The width difference in the $B_s - \bar{B}_s$ system: towards NNLO

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$B_s - \bar{B}_s$ mixing induces different masses and widths for the two $B_s$ mass eigenstates:

The width difference $\Delta \Gamma$ stems from the absorptive part of the box diagram, involving $u, c$ quarks on the internal lines.
Leading contribution to $\Delta \Gamma$:  

$\Delta \Gamma$ stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays.

Heavy Quark Expansion (HQE):  
Exploit $m_b \gg \Lambda_{QCD}$ to express $\Delta \Gamma$ in terms of short-distance coefficients and matrix elements of local $|\Delta B| = 2$ operators.

$\Rightarrow$ expansion of $\Delta \Gamma$ in $\alpha_s(m_b)$ and $\Lambda_{QCD}/m_b$. 
Operators at leading order in $\Lambda_{QCD}/m_b$ (leading power):

\[ Q = (\bar{s}_i b_i)_{V-A} (\bar{s}_j b_j)_{V-A}, \quad \tilde{Q}_S = (\bar{s}_i b_j)_{S-P} (\bar{s}_j b_i)_{S-P}. \]

\(i, j\): colour indices, \( V \pm A = \gamma_\mu(1 \pm \gamma_5), \quad S \pm P = (1 \pm \gamma_5). \)

Matrix elements:

\[ \langle B_s | Q(\mu_2) | \bar{B}_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu_2) \]

\[ \langle B_s | \tilde{Q}_S(\mu_2) | \bar{B}_s \rangle = \frac{1}{3} M_{B_s}^2 f_{B_s}^2 \tilde{B}'_S(\mu_2). \]

Here \( f_{B_s} \) is the \( B_s \) decay constant and \( \mu_2 = \mathcal{O}(m_b) \) is the renormalization scale at which the matrix elements are calculated.
The HQE gives

\[
\Delta \Gamma = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cs} V_{cb}|^2 \left| G' \langle B_s | Q | B_s \rangle + \tilde{G}_S \langle B_s | \tilde{Q}_S | B_s \rangle \right|
\]

with the perturbative coefficients \( G', \tilde{G}_S \).

The coefficients \( G', \tilde{G}_S \) emerging from the calculation correspond to the choice \( m_b = m_b^{\text{pole}} \) in the prefactor. Subsequently one may switch to the \( \overline{\text{MS}} \) definition \( \tilde{m}_b \) through e.g.

\[
\tilde{G}_S^{\overline{\text{MS}}} \equiv \frac{m_b^{\text{pole}}}{\tilde{m}_b^2} \tilde{G}_S
\]

and expanding in \( \alpha_s \) to the order in which \( G', \tilde{G}_S \) are calculated.
Experiment (HFLAV 2018):

\[
\Delta \Gamma^{\text{exp}} = (0.088 \pm 0.006) \text{ps}^{-1}
\]

average from LHCb, ATLAS, CMS, and CDF data.

Theory prediction with QCD corrections at next-to-leading order (NLO):

\[
\Delta \Gamma = \left( 0.091 \pm 0.020_{\text{scale}} \pm 0.006_{B,\Bar{B}_S} \pm 0.017_{\Lambda_{QCD}/m_b} \right) \text{GeV} \quad \text{(pole)}
\]

\[
\Delta \Gamma = \left( 0.104 \pm 0.008_{\text{scale}} \pm 0.007_{B,\Bar{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b} \right) \text{GeV} \quad \text{(MS)}
\]

Scale and scheme dependences exceed the experimental error.

\Rightarrow \text{ need NNLO!}
The NNLO calculation involves propagator-type three-loop diagrams with the two masses $m_c$ and $m_b$.

First step: diagrams with closed fermion loop large-$N_f$ limit.

One can neglect the charm mass in the charm lines attached to a weak vertex. This inflicts an error of order $\frac{\bar{m}_c^2(m_b)}{\bar{m}_b^2(m_b)} = 0.048$ on the NNLO correction.

However, the charm mass in the fermion loop cannot be neglected, there are terms of order $m_c/m_b$.

Method: reduction of the three-loop diagrams to master integrals with FIRE (A.V. Smirnov 2008), calculation of the master integrals in terms of an expansion in $m_c/m_b$. 
NNLO charm-loop contribution to the coefficient multiplying $C_2^2$ (with $C_2$ being the usual $W$-exchange Wilson coefficient in the weak hamiltonian) and $\langle Q \rangle$:

$$F_{22}^{(2),Nv}(z) =$$

$$13.1272 \log \frac{\mu_1}{m_b} + 2.14815 \log \frac{\mu_2}{m_b} - 3.55556 \log \frac{\mu_1}{m_b} \log \frac{\mu_2}{m_b}$$

$$+ 6.66667 \log^2 \frac{\mu_1}{m_b} + 1.77778 \log^2 \frac{\mu_2}{m_b} + 20.858 - 52.6379 \sqrt{z}$$

$$- z (18.1739 + 32 \log z) + 35.0919 z^{3/2}$$

$$+ z^2 \left( -2.83333 \log^2 z - 16.6481 \log z + 13.9138 \right)$$

$$+ z^3 \left( -1.48148 \log^2 z + 9.29383 \log z + 0.204084 \right) + \mathcal{O}(z^4)$$

with $z \equiv \frac{m_c^2}{m_b^2}$.

$\mu_1$ and $\mu_2$ are the renormalisation scales at which the $|\Delta B| = 1$ and $|\Delta B| = 2$ operators are defined, respectively.
Results

\[ \Delta \Gamma^{NLO} = (0.091 \pm 0.020_{\text{scale}}) \text{ GeV} \quad (\text{pole}) \]
\[ \Delta \Gamma^{NLO} = (0.104 \pm 0.015_{\text{scale}}) \text{ GeV} \quad (\overline{\text{MS}}) \]

\[ \Delta \Gamma^{NNLO} = (0.108 \pm 0.021_{\text{scale}}) \text{ GeV} \quad (\text{pole}) \]
\[ \Delta \Gamma^{NNLO} = (0.103 \pm 0.015_{\text{scale}}) \text{ GeV} \quad (\overline{\text{MS}}) \]

Naive non-abelianisation (NNA): trade \( N_f \) for \( \beta_0 \):

\[ \Delta \Gamma^{NNA} = (0.071 \pm 0.020_{\text{scale}}) \text{ GeV} \quad (\text{pole}) \]
\[ \Delta \Gamma^{NNA} = (0.099 \pm 0.012_{\text{scale}}) \text{ GeV} \quad (\overline{\text{MS}}). \]
Thus a **full NNLO** calculation is needed. To this end we have applied for long-term funding for staff and special computing resources. If approved, we envisage the following timeline:

**CKM 2020:** $\alpha_s/m_b$ corrections to $\Delta \Gamma$

**CKM 2022:** NNLO corrections to $\Delta \Gamma$

**CKM 2024:** NNLO corrections to semileptonic CP asymmetry in $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing
Conclusions

- The NLO prediction for $\Delta \Gamma$ has larger errors than the experimental value.
- Large-$N_f$ terms of the NNLO corrections reduce the scheme dependence of the NLO result (but not the scale dependence).
- The NLO result in the MS scheme receives smaller large-$N_f$ NNLO corrections than the pole-scheme result.
- A full NNLO calculation is desirable.
  $\Rightarrow$ need stable long-term funding.