Review of R-Parity

The superpotential of the MSSM can be separated into two parts:

$$
\begin{aligned}
W_{R_p} &= h^e_{ij} L_i H_1 \bar{E}_j + h^d_{ij} Q_i H_1 \bar{D}_j \\
&\quad + h^u_{ij} Q_i H_2 \bar{U}_j + \mu H_1 H_2, \\
W_{\not R_P} &= \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k \\
&\quad + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k + \kappa_i L_i H_2.
\end{aligned}
$$

W_{R_p} is what is usually meant by the MSSM.

$\mathcal{Q}$: Why ban $W_{\not R_P}$?

$\mathcal{A}$: "Proton decay"

Definition of R-Parity

$\mathcal{Q}$: How is $W_{\not R_P}$ normally banned?

$\mathcal{A}$: By defining discrete symmetry R_p

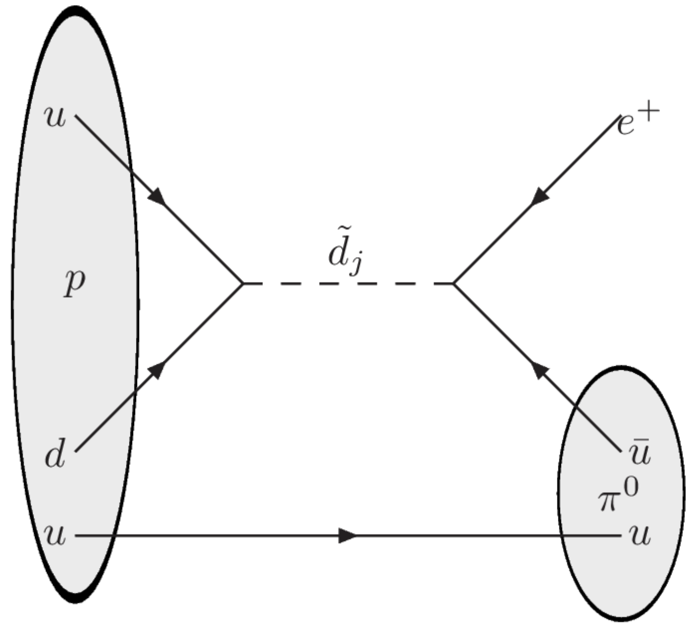
$$R_p = (-\mathbf{1})^{3B+L+2S}.$$

$\rightarrow$ SM fields have $R_p = +1$ and superpartners have $R_p = -1$. There are two important consequences:

- Because initial states in colliders are R_p EVEN, we can only pair produce SUSY particles
- The *lightest superpartner is stable*

Proton decay

$R\!\!\!/_p$ terms are either $L\!\!\!/$ or $B\!\!\!/$. $\Gamma(p \to e^+\pi^0) \approx \frac{{\lambda'}^2_{11j}{\lambda''}^2_{11j}}{16\pi^2 \tilde{m}^4_{d_j}} M_p^5$

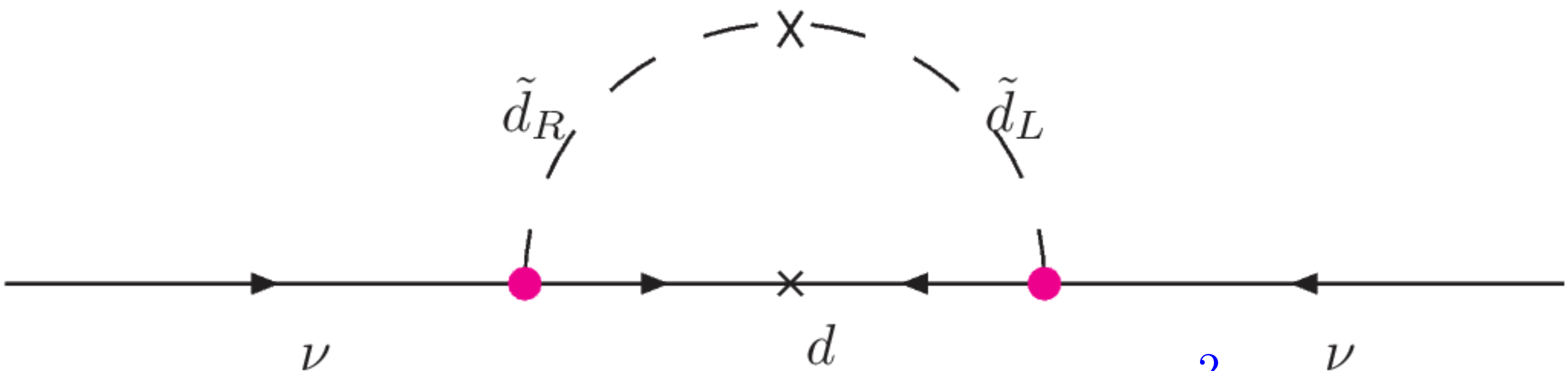


$$\tau(p \to e^+\pi^0) > 1 \times 10^{34}\, yr$$

$$\Rightarrow \quad \lambda'_{11j} \cdot \lambda''_{11j} \lesssim 10^{-27} \left(\frac{\tilde{m}_{d_j}}{1 \text{ TeV}} \right)^2 .$$

Motivation for $\not{R}_p$

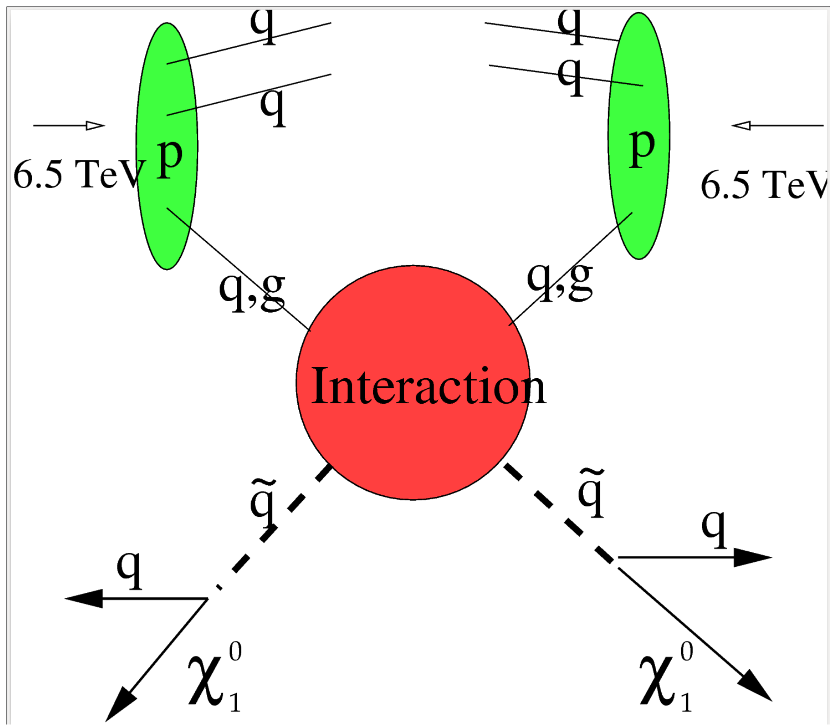
- It has additional search possibilities.
- Dark matter changes character: gravitino or hidden
- Neutrino masses and mixings testable at LHC



$$(m_\nu)_{11} = \frac{3}{32\pi^2} m_d {\lambda'_{111}}^2 \sin 2\theta_d \ln \frac{m^2_{\tilde{d}_L}}{m^2_{\tilde{d}_R}}$$

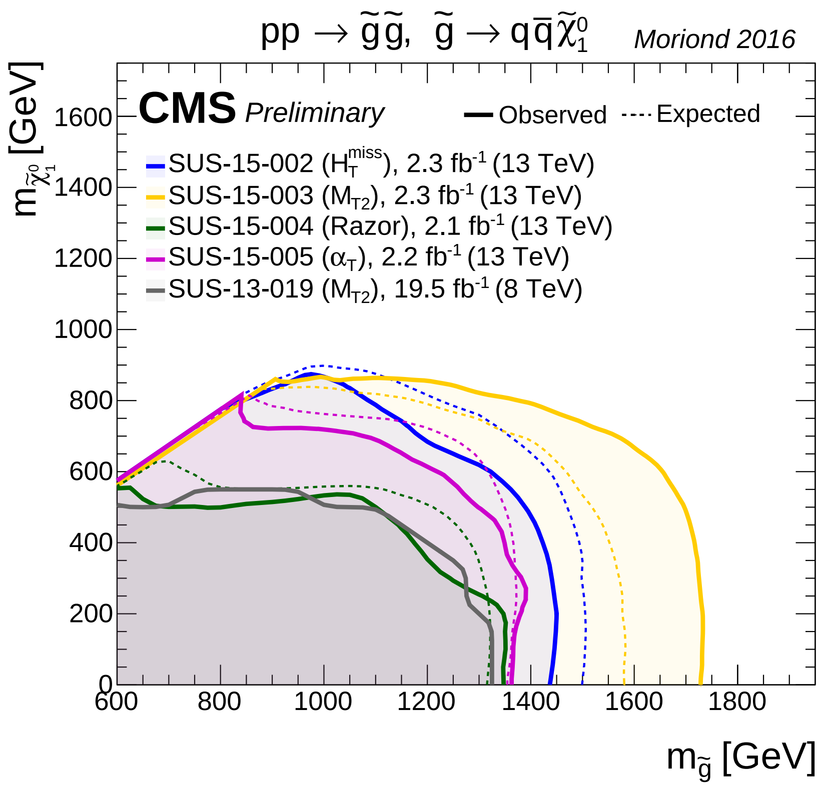
Collider SUSY Production

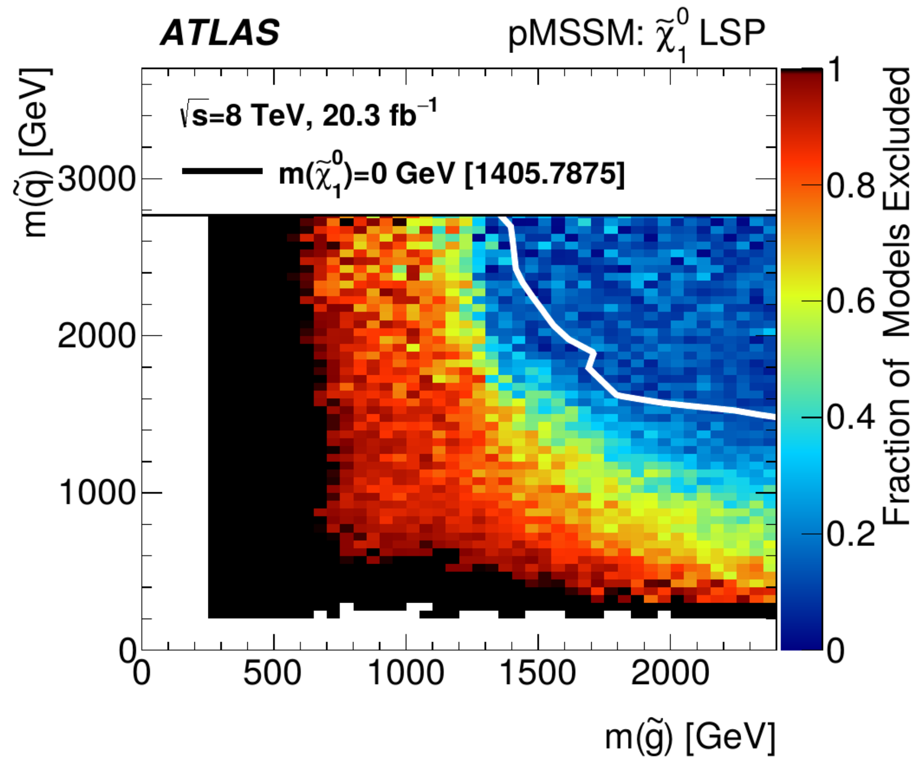
Strong sparticle production and decay to dark matter particles.



Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC

Search limits: be careful





Left hand plot: based on simplified models. More realistic models (see right panel) may evade these limits by having many different decay modes.

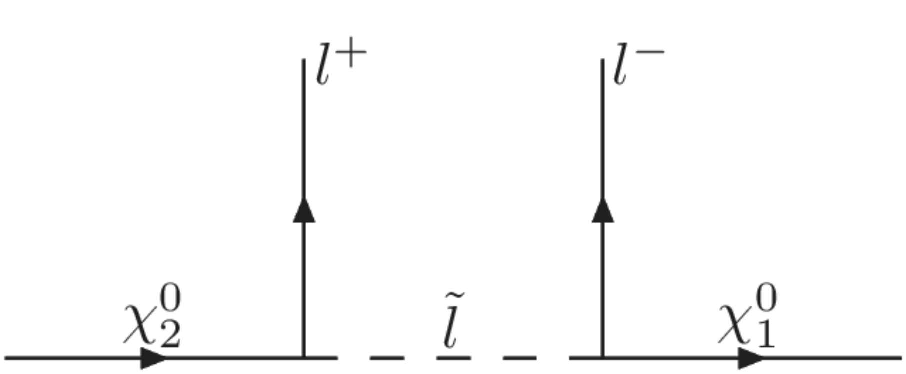
Narrow Width Approximation

Take some scalar propagator mod-squared:

$$
\begin{aligned}
D(p^2) &= \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2}. \\
\lim_{\Gamma/m \to 0} D(p^2) &= \pi/(m\Gamma)\delta(p^2 - m^2).
\end{aligned}
$$

Thus (as is often the case in the MSSM), for particles with narrow widths, we may approximate them assuming they have $p^2 = m^2$, ie they are *on-shell*. The next order in perturbation theory is $\mathcal{O}(m/\Gamma)$.

Cascade Decay



$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$
$$p^\mu_{l\pm} = (|\underline{p}_{l\pm}|, \underline{p}_{l\pm})$$
$$p^\mu_{\chi^0_{1,2}} = \left(\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2}, \underline{p}_{\chi^0_{1,2}}\right)$$

Work in $\tilde{l}$ rest frame. The invariant mass of the l^+l^- pair is

$$m_{ll}^2 = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p_{l^+}^2 + p_{l^-}^2 + 2 p_{l^+} \cdot p_{l^-}$$
$$= 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos\theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

Momentum conservation:

$$\Rightarrow \underline{p}_{\chi_2^0} + \underline{p}_{l^+} = \underline{0}, \qquad \underline{p}_{l^-} + \underline{p}_{\chi_1^0} = \underline{0}.$$

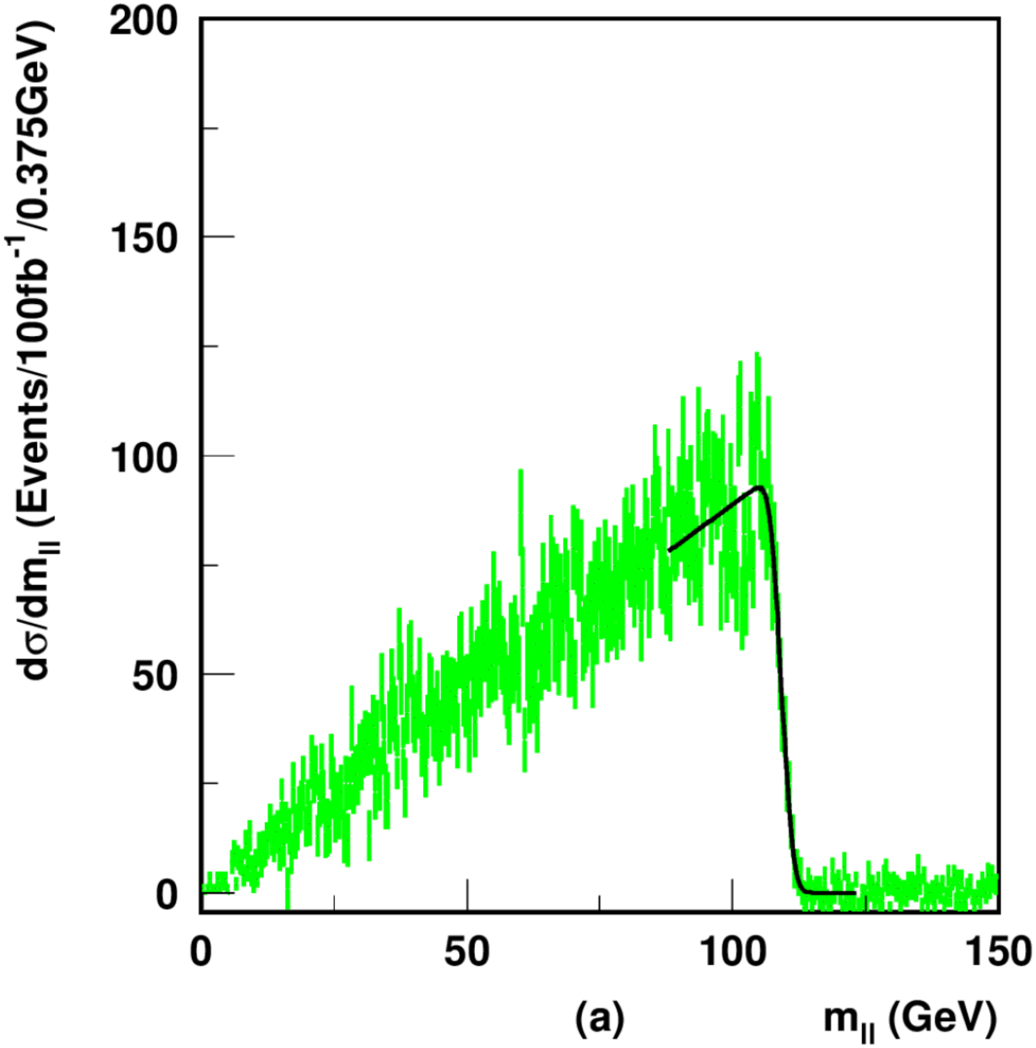
Energy conservation: $\sqrt{m_{\chi_2^0}{}^2 + |\underline{p}_{\chi_2^0}|^2} = m_{\tilde{l}} + |\underline{p}_{l+}|$,

$\Rightarrow |\underline{p}_{l+}| = \frac{m^2_{\chi_2^0} - m^2_{\tilde{l}}}{2m_{\tilde{l}}}$. Similarly $|\underline{p}_{l-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi_1^0}}{2m_{\tilde{l}}}$.

Substituting into the original m_{ll}, we get

$$m^2_{ll} \leq \frac{(m^2_{\chi_2^0} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi_1^0})}{m^2_{\tilde{l}}}$$

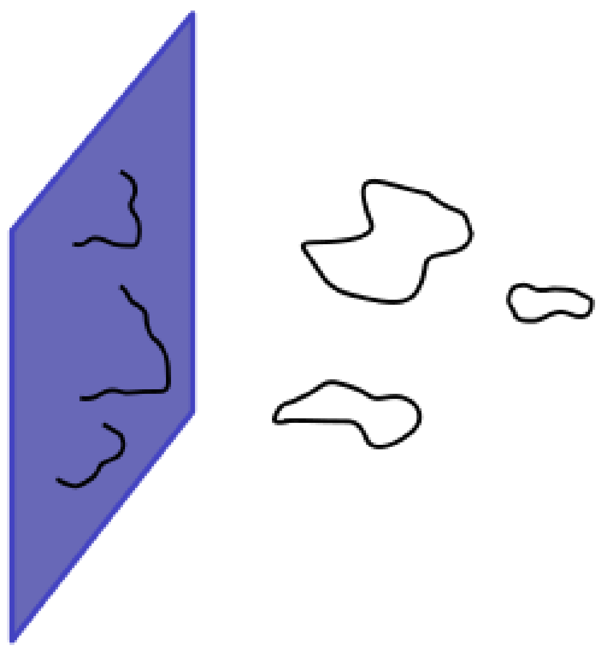
A Sharp Invariant Feature



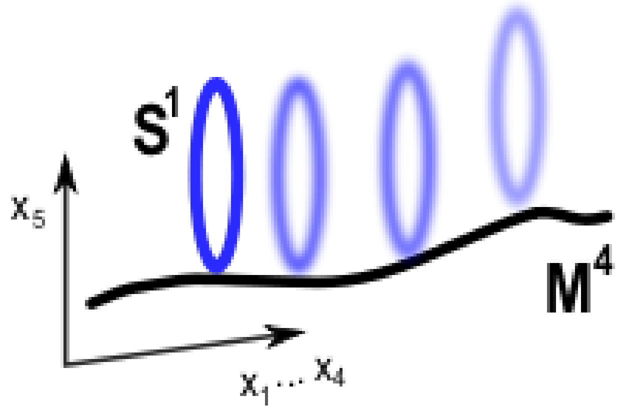
Extra Dimensions

Superstring theory requires them, but: why are we not aware of them?

- We are stuck on a brane



- Or they are curled up on themselves, tightly



Scalar field in 5d

Consider a massless 5D scalar field $\varphi(x^M)$, $M = 0, 1, ..., 4$ with action

$$\mathcal{S}_{5\mathrm{D}} = \int d^5x \; \partial^M \varphi \, \partial_M \varphi \; .$$

Set the extra dimension $x^4 = y$ defining a circle of radius r with $y \equiv y + 2\pi r$.

Our spacetime is now $\mathbb{M}_4 \times S^1$. Periodicity in y direction implies discrete Fourier expansion

$$\varphi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \varphi_n(x^\mu) \exp\left(\frac{iny}{r}\right) .$$

Notice that the Fourier coefficients are functions of the standard 4D coordinates and therefore are (an infinite number of) 4D scalar fields. The equations of motion for the Fourier modes are (in general massive) wave equations

$$\partial^M \partial_M \varphi = 0 \Rightarrow \sum_{n=-\infty}^{\infty} \left(\partial^\mu \partial_\mu - \frac{n^2}{r^2}\right) \varphi_n(x^\mu) \exp\left(\frac{iny}{r}\right) = 0$$

$$\Longrightarrow \quad \boxed{\partial^\mu \partial_\mu \varphi_n(x^\mu) \; - \; \frac{n^2}{r^2}\, \varphi_n(x^\mu) = 0 \; .}$$

These are then an infinite number of Klein Gordon equations for massive 4D fields. This means that each Fourier mode φ_n is a 4D particle with mass $m_n^2 = \frac{n^2}{r^2}$. Only the zero mode ($n = 0$) is massless. One can visualize the states as an infinite tower of massive states (with increasing mass proportional to n). This is called *Kaluza Klein tower* and the massive states ($n \neq 0$) are called *Kaluza Klein-* or *momentum states*, since they come from the momentum in the extra dimension:

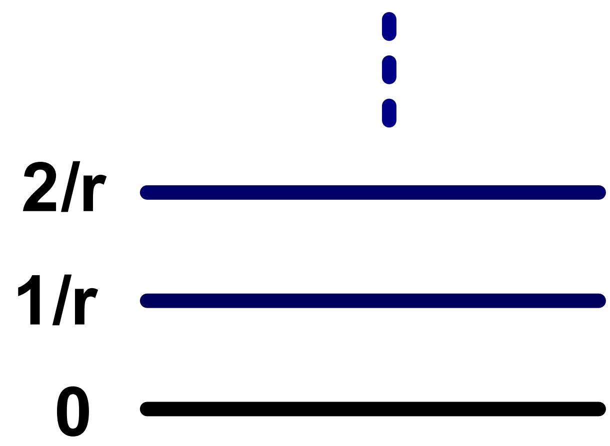


Figure 1: The Kaluza Klein tower of massive states due to an extra S^1 dimension. Masses $m_n = |n|/r$ grow linearly with the fifth dimension's wave number $n \in \mathbb{Z}$.

In order to obtain the effective action in 4D for all these particles, let us plug the mode expansion of φ into the original 5D action,

$$\begin{aligned}\mathcal{S}_{5\mathrm{D}} &= \int d^4x \int dy \sum_{n=-\infty}^{\infty} \Big(\partial^\mu \varphi_n(x^\mu)\, \partial_\mu \varphi_n(x^\mu)^* \\ &\quad -\frac{n^2}{r^2}|\varphi_n|^2\Big) \\ &= 2\,\pi\, r \int d^4x \left(\partial^\mu \varphi_0(x^\mu)\, \partial_\mu \varphi_0(x^\mu)^* + \ldots\right) \\ &= 2\pi r \mathcal{S}_{4\mathrm{D}} + \ldots\end{aligned}$$

This means that the 5D action reduces to one 4D action for a massless scalar field plus an infinite sum of massive scalar actions in 4D. If we are only interested in energies smaller than the $\frac{1}{r}$ scale, we may concentrate only on the 0 mode action.

Vector Field

Vector fields go a similar way: $A_M = A_\mu, A_4 = \phi, \ldots$. Consider the action

$$\mathcal{S}_{5\mathrm{D}} = \int d^5x \, \frac{1}{g^2_{5\mathrm{D}}} \, F_{MN} \, F^{MN}$$

with field strength

$$F_{MN} = \partial_M A_N \; - \; \partial_N A_M$$

implying

$$\partial^M \partial_M A_N \; - \; \partial^M \partial_N A_M = 0 \; .$$

Choose a gauge, e.g. transverse

$$\partial^M A_M = 0, A_0 = 0 \Rightarrow \partial^M \partial_M A_N = 0 \ ,$$

then this obviously becomes equivalent to the scalar field case (for each component A_M) indicating an infinite tower of massive states for each massless state in 5D. In order to find the 4D effective action we once again plug this into the 5D action:

$$\begin{aligned} \mathcal{S}_{5\mathrm{D}} &\mapsto \mathcal{S}_{4\mathrm{D}} \\ &= \int d^4x \left(\frac{2\pi r}{g_{5\mathrm{D}}^2} \, F_{(0)}{}^{\mu\nu} \, F_{(0)\mu\nu} + \frac{2\pi r}{g_{5\mathrm{D}}^2} \, \partial_\mu \rho_0 \, \partial^\mu \rho_0 + \ldots \right) , \end{aligned}$$

Therefore we end up with a 4D theory of a gauge particle (massless), a massless scalar and infinite towers of massive vector and scalar fields. Notice that the gauge couplings of 4- and 5 dimensional actions (coefficients of $F_{MN}F^{MN}$ and $F_{\mu\nu}F^{\mu\nu}$) are related by

$$\frac{1}{g_4^2} = \frac{2\pi r}{g_5^2}.$$

In D spacetime dimensions, this generalizes to

$$\frac{1}{g_4^2} = \frac{V_{D-4}}{g_D^2}$$

where V_n is the volume of the n dimensional compact space (e.g. an n sphere of radius r).

Electric (and gravitational) potential

Gauss' law implies for the electric field $\vec{E}$ and its potential Φ of a point charge Q:

$$\oint_{S^2} \vec{E} \cdot d\vec{S} \;=\; Q \Rightarrow \|\vec{E}\| \;\propto \frac{1}{R^2}, \Phi \propto \frac{1}{R} : \; 4D$$

$$\oint_{S^3} \vec{E} \cdot d\vec{S} \;=\; Q \Rightarrow \|\vec{E}\| \;\propto \frac{1}{R^3}, \Phi \propto \frac{1}{R^2} : \; 5D$$

So in D spacetime dimensions

$$\|\vec{E}\| \propto \frac{1}{R^{D-2}}, \Phi \propto \frac{1}{R^{D-3}}.$$

If one dimension is compactified (radius r) like in $\mathbb{M}_4 \times S^1$, then

$$\|\vec{E}\| \propto \begin{cases} \frac{1}{R^3} & : R < r \\ \frac{1}{R^2} & : R \gg r \end{cases}.$$

Analogous arguments hold for gravitational fields and their potentials.

Gravitation

The spin $2\bar{h}$ graviton G_{MN} becomes the 4D graviton $g_{\mu\nu}$, some gravivectors $G_{\mu n}$ and some graviscalars G_{mn}, where $m, n = 4, \ldots, D-1$ along with their infinite towers.

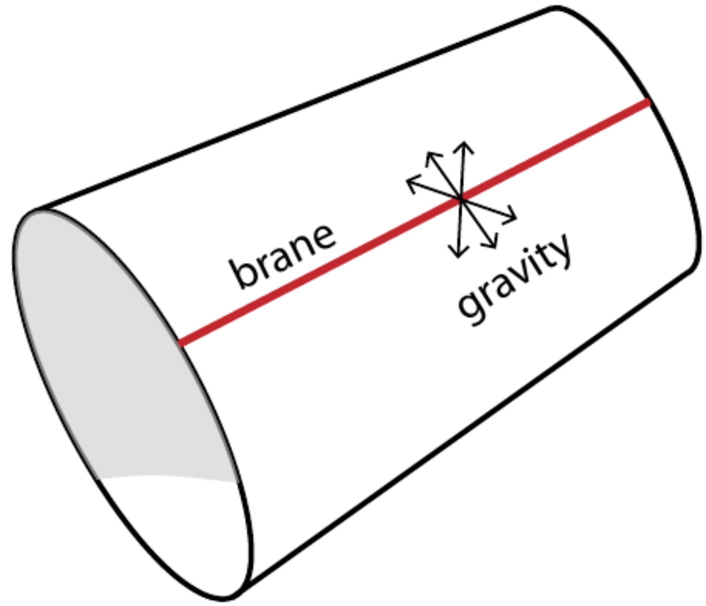
$$M_{Pl}^2 = M_D^{D-2} V_{D-4} \sim M_D^{D-2} r^{D-4}$$

is a derived quantity. Fixing D, we can fix M_D and r to get the correct result of $M_{Pl} \sim 10^{19}$ GeV. So far, we require $M_D > 1$ TeV and $r < 10^{-16}$ cm from Standard Model measurements since no signature of extra dimensions has been set yet.

Brane Worlds

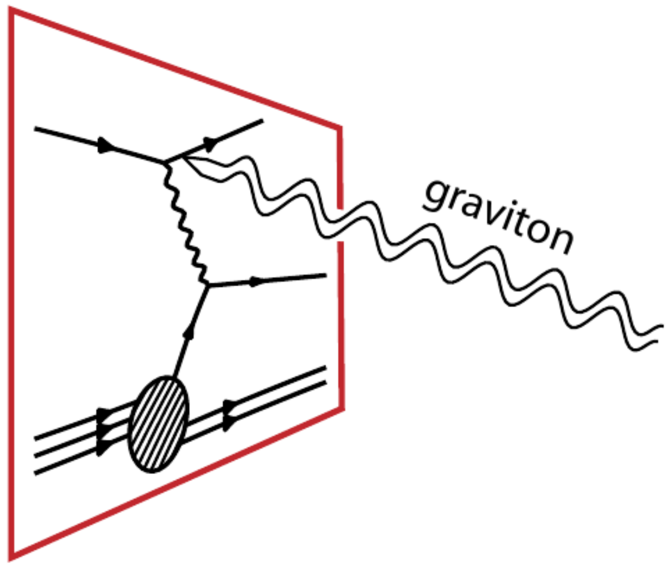
We are trapped on a 3+1 surface in a $D+1$ dimensional bulk space-time. There are two cases here: *large extra dimensions* and $\mathrm{warped\ space\text{-}times}$. Here, since gravity is so weak, the constraints on it are much weaker: $r < 0.1$ mm or so, much larger than the 10^{-16}cm of the Standard Model.

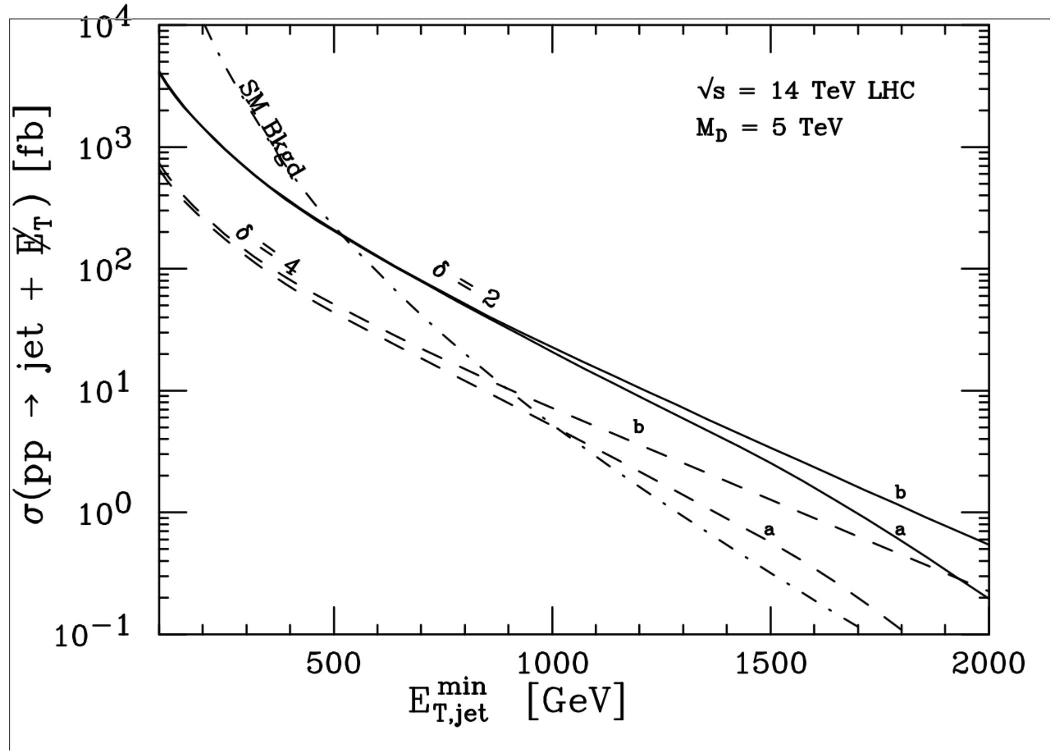
large extra dimensions: Let's try to solve the hierarchy problem: put $M_D \sim 1$ TeV. The idea is that this *is* the fundamental scale: there is no high scale associated with M_{Pl} fundamentally - it is an illusion.



With 5D, $M_{Pl^2} = M_D^{D-2} V_{D-4} \Rightarrow r \sim 10^8$ km, clearly ruled out. Already with 6D though, $r = 0.1$ mm - consistent with experiment. This really then changes the hierarchy problem to the question "why are the extra dimensions so large compared with 10^{-16}cm?"

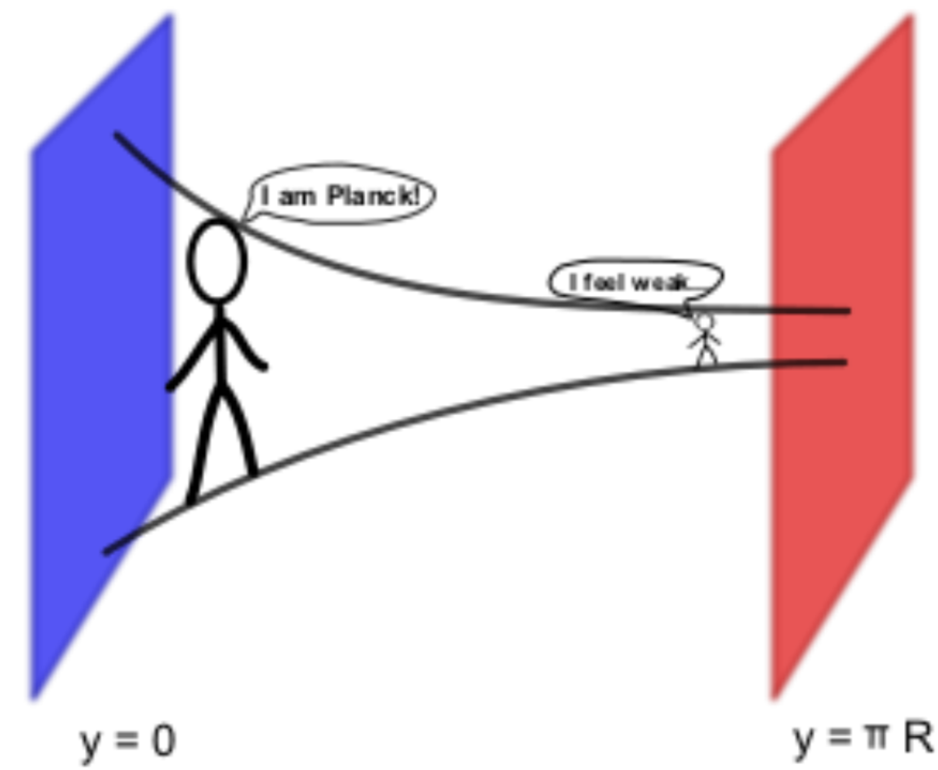
Graviton phenomenology: each Kaluza-Klein mode couples weakly $\propto 1/M_{Pl}$, but there are so many modes that after summing over them, you end up with $1/M_D$ suppression only! One can approximate them by a *continuum* of modes with a cut-off. The graviton tower propagates into the bulk and takes away missing momentum leading to a $pp \rightarrow j + \vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ signature by eg:





warped (or 'Randall-Sundrum' space-times: This is where the metric exponentially warps along the extra dimension y:

$$ds^2 = e^{-|ky|}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dy^2.$$



The metric changes from $y = 0$ to $y = \pi r$ via $\eta_{\mu\nu} \mapsto e^{-k\pi r}\eta_{\mu\nu}$. Here, we set $M_D = M_{Pl}$, but this gets warped down to the weak brane:

$$\Lambda_\pi \sim M_{Pl} e^{-k\pi r} \sim \text{ TeV}$$

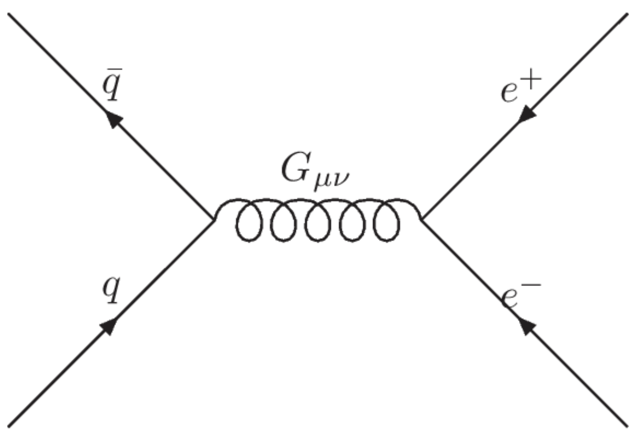
if $r \sim 10/k$. Here, k is of order M_{Pl} and so we have a *small extra dimension*, but the warping explains the smallness of the weak scale. Note that we still have to stabilise the separation between the branes, which can involve extra tuning unless extra physics is added.

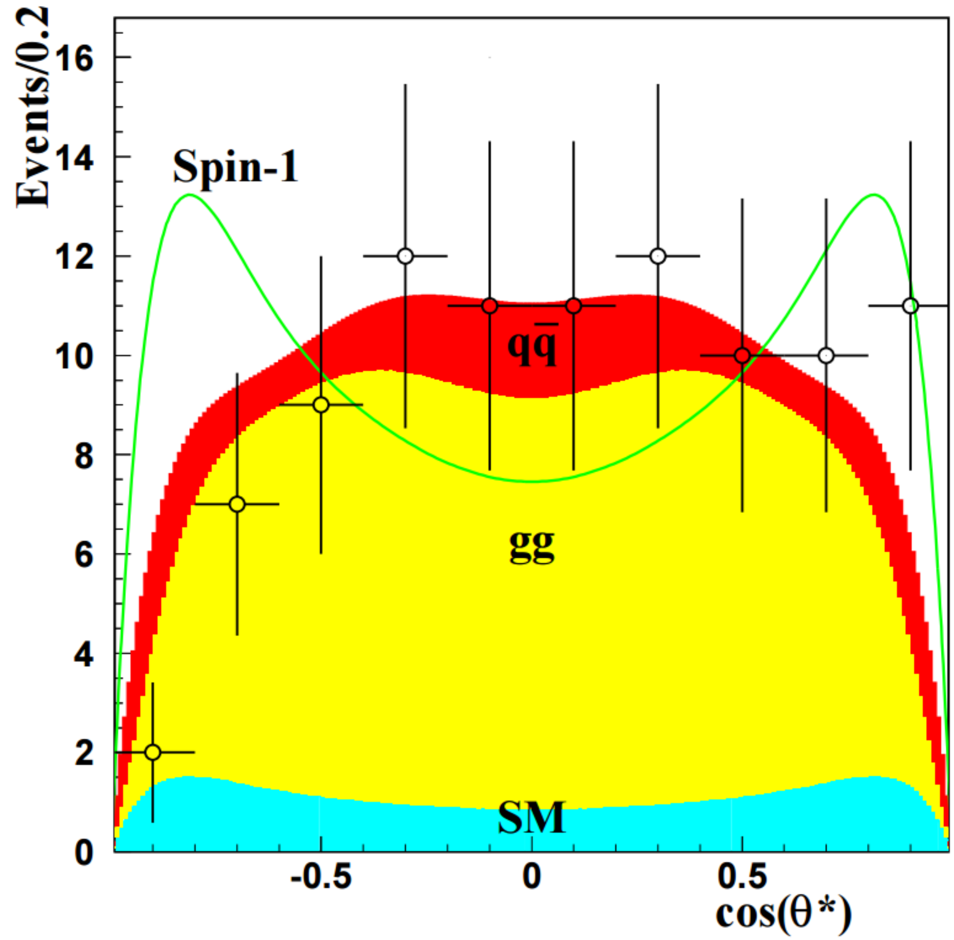
The interaction Lagrangian is

$$\mathcal{L}_I = -G^{\mu\nu} T_{\mu\nu} / \Lambda_\pi$$

, where $T_{\mu\nu}$ is the *stress energy tensor*, containing products of the other Standard Model fields. $\Lambda_\pi \sim \text{ TeV}$, so the interaction leads to weak cross-sections, not gravitationally suppressed ones. Thus, you can produce the resonance: you'll tend to produce the lightest one more often in the LHC. The ratios of masses of higher modes are given by zeros of Bessell functions, so they are not as regular as in large extra dimensions.

Randall-Sundrum phenomenology: one looks for the TeV scale first resonances, which are weakly coupled to Standard Model states. If only gravity travels in the extra dimensions, then it is the 'RS graviton': it has universal coupling to all particles and so can decay into $q\bar{q}$, WW, ZZ, $\gamma\gamma$, gg, l^+l^- or h^0h^0 with similar branching ratios.





Flavour considerations imply that this isn't the end of the story: one requires additional flavour structure, or the model violates flavour bounds from experiment. One common way of doing this is to allow the other particles into the bulk, but have different profiles of fermions with the weak brane, where the Higgs field is localised (proportional to their Yukawa couplings). Then, one can look for the first Kaluza Klein modes of gauge bosons and fermions, too.

Effective Field Theories

At low momenta p^μ, we can model the effects of particles with a much heavier mass $M^2 \gg p^2$ with *effective field theory*. This squeezes a propagator down to a point:

$$\frac{1}{p^2 - M^2} \approx -\frac{1}{M^2},$$

in a fairly model independent way.

Couplings

Thus, for example a W coupling like

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\bar{e}\gamma^\rho(1-\gamma_5)W_\rho\nu_e - \frac{g}{2\sqrt{2}}\bar{\nu_\mu}\gamma^\rho(1-\gamma_5)W_\rho\mu$$

becomes

$$\mathcal{L} \approx -\frac{G_F}{\sqrt{2}}\left(\bar{e}\gamma^\rho(1-\gamma_5)\nu_e\right)\left(\bar{\nu_\mu}\gamma^\rho(1-\gamma_5)W_\rho\mu\right),$$

where $G_F = \sqrt{2}g^2/(8M_W^2)$.