The width difference in the B_s–$\bar{B}_s$ system: towards NNLO

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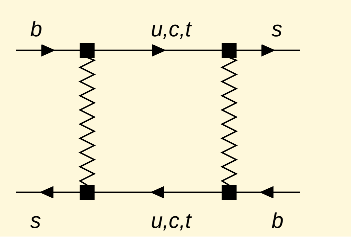
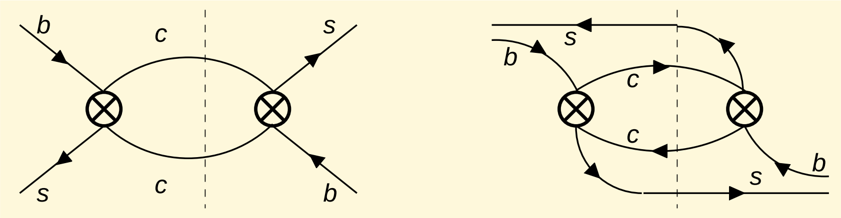
Meson-antimeson mixing

$B_s - \bar{B}_s$ mixing induces different masses and widths for the two B_s mass eigenstates:

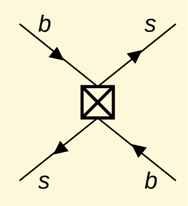


The width difference $\Delta\Gamma$ stems form the absorptive part of the box diagram, involving u,c quarks on the internal lines.

Leading contribution to $\Delta\Gamma$:



$\Delta\Gamma$ stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays.



Heavy Quark Expansion (HQE):

Exploit $m_b \gg \Lambda_{QCD}$ to express $\Delta\Gamma$ in terms of short-distance coefficients and matrix elements of local $|\Delta B| = 2$ operators.

$\Rightarrow$ expansion of $\Delta\Gamma$ in $\alpha_s(m_b)$ and Λ_{QCD}/m_b.

Operators at leading order in Λ_{QCD}/m_b (leading power):

$$Q = (\bar{s}_i b_i)_{V-A}\,(\bar{s}_j b_j)_{V-A}, \qquad \tilde{Q}_S = (\bar{s}_i b_j)_{S-P}\,(\bar{s}_j b_i)_{S-P}.$$

i, j: colour indices, $V \pm A = \gamma_\mu(1 \pm \gamma_5)$, $S \pm P = (1 \pm \gamma_5)$.

Matrix elements:

$$\langle B_s | Q(\mu_2) | \overline{B}_s \rangle = \frac{8}{3} M_{B_s}^2\, f_{B_s}^2 B(\mu_2)$$

$$\langle B_s | \widetilde{Q}_S(\mu_2) | \overline{B}_s \rangle \;=\; \frac{1}{3} M_{B_s}^2\, f_{B_s}^2 \widetilde{B}'_S(\mu_2).$$

Here f_{B_s} is the B_s decay constant and $\mu_2 = \mathcal{O}(m_b)$ is the renormalization scale at which the matrix elements are calculated.

The HQE gives

$$\Delta\Gamma = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cs}^* V_{cb}|^2 \left| G' \langle B_s|Q|\bar{B}_s\rangle + \widetilde{G}_S \langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle \right|$$

with the perturbative coefficients $G', \widetilde{G}_S$.

The coefficients $G', \widetilde{G}_S$ emerging from the calculation correspond to the choice $m_b = m_b^{\rm pole}$ in the prefactor. Subsequently one may switch to the $\overline{\rm MS}$ definition $\bar{m}_b$ through e.g.

$$\widetilde{G}_S^{\overline{\rm MS}} \equiv \frac{m_b^{\rm pole\,2}}{\bar{m}_b^2} \widetilde{G}_S$$

and expanding in α_s to the order in which $G', \widetilde{G}_S$ are calculated.

Experiment (HFLAV 2018):

$$\Delta\Gamma^{\mathrm{exp}} = (0.088 \pm 0.006)\,\mathrm{ps}^{-1}$$

average from LHCb, ATLAS, CMS, and CDF data.

Theory prediction with QCD corrections at next-to-leading order (NLO):

$$\Delta\Gamma = \left(0.091 \pm 0.020_{\mathrm{scale}} \pm 0.006_{B,\widetilde{B}_S} \pm 0.017_{\Lambda_{QCD}/m_b}\right)\ \mathrm{GeV} \qquad \text{(pole)}$$

$$\Delta\Gamma = \left(0.104 \pm 0.008_{\mathrm{scale}} \pm 0.007_{B,\widetilde{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b}\right)\ \mathrm{GeV} \qquad (\overline{\mathrm{MS}})$$

Scale and scheme dependences exceed the experimental error.

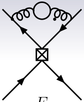
⇒ need NNLO!

NNLO

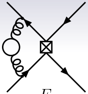
The NNLO calculation involves propagator-type three-loop diagrams with the two masses m_c and m_b.

First step: diagrams with closed fermion loop large-N_f limit.

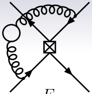
H.M. Asatrian, A. Hovhannisyan, A. Yeghiazaryan, UN, JHEP 1710 (2017) 191



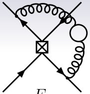
E_1



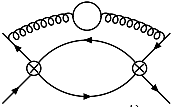
E_2



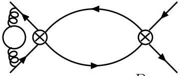
E_3



E_4



D_1



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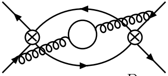
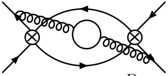
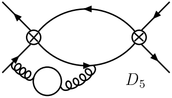


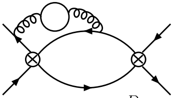
D_3



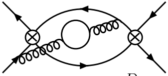
D_4



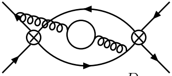
D_5



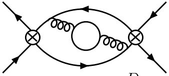
D_6



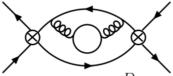
D_7



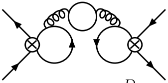
D_8



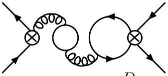
D_9



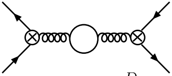
D_{10}



D_{11}



D_{12}



D_{13}

One can neglect the charm mass in the charm lines attached to a weak vertex. This inflicts an error of order $\frac{\bar{m}_c^2(m_b)}{\bar{m}_b^2(m_b)} = 0.048$ on the NNLO correction.

However, the charm mass in the fermion loop cannot be neglected, there are terms of order m_c/m_b.

Method: reduction of the three-loop diagrams to master integrals with FIRE (A.V. Smirnov 2008), calculation of the master integrals in terms of an expansion in m_c/m_b.

Sample result

NNLO charm-loop contribution to the coefficient multiplying C_2^2 (with C_2 being the usual W-exchange Wilson coefficient in the weak hamiltonian) and $\langle Q \rangle$:

$$
\begin{aligned}
F_{22}^{(2),N_V}(z) \; = \\
& 13.1272 \log \frac{\mu_1}{m_b} + 2.14815 \log \frac{\mu_2}{m_b} - 3.55556 \log \frac{\mu_1}{m_b} \log \frac{\mu_2}{m_b} \\
& + 6.66667 \log^2 \frac{\mu_1}{m_b} + 1.77778 \log^2 \frac{\mu_2}{m_b} + 20.858 - 52.6379\sqrt{z} \\
& - z(18.1739 + 32 \log z) + 35.0919 z^{3/2} \\
& + z^2 \left(-2.83333 \log^2 z - 16.6481 \log z + 13.9138 \right) \\
& + z^3 \left(-1.48148 \log^2 z + 9.29383 \log z + 0.204084 \right) + \mathcal{O}(z^4)
\end{aligned}
$$

with $z \equiv \dfrac{m_c^2}{m_b^2}$.

μ_1 and μ_2 are the renormalisation scales at which the $|\Delta B| = 1$ and $|\Delta B| = 2$ operators are defined, respectively.

Results

$$\Delta\Gamma^{NLO} = (0.091 \pm 0.020_{\rm scale})\ \text{GeV} \qquad (\text{pole})$$
$$\Delta\Gamma^{NLO} = (0.104 \pm 0.015_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS})$$

$$\Delta\Gamma^{NNLO} = (0.108 \pm 0.021_{\rm scale})\ \text{GeV} \qquad (\text{pole})$$
$$\Delta\Gamma^{NNLO} = (0.103 \pm 0.015_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS})$$

Naive non-abelianisation (NNA): trade N_f for β_0:

$$\Delta\Gamma^{NNA} = (0.071 \pm 0.020_{\rm scale})\ \text{GeV} \qquad (\text{pole})$$
$$\Delta\Gamma^{NNA} = (0.099 \pm 0.012_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS}).$$

Thus a full NNLO calculation is needed. To this end we have applied for long-term funding for staff and special computing resources. If approved, we envisage the following timeline:

CKM 2020: α_s/m_b corrections to $\Delta\Gamma$

CKM 2022: NNLO corrections to $\Delta\Gamma$

CKM 2024: NNLO corrections to semileptonic CP asymmetry in $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mixing

Conclusions

- The NLO prediction for $\Delta\Gamma$ has larger errors than the experimental value.
- Large-N_f terms of the NNLO corrections reduce the scheme dependence of the NLO result (but not the scale dependence).
- The NLO result in the $\overline{\mathrm{MS}}$ scheme receives smaller large-N_f NNLO corrections than the pole-scheme result.
- A full NNLO calculation is desirable.
  $\Rightarrow$ need stable long-term funding.