

ESHEP 2018, MARATEA (ITALY)

COSMOLOGY AND DARK MATTER

LECTURE 1

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> Motivations

WHY SHOULD A PARTICLE PHYSICIST CARE ABOUT COSMOLOGY?

- cosmology provides insight on particle physics at **energy scales** impossible to probe on Earth
- cosmology provides alternative/competitive **constraints** on particle physics properties (neutrinos, dark matter ...)
- cosmology provides **motivations/completions** of models for physics beyond the Standard Model (inflation, baryogenesis, dark matter...)

> Outline

- **LECTURE 1:**

The Universe around us. Dynamics. Energy Budget.

The Standard Model of Cosmology:
the 3 pillars (Expansion, Nucleosynthesis, CMB).

- **LECTURE 2:**

Dark Energy.

Dark Matter as a thermal relic. Searches for WIMPs.

- **LECTURE 3:**

Shortcomings of Big Bang cosmology. Inflation. Baryogenesis

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> Warm-up

Natural Units

$$[\text{Energy}] = [\text{Mass}] = [\text{Temperature}] = [\text{Length}]^{-1} = [\text{Time}]^{-1}$$

$$\hbar = c = k_B = 1$$

from which it follows that:

$$1 = \hbar \cdot c \simeq 197.33 \text{ MeV} \cdot \text{fm} = 1.9733 \times 10^{-14} \text{ GeV} \cdot \text{cm}$$

$$\begin{aligned} 1 \text{ GeV}^{-1} &= 1.9733 \times 10^{-14} \text{ cm} = 1.9733 \times 10^{-14} \frac{\text{cm}}{c} \\ &= 6.5822 \times 10^{-25} \text{ s} \end{aligned}$$

the fundamental mass scale of
gravitational interactions

$$M_P = 1.22 \times 10^{19} \text{ GeV}$$

astronomical distances

$$1 \text{ pc} = 3.08 \times 10^{18} \text{ cm}$$

> The 3 Pillars of Standard Cosmology

From observations of the Universe around us:

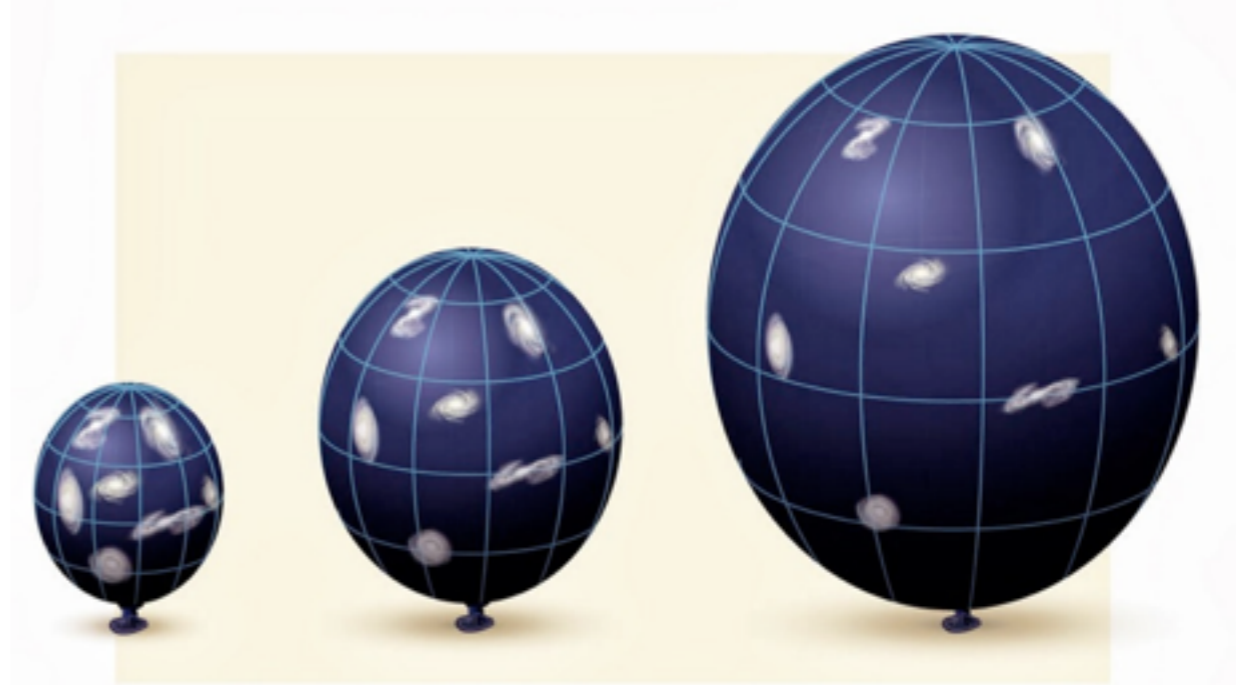
- The Universe is **expanding**
- The abundances of light element are in agreement with **Big Bang Nucleosynthesis**
- The Universe is filled by blackbody radiation (**Cosmic Microwave Background**)

> Expansion

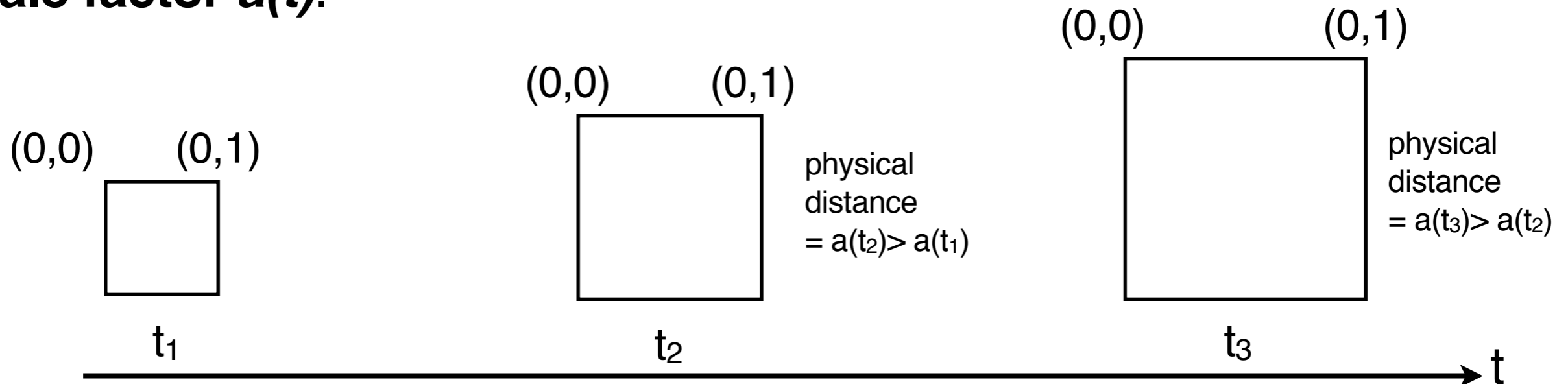
in the past, the relative physical distance between any two points was smaller

distant galaxies are observed with red-shifted spectra

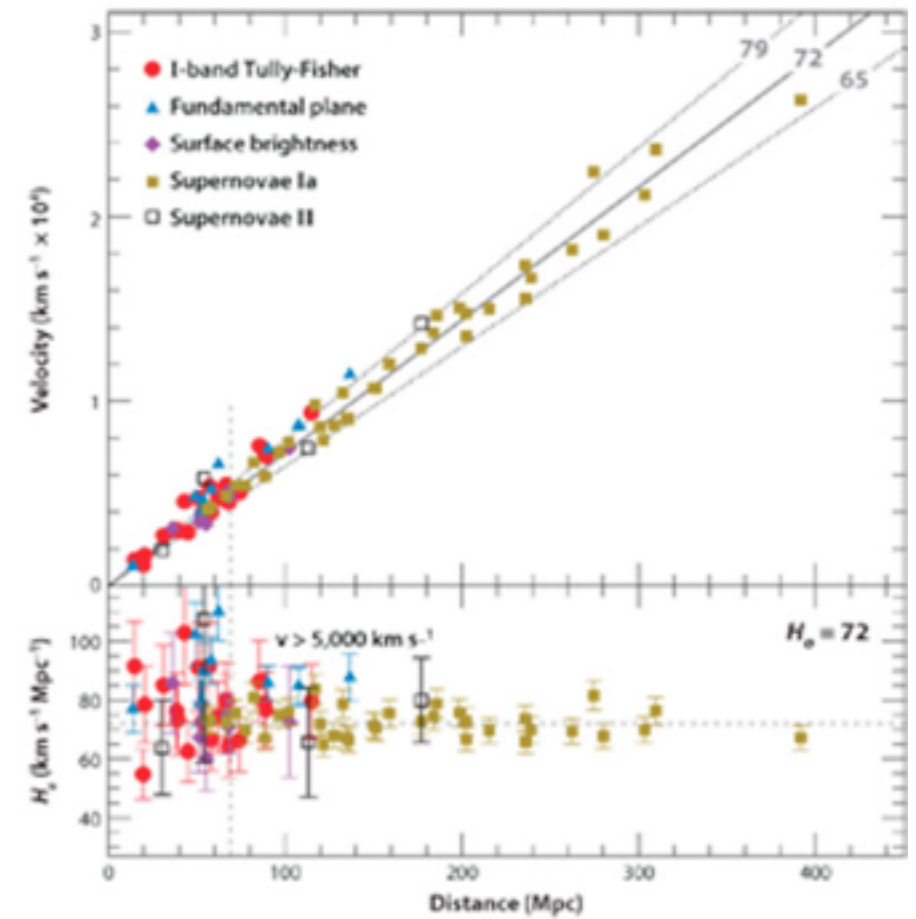
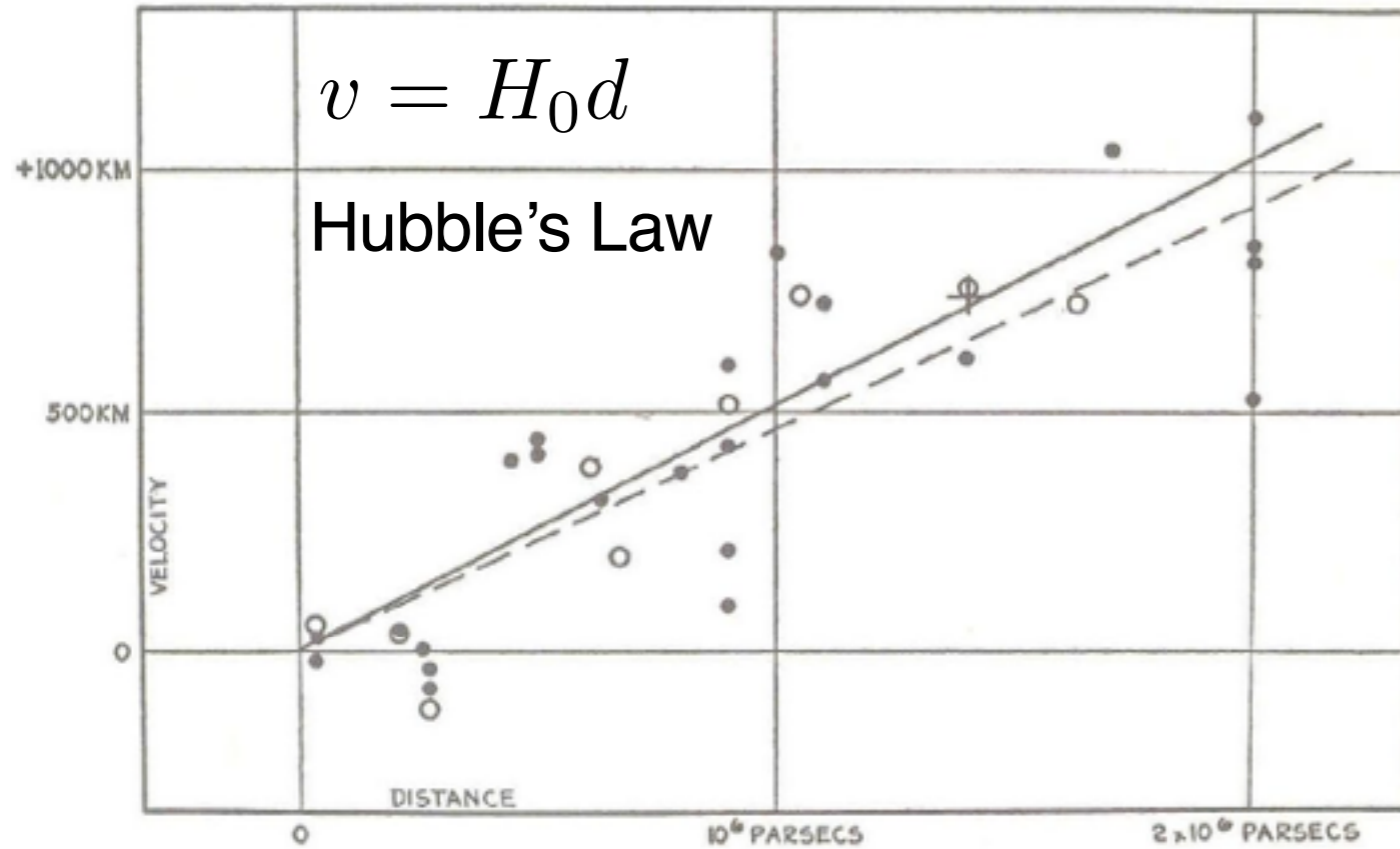
➔ Universe is expanding



in an expanding universe the physical distances between 2 points get larger. they are proportional to a factor measuring the expansion of the universe: the **scale factor $a(t)$** .



> Kinematics



for a photon of given wavelength:
$$v = \frac{\lambda_{\text{observation}} - \lambda_{\text{emission}}}{\lambda_{\text{emission}}} \equiv z$$

Hubble's Law $z = H_0 d$ (for $z \ll 1$)

$$1 + z \equiv \frac{\lambda_{\text{observation}}}{\lambda_{\text{emission}}} = \frac{a(t_0)}{a(t_e)}$$

redshift measures how much the Universe has expanded since the emission.

> Kinematics

$$1 + z \equiv \frac{\lambda_{\text{observation}}}{\lambda_{\text{emission}}} = \frac{a(t_0)}{a(t_e)}$$

Hubble law

Taylor-expand around today t_0 : $\frac{a(t)}{a(t_0)} = 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2 + \dots$

$$H_0 \equiv \left. \frac{\dot{a}}{a} \right|_{t_0} \quad \text{local expansion rate today ("Hubble constant")}$$

$$q_0 \equiv - \left. \frac{\ddot{a}}{a H_0^2} \right|_{t_0} = - \left. \frac{\ddot{a} a}{\dot{a}^2} \right|_{t_0} \quad \text{deceleration parameter (deviation from Hubble law)}$$

Measurements:

$$1/H_0 \simeq 1.4 \times 10^{10} \text{ yrs} \simeq 4.3 \text{ Gpc} \simeq 1.3 \times 10^{26} \text{ m} \quad \begin{array}{l} \text{"Hubble time"} \\ \text{"Hubble length"} \end{array}$$

$$h \equiv \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \simeq 0.67$$

> The Universe Around Us

HOMOGENEITY: the distribution of matter in the Universe looks homegenous (= \sim constant density) on large scales (> 100 Mpc)

(i.e.: 2-point function of galaxies and galaxy clusters $\ll 1/H_0$)

ISOTROPY: if the expansion of the universe were not isotropic, we would observe large temperature anisotropies in the CMB (more later...).

Rotation invariance is around any point of the Universe, so it is isotropic.

Cosmological Principle: the Universe is **homogeneous** and **isotropic**
(no observer is special, no preferred directions)

> FRW

Build a metric for a **homogeneous** and **isotropic** Universe

Friedmann-Robertson-Walker

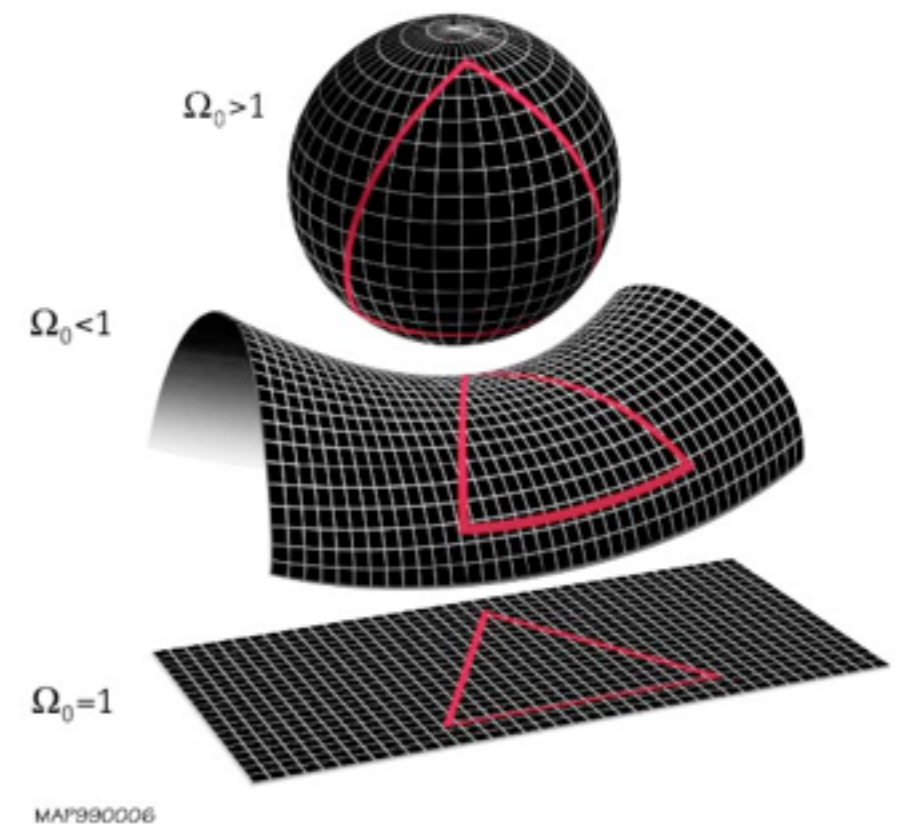
$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

$$k = \begin{cases} +1 & \text{positive spatial curvature} \\ 0 & \text{zero spatial curvature} \\ -1 & \text{negative spatial curvature} \end{cases}$$

scalar curvature of spatial slices

$$|{}^3\mathcal{R}| = \frac{6|k|}{a^2} \equiv \frac{6}{R_{\text{curv}}^2}$$

R_{curv} is a sort of curvature radius of the Universe



> Dynamics

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Einstein's Equations

Energy-momentum

tensor for a perfect fluid: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$

p : pressure

ρ : energy density

in the *comoving frame* (moving with the fluid)

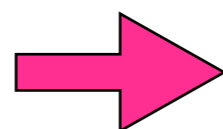
$$u^0 = 1, \vec{u} = 0 \quad T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

0-component of Energy-Momentum conservation ($\nabla_\mu T^{\mu\nu} = 0$)
in expanding Universe

$$\nabla_\mu T^{\mu 0} = 0 \implies \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

← same equation

$dU + pdV = TdS$ 1st law of thermodyn.
entropy cons. $dS=0$



$$d(a^3 \rho) + pd(a^3) = 0$$

> Fluids

$$p = w\rho \quad (\dot{w} = 0)$$

$$\dot{\rho} + 3(1+w)\frac{\dot{a}}{a}\rho = 0 \quad \Rightarrow \quad \frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \quad \Rightarrow \quad \rho \propto a^{-3(1+w)}$$

radiation

$$w = 1/3 \quad \Rightarrow \quad \rho \propto a^{-4} \quad \text{(radiation domination - RD)}$$

dust

$$w = 0 \quad \Rightarrow \quad \rho \propto a^{-3} \quad \text{(matter domination - MD)}$$

vacuum energy

$$w = -1 \quad \Rightarrow \quad \rho \propto \text{const} \quad \text{(vacuum energy domination)}$$

- rest-mass energy must be constant
- volume $\sim a^3$ \Rightarrow energy density of matter $\sim a^{-3}$

for radiation, energy density has a further $1/a$ due to redshift so $\sim a^{-4}$

> Friedmann Equation

- (00) component of the Einstein Eq. for FRW:

$$H^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad \text{Friedmann Equation}$$

Hubble parameter $H \equiv \dot{a}/a$ is not a constant

- (ii) component of the Einstein Eq. $\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3}$

does not give anything new wrt Friedmann equation + energy-mom conservation

$$H^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}} - \frac{k}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\rho_{\Lambda} \equiv \frac{\Lambda}{8\pi G_N}$$

$$\rho_{\text{tot}} \equiv \rho + \rho_{\Lambda}$$

this system of eqns encodes the evolution of the Universe and its constituents

> Cosmological Dynamics

what is the evolution of the scale factor?

$$H^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}} - \frac{k}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

critical density $\rho_c \equiv \frac{3H^2}{8\pi G_N}$

today $\rho_c \simeq 1.88 \times 10^{-29} h^2 \text{ g cm}^{-3}$
 $\simeq 1.05 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$.

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$

$$\Omega_k \equiv -\frac{k}{a^2 H^2}$$

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3H^2}$$

$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$$

$$\left\{ \begin{array}{l} \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\frac{a}{a_0}\right)^{-3(1+w)} \\ y \equiv a/a_0 \end{array} \right. \xrightarrow{\text{(exercise)}} \dot{y} = H_0 y^{-\frac{3(1+w)}{2} + 1} \xrightarrow{} y^{\frac{1}{2} + \frac{3}{2}w} dy = H_0 dt$$

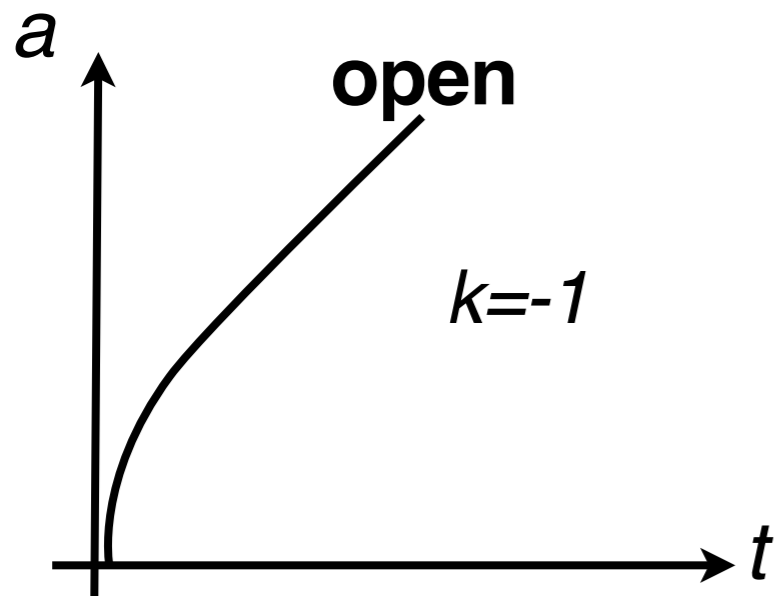
$$a(t) \propto t^{\frac{2}{3(1+w)}} \quad (w \neq -1) \xrightarrow{} \text{dust } (w=0): \quad a(t) \propto t^{2/3}$$

$$a(t) \propto e^{H_0 t} \quad (w = -1) \text{ vacuum energy} \xrightarrow{} \text{radiation } (w=1/3): \quad a(t) \propto t^{1/2}$$

$$\text{dust } (w=0): \quad a(t) \propto t^{2/3}$$

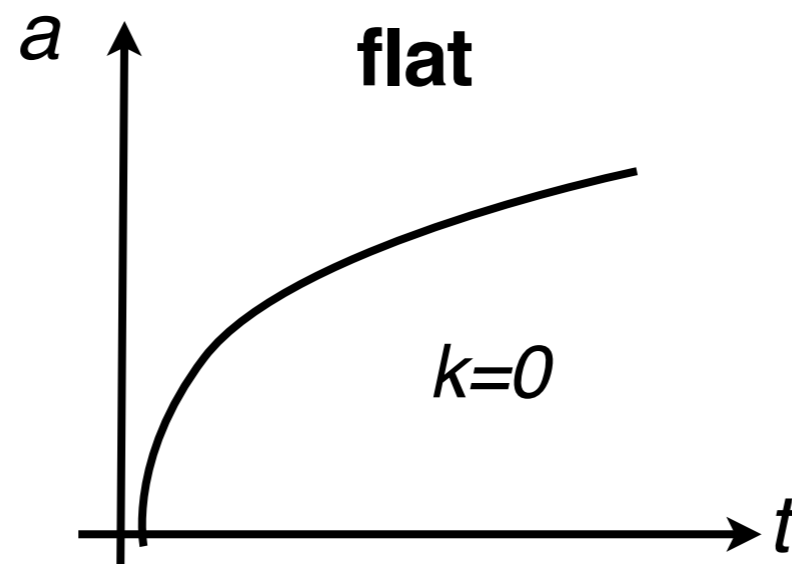
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> Cosmological Dynamics

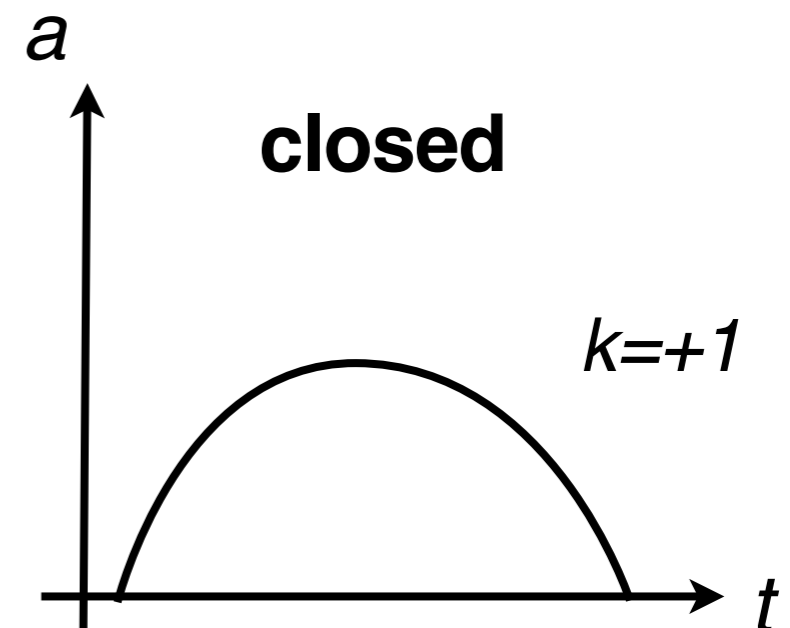


$$H^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}} - \frac{k}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$



$$R_{\text{curv}} = \frac{1}{H \sqrt{|\Omega_k|}}$$



$\rho_{\text{tot}} = \rho_c \leftrightarrow k=0 \leftrightarrow \mathbf{FLAT}$ universe

> Energy Budget

Picture of the Universe today (from PLANCK 2015)

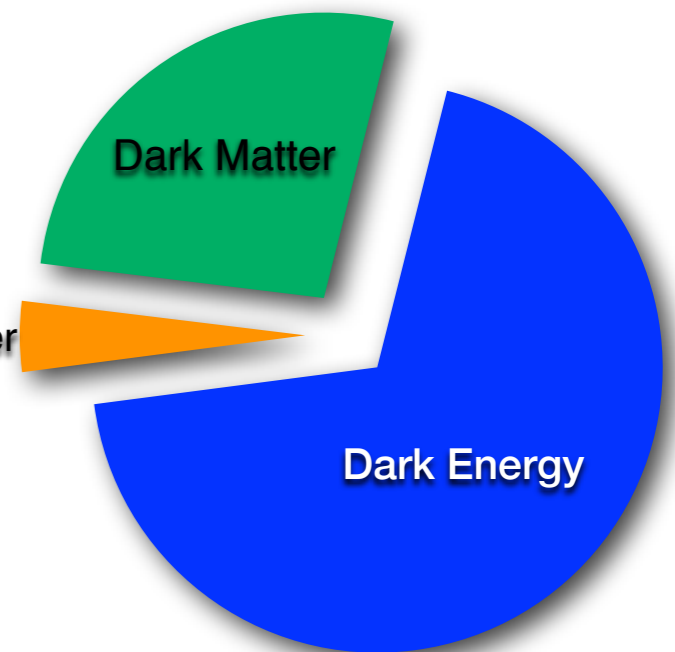
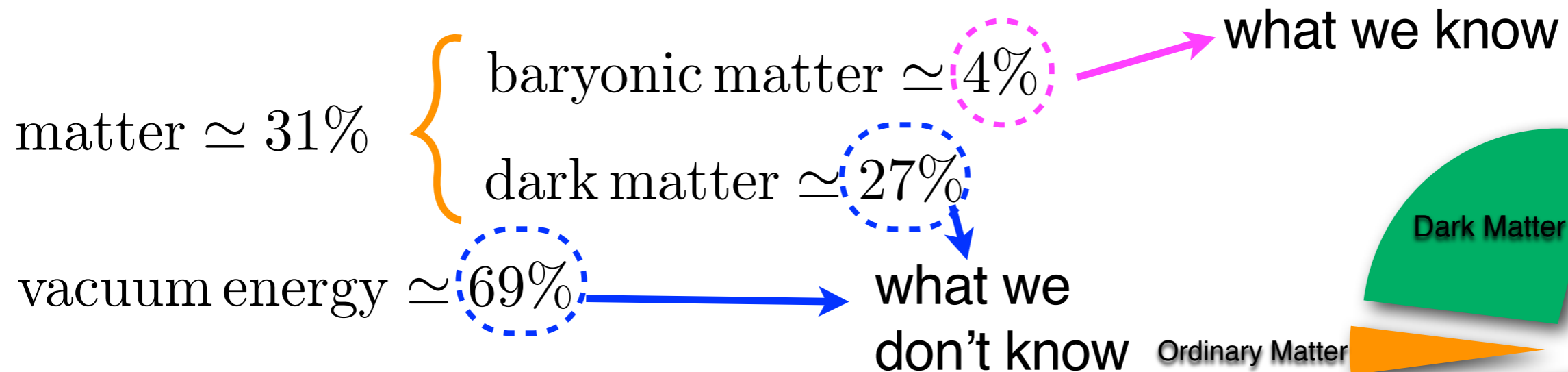
$$h = 0.6774 \pm 0.0046$$

$$\Omega_m h^2 = 0.14170 \pm 0.00097 \quad \left\{ \begin{array}{l} \Omega_b h^2 = 0.02230 \pm 0.00014 \\ \Omega_{\text{CDM}} h^2 = 0.1188 \pm 0.0010 \end{array} \right.$$

$$\Omega_k = 0.0008^{+0.0040}_{-0.0039}$$

$$\Omega_\Lambda = 0.6911 \pm 0.0062$$

curvature $\simeq 0\%$ \longrightarrow FLAT universe



Energy budget of the Universe

> Age of the Universe

almost irrelevant contribution from very early (uncertain) times:

we can compute the age of the universe from when it started RD or MD eras

$$\frac{da}{dt} = Ha \quad H^2 = H_0^2 \left[\Omega_r^0 \left(\frac{a_0}{a} \right)^4 + \Omega_m^0 \left(\frac{a_0}{a} \right)^3 + \Omega_k^0 \left(\frac{a_0}{a} \right)^2 + \Omega_\Lambda^0 \right]$$

$$\longrightarrow dt = \frac{da}{aH_0 \left[\Omega_r^0 \left(\frac{a_0}{a} \right)^4 + \Omega_m^0 \left(\frac{a_0}{a} \right)^3 + \Omega_k^0 \left(\frac{a_0}{a} \right)^2 + \Omega_\Lambda^0 \right]^{1/2}}$$

Age of the Universe estimate: $t \sim 13$ Gyrs

MD universe

$$\Omega_r^0 = \Omega_k^0 = \Omega_\Lambda^0 = 0, \Omega_m^0 = 1 \xrightarrow{\text{exercise}} t_0 = \frac{2}{3} \frac{1}{H_0} \simeq 9 \times 10^9 \text{ yrs} \quad \text{too young!}$$

$$\Omega_r^0 = 0, \Omega_k^0 = 0, \Omega_m^0 = 1 - \Omega_\Lambda^0 = 0.3 \xrightarrow{\text{exercise}} t = \frac{2}{3H_0 \sqrt{\Omega_\Lambda^0}} \sinh^{-1} \sqrt{\frac{\Omega_\Lambda^0}{1 - \Omega_\Lambda^0} \left(\frac{a}{a_0} \right)^3}$$

$$\longrightarrow t_0 = \frac{2}{3H_0 \sqrt{\Omega_\Lambda^0}} \sinh^{-1} \sqrt{\frac{\Omega_\Lambda^0}{1 - \Omega_\Lambda^0}} \simeq 0.96 \frac{1}{H_0} \longrightarrow t_0 \simeq 1.3 \times 10^{10} \text{ yrs}$$

OK! Λ makes the universe older

> Distance-Redshift Relation

Friedmann:
$$H^2 = H_0^2 \left[\Omega_r^0 \left(\frac{a_0}{a} \right)^4 + \Omega_m^0 \left(\frac{a_0}{a} \right)^3 + \Omega_k^0 \left(\frac{a_0}{a} \right)^2 + \Omega_\Lambda^0 \right]$$

$a_0/a = 1 + z$

$\longrightarrow H^2 = H_0^2 \left[\Omega_r^0 (1 + z)^4 + \Omega_m^0 (1 + z)^3 + \Omega_k^0 (1 + z)^2 + \Omega_\Lambda^0 \right]$

light $ds = 0 \longrightarrow dr = dt/a$

$\frac{da}{dt} = Ha \longrightarrow dr = \frac{da}{a^2 H}$

$\longrightarrow r(a) = \frac{1}{H_0} \int_a^{a_0} \frac{da'}{a'^2 \left[\Omega_r^0 \left(\frac{a_0}{a'} \right)^4 + \Omega_m^0 \left(\frac{a_0}{a'} \right)^3 + \Omega_k^0 \left(\frac{a_0}{a'} \right)^2 + \Omega_\Lambda^0 \right]^{1/2}}$

redshift-distance relation $r(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\left[\Omega_r^0 (1 + z')^4 + \Omega_m^0 (1 + z')^3 + \Omega_k^0 (1 + z')^2 + \Omega_\Lambda^0 \right]^{1/2}}$

Luminosity distance: $\text{Flux} = \frac{\text{Luminosity}}{4\pi r(z)^2 (1+z)^2} \equiv \frac{\text{Luminosity}}{4\pi d_L^2} \longrightarrow d_L = (1+z)r(z)$

redshift of energy+dilated time

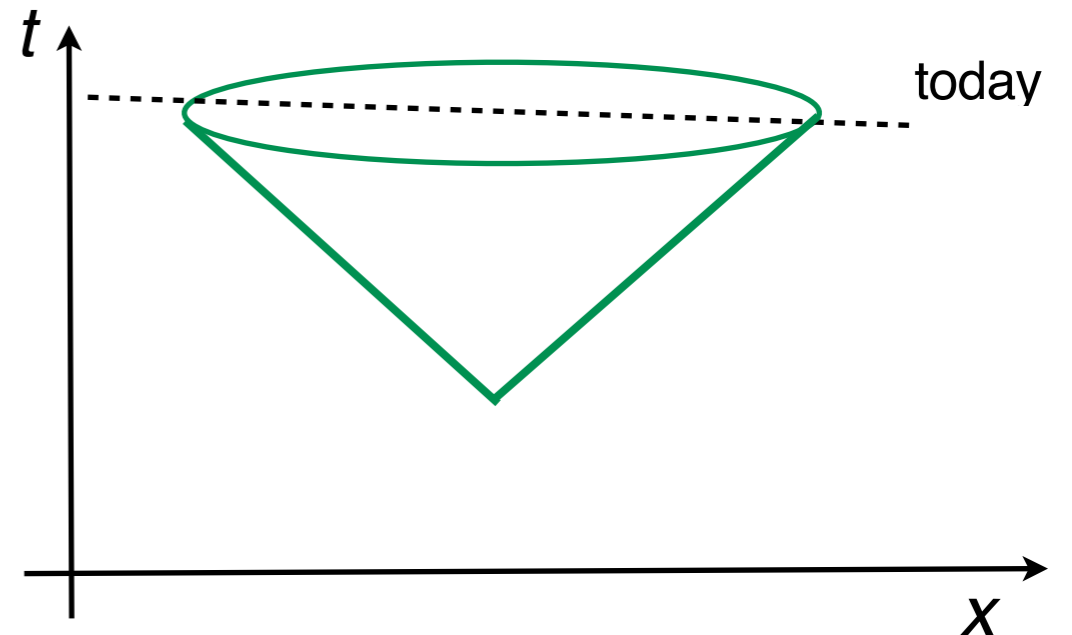
depends on Universe content

> Particle Horizon

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

the boundary between the visible universe and the part of the universe from which light signals have not reached us

measure the portion of the Universe in causal contact



$$d_H(t) = \frac{a(t)}{H_0} \int_0^{a(t)} \frac{da'}{a'^2 \left[\Omega_r^0 \left(\frac{a_0}{a}\right)^4 + \Omega_m^0 \left(\frac{a_0}{a}\right)^3 + \Omega_k^0 \left(\frac{a_0}{a}\right)^2 + \Omega_\Lambda^0 \right]^{1/2}}$$

dust

$$d_H(t) = 3t = \frac{2}{H(t)} \propto a^{3/2}$$

radiation

$$d_H(t) = 2t = \frac{1}{H(t)} \propto a^2$$

vacuum energy

$$d_H(t) = \infty$$

(no horizon!)

> Equilibrium Thermodynamics

equilibrium number density and energy density

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3 p$$

d.o.f.
distrib. function

$$f(\vec{p}) = \left[e^{E/T} \pm 1 \right]^{-1} \quad \begin{array}{l} \text{FD} \\ \text{BE} \end{array}$$

in extreme cases, these integrals can be solved analytically (**do it!**)

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3 p$$

- for NON-RELATIVISTIC species ($T \ll m$)

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \quad \rho = n \cdot m$$

- for ULTRA-RELATIVISTIC species ($T \gg m$)

$$n_{\text{BE}} = \frac{\zeta(3)}{\pi^2} g T^3 \quad \rho_{\text{BE}} = \frac{\pi^2}{30} g T^4$$

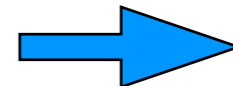
$$n_{\text{FD}} = (3/4) n_{\text{BE}} \quad \rho_{\text{FD}} = (7/8) \rho_{\text{BE}}$$

total number of massless d.o.f. at temperature T
(species "i" has thermal distr. of temperature T_i)

$$g_* = \sum_{i \in \text{bosons}} g_i \left(\frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{j \in \text{fermions}} g_j \left(\frac{T_j}{T} \right)^4$$

Hubble rate
in RD era:

$$H^2 = \frac{8\pi G_N}{3} \rho_{\text{rel}} = \frac{8\pi G_N}{3} \frac{\pi^3}{30} g_* T^4$$



$$H \simeq 1.66 \sqrt{g_*} \frac{T^2}{M_P}$$

> Temperature-Expansion Relation

$$dU + pdV = TdS \quad \text{1st law of thermodynamics}$$

entropy density

$$s(T) \equiv \frac{S(V, T)}{V} = \frac{\rho(T) + p(t)}{T} = \frac{4}{3} \frac{\rho(T)}{T}$$

radiation
domination $w=1/3$

$$\rho(T) \propto T^4 \quad \longrightarrow \quad s \propto T^3$$

conservation of entropy in comoving volume in thermal equilibrium:

$$S(V, T) = s(T)V = \text{const.} \implies T^3 a^3 = \text{const.}$$

$$\implies T \propto 1/a$$

> Exercises

Observation of a high-redshift object: $z=10$

- how long ago was the light we observe today emitted from there?
- what is the present distance of that object?
- what was the distance of the object when the light was emitted?

> The Universe Around Us

SUMMARY:

- expanding $1/H_0 \simeq 1.4 \times 10^{10}$ yrs $\simeq 4.3$ Gpc $\simeq 1.3 \times 10^{26}$ m

- homogeneous

- isotropic

- flat $\rho_c \simeq 1.88 \times 10^{-29} h^2 \text{ g cm}^{-3}$
 $\simeq 1.05 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$.

curvature $\simeq 0\%$

Energy budget

matter $\simeq 31\%$ $\left\{ \begin{array}{l} \text{baryonic matter} \simeq 4\% \\ \text{dark matter} \simeq 27\% \end{array} \right.$

vacuum energy $\simeq 69\%$



**KEEP
CALM
AND
TAKE A DEEP
BREATH**

> Big Bang Nucleosynthesis

in hot Universe, no atoms/nuclei because of high energy photons around

as Universe cools down below binding energies of nuclei,
light elements begin to form

T ~ MeV

- all elements heavier than ${}^7\text{Li}$ are produced in stars or astrophysical processes
- lighter elements (${}^2\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$) have a **cosmological** origin.
- predictions depending on the baryon densities at $T \sim \text{MeV}$

> Big Bang Nucleosynthesis

predictions for abundances (spanning **9 orders of magnitude**) well fitted by a **single** parameter:

baryon-to-photon ratio $\eta = n_b/n_\gamma \sim 10^{-10}$

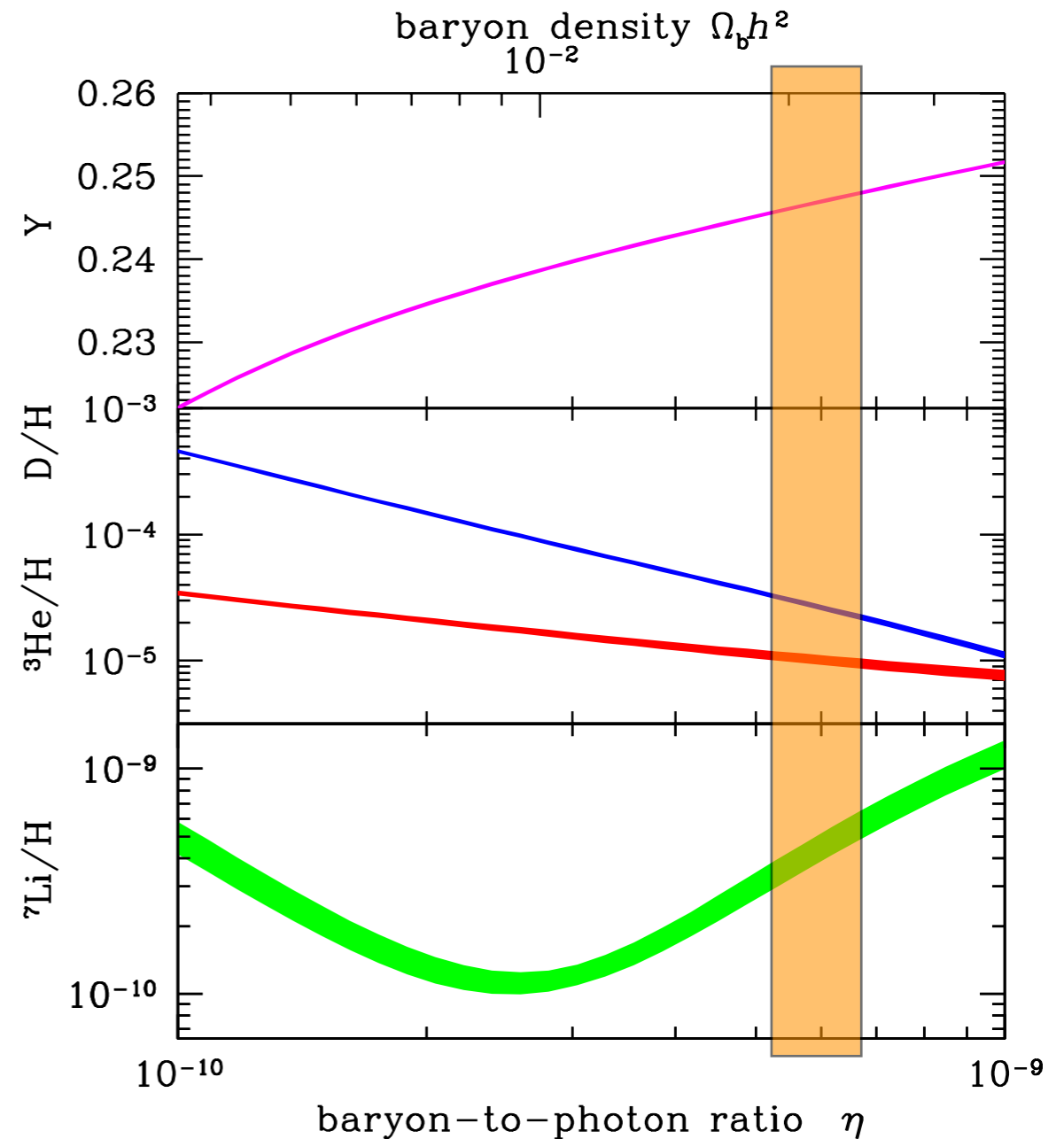
Great Success of Standard Cosmology!

scaling a^{-3} and n_γ are known, so measure light element abundances

→ measure η

→ measure Ω_b today

BBN provides a compelling argument for non-baryonic DM



[arXiv:1505.01076]

> Big Bang Nucleosynthesis

experimental measurements of primordial ^4He fraction in metal-poor stars and gas clouds:

$$Y(^4\text{He}) \equiv \frac{4n_{\text{He}}}{n_n + n_p} \simeq 0.25$$

Need to compute neutron-to-proton ratio n_n/n_p and Helium-4 abundance n_{He} .

- at very early times ($t \ll 1$ s, $T \gg 1$ MeV), the reactions

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

$$\nu_e + n \leftrightarrow p + e^-$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$
 are in equilibrium so $n_n = n_p$.

- at $T \sim 1$ MeV $\ll m_n, m_p$, mass difference starts to be important

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}$$

$$g_n = g_p = 2$$

$$\longrightarrow \frac{n_n}{n_p} = e^{-(m_n - m_p)/T} = e^{-(1.29 \text{ MeV})/T}$$

- equilibrium is broken by expansion. Scattering rate $<$ expansion rate, so neutrons are not created or destroyed by scatterings. Neutron abundance gets "frozen in".

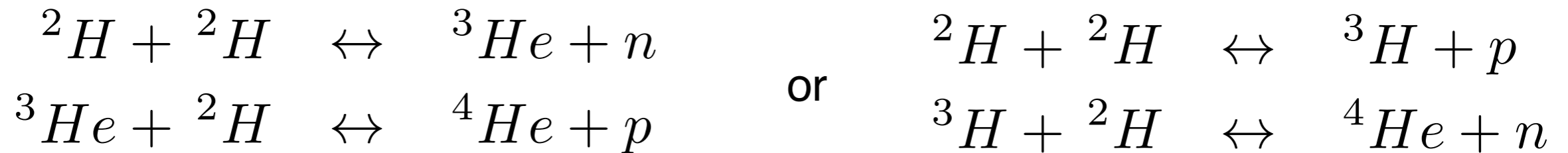
$$\Gamma(\nu_e + n \leftrightarrow p + e^-) \sim 2.1 \left(\frac{T}{1 \text{ MeV}} \right)^5 \text{ s}^{-1} < H \simeq 6 T^2 / M_P \longrightarrow T \lesssim 0.8 \text{ MeV}$$

At $T = 0.8$ MeV $n_n/n_p \simeq e^{-1.29/0.8} \simeq 0.2$. After that, some n decay, and at the end of BBN ($T \sim 0.01$ MeV) there are slightly fewer neutrons $n_n/n_p \sim 0.14$.

> Big Bang Nucleosynthesis

Now we need to find n_{He} .

Helium nuclei are produced by **deuterium ^2H** by the chains



Deuterium is produced by $p + n \leftrightarrow {}^2\text{H} + \gamma$ which is fast for $T > 0.1$ MeV.
Photo-dissociation prevents deuterium formation.

For $T < 0.1$ MeV, deuterium is produced and feeds reactions for He production.

Most neutrons are consumed to make Helium-4 nuclei, so $n_{\text{He}} \sim n_n/2$.

$$n_n/n_p \simeq 0.14 \longrightarrow Y({}^4\text{He}) = \frac{4(n_n/2)}{n_n + n_p} = \frac{2n_n/n_p}{1 + n_n/n_p} \simeq 0.25!$$

Impressive success of BBN is the earliest and most stringent test of standard cosmology!

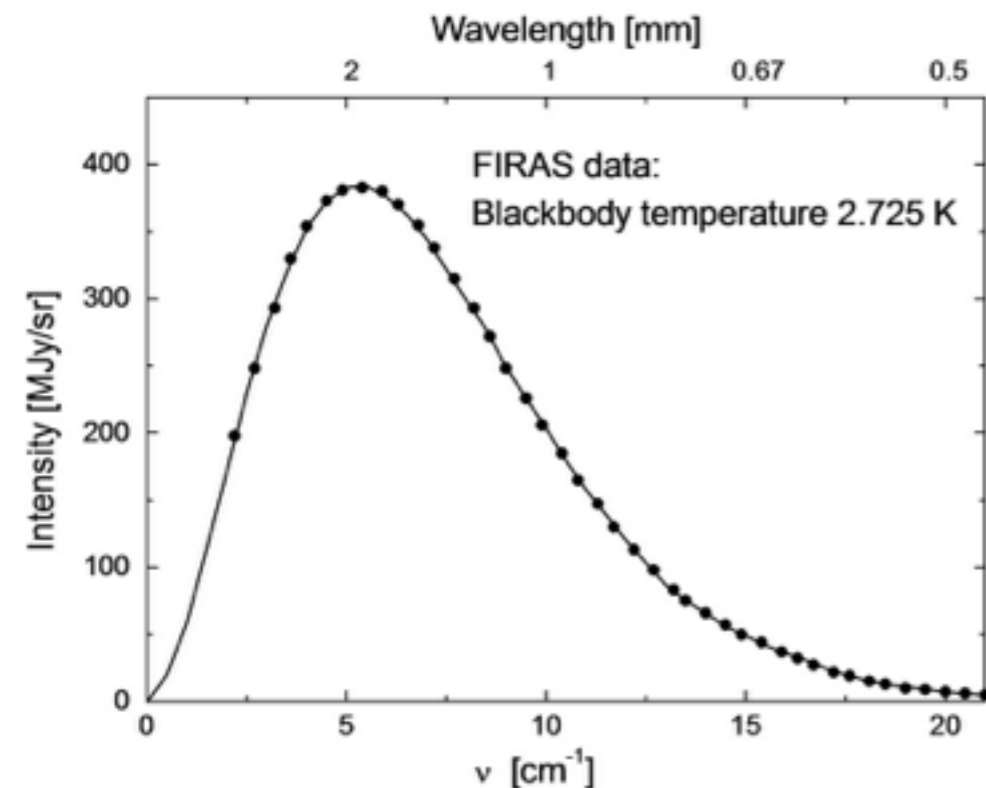
> Cosmic Microwave Background

1965: Arno Penzias and Robert Wilson published a paper where they admitted to have *failed* to eliminate a background *noise* coming from all directions (corresponding to a residual $T=3.5\pm 1.0$ K).

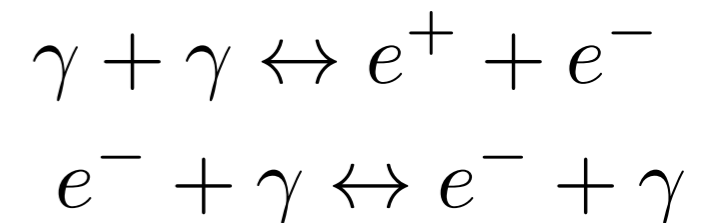
1975: Penzias & Wilson shared Nobel prize in physics.



>> WHAT WAS THAT NOISE? <<



Very early Universe ($T > m_e$):
EM radiation kept in thermal equilibrium by pair-productions and Compton scatterings



photons were continuously created/absorbed or annihilated/emitted



The Universe was an almost perfect
BLACK-BODY

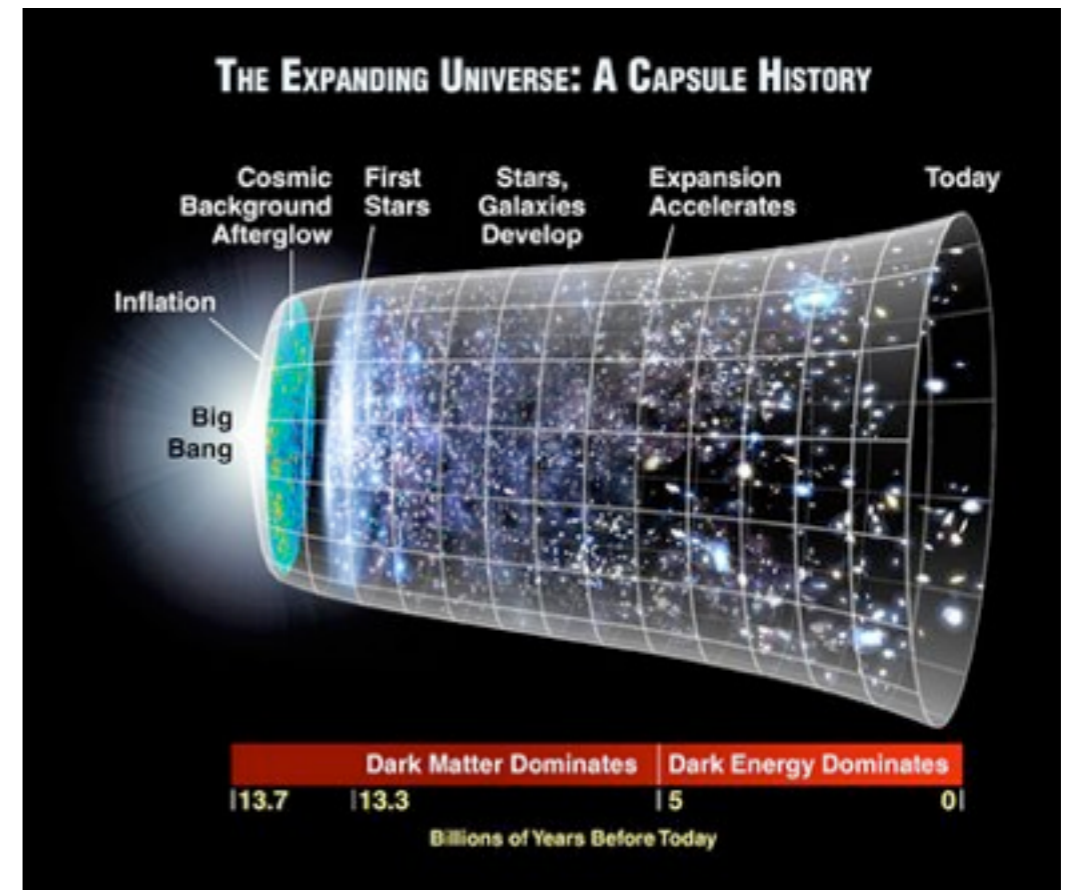
> Cosmic Microwave Background

electrons linked to protons by H production $p + e^- \leftrightarrow H + \gamma$

when this went out of equilibrium (LAST SCATTERING), photons “decoupled” from matter

then photons propagated un-scattered until today

CMB photons *last scattered* off electrons at $z \sim z_{dec}$ and then travelled freely.



They cooled down with expansion to $T_0 = 2.7 \text{ K} = 2.3 \times 10^{-4} \text{ eV}$

The CMB is a residual EM radiation from the Big Bang.

Snapshot of the Universe when it was a baby (only 300,000 yrs old)

The most powerful probe of the early Universe

> Cosmic Microwave Background

photon energy distribution (Planck)

$$n(\omega, T)d\omega = \frac{1}{c^3} \frac{1}{e^{\hbar\omega/(k_B T)} - 1} \frac{2d^3\omega}{(2\pi)^3} = \frac{1}{\pi^2 c^3} \frac{\omega^2 d\omega}{e^{\hbar\omega/(k_B T)} - 1}$$

photons polarizations

differential energy spectrum (black-body)

$$u(\omega, T)d\omega = (\hbar\omega)n(\omega, T)d\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\hbar\omega/(k_B T)} - 1}$$

Total energy density

$$\rho_\gamma = \int_0^\infty u(\omega, T)d\omega = \frac{\hbar}{\pi^2 c^3} \left(\frac{k_B T}{\hbar} \right)^4 \int_0^\infty \frac{\xi^3 d\xi}{e^\xi - 1}$$

$\pi^4/15$

$$\rho_\gamma = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} T^4 \equiv \sigma T^4 \quad (\text{Stefan-Boltzmann relation})$$

$\sigma = 4.72 \times 10^{-3} \text{ eV cm}^{-3} \text{ K}^{-4}$

With $T=2.73 \text{ K}$:

$$\rho_\gamma \simeq 0.26 \text{ eV cm}^{-3} \longrightarrow \Omega_r^0 h^2 = \frac{\rho_\gamma}{\rho_c/h^2} \simeq 4 \times 10^{-5}$$

Present radiation energy density is **negligible!**

> Cosmic Microwave Background

let's compute the time when CMB formed (the time of decoupling z_{dec})

$$p + e^- \leftrightarrow H + \gamma \quad \left\{ \begin{array}{l} \mu_p + \mu_e = \mu_H \quad \text{chemical equilibrium} \\ n_p = n_e \quad \text{charge neutrality} \\ n_p + n_H = n_B = \eta_B n_\gamma \quad \text{tot. number of baryons} \quad (n_\gamma \simeq (1/4)T^3) \end{array} \right.$$

free electron fraction (ionization fraction): $X_e \equiv \frac{n_e}{n_p + n_H} = \frac{n_e}{n_e + n_H}$

$$m_i \gg T \quad n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{(\mu_i - m_i)/T} \quad g_p = g_e = 2, g_H = 4$$

$$\begin{aligned} \frac{X_e^2}{1 - X_e} &= \frac{n_e + n_H}{n_H} \frac{n_e n_p}{(n_e + n_H)^2} = \frac{1}{\eta_B n_\gamma} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{\mu_p + \mu_e - \mu_H} e^{-B/T} = \dots \\ &= \frac{1}{4\eta_B} \left(\frac{m_e}{T} \right)^{3/2} e^{-B/T} \quad (B \equiv m_p + m_e - m_H \simeq 13.6 \text{ eV}) \end{aligned}$$

$$B \lesssim T \implies \frac{X_e^2}{1 - X_e} \simeq 10^9 (m_e/T)^{3/2} \simeq 10^{15} \implies X_e \simeq 1 \quad (\text{all Hydrogen ionized})$$

$$B > T \implies X_e \ll 1 \implies X_e \simeq \left[\frac{1}{4\eta_B} \left(\frac{m_e}{T} \right)^{3/2} e^{-B/T} \right]^{1/2}$$

> Cosmic Microwave Background

Decoupling occurs when rate for photon-electron scattering < expansion rate

$$\Gamma \simeq H \quad \Gamma \simeq n_e \sigma_T$$

$$n_e \sigma_T \simeq H$$

Thomson scattering cross section

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 \simeq 6.65 \times 10^{-25} \text{ cm}^2$$

assume matter-dominated universe

$$H \simeq H_0 a^{-3/2} = H_0 (T/T_0)^{3/2}$$

$$n_e = X_e n_B$$

$$n_B = \frac{\Omega_b^0 \rho_c}{m_p a^3} = \frac{\Omega_b^0 \rho_c}{m_p} \left(\frac{T}{T_0} \right)^3 \simeq 2.2 \times 10^{-7} \text{ cm}^{-3} \left(\frac{T}{T_0} \right)^3$$

$$\longrightarrow n_e \sigma_T \simeq X_e 1.5 \times 10^{-31} \text{ cm}^{-1} \left(\frac{T}{T_0} \right)^3$$

$$\longrightarrow \left(\frac{T}{T_0} \right)^{3/2} \simeq \frac{1}{2 \times 10^{-3} X_e} \xrightarrow{\text{solve implicit eq.}} T \simeq 1000 T_0 \simeq 0.2 \text{ eV}$$

$$\longrightarrow 1 + z_{\text{dec}} = T/T_0 \simeq 1000 \longrightarrow t_{\text{dec}} = (2/3) H_0^{-1} (1 + z_{\text{dec}})^{-3/2} \simeq 300,000 \text{ yrs}$$

photons stop scattering off electrons at $z \sim z_{\text{dec}}$ but electrons keep scatter off photons until much later. WHY? there are many more photons than baryons!

> CMB Anisotropies

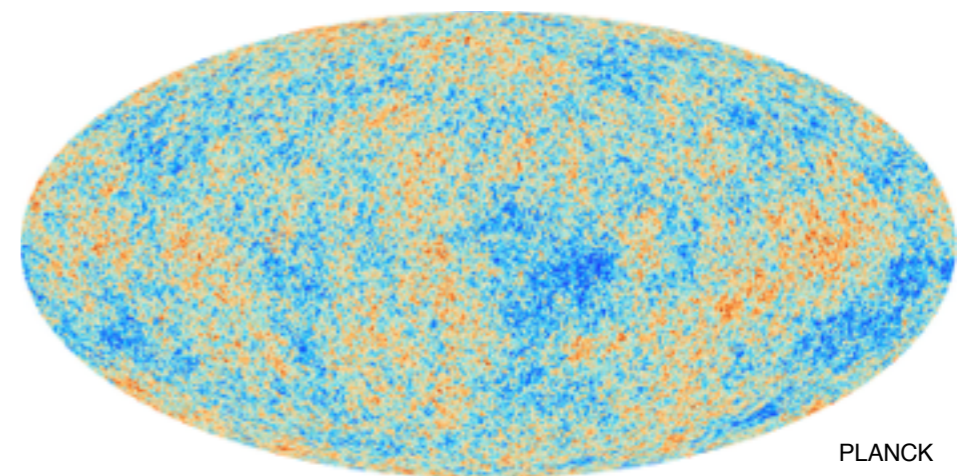
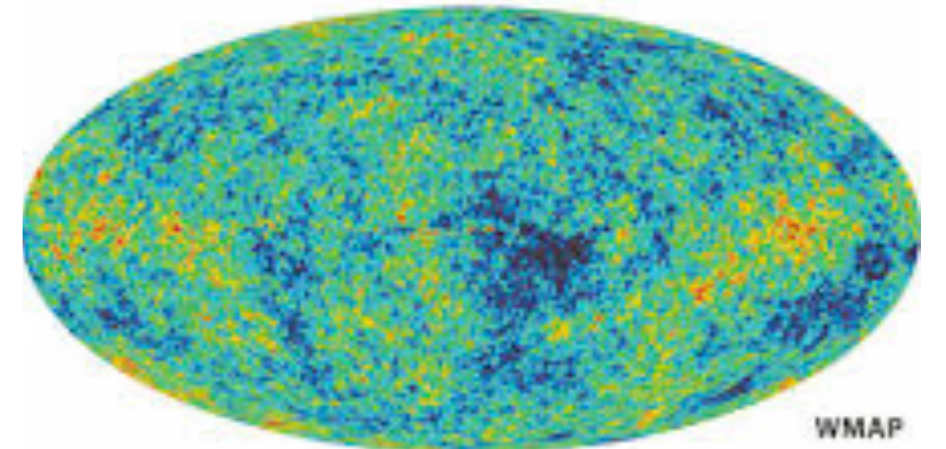
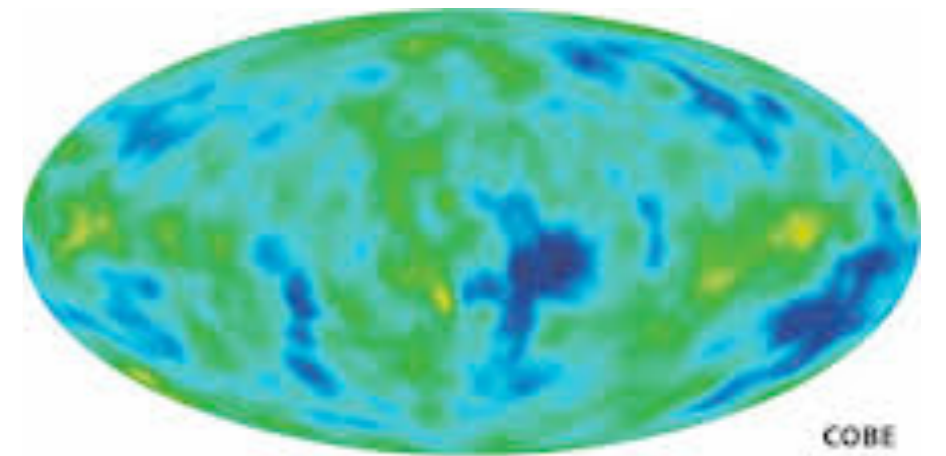
CMB Radiation is not perfectly isotropic

temperature anisotropies

$$\frac{\Delta T}{T} \simeq 10^{-5}$$

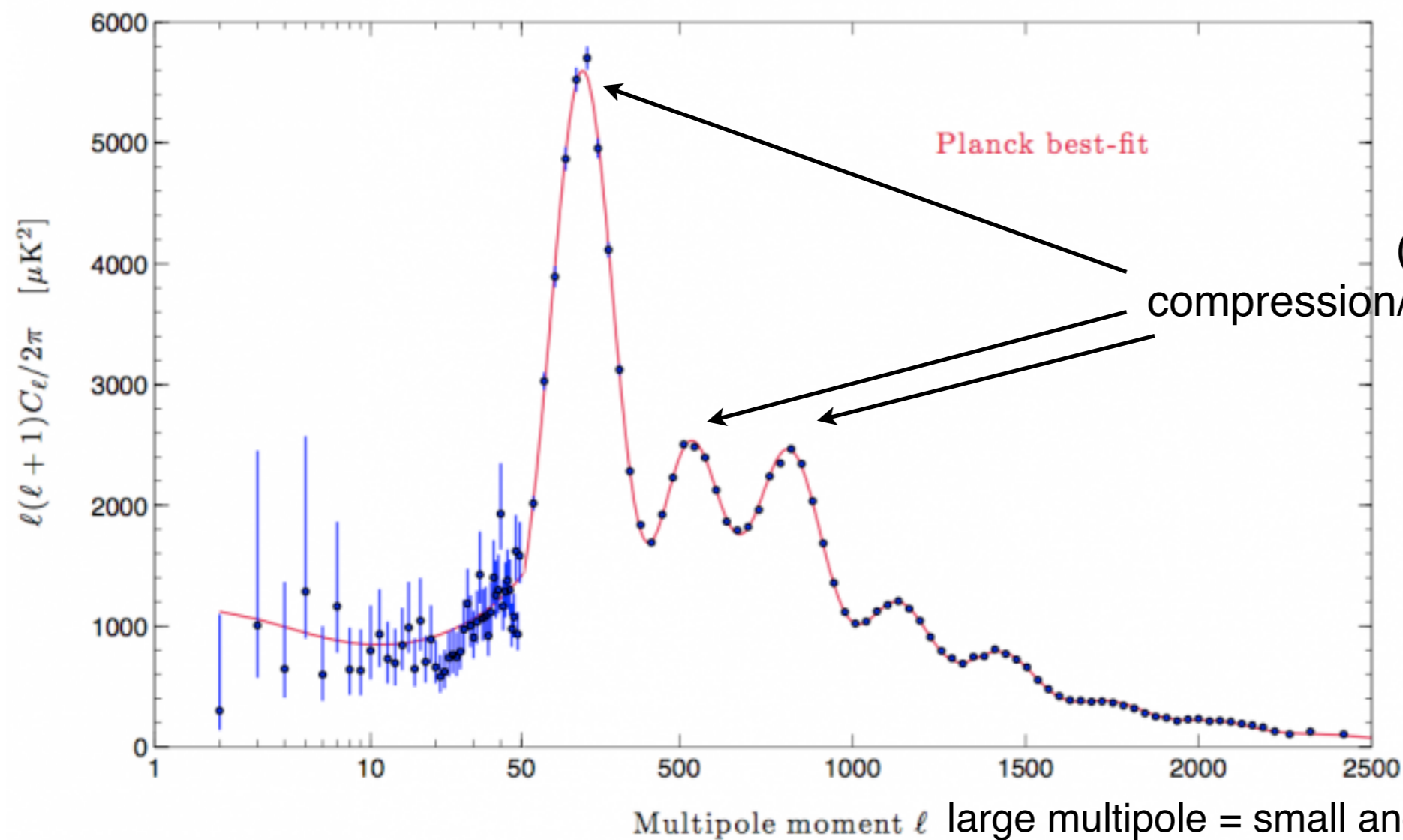
A lot of information from temperature maps.

Two-point correlation functions crucially depend on cosmological parameters (H_0 , Ω_{tot} , Ω_B etc).



> CMB Anisotropies

snapshot of the oscillating photon field at $z \sim 1100$



(acoustic) peaks:
compression/rarefaction of the baryon fluid

competing forces:
- gravity (attractive)
- photon pressure (repulsive)

fit with cosmological model provides
energy content of the Universe

$$\Omega_{tot} \sim 1.0, \quad \Omega_{matter} \sim 0.3, \quad \Omega_{radiation} \sim 0.0 \quad \longrightarrow \quad \Omega_{\Lambda} \sim 0.7$$

“ Λ CDM MODEL”

Great triumph of cosmology!

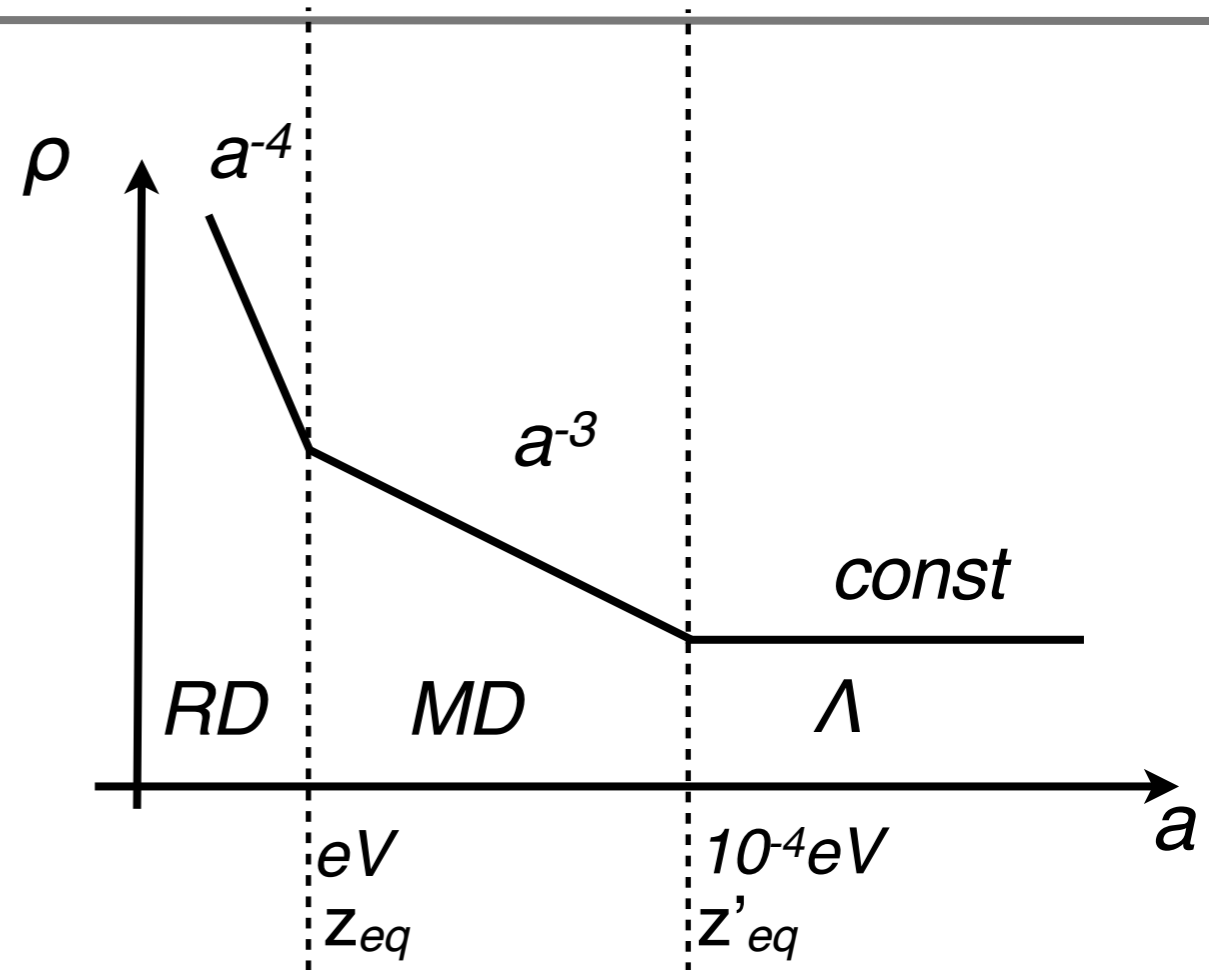
> Phases of the Universe

$$\rho_r = \rho_c \Omega_r^0 / a^4$$

$$\rho_m = \rho_c \Omega_m^0 / a^3$$

$$\Omega_r^0 \equiv \rho_{\text{CMB}} / \rho_c \simeq 8.5 \times 10^{-5}$$

$$a = (1 + z)^{-1}$$



Matter-radiation equality

$$\rho_r = \rho_m \implies z_{\text{eq}} \simeq 3300$$

Matter- Λ equality

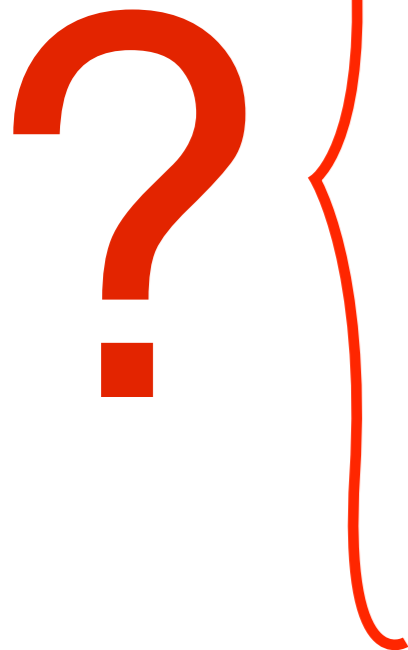
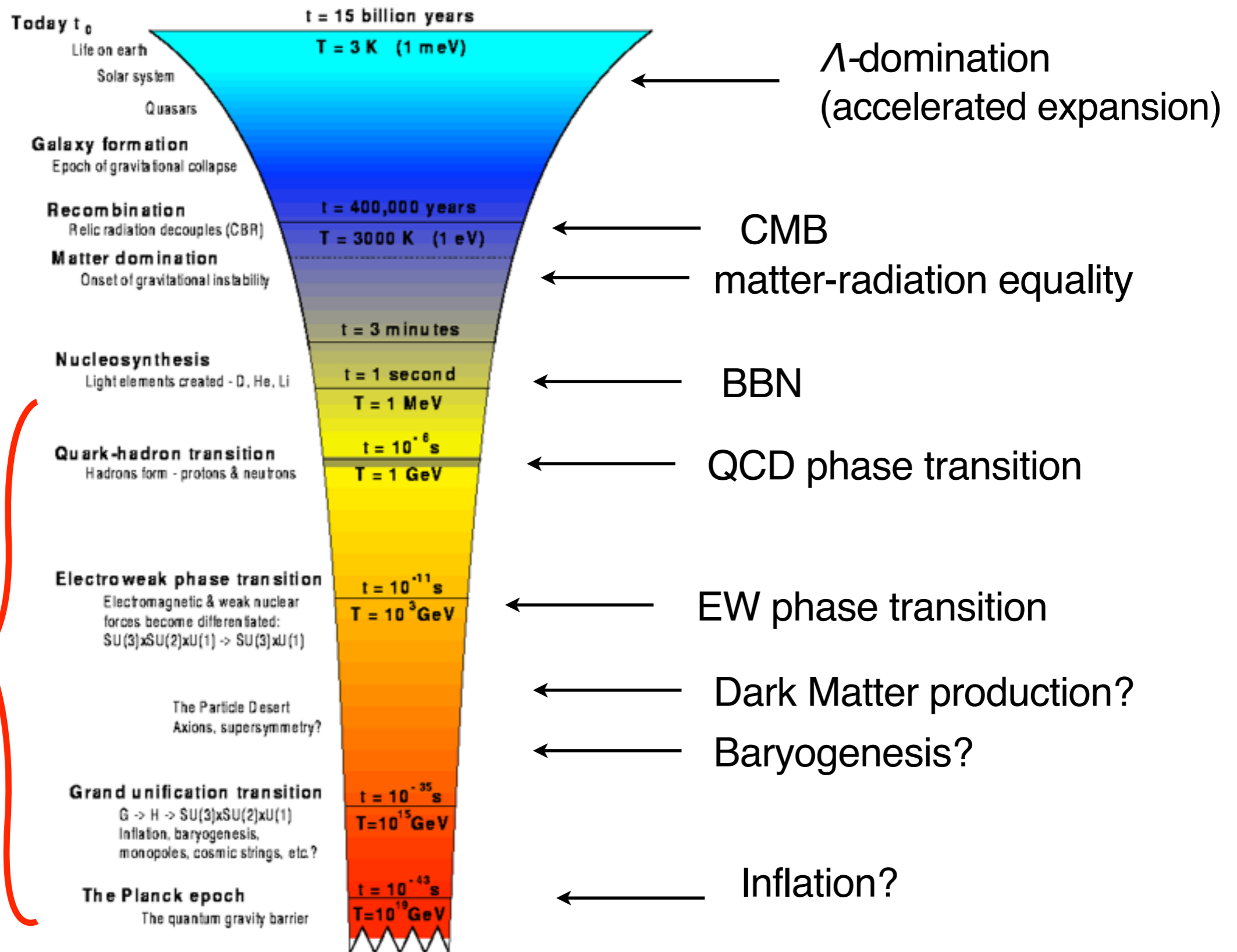
$$\rho_m = \rho_\Lambda \implies z'_{\text{eq}} \simeq 0.3$$

Exercise: compute z_{eq} , z'_{eq}

Λ -domination started very recently:

$$z'_{\text{eq}} \simeq 0.3 \implies T_\Lambda = (1 + z'_{\text{eq}}) T_0 \simeq 3.5 \text{ K} \quad \text{Coincidence (or "WHY NOW") problem!}$$

> Thermal History of the Universe



> Beyond the SM of Cosmology?

Open problems of the Standard Model of Cosmology

- what is Dark Energy?
- why cosmological constant so small?
- what is Dark Matter?
- what happened in the first 3 minutes (before BBN)?
- why matter and not antimatter?



> Summary of Lecture 1

- our Universe is homogeneous, isotropic and it is expanding.

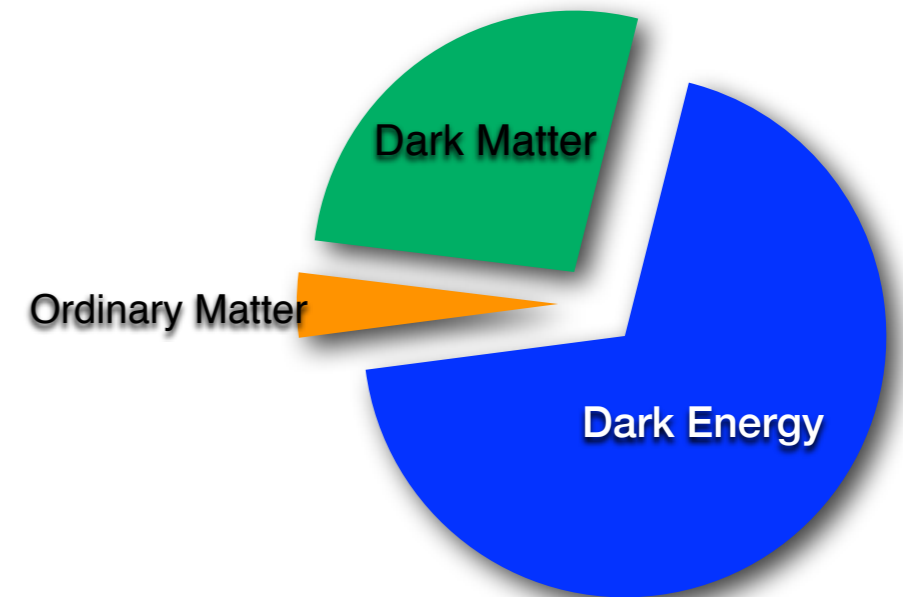
- Energy budget

Λ CDM

baryonic matter $\simeq 4\%$

dark matter $\simeq 27\%$

vacuum energy $\simeq 69\%$



- **Big Bang Nucleosynthesis** is the earliest epoch we have tested (successfully)
- **CMB** temperature and anisotropies provide great deal of info about the Universe today
- Still the picture is not complete... many open problems on the table!