Centrality dependence of freeze-out temperature fluctuations in Pb-Pb collisions at the LHC

Dariusz Prorok

Institute of Theoretical Physics

University of Wrocław

XIII Polish Workshop on RHIC
Wrocław, January 7, 2017

Source

Results are from

DP, J. Phys. G**43** (2016) 055101

DP, in preparation

How (when) the statistical system ends?

Generally, the freeze-out moment (the end of a statistical system) is defined as the moment when hadrons cease to interact and start to stream freely to detectors.

The whole experimental information we get (the data) is from this particular moment, this is like a photo taken at this (and only this) moment.

We model the freeze-out by imposing the condition:

$$\boxed{T(\vec{r}, t) = T_{f.o.} = constant\ .}$$

This defines the 3dim freeze-out hypersurface.

The Single-Freeze-Out Model

W. Broniowski and W. Florkowski, PRL **87** (2001) 272302; PRC **65** (2002) 064905

1. The freeze-out hypersurface and the Hubble-like expansion

$$\tau_f = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2} = const, \qquad u^\mu = \frac{x^\mu}{\tau_f}$$

   with condition $r = \sqrt{r_x^2 + r_y^2} < \rho_{max}$.

2. Contributions from resonance decays to the measured particle multiplicities and momentum distributions are taken into account completely.

Parameters of the Single-Freeze-Out Model

- Statistical parameters $T_f,\ \mu_B$
- Geometric parameters $\tau_f,\ \rho_{max}$
- All four parameters T_f, μ_B, ρ_{max} and τ_f are fitted to the spectra simultaneously in this version of the model [for RHIC: DP, APPB **40**, 2825 (2009)].
- For LHC $\mu_B = 0$ (and $\mu_S = \mu_Q = 0$), so there are three parameters: T_f, ρ_{max} and τ_f.

The invariant distribution of particle species i

$$\boxed{\frac{dN_i}{d^2p_T\ dy} = \int p^\mu d\sigma_\mu\ f_i(p \cdot u)}$$

f_i - final distribution of the ith particle, *i.e.* with contributions from resonance decays:

$d\sigma_\mu$ - normal vector to the freeze-out hypersurface

$$f_i = f_i^{prim} + \sum_{decay} f_i^{decay}$$

The distribution describes production from one collision ($\equiv$ one event).

What are data "points" really?

$$\frac{1}{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} \Longrightarrow \frac{1}{N_{ev}(i)}\frac{1}{2\pi p_T}C_{ij}(p_T)\frac{N^j(i,p_T)}{\Delta p_T \Delta y}$$

$N_{ev}(i)$ - the number of events in the ith centrality bin
p_T - the value in the middle of the p_T bin Δp_T
$N^j(i,p_T)$ - the number of counts of particle species j in the centrality bin i and the p_T bin Δp_T
$C_{ij}(p_T)$ - the total correction factor
N_k^j - the number of counts of j's from kth event $\in (i, \Delta p_T)$

$$N^j = \sum_{k=1}^{N_{ev}} N_k^j \Rightarrow \frac{1}{2\pi p_T}C_{ij}\frac{N^j}{\Delta p_T \Delta y} = \sum_{k=1}^{N_{ev}} \frac{1}{2\pi p_T}C_{ij}\frac{N_k^j}{\Delta p_T \Delta y}$$

$$\frac{1}{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = \frac{1}{N_{ev}}\sum_{k=1}^{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N_k}{dp_T dy}$$

What are data "points" really? *cont.*

$$\frac{1}{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = \frac{1}{N_{ev}}\sum_{k=1}^{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N_k}{dp_T dy}$$

$$\boxed{\frac{1}{2\pi p_T}\frac{d^2N_k}{dp_T dy} \leftarrow k\text{th observation of } \frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}}$$

for given particle species and a centrality and p_T bin

Data "points" are sample means!

The weak law of large numbers

x - a random variable with the p.d.f. $f(x)$
$a(x)$ - a continuous function of x (also a random variable)
$a_k = a(x_k)$ - kth observation of a
$(a_1,\ a_2,\ ...,a_n)$ - a sample of size n

If the variance of $a(x)$ exists, then

$$\frac{1}{n}\sum_{k=1}^{n} a_k \quad \longrightarrow \quad E[a] = \int a(x)f(x)dx\ ,$$

when $n \to \infty$.

The theoretical equivalent of data

The weak law of the large numbers

$$\Downarrow$$

The correct equivalent of the data "point" is:

$$\boxed{\left\langle \frac{dN_i}{d^2p_T\ dy} \right\rangle_\theta = \int \frac{dN_i}{d^2p_T\ dy} f(\theta) d\theta\ .}$$

$\theta = \beta_f, \tau_f \text{ or } \rho_{max}$ is a random variable now!

Only $\theta = \beta_f (= 1/T_f)$ works!

Probability distributions considered

log-normal p.d.f.

$$f(\beta_f;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\frac{1}{\beta_f}\exp\left\{-\frac{(\ln\beta_f-\mu)^2}{2\sigma^2}\right\}$$

triangular p.d.f.

$$f(\beta_f;\breve{\beta}_f,\Gamma) = \begin{cases} \frac{\Gamma-|\beta_f-\breve{\beta}_f|}{\Gamma^2} & ,|\ \beta_f-\breve{\beta}_f\ |\le\Gamma \\ 0 & ,|\ \beta_f-\breve{\beta}_f\ |>\Gamma \end{cases}$$

Removal of some resonance states

The most weakly bound resonances are removed, with the full width $\Gamma > 250$ MeV (and masses below 1600 MeV).

Removed resonances: $f_0(500)$, $h_1(1170)$, $a_1(1260)$, $\pi(1300)$,$f_0(1370)$, $\pi_1(1400)$, $a_0(1450)$, $\rho(1450)$, $K_0^*(1430)$ and $N(1440)$.

The removal of $f_0(500)$ state has the theoretical justification:
W. Broniowski, F. Giacosa and V. Begun, Phys. Rev. C **92**, 034905 (2015).

The removal itself is not enough, without randomization
$\chi^2/n_{dof} = 1.5$, *p-value* $= 10^{-6}$ $(n_{dof} = 235)$,
for most central Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76 TeV.

unacceptable!

Fit to the spectra for the most central class

Fitting the average invariant distributions with the removal, acceptable!

log-normal p.d.f.			
τ_f (fm)	ρ_{max} (fm)	μ	σ
13.80 $\pm$ 0.40	20.48 $\pm$ 0.60	-4.7439 $\pm$ 0.0235	0.1764 $\pm$ 0.0090
$E[T_f]$ (MeV)	$\sqrt{V[T_f]}$ (MeV)	χ^2/n_{dof}	*p-value* (%)
116.7 $\pm$ 3.0	20.7 $\pm$ 1.6	1.048	29

triangular p.d.f.			
τ_f (fm)	ρ_{max} (fm)	$\breve{\beta}_f$ (MeV^{-1})	Γ (MeV^{-1})
14.42	21.45	0.0092482	0.0040906
$E[T_f]$ (MeV)	$\sqrt{V[T_f]}$ (MeV)	χ^2/n_{dof}	*p-value* (%)
111.6	22.6	1.026	38

Centrality dependence

Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV, log-normal p.d.f., $n_{dof} = 234$

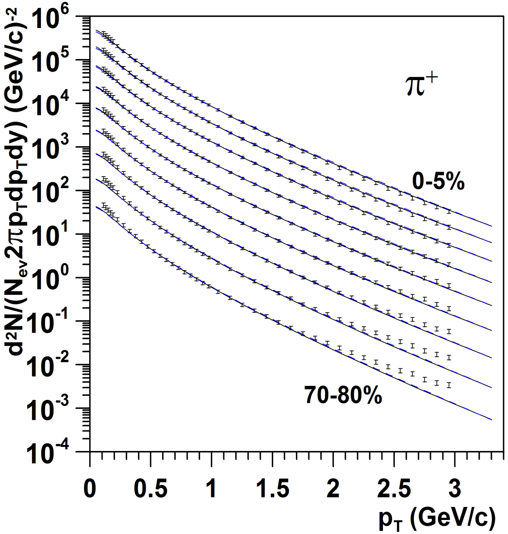
Cent. [%]	τ_f [fm]	ρ_{max} [fm]	$E[T_f]$ [MeV]	$\sqrt{V[T_f]}$ [MeV]	χ^2/n_{dof}	p-v [%]
0-5	13.8 ± 0.4	20.5 ± 0.6	116.7 ± 3.0	20.7 ± 1.6	1.05	29
-10	12.6 ± 0.4	18.7 ± 0.5	119.1 ± 3.0	20.2 ± 1.7	0.84	96
-20	11.1 ± 0.3	16.4 ± 0.5	122.2 ± 3.0	19.6 ± 1.7	0.59	100
-30	9.3 ± 0.3	13.6 ± 0.4	126.5 ± 3.2	18.7 ± 1.9	0.34	100
-40	7.8 ± 0.2	11.0 ± 0.3	131.4 ± 3.3	17.3 ± 2.0	0.30	100
-50	6.6 ± 0.2	9.0 ± 0.3	133.7 ± 3.4	16.7 ± 2.2	0.61	100
-60	5.5 ± 0.2	7.2 ± 0.2	134.7 ± 3.6	16.7 ± 2.3	1.37	0.01
-70	4.6 ± 0.2	5.8 ± 0.2	133.4 ± 3.6	17.4 ± 2.2	2.82	0
-80	3.6 ± 0.1	4.4 ± 0.2	132.5 ± 3.6	17.6 ± 2.2	4.36	0

C. dependence, no fluctuations, no removal

Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV, $n_{dof} = 235$

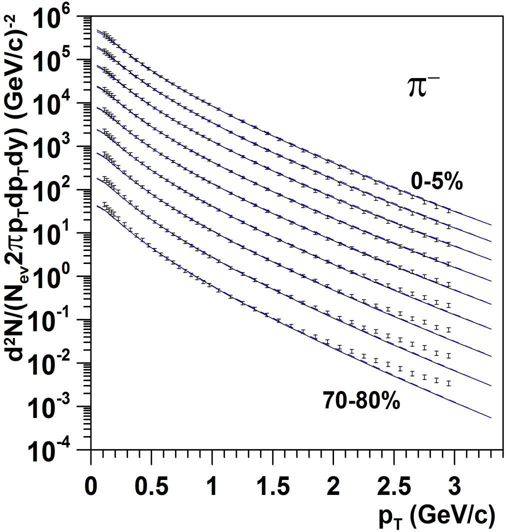
Cent. [%]	τ_f [fm]	ρ_{max} [fm]	T_f [MeV]	χ^2/n_{dof}	p-v [%]
0-5	10.0 $\pm$ 0.1	14.7 $\pm$ 0.9	147.0 $\pm$ 0.5	1.74	$1.6 \cdot 10^{-9}$
-10	9.3 $\pm$ 0.1	13.7 $\pm$ 0.8	147.3 $\pm$ 0.5	1.45	$9.2 \cdot 10^{-4}$
-20	8.4 $\pm$ 0.1	12.3 $\pm$ 0.8	148.0 $\pm$ 0.6	1.09	17.2
-30	7.3 $\pm$ 0.1	10.6 $\pm$ 0.7	149.3 $\pm$ 0.6	0.69	99.99
-40	6.3 $\pm$ 0.1	8.9 $\pm$ 0.6	150.5 $\pm$ 0.6	0.49	100
-50	5.4 $\pm$ 0.1	7.3 $\pm$ 0.5	152.0 $\pm$ 0.6	0.66	99.999
-60	4.5 $\pm$ 0.1	5.9 $\pm$ 0.5	153.0 $\pm$ 0.7	1.24	0.65
-70	3.7 $\pm$ 0.1	4.6 $\pm$ 0.4	153.5 $\pm$ 0.7	2.47	0
-80	2.9 $\pm$ 0.1	3.4 $\pm$ 0.3	154.1 $\pm$ 0.8	3.86	0

Spectra of positive pions: Pb-Pb@2.76 TeV



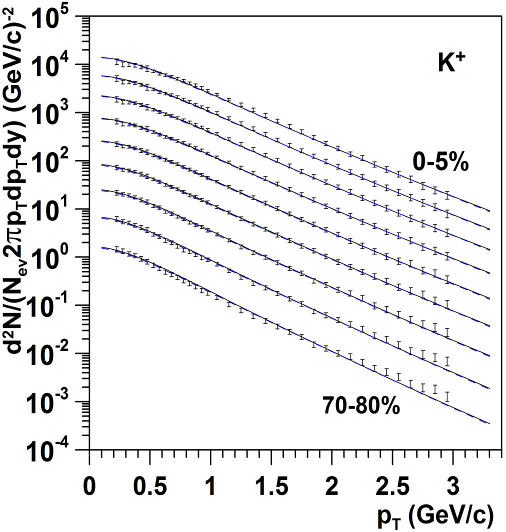
Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Spectra of negative pions: Pb-Pb@2.76 TeV



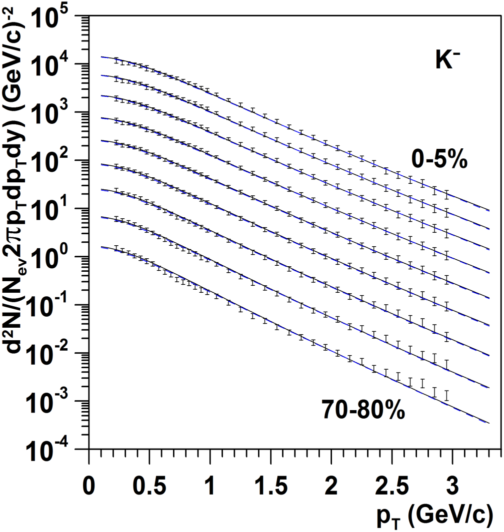
Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Spectra of positive kaons: Pb-Pb@2.76 TeV



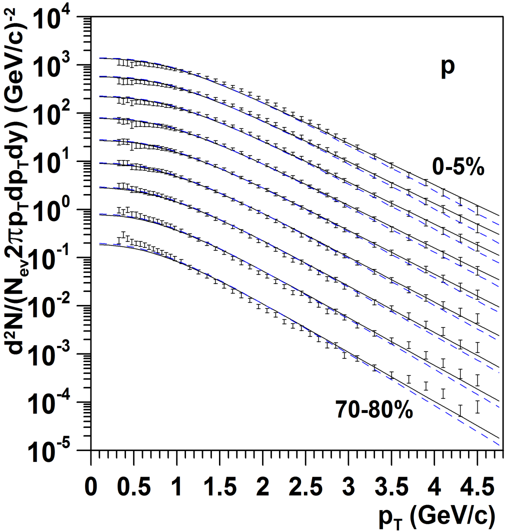
Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Spectra of negative kaons: Pb-Pb@2.76 TeV



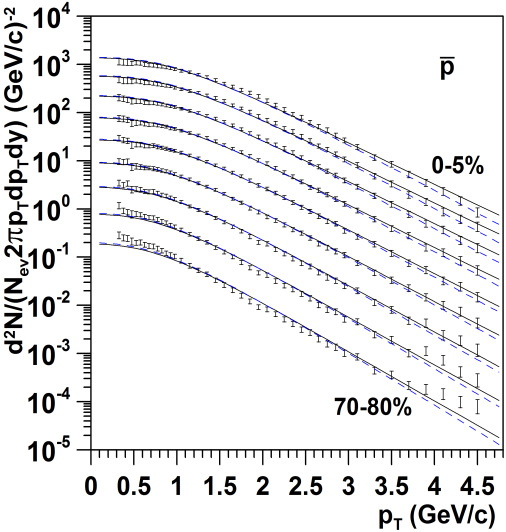
Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Spectra of protons: Pb-Pb@2.76 TeV



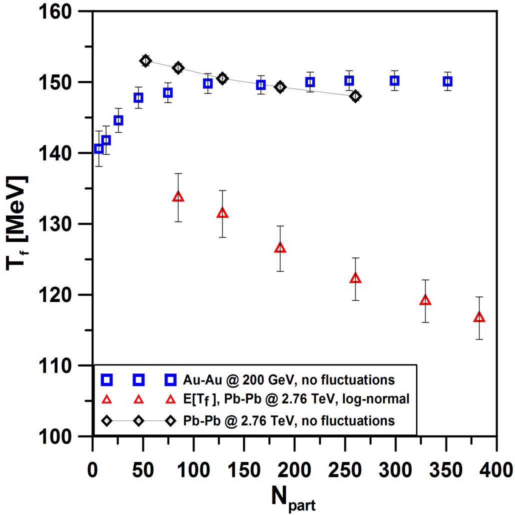
Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Spectra of antiprotons: Pb-Pb@2.76 TeV



Lines - the model with the log-normal p.d.f. of β_f, blue dashed lines - the SFOM without randomization and with all hadronic resonances included.

Freeze-out temperature vs centrality



Au-Au: DP, APPB **40**, 2825 (2009)

Conclusions

- The 2 most central bins of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV are inhomogeneous, during each event the thermal system is created indeed and with approximately the same size at its end, however with different temperature. And the final shape of the spectra is the consequence of summing emissions from many different sources.

- The centrality bins of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV can be divided into 3 groups: the first, the 2 most central bins where the freeze-out temperature fluctuates significantly; the second, the mid central bins where the situation might look similar to that at the RHIC, the same freeze-out temperature, $T_f \sim 150$ MeV, only ρ_{max} factor ~ 1.5 greater (τ_f approx. the same) what causes that the volume is greater ~ 2.5 times; the third, the peripheral bins where nothing helps.

Conclusions, *cont.*

- The distribution of the freeze-out temperature means the distribution within a bin here. But the significant part of the freeze-out temperature fluctuations might be of *non-thermal* origin, so this would represent the possible variation of the freeze-out conditions event-by-event within the bin.

- A great deal of data in high energy physics are averages, so in any theoretical modeling (of these data) one should be aware of possible misinterpretations when an average is compared with a prediction for a single event.

$z = 0$ plane

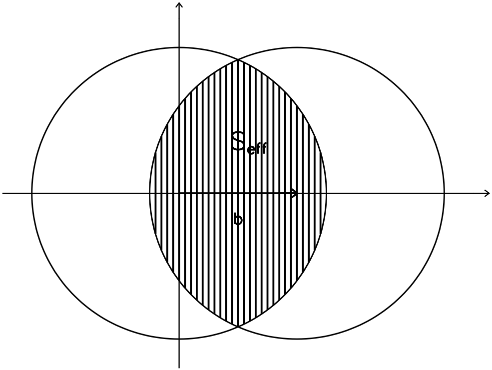
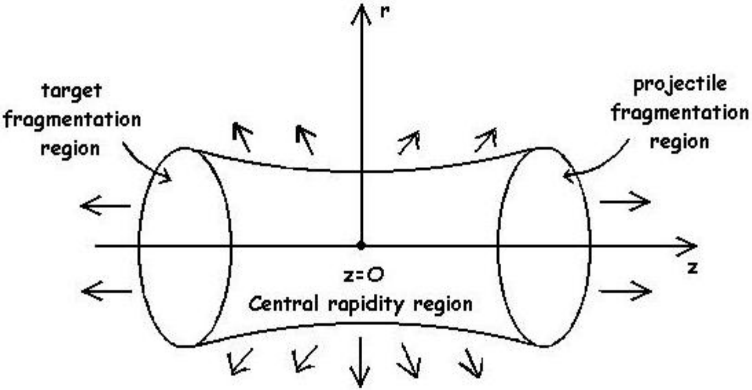


Figure: View of an AA collision at impact parameter b. The region where the nuclei overlap has been hatched and its area equals S_{eff}.

A central collision



Yields in the Single-Freeze-Out Model

$$\boxed{\frac{dN_i}{dy} = \pi \rho_{max}^2 \tau_f n_i}$$

V. Begun, W. Florkowski and M. Rybczynski,
Phys. Rev. C **90**, 054912 (2014)

The average particle yield per unit rapidity

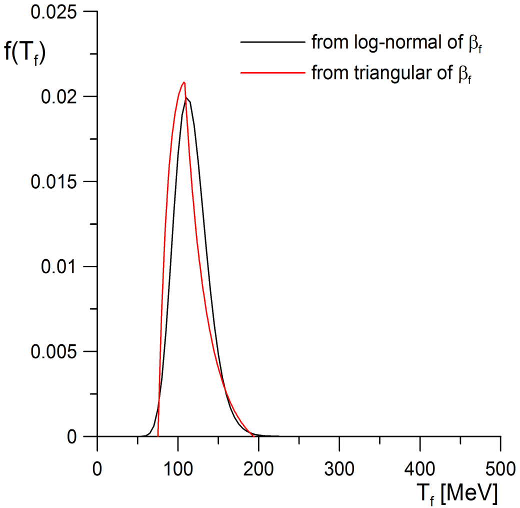
$$\boxed{\left\langle \frac{dN_i}{dy} \right\rangle_{\beta_f} = \pi \rho_{max}^2 \tau_f \left\langle n_i \right\rangle_{\beta_f}}$$

Midrapidity particle yields and ratios

		Model: $\left\langle \frac{dN_i}{dy} \right\rangle_{\beta_f}$	
Species	Data	triangular p.d.f.	log-normal p.d.f.
π^+	733 ± 54.0	745.3	739.2
π^-	732 ± 52.0	745.3	739.2
K^+	109 ± 9.0	106.9	107.5
K^-	109 ± 9.0	106.9	107.5
p	34 ± 3.0	33.0	32.9
$\bar{p}$	33 ± 3.0	33.0	32.9

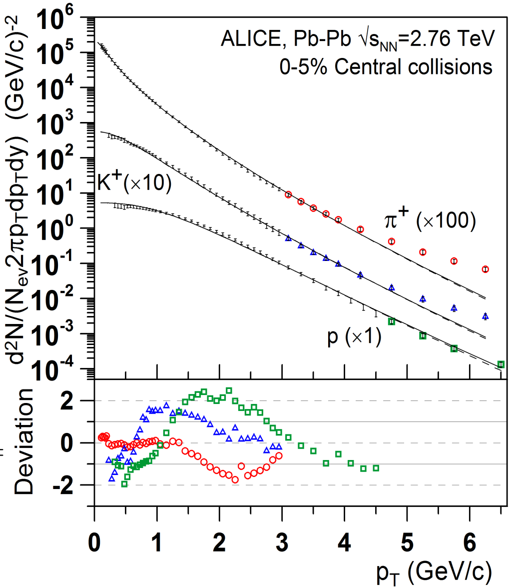
	Ratios		
	Data	triangular p.d.f.	log-normal p.d.f.
p/π	0.046 ± 0.003	0.044	0.045
K/π	0.149 ± 0.010	0.143	0.145

Distributions of the freeze-out temperature



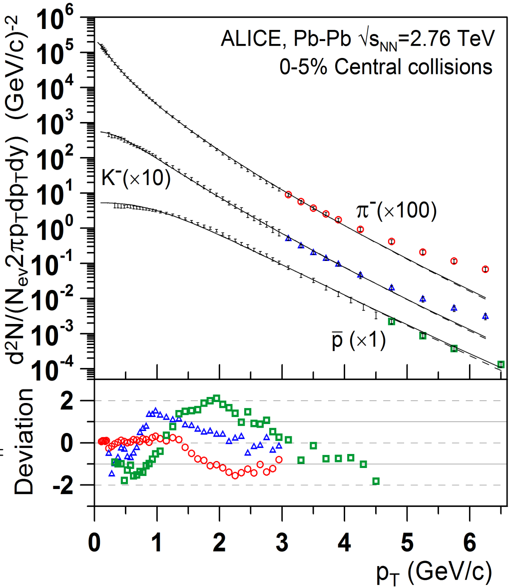
0-5% centrality Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV

Spectra of positive pions, kaons and protons



$$\frac{f_{exp}-f_{mod}}{\sigma_{exp}} =$$

Spectra of negative pions, kaons and $\overline{\text{protons}}$



$$\frac{f_{exp}-f_{mod}}{\sigma_{exp}} = \text{Deviation}$$

The least-squares (LS) test statistic

$$\chi^2_{LS}(\vec{Y};\vec{\theta}) = \sum_{i,j=1}^{N} (Y_i - \Lambda(X_i;\vec{\theta}))[V^{-1}]_{ij}(Y_j - \Lambda(X_j;\vec{\theta}))$$

$$\chi^2_{LS}(\vec{Y};\vec{\theta}) = \sum_{i=1}^{N} \frac{(Y_i - \Lambda(X_i;\vec{\theta}))^2}{\sigma_i^2}$$

$\Lambda(X;\vec{\theta})$ - the true value function

$\vec{\theta} = (\theta_1, ..., \theta_m)$ - unknown parameters

V - covariance matrix

$n_{dof} = N - m$ - the number of degrees of freedom

σ_i^2 - the variance of Y_i

p-value of the test statistic

The probability of obtaining the value of the test statistic equal to or greater then the value just obtained for the present data set (*i.e.* χ^2_{min}), when repeating the whole experiment many times:

$$\boxed{p = P(\chi^2 \geq \chi^2_{min}; n_{dof}) = \int_{\chi^2_{min}}^{\infty} f(z; n_{dof}) dz \ ,}$$

$f(z; n_{dof})$ - the χ^2 p.d.f.

χ^2 (chi-square) distribution

$0 \leq z \leq +\infty$,

$n = 1, 2, \ldots$ - the number of degrees of freedom

$$\boxed{f(z;n) = \frac{1}{2^{n/2}\Gamma(n/2)} z^{n/2-1} \cdot e^{-z/2}}$$

$$\Gamma(n) = (n-1)!\ , \quad \Gamma(x+1) = x\Gamma(x), \quad \Gamma(1/2) = \sqrt{\pi}$$

$$E[z] = n, \qquad V[z] = 2n$$

What does SHM mean?

For the boost invariant system:

$$\boxed{\frac{(dN_i/dy)_{y=0}}{(dN_j/dy)_{y=0}} = \frac{N_i}{N_j} = \frac{n_i}{n_j}}, \qquad y = \frac{1}{2}\ln\frac{E+p_L}{E-p_L}$$

$$n_i(T,\mu_B) = n_i^{prim}(T,\mu_B) + \sum_a \varrho(i,a)\, n_a^{prim}(T,\mu_B)\ ,$$

$n_i^{prim}(T,\mu_B)$ - the thermal density of particle species i at the freeze-out

$\varrho(i,a)$ - the final fraction of particle species i which can be received from all possible decays (cascades) of particle a, the sum is over all kinds of resonances in the hadron gas

Primordial distributions

At the freeze-out the momentum distributions are frozen and these are primordial distributions:

$$\boxed{f_i^{prim} = \frac{(2s_i+1)}{(2\pi\hbar c)^3}\,\frac{1}{\exp\left\{\frac{E_i-\mu_i}{T}\right\}\pm 1}}$$

$$\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$$

$$\boxed{n_i^{prim} = \int d\vec{p}\; f_i^{prim}(\vec{p})}$$

$$\sum S_i n_i = 0\;, \qquad \frac{\sum Q_i n_i}{\sum B_i n_i} = \frac{Z}{A}$$

The Cooper-Frye formula

$$\sigma^\mu = \sigma^\mu(\alpha, \eta, \phi) \quad - \quad \text{a freeze-out hypersurface}$$

$$j^\mu \quad - \quad \text{a particle density current} = \text{a fluid 4-flow}$$

$dQ = j^\mu \, d\sigma_\mu$ - the amount of the fluid (the number of particles) passing through the hypersurface element $d\sigma_\mu$

$$d\sigma_\mu = \epsilon_{\mu\nu\beta\gamma} \frac{\partial \sigma^\nu}{\partial \alpha} \frac{\partial \sigma^\beta}{\partial \eta} \frac{\partial \sigma^\gamma}{\partial \phi} \, d\alpha \, d\eta \, d\phi$$

$j^\mu = f(x, p) \, d\vec{p} \, \frac{p^\mu}{E}$ - the particle density current with momenta in $[\vec{p}, \vec{p} + d\vec{p}]$

The Cooper-Frye formula, *cont.*

$$dN = \int_\sigma f(x,p)\, d\vec{p}\, \frac{p^\mu}{E}\, d\sigma_\mu$$

the total number of particles with momenta in $[\vec{p}, \vec{p} + d\vec{p}]$ emitted (decoupled) from the hypersurface σ^μ,
$d\sigma_\mu$ is the normal vector to the hypersurface.

$$\boxed{E\, \frac{dN}{d^3p} = \frac{dN}{d^2p_T\, dy} = \int_\sigma f(x,p)\, p^\mu\, d\sigma_\mu}$$

F.Cooper, G.Frye and E.Schonberg, Phys.Rev.**D11**, 192 (1975)

Parameterization of the hypersurface

$$t = \tau \cosh \alpha_{\parallel} \cosh \alpha_{\perp}, \quad r_x = \tau \sinh \alpha_{\perp} \cos \phi,$$
$$r_y = \tau \sinh \alpha_{\perp} \sin \phi, \quad r_z = \tau \sinh \alpha_{\parallel} \cosh \alpha_{\perp}.$$

$$\frac{dN_i}{d^2 p_T \, dy} = \tau^3 \int\limits_{-\infty}^{+\infty} d\alpha_{\parallel} \int\limits_{0}^{\rho_{max}/\tau} \sinh \alpha_{\perp} d(\sinh \alpha_{\perp}) \int\limits_{0}^{2\pi} d\xi \, (p{\cdot}u) \, f_i(p{\cdot}u) \, ,$$

$$p \cdot u = m_T \cosh\left(\alpha_{\parallel} - y\right) \cosh \alpha_{\perp} - p_T \cos \xi \sinh \alpha_{\perp} \, .$$

The distribution in the presence of the flow

$$\boxed{f_i(\vec{r},\vec{q},t) = \frac{(2s_i+1)}{(2\pi\hbar c)^3}\,\frac{1}{\exp\left\{\frac{q_\nu u^\nu(\vec{r},t)-\mu_i(\vec{r},t)}{T(\vec{r},t)}\right\}\pm 1}}$$