How Much Information is in a Jet?

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with Kaustuv Datta,

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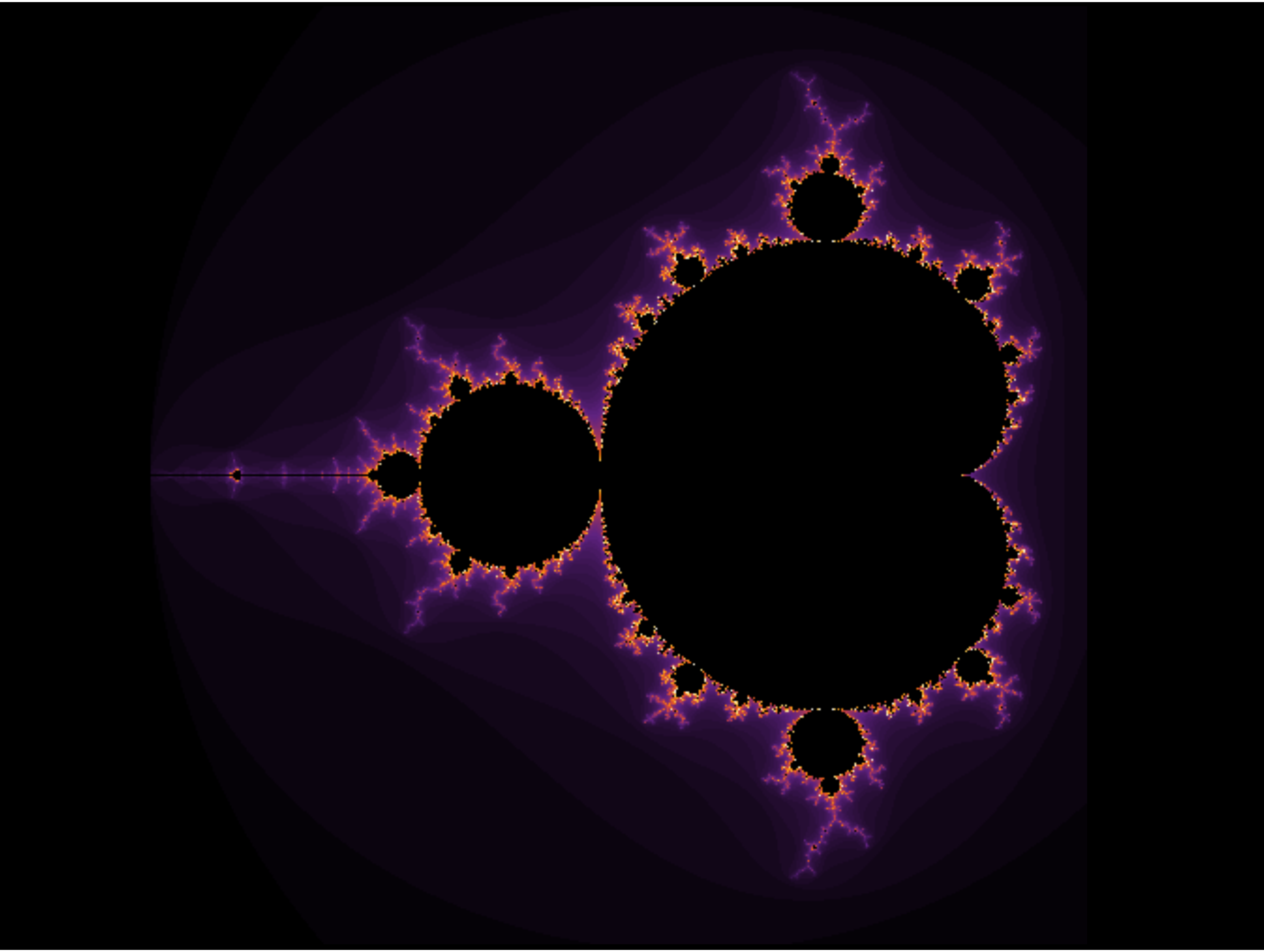
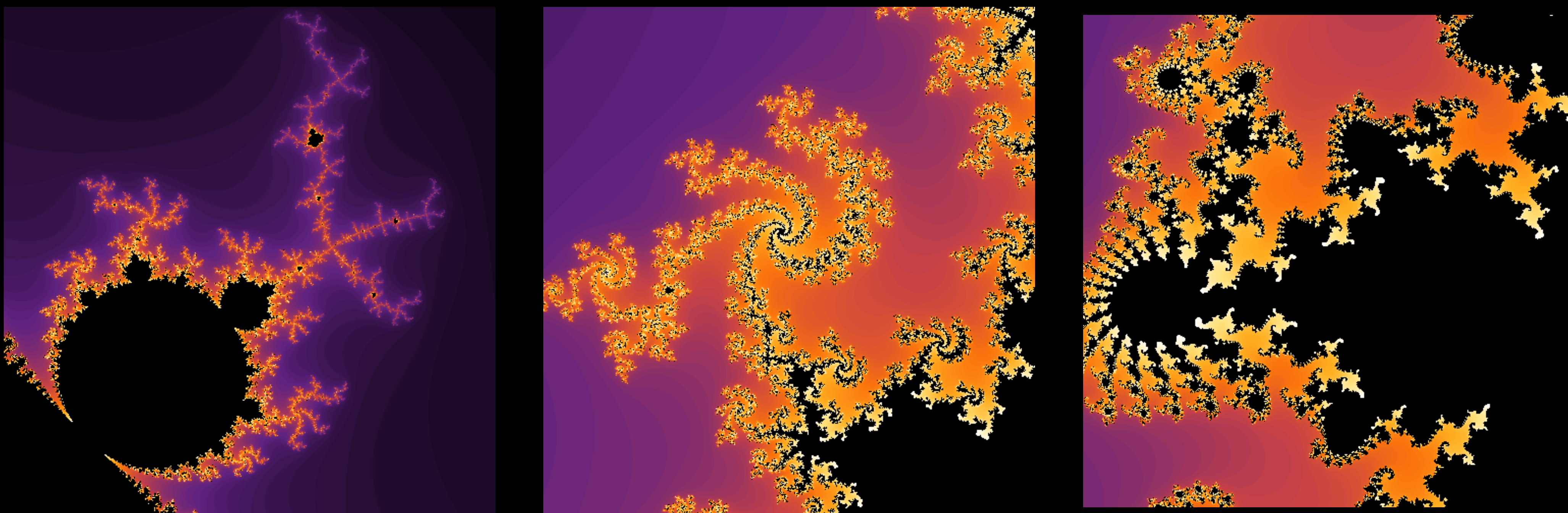
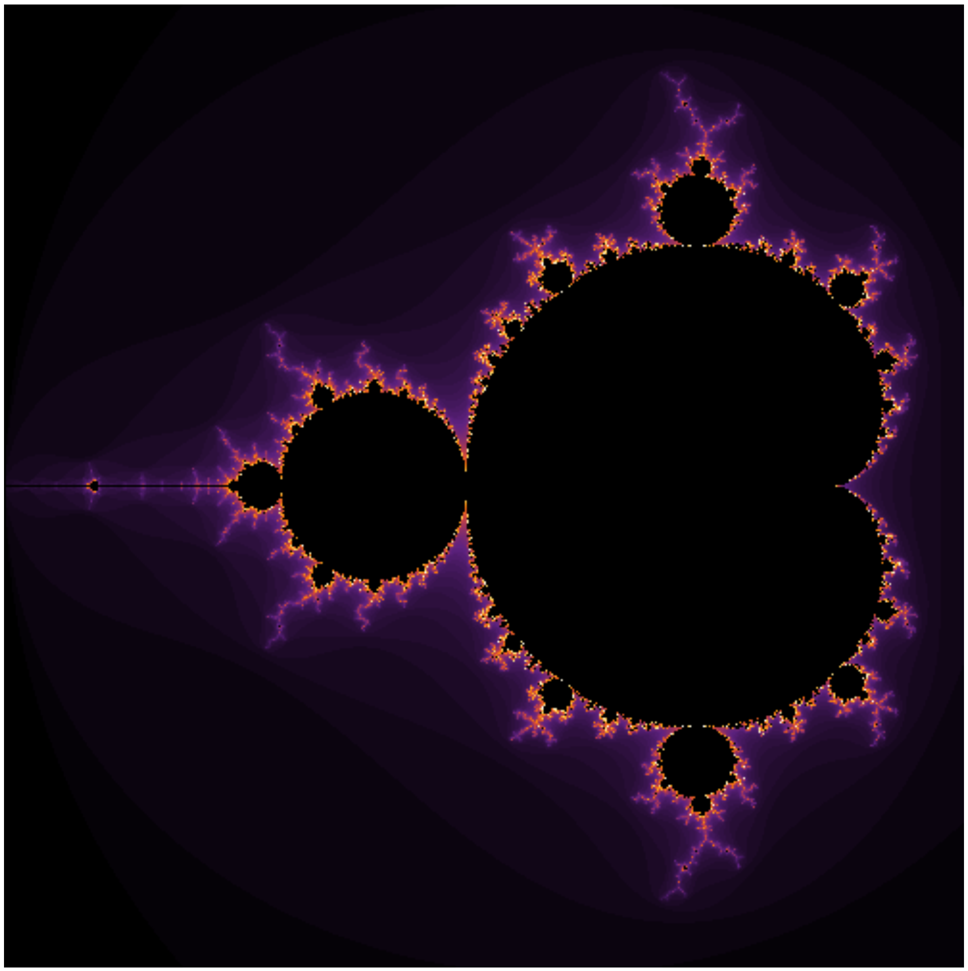




Complexity and Information



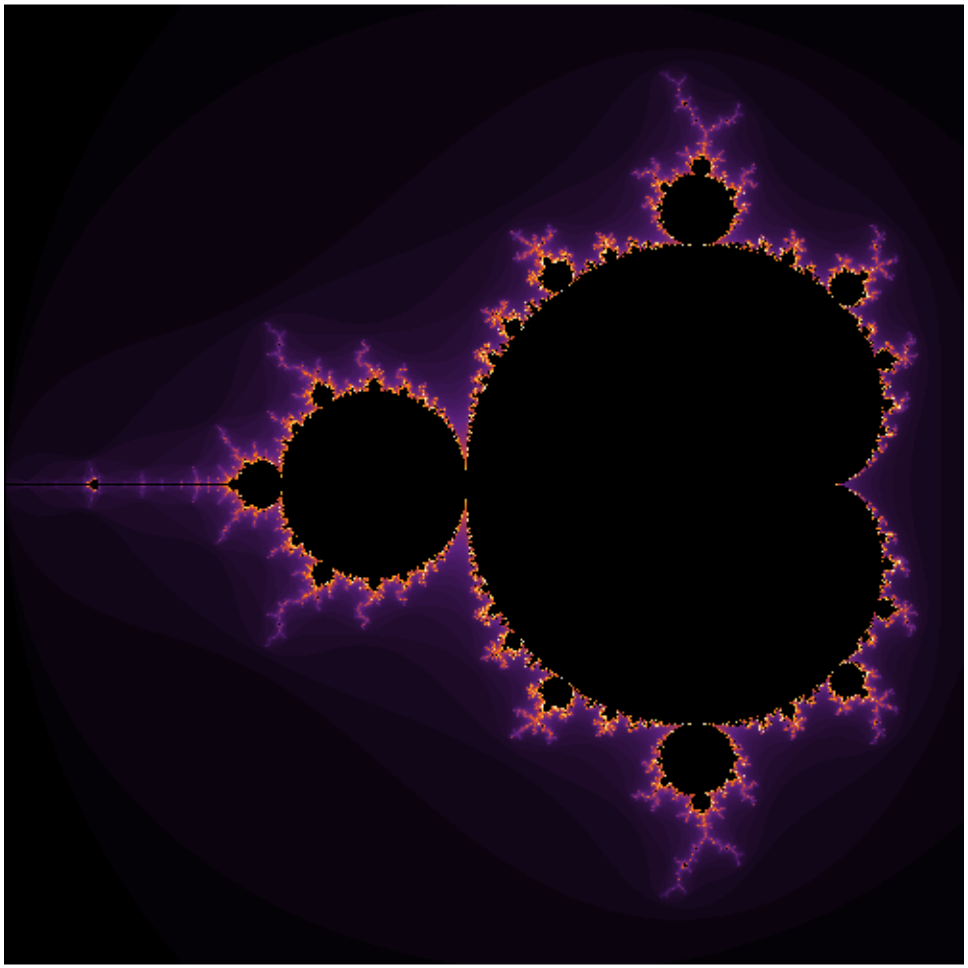
This image:

500 x 500 = 250,000 pixels

8-bit color in each pixel

Total information in bits
≈ 2 Mbits

Complexity and Information



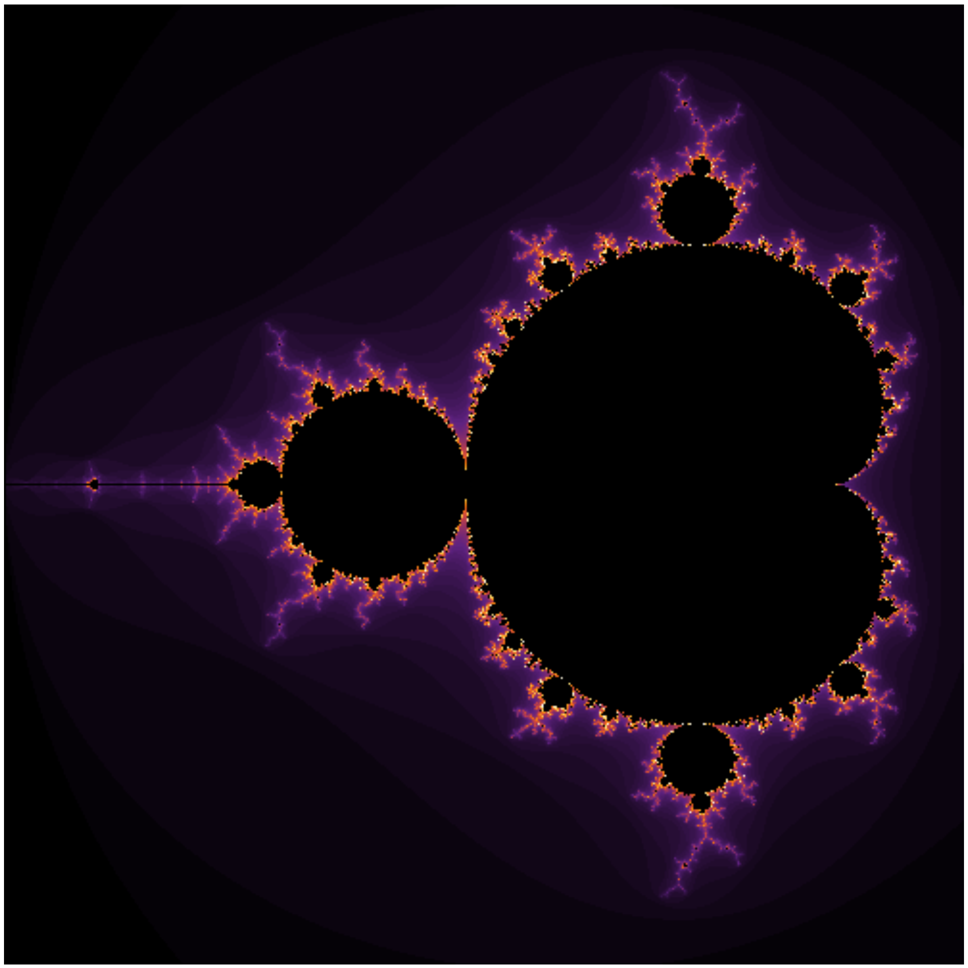
Mandelbrot set:
Defined by recursively applying

$$f(z) = z^2 + c$$

Complexity does not mean explosion of information content

Fractals can have *apparent* arbitrary complexity from simple rules

Complexity and Information

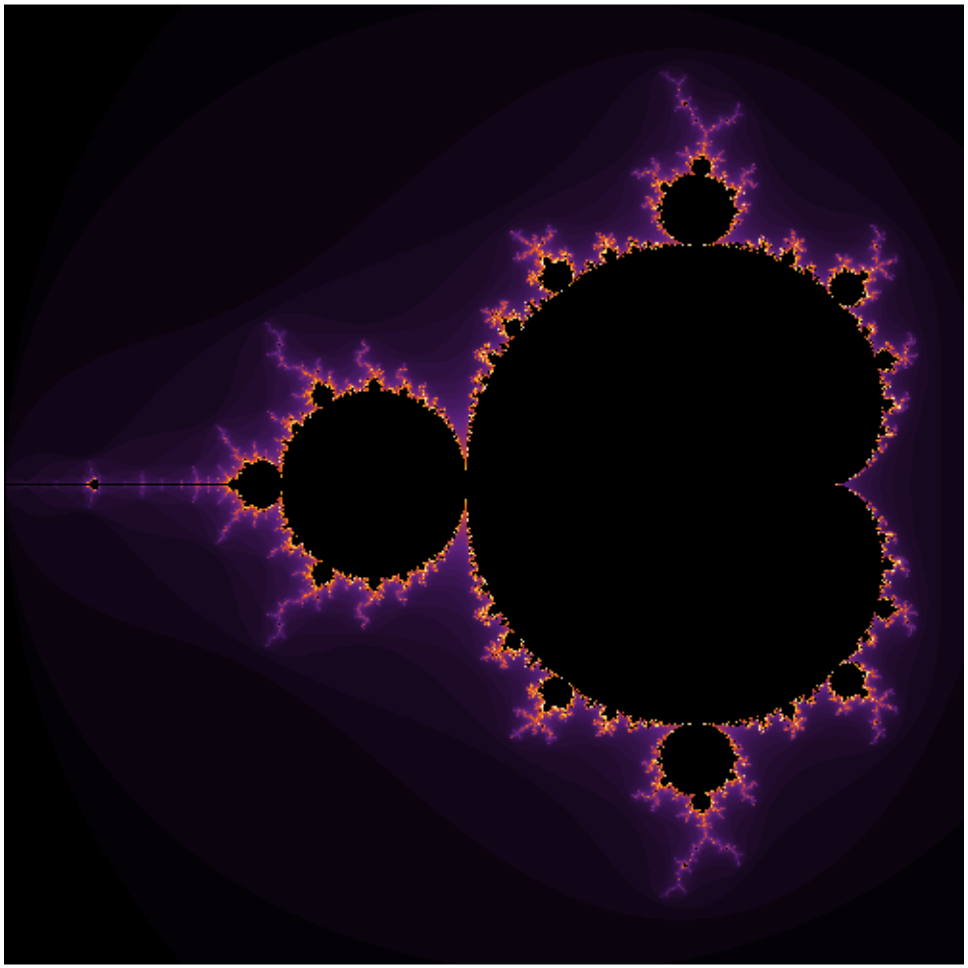


Kolmogorov Complexity:
The information of the simplest computer program that can construct the object

Example pseudo-program:

For each pixel c_i
  For $n < n_{max}$, do
    $z_0 = 0$;
    $z_{n+1} = z_n^2 + c_i$;
  Color pixel c_i from z_{nmax}

Complexity and Information



Number of bits in image:
~2 Mbits

Number of bits in program
(Kolmogorov complexity):
~100s of bits

Takeaway:
Just because something looks complex, doesn't mean it is

Caveats

I am a theoretical physicist

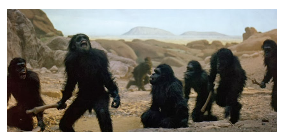
I don't know much about machine learning
(nor do I want to know much)

Motto: "What I cannot understand, I should not create."
~Feynman^{-1}

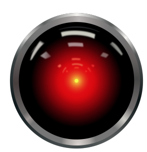
Machine Learning on Jets

My nightmare as a physicist:

Hundreds of input variables → → Mysterious output



Any organizing principle?
Can the input be simplified?
Is there any hope for a **human** to understand the output?

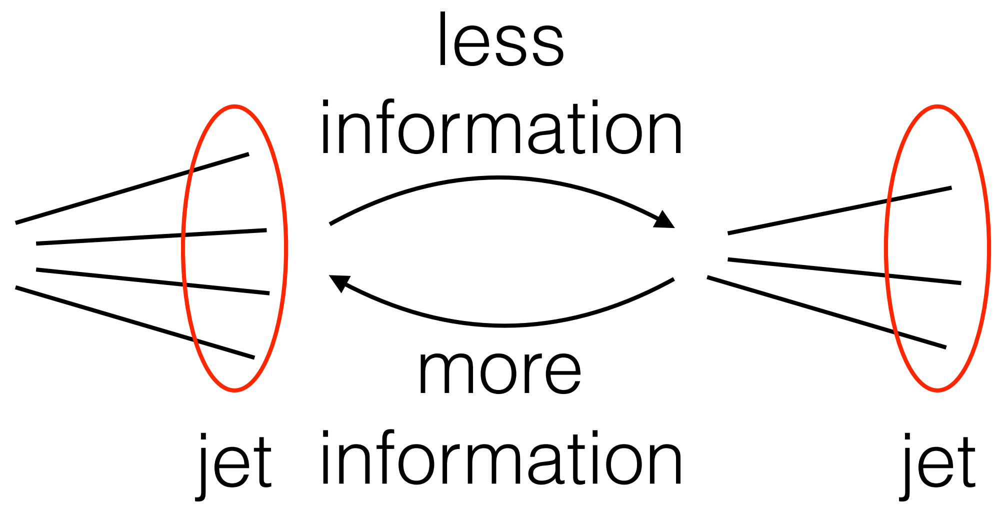


Human Learning on Jets

To make progress, use the guiding principles:

Systematic Improvability

Including more or less information in jet description is well-defined



Direct Calculability (technical)

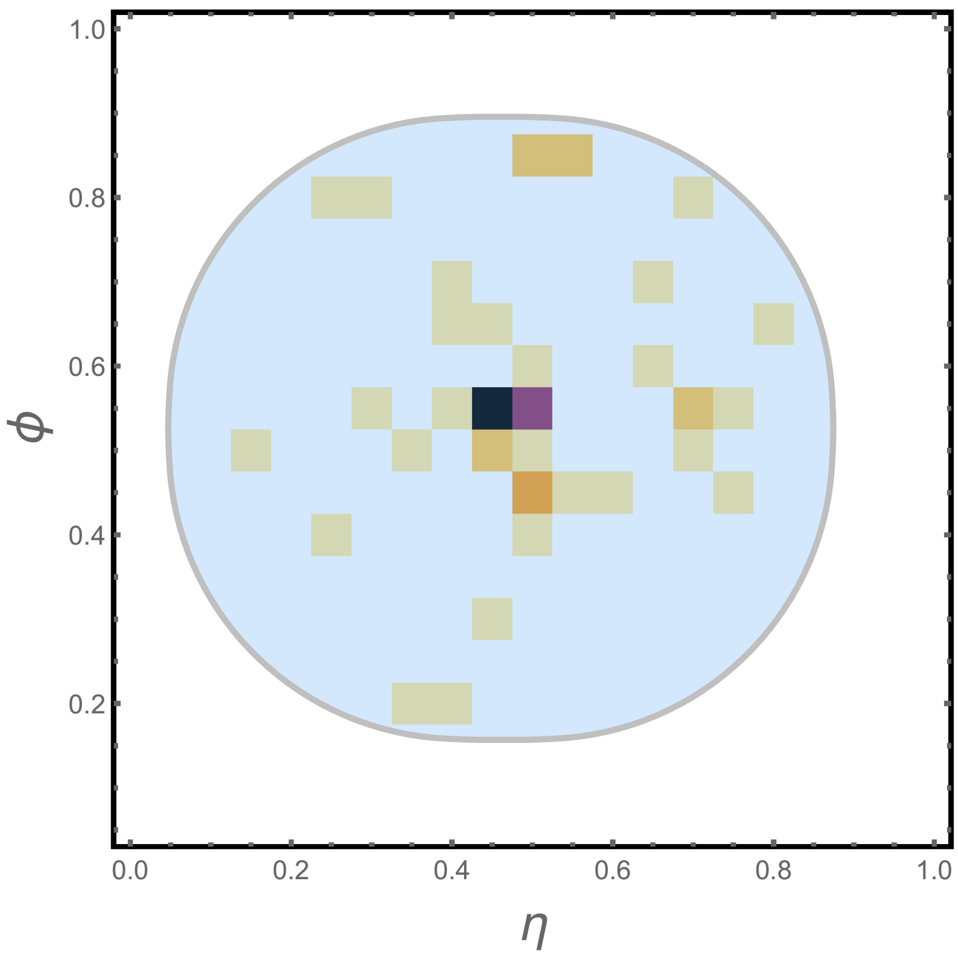
$$\tau_N^{(\alpha)} = \frac{1}{p_{TJ}} \sum_{i \in \text{jet}} p_{Ti} \min\left\{\Delta R_{i1}^{\alpha}, \ldots, \Delta R_{iN}^{\alpha}\right\}$$

Sensitive to radiation off of *N* axes in the jet

“Infrared and collinear safe”

Human Learning on Jets

Systematically resolve more structure in the jet



Full Jet

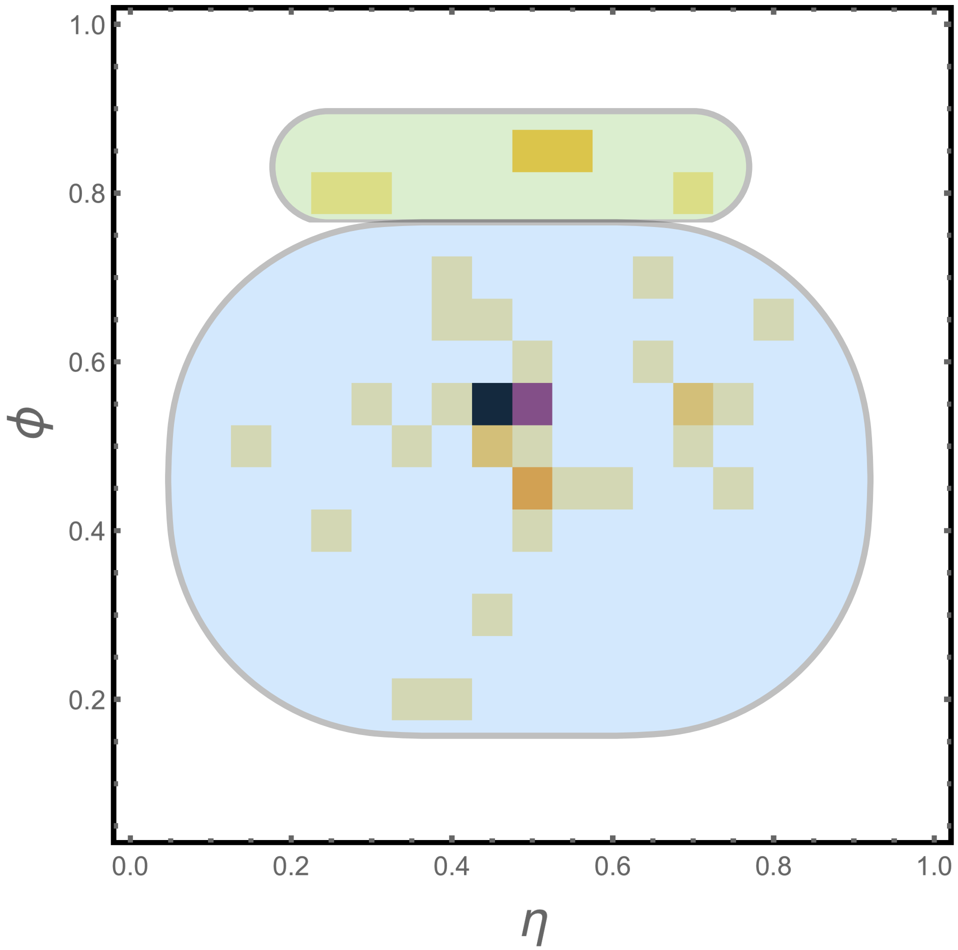
Net p_T, η, ϕ selected for

1 useful quantity:
jet invariant mass

Restrict m_J in a range about the mass of interest

Human Learning on Jets

Systematically resolve more structure in the jet



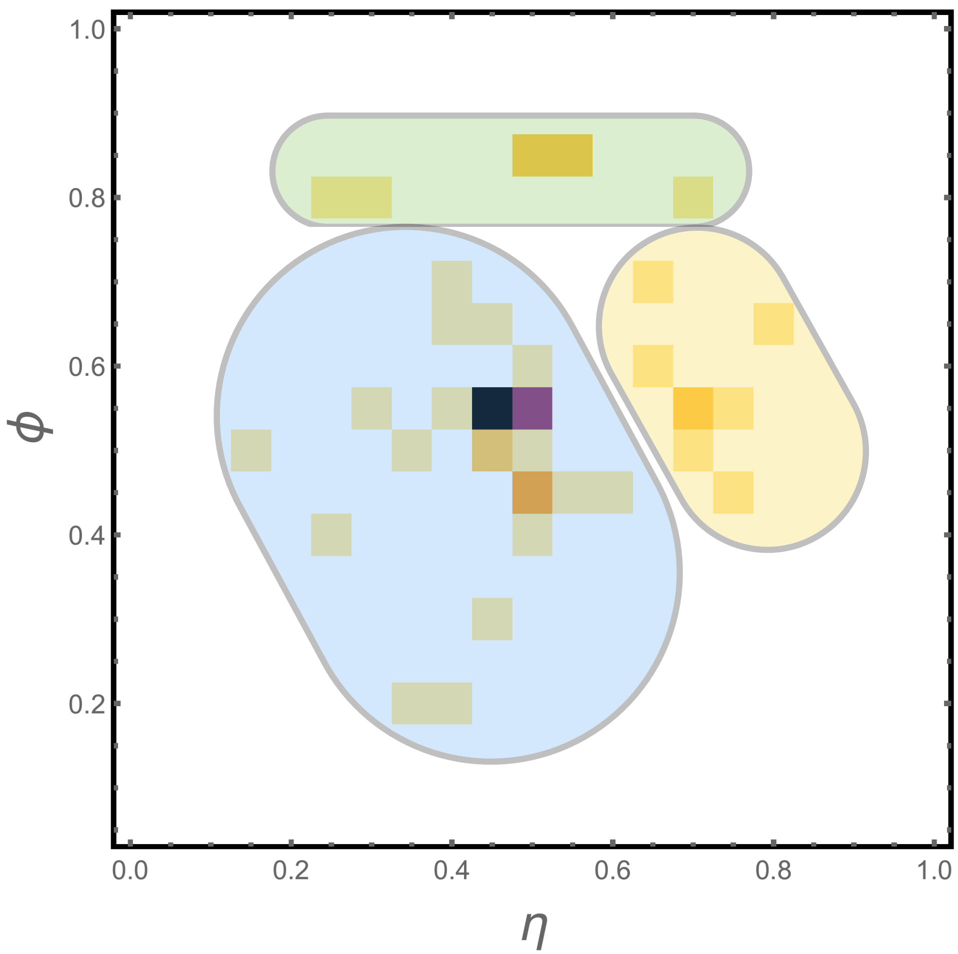
Two Subjets

Net p_T, η, ϕ, m_J selected for

2 useful quantities:
relative p_T fraction
relative angle

Human Learning on Jets

Systematically resolve more structure in the jet



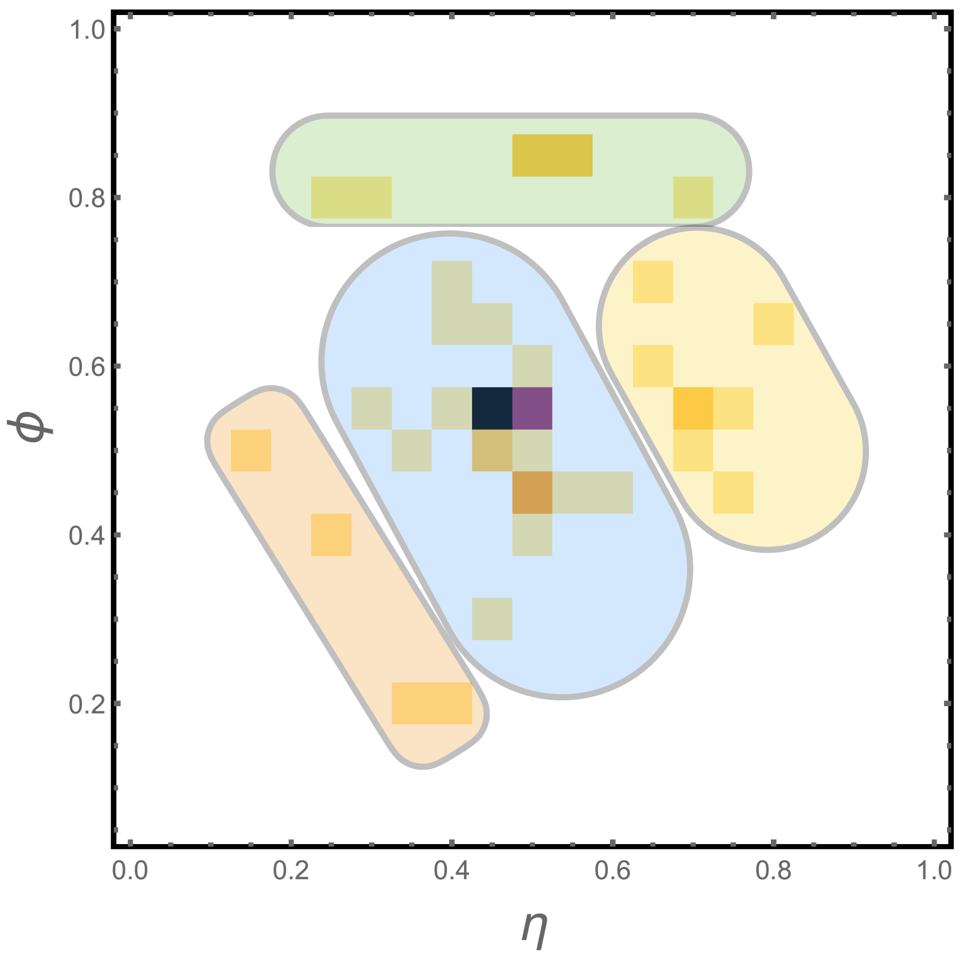
Three Subjets

Net p_T, η, ϕ, m_J selected for

5 useful quantities:
2 relative p_T fractions
3 relative angles

Human Learning on Jets

Systematically resolve more structure in the jet



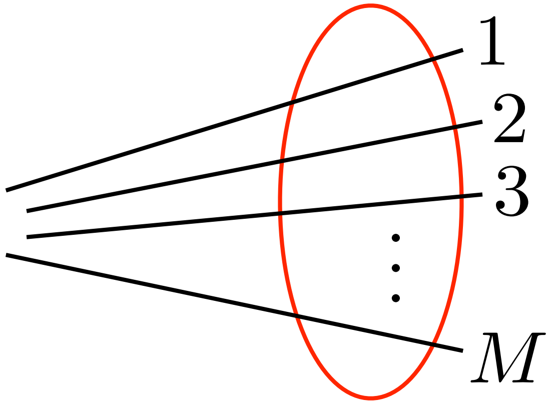
Four Subjets

Net p_T, η, ϕ, m_J selected for

8 useful quantities:
3 relative p_T fractions
5 relative angles

Can continue to resolve arbitrary structure

Human Learning on Jets



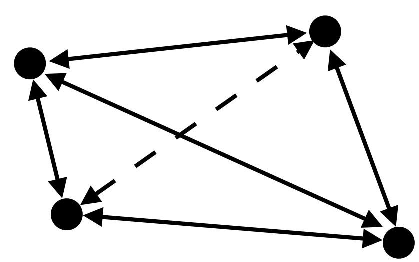
jet

Measure observables to resolve M-body phase space

$$\sigma \sim \int \prod_{i=1}^{M} \left[\frac{d^4 p_i}{(2\pi)^4} 2\pi \delta(p_i^2 - m_i^2) \right] \delta^{(4)} \left(Q - \sum_{i=1}^{M} p_i \right) |\mathcal{M}|^2$$

3M - 4 dimensional phase space

In general:
M - 1 relative p_T fractions
2M - 3 relative angles



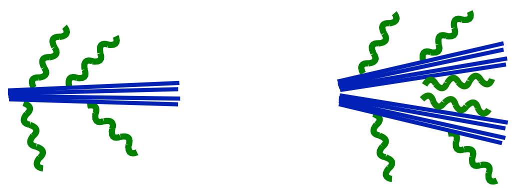
4 particle example

M-body Phase Space Machine Learning

Measure observables sensitive to
2-, 3-, 4-, 5-, and 6-body phase space + jet mass

Analyzed with a deep neural network on GPUs

Calculated ROC curves for QCD vs. Z boson



If information is finite, should see saturation

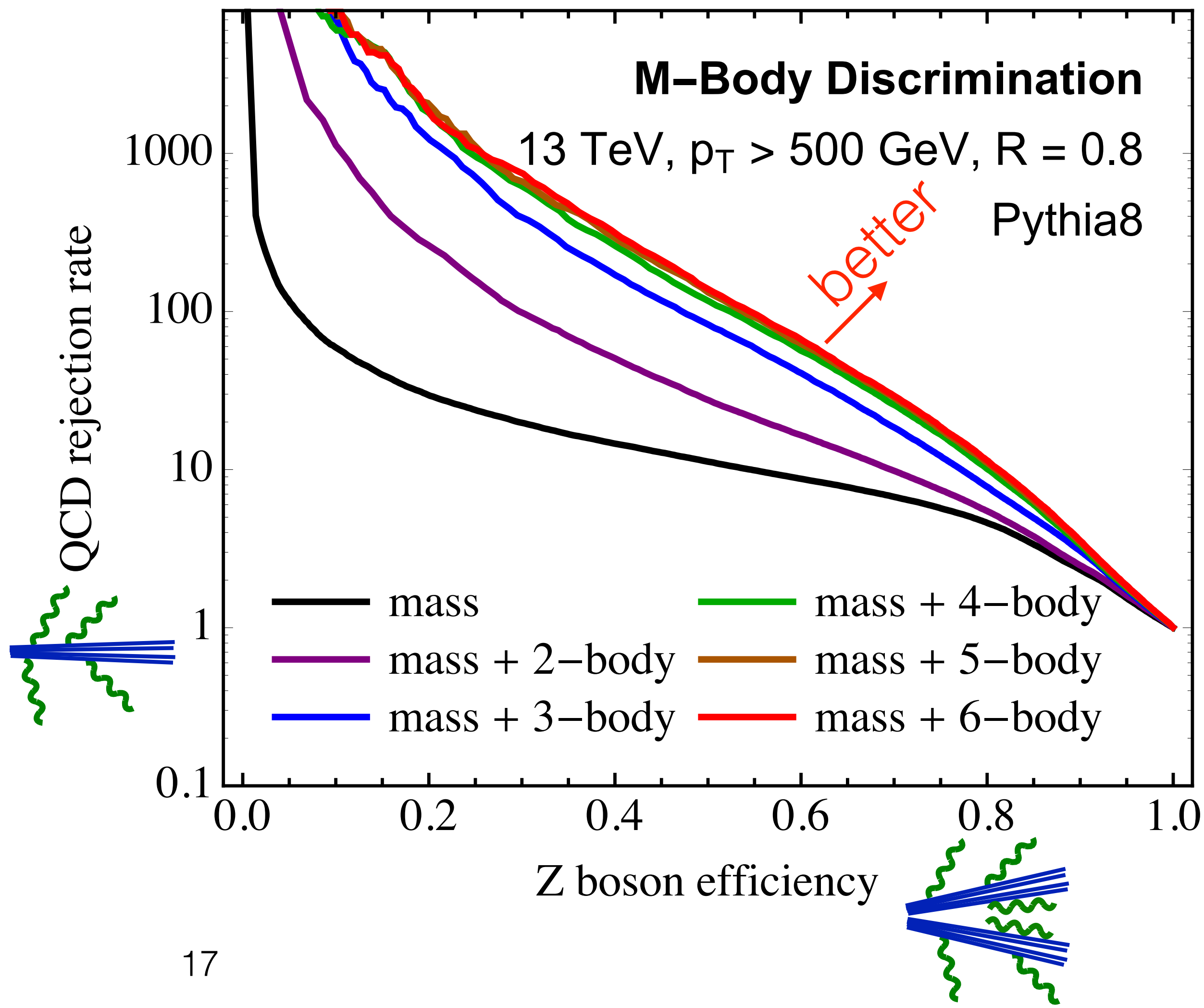
M-body Phase Space Machine Learning

Measure observables sensitive to
2-, 3-, 4-, 5-, and 6-body phase space + jet mass

Results:

Saturation observed at 4-body phase space!

4-body phase space = 8 dimensional



Why does this approach work?

Apparently there's very little information useful for discrimination

Why?

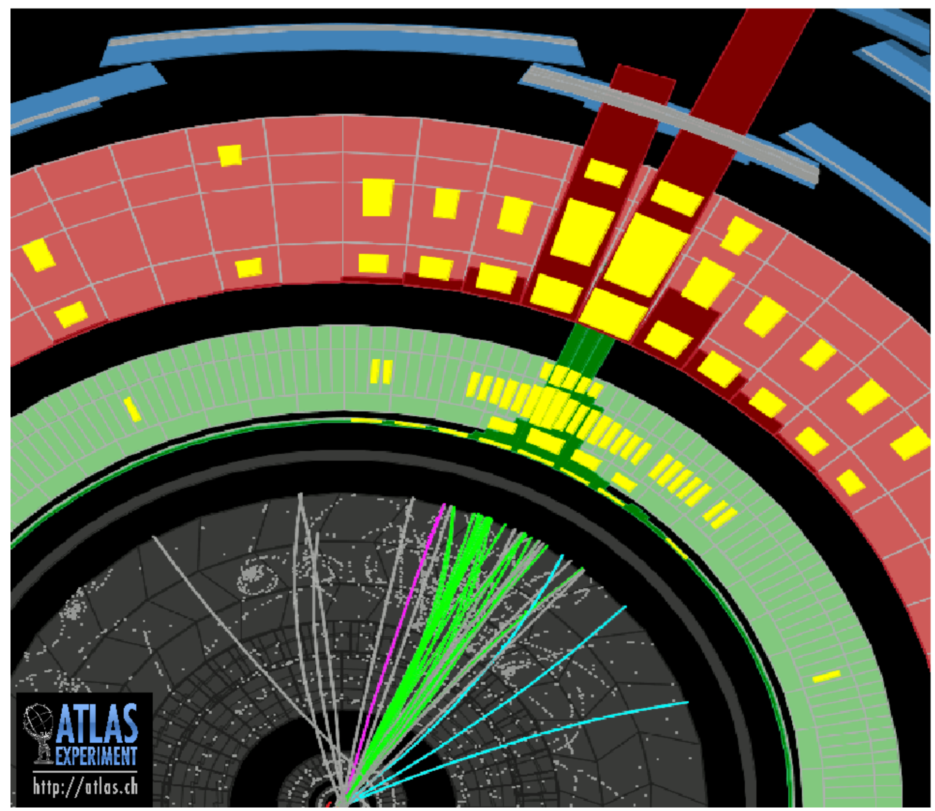
This jet has 30 particles

Information to define all particles:

$3 \times 32 \times 30 \approx 3000$ bits

(p_T,η,ϕ) — 9 digits — particles

ATLAS 2011



Why does this approach work?

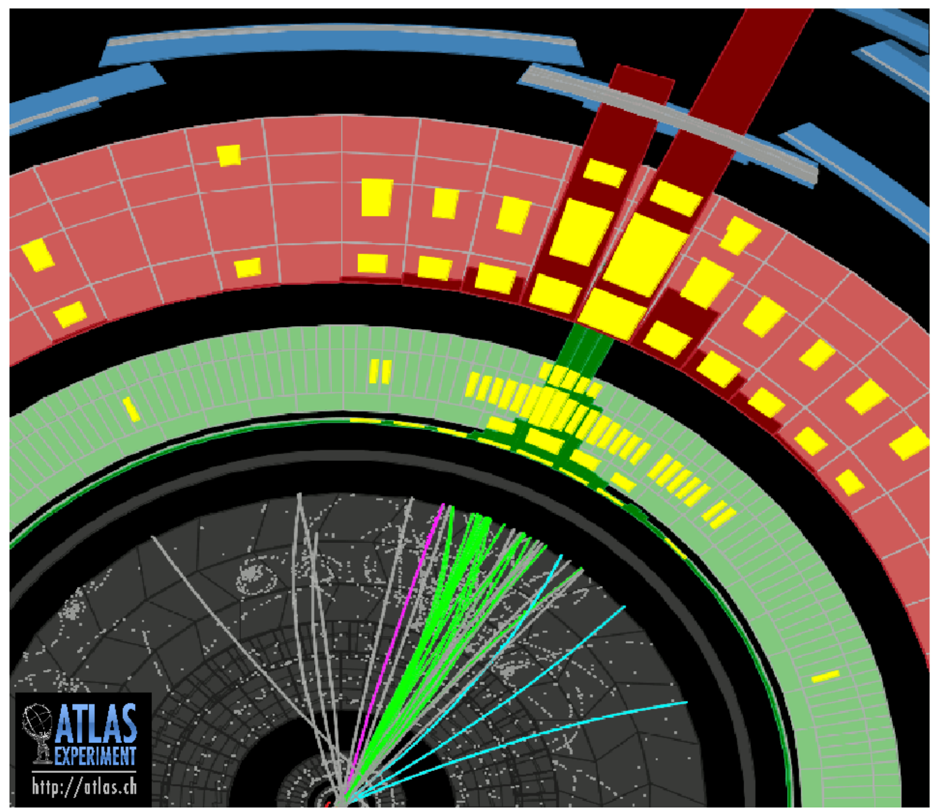
Essentially all particle production in QCD is governed by the surprisingly simple DGLAP equation:

$$Q^2 \frac{df_i(x,Q^2)}{dQ^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ij\leftarrow k}\left(\frac{x}{z}\right) f_k(z,Q^2)$$

Recursive just like Mandelbrot set

Corresponding Kolmogorov complexity will be small

ATLAS 2011



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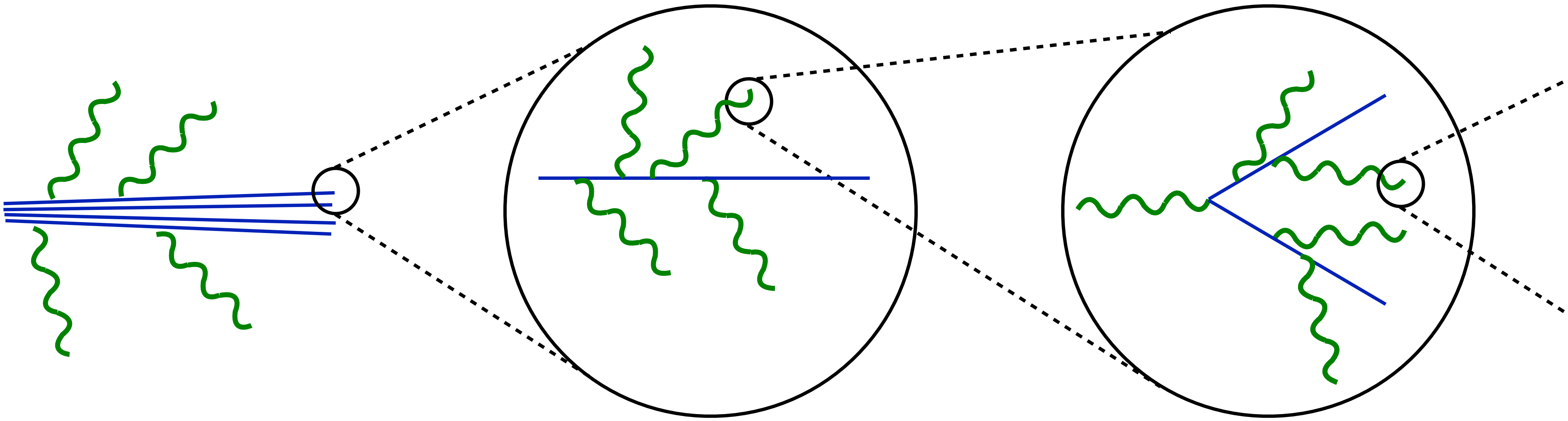

Seemingly-complex, fractal-like substructure of a jet

Conclusions

There isn't that much information in a jet:
particle production is recursive

Need to use techniques that exploit this feature

Resolving 4 subjets is sufficient to saturate possible QCD vs. Z boson discrimination