij}^* = Y_{ij}.\]

- in the SM the CP violation controlled by one parameter: \(\eta\), "the CKM phase"

- CPT conserved in Lorentz invariant QFTs
  - CP violation = T violation
JARLSKOG INVARIANT

- for existence of CPV in the SM crucial that 3 generations
  - if 2 generations of quarks: CKM matrix can be made real
    - $\Rightarrow$ no physical phase, no CPV
- if $Y_u, Y_d$ can be made diagonal with the same left-handed rotation (= they are "aligned"):
  - $\Rightarrow V_{\text{CKM}}=1 \Rightarrow$ no flavor violation $\Rightarrow$ no CPV
- all the above statements can be encoded in a single parameter: the Jarlskog invariant

$$J_Y \equiv \text{Im} \left( \det \left[ Y_d Y_d^\dagger, Y_u Y_u^\dagger \right] \right).$$

Cecilia Jarlskog in early 1980s
TEST CKM STRUCTURE

- all flavor transitions in SM depend only on 4 fundamental parameters $\lambda, A, \rho, \eta$
- overconstrain the system by making many measurements
- one way to visualise is through the standard CKM unitarity triangle
STANDARD CKM UNITARITY TRIANGLE

- a test of CKM matrix unitarity

\[ V_{\text{CKM}}^\dagger V_{\text{CKM}} = V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1. \]

\[ V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2 / 2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2 / 2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4) \]

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\[ \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \]

\[ - (\bar{\rho} + i\bar{\eta}) + 1 + ( -1 + \bar{\rho} + i\bar{\eta} ) = 0, \]

\[ \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \]
Standard CKM Unitarity

- \((\rho, \eta)\)

\[
\begin{vmatrix}
\frac{V_{ud}^* V_{ub}^*}{V_{cd}^* V_{cb}^*} \\
\frac{V_{td}^* V_{tb}^*}{V_{cd}^* V_{cb}^*}
\end{vmatrix}
\]

- \(\alpha = \phi_2\)
- \(\gamma = \phi_3\)
- \(\beta = \phi_1\)

(0,0) \rightarrow (1,0)

**Figure 12.1:** Sketch of the unitarity triangle.

\[V_{CKM} = \begin{pmatrix}
1 - \lambda^2 / 2 & A\lambda^2 & A\lambda^3 (1 - \rho - i\eta) \\
A\lambda^2 & 1 & A\lambda \\
A\lambda^3 (1 - \rho - i\eta) & A\lambda & 1 + O(\lambda^4)
\end{pmatrix}\]

\[V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0\]

\[
\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + 1 + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0
\]

\[-(\bar{\rho} + i\bar{\eta}) + 1 + (-1 + \bar{\rho} + i\bar{\eta}) = 0,\]

\[
\bar{\rho} + i\bar{\eta} = -\frac{V_{ud} V_{ub}^*}{(V_{cd} V_{cb}^*)}
\]
THE PLAYERS

• B-factories
  - Belle (1999-2010): $\sim 1.5 \times 10^9 B$ mesons
  - Babar (1999-2008): $\sim 0.9 \times 10^9 B$ mesons

• (super)B-factories
  - LHCb(2010-2030?): $\sim$ up to $10^{11}$ (useful) $B$’s
  - Belle-II (2018- 2024?): $\sim 8 \times 10^{10} B$ mesons

• kaon physics experiments
  - in the past (2000s): KLOE, NA62
  - present: NA62 at CERN, KOTO at J-PARC
THE PLAYERS

• B-factories
  • Belle (1999-2010): ~ $1.5 \times 10^9$ B mesons
  • Babar (1999-2008): ~ $0.9 \times 10^9$ B mesons
• (super) B-factories
  • LHCb (2010-2030?): ~ up to $10^{11}$ B mesons
  • Belle-II (2018-2024?): ~ $8 \times 10^{10}$ B mesons

B physics experiencing deflation:
in 2000s: ~50¢/B meson
in 2020s: <1¢/B meson

• kaon physics experiments
  • in the past (2000s): KLOE, NA62
  • present: NA62 at CERN, KOTO at J-PARC
1995
2009
2016
2016
THE FUTURE: TREE PROCESSES @ BELLE 2

Charles et al, 1309.2293
THE FUTURE: TREE PROCESSES @ BELLE 2

SM standard candle
THE UPHSHOT

• CPV an inherently quantum mechanical effect
  • governed by a phase in Lagrangian
• KM mechanism the dominant origin of CPV
  • measurements point to a consistent picture

\[ A = 0.825(9), \quad \lambda = 0.2251(3), \quad \bar{\rho} = 0.160(7), \quad \bar{\eta} = 0.350(6). \]

• since \( \bar{\rho} \approx \bar{\eta} \) the CKM weak phase is large, O(1)

\[ e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} = \arg(V_{ub}^*), \]

• tests will be significantly improved in the near future
Jarlskog Invariant

- since nonzero CPV means Jarlskog invariant is non-zero
  \[ J_Y \equiv \text{Im} \left( \det \left[ Y_d Y_d^\dagger, Y_u Y_u^\dagger \right] \right). \]
- explicitly it is
  \[ J_Y = J_{\text{CP}} \prod_{i>j} \frac{m_i^2 - m_j^2}{v^2/2} \approx O(10^{-22}). \]

\[ J_{\text{CP}} = \text{Im} \left[ V_{us} V_{cb} V_{ub}^* V_{cs}^* \right] = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin \delta_{\text{KM}} \sim \lambda^6 A^2 \eta \sim O(10^{-5}). \]

\[ \prod_{i>j} \frac{m_i^2 - m_j^2}{v^2/2} = \frac{(m_t^2 - m_c^2)}{v^2/2} \cdot \frac{(m_t^2 - m_u^2)}{v^2/2} \cdot \frac{(m_c^2 - m_u^2)}{v^2/2} \cdot \frac{(m_b^2 - m_s^2)}{v^2/2} \cdot \frac{(m_b^2 - m_d^2)}{v^2/2} \cdot \frac{(m_s^2 - m_d^2)}{v^2/2}. \]

- \( J_Y = 0 \), if any of the mixing angles zero or if \( \eta = 0 \)
- \( J_Y = 0 \), if any of up or down quark masses are degenerate
  - origin of the so called GIM mechanism: FCNCs in the SM vanish for equal masses \( \Rightarrow \) extra cancellations in SM amplitudes
CONCLUSIONS

- have looked at the flavor structure in the SM
- experiments shows it is predominantly due to Kobayashi-Maskawa mechanism
BACKUP SLIDES