QCD.4

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QCD at higher and all orders

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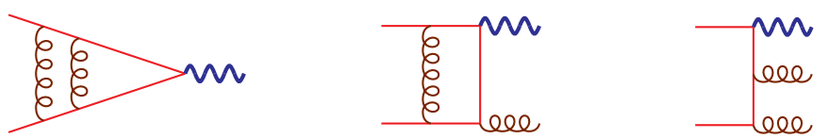
Plan of lecture 4

- Higher order corrections for LHC processes
- Resummation for specific distributions
- Parton shower
- Merging, Matching, and both

Higher order QCD corrections in hadron collisions

- Drell Yan process and Di-boson production known including up to two loop QCD at fully differential level

[Anastasiou, Dixon, Melnikov, Petriello 2004]



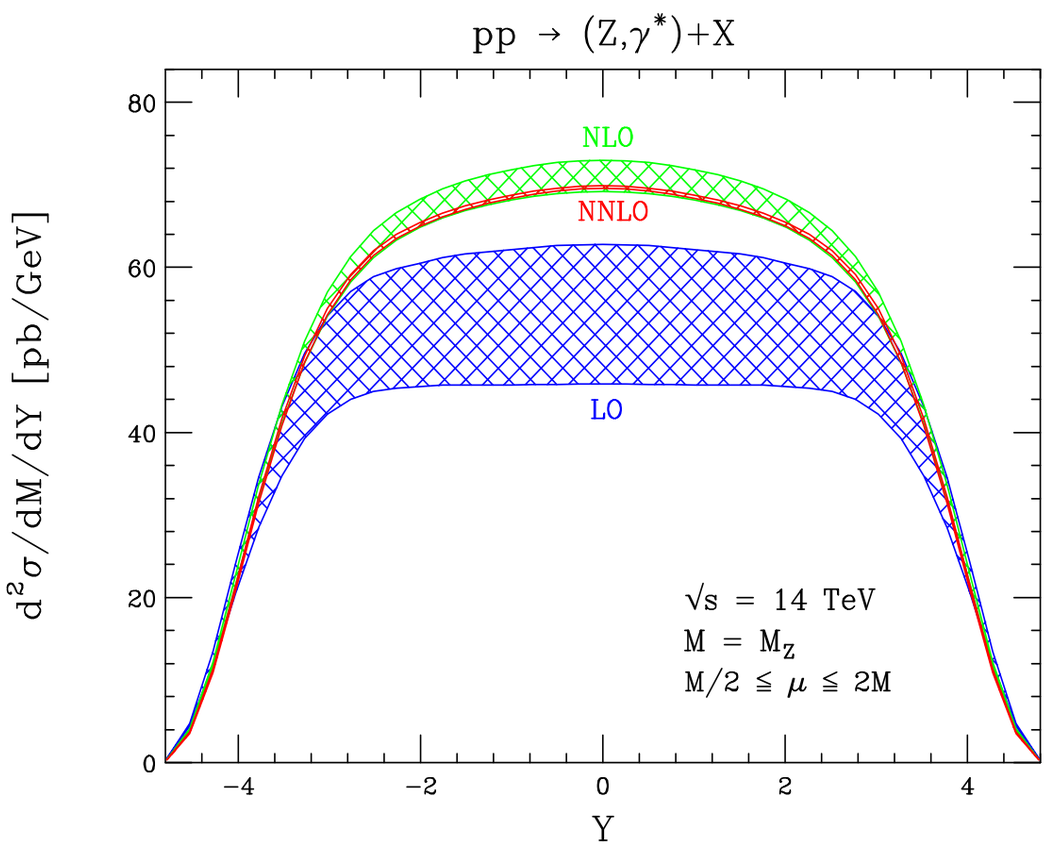
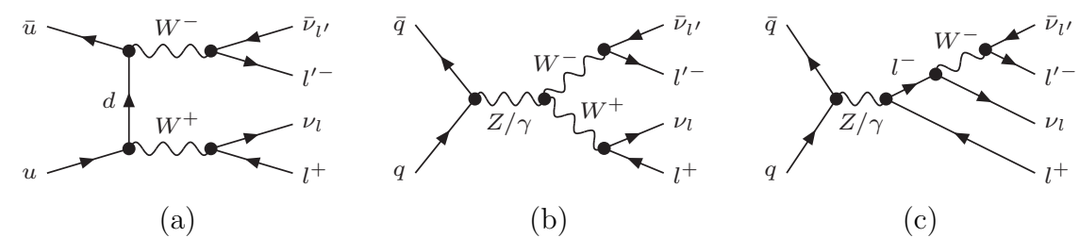
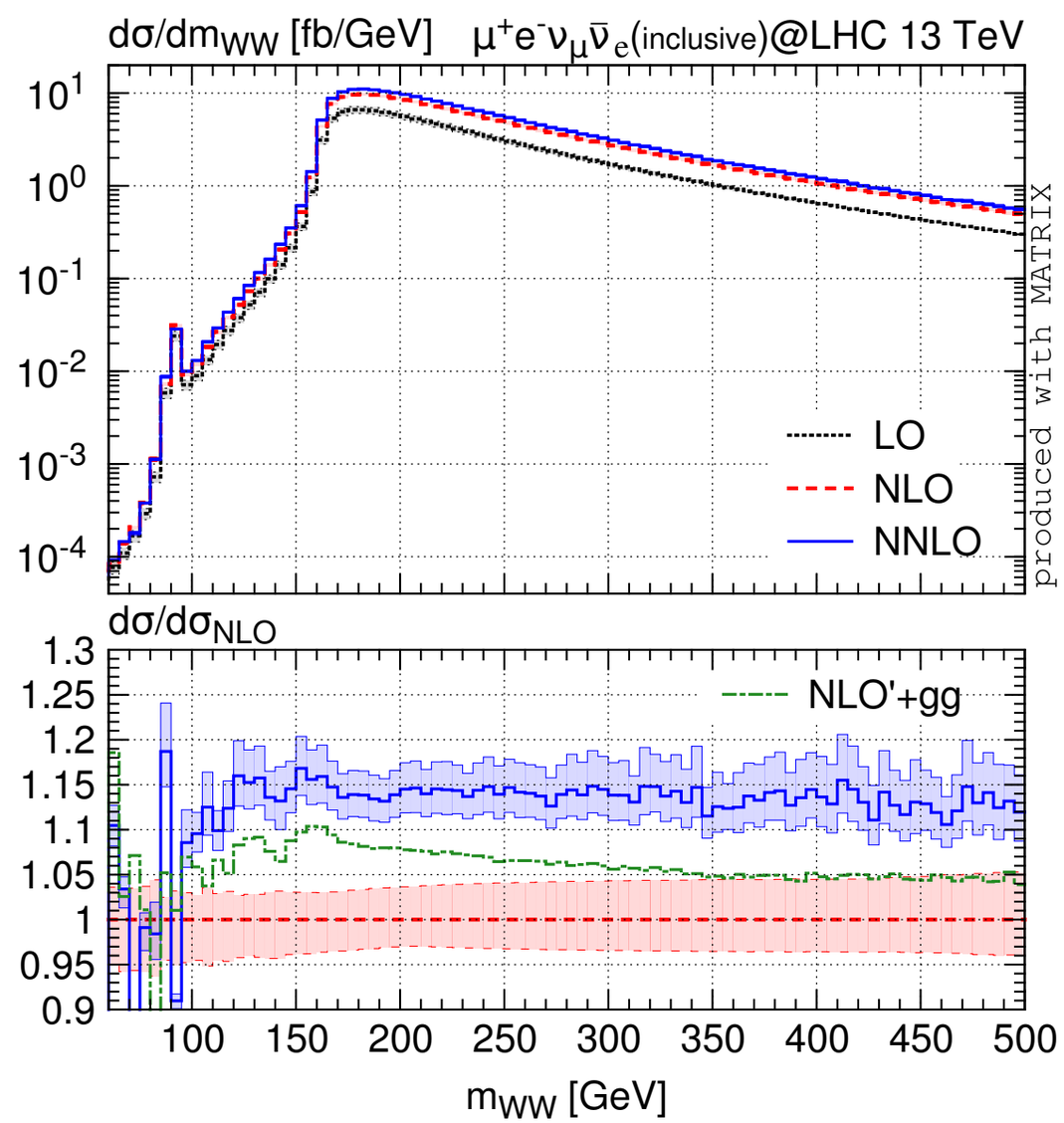


Figure 3: The CMS rapidity distribution of an on-shell Z boson at the LHC. The LO, NLO, and NNLO results have been included. The bands indicate the variation of the renormalization and factorization scales in the range $M_Z/2 \leq \mu \leq 2M_Z$.

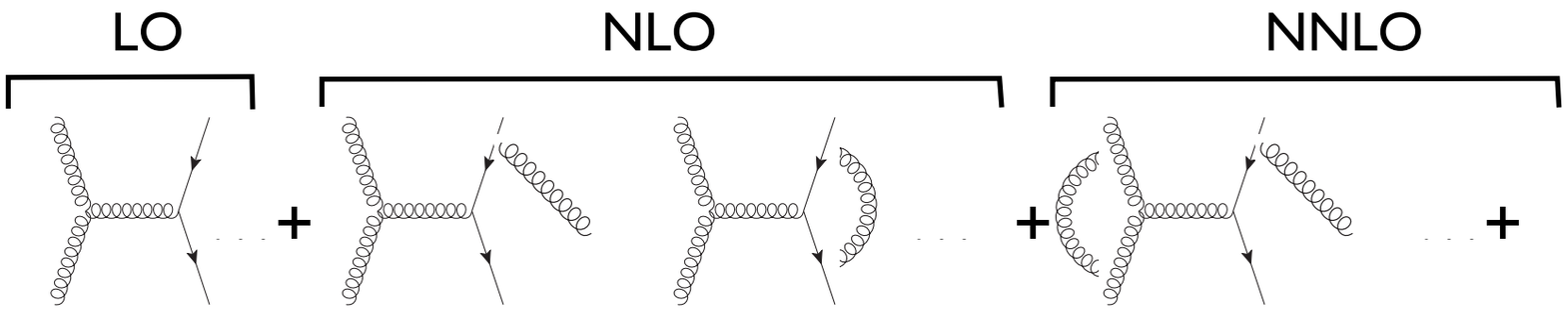
[Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann 2016]



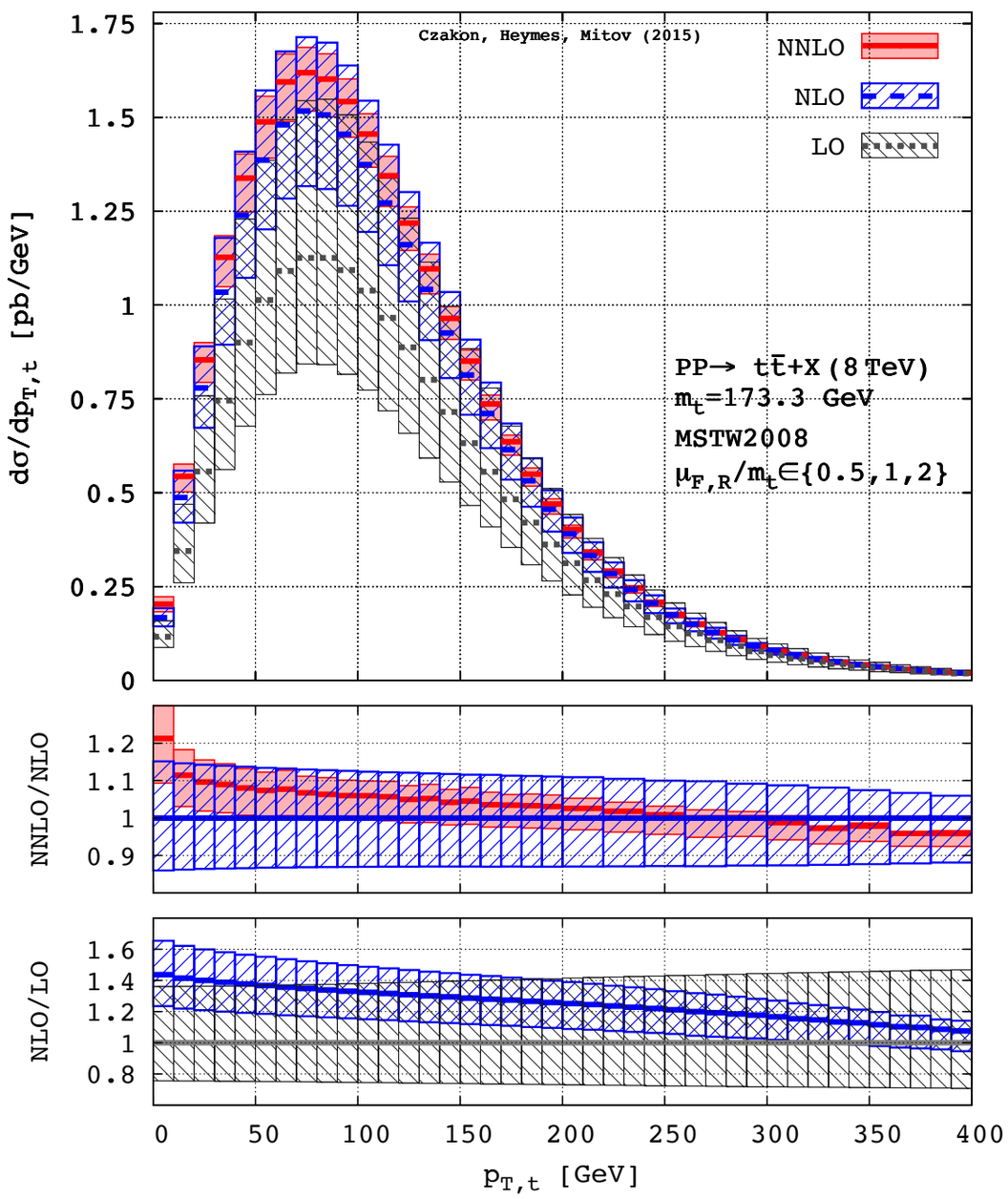


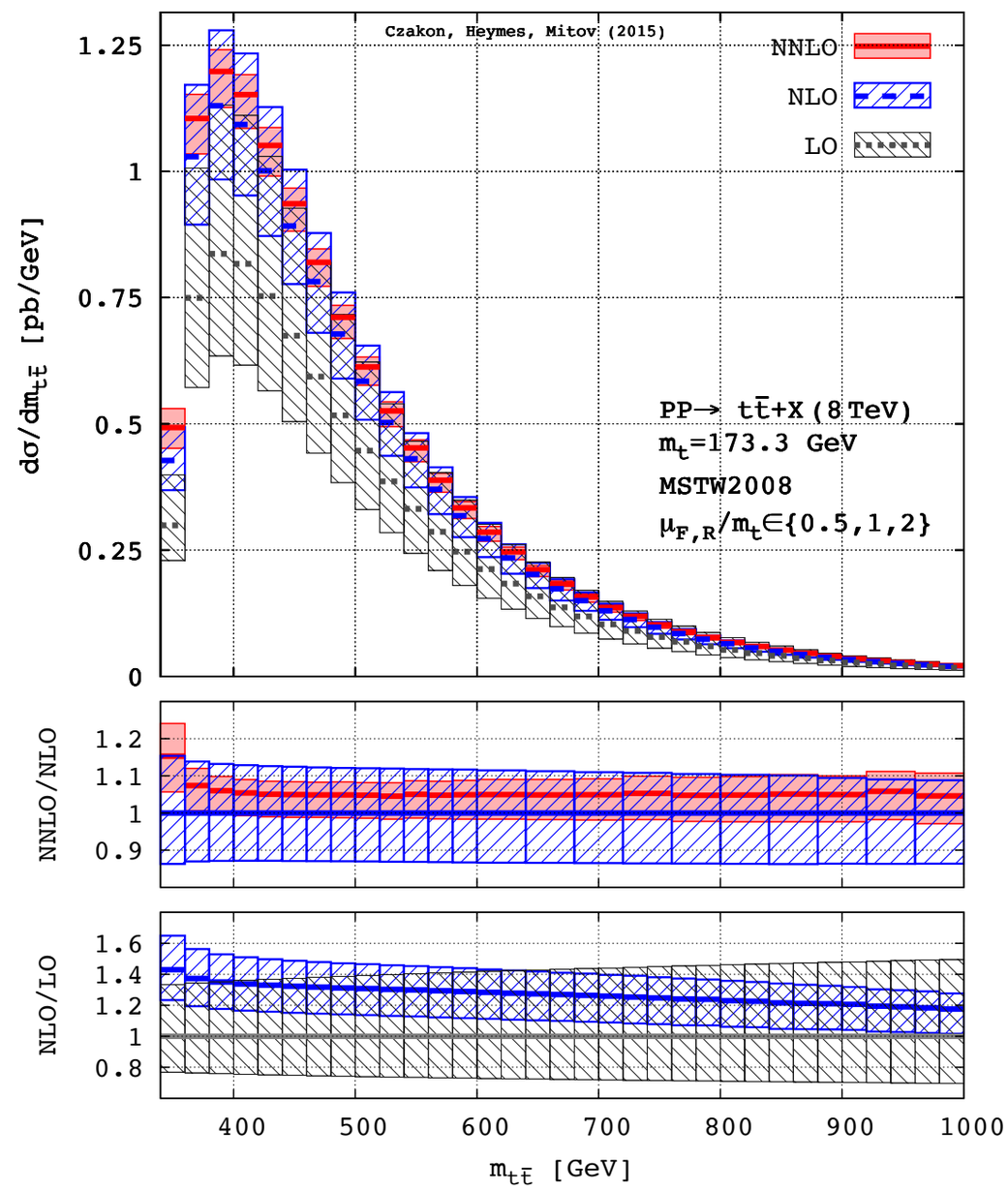
Higher order QCD corrections in hadron collisions

- stable top anti-top quark production is known including up to second order QCD corrections [Czakon, Fielder, Mitov 2013]



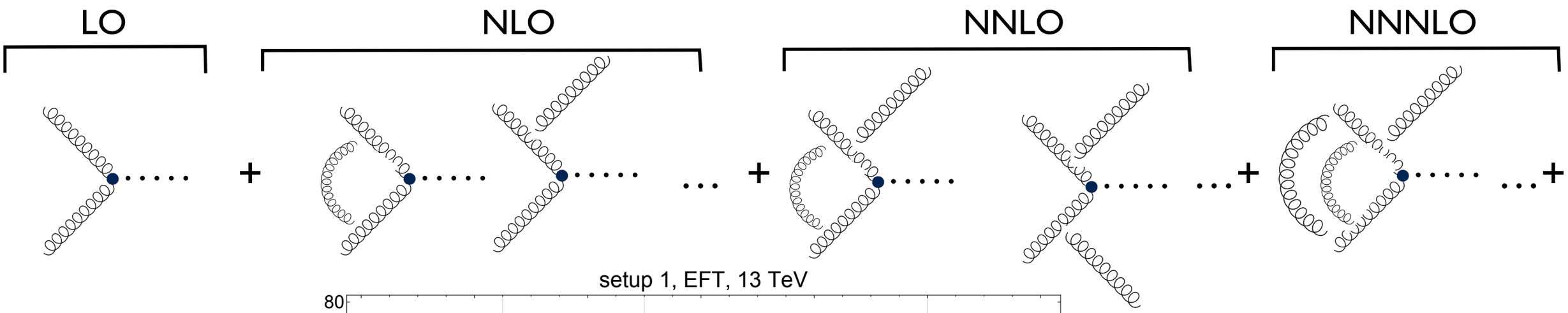
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$^{+0.259(3.7\%)}_{-0.374(5.3\%)}$	$^{+0.169(2.4\%)}_{-0.121(1.7\%)}$
LHC 7 TeV	167.0	$^{+6.7(4.0\%)}_{-10.7(6.4\%)}$	$^{+4.6(2.8\%)}_{-4.7(2.8\%)}$
LHC 8 TeV	239.1	$^{+9.2(3.9\%)}_{-14.8(6.2\%)}$	$^{+6.1(2.5\%)}_{-6.2(2.6\%)}$
LHC 14 TeV	933.0	$^{+31.8(3.4\%)}_{-51.0(5.5\%)}$	$^{+16.1(1.7\%)}_{-17.6(1.9\%)}$





Higher order QCD corrections in hadron collisions

- differential (inclusive) Higgs boson production in gluon fusion and vector boson fusion is known including up to two (three) loops by several (1) groups ([Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2015])



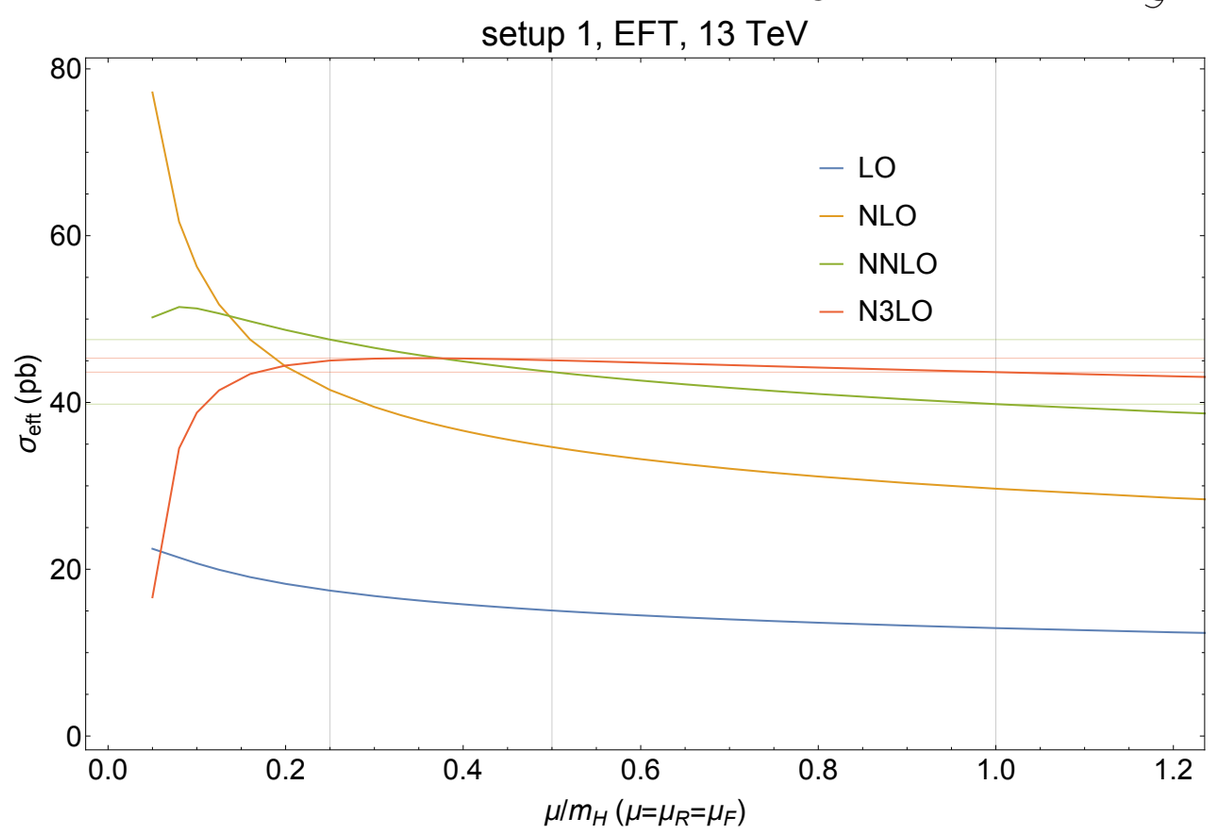


Figure 8: The dependence of the cross-section on a common renormalization and factorization scale $\mu = \mu_F = \mu_R$.

Rapid progress of the theory community on such calculations:

➡ on both the calculation of difficult integrals and the construction of general subtraction schemes

➡ all the complicated calculations carried out so far have proved useful to match the experimental precision!

All order corrections

- Note however that logarithmic enhancement in the computation of observables are very frequent

- They require resummation

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➡ UV log: Running, the Renormalisation Group Equation resume this log

$$\alpha_S(m_Z^2) = \frac{\alpha_S(m_\tau^2)}{1 + b_0 \alpha_S(m_\tau^2) \log \frac{m_Z^2}{m_\tau^2}} \neq \alpha_S(m_\tau^2) \left(1 - b_0 \alpha_S(m_\tau^2) \log \frac{m_Z^2}{m_\tau^2}\right)$$

$$\left.\begin{array}{l} \alpha_S(m_\tau^2) \sim 0.36 \\ \\ \alpha_S(m_Z^2) \sim 0.12 \end{array}\right\} \longrightarrow \quad b_0 \alpha_S(m_\tau^2) \log \frac{m_Z^2}{m_\tau^2} \sim 2$$

All order corrections

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- They require resummation:

➡ UV log: Running, the Renormalisation Group Equation resume this log

➡ collinear log: Altarelli-Parisi Equation resume these logs

$$\frac{d}{d\log\mu^2} f_i^H(x,\mu) = \int_x^1 \frac{dz}{z} \sum_j P_{ij}(\alpha_S(\mu), z) f_j^H(x/z, \mu)$$

$$P_{ij}(\alpha_S(\mu), z) = \frac{\alpha_S(\mu)}{2\pi} P_{ij}^0(z) + \left(\frac{\alpha_S(\mu)}{2\pi}\right)^2 P_{ij}^1(z) + ...$$

All order corrections

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  ➡ UV log: Running, the Renormalisation Group Equation resume this log

  ➡ collinear log: Altarelli-Parisi Equation resume these logs

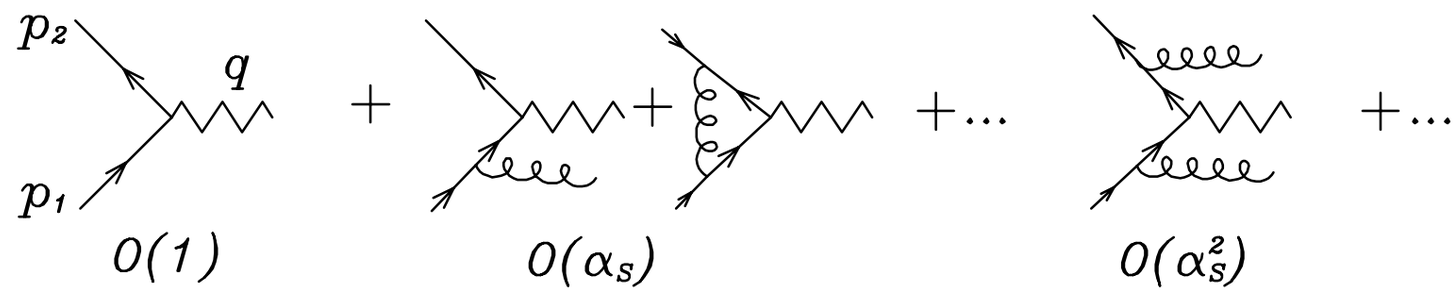
  ➡ other logs: poles cancels, but large logarithmic contribution can remain if the measurement constraint produce an unbalanced cancellation of real and virtual contribution (suppression or emphasis of the real contribution)

- basic example: Sterman-Weinberg jets

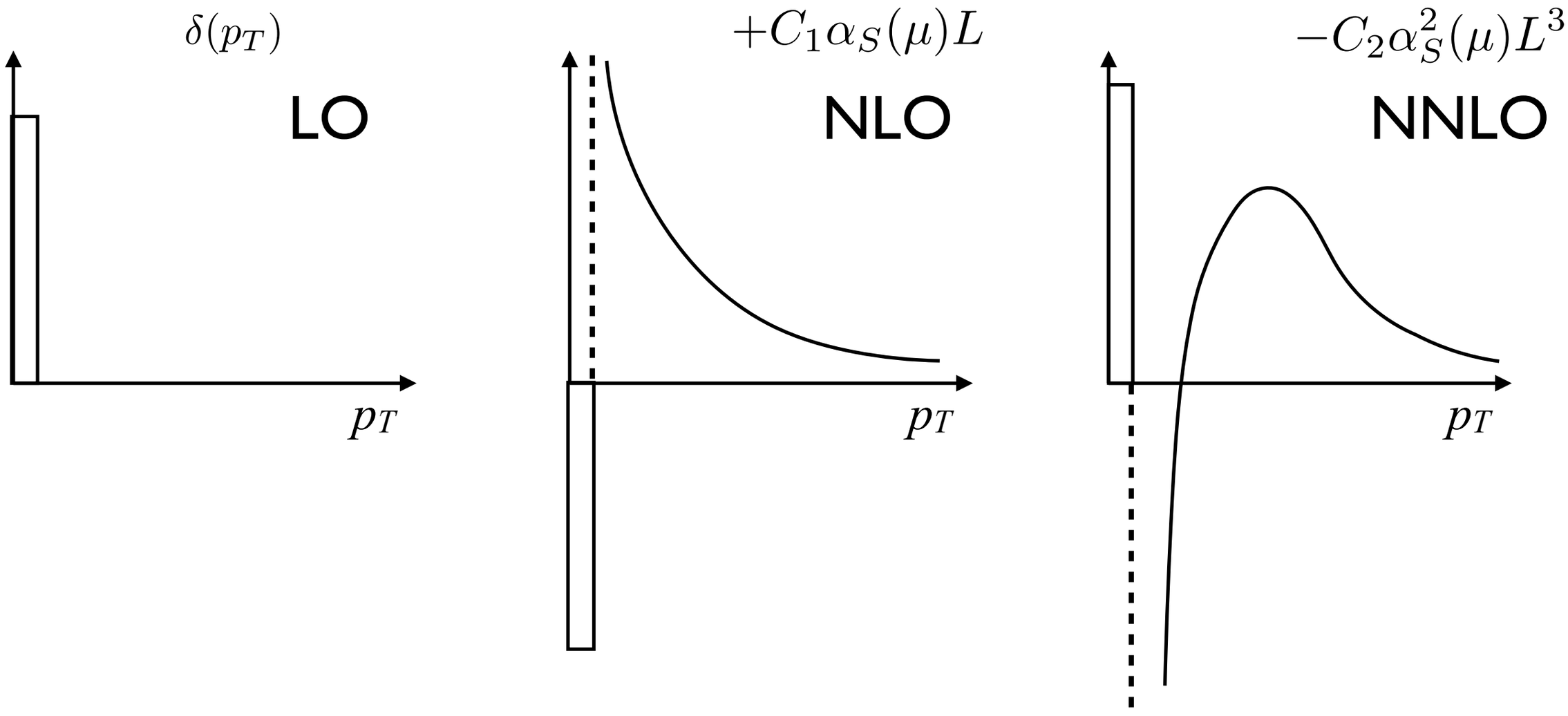
- But this situation can happen for every observable quantity

Higher order corrections

- Pt of a Z boson (or W/H)



$$L = \log \frac{M^2}{p_T^2}$$



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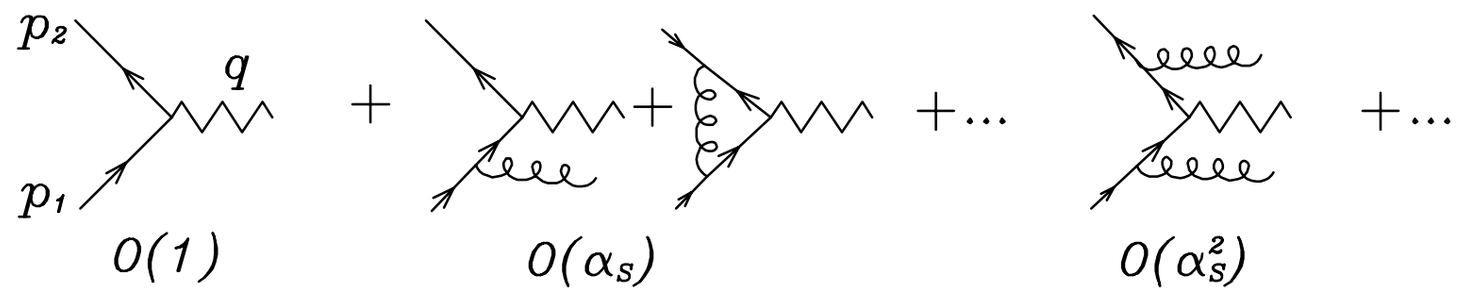
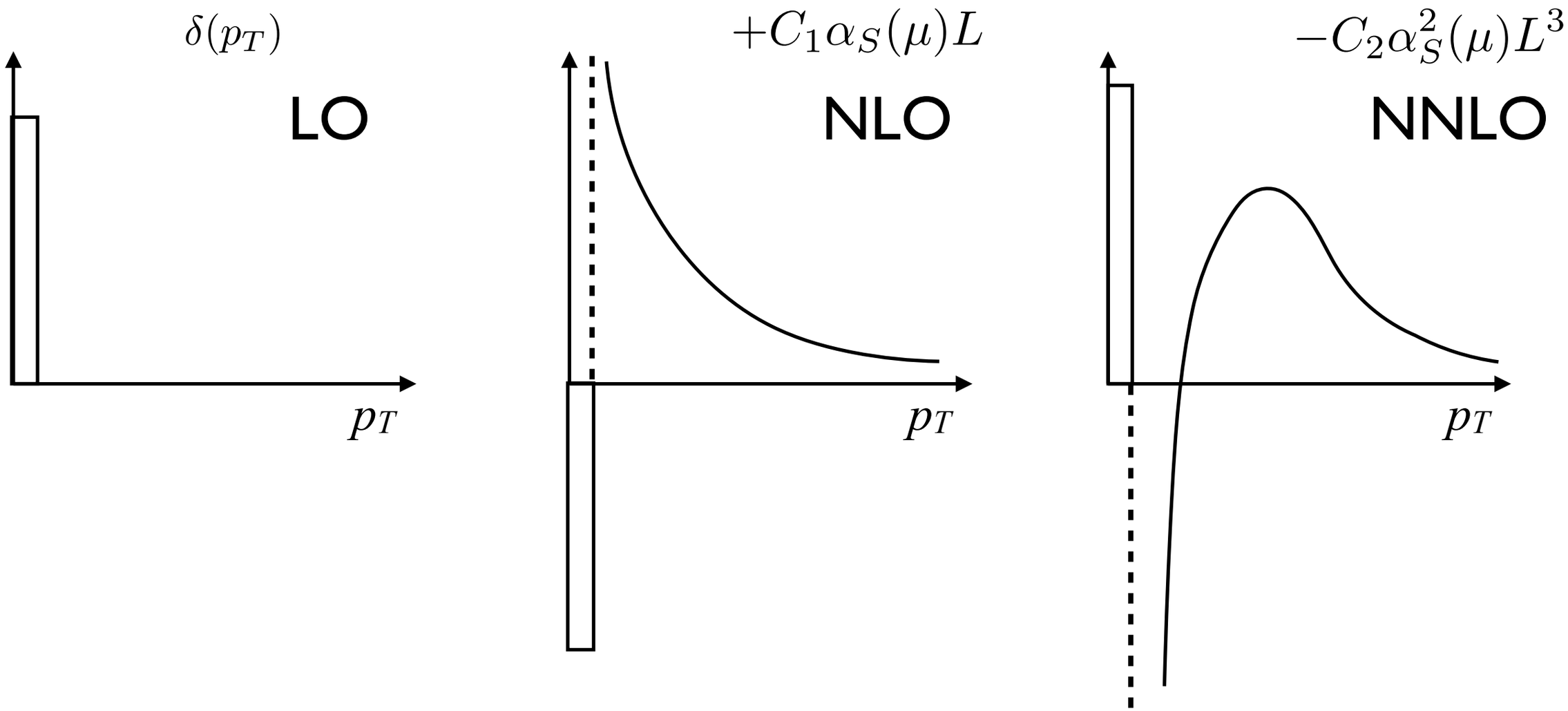


$$L = \log \frac{M^2}{p_T^2}$$

Indeed all logs $\alpha_S^n \log^m \frac{M^2}{p_T^2}$
with $0 \leq m \leq 2n-1$



Higher order corrections

- But also, for other measurements that are very well defined experimentally:
  - small mass limit of a jet, or small jet radius
  - distributions of heavy particles produced close to threshold
  - …
- Prediction for all these observables need resummation of large logs in the region where $\alpha_S L^2 \sim 1$

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Dominant Log

1			LO
$\alpha_S L^2$	$\alpha_S L$	α_S	NLO
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\alpha_S^2 L^2$ …	NNLO
$\vdots$	$\vdots$		
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	……	nNNLO
LL	NLL		

$1/L$ effect (small correction)

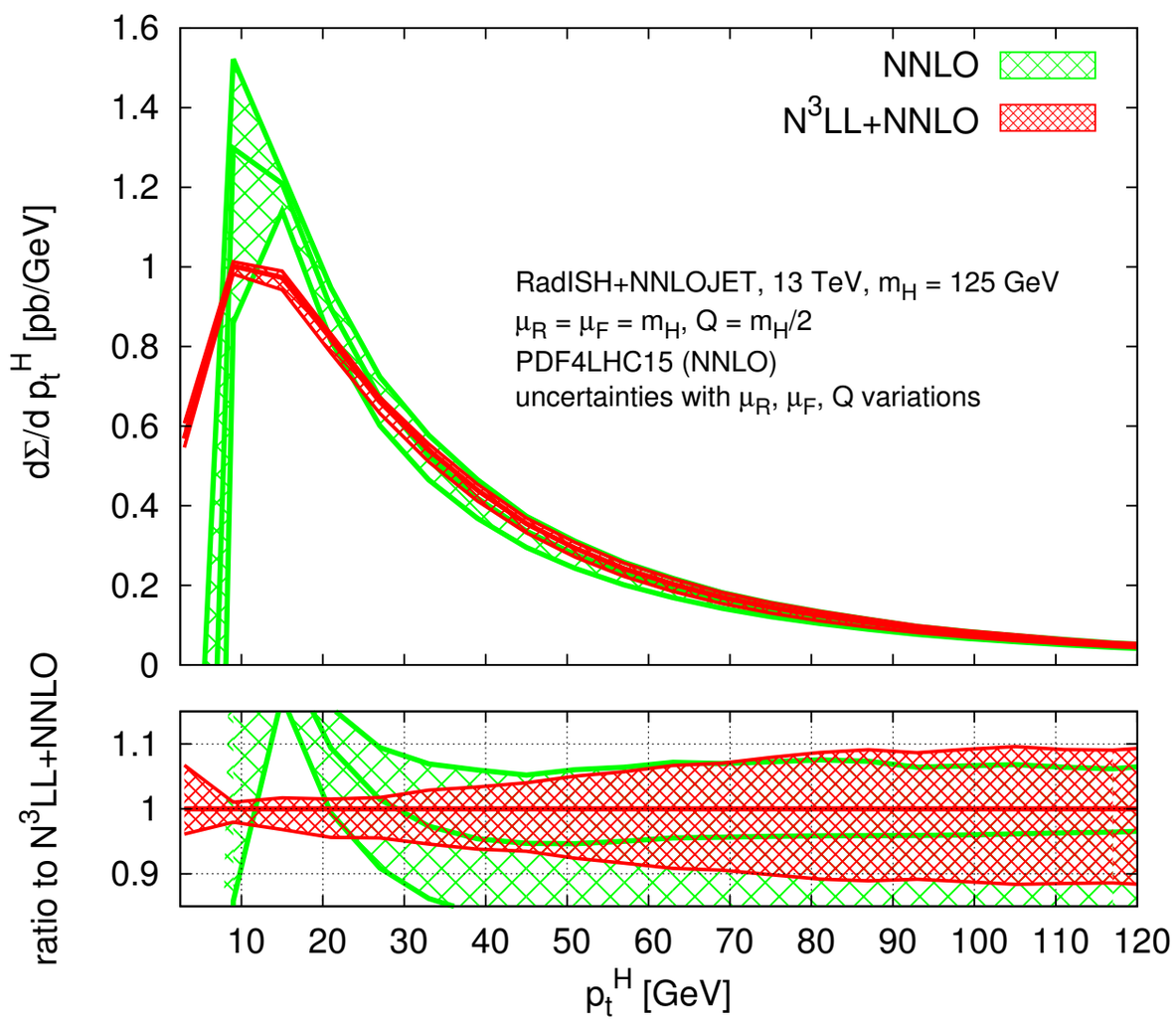
- one resum classes of contributions
- resumed results can be combined with horizontal contribution via matching (removing double counting)
- whether the series converges or not and if it has an exponential form depends on the quantity being measured: es. ycut log in e+e- annihilation does not exponentiate for the JADE algorithm but it does for k_T!

Resummation for specific distributions: Higgs boson production

- pt of the Higgs boson (EFT production) decaying into two photons

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]

- Inclusive results



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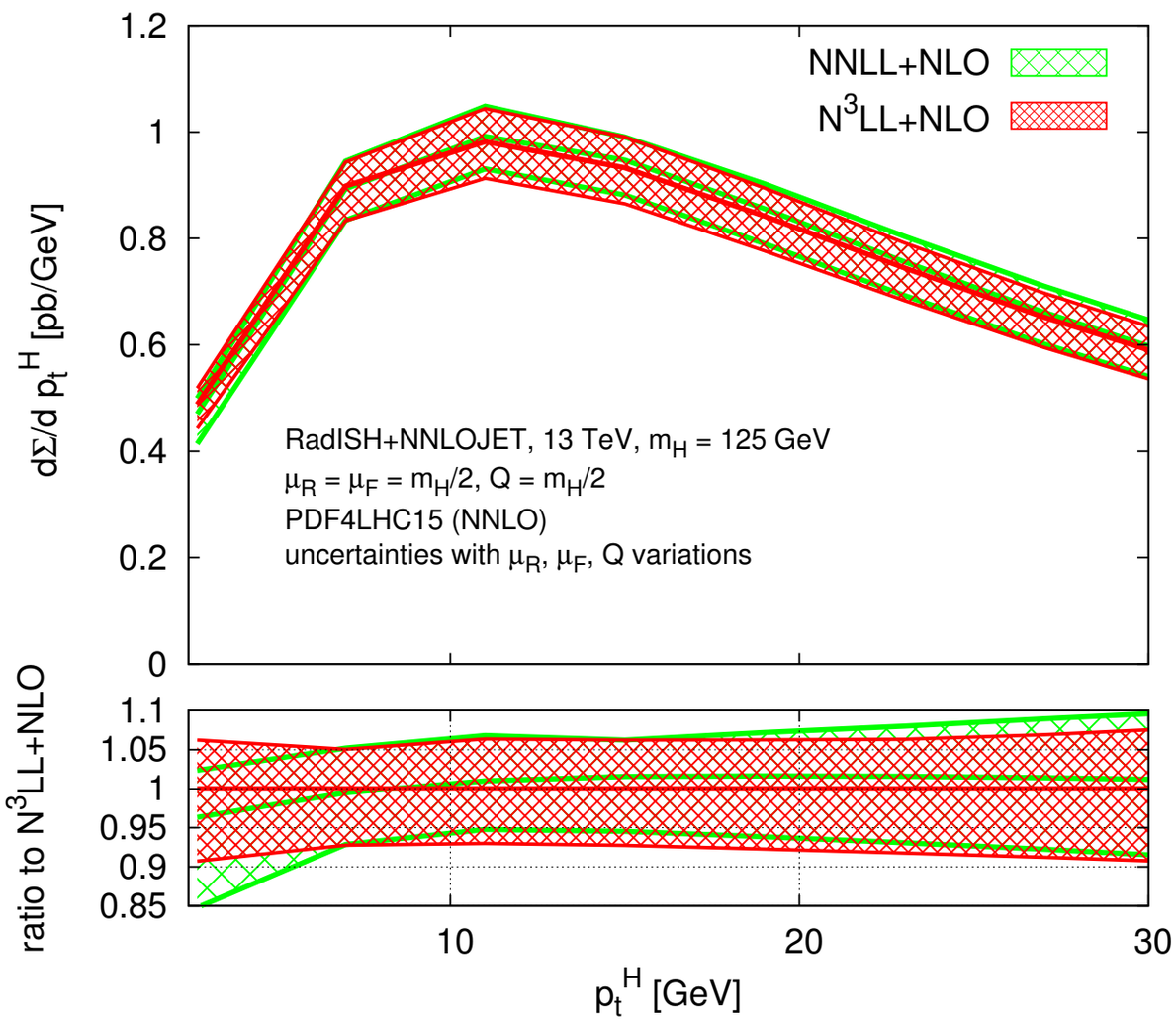

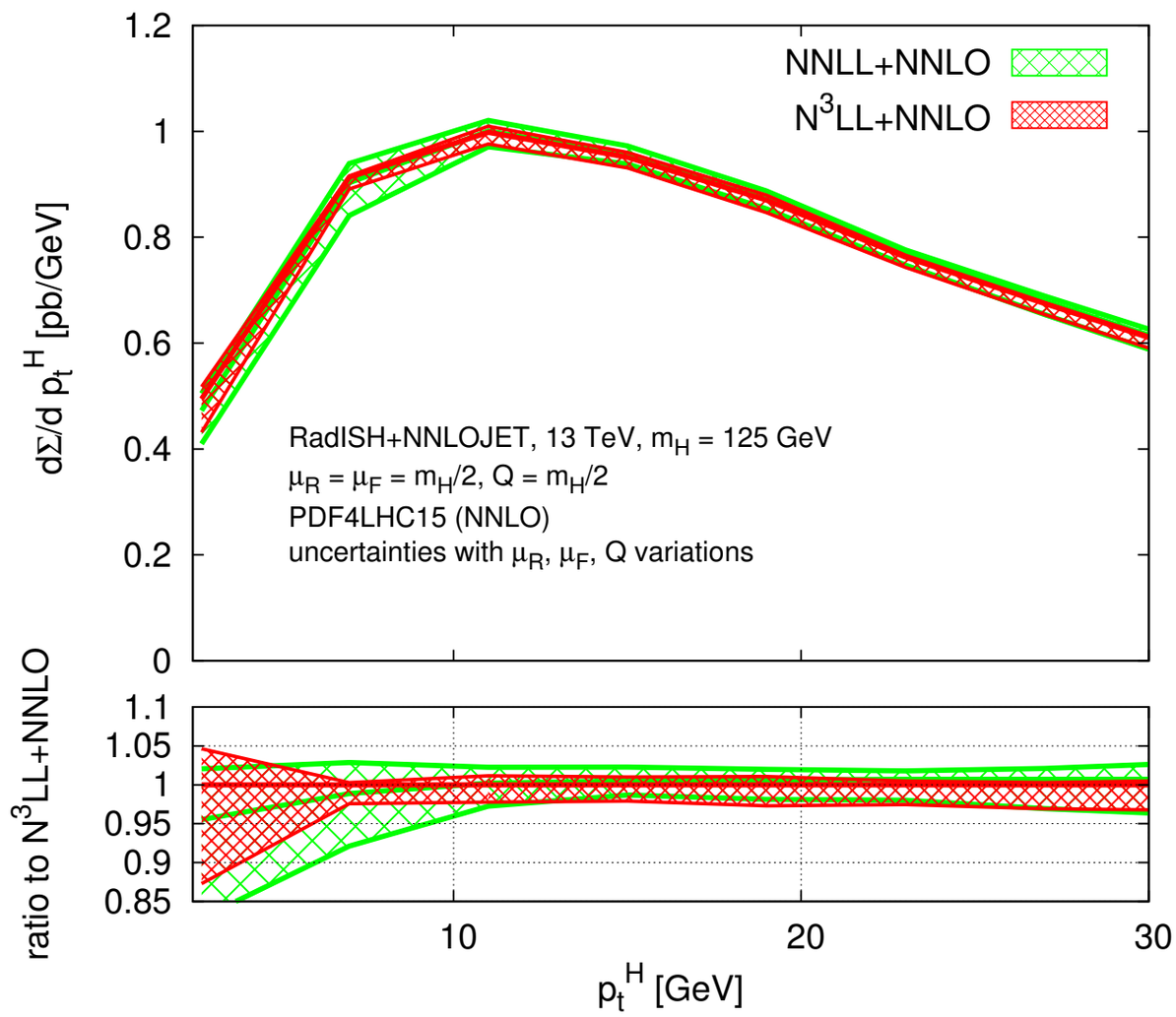

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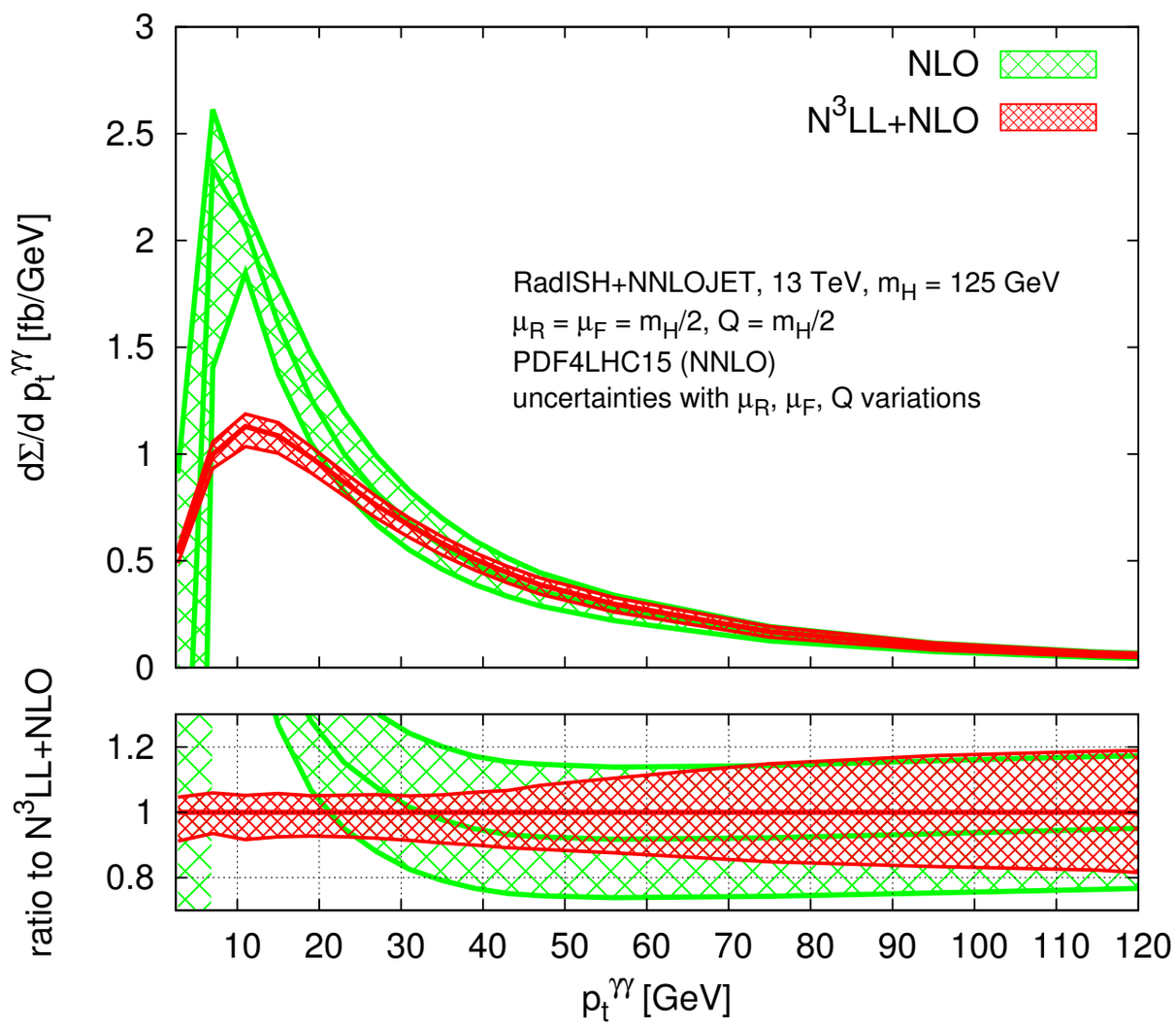
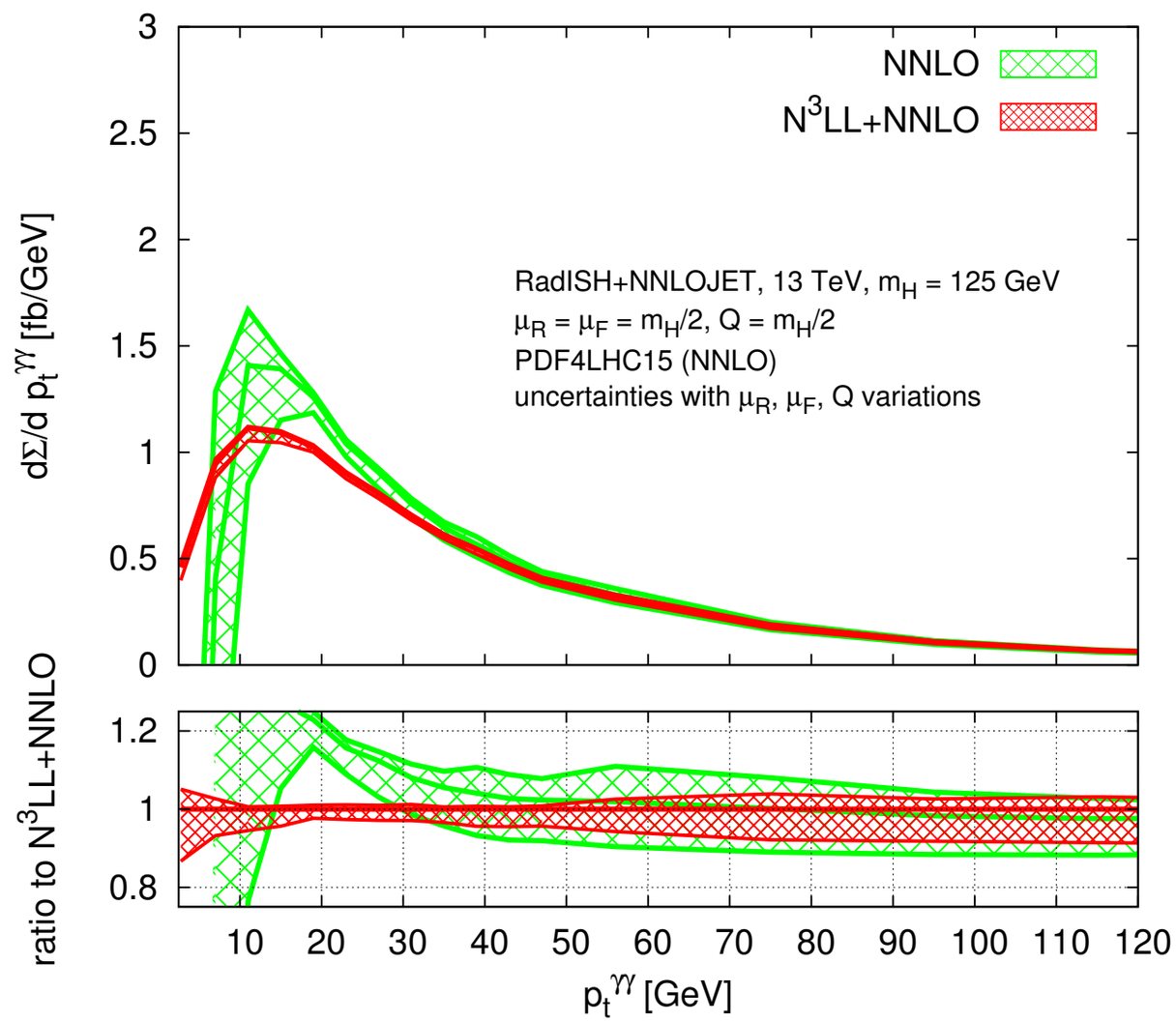
- results with experimental cuts on photons

$$\min(p_t^{\gamma_1}, p_t^{\gamma_2}) > 31.25 \text{ GeV}, \qquad \max(p_t^{\gamma_1}, p_t^{\gamma_2}) > 43.75 \text{ GeV}$$

$$0 < |\eta^{\gamma_{1,2}}| < 1.37 \;\text{ or }\; 1.52 < |\eta^{\gamma_{1,2}}| < 2.37, \qquad |Y_{\gamma\gamma}| < 2.37$$

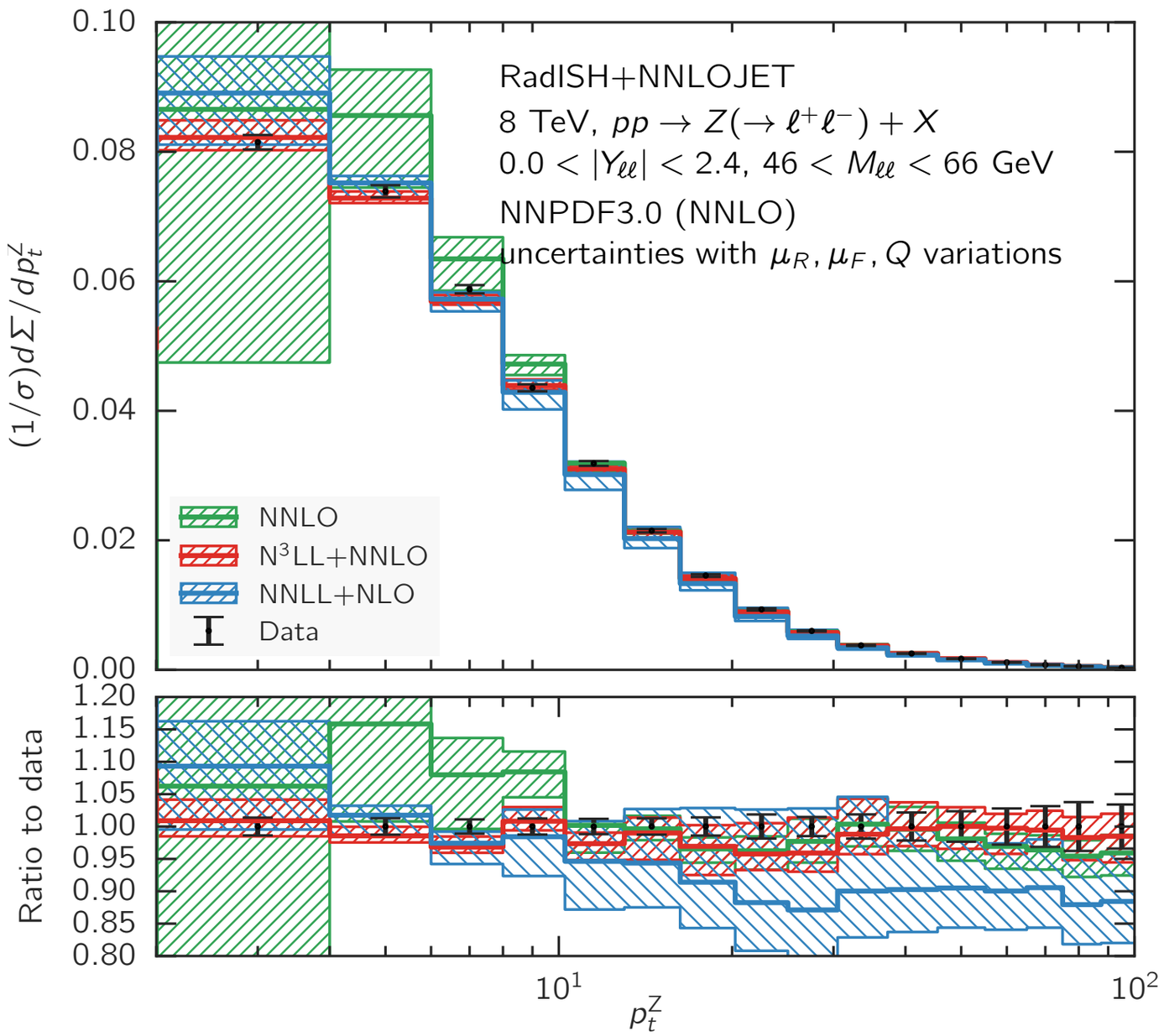




Resummation for specific distributions: Drell-Yan production

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]

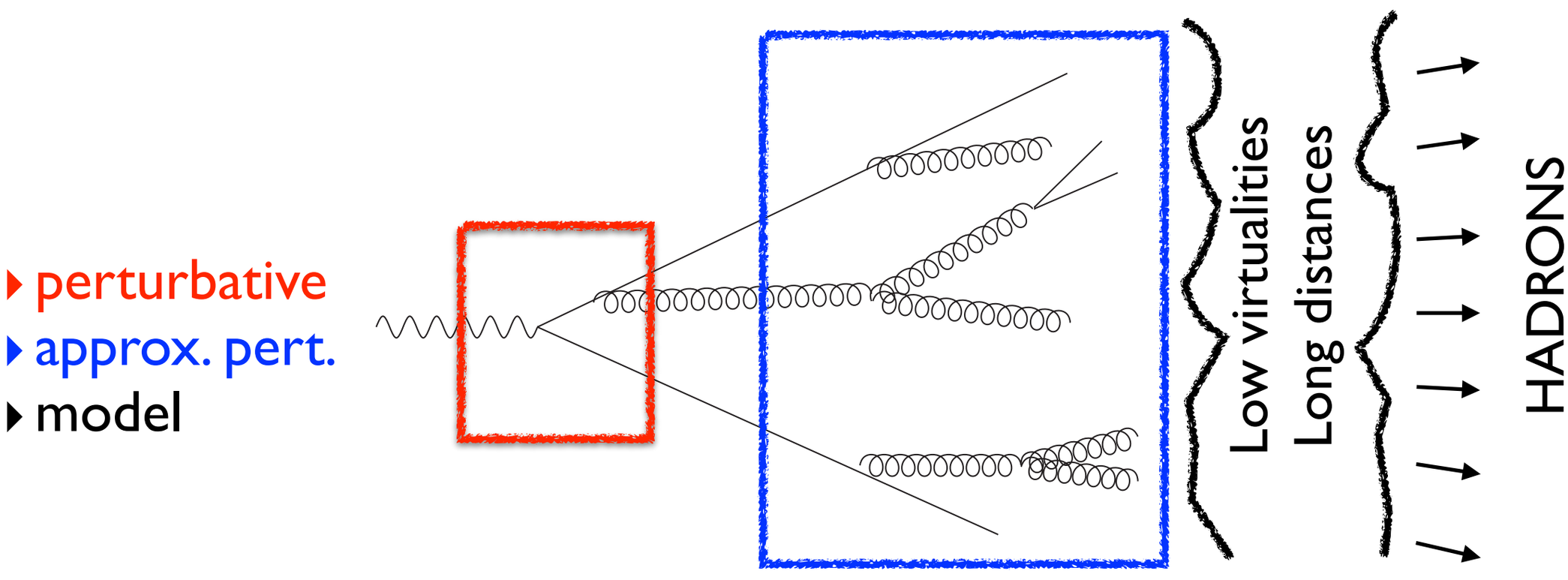
- pt of the Z boson



Parton Shower

- the leading soft/collinear logarithmic enhancement can be resummed for all observables
- starting from an hard 2 to 2 process it is possible to build an approximation for all the diagrams in which subsequent multiple emissions has distributed the available energy to a shower of partons
- consider final state radiation in e+e- annihilation(simplifying example)

➡ start from the hard qqbar event. It happens at a certain fixed starting hard scale Q
➡ we know that collinear emission from each leg is enhanced



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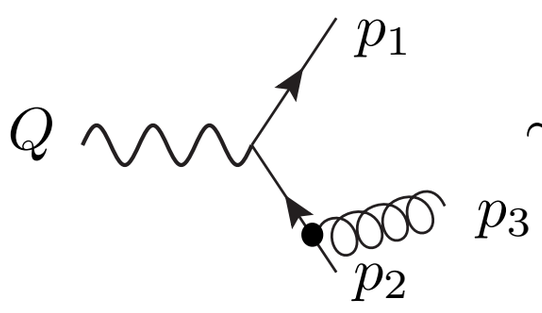
➡ we know that collinear emission from each leg is enhanced
➡ we look for an iterative formula to approximately describe the sequence of parton splittings

$$d\sigma_R = \sigma_0 dW_{13} + \sigma_0 dW_{23} + \ finite\, terms$$



$$\sim \sigma_0 dW_{23}$$

- qg splitting probability:

$$P_{qg}(x_3) = C_F \frac{\alpha_S}{2\pi} \frac{1 + (1 - x_3)^2}{x_3}$$

$$dW_{23} = \frac{dx_1}{1 - x_1} dx_3 P_{qg}(x_3) = \frac{d\cos\theta_{23}}{1 - \cos\theta_{23}} dx_3 P_{qg}(x_3) \xrightarrow[E_3 \to 0]{\theta_{23} \to 0} \frac{d\theta_{23}^2}{\theta_{23}^2} \frac{dE_3}{E_3}$$

Parton Shower

$$d\sigma_R \sim \sigma_0 \sum_i \frac{d\theta^2}{\theta^2} dz P_{ji}(z)$$

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}q^2}{q^2} = \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

- probability diverges, let's think for example to the invariant mass of the internal propagator as a cut off

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Note! Indeed, to describe the collinear limit we could have used also other variables that are good as well, we get the same expression if they are proportional to θ^2 for example $q^2 = z(1-z)\,\theta^2\,E^2$ or $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$ as a result we would get identical results for the collinear limit, but different extrapolation away from it. Let's move to the virtuality of the internal line.

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- but still the probability for a given emission path built reiterating these emissions can well exceed unity! while the sum of the probabilities of all shower configuration must yield 1
- indeed, approaching the strict collinear limit, the fixed order theory prediction is challenged and this enhancement are not really there as such
- in turn we need to dump this emission probability with a factor that has to restore unitarity keeping into account the missing virtual corrections and resum the logarithmic enhancement

Parton Shower

- to simplify the things, let's consider a single kind of splitting, to be definite, that we have just gluons

$$d\sigma_R \sim \sigma_0 P_{gg}(z) \frac{dq^2}{q^2} dz$$

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- we normalise the emission probability multiplying times the inclusive probability of **<u>no emission</u>** generating a virtuality above q^2 and given the maximum possible virtuality Q^2 (Sudakov form factor)

$$d\sigma_{1st-emission} = P_{gg}(z) \frac{dq^2}{q^2} dz \Delta_g(Q^2, q^2)$$

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- of course, this "no emission" probability has to be related to the emission probability, it has to be a function of it. Given that for smaller and smaller q^2 the emission probability diverges, in this limit the inclusive "no emission" probability has to go to zero rapidly, so regularising the divergence

Parton Shower

- now let's compute Δ (here comes the resummation!)
  The probability of emission giving a virtuality among q^2 to $q^2+\mathrm{d}q^2$ is:

$$dW_g = \frac{dq^2}{q^2}\int_{z_{min}}^{z_{max}} dz P_{gg}(z)$$

- so, for such infinitesimal range of virtuality $\mathrm{d}q^2$ the "no emission" probability is

$$dP(\text{no emission}) = 1 - dW_g = 1 - \frac{dq^2}{q^2}\int_{z_{min}}^{z_{max}} dz P_{gg}(z)$$

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- to get the "no emission" probability over the finite range from q^2up to Q^2, we don't integrate this, but take the product over "all" the infinitesimal regions, so collecting the probability of "no multiple emission" of every number of gluons

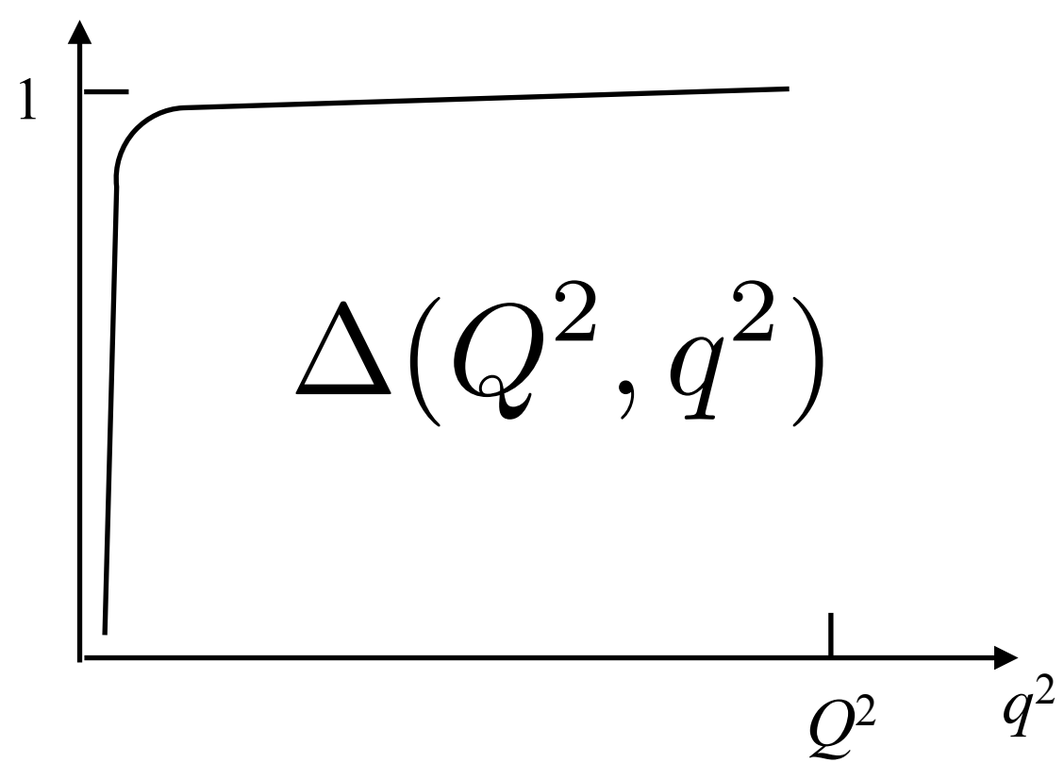
$$\Delta_g(Q^2, q^2) = \prod dP(\text{no emission}) = \exp\left\{-\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\int_{z_{min}}^{z_{max}} dz P_{gg}(z)\right\}$$

Parton Shower

- the parton shower is then produced generating radiation with distribution

$$d\sigma_{1st\ emission} = P_{gg}(z)dz\frac{dq^2}{q^2}\exp\left\{-\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\int_{z_{min}}^{z_{max}}dzP_{gg}(z)\right\}$$

- this formula can now be iterated updating the upper Q^2 scale to the extracted q^2 at each step until the virtuality of the internal line reaches the cut-off scale



Parton Shower

Observations:

- a large number of technicalities: momentum reshuffling after each emission, management of multiple splitting processes, hadrons in the initial state…

  ➡ hadronization models are just models that parametrise data in terms of a large number of parameters, inspired by QCD but in principle there could well be no physics at all in their parameters

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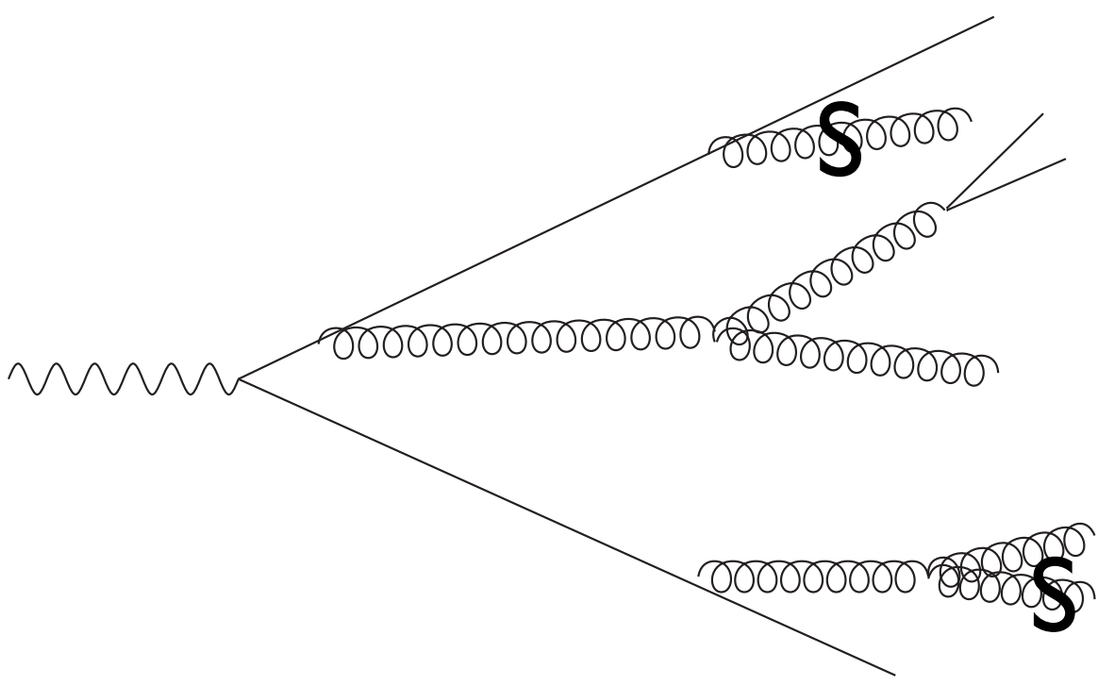
- and choices: evolution variable, scale in the strong coupling, cut-off scale…



Parton Shower

Observations:

- a large number of technicalities: momentum reshuffling after each emission, management of multiple splitting processes, hadrons in the initial state…

  ➡ hadronization models are just models that parametrise data in terms of a large number of parameters, inspired by QCD but in principle there could well be no physics at all in their parameters

  ➡ initial state has to take into account the structure of the colliding hadrons, more technical complications but is done with the same logic (space-like shower)

- and choices: evolution variable, scale in the strong coupling, cut-off scale…

  ➡ wide angle soft emission and coherence: evolution variable θ angle or kt

  ➡ cut-off scale dependence. Note that soft radiation has to be more and more collinear and so we can imagine that although soft radiation is long range, hadronization can be considered a local process

- and even more needed for a full description of the possible kind of collisions: multiple interactions in hadron-hadron collisions, other non perturbative phenomena…

Jet shape at the LHC

- require resummation of large soft and collinear logarithmic enhancement

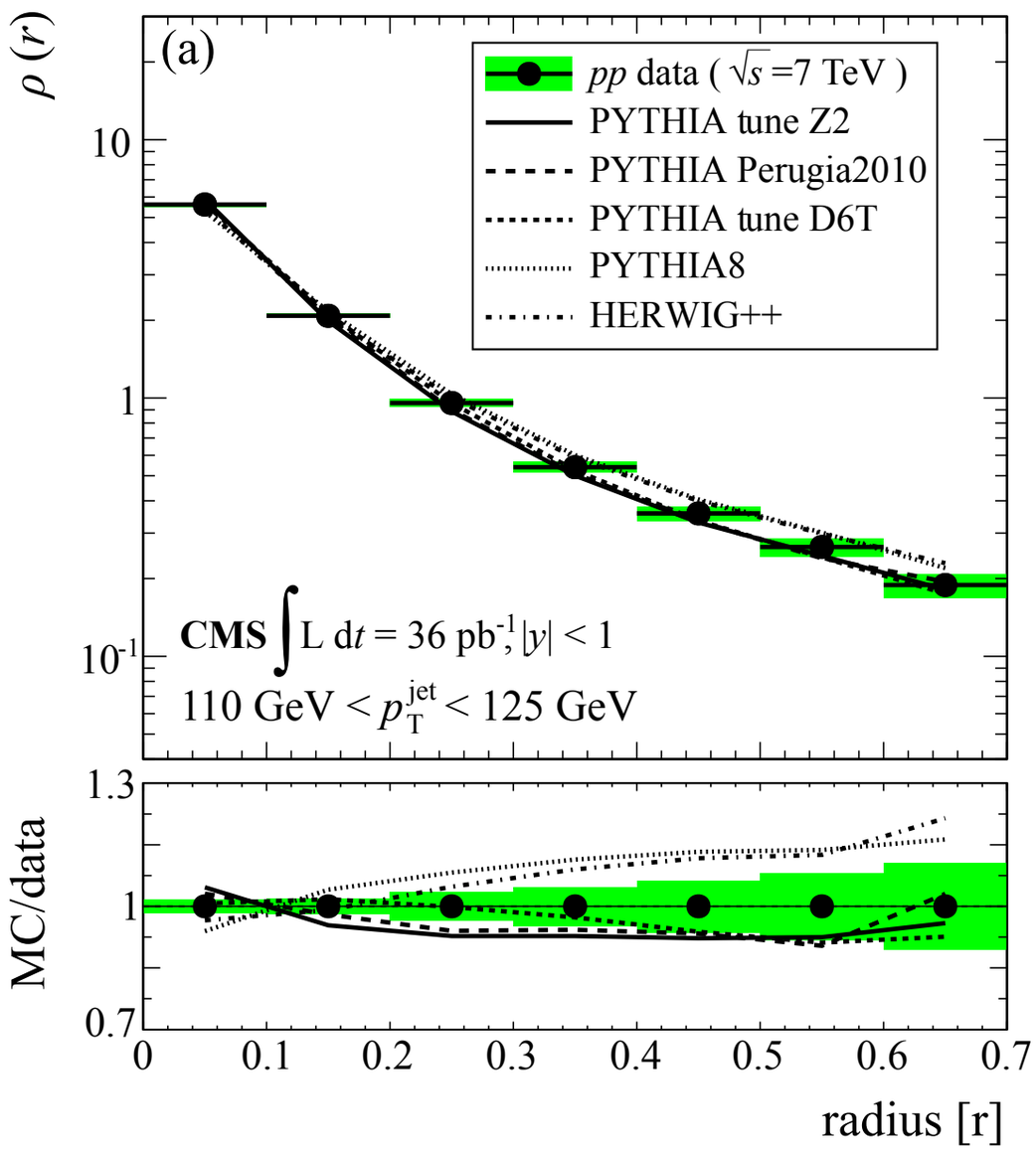
  ➡ parton shower description

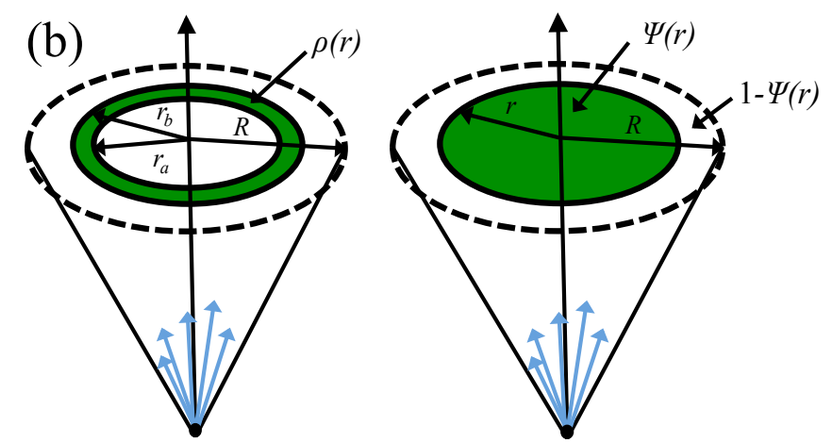
- fraction, ρ, of the jet transverse momentum contained within a ring of radius *r* around the jet core

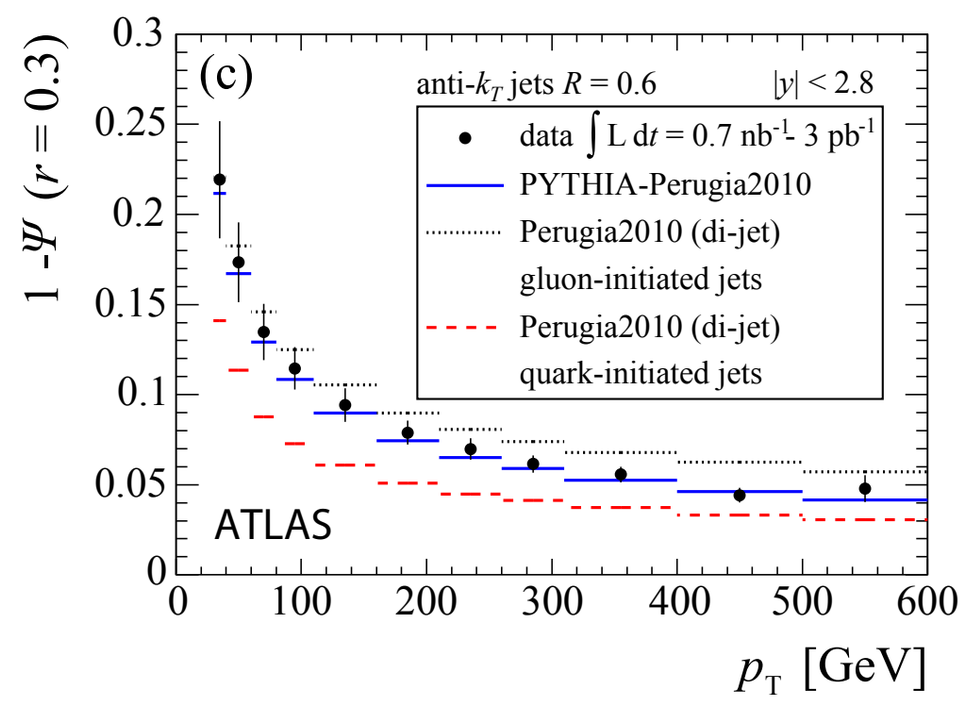
$$\rho(r) = \frac{1}{N}\sum_{\text{jets}} \frac{1}{\Delta r} \frac{\sum_{r_a \le r_i < r_b} p_{\mathrm{T},i}}{\sum_{r_i \le R} p_{\mathrm{T},i}}$$

$$r_i = \sqrt{(\Delta_{i,\text{jet}} y)^2 + (\Delta_{i,\text{jet}} \phi)^2}$$

$$r_a = r - \Delta r/2 \qquad r_b = r + \Delta r/2$$







Courtesy of [Tancredi Carli, Klaus Rabbertz, and Steffen Schumann 2015]

Jet shape at the LHC

- works also for lower pt jets

$$\rho(r) = \frac{1}{N} \sum_{\text{jets}} \frac{1}{\Delta r} \frac{\sum_{r_a \le r_i < r_b} p_{\mathrm{T},i}}{\sum_{r_i \le R} p_{\mathrm{T},i}}$$

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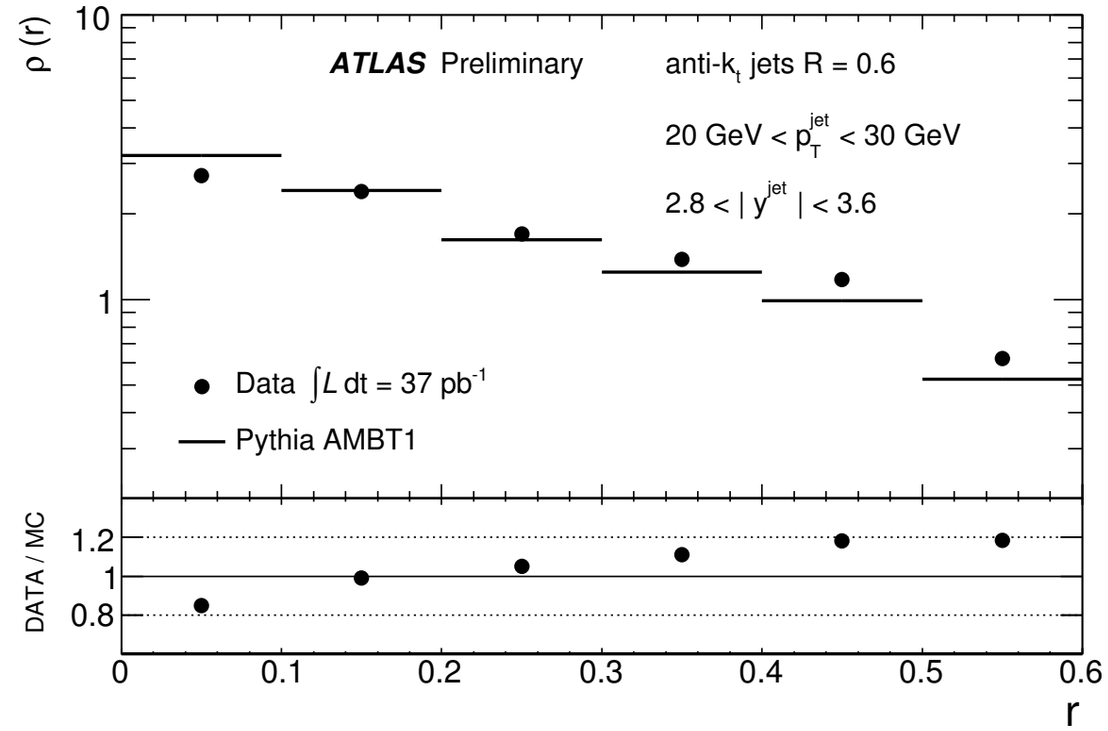
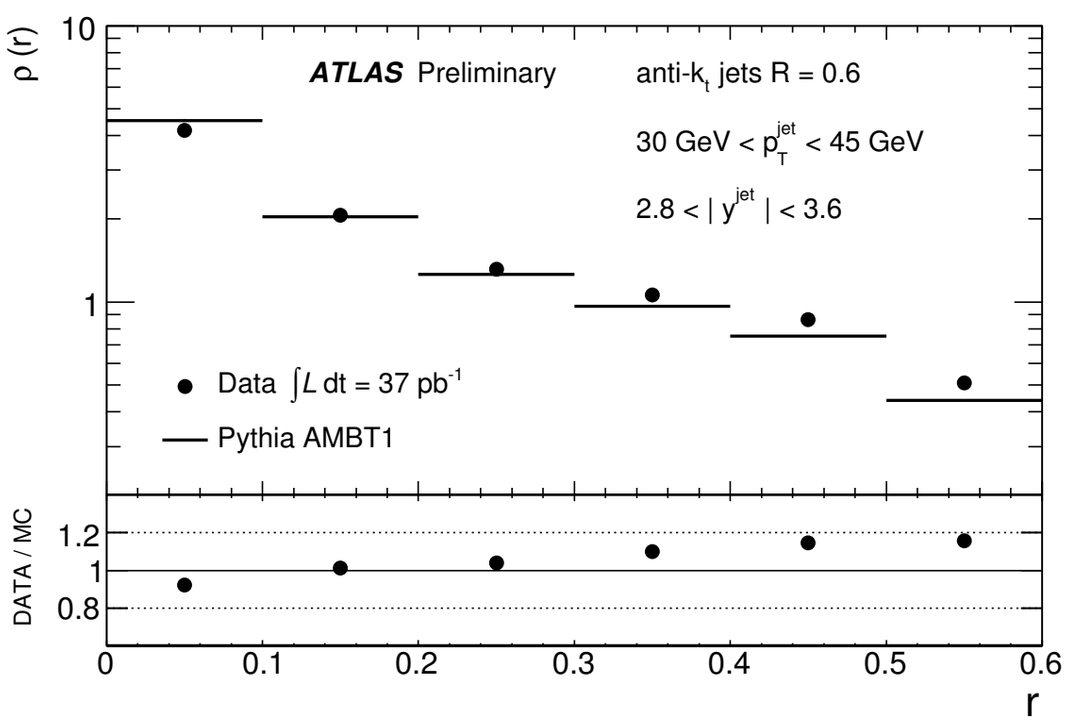
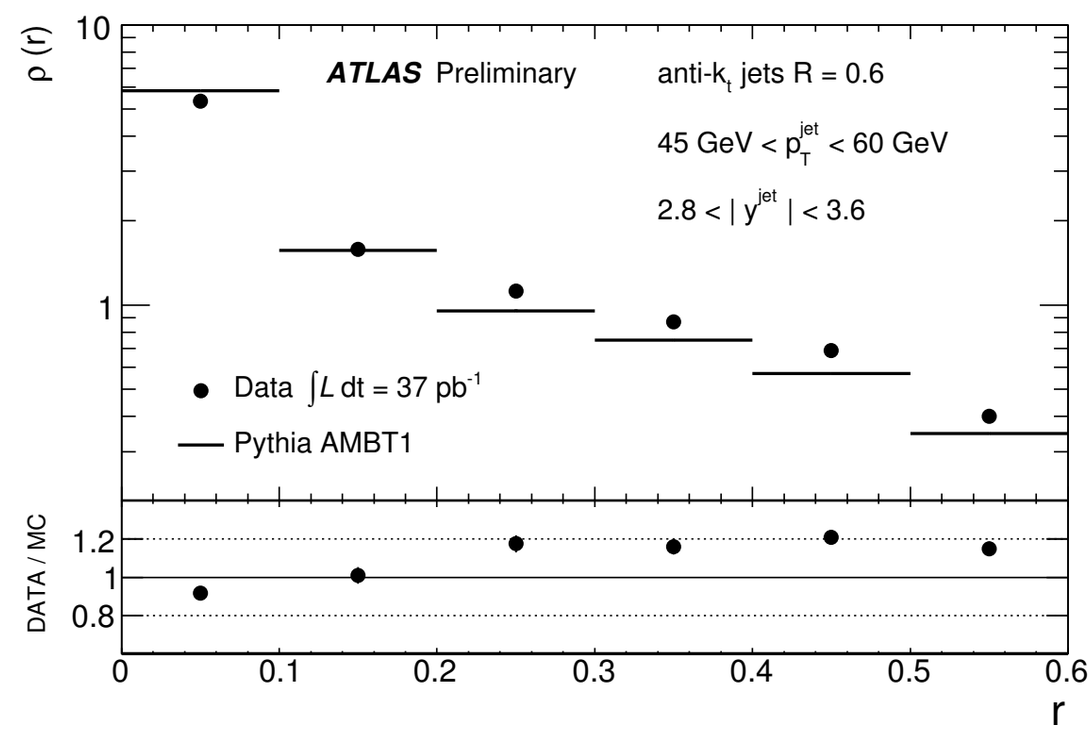






Parton Shower

Note that:

- soft and collinear approximations that underlie PS may fail to reproduce the full pattern of hard wide-angle emissions

- PS generate cross sections for the requested hard process that are correct at LO

Nevertheless:

- PS is a fundamental tool to study the sensitivity of the experiments in high energy physics

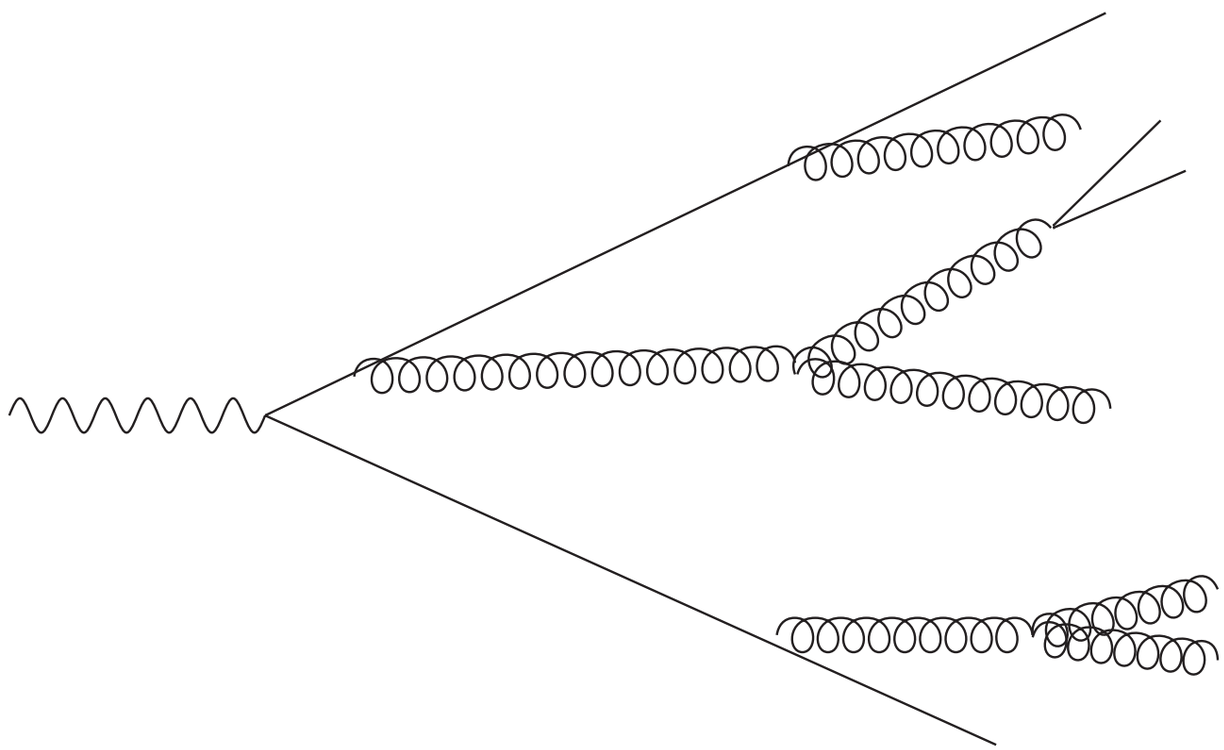
- one would like to include all possible hard processes and all of them computed including higher order corrections in a framework like this

  ➡ we are on this path…

Merging

Multi-jet final states can be described at LO+LL using LO matrix elements with up to n parsons in the final state. Let's consider e+e- at an Energy scale Q for simplicity

[Catani, Kuhn, Krauss, Webber 2001]



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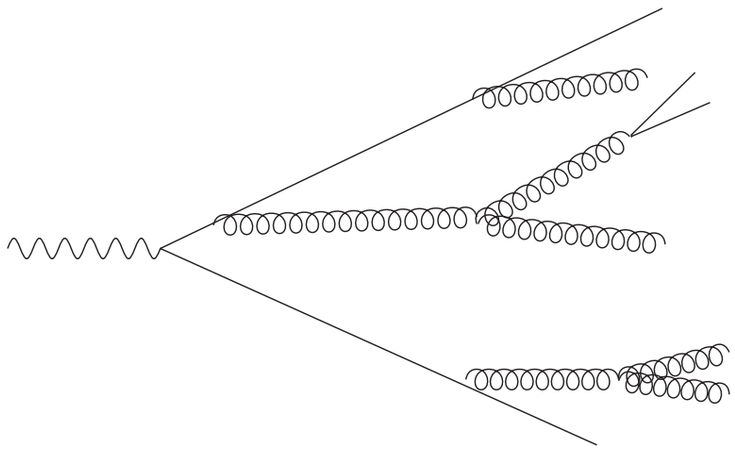

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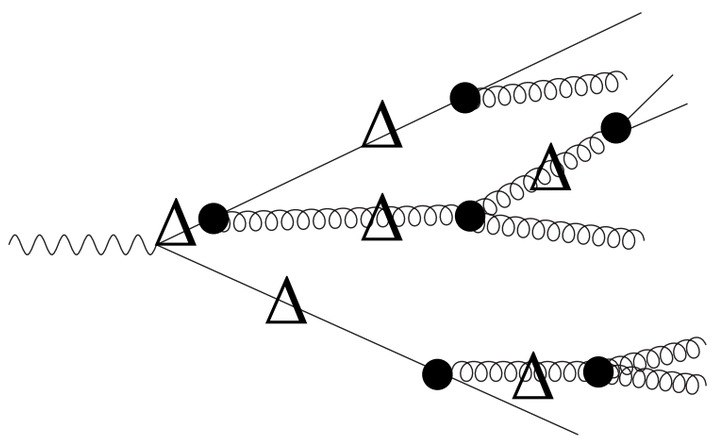

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   a. If the event is accepted, assign a color configuration and start the shower from each leg of the n parton event at scale t at which it has been generated, vetoing radiation harder then Q_1
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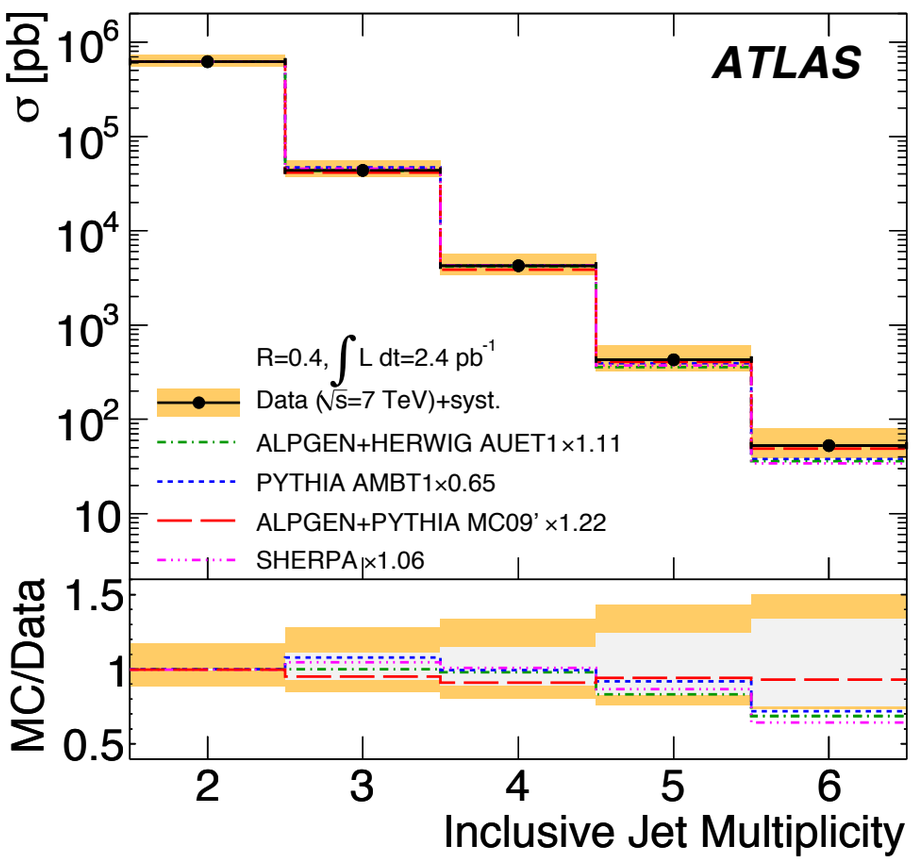
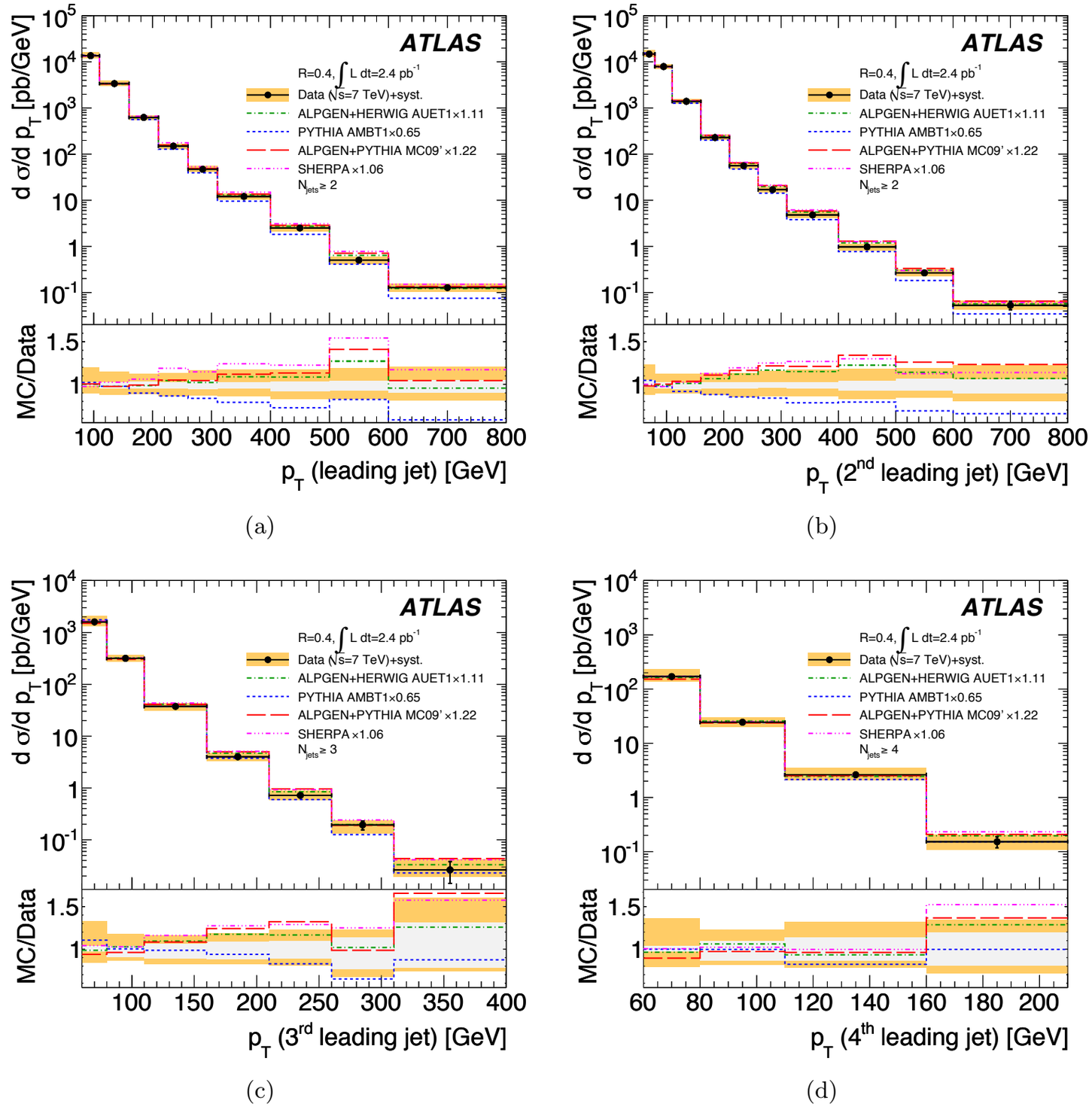
Needs modifications when there are hadrons in the initial state, but the logic is the same: the point is always to regularise matrix elements factorising the phase space into hard (ME domain) and soft (PS domain) and to accept or reject an event according to the PS weight of the event

Merging





Successful but do not expect too much

- 5% uncertainty on the strong coupling results on 30% uncertainty for 6 jets production, and then there is scale variations (both renormalization and factorisation), hadronization effects…
- description with just MC programs is more an exercise to tune them than a real test for QCD

Matching: NLO and PS

First proposals have been the MC@NLO [Frixione and Webber 2002] and the POWHEG [Nason 2004] methods. Let's take for example the POWHEG method that stands for positive weight hardest emission generator. See the original references for the proofs, we just sketch the construction

- we start considering the first emission in a parton shower approach

$$d\sigma_{1st\ emission} = \sigma_0 P_{gg}(z) \frac{dk_T^2}{k_T^2} dz \Delta(Q^2, q^2)$$

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- to have NLO prediction on inclusive variables one start generating the "underlining born" kinematic configuration with probability given by NLO accuracy for parton level generation

$$\sigma_0 \Longrightarrow \bar{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \left[\int d\Phi_{\rm rad} \left[R(\mathbf{\Phi}_{n+1}) - C(\mathbf{\Phi}_{n+1}) \right] + \int \frac{dz}{z} \left[G_\oplus(\mathbf{\Phi}_{n,\oplus}) + G_\ominus(\mathbf{\Phi}_{n,\ominus}) \right] \right]^{\bar{\mathbf{\Phi}}_n = \mathbf{\Phi}_n},$$

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- then one attach the hardest radiation with probability given by a Sudakov form factor build upon the ratio of Real and Born matrix elements:

$$d\sigma = \bar{B}(\mathbf{\Phi}_n)\, d\mathbf{\Phi}_n \left\{ \Delta\left(\mathbf{\Phi}_n, p_{\rm T}^{\rm min}\right) + \Delta\left(\mathbf{\Phi}_n, k_{\rm T}\left(\mathbf{\Phi}_{n+1}\right)\right) \frac{R\left(\mathbf{\Phi}_{n+1}\right)}{B(\mathbf{\Phi}_n)} d\Phi_{\rm rad} \right\}_{\bar{\mathbf{\Phi}}_n = \mathbf{\Phi}_n}$$

$$\Delta\left(\mathbf{\Phi}_n, p_{\rm T}\right) = \exp\left\{ - \int \frac{\left[d\Phi_{\rm rad}\, R(\mathbf{\Phi}_{n+1})\, \theta(k_{\rm T}\left(\mathbf{\Phi}_{n+1}\right) - p_{\rm T}) \right]^{\bar{\mathbf{\Phi}}_n = \mathbf{\Phi}_n}}{B(\mathbf{\Phi}_n)} \right\}$$

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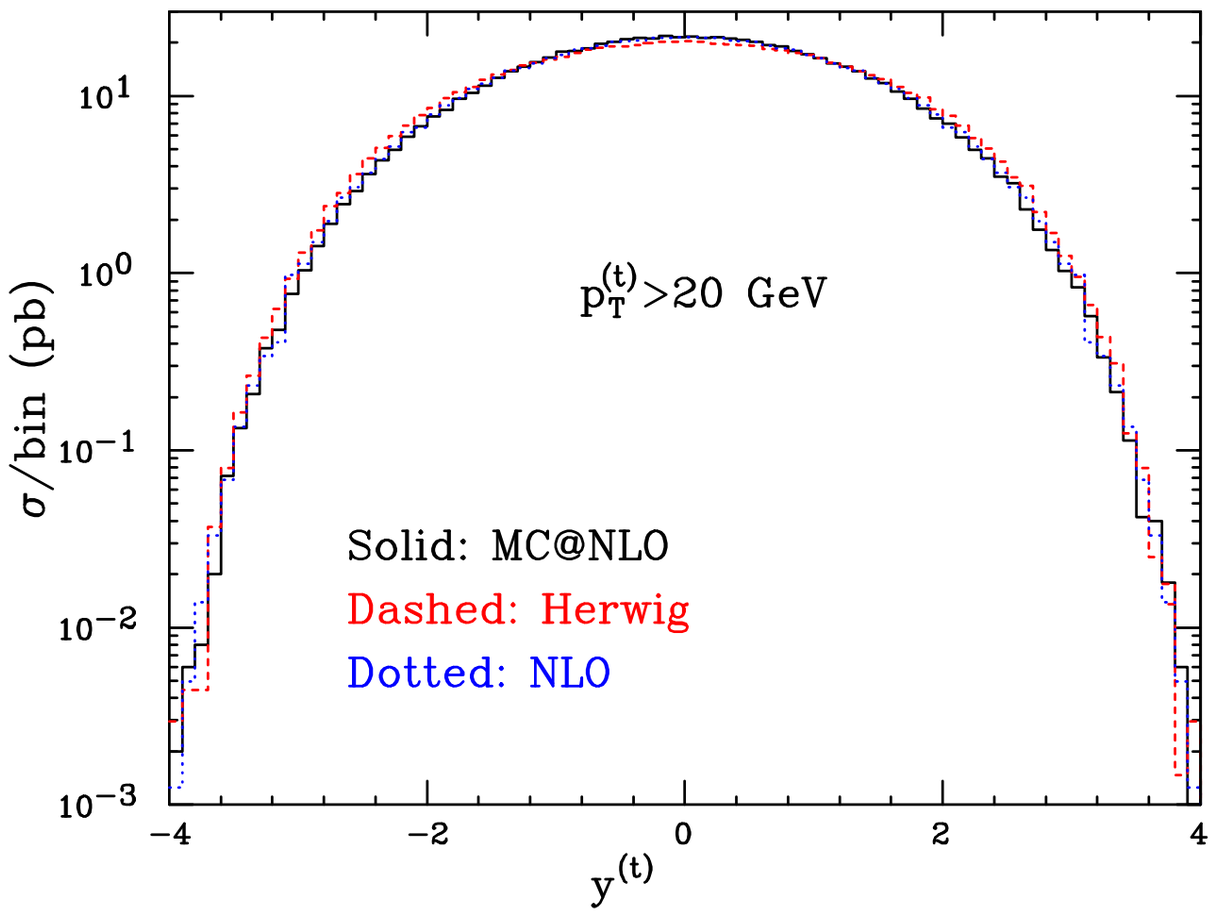
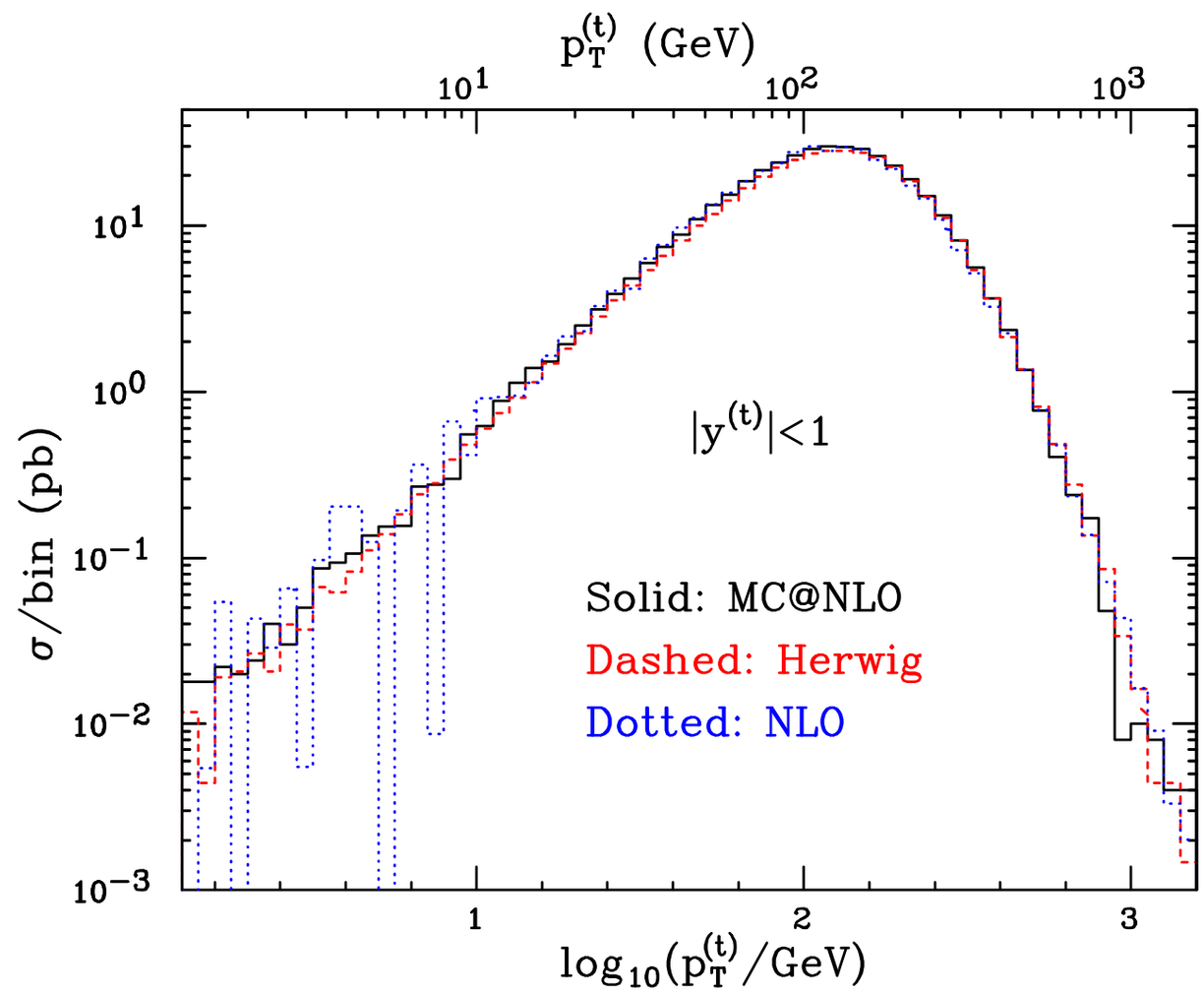
- finally, assign a color configuration and shower the event vetoing emissions harder then the first one to avoid double counting

NLO + PS: $t\bar{t}$ @LHC14

[Frixione, Nason, Webber 2003]

- Inclusive variables have NLO precision





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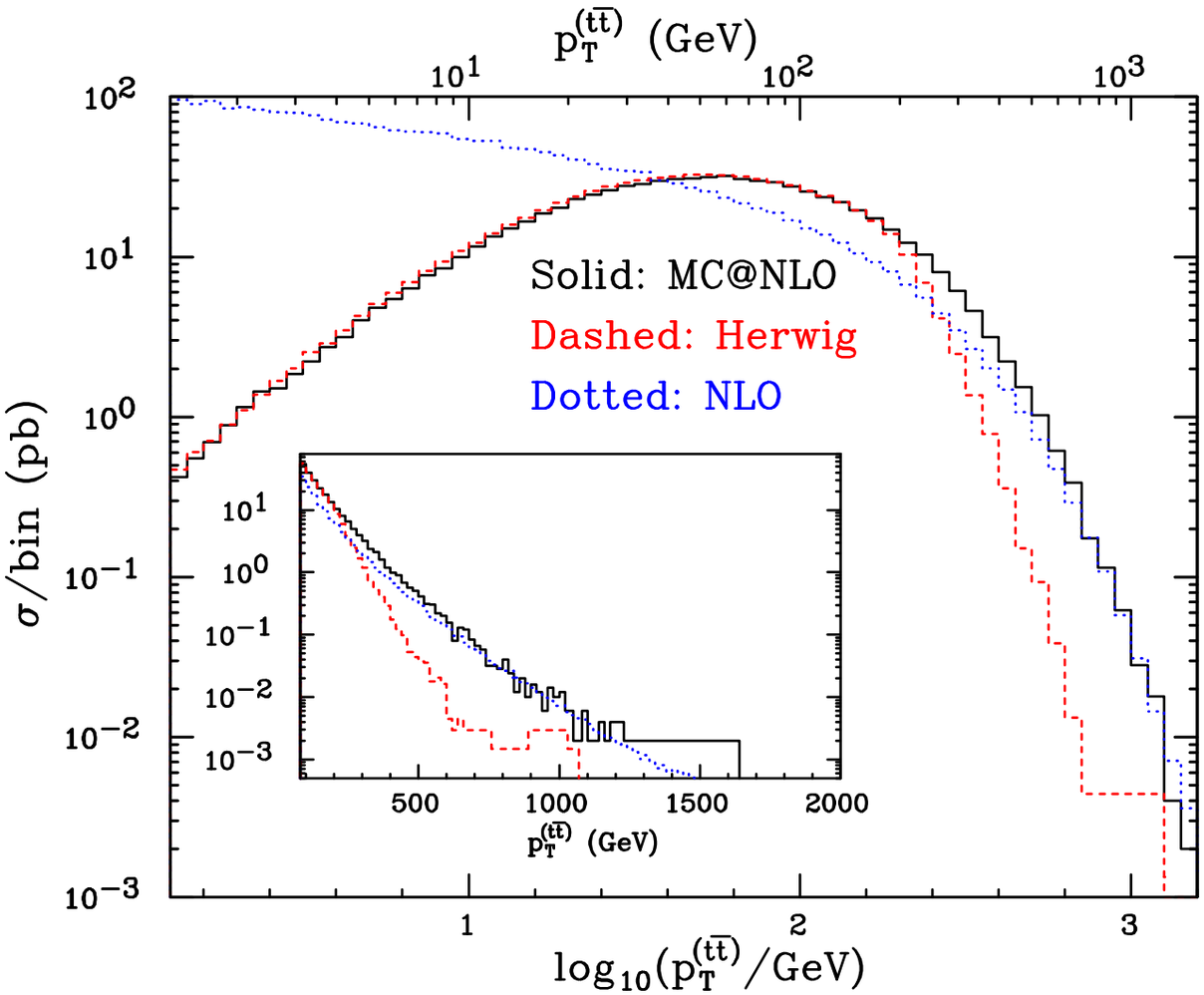
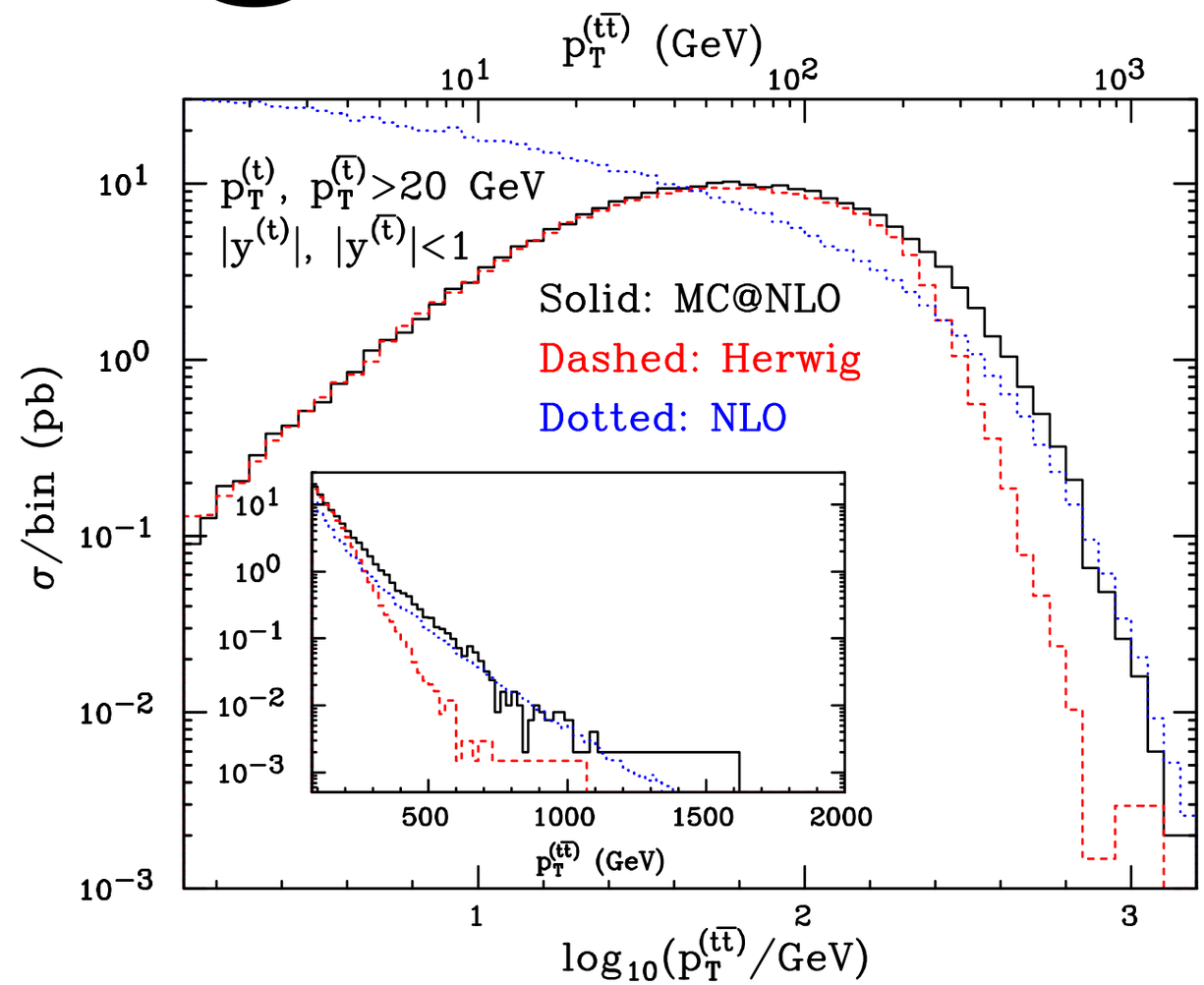




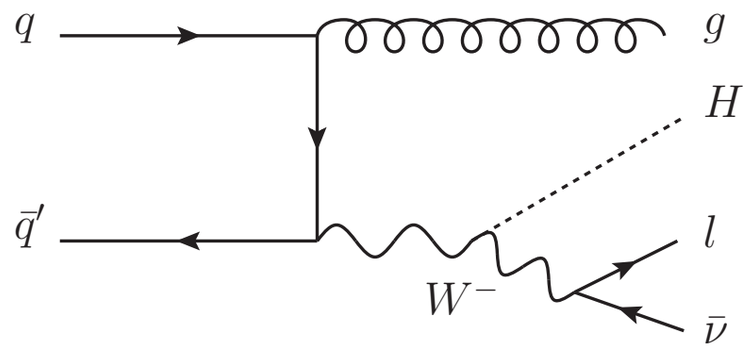
- Complementary behaviour of the NLO and MC approaches regardless of the cuts on the rapidities and transverse momenta of the top quark of the heavy quarks
- In the tail of the transverse momentum pt distribution of the top pairs, the NLO cross section is much larger than the MC one, simply because hard emissions are correctly treated only in the former.
- For small transverse momentum of the top pair the difference between the two histograms shows the effect of all-order resummation, clearly, no meaningful comparison between NLO and data can be attempted in this region

[Frixione, Nason, Webber 2003]

→ smooth interpolation performed by MC@NLO

Merging and Matching

- NLO+PS generators for a process that has at least 1 jet in the final state in the leading order, has divergences when this radiation became unresolved



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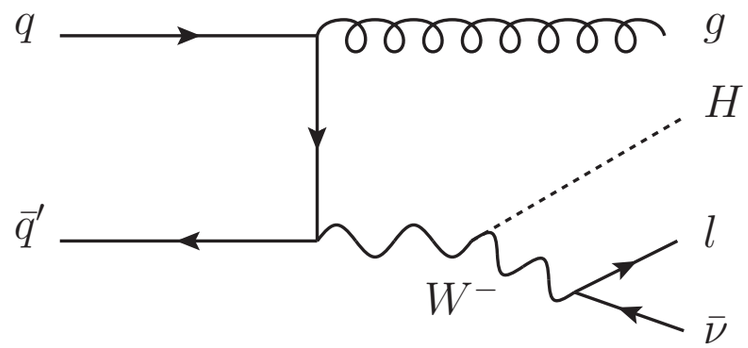

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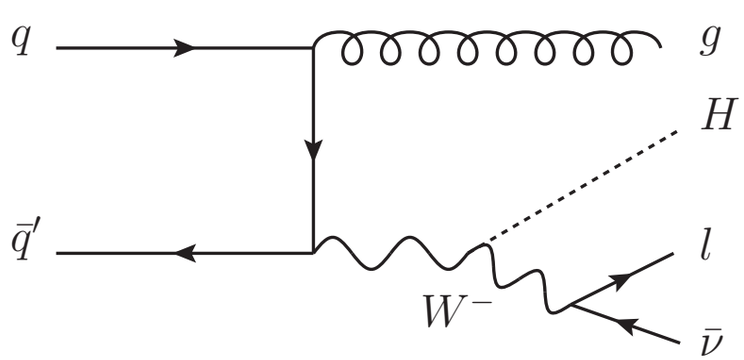
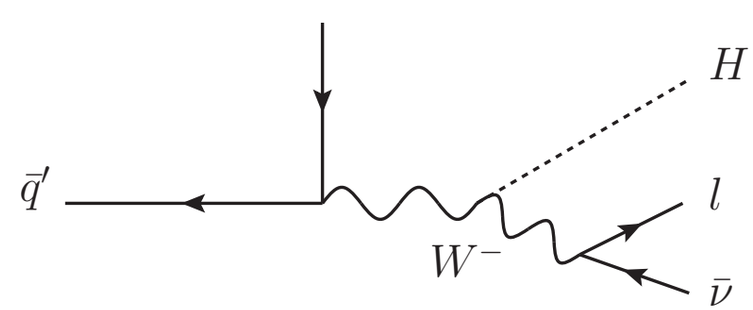
- BTW, an optimal choice of this factor can be done, such that the formal accuracy of the inclusive generation is NLO accurate [Hamilton, Nason, Oleari, Zanderighi 2013]





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  ➡ this method is called MINLO that stands for Multi-scale improved NLO

  ➡ you can generate simultaneously WH+0/1jet or $Wb\bar{b}$+0/1 jet or whatever other process +0/1 jets with this technique

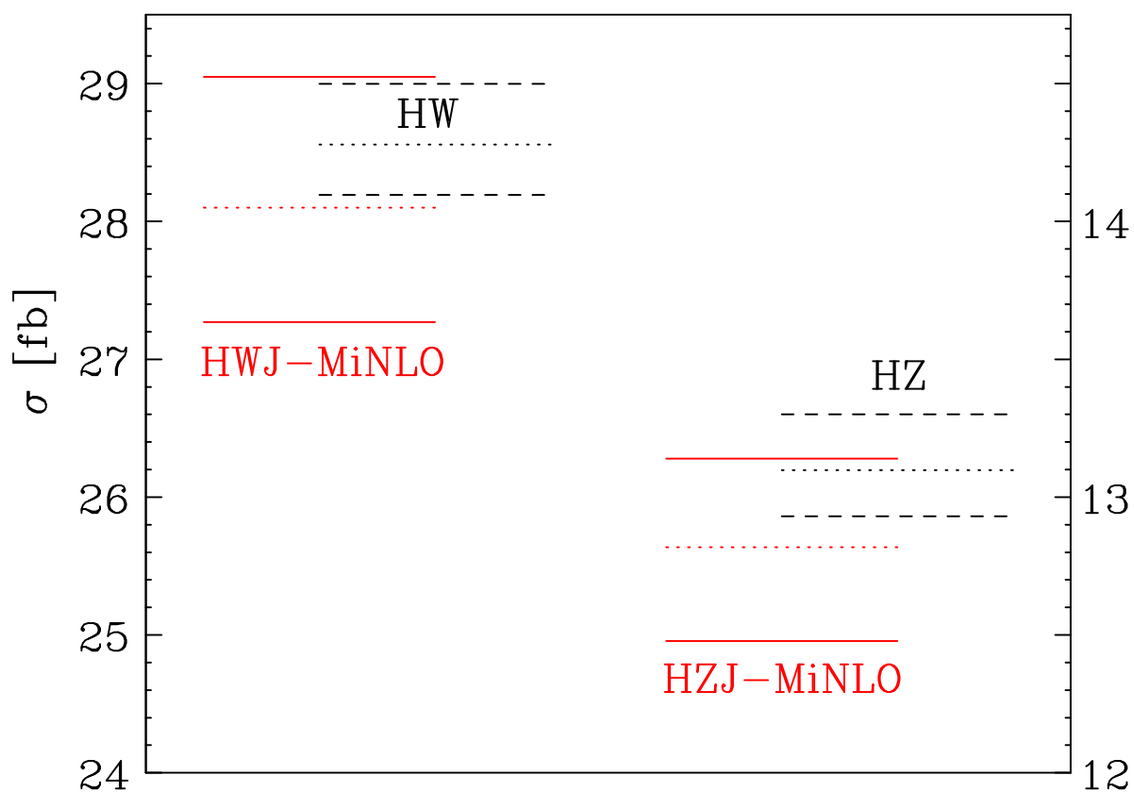
  ➡ do not require a merging scale as for CKKW

- this is not the only way to do that, several techniques used by aMC@NLO and SHERPA collaborations to merge NLO computations with more jets in the final state

Merging and Matching: HV+0/1j @LHC8

[Luisoni, Nason, Oleari et al 2013]

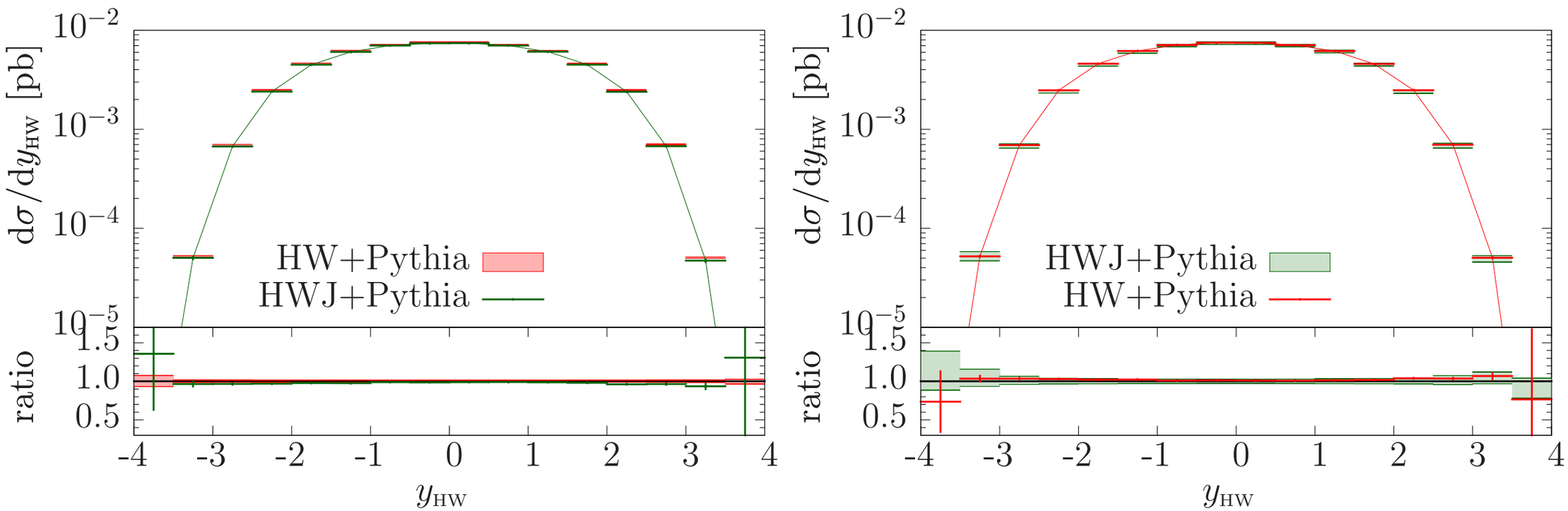
- total cross sections



Merging and Matching: HV+0/1j @LHC8

[Luisoni, Nason, Oleari et al 2013]

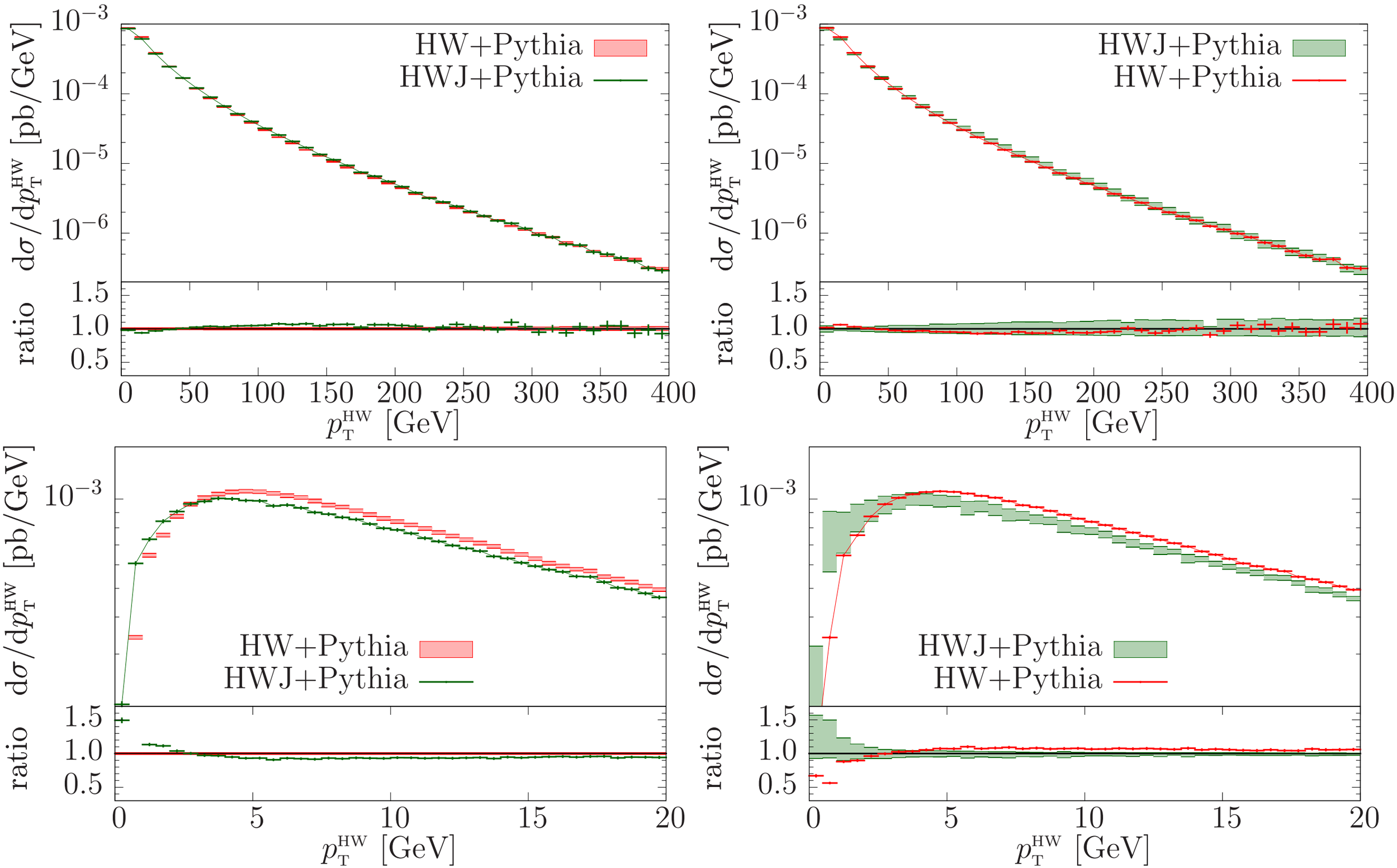
- differential distributions: HW rapidity



Merging and Matching: HV+0/1j @LHC8

[Luisoni, Nason, Oleari et al 2013]

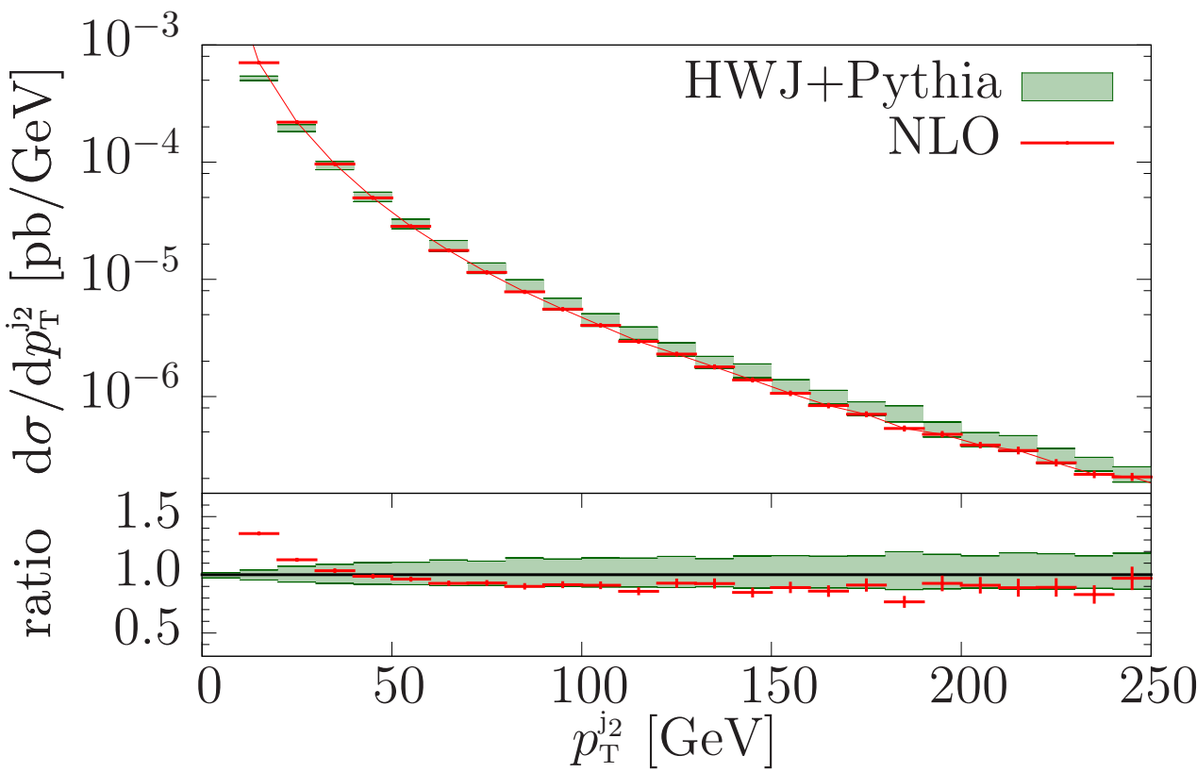
- differential distributions: HW pt

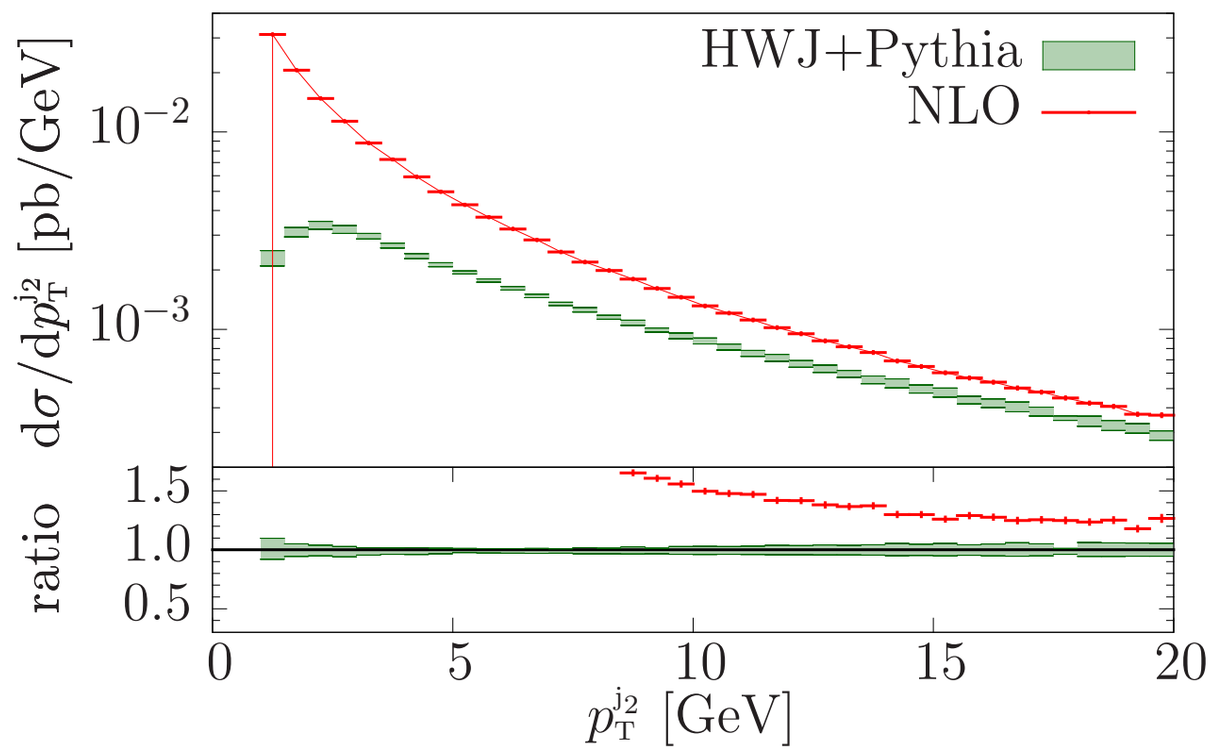


Merging and Matching: HV+0/1j @LHC8

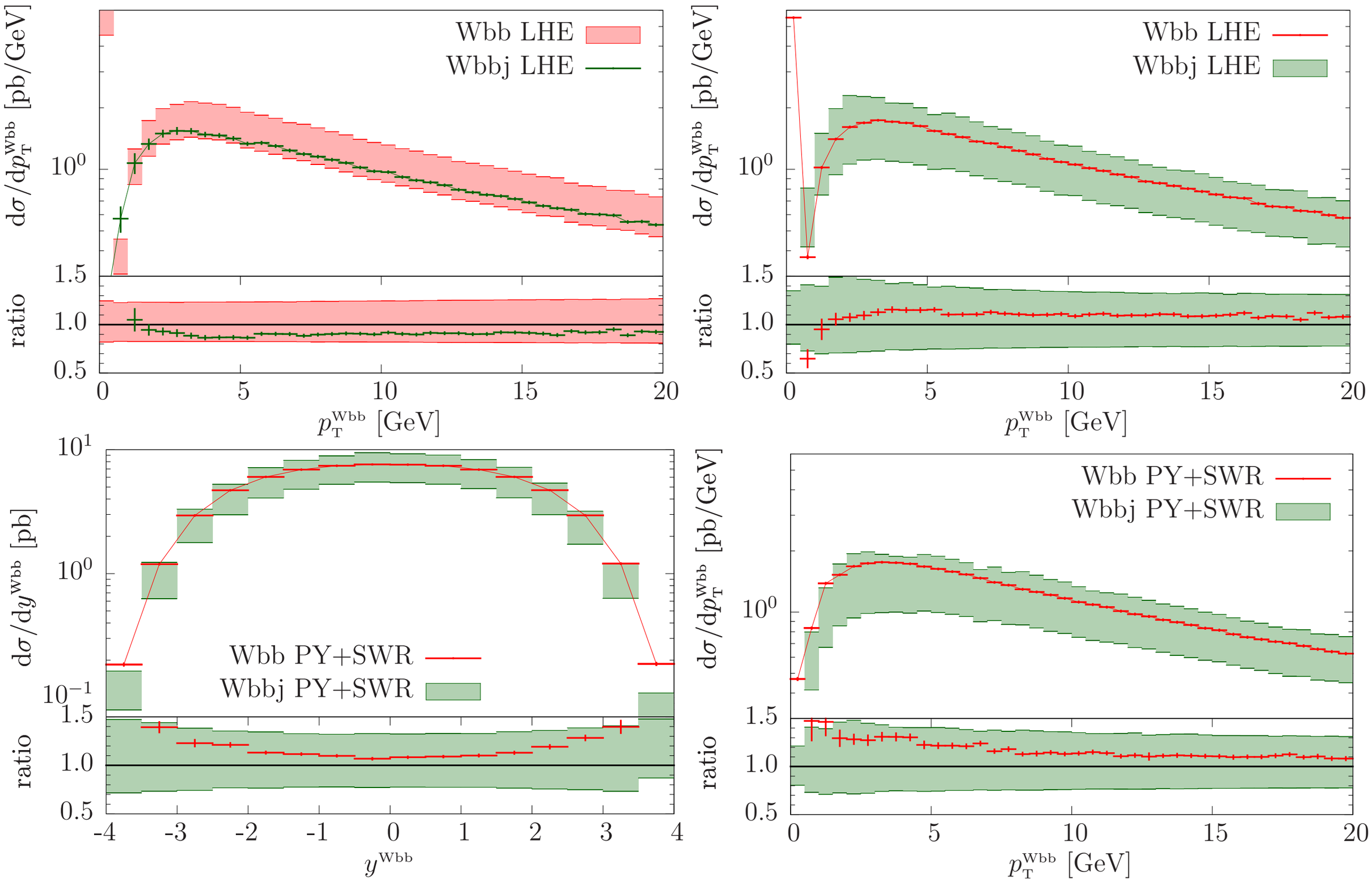
[Luisoni, Nason, Oleari et al 2013]

- differential distributions: second jet pt





Merging and Matching: $Wb\bar{b}$+0/1j @LHC7



[Luisoni, Oleari et al 2015]

Matching NNLO with PS

- Reweighing NLO+PS generators for a process that has exactly 1 jet in the final state in the leading order (like H/W/Z+1jet, and in principle every process producing a colourless final state plus 1jet) with a parton level NNLO computation for the fully inclusive production of such a colourless final state, one obtain NNLO+PS matching!

[Hamilton, Nason, Re, Zanderighi 2013]

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- in practice:

  a. produce a sample of events for H+1jet, for example, including the MINLO Sudakov
  b. bin the sample in the fully inclusive variables (just rapidity for H, azimuth is irrelevant)
  c. reweight the events such that the rapidity distribution is the one of the full NNLO computation

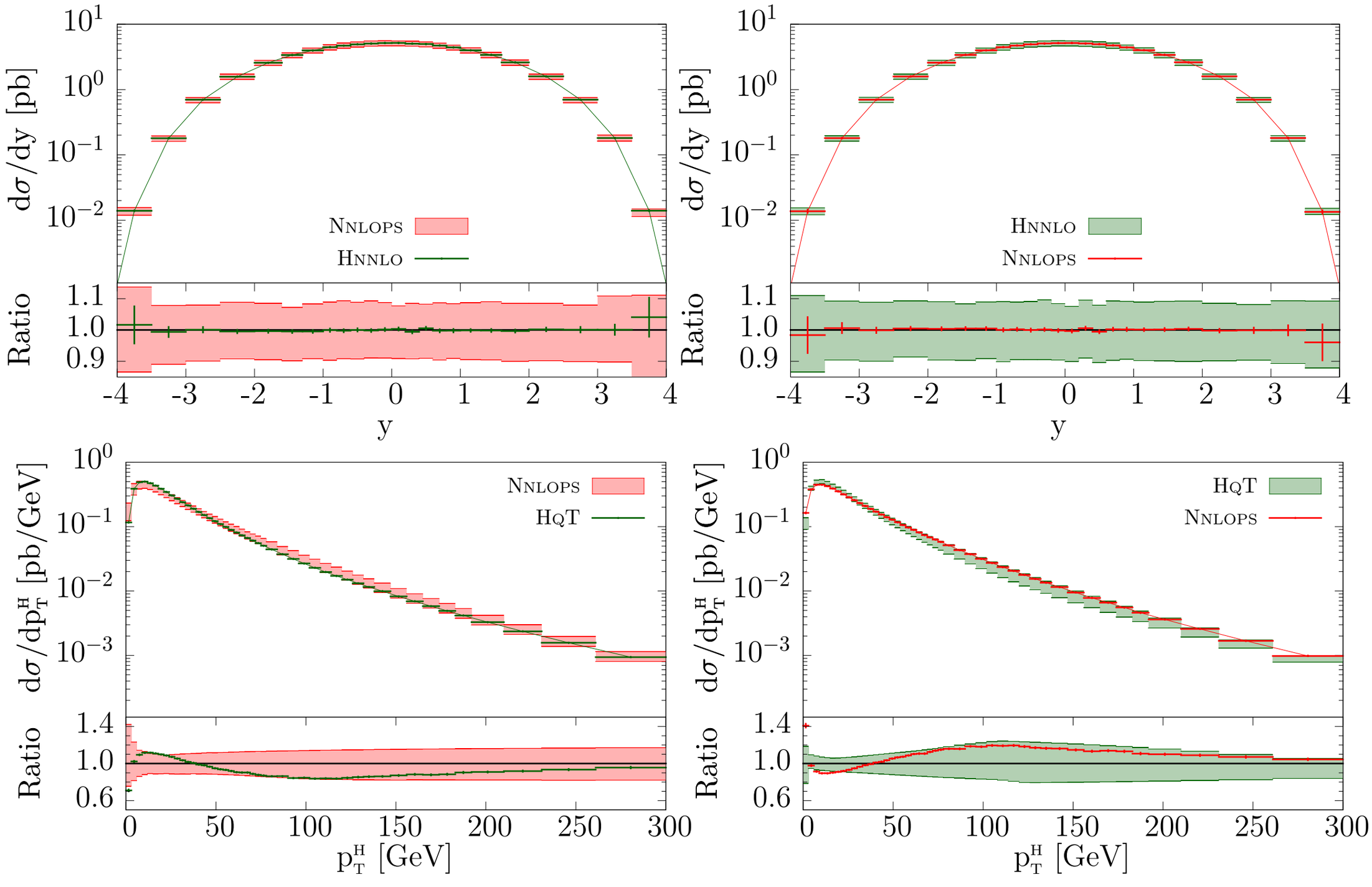
- for more complicated Born processes it's a difficult task:

  ➡ more variables and so need to reweight multi-dimensional histograms!

- can be extended to more complex final state, but some parts of the MINLO Sudakov suppression factor for the general case are still missing

NNLO+PS: Higgs production @LHC8

[Hamilton, Nason, Re, Zanderighi 2013]



HNNLO: [Catani, Grazzini 2007]

HqT: [Bozzi, Catani, De Florian, Grazzini 2005], [De Florian, Ferrera, Grazzini, Tommasini 2011]