Plan of lecture 4

• Higher order corrections for LHC processes

• Resummation for specific distributions

• Parton shower

• Merging, Matching, and both
Higher order QCD corrections in hadron collisions

• Drell Yan process and Di-boson production known including up to two loop QCD at fully differential level

[Anastasiou, Dixon, Melnikov, Petriello 2004]

•

[Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann 2016]

\[
\begin{align*}
\text{pp} & \rightarrow (Z,\gamma^*) + X \\
\end{align*}
\]

\[
\begin{align*}
\frac{d^2\sigma}{dM/dY} & \text{ [pb/GeV]} \\
\end{align*}
\]

\[
\begin{align*}
\sqrt{s} = 14 \text{ TeV} \\
M = M_Z \\
M/2 \leq \mu \leq 2M \\
\end{align*}
\]

\[
\begin{align*}
\text{Figure 3: The CMS rapidity distribution of an on-shell Z boson at the LHC. The LO, NLO, and NNLO results have been included. The bands indicate the variation of the renormalization and factorization scales in the range } M_Z/2 \leq \mu \leq 2M_Z.
\end{align*}
\]
Higher order QCD corrections in hadron collisions

- stable top anti-top quark production is known including up to second order QCD corrections [Czakon, Fielder, Mitov 2013]

LO \hspace{1cm} NLO \hspace{1cm} NNLO

![Diagram showing LO, NLO, and NNLO contributions](image)

<table>
<thead>
<tr>
<th>Collider</th>
<th>(\sigma_{\text{tot}} , [\text{pb}])</th>
<th>scales [pb]</th>
<th>pdf [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron</td>
<td>7.009</td>
<td>+0.259(3.7%)</td>
<td>+0.169(2.4%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.374(5.3%)</td>
<td>-0.121(1.7%)</td>
</tr>
<tr>
<td>LHC 7 TeV</td>
<td>167.0</td>
<td>+6.7(4.0%)</td>
<td>+4.6(2.8%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10.7(6.4%)</td>
<td>-4.7(2.8%)</td>
</tr>
<tr>
<td>LHC 8 TeV</td>
<td>239.1</td>
<td>+9.2(3.9%)</td>
<td>+6.1(2.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-14.8(6.2%)</td>
<td>-6.2(2.6%)</td>
</tr>
<tr>
<td>LHC 14 TeV</td>
<td>933.0</td>
<td>+31.8(3.4%)</td>
<td>+16.1(1.7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-51.0(5.5%)</td>
<td>-17.6(1.9%)</td>
</tr>
</tbody>
</table>

**TABLE II:** Pure NNLO theoretical predictions for various hadron colliders. The leading-order central value is given without uncertainties, and the next-to-leading-order (NLO) and next-to-next-to-leading order (NNLO) central values are essentially identical in the region of the top quark mass. Because of a slight difference in the scale variations, we also present the absolute prediction for the top-quark mass, compared to the latest combination of Tevatron measurements.

In Figs. 3, 4, and 5 we show the scale variation for each of these distributions. We stress that the bin sizes we choose are for illustrative purposes only and should not be taken as a precise representation of the data.

**FIG. 3:** Theoretical prediction for the Tevatron as a function of the top quark mass, compared to the latest combination of Tevatron measurements. The coeﬁcients which allow NNLL soft-gluon re-...
Higher order QCD corrections in hadron collisions

- differential (inclusive) Higgs boson production in gluon fusion and vector boson fusion is known including up to two (three) loops by several (1) groups ([Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2015])

![Diagram of Higgs boson production](image)

**Figure 8:** The dependence of the cross-section on a common renormalization and factorization scale $\mu = \mu_F = \mu_R$.

Rapid progress of the theory community on such calculations:

- on both the calculation of difficult integrals and the construction of general subtraction schemes
- all the complicated calculations carried out so far have proved useful to match the experimental precision!
All order corrections

- Note however that logarithmic enhancement in the computation of observables are very frequent
- They require resummation
All order corrections

• Note however that logarithmic enhancement in the computation of observables are very frequent

• They require resummation:

  ➡ UV log: Running, the Renormalisation Group Equation resume this log

\[
\alpha_S(m_Z^2) = \frac{\alpha_S(m_{\tau}^2)}{1 + b_0\alpha_S(m_{\tau}^2) \log \frac{m_Z^2}{m_{\tau}^2}} \neq \alpha_S(m_{\tau}^2) \left( 1 - b_0\alpha_S(m_{\tau}^2) \log \frac{m_Z^2}{m_{\tau}^2} \right)
\]

\[
\begin{align*}
\alpha_S(m_{\tau}^2) & \sim 0.36 \\
\alpha_S(m_Z^2) & \sim 0.12
\end{align*}
\]

\[
b_0\alpha_S(m_{\tau}^2) \log \frac{m_Z^2}{m_{\tau}^2} \sim 2
\]
All order corrections

• Note however that logarithmic enhancement in the computation of observables are very frequent

• They require resummation:
  ➡ UV log: Running, the Renormalisation Group Equation resume this log
  ➡ collinear log: Altarelli-Parisi Equation resume these logs

\[
\frac{d}{d \log \mu^2} f_i^H(x, \mu) = \int_x^1 \frac{dz}{z} \sum_j P_{ij}(\alpha_S(\mu), z) f_j^H(x/z, \mu)
\]

\[
P_{ij}(\alpha_S(\mu), z) = \frac{\alpha_S(\mu)}{2\pi} P_{ij}^0(z) + \left( \frac{\alpha_S(\mu)}{2\pi} \right)^2 P_{ij}^1(z) + \ldots
\]
All order corrections

• Note however that logarithmic enhancement in the computation of observables are very frequent

• They require resummation:

  ➡ UV log: Running, the Renormalisation Group Equation resume this log

  ➡ collinear log: Altarelli-Parisi Equation resume these logs

  ➡ other logs: poles cancels, but large logarithmic contribution can remain if the measurement constraint produce an unbalanced cancellation of real and virtual contribution (suppression or emphasis of the real contribution)

• basic example: Sterman-Weinberg jets

• But this situation can happen for every observable quantity
Higher order corrections

- Pt of a Z boson (or W/H)

\[ L = \log \frac{M^2}{p_T^2} \]

\[ \delta(p_T) \]

LO

NLO

NNLO

\[ + C_1 \alpha_S(\mu)L \]

\[ - C_2 \alpha_S^2(\mu)L^3 \]
Higher order corrections

- Pt of a Z boson (or W/H)

\[ L = \log \frac{M^2}{p_T^2} \]

\[ \delta(p_T) \]

LO

\[ + C_1 \alpha_S(\mu) L \]

NLO

\[ - C_2 \alpha_S^2(\mu) L^3 \]

NNLO

Indeed all logs \( \alpha_S^n \log^m \frac{M^2}{p_T^2} \)

with \( 0 \leq m \leq 2n - 1 \)
Higher order corrections

• But also, for other measurements that are very well defined experimentally:
  - small mass limit of a jet, or small jet radius
  - distributions of heavy particles produced close to threshold
  - ...
• Prediction for all these observables need resummation of large logs in the region where $\alpha_s L^2 \sim 1$
Higher order corrections

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  - ...

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### Dominant Log

<table>
<thead>
<tr>
<th>$\alpha_S L$</th>
<th>$\alpha_S L^2$</th>
<th>$\alpha_S L^4$</th>
<th>\ldots</th>
<th>$\alpha_S L^{2n}$</th>
<th>$\alpha_S L^{2n-1}$</th>
<th>\ldots</th>
<th>$\alpha_S L^{2n}$</th>
<th>$\alpha_S L^{2n-1}$</th>
<th>\ldots</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>NLO</td>
<td>NNLO</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

1/$L$ effect (small correction)

• one resum classes of contributions

• resumed results can be combined with horizontal contribution via matching (removing double counting)

• whether the series converges or not and if it has an exponential form depends on the quantity being measured: es. ycut log in e+e- annihilation does not exponentiate for the JADE algorithm but it does for $k_T$!
Resummation for specific distributions: Higgs boson production

- $p_t$ of the Higgs boson (EFT production) decaying into two photons

[Bozoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]

- Inclusive results
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[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]

- results with experimental cuts on photons

$$\min(p_t^{\gamma_1}, p_t^{\gamma_2}) > 31.25 \text{ GeV}, \quad \max(p_t^{\gamma_1}, p_t^{\gamma_2}) > 43.75 \text{ GeV}$$

$$0 < |\eta^{\gamma_1,2}| < 1.37 \quad \text{or} \quad 1.52 < |\eta^{\gamma_1,2}| < 2.37, \quad |Y_{\gamma\gamma}| < 2.37$$
**Resummation for specific distributions: Drell-Yan production**

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]

- $p_T$ of the $Z$ boson

![Graph showing $p_T$ distribution for Drell-Yan pair production](image)

RadISH+NNLOJET

8 TeV, $pp \to Z(\rightarrow \ell^+\ell^-) + X$

$0.0 < |y_{\ell\ell}| < 2.4, 46 < M_{\ell\ell} < 66$ GeV

NNPDF3.0 (NNLO)

uncertainties with $\mu_R, \mu_F, Q$ variations

---

Figure 9. Comparison of the normalised transverse momentum distribution for Drell-Yan pair production at NNLO (green), NNLL+NLO (blue) and $N^3$LL+NNLO (red) at $p_s=8$ TeV, $pp \to Z(\rightarrow \ell^+\ell^-) + X$ integrated over the full lepton-pair rapidity range ($0 < |Y_{\ell\ell}| < 2.4$), in three different lepton-pair invariant-mass windows. For reference, the ATLAS data is also shown, and the lower panel shows the ratio of each prediction to data.
• the leading soft/collinear logarithmic enhancement can be resummed for all observables

• starting from an hard 2 to 2 process it is possible to build an approximation for all the diagrams in which subsequent multiple emissions has distributed the available energy to a shower of partons

• consider final state radiation in e+e- annihilation(simplifying example)

  ➡ start from the hard qqbar event. It happens at a certain fixed starting hard scale Q
  ➡ we know that collinear emission from each leg is enhanced

- perturbative
- approx. pert.
- model
Parton Shower

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• consider final state radiation in e+e- annihilation(simplifying example)
  ➡ start from the hard qqbar event. It happens at a certain fixed starting hard scale $Q$
  ➡ we know that collinear emission from each leg is enhanced
  ➡ we look for an iterative formula to approximately describe the sequence of parton splittings

$$d\sigma_R = \sigma_0 dW_{13} + \sigma_0 dW_{23} + \text{finite terms}$$

$$dW_{23} = \frac{d x_1}{1 - x_1} dx_3 P_{qg}(x_3) = \frac{d \cos \theta_{23}}{1 - \cos \theta_{23}} dx_3 P_{qg}(x_3)$$

• $qg$ splitting probability:

$$P_{qg}(x_3) = C_F \frac{\alpha_S}{2\pi} \frac{1 + (1 - x_3)^2}{x_3}$$

$$\theta_{23} \to 0$$

$$E_3 \to 0$$

$$\frac{d\theta_{23}^2}{\theta_{23}^2} \frac{dE_3}{E_3}$$
Parton Shower

\[ d\sigma_R \sim \sigma_0 \sum_i \frac{d\theta^2}{\theta^2} dzP_{ji}(z) \]

\[ \frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_{\perp}^2}{k_{\perp}^2} \]

• probability diverges, let’s think for example to the invariant mass of the internal propagator as a cut off
Parton Shower

\[ d\sigma_R \sim \sigma_0 \sum_i \frac{d\theta^2}{\theta^2} dz P_{ji}(z) \]

Note! Indeed, to describe the collinear limit we could have used also other variables that are good as well, we get the same expression if they are proportional to \( \theta^2 \) for example \( q^2 = z(1 - z) \theta^2 E^2 \) or \( k_{\perp}^2 = z^2(1 - z)^2 \theta^2 E^2 \) as a result we would get identical results for the collinear limit, but different extrapolation away from it. Let’s move to the virtuality of the internal line.

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• probability diverges, let's think for example to the invariant mass of the internal propagator as a cut off

• but still the probability for a given emission path built reiterating these emissions can well exceed unity! while the sum of the probabilities of all shower configuration must yield 1
Parton Shower

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- in turn we need to dump this emission probability with a factor that has to restore unitarity keeping into account the missing virtual corrections and resum the logarithmic enhancement
Parton Shower

• to simplify the things, let’s consider a single kind of splitting, to be definite, that we have just gluons

\[ d\sigma_R \sim \sigma_0 P_{gg}(z) \frac{dq^2}{q^2} dz \]
Parton Shower

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\[ d\sigma_R \sim \sigma_0 P_{gg}(z) \frac{dq^2}{q^2} dz \]

• we normalise the emission probability multiplying times the inclusive probability of no emission generating a virtuality above \( q^2 \) and given the maximum possible virtuality \( Q^2 \) (Sudakov form factor)

\[ d\sigma_{1st\text{-}emission} = P_{gg}(z) \frac{dq^2}{q^2} dz \Delta_g(Q^2, q^2) \]
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• of course, this “no emission” probability has to be related to the emission probability, it has to be a function of it. Given that for smaller and smaller \( q^2 \) the emission probability diverges, in this limit the inclusive “no emission” probability has to go to zero rapidly, so regularising the divergence
Parton Shower

- now let’s compute $\Delta$ (here comes the resummation!)
  The probability of emission giving a virtuality among $q^2$ to $q^2+dq^2$ is:

\[ dW_g = \frac{dq^2}{q^2} \int_{z_{min}}^{z_{max}} dz P_{gg}(z) \]

- so, for such infinitesimal range of virtuality $dq^2$ the “no emission” probability is

\[ dP(\text{no emission}) = 1 - dW_g = 1 - \frac{dq^2}{q^2} \int_{z_{min}}^{z_{max}} dz P_{gg}(z) \]
Parton Shower

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- to get the “no emission” probability over the finite range from $q^2$ up to $Q^2$, we don’t integrate this, but take the product over “all” the infinitesimal regions, so collecting the probability of “no multiple emission” of every number of gluons

$$\Delta_g(Q^2, q^2) = \prod dP(\text{no emission}) = \exp \left\{ - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{z_{\text{min}}}^{z_{\text{max}}} dz P_{gg}(z) \right\}$$
Parton Shower

- the parton shower is then produced generating radiation with distribution

\[ d\sigma_{1st\ emission} = P_{gg}(z)dz\frac{dq^2}{Q^2} \exp \left\{ -\int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{z_{min}}^{z_{max}} dz P_{gg}(z) \right\} \]

- this formula can now be iterated updating the upper \( Q^2 \) scale to the extracted \( q^2 \) at each step until the virtuality of the internal line reaches the cut-off scale
Parton Shower

Observations:

• a large number of technicalities: momentum reshuffling after each emission, management of multiple splitting processes, hadrons in the initial state…

➡ hadronization models are just models that parametrise data in terms of a large number of parameters, inspired by QCD but in principle there could well be no physics at all in their parameters

➡ initial state has to take into account the structure of the colliding hadrons, more technical complications but is done with the same logic (space-like shower)
Parton Shower

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• and choices: evolution variable, scale in the strong coupling, cut-off scale…
Parton Shower

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• and choices: evolution variable, scale in the strong coupling, cut-off scale…

➤ wide angle soft emission and coherence: evolution variable $\theta$ angle or kt

➤ cut-off scale dependence. Note that soft radiation has to be more and more collinear and so we can imagine that although soft radiation is long range, hadronization can be considered a local process

• and even more needed for a full description of the possible kind of collisions: multiple interactions in hadron-hadron collisions, other non perturbative phenomena…
Jet shape at the LHC

- require resummation of large soft and collinear logarithmic enhancement
  - parton shower description

- fraction, $\rho$, of the jet transverse momentum contained within a ring of radius $r$ around the jet core

$$\rho(r) = \frac{1}{N} \sum_{\text{jets}} \frac{1}{\Delta r} \frac{\sum_{r_a \leq r_i < r_b} p_{T,i}}{\sum_{r_i \leq R} p_{T,i}}$$

$$r_i = \sqrt{(\Delta_i,y)^2 + (\Delta_i,\phi)^2}$$

$$r_a = r - \Delta r/2 \quad r_b = r + \Delta r/2$$

![Graph](image)

**Figure 19:**
- CMS integral of differential jet shape up to a radius $r$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 18:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 17:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 16:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 15:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 14:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 13:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 12:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 11:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 10:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 9:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 8:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 7:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 6:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 5:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 4:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 3:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$

**Figure 2:**
- Distribution of differential jet shape for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$
- MC/data ratio for $|y| < 1$ and $110 \text{ GeV} < p_T^{\text{jet}} < 125 \text{ GeV}$

**Figure 1:**
- CMS integral of differential jet shape up to a radius $r$
- ATLAS measurement of $1 - \Psi(r)$ for $r = 0.3$, $|y| < 2.8$
Jet shape at the LHC

- works also for lower pt jets

$$\rho(r) = \frac{1}{N} \sum_{\text{jets}} \frac{1}{\Delta r} \sum_{r_a \leq r_i < r_b} \frac{1}{\sum r_i \leq R} p_{T,i}$$

$$r_i = \sqrt{(\Delta_{i,jet} y)^2 + (\Delta_{i,jet} \phi)^2}$$

$$r_a = r - \Delta r/2 \quad r_b = r + \Delta r/2$$
Parton Shower

Note that:

• soft and collinear approximations that underlie PS may fail to reproduce the full pattern of hard wide-angle emissions

• PS generate cross sections for the requested hard process that are correct at LO

Nevertheless:

• PS is a fundamental tool to study the sensitivity of the experiments in high energy physics

• one would like to include all possible hard processes and all of them computed including higher order corrections in a framework like this

⇒ we are on this path…
Merging

Multi-jet final states can be described at LO+LL using LO matrix elements with up to n partons in the final state. Let’s consider $e^+e^-$ at an Energy scale $Q$ for simplicity

[Catani, Kuhn, Krauss, Webber 2001]
Merging

Multi-jet final states can be described at LO+LL using LO matrix elements with up to n parsons in the final state. Let’s consider e+e- at an Energy scale $Q$ for simplicity

[Catani, Kuhn, Krauss, Webber 2001]

1. Define a scale $Q_1$ for the kT algorithm resolution parameter $y_{ini} = Q_1^2/Q^2$ that acts as a cut-off regulator to compute x-jet cross section using exact tree level matrix element with x-parton in the final state. Do it for x from 2 to n (typically 5 or 6)
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2. Select the jet multiplicity (n) and partonic subprocess (i) with probability:

$$P^{(0)}(n, i) = \frac{\sigma^{(0)}_{n,i}}{\sum_{k=N}^{k,j} \sigma^{(0)}_{k,j}}$$
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2. Select the jet multiplicity (\( n \)) and partonic subprocess (\( i \)) with probability:

\[
P^{(0)}(n, i) = \frac{\sigma^{(0)}_{n,i}}{\sum_{k=N}^{\Delta} \sigma^{(0)}_{k,j}}
\]

3. Generate an event with probability given by the exact matrix element
4. Reconstruct the PS probability of this event using the kT algorithm backward recombining parsons until only 2 remain and build a weight combining Sudakov from factors and Splitting probabilities with \( \alpha_s \) evaluated at the scale of the branching process for each branch
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5. Accept or reject the event according to the combined PS weight:
   a. If the event is accepted, assign a color configuration and start the shower from each leg of the n parton event at scale $\tau$ at which it has been generated, vetoing radiation harder then $Q_1$
   b. If the event is rejected go back to 1.
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   b. If the event is rejected go back to 1.

Needs modifications when there are hadrons in the initial state, but the logic is the same: the point is always to regularise matrix elements factorising the phase space into hard (ME domain) and soft (PS domain) and to accept or reject an event according to the PS weight of the event
Merging

Successful but do not expect too much

- 5% uncertainty on the strong coupling results on 30% uncertainty for 6 jets production, and then there is scale variations (both renormalization and factorisation), hadronization effects...
- Description with just MC programs is more an exercise to tune them than a real test for QCD
Matching: NLO and PS

First proposals have been the MC@NLO [Frixione and Webber 2002] and the POWHEG [Nason 2004] methods. Let’s take for example the POWHEG method that stands for positive weight hardest emission generator. See the original references for the proofs, we just sketch the construction

- we start considering the first emission in a parton shower approach

\[d\sigma_{1st\ emission} = \sigma_0 P_{gg}(z) \frac{dk_T^2}{k_T^2} \, dz \Delta(Q^2, q^2)\]
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- to have NLO prediction on inclusive variables one start generating the “underlining born” kinematic configuration with probability given by NLO accuracy for parton level generation

\[
\sigma_0 \implies \bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n)
\]

\[
\quad + \int d\Phi_{\text{rad}} [R(\Phi_{n+1}) - C(\Phi_{n+1})] + \int \frac{dz}{z} [G_{\oplus}(\Phi_{n,\oplus}) + G_{\ominus}(\Phi_{n,\ominus})]
\]

\[
\Phi_{n}=\Phi_n
\]
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• we start considering the first emission in a parton shower approach

\[ d\sigma_{1st\ emission} = \sigma_0 P_{gg}(z) \frac{d^2 k_T}{k_T^2} dz \Delta(Q^2, q^2) \]

• to have NLO prediction on inclusive variables one start generating the “underlining born” kinematic configuration with probability given by NLO accuracy for parton level generation

\[
\sigma_0 \implies \tilde{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) \\
+ \left[ \int d\Phi_{\text{rad}} [R(\Phi_{n+1}) - C(\Phi_{n+1})] + \int \frac{dz}{z} [G_{\oplus}(\Phi_{n,\oplus}) + G_{\ominus}(\Phi_{n,\ominus})] \right]_{\Phi_n=\Phi_n}
\]

• then one attach the hardest radiation with probability given by a Sudakov form factor build upon the ratio of Real and Born matrix elements:

\[
d\sigma = \tilde{B}(\Phi_n) d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\text{min}}) + \Delta(\Phi_n, k_T(\Phi_{n+1})) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\text{rad}} \right\} \bigg|_{\Phi_n=\Phi_n}
\]

\[
\Delta(\Phi_n, p_T) = \exp \left\{ -\int \frac{[d\Phi_{\text{rad}} R(\Phi_{n+1}) \theta(k_T(\Phi_{n+1}) - p_T)]}{B(\Phi_n)} \bigg|_{\Phi_n=\Phi_n} \right\}
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\[
\sigma_0 \quad \Longrightarrow \quad \tilde{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) \\
\quad \quad \quad + \left[ \int d\Phi_{\text{rad}} [R(\Phi_{n+1}) - C(\Phi_{n+1})] + \int \frac{dz}{z} \left[ G_\oplus(\Phi_{n,\oplus}) + G_\ominus(\Phi_{n,\ominus}) \right] \right]_{\Phi_n=\Phi_n}
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- then one attach the hardest radiation with probability given by a Sudakov form factor build upon the ratio of Real and Born matrix elements:

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d\sigma = \tilde{B}(\Phi_n) d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \Delta(\Phi_n, k_T(\Phi_{n+1})) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\text{rad}} \right\}_{\Phi_n=\Phi_n}
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\[
\Delta(\Phi_n, p_T) = \exp \left\{ - \int \frac{d\Phi_{\text{rad}} R(\Phi_{n+1}) \theta(k_T(\Phi_{n+1}) - p_T)}{B(\Phi_n)} \right\}_{\Phi_n=\Phi_n}
\]

- finally, assign a color configuration and shower the event vetoing emissions harder then the first one to avoid double counting
NLO + PS: $t\bar{t}$ @ LHC 14

- Inclusive variables have NLO precision

[Frxiione, Nason, Webber 2003]
NLO + PS: $t\bar{t}$ @ LHC14

- Complementary behaviour of the NLO and MC approaches regardless of the cuts on the rapidities and transverse momenta of the top quark of the heavy quarks
- In the tail of the transverse momentum $p_T$ distribution of the top pairs, the NLO cross section is much larger than the MC one, simply because hard emissions are correctly treated only in the former.
- For small transverse momentum of the top pair the difference between the two histograms shows the effect of all-order resummation, clearly, no meaningful comparison between NLO and data can be attempted in this region.

[Fiuxione, Nason, Webber 2003]
Merging and Matching

• NLO+PS generators for a process that has at least 1 jet in the final state in the leading order, has divergences when this radiation became unresolved.

\[ q \rightarrow \bar{q}' \rightarrow W^{-} \rightarrow H \rightarrow g \]

Figure 1: A sample of leading-order Feynman diagrams for $HWj$ and $HZj$ production.

Figure 2: A sample of one-loop Higgs-Strahlung diagrams, with no closed fermionic loop.

(b) A sample of diagrams belonging to the second class of virtual corrections are illustrated in fig. 3. In this figure we have plotted Higgs-Strahlung-type diagrams when a closed quark loop is present, and the Z boson couples to the internal quark. No such diagrams are present for $W$ production, since the flavour running in the loop must be conserved. These contributions vanish by charge-conjugation invariance (Furry's theorem), when they couple to a vector current. For axial currents, they can cancel in pairs of up-type and down-type quarks, because they have opposite axial coupling, as long as the loop of different flavours can be considered massless. Thus, the up-quark contribution cancels with the down-quark, and, since we treat the charm as massless, its contribution cancels with the strange one. Only the difference between the diagrams with a massive top
• NLO+PS generators for a process that has at least 1 jet in the final state in the leading order, has divergences when this radiation became unresolved

• they can be made finite by multiplying the probability for the first emission by an extra Sudakov form factor, of course

\[ q \rightarrow g \rightarrow H \rightarrow l \]

\[ \bar{q}' \rightarrow W^- \rightarrow \bar{\nu} \]
Merging and Matching

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• BTW, an optimal choice of this factor can be done, such that the formal accuracy of the inclusive generation is NLO accurate

[Hamilton, Nason, Oleari, Zanderighi 2013]
Merging and Matching

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  [Hamilton, Nason, Oleari, Zanderighi 2013]

  ➡ this method is called MINLO that stands for Multi-scale improved NLO

  ➡ you can generate simultaneously WH+0/1 jet or Wb\bar{b}+0/1 jet or whatever other process +0/1 jets with this technique

  ➡ do not require a merging scale as for CKKW

• this is not the only way to do that, several techniques used by aMC@NLO and SHERPA collaborations to merge NLO computations with more jets in the final state
Merging and Matching: HV+0/1j @LHC8

- total cross sections

Figure 8: Total cross section variation for HVJ-MiNLO (solid red) and HV (dashed black). The maximum and minimum values for the total cross section are taken from tabs. 1 and 2. The total cross section with central scales is drawn in dotted lines.

Since the improved MiNLO prescription applied here achieves NLO accuracy for observables inclusive in the HV production, we begin showing results for the most inclusive quantity, i.e. the total cross section. In tabs. 1 and 2 we collect the results for the total cross sections obtained with the HVJ-MiNLO and the HV programs, both at full NLO level, for different scale combinations. The scale variation in the HVJ-MiNLO results is obtained by multiplying the factorization scale and each of the several renormalization scales that appear in the procedure by the scale factors $(K_R, K_F)$, respectively, where $(0.5, 0.5)$, $(0.5, 1)$, $(1, 0.5)$, $(1, 1)$, $(2, 1)$, $(1, 2)$, $(2, 2)$.

The Sudakov form factor is also changed according to the prescription described in ref. [22].

For ease of visualization, in fig. 8 we have plotted the maximum and minimum values for the HVJ-MiNLO (red lines) and HV (black lines) cross sections in solid and dashed lines, respectively. We have also plotted the central-scale cross section in dotted lines. Notice that we expect agreement only up to terms of higher order in $\alpha_S$, since the HVJ-MiNLO results include terms of higher order, and also since the meaning of the scale choice is different in the two approaches. For similar reasons, we do not expect the scale variation bands to be exactly the same in the two approaches. From the tables and the figure, it is clear that the standard HV NLO+PS results and the HVJ-MiNLO one are fairly consistent: the HVJ-MiNLO independent scale variation is in general larger than the HV one, and it shrinks if a symmetric scale variations is performed, as illustrated in the last column of the tables.
Merging and Matching: HV+0/1j @LHC8

- differential distributions: HW rapidity

Figure 9: Comparison between the HW+PYTHIA result and the HWJ-MiNLO+PYTHIA result for the HW rapidity distribution at the LHC at 8 TeV. The left plot shows the 7-point scale-variation band for the HW generator, while the right plot shows the HWJ-MiNLO 7-point band.

Turning now to less inclusive quantities, we plot in fig. 9 the rapidity distribution of the HW system obtained with the HW and HWJ-MiNLO generator. We remind that this quantity is predicted at NLO by both generators, and in fact the agreement is very good.

The uncertainty band of the HW generator is shown on the left while that of the HWJ-MiNLO generator is shown on the right.

In fig. 10 we show another inclusive quantity, i.e. the charge depleting transverse momentum from the W decay. Also in this case we find perfect agreement between the two generators.

In figs. 11 and 12 we compare the HW and HWJ-MiNLO generators for the transverse momentum of the HW system. In this case we do observe small differences, that are however perfectly acceptable if we remember that this distribution is only computed at leading order by the HW generator, while it is computed at NLO accuracy by the HWJ-MiNLO generator. It can also be noted that the uncertainty band for the HW generator is uniform, while it depends upon the transverse momentum for the HWJ-MiNLO one. In fact, the uniformity of the scale-variation band in the HW case is well understood: in POWHEG, the scale uncertainty manifests itself only in the $\bar{B}$ function, while the shape of the transverse-momentum distribution is totally insensitive to it.

The transverse momentum of the second jet computed with the HWJ-MiNLO generator

[Luisoni, Nason, Oleari et al 2013]
Merging and Matching: HV+0/1j @LHC8

- differential distributions: HW pt

[Luisoni, Nason, Oleari et al 2013]
Merging and Matching: HV+0/1j @LHC8

- differential distributions: second jet pt

![Graph showing differential distributions for second jet pt](image)
5. Comparison with ATLAS and CMS data

This is clearly displayed in fig. 12, where the scale variation of the differential cross sections as a function of the transverse momentum distribution of the system, generated by the event generators. Wbb PY+SWR and Wbbj PY+SWR, is completed by a Monte Carlo program such as MiNLO, Sudakov form factor, attached to the hardest jet accompanying the jet production. The finite contribution to the differential cross section as a function of the rapidity distribution of the system, generated by the event generator, and at NLO by the Wbb PY+SWR generator, and at NLO by the Wbbj PY+SWR generator, is due to events that have not radiated at NLO level, but at this level, b hadronization has switched on. In the LHE level, this peak is diluted away when the whole shower is completed by a Monte Carlo program such as MiNLO. Again the agreement is very good. The finite contribution to the ratio of the differential cross sections as a function of rapidity and transverse momentum distribution of the system, generated by the event generator, and at NLO by the Wbb PY+SWR generator, and at NLO by the Wbbj PY+SWR generator, is due to events that have not radiated at NLO level, but at this level, b hadronization has switched on. In the LHE level, this peak is diluted away when the whole shower is completed by a Monte Carlo program such as MiNLO.

[Luisoni, Oleari et al 2015]
Matching NNLO with PS

• Reweighing NLO+PS generators for a process that has exactly 1 jet in the final state in the leading order (like H/W/Z+1jet, and in principle every process producing a colourless final state plus 1 jet) with a parton level NNLO computation for the fully inclusive production of such a colourless final state, one obtain NNLO+PS matching!

[Hamilton, Nason, Re, Zanderighi 2013]
Matching NNLO with PS

- Reweighing NLO+PS generators for a process that has exactly 1 jet in the final state in the leading order (like H/W/Z+1jet, and in principle every process producing a colourless final state plus 1jet) with a parton level NNLO computation for the fully inclusive production of such a colourless final state, one obtain NNLO+PS matching!

- in practice:
  
a. produce a sample of events for H+1jet, for example, including the MINLO Sudakov
b. bin the sample in the fully inclusive variables (just rapidity for H, azimuth is irrelevant)
c. reweight the events such that the rapidity distribution is the one of the full NNLO computation

- for more complicated Born processes it's a difficult task:
  
  ➡ more variables and so need to reweight multi-dimensional histograms!

- can be extended to more complex final state, but some parts of the MINLO Sudakov suppression factor for the general case are still missing

[Hamilton, Nason, Re, Zanderighi 2013]
**NNLO+PS: Higgs production @LHC8**

[HNNLO: [Catani, Grazzini 2007]

[HqT: [Bozzi, Catani, De Florian, Grazzini 2005], [De Florian, Ferrera, Grazzini, Tommasini 2011]]