QCD.3

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Hadronic collisions and jets

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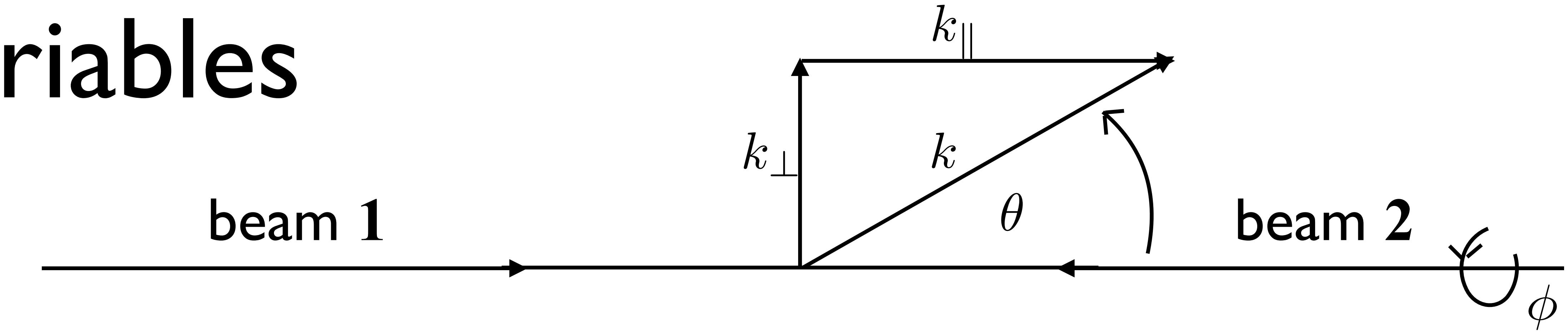
Hadronic collisions

- $p\bar{p}$ and pp colliders are discovery machines

Accelerator	**SppS**	**TEVATRON**	**LHC**
Energy	200÷900GeV	1.96TeV	13TeV
Discovery	W/Z,Jets	top	Higgs

- QCD is ubiquitous in hadronic collisions (many tests) and
- essential to establish signal and background rates

Variables



- Transverse plane: plane orthogonal to the beams
- Azimuthal angle ϕ: azimuth around the beam direction
- Transverse momentum: projection of momentum on the transverse plane $|k_\perp| = |k| \sin\theta$
- Transverse energy: $|E_T| = E \sin\theta$
- Transverse mass: $|m_T| = \sqrt{k_\perp^2 + m^2}$
- Rapidity:

$$y = \frac{1}{2} \log \frac{k^0 + k_\parallel}{k^0 - k_\parallel}$$

Under boost:

$$y \to y + \log\gamma$$

For massless particles:

$$y = \frac{1}{2} \log \frac{1+\cos\theta}{1-\cos\theta} = -\log\tan\frac{\theta}{2}$$

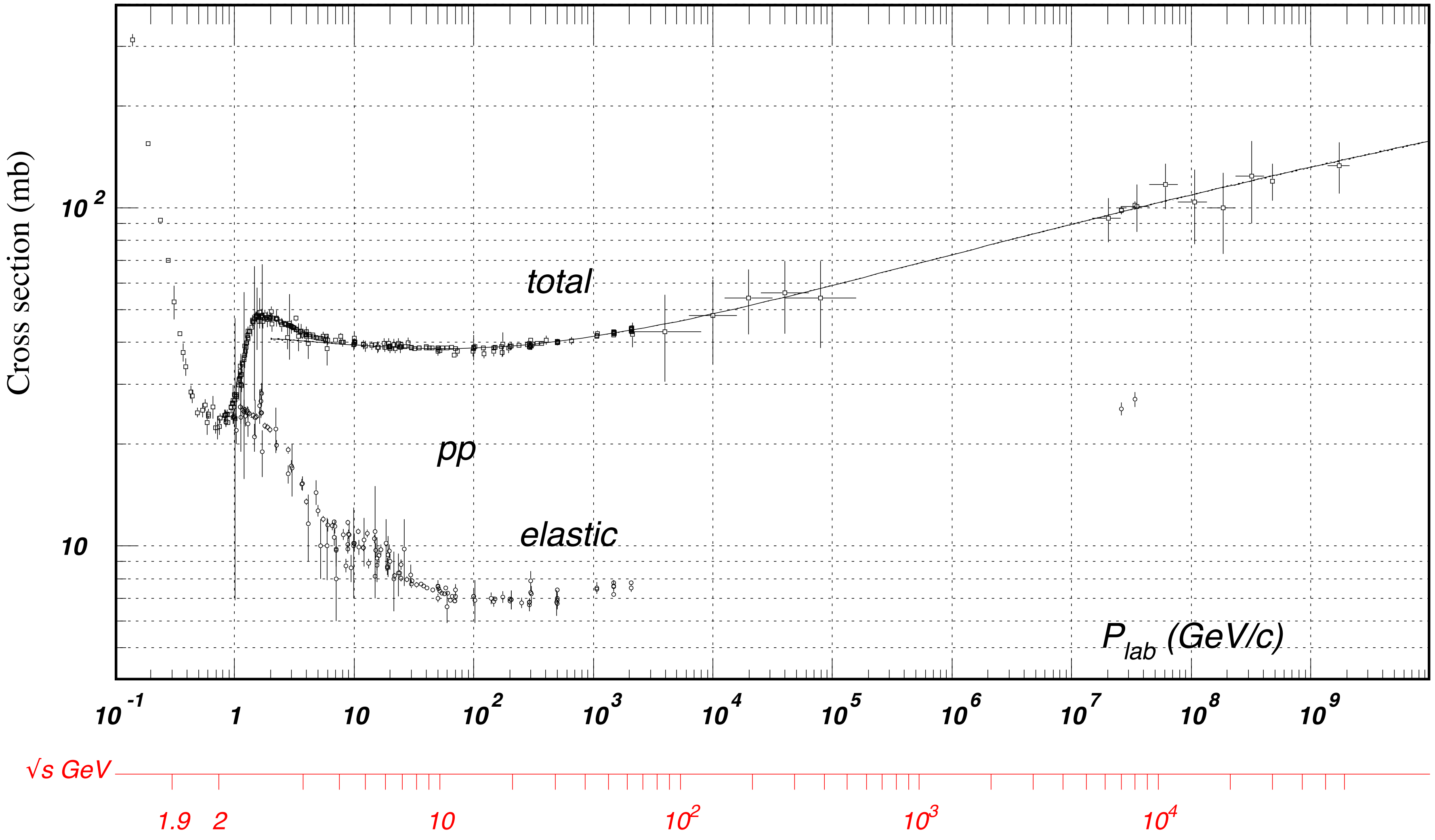
- Pseudorapidity:

$$\eta = -\log\tan\frac{\theta}{2}$$

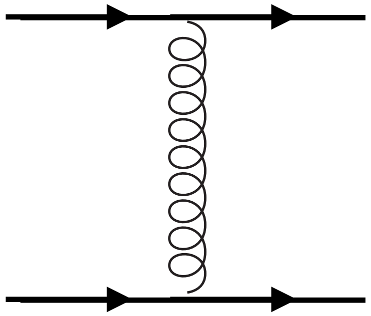
same as rapidity for massless particles

Total cross section

$$\sigma \simeq \frac{1}{(\text{Few hundred MeV})^2}$$



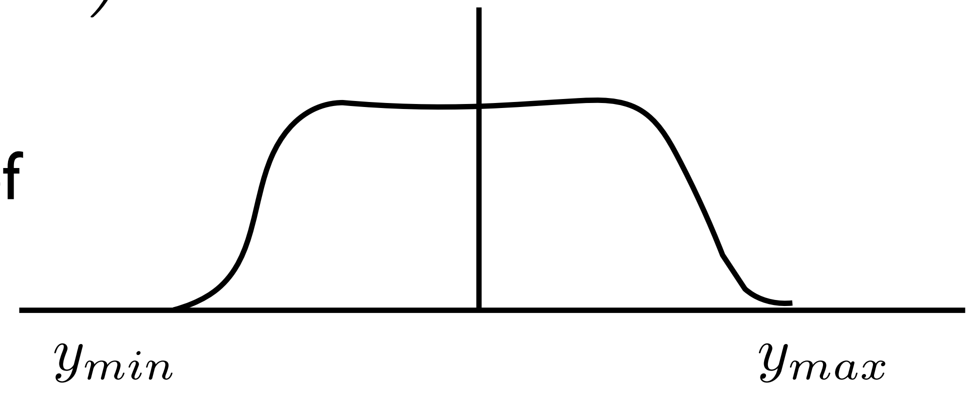
- not computable in QCD, although this behaviour is consistent with QCD if we consider quark quark scattering down to a minimum Q^2



$$\frac{d\sigma}{dQ^2} \approx \frac{1}{Q^4} \Longrightarrow \int_{Q^2_{min}}^{Q^2_{maz}} \frac{d\sigma}{dQ^2} dQ^2 \approx \frac{1}{Q^2_{min}}$$

Collisions are almost all soft (Minimum Bias)

1. average charged multiplicity at the LHC ~ 80, with huge fluctuations, grows (a bit more then) logarithmically with energy
2. transverse momentum distribution ~ $\exp\left(-a\sqrt{m^2+k_\perp^2}\right)$ with $\langle k_\perp \rangle \approx 600\mathrm{MeV}$ growing slowly with energy
3. rapidity distribution rather flat: 5~6 particles per unit of central rapidity
   $|y_{max}| \approx 9$ for $m = 1\mathrm{GeV}, \quad E_{CM} = 13\mathrm{TeV}$



4. these features are not predictable in QCD, but compatible with QCD

➡ just start from heavy quark pair production ($m >> \Lambda$)

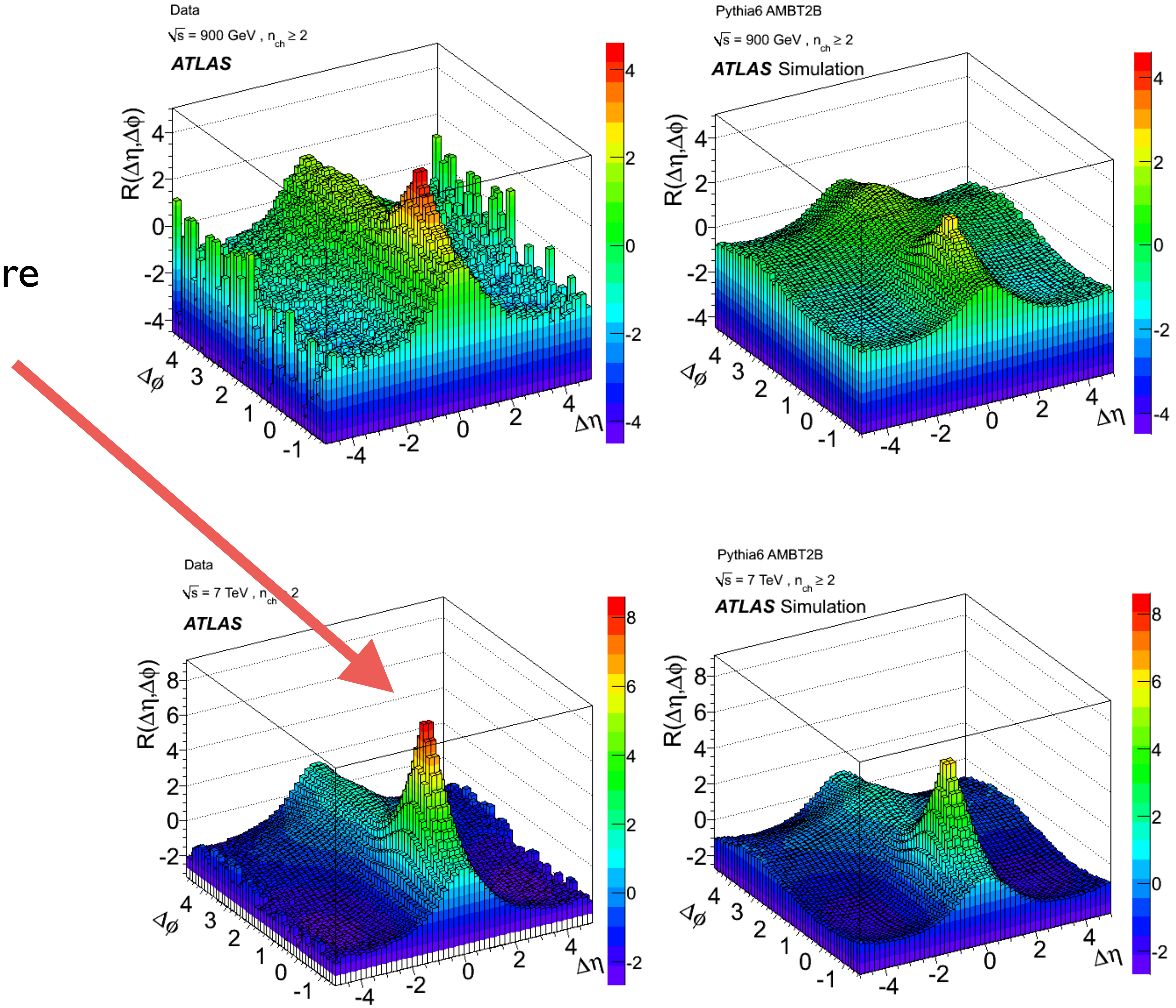
➡ similar rapidity dist. with a smaller range that increases reducing the heavy quark mass

➡ typical $\langle k_\perp \rangle$ of the order of the mass of the produced object

Collisions are almost all soft (Minimum Bias)

- $p_{\mathrm{T}} > 100\,\mathrm{MeV}$,
- $|\eta| < 2.5$,
- …other cuts

1. emergence of jet like structure at Δη=Δφ=0



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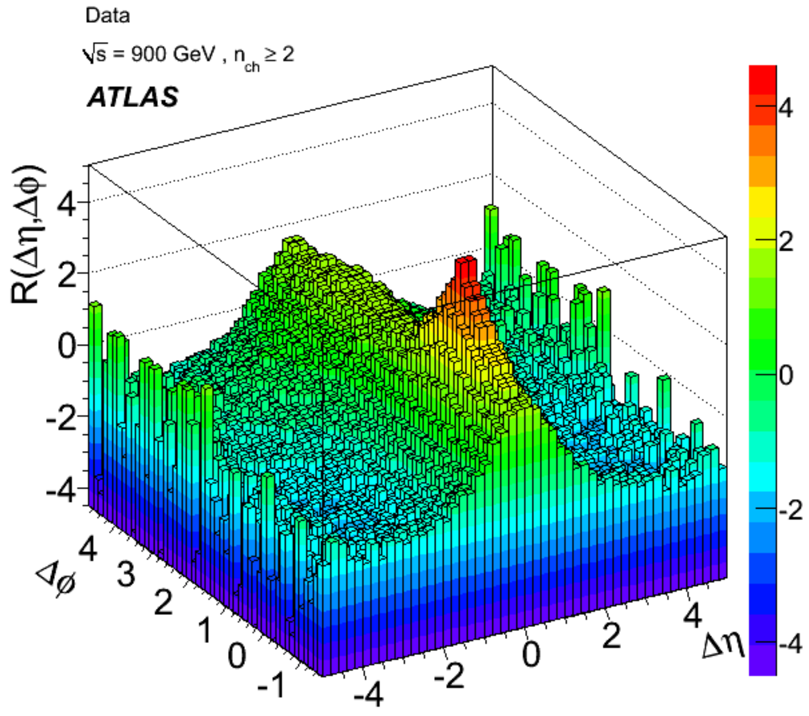
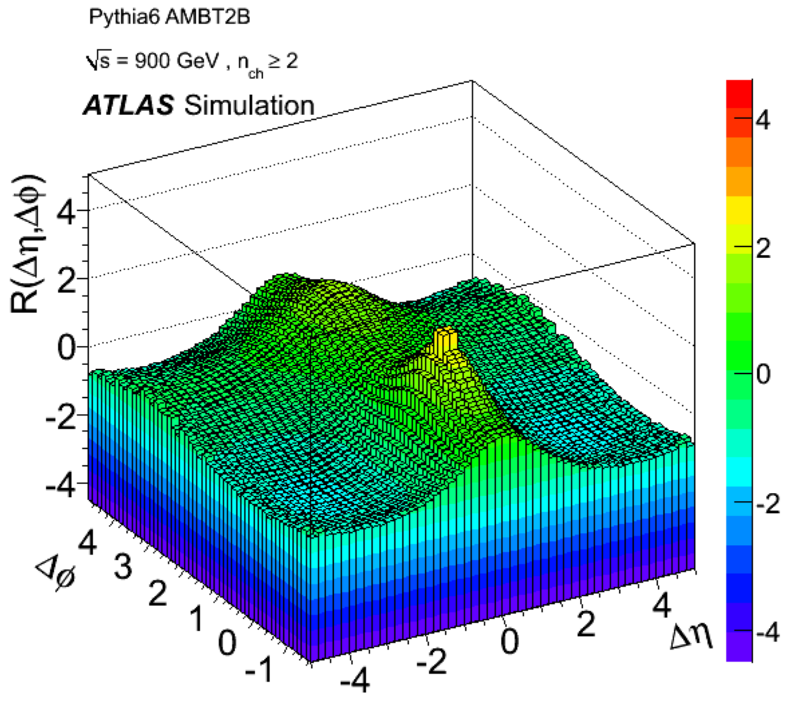
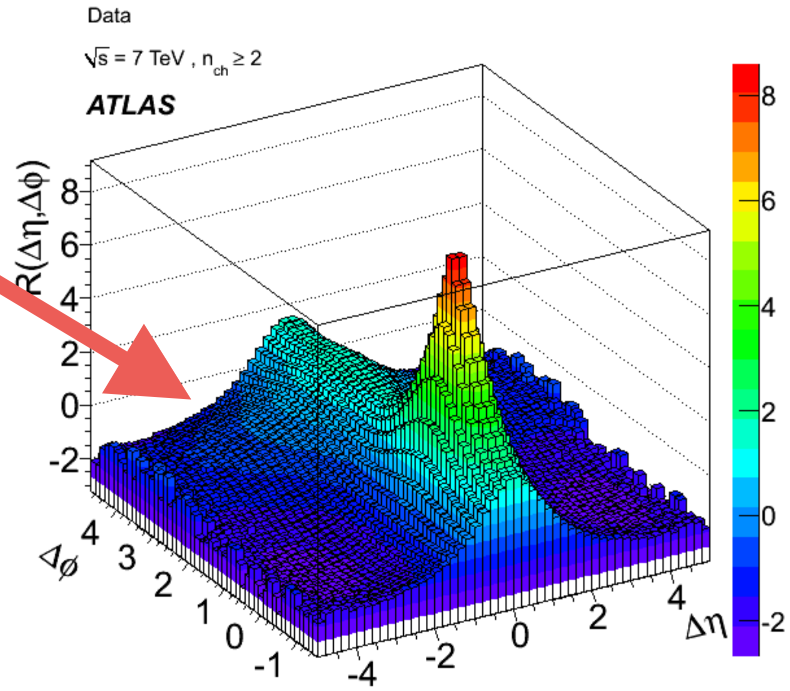
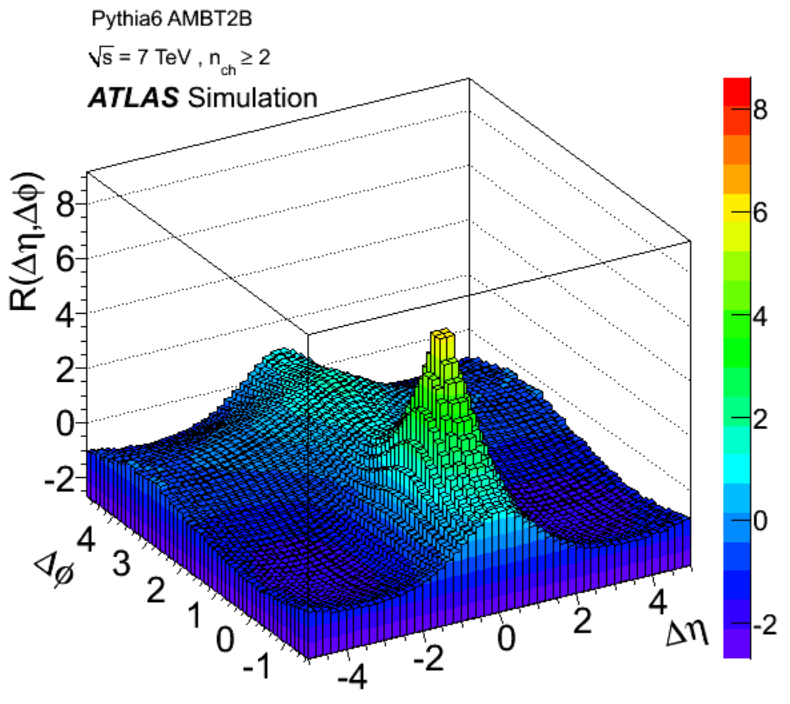
1. emergence of jet like structure at Δη=Δφ=0

2. emergence of recoil of one particle against the other: back to back jet like structure at Δη=π for all Δφ









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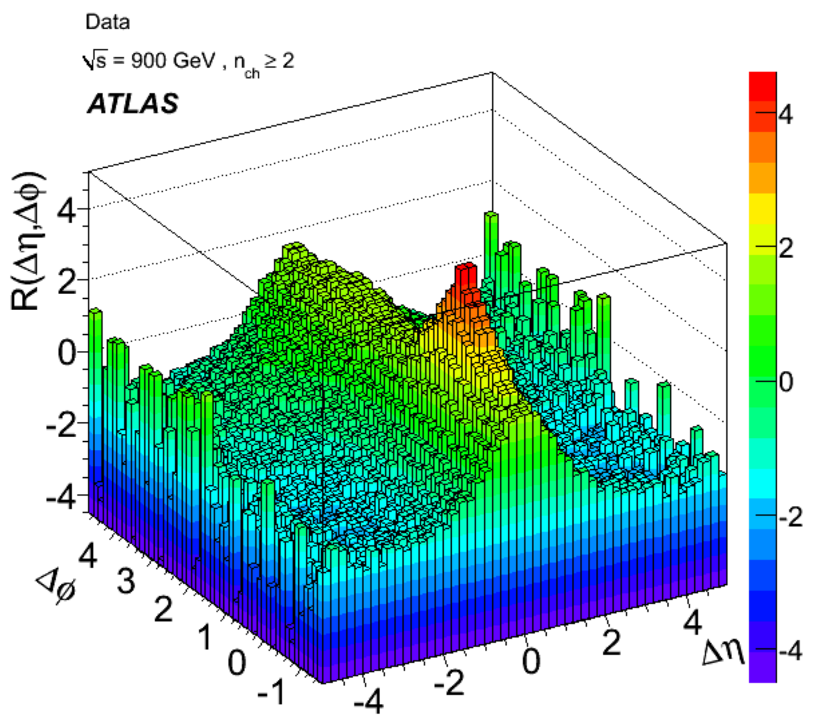
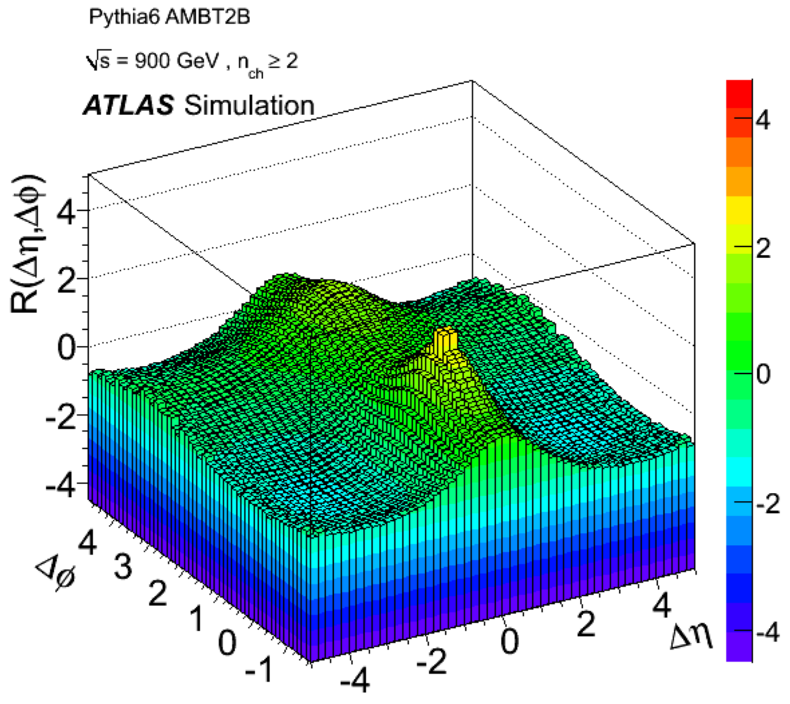
1. emergence of jet like structure at Δη=Δφ=0

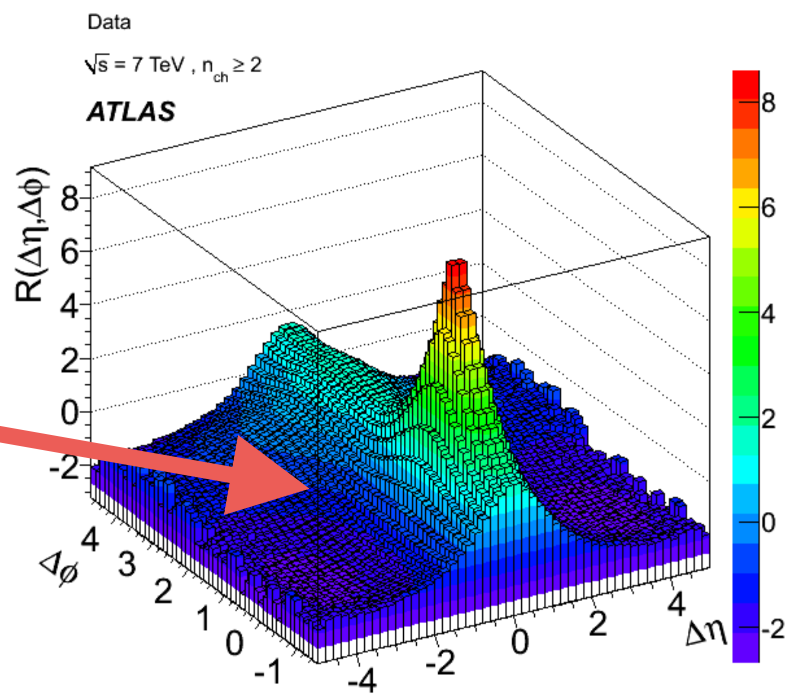
2. emergence of recoil of one particle against the other: back to back jet like structure at Δη=π for all Δφ

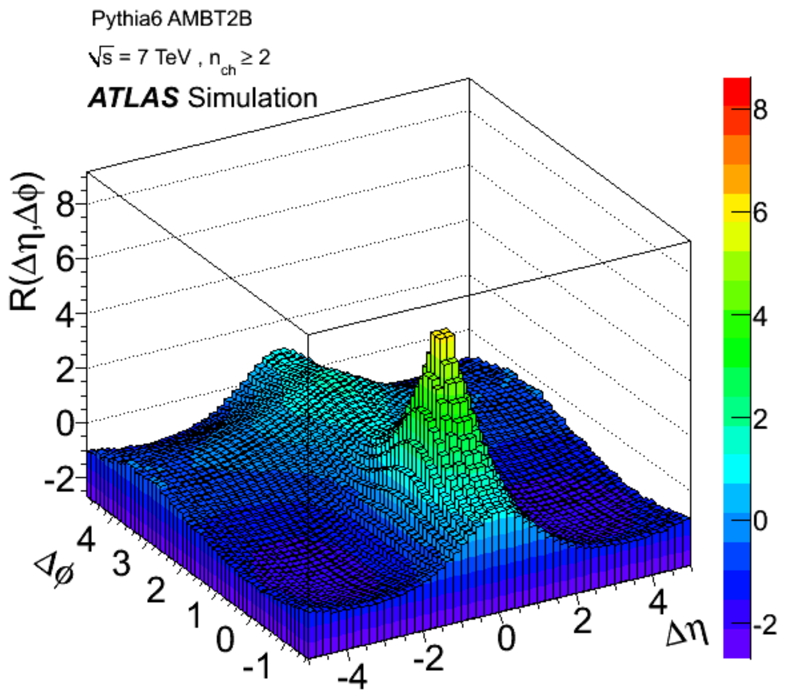
3. decay of resonances or other colour structures: width of Δη of about 2 and for all Δφ

- 1. 2. and 3. are not well reproduced by simulations programs and more investigation is needed here









About Jets

- Very different situation with respect to e+e- where almost all events are back to back

- we expect something similar, although now the higher the energy of the jets, the smaller fraction of events

- to identify jets something has to be done to distinguish them from the fluctuation of the minimum bias

- indeed, trigger is crucial to make discoveries!

Trigger

- discovery the Z at UA2 ($p\bar{p}$ @ 630GeV)

$$\frac{\sigma(Z)}{\sigma_{TOT}} \cdot Br(Z \to l^+l^-) \approx \frac{\frac{1}{3}\left(\frac{1}{300\mathrm{GeV}}\right)^2}{\left(\frac{1}{300\mathrm{MeV}}\right)^2} \cdot 0.1 \approx \frac{1}{3}10^{-7}$$

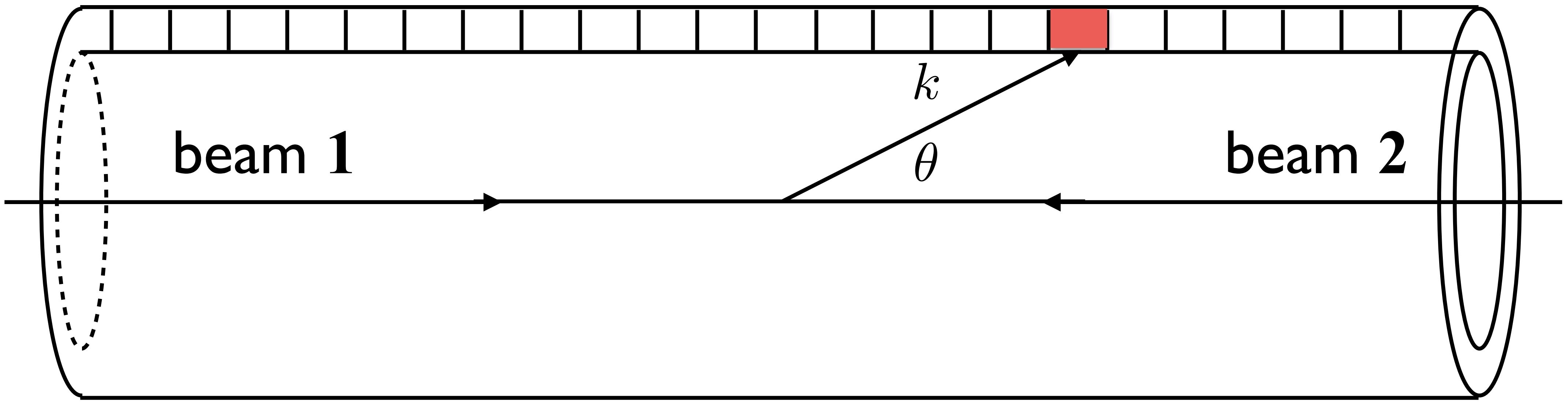
- 1 year has about $3 \cdot 10^7 s$

  ➡ to have 1 Z decaying to charged lepton per year you need of 1 proton collision per second, but you need more, O(1000)! so you need to register of the order of few thousands per second

- in general one store events characterised by an high transverse momentum transfer, that is the signal of a short distance interaction

Discovery of jets in hadronic collisions

- one could think to trigger events on the base of the presence of very high energetic particles in an event, but that's not a very good idea



$$E_T = \sum_i E_T^i \sin\theta_i$$

- minimum bias event can give very high energetic particles at small angles
- better to use a "transverse energy trigger" (energy suppressed by the angle)
- but, still counting charged (~25) as well neutral hadrons, minimum bias events at UA2 had:

$$\langle E_T \rangle \approx 50 \cdot 500\text{MeV} \approx 25\text{GeV}$$

Discovery of jets in hadronic collisions: UA**1**/**2**

- to stay away from minimum bias events: look at events with $E_T > 70\mathrm{GeV}$

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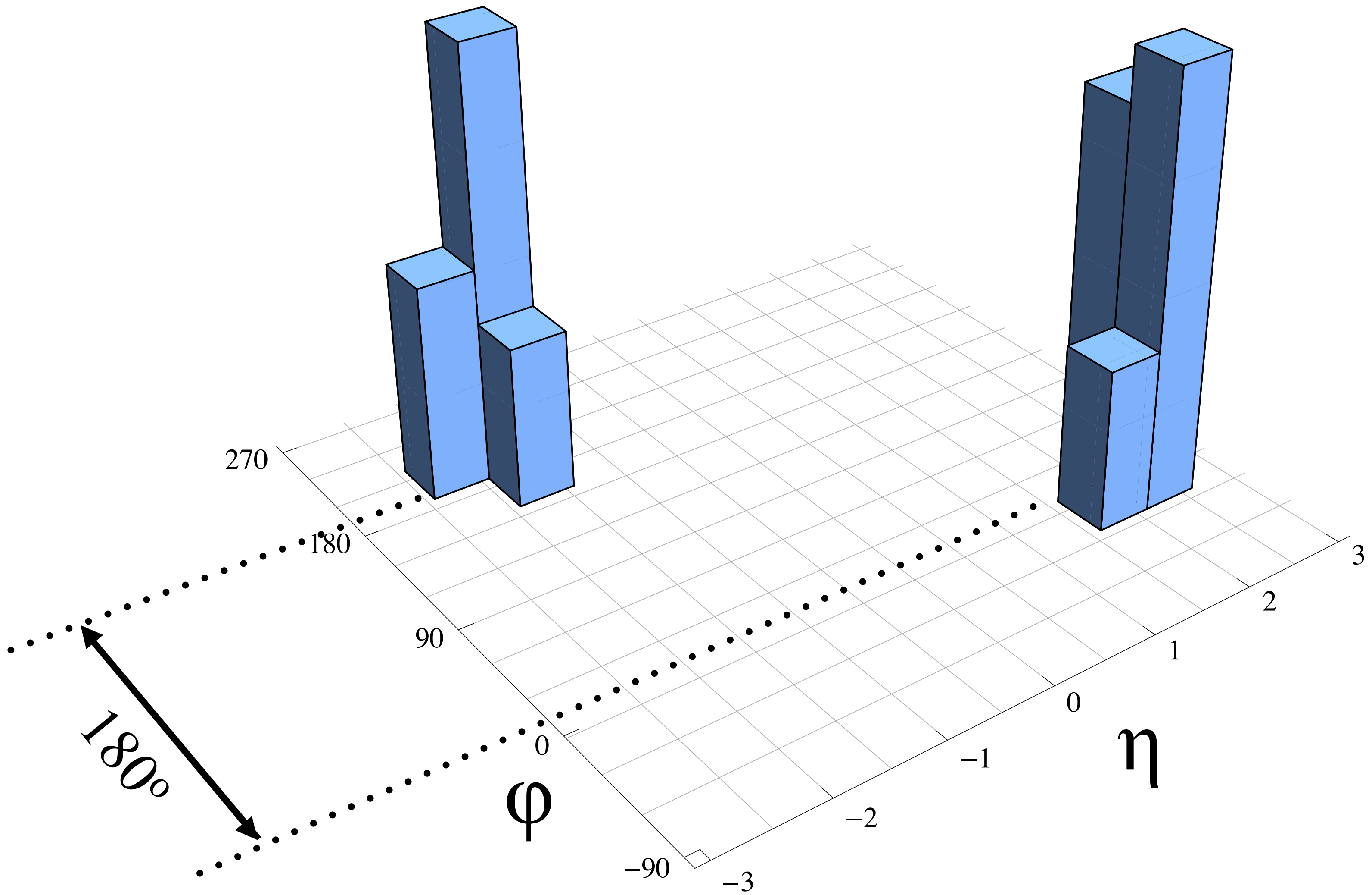
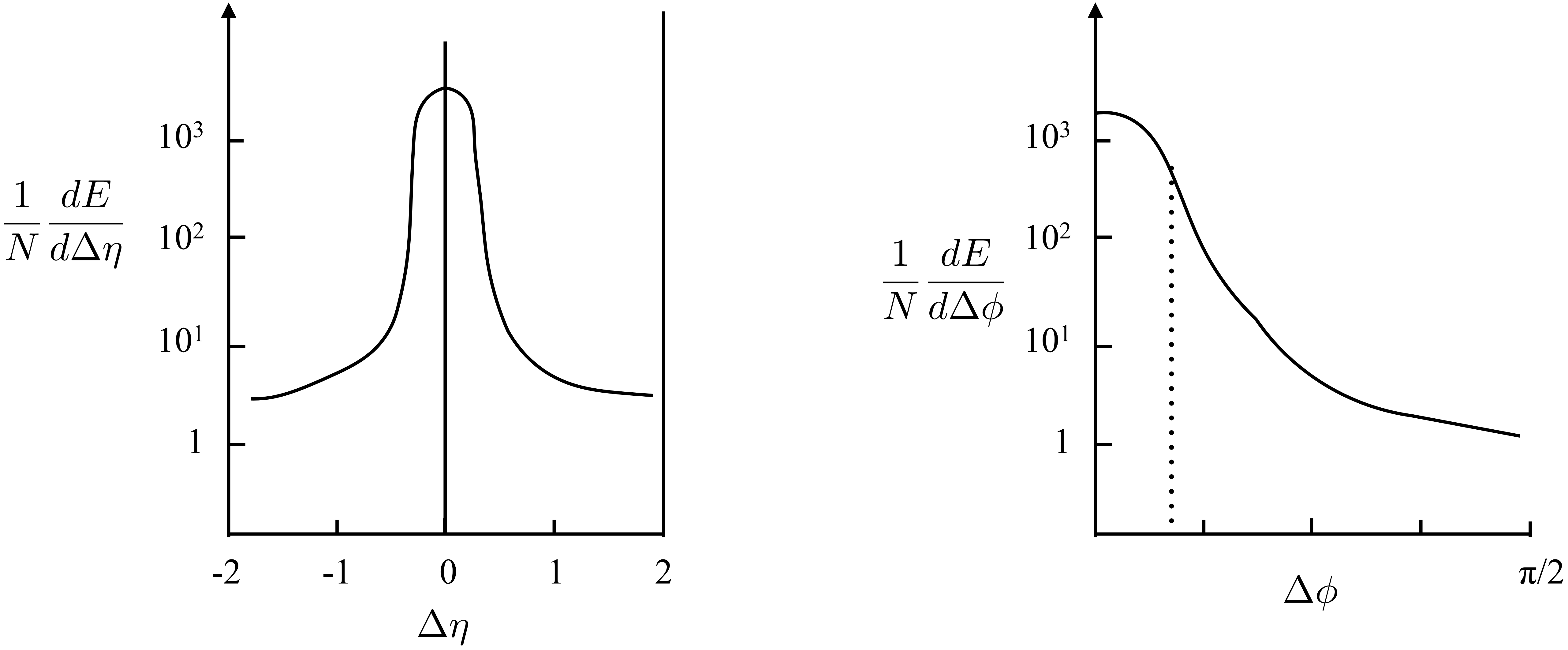


- all events are jet like!
- most events have 2 jets
- jets are a property of nature!

Discovery of jets in hadronic collisions: UA1/2

- Define a jet trigger: require clusters of energy in the calorimeter with some criteria
- Jet profile:



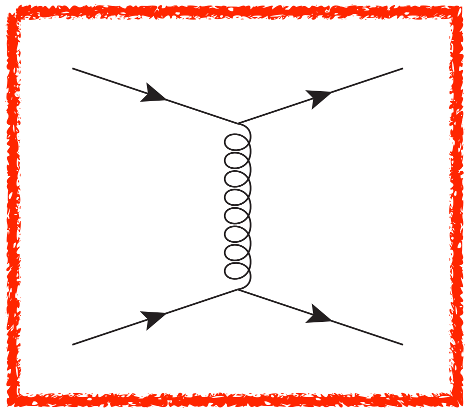
with $\Delta\phi,\, \Delta\eta \approx 0.2,\, 0.3$ for 40GeV Jets

Back to theory

- we start from the improved parton model for the single inclusive jet diff. cross section

$$\frac{d^2\sigma}{dy\,dp_\perp^2} = \sum_{i,j,k} \int dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d^2\hat{\sigma}_{ij\to kX}}{dy\,dp_\perp^2}$$

different flavours



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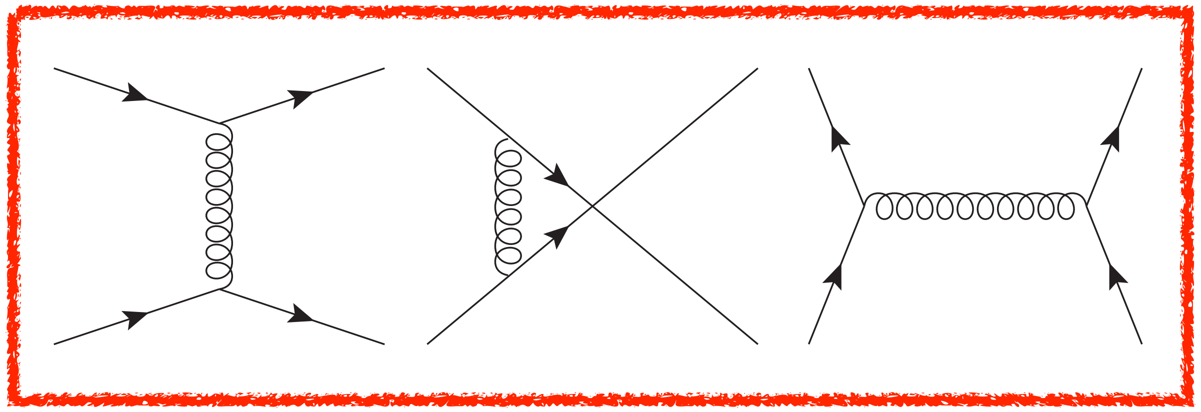
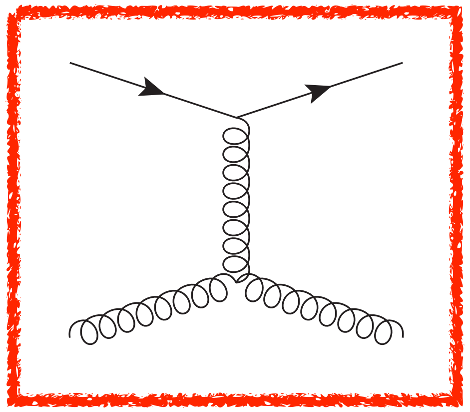
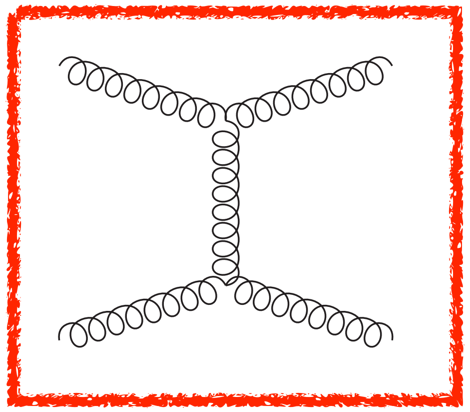


quark-gluon



gluon-gluon



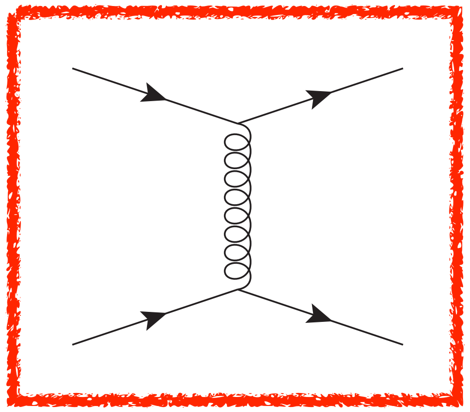
- predictions:
  ➡ starting from parton densities measured in DIS

Back to theory

- next: the improved parton formula model for the 2 jet differential cross section

$$\frac{d^3\sigma}{dy_1 dy_1 dp_\perp^2} = \frac{1}{16\pi S^2} \sum_{i,j,k,l} \frac{f_i(x_1,\mu)}{x_1} \frac{f_j(x_2,\mu)}{x_2} \overline{\sum} |M_{i,j,k,l}|^2 \frac{1}{1+\delta_{kl}}$$

different flavours



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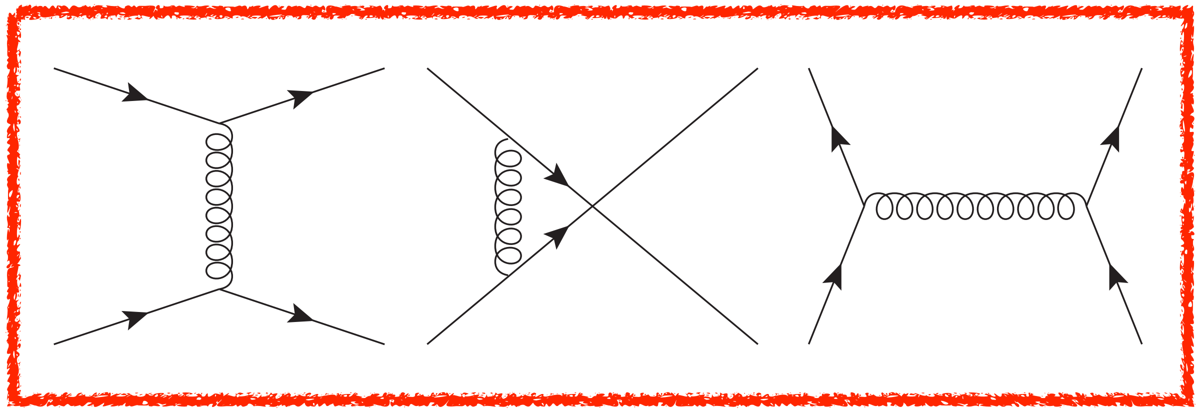

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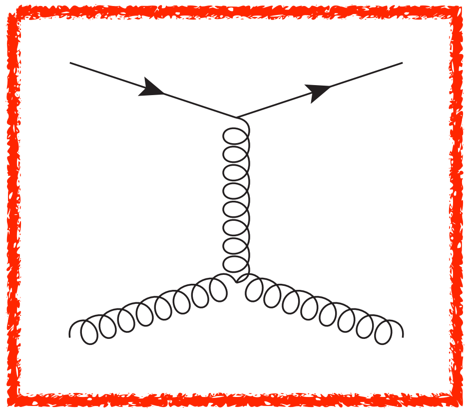

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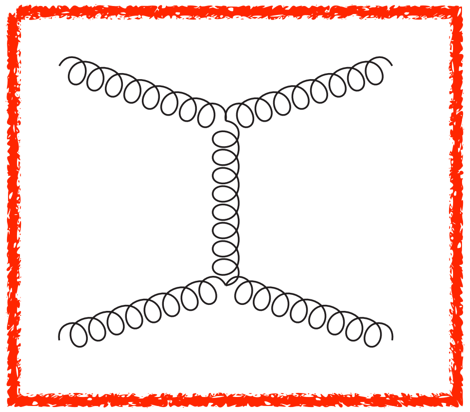

- from the definitions of rapidity and transverse momentum we have that measuring y_1, y_2 and pt fix the values of x_1 and x_2

$$y^* = \frac{y_1 - y_2}{2} \qquad y_0 = \frac{y_1 + y_2}{2} = \frac{1}{2}\log\frac{x_1}{x_2} \qquad x_T = \frac{2\,p_T}{\sqrt{s}}$$

$$x_1 = x_T e^{y_0} \cosh y^* \qquad x_2 = x_T e^{-y_0} \cosh y^*$$

- small $y_0 \to x_1 \sim x_2$

Back to theory

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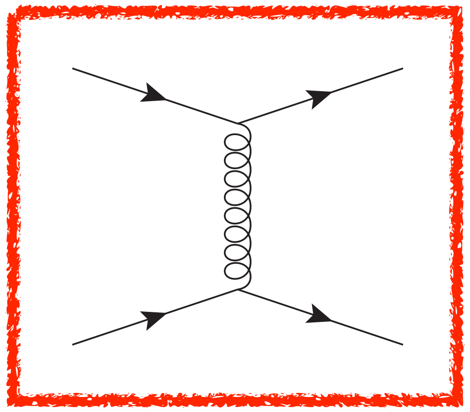
<u>different flavours</u>



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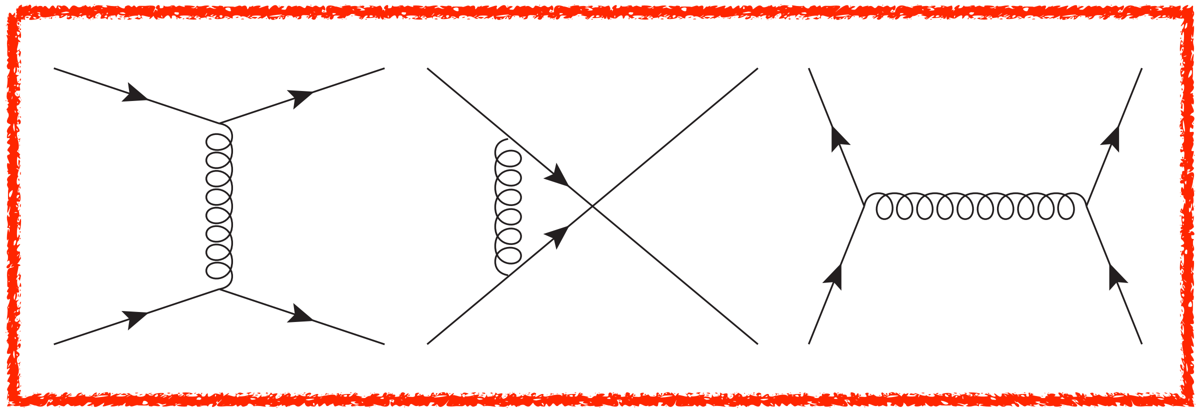

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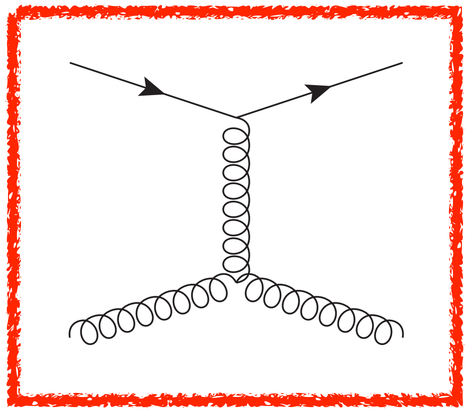

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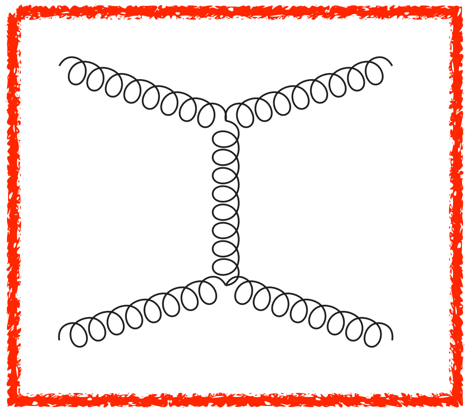


- from the definitions of rapidity and transverse momentum we have that measuring y_1, y_2 and pt fix the values of x_1 and x_2

- use parton densities extracted from DIS measurements

- predictions:
  ➡ jets are back to back in azimuth
  ➡ at fixed x_1 and x_2 (pt dependence or) the angular distribution of jets in jets CM frame

Theory vs Experiment

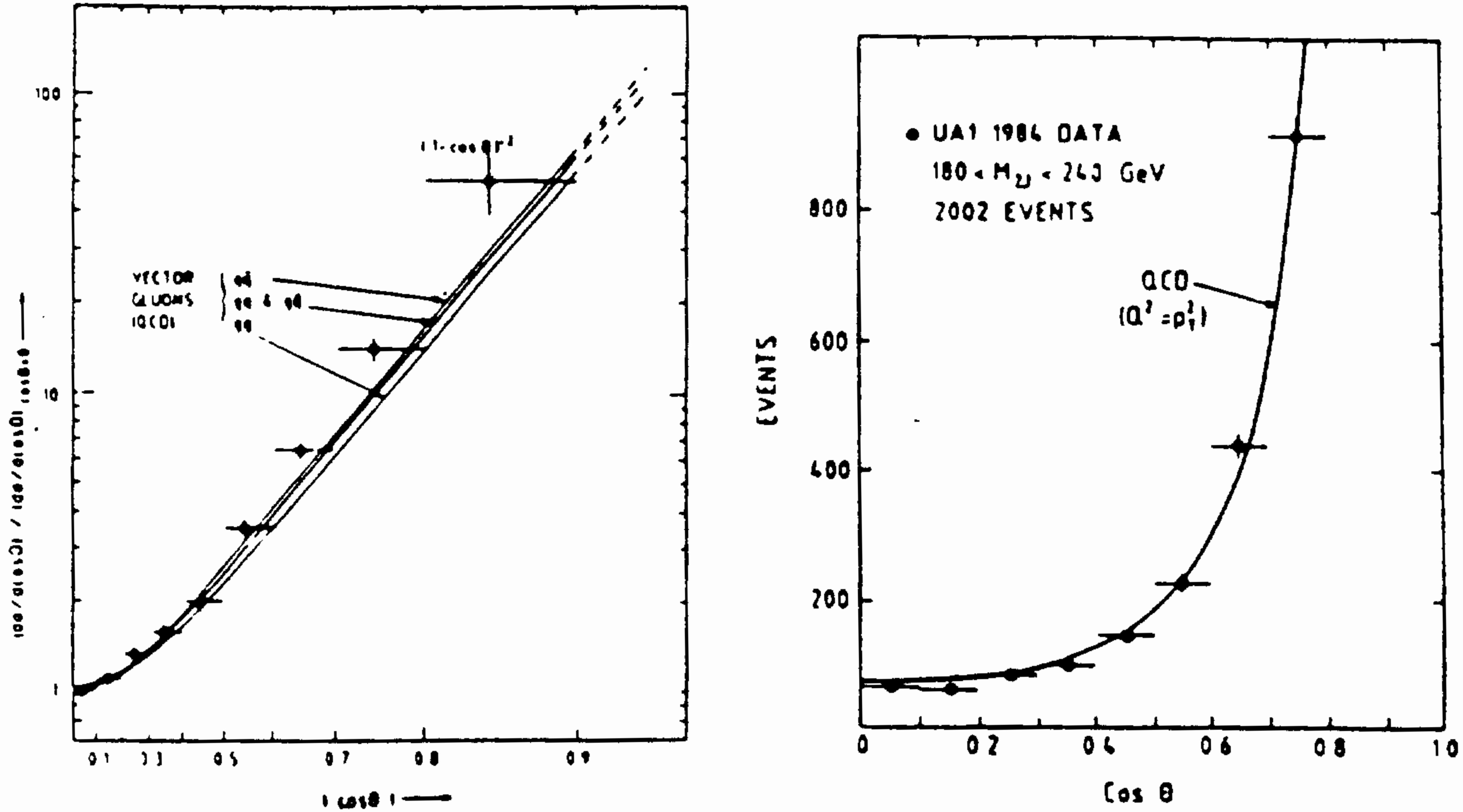
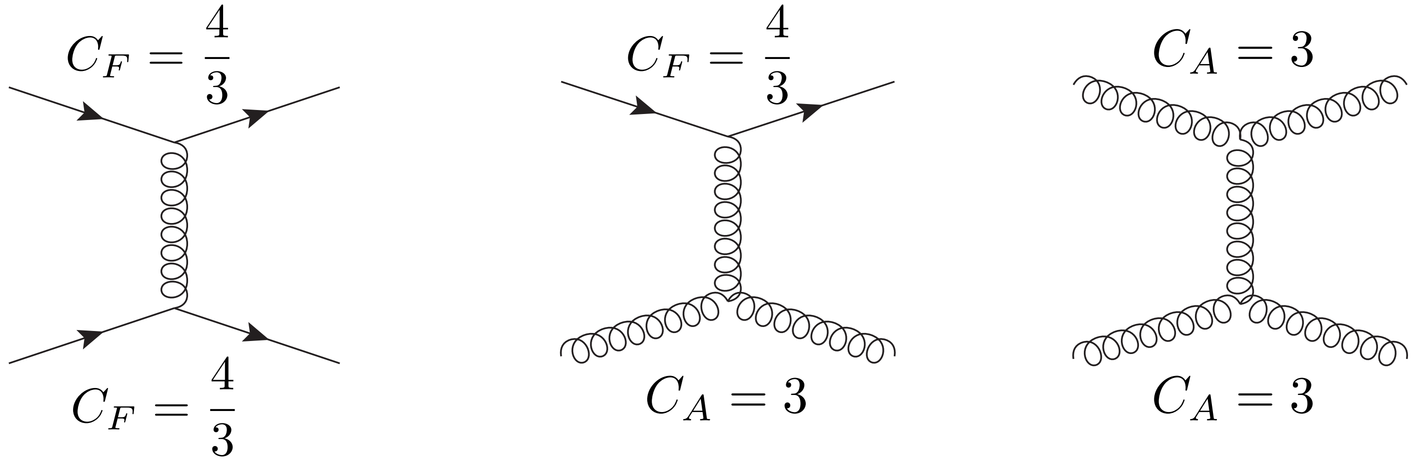


Figure 12: a) The two jet angular distribution plotted to display the power law dependence as $cos\theta \rightarrow 0$. b) The two jet angular distribution for $m_{2J} = 180 - 240$ GeV.

Back to theory

- polar angular prediction (approximately) independent upon structure functions



$$\sigma_{qq\to qq} \quad : \quad \sigma_{qg\to qg} \quad : \quad \sigma_{gg\to gg}$$

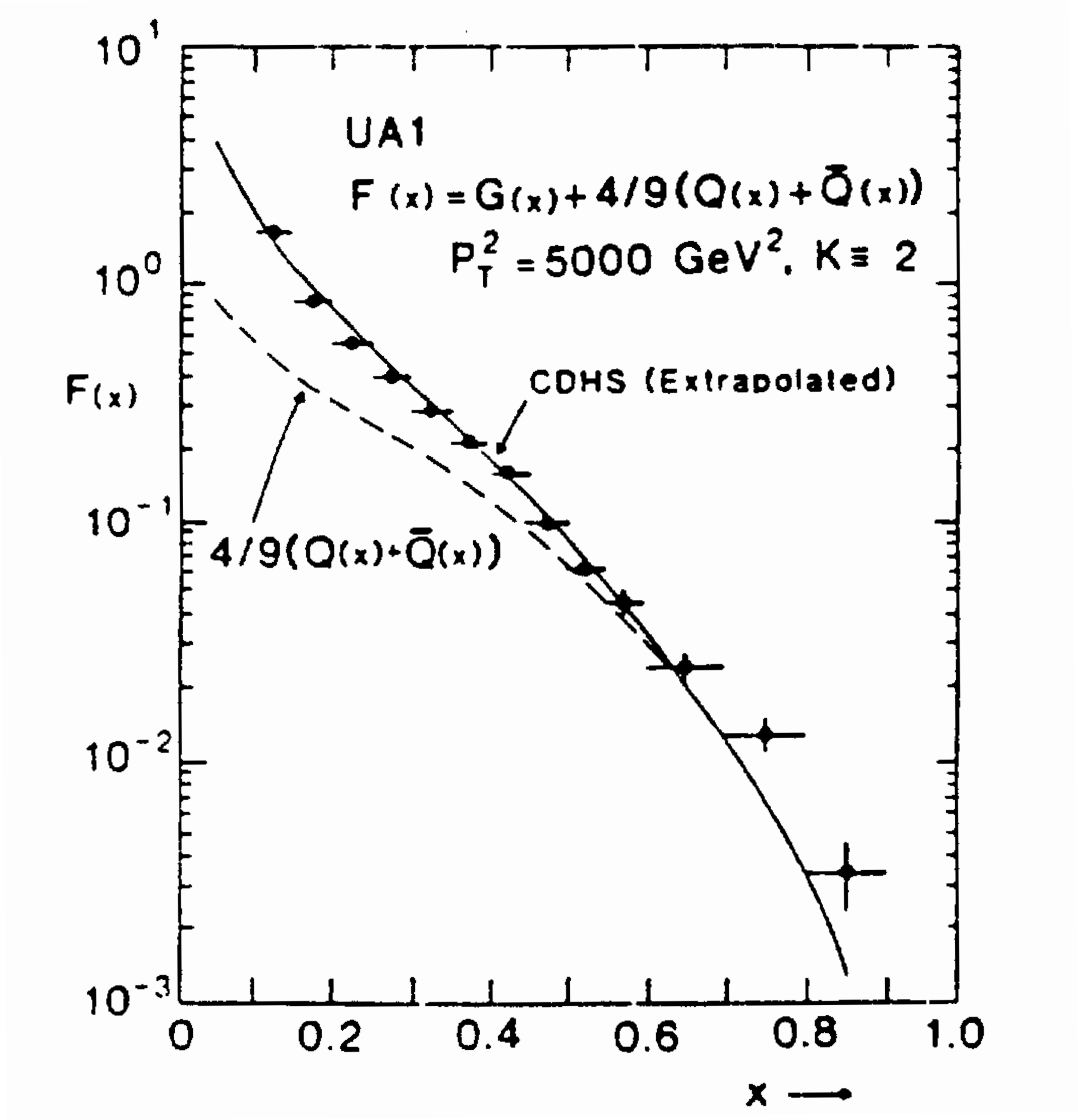
$$\frac{4}{3}\times\frac{4}{3} \quad : \quad 3\times\frac{4}{3} \quad : \quad 3\times 3$$

$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta} = \frac{F(x_1)}{x_1}\frac{F(x_2)}{x_2}\frac{d\hat{\sigma}_{gg\to gg}}{d\cos\theta}$$

$$F(x) = f_g(x) + \frac{4}{9}\sum_q \left[f_q(x) + f_{\bar{q}}(x)\right]$$

- can be used to asses the relevance of gluons

Theory vs Experiment



Higher order calculation

- cross section normalisation
- unphysical scale dependence
- jet radius dependence
- more detailed comparison of differential distributions

Require higher order corrections

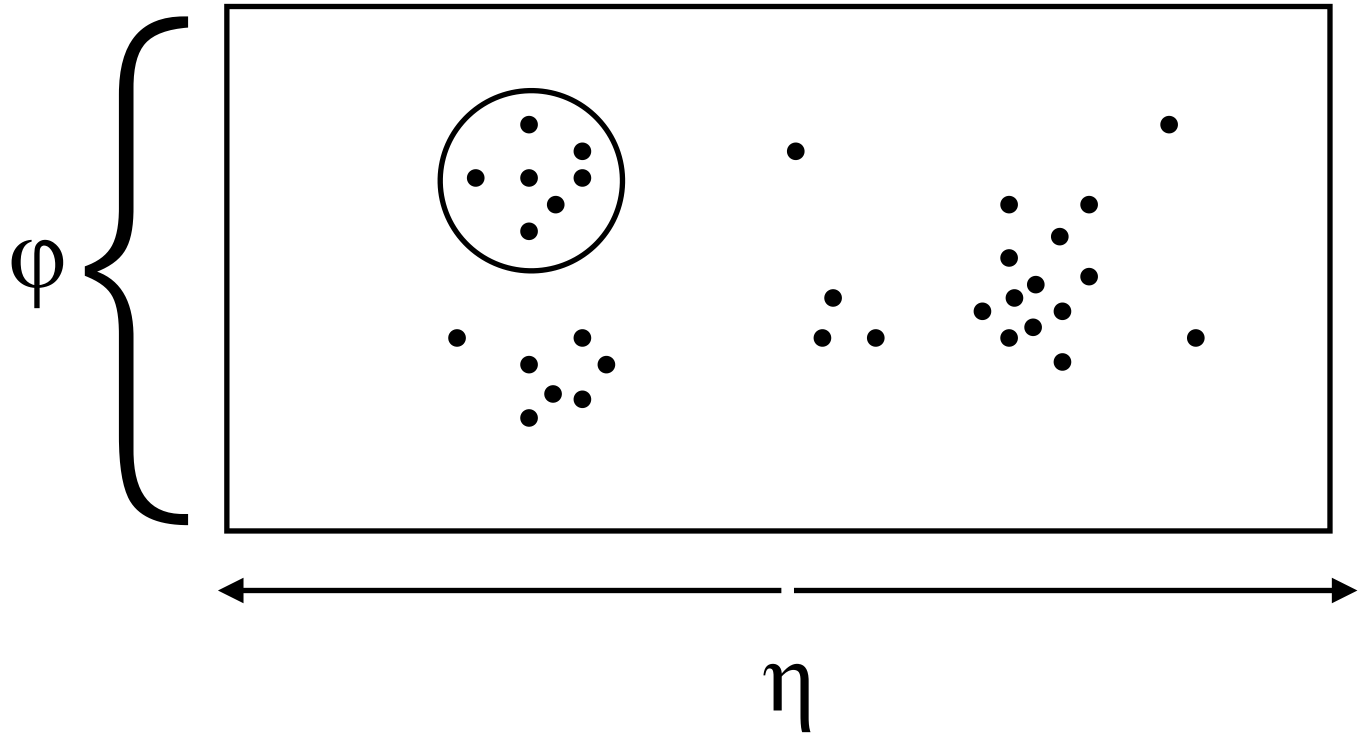
However

- not all measurements can be compared to theory
- QCD perturbation theory can provide predictions only for InfraRed and Collinear safe variables
- Care must be given for theoretically viable jet definitions…

Jet algorithm

- cone algorithm: draw a cone and collect all the particles contained

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$



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how to fix the jet axis?

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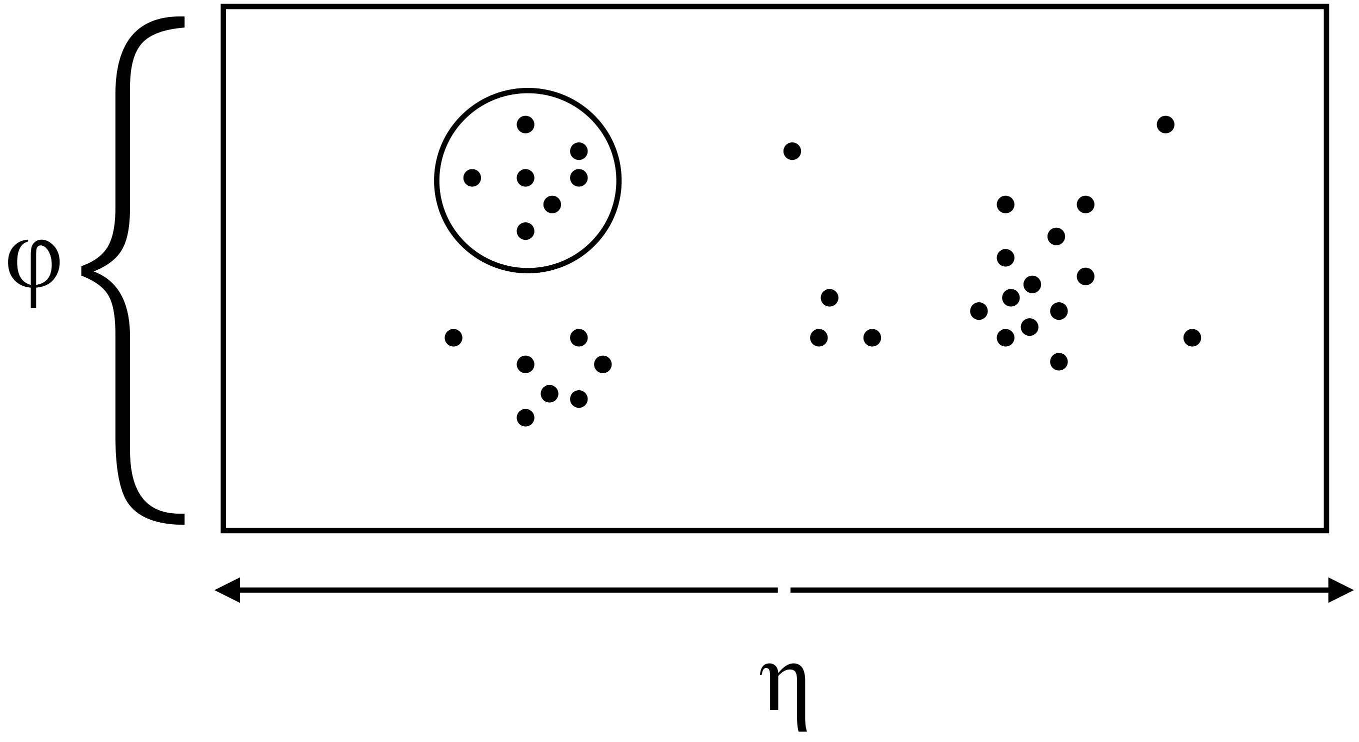


Jet algorithm

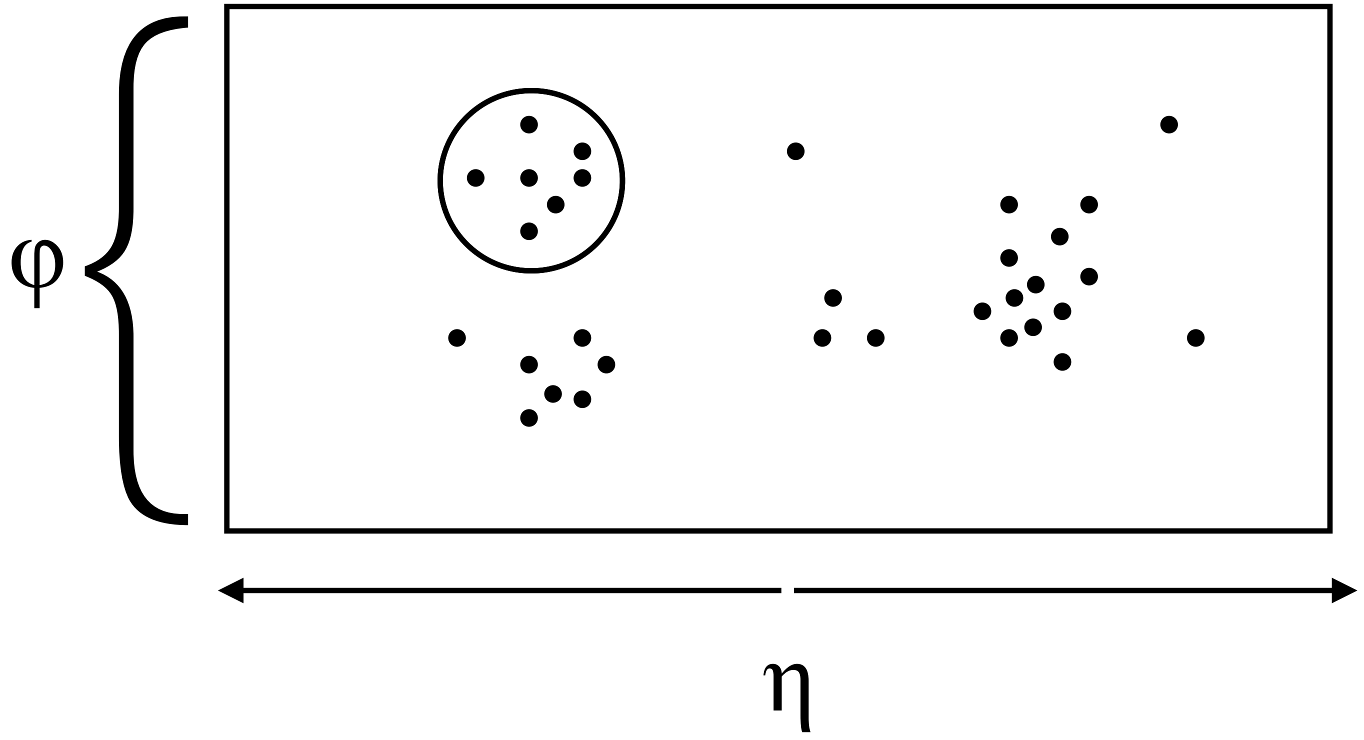
- cone algorithm: draw a cone and collect all the particles contained

how to fix the jet axis?

1. Highest E_T seed-algorithm

   A. select the highest E_T particle
   B. build the cone and remove the particles
   C. restart from A.

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$



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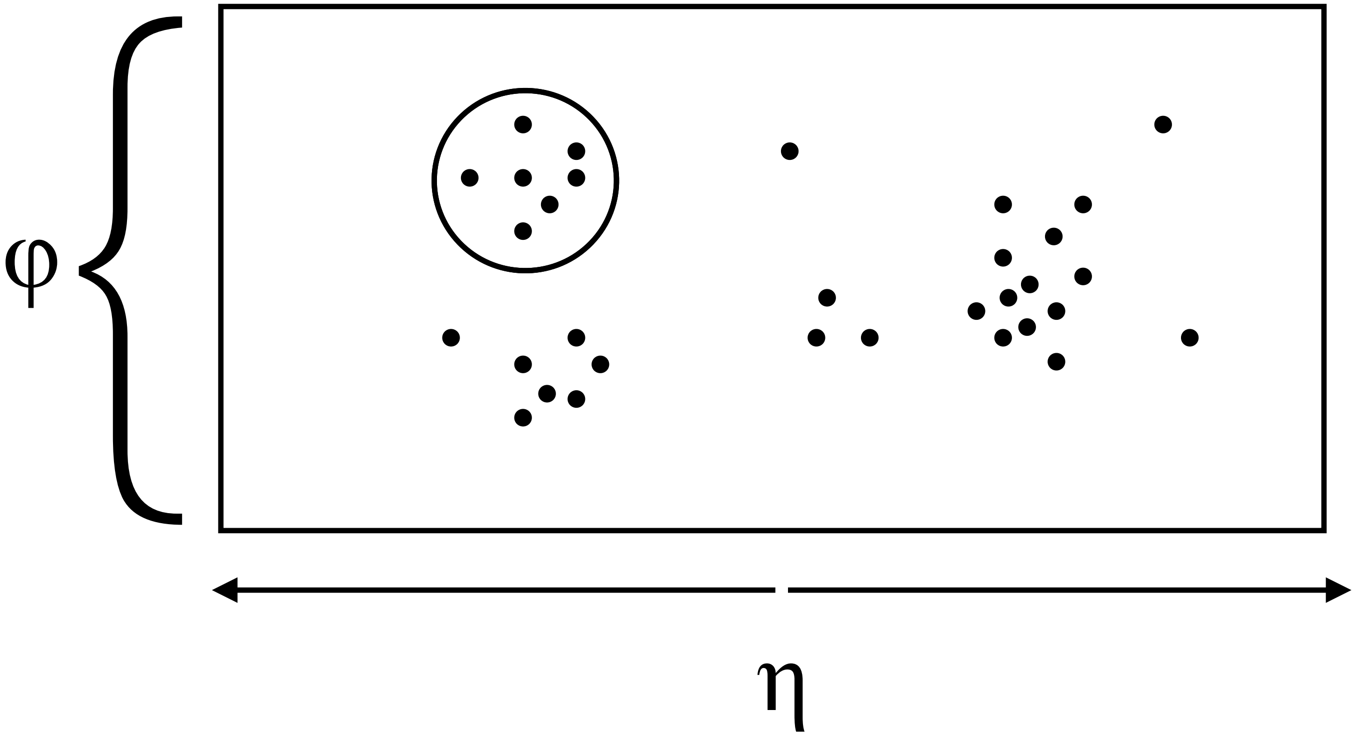
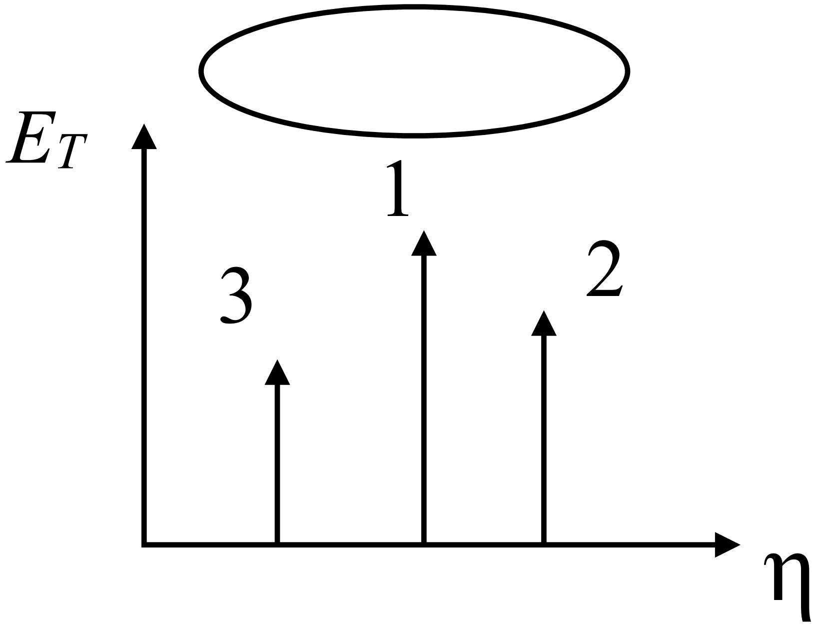


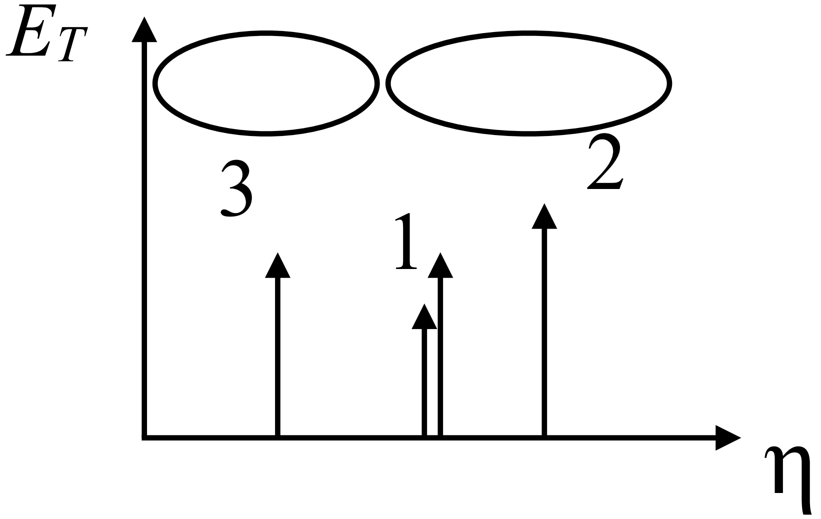
- jet number is a Collinear unsafe observable: sensitive to a collinear splitting



$$E_{T_1} > E_{T_2} > E_{T_3}$$

$$d_{21}, d_{31} < R$$

$$\frac{E_{T_1}}{2} < E_{T_2}$$



Jet algorithm

- cone algorithm: draw a cone and collect all the particles contained

how to fix the jet axis?

2. Iterative cone algorithm

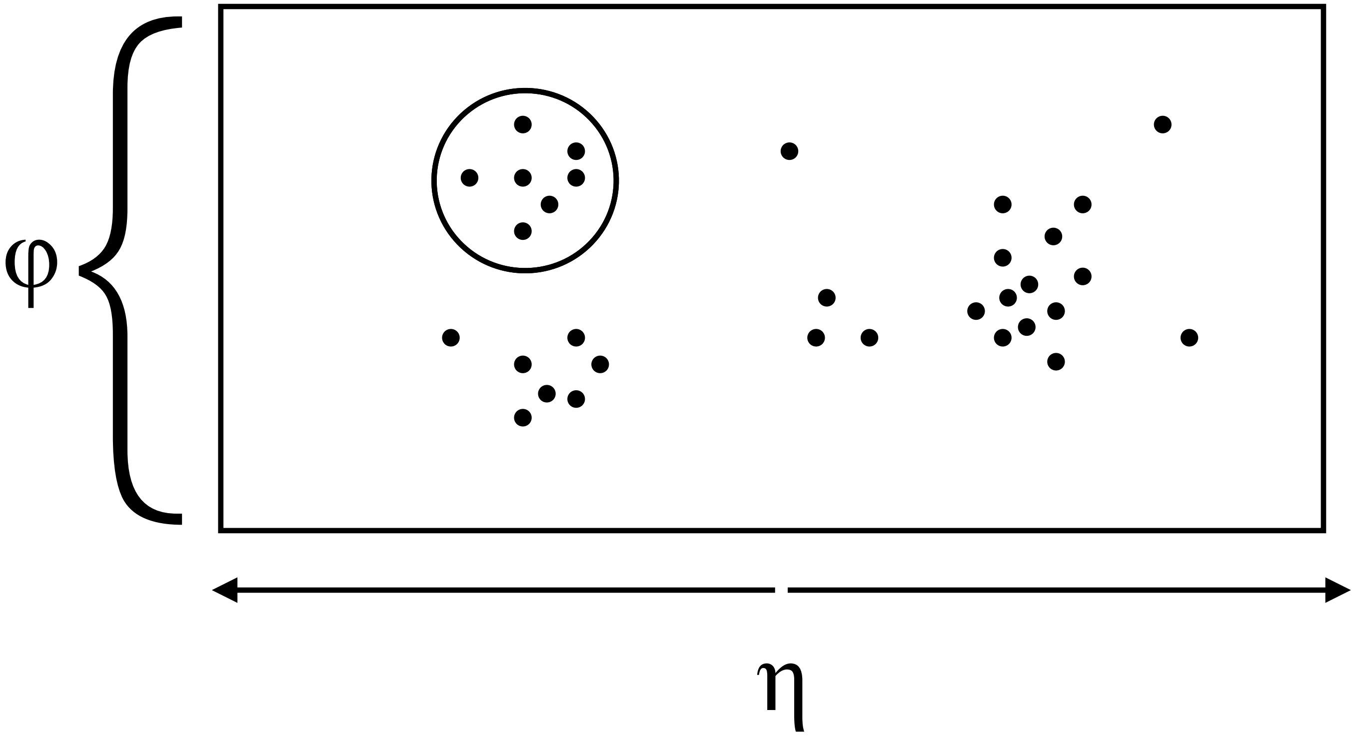
A. all particles above a certain threshold are seeds
B. combine all the particles in a cone to define a new proto-jet axis

$$\phi_c = \frac{\sum_{i\ \epsilon\ \text{cone}} E_{T_i}\phi_i}{\sum_{j\ \epsilon\ \text{cone}} E_{T_j}} \qquad \eta_c = \frac{\sum_{i\ \epsilon\ \text{cone}} E_{T_i}\eta_i}{\sum_{j\ \epsilon\ \text{cone}} E_{T_j}}$$

check the particles in the new cone, if they are the same a potto-jet has been formed (stable cone

C. if proto-jet have small (large) overlap split (merge) them in some way (technical parameters)

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$



Jet algorithm

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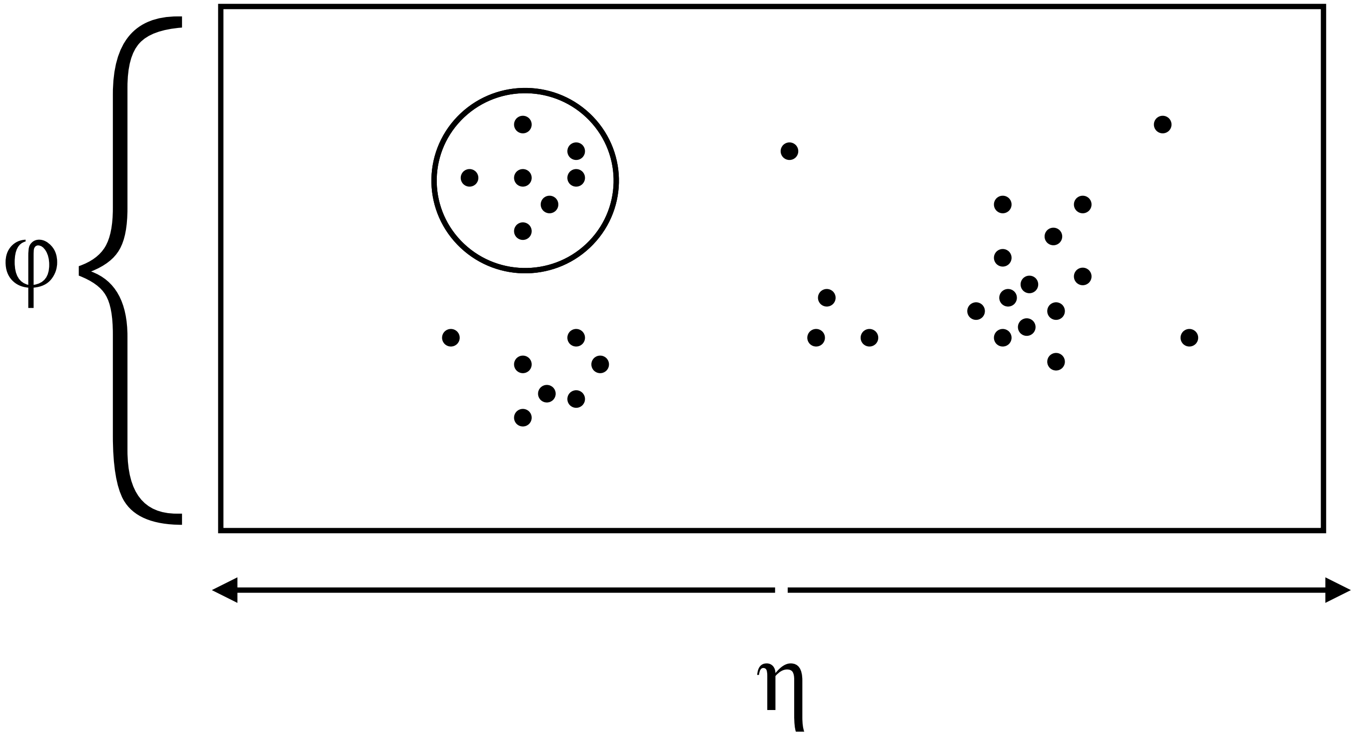

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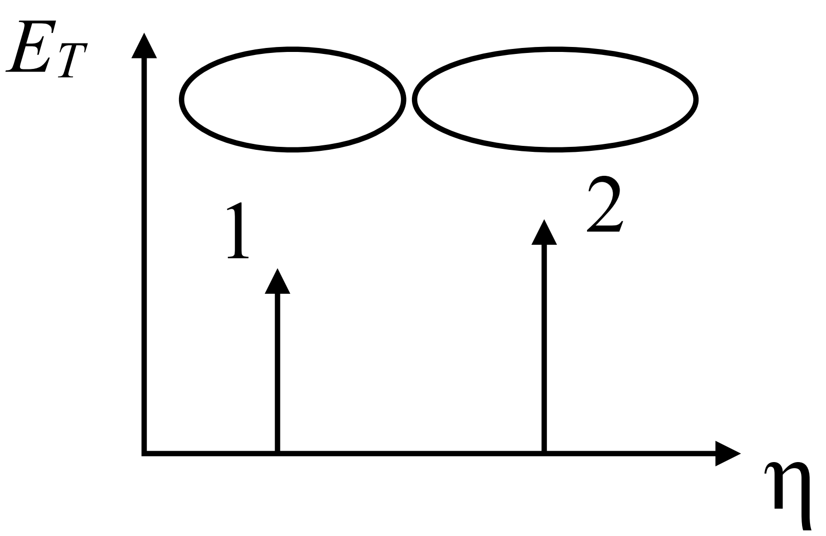
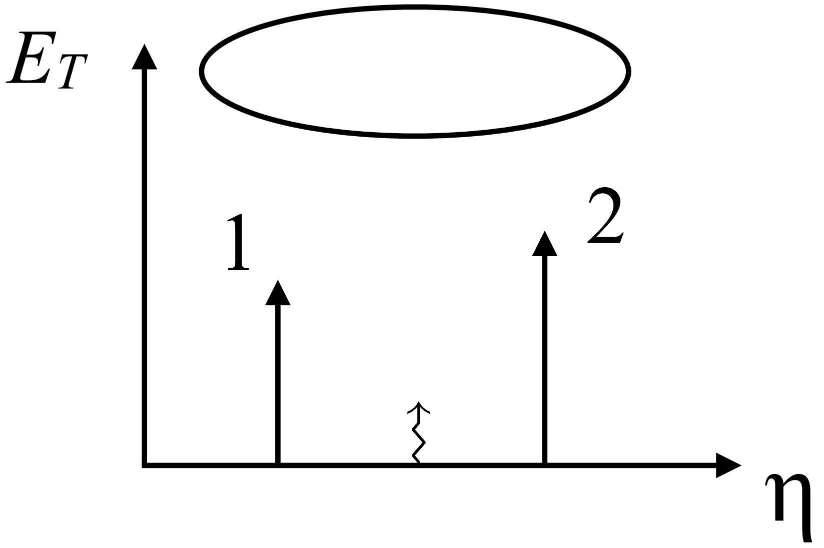
check the particles in the new cone, if they are the same a proto-jet has been formed (<u>stable cone</u>)

C. if proto-jet have small (large) overlap split (merge) them in some way (technical parameters)

- jet number is collinear safe but is an InfraRed unsafe observable: sensitive to soft emissions



$$R < R_{12} < 2R$$



Jet algorithm

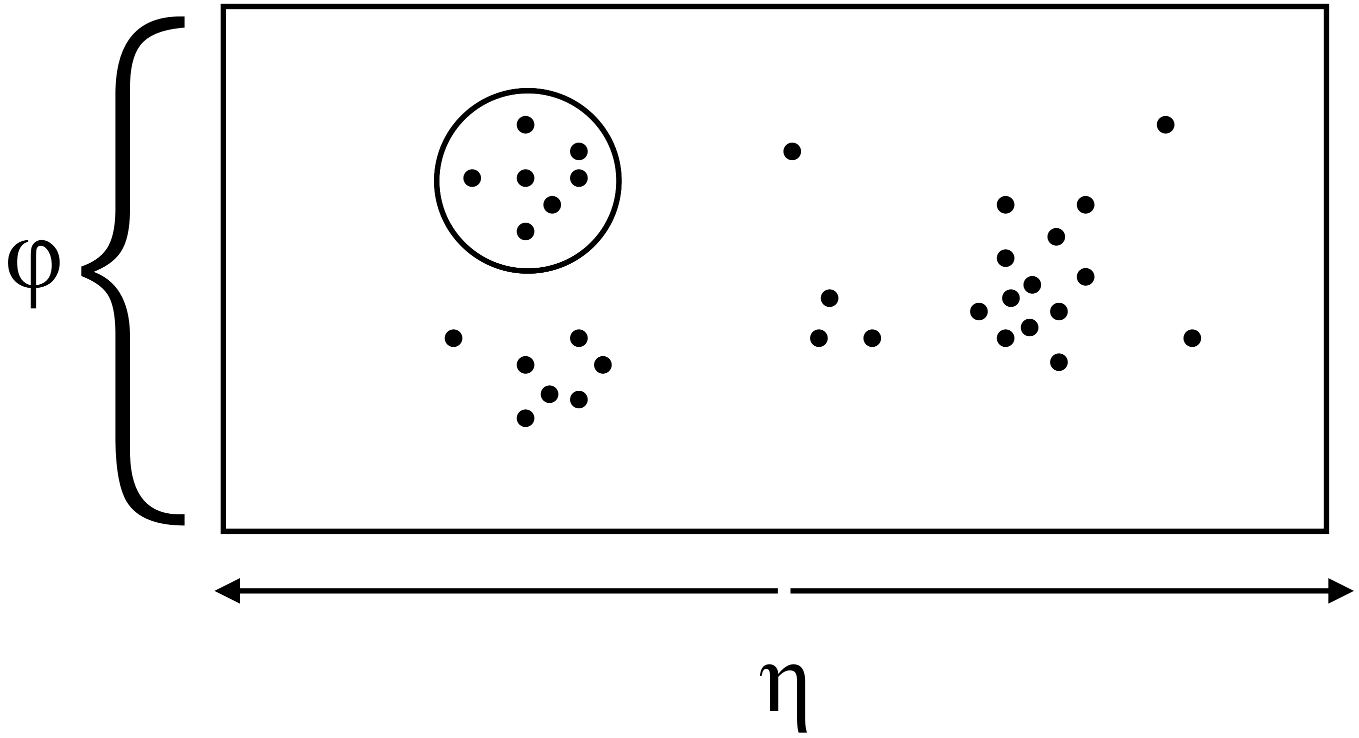
- cone algorithm: draw a cone and collect all the particles contained

how to fix the jet axis?

3. Mid point algorithm

   A. Find stable cones
   B. Add mid point among them as new seeds

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$



- the problem is not really solved, it present again at higher multiplicity of the theory computation

Jet algorithm

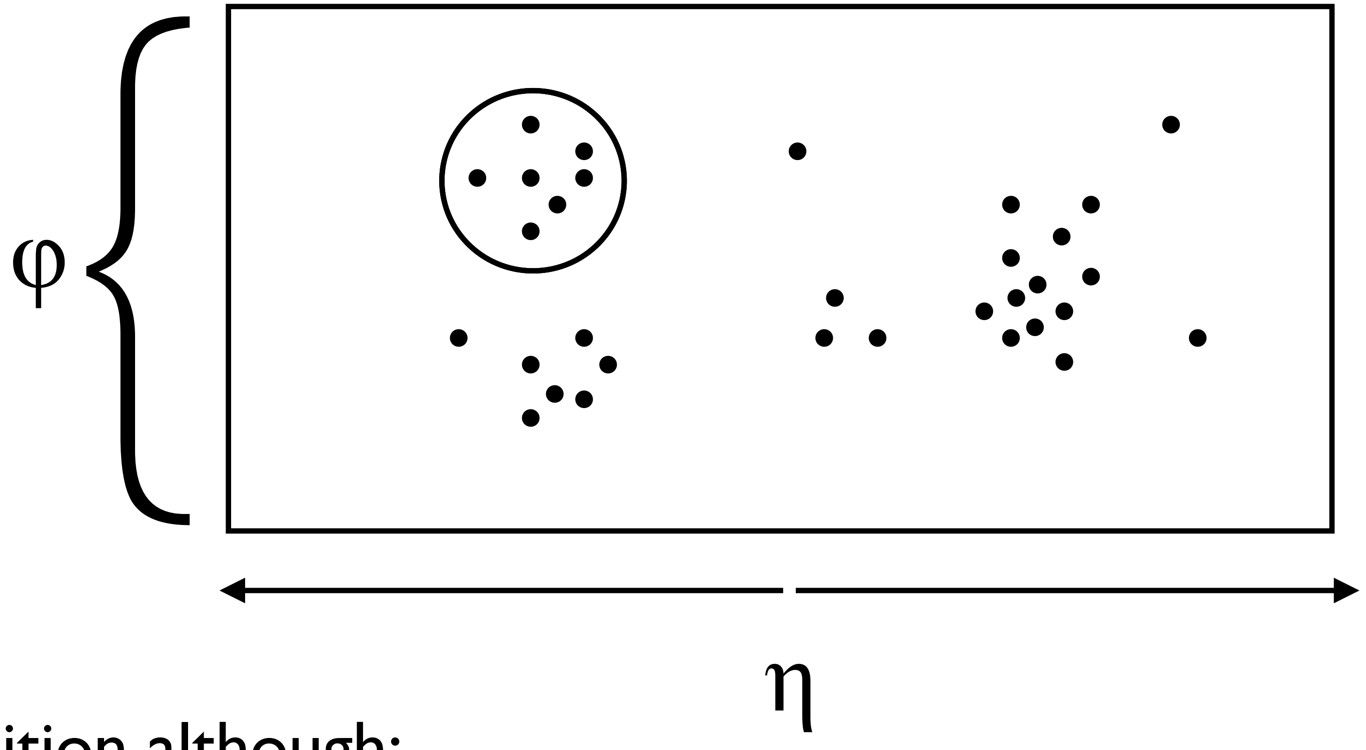
- cone algorithm: draw a cone and collect all the particles contained

how to fix the jet axis?

4. Seedless cone algorithm

A. Consider a subset of particles
B. Combine momenta and search for a stable cone
C. Make it for all the subsets
D. Use split-merge

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$



- solution of the problem: fully IR/C safe definition although:

➡ quite unpractical!

➡ Dark towers: it happen that single particles with sufficiently high energy are not assigned to any jet

Jet algorithm

- clustering algorithm: sequential recombination

A. $\forall \{i,j\}$ compute distance among them $d_{i,j}$

$\forall \{i\}$ compute distance from the beam d_{iB}

B. if the smaller is $d_{i,j} \implies$ combine i and $j \implies p_{ij} = p_i + p_j$

if the smaller is $d_{iB} \implies$ i is promoted to Jet

- fully exhaustive and unambiguous, fast, no dark towers
  - distances

$$d_{iB} = (p_{T_i})^p$$

$$d_{ij} = \min\left\{(p_{T_i})^p, (p_{T_j})^p\right\} \frac{R_{ij}^2}{R^2}$$

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

  - p parameter

$p = 1$: kt-alg
$p = 0$: C/A-alg
$p = -1$: anti-kt-alg

  - IR/C safety
    - ✓ in the collinear limit $R_{ij} \to 0$
    - ✓ soft radiation or assigned to a jet or form a jet by itself, but a jet below any measurable threshold

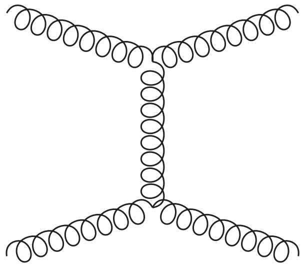
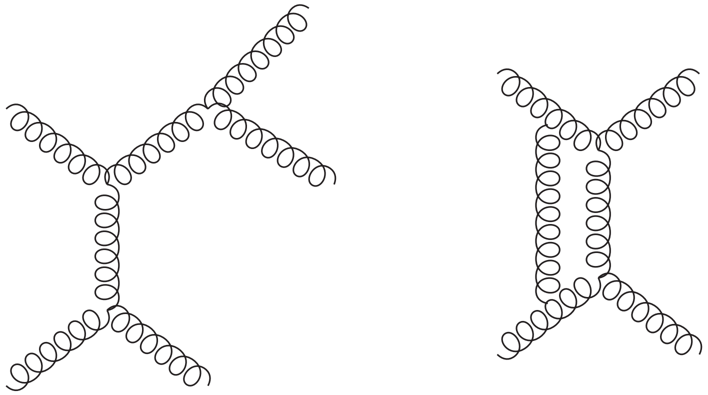
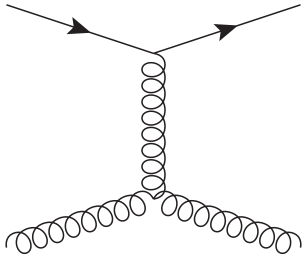
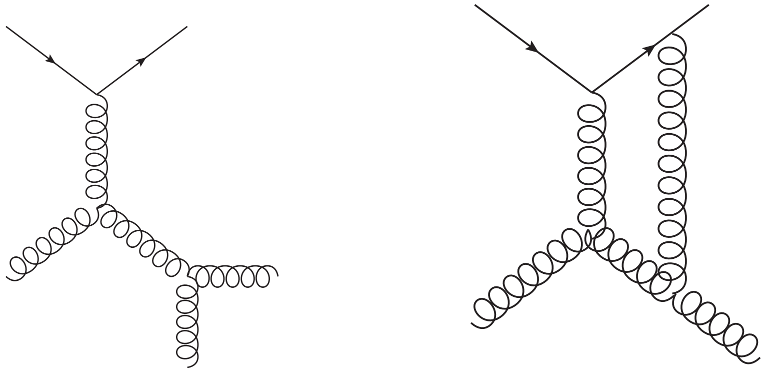
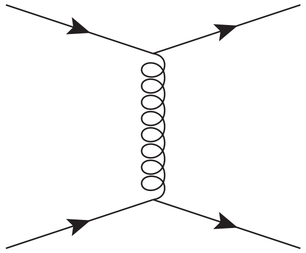
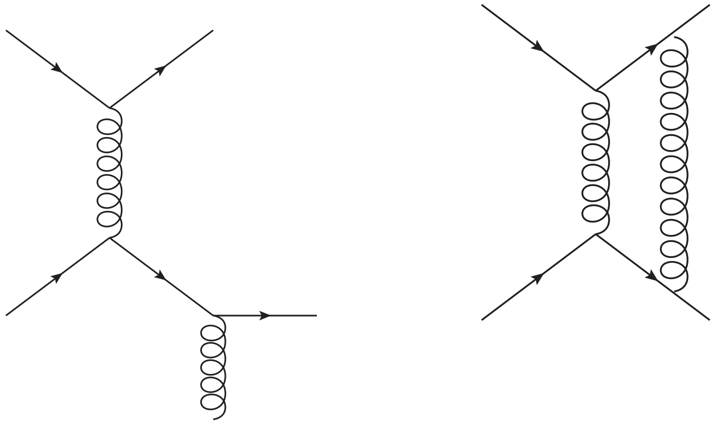
➡ R acts as the radius for the cone algorithm

➡ with anti-kt jet contours are better defined (sharp boundary), it is less sensitive to the detailed structure of the soft component of the event: <u>standard at LHC</u>

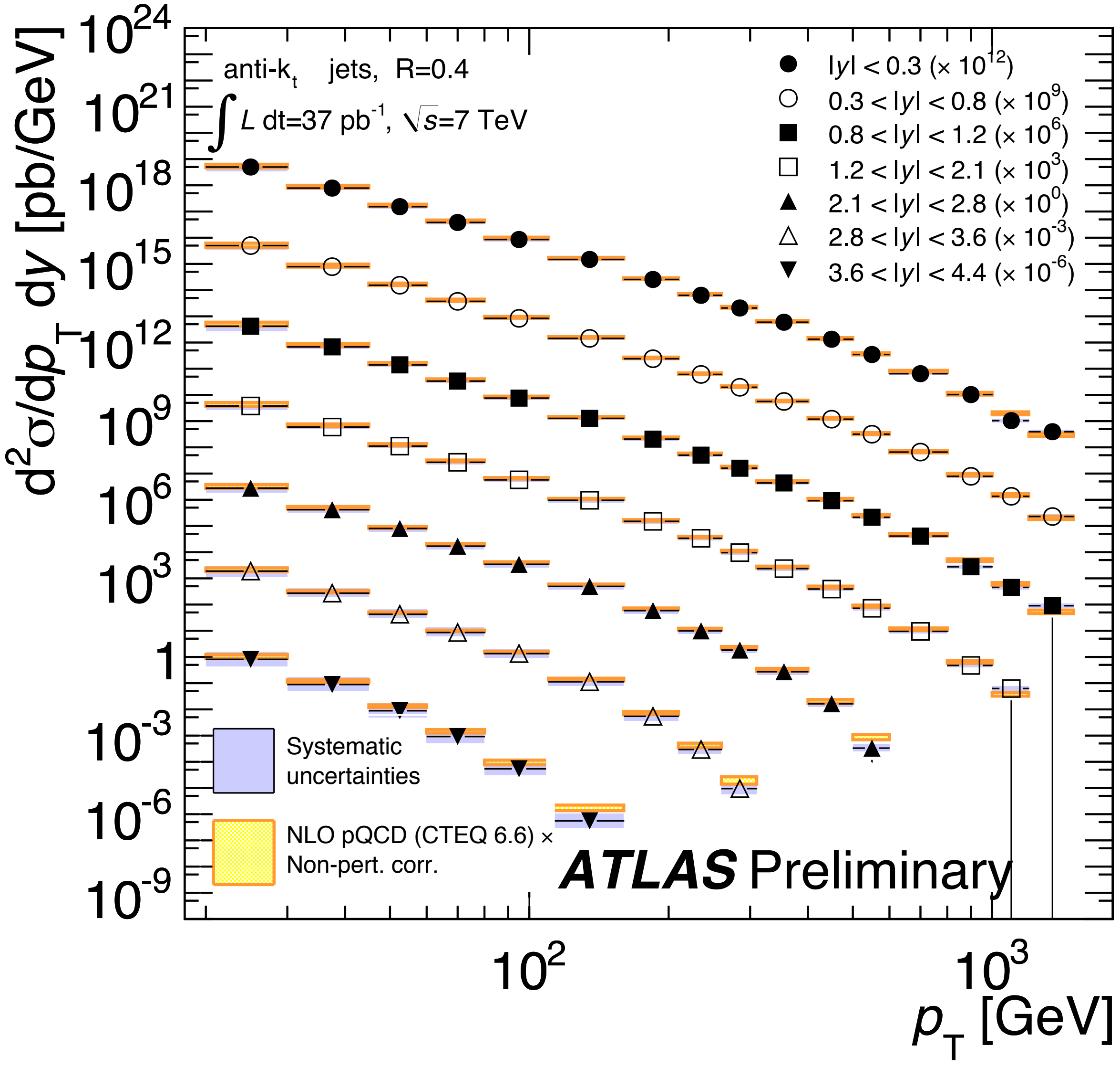
NLO computation

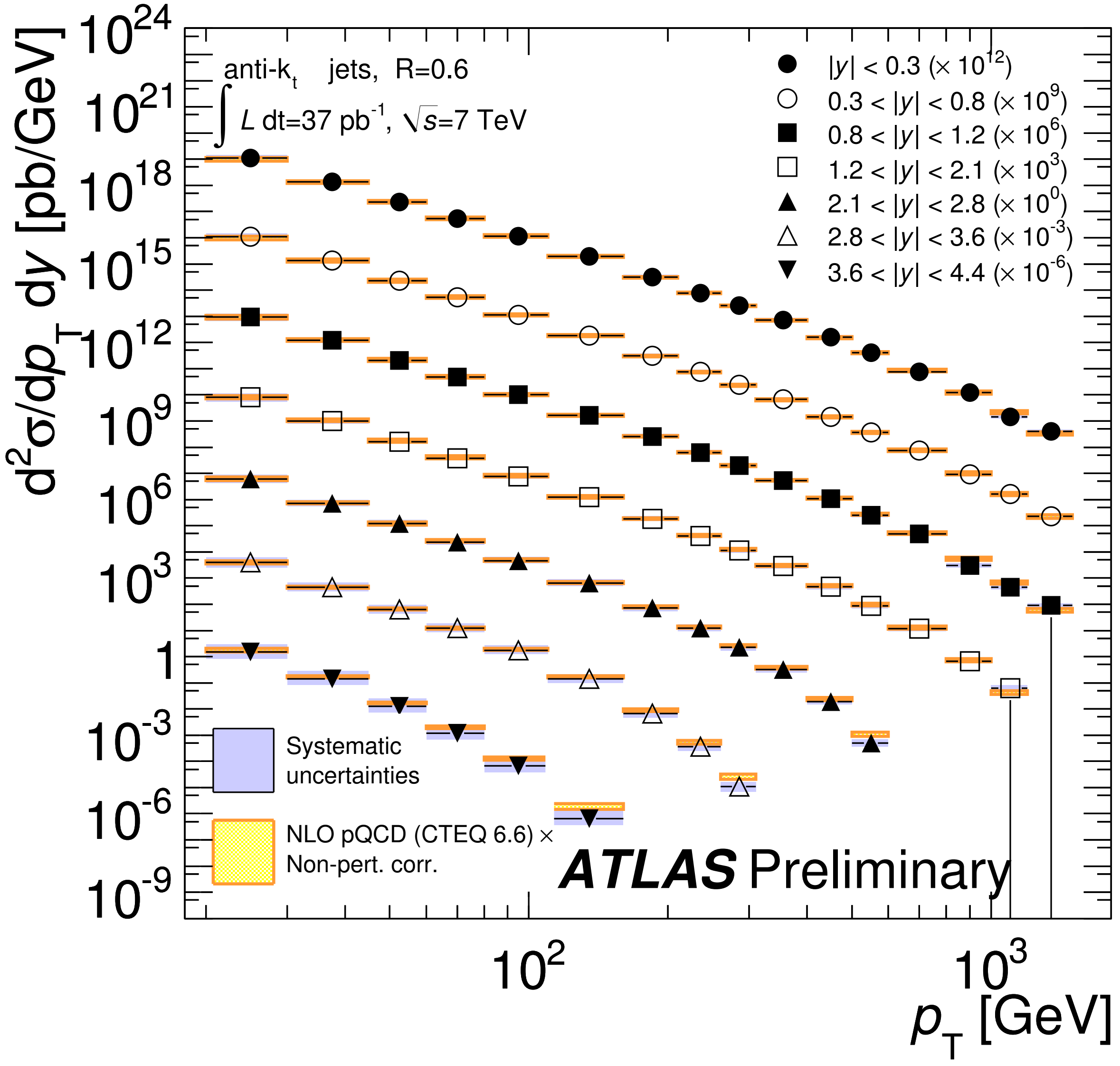
- known since a while

[R.K.Ellis, Sexton (1985)]
[Aversa, Chiappetta, Greco, Guillet (1989)]
[S.Ellis, Kunszt, Soper (1990)]

	LO: $O\left(\alpha_S^2\right)$	NLO: $O\left(\alpha_S^3\right)$
gg		
qg		
qq		

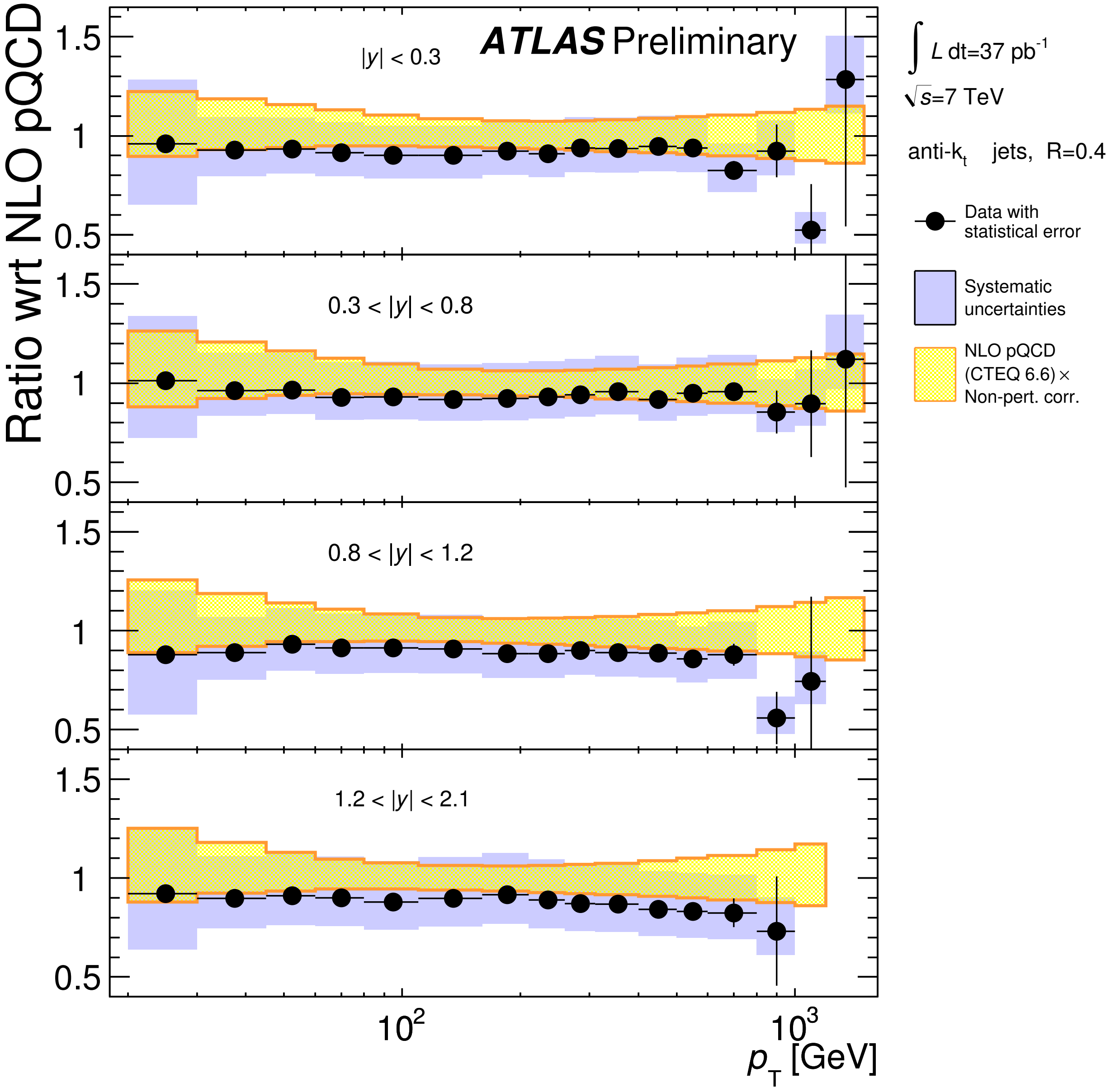
Inclusive jet cross section at the LHC

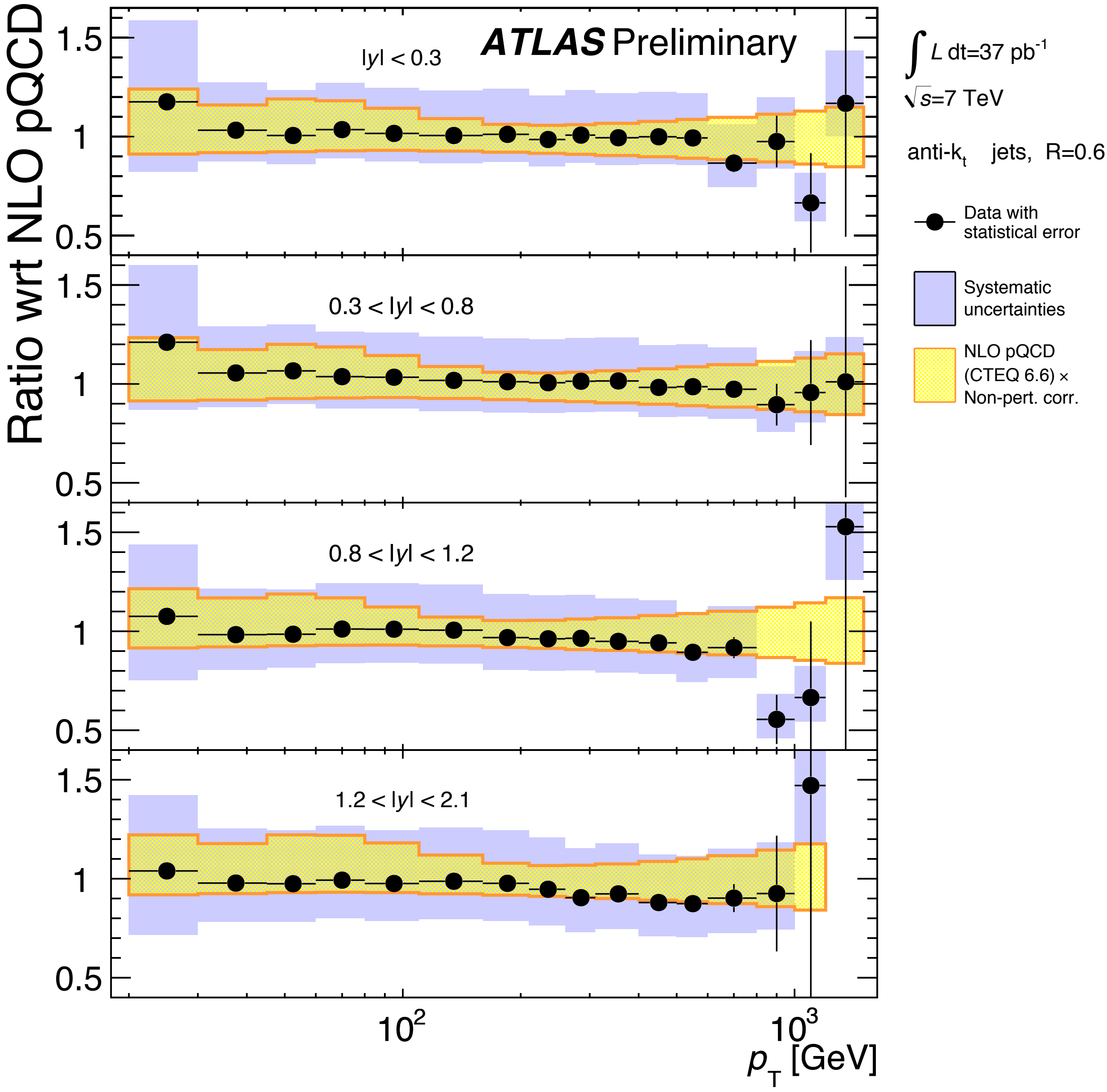




- double differential
- R dependence
- normalisation error in the prediction from scale variations, pdf and strong coupling
- non perturbative corrections estimation: using MC generators to evaluate the ratio of cross sections with and without hadronisation and underlying event, and multiplying bin-by-bin the NLO parton-level cross sections by this ratio. Its uncertainty is estimated as the maximum spread of the correction factors obtained from different Monte Carlo tools.

Inclusive jet cross section at the LHC



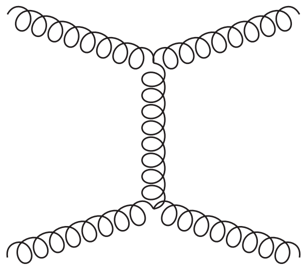
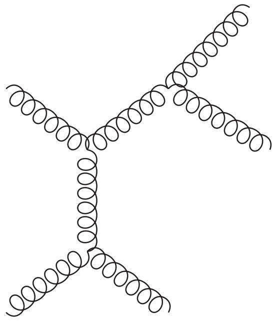
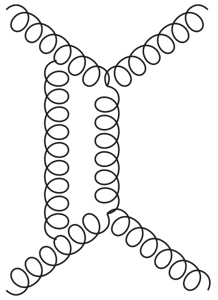
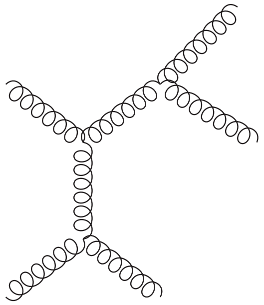
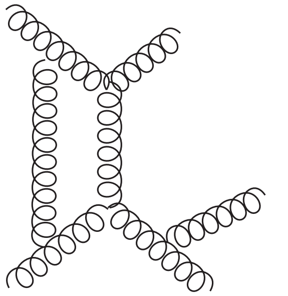
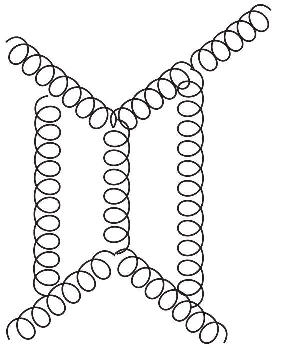
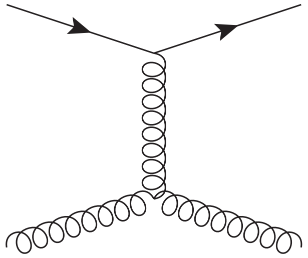
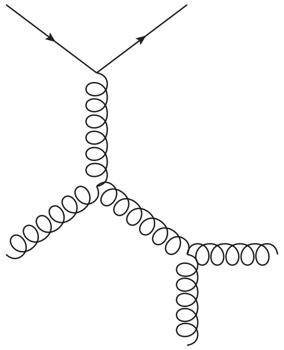
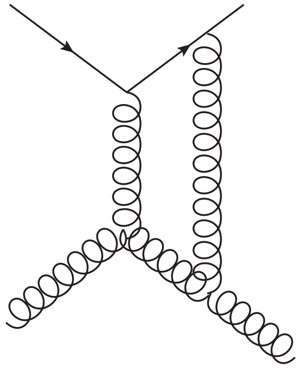
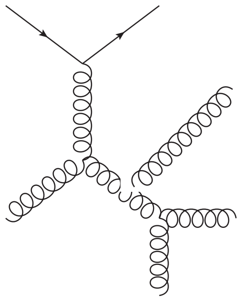
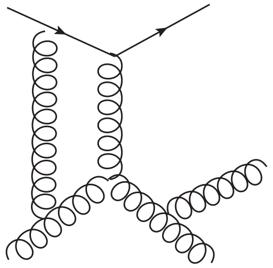
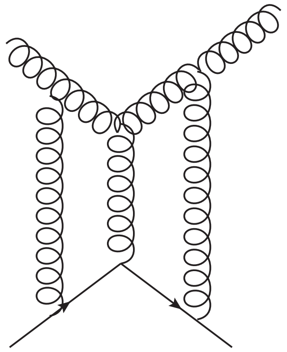
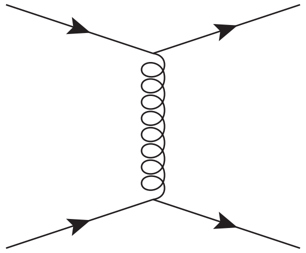
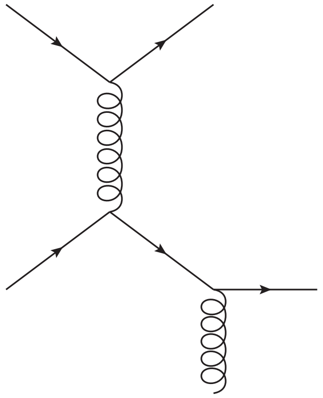
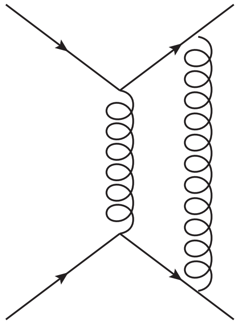
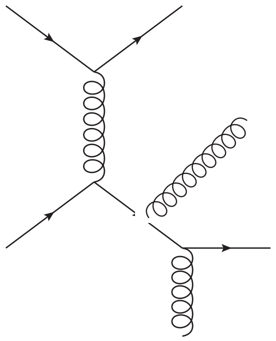
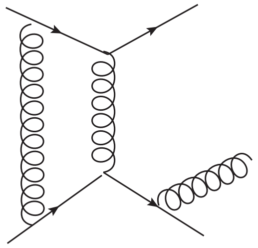
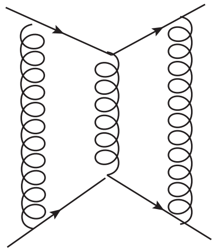


- pQCD+NP correction and EXP uncertainties of similar size

Theory prediction for high pt jets with differential NNLO computation

- Leading colour

[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 2017]

	LO: $O\left(\alpha_S^2\right)$	NLO: $O\left(\alpha_S^3\right)$		NNLO: $O(\alpha_S^4)$		
	$2 \to 2$	$2 \to 3$	$2 \to 2$	$2 \to 4$	$2 \to 3$	$2 \to 2$
gg						
qg						
qq						

Theory prediction for high pt jets with differential NNLO computation

- Leading colour

[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 2017]

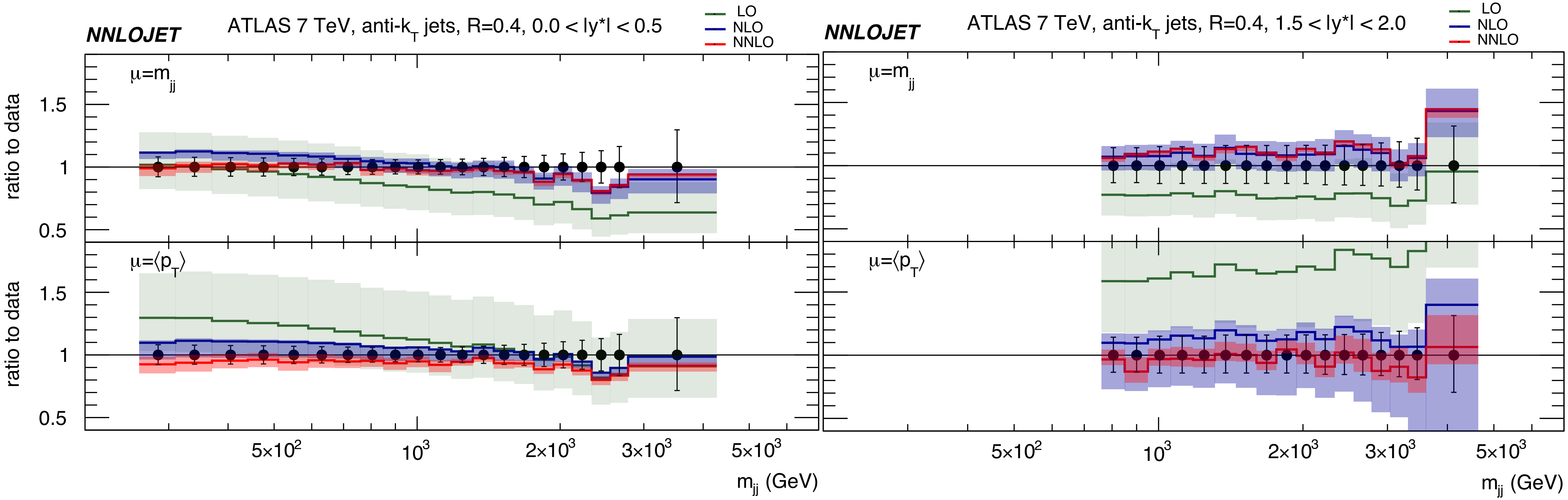
- scale choice

$$p_{T_1} > 100\text{GeV} \qquad p_{T_2} > 50\text{GeV}$$

$$\mu = m_{jj}$$

$$\mu = \langle p_T \rangle = \frac{1}{2}\left(p_{T_1} + p_{T_2}\right)$$

Shows a slightly better convergence for smaller and larger y* values



Theory prediction

[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 2017]

- scale dependence

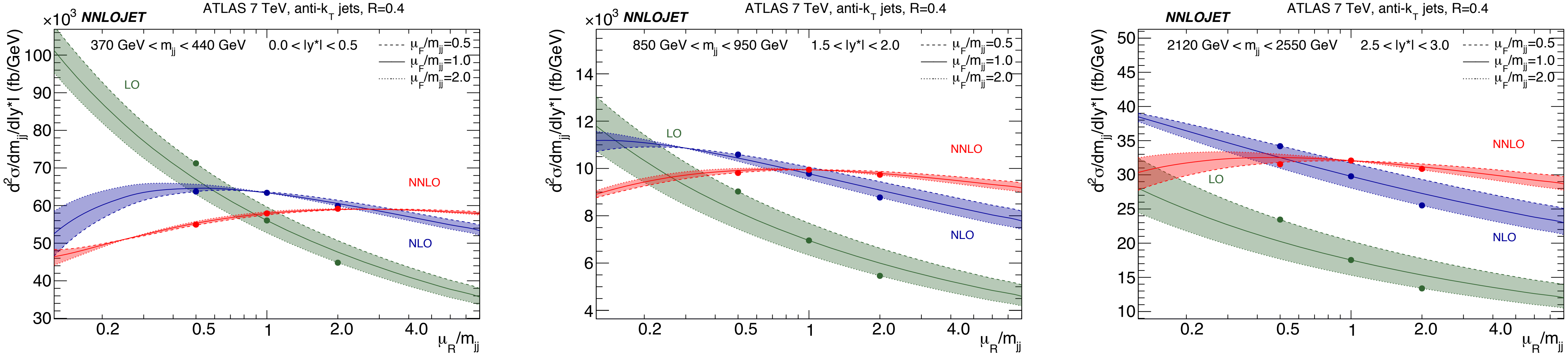


FIG. 5: The cross section evaluated in three bins as a function of μ_R/m_{jj}: 370 GeV $< m_{jj} <$ 440 GeV, $0.0 < |y^*| < 0.5$ (left); 850 GeV $< m_{jj} <$ 950 GeV, $1.5 < |y^*| < 2.0$ (centre); 2120 GeV $< m_{jj} <$ 2550 GeV, $2.5 < |y^*| < 3.0$ (right). The variation of the cross section, for fixed μ_F/m_{jj}, from the central value is shown as solid lines, computed from the renormalization group equation, for LO (green), NLO (blue) and NNLO (red). The thickness of the bands shows the variation due to factorization scale, with boundaries given by $\mu_F/m_{jj} = 0.5$ (dashed) and $\mu_F/m_{jj} = 2.0$ (dotted). The points show the NNLOJET result evaluated at $\mu_R/m_{jj} = \mu_F/m_{jj} = \{0.5, 1, 2\}$.

- flatter and thiner from one order to the next
- non overlapping (left plot) signal accidental cancellations of scale dependence (higer order or larger scale variation needed)

Theory prediction for high pt jets

[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 2017]

- excellent agreement over about 7 orders of magnitude

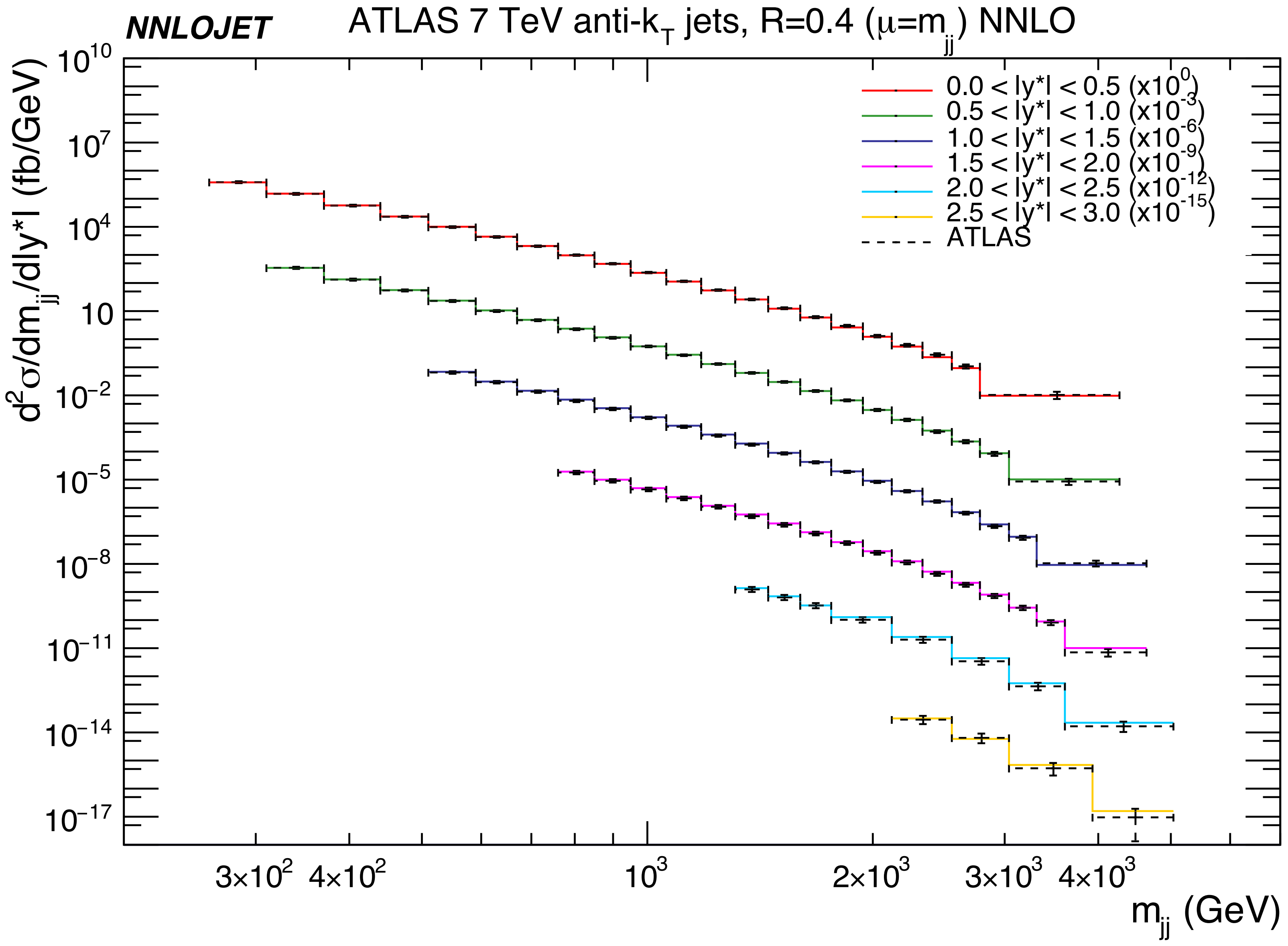


FIG. 2: The dijet cross section as a function of invariant mass, m_{jj}, for the six bins of $|y^*|$, compared to ATLAS 7 TeV 4.5 fb^{-1} data.