Partial correlation analysis in ultra-relativistic nuclear collisions

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Outline

- Partial correlations (PC) analysis, physical and control random variables (meaning of centrality)
- PC in a superposition approach – placing constraints on sources
- Test on a hydro solution: a working scheme

Partial correlations

Kindergarden

Sample of children:

1. weight
2. intelligence

Pearson's correlation matrix:

$$\rho = \begin{pmatrix} 1 & 0.62 \\ 0.62 & 1 \end{pmatrix}$$

$\rightarrow \rho(\text{weight}, \text{intelligence}) \simeq 0.6$ – large

Hints to wrong conclusions

[W. Krzanowski, *Principles of Multivariate Analysis*, Oxford U. Press, 2000]

Kindergarden

Sample of children:

1. weight
2. intelligence
3. age – control (external, nuisance) variable

Pearson's correlation matrix:

$$\rho = \begin{pmatrix} 1 & 0.62 & 0.84 \\ 0.62 & 1 & 0.74 \\ 0.84 & 0.74 & 1 \end{pmatrix}$$

$\rightarrow \rho(\text{weight}, \text{intelligence}) \simeq 0.6$ – large

Partial correlation (defined shortly) gives $\rho(\text{weight}, \text{intelligence} \bullet \text{age}) \simeq 0$

[W. Krzanowski, *Principles of Multivariate Analysis*, Oxford U. Press, 2000]

Partial correlation

Two physical variables X, Y and one control variable Z:

$$\mathrm{c}(X, Y \bullet Z) \quad = \quad \mathrm{c}(X, Y) - \frac{\mathrm{c}(X, Z)\mathrm{c}(Z, Y)}{\mathrm{v}(Z)}$$

Pearson's-like partial correlation coefficient is

$$\rho(X, Y \bullet \mathbf{Z}) = \frac{c(X, Y \bullet Z)}{\sqrt{c(X, X \bullet Z)c(Y, Y \bullet Z)}} = \frac{\rho(X, Y) - \rho(X, Z)\rho(Z, Y)}{\sqrt{1 - \rho(X, Z)^2}\sqrt{1 - \rho(Z, Y)^2}}$$

$\rho(\text{weight}, \text{intelligence} \bullet \text{age}) \simeq 0$

One often uses the correlation = covariance scaled with the multiplicities:

$$\mathrm{C}(X, Y) = \frac{\mathrm{c}(X, Y)}{\langle X \rangle \langle Y \rangle}, \quad \mathrm{V}(X) \equiv \mathrm{c}(X, X) = \frac{\mathrm{v}(X)}{\langle X \rangle^2}$$

Then

$$\mathrm{C}(X, Y \bullet Z) = \mathrm{C}(X, Y) - \frac{\mathrm{C}(X, Z)\mathrm{C}(Z, Y)}{\mathrm{V}(Z)}$$

Relation to conditional covariance

$\mathrm{c}(X_i, X_j|\mathbf{Z})$ - evaluate at fixed $\mathbf{Z}$ and then average over $\mathbf{Z}$

[Lawrance 1976]: if a sample satisfies $E(\mathbf{X}|\mathbf{Z}) = \alpha + \mathbf{BZ}$, with α a constant and $\mathbf{B}$ a constant matrix $\Rightarrow$

$$c(X_i, X_j \bullet \mathbf{Z}) = c(X_i, X_j|\mathbf{Z})$$

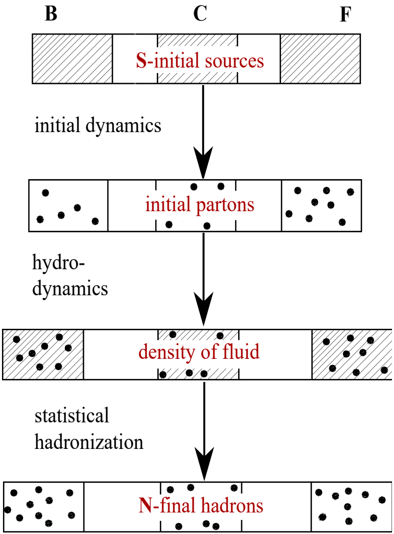
$\Leftarrow$ shown by [Baba et al. 2005]

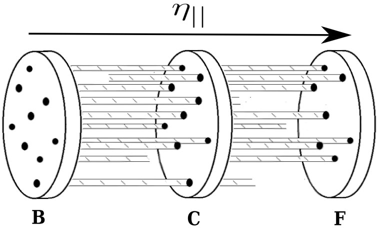
Application of conditinal covariance by [STAR 2008], where Z is hadron multiplicity in the reference bin R:

1. Divide R into very narrow subsamples (centrality classes) according to Z
2. Evaluate the covariance between X_i and X_j in each subsample
3. Average obtained covariances over the subsamples

Superposition model

Superposition model





overlaid distribution of partons

deterministic

overlaid distribution of hadrons

overlaid detector efficiency

Superposition model (cont.)

$$N_A = \sum_{i=1}^{S_A} m_i, \quad A = F, B, C$$

$$\begin{aligned}
\langle N_A \rangle &= \langle S_A \rangle \langle m \rangle \\
\mathrm{v}(N_A) &= \langle m \rangle^2 \mathrm{v}(S_A) + \mathrm{v}(m) \langle S_A \rangle \\
\mathrm{c}(N_A, N_{A'}) &= \langle m \rangle^2 \mathrm{c}(S_A, S_{A'}), \quad A \neq A' \\
\\
\mathrm{c}(N_A, S_{A'}) &= \langle m \rangle \mathrm{c}(S_A, S_{A'})
\end{aligned}$$

$$C(S_A, S_{A'}) = C(N_A, N_{A'}) - \delta^{AA'} \frac{\omega(m)}{\langle N_A \rangle} \equiv \overline{C}(N_A, N_{A'})$$

$\omega(m) = \frac{\mathrm{v}(m)}{\langle m \rangle}$ (for Poisson $\omega(m) = 1$)

Partial correlations in the superposition model

Multiplicities in **F,B** are physical, multiplicity in **C** is a control variable

N_C constraint:

$$C(S_F, S_B \bullet N_C) = \overline{C}(N_F, N_B) - \frac{\overline{C}(N_F, N_C)\overline{C}(N_B, N_C)}{\mathrm{v}(N_C)}$$

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Only measured quantities (hadron multiplicities) on r.h.s.!

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$C(S_F, S_B \bullet N_C)$ vs $C(S_F, S_B \bullet S_C) \leftrightarrow \mathrm{v}(N_C)$ vs $\overline{\mathrm{v}}(N_C)$

Method allows us to impose constraints at the level of initial sources, based on experimentally available info

Test of the method

Test on actual simulations

- Wounded quark model with GLISSANDO at centrality $30-40\%$
- Bzdak-Teaney model with triangular emission functions
- 3+1D viscous hydrodynamics
- Statistical hadronization via THERMINATOR
- Results for
  1. all charged particles - $\pi^{\pm}$, $K^{\pm}$, p and $\overline{p}$,
  2. primordial particles - before resonance decays
  3. π^{+}
- Wide acceptance, $|\eta_{||}| \leq 5.1$, divided into 51 bins with $\Delta\eta = 0.2$

Bzdak-Teaney (BT) model

Use the triangle emission profiles, then:

$$C(S_F, S_B) = \frac{\mathrm{v}(Q_+)}{\langle Q_+\rangle^2} + \frac{\mathrm{v}(Q_-)}{\langle Q_+\rangle^2} u_F u_B,$$

where $u_{F,B} = \eta_{F,B}/y_b$, $Q_\pm = Q_A \pm Q_B$ – numbers of wounded quarks

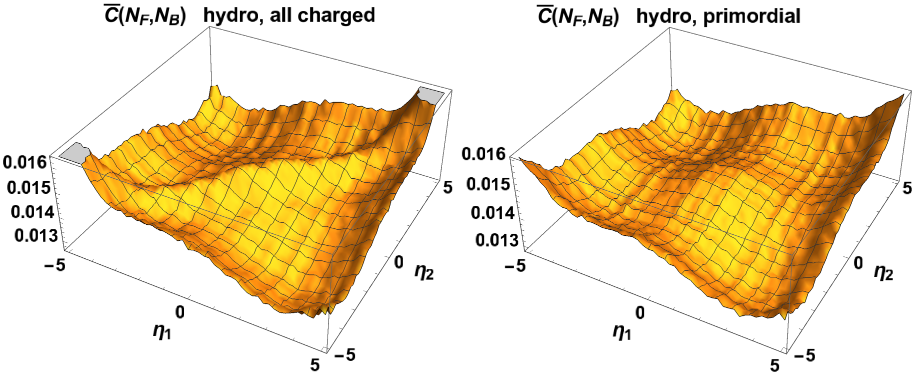
In the central (reference) bin S_C we have $\eta = 0$, which yields

$$C(S_{F,B}, S_C) = C(S_C, S_C) = \frac{\mathrm{v}(Q_+)}{\langle Q_+\rangle^2}$$

$$C(S_F, S_B \bullet S_C) = \frac{\mathrm{v}(Q_-)}{\langle Q_+\rangle^2} u_F u_B$$

(the same result follows via the condition fixing $Q_+ \to \mathrm{v}(Q_+) = 0$)

Scaled covariance

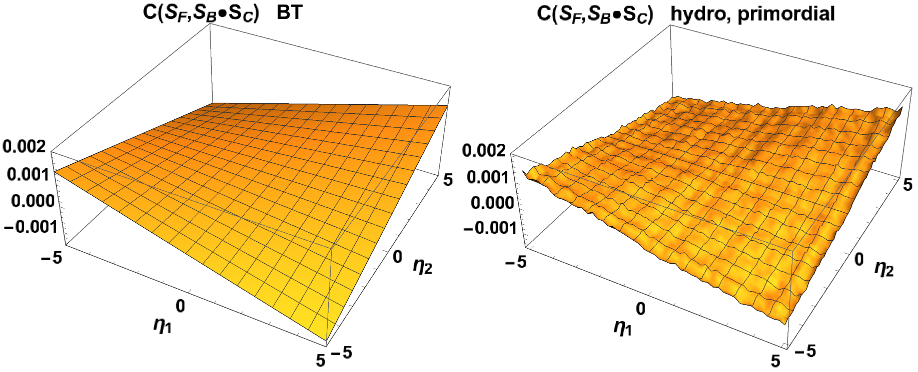


Covariance matrices with the auto-correlations removed
Hallmark ridge along the diagonal from resonance decays

(looks as nothing ...)

Partial: BT vs primordial

$C : -0.1 < \eta < 0.1$



Remarkable agreement of BT and primordial partial correlations

$$C(S_F, S_B \bullet S_C) = \overline{C}(N_F, N_B) - \frac{\overline{C}(N_F, N_C)\overline{C}(N_B, N_C)}{\overline{\mathrm{v}}(N_C)}$$

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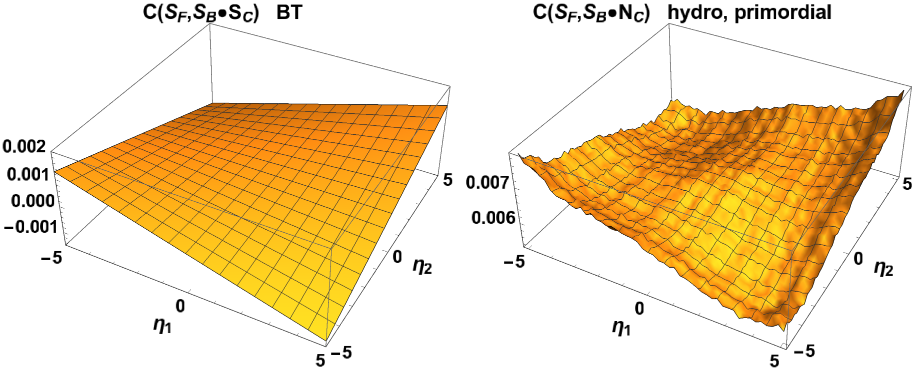

No agreement for the N_C constraint

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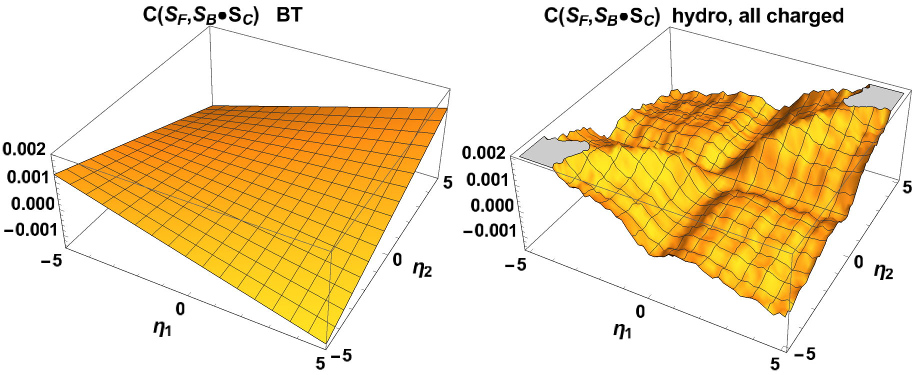
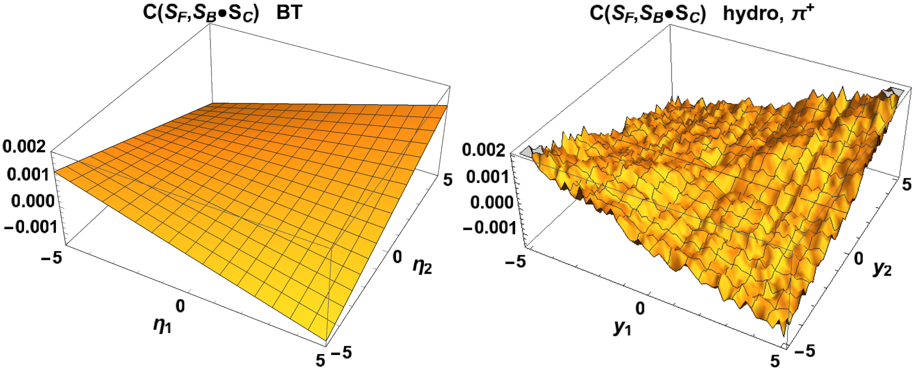
Partial: BT vs all charged



Short-range correlations spoil the agreement

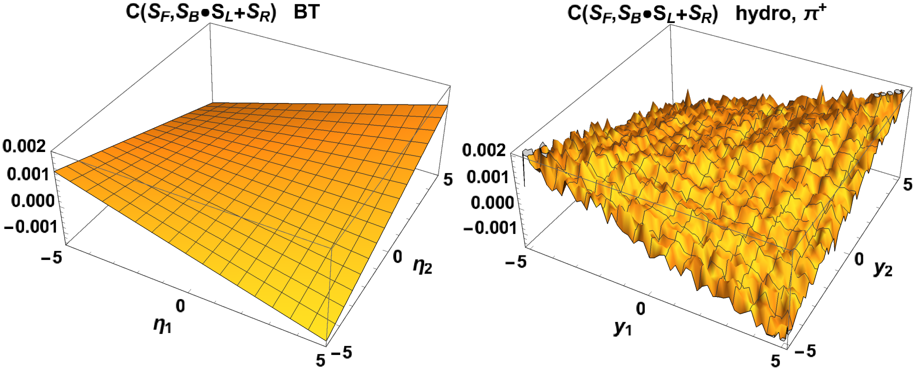
Partial: BT vs π^+



Reduce correlations from resonance decays - no direct decays to $\pi^+\pi^+$

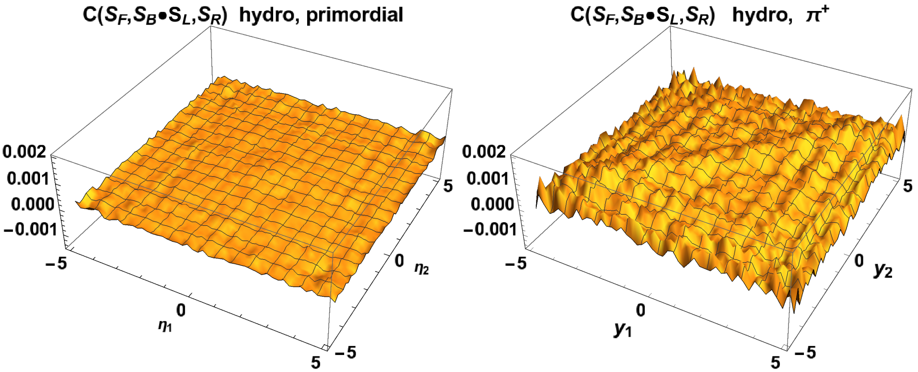
Left+right constraint

$L: -6.1 < \eta < -5.1, \quad R: 5.1 < \eta < 6.1$



(for BT the same effect as from the central constraint)

Independent left- and right constraints



This correlation vanishes in BT
(fixes both Q_A and Q_B, so nothing is left to fluctuate)

Conclusions

Conclusions

- Partial correlations+superposition model – possibility of imposing constraints at the level of sources, gaining insight into the initial stage
- Contraining (event strictly) the number of particles leaves the fluctuation of sources!
- Feasibility of the method demonstrated on simulated data (wounded quarks, hydrodynamics, THERMINATOR) - would be great to use on actual data!
- Need to reduce the short-range correlations (e.g., by looking at π^+), nice to have a large pseudorapidity acceptance
- Several simultaneous constraints possible, generalization of the concept of centrality

Conclusions

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Thank you!

Definition of partial covariance

n physical variables $\mathbf{X} = (X_1, \ldots, X_n)$, m control variables $\mathbf{Z} = (Z_1, \ldots, Z_m)$
X_i, Z_j are vectors in the space of events, i.e., $X_1 = (X_1^{(1)}, X_1^{(2)} \ldots X_1^{(N_{\rm ev})})$
$\langle \mathcal{O} \rangle \equiv \frac{1}{N_{\rm ev}} \sum_{k=1}^{N_{\rm ev}} \mathcal{O}^{(k)}$

Partial covariance:

$$c(\mathbf{X},\mathbf{X} \bullet \mathbf{Z}) \equiv c(\mathbf{X},\mathbf{X}) - c(\mathbf{X},\mathbf{Z}) c^{-1}(\mathbf{Z},\mathbf{Z}) c(\mathbf{Z},\mathbf{X})$$

where $c(\mathbf{A},\mathbf{B})$ is the usual covariance $c(A_i, B_j) = \langle A_i B_j \rangle - \langle A_i \rangle \langle B_j \rangle$.
Diagonalizing $c(\mathbf{Z},\mathbf{Z})$ (orthonormal eigenvectors U_k) yields

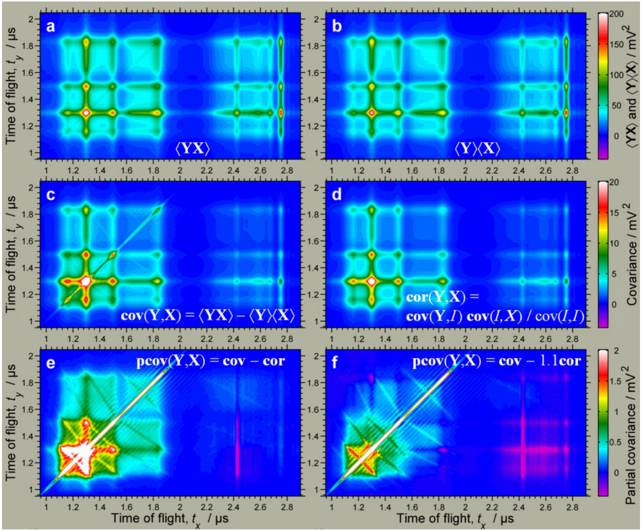
$$\begin{aligned} c\,(X_i, X_j \bullet \mathbf{Z}) &= \mathrm{c}(X_i, X_j) - \sum_{k=1}^{m} \mathrm{c}(X_i, U_k)\mathrm{c}(U_k, X_j) \\ &= c\,(X_i - c(X_i, U_k) U_k, X_j - c(X_j, U_{k'}) U_{k'}) \end{aligned}$$

Components of $\mathbf{X}$ belonging to the space spanned by $\mathbf{Z}$ are projected out

[H. Cramer, *Mathematical methods of statistics*, Princeton U. Press, 1946]

Example: Coulomb explosion of N_2 molecule at FEL

- (a) correlated product
- (b) uncorrelated product
- (c) covariance map
- (d) spurious correlations
- (e) partial covariance
- (f) + corrections



L. J. Frasinski, 2016]