Heavy meson mixing and lifetimes from sum rules

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Based on work in collaboration with
D. King, M. Kirk and A. Lenz
Mixing in the SM

\[ i \frac{d}{dt} \left( \begin{array}{c} B_s^0(t) \\ \bar{B}_s^0(t) \end{array} \right) = \left( \hat{M}^s - \frac{i}{2} \hat{\Gamma}^s \right) \left( \begin{array}{c} B_s^0(t) \\ \bar{B}_s^0(t) \end{array} \right) \]

Factorizes into perturbative Wilson coefficients and hadronic matrix elements:

\[ M_{12}^q = \frac{G_F^2}{16\pi^2} \lambda_t^2 M_W^2 S_0(x_t) \hat{n}_B \frac{\langle \bar{B}_q|Q_1|B_q \rangle}{2M_{B_q}} \]

\[ \Gamma_{12}^q = -\frac{G_F^2 m_b^2}{24\pi M_{B_q}} \sum_{x=u,c} \sum_{y=u,c} \left[ G_{1}^{q,xy} \langle \bar{B}_q|Q_1|B_q \rangle - G_{2}^{q,xy} \langle \bar{B}_q|Q_2|B_q \rangle \right] + \mathcal{O}(1/m_b) \]

Full basis of dimension-six operators (SM + BSM):

\[ Q_1 = \bar{b}_i \gamma_\mu (1 - \gamma^5) q_i \bar{b}_j \gamma^\mu (1 - \gamma^5) q_j, \]
\[ Q_2 = \bar{b}_i (1 - \gamma^5) q_i \bar{b}_j (1 - \gamma^5) q_j, \quad Q_3 = \bar{b}_i (1 - \gamma^5) q_j \bar{b}_j (1 - \gamma^5) q_i, \]
\[ Q_4 = \bar{b}_i (1 - \gamma^5) q_i \bar{b}_j (1 + \gamma^5) q_j, \quad Q_5 = \bar{b}_i (1 - \gamma^5) q_j \bar{b}_j (1 + \gamma^5) q_i. \]
Matrix elements can be determined on the lattice. Currently dominated by one result FNAL/MILC 16

We want an independent determination!

\[
\langle Q(\mu) \rangle = A_Q f_B^2 M_B^2 B_Q(\mu)
\]

\[
A_{Q_1} = 2 + \frac{2}{N_c},
\]

\[
A_{Q_2} = \frac{M_B^2}{(m_b + m_q)^2} \left( -2 + \frac{1}{N_c} \right),
\]

\[
A_{Q_4} = \frac{2M_B^2}{(m_b + m_q)^2} + \frac{1}{N_c},
\]

\[
A_{Q_3} = \frac{M_B^2}{(m_b + m_q)^2} \left( 1 - \frac{2}{N_c} \right),
\]

\[
A_{Q_5} = 1 + \frac{2M_B^2}{N_c(m_b + m_q)^2},
\]
HQET sum rules: decay constant

Sum rules give results which are truly independent from the lattice. Based on:

- Analyticity of correlation functions
- Quark-hadron duality

First consider the sum rule for the decay constant. Based on the two-point correlator:

\[ \Pi(\omega) = i \int d^d x e^{i p x} \left\langle 0 \left| T \left[ \tilde{j}_+^\dagger(0) \tilde{j}_+(x) \right] \right| 0 \right\rangle \]

\[ \tilde{j}_+ = \bar{q} \gamma^5 h^{(+)} \quad \omega = p \cdot v \]

Use Cauchy’s residue theorem to derive a dispersion relation:

\[ \Pi(\omega) = \frac{1}{2\pi i} \oint_C d\eta \frac{\Pi(\eta)}{\eta - \omega} \]
HQET sum rules: decay constant

Deform the contour:

\[ \Pi(\omega) = \int_0^\infty d\eta \frac{\rho \Pi(\eta)}{\eta - \omega} + \int d\eta \frac{\Pi(\eta)}{\eta - \omega} \]
HQET sum rules: decay constant

Deform the contour:

\[
\Pi(\omega) = \int_0^\infty d\eta \frac{\rho \Pi(\eta)}{\eta - \omega} + \int d\eta \frac{\Pi(\eta)}{\eta - \omega}
\]

Can be computed with an OPE when \(\omega\) is far away from the physical cut
HQET sum rules: decay constant

Deform the contour:

\[ \Pi(\omega) = \int_0^\infty d\eta \frac{\rho_{\Pi}(\eta)}{\eta - \omega} + \int d\eta \frac{\Pi(\eta)}{\eta - \omega} \]

Can be computed with an OPE when \( \omega \) is far away from the physical cut

Discontinuity

\[ \rho_{\Pi}^{\text{had}}(\omega) = F^2(\mu)\delta(\omega - \Lambda) + \rho_{\Pi}^{\text{cont}}(\omega) \]

HQET decay constant
HQET sum rules: decay constant

Applying a Borel transform and a cutoff on the continuum part we obtain:

\[ F^2(\mu) e^{-\frac{\Lambda^2}{\mu^2}} = \int_0^{\omega_c} d\omega e^{-\frac{\omega}{\mu^2}} \rho_{\Pi}^{\text{OPE}}(\omega) \]

Reference | Method | \( N_f \) | \( f_{B^+}(\text{MeV}) \) | \( f_{B_s}(\text{MeV}) \) | \( f_{B_s}/f_{B^+} \) |
--- | --- | --- | --- | --- | --- |
ETM 13 [85] *,† | LQCD | 2+1+1 | 196(9) | 235(9) | 1.201(25) |
HPQCD 13 [86] | LQCD | 2+1+1 | 184(4) | 224(5) | 1.217(8) |
Average | LQCD | 2+1+1 | 184(4) | 224(5) | 1.217(8) |
Aoki 14 [87] *,‡ | LQCD | 2+1 | 218.8(6.5)(30.8) | 263.5(4.8)(36.7) | 1.193(20)(44) |
RBC/UKQCD 14 [88] | LQCD | 2+1 | 195.6(6.4)(13.3) | 235.4(5.2)(11.1) | 1.223(14)(70) |
HPQCD 12 [89] * | LQCD | 2+1 | 191(1)(8) | 228(3)(10) | 1.188(12)(13) |
HPQCD 12 [89] * | LQCD | 2+1 | 189(3)(3)* | $-$ | $-$ |
HPQCD 11 [90] | LQCD | 2+1 | $-$ | 225(3)(3) | $-$ |
Fermilab/MILC 11 [69] | LQCD | 2+1 | 196.9(5.5)(7.0) | 242.0(5.1)(8.0) | 1.229(13)(23) |
Average | LQCD | 2+1 | 189.9(4.2) | 228.6(3.8) | 1.210(15) |
Our average | LQCD | Both | 187.1(4.2) | 227.2(3.4) | 1.215(7) |
Wang 15 [71] ‡ | QCD SR | $-$ | 194(15) | 231(16) | 1.19(10) |
Baker 13 [91] | QCD SR | $-$ | 186(14) | 222 (12) | 1.19(4) |
Lucha 13 [92] | QCD SR | $-$ | 192.0(14.6) | 228.0(19.8) | 1.184(24) |
Gelhausen 13 [72] | QCD SR | $-$ | 207(17) | 242(17) | 1.17(3) |
Narison 12 [73] | QCD SR | $-$ | 206(7) | 234(5) | 1.14(3) |
Hwang 09 [75] | LFQM | $-$ | 270.0(42.8) | $-$ | 1.32(8) |

Sum rules are in good agreement with lattice, but have larger uncertainties

[PDG '16]

[Broadhurst, Grozin '92; Bagan, Ball, Braun, Dosch '92; Neubert '92]
HQET sum rules: Bag parameters

Consider the three-point correlator:

\[ K_{\tilde{Q}}(\omega_1, \omega_2) = \int d^d x_1 d^d x_2 e^{i p_1 \cdot x_1 - i p_2 \cdot x_2} \langle 0 | T [ \tilde{j}_+(x_2) \tilde{Q}(0) \tilde{j}_-(x_1) ] | 0 \rangle \]

Going through the same steps one obtains the sum rule: [Chetyrkin, Kataev, Krasulin, Pivovarov '86]

\[ F^2(\mu) \langle \tilde{Q}(\mu) \rangle e^{-\frac{\Lambda}{t_1} - \frac{\Lambda}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} \rho_{\tilde{Q}}^{\text{OPE}}(\omega_1, \omega_2) \]

\[ \rho_{\tilde{Q}}^{\text{OPE}}(\omega_1, \omega_2) = \rho_{\tilde{Q}}^{\text{pert}}(\omega_1, \omega_2) + \rho_{\tilde{Q}}^{\langle \bar{q}q \rangle}(\omega_1, \omega_2)\langle \bar{q}q \rangle + \rho_{\tilde{Q}}^{\langle \alpha_s G^2 \rangle}(\omega_1, \omega_2)\langle \alpha_s G^2 \rangle + \ldots \]

In practise we compute the correlator and then take its double discontinuity.
Three-point correlator

NLO accuracy in the perturbative part requires a three-loop calculation:

\[
\rho_{Q_i}^{\text{pert}}(\omega_1, \omega_2) = A_{\tilde{Q}_i} \rho_{\Pi}(\omega_1) \rho_{\Pi}(\omega_2) + \frac{\omega_1^2 \omega_2^2}{\pi^4} \frac{\alpha_s}{4\pi} r_{\tilde{Q}_i} \left( \frac{\omega_2}{\omega_1}, L_\omega \right)
\]

Factorizable contribution, reproduces the vacuum saturation approximation B=1 (VSA)

Master integrals:
[Grozin, Lee '08]

Operator Q1:
[Grozin, Mannel, Klein, Pivovarov '16]

All dimension six operators:
[Kirk, Lenz, TR '17]

Non-factorizable contribution

\[
\begin{align*}
r_{\tilde{Q}_1}(x, L_\omega) &= 8 - \frac{a_2}{2} - \frac{8\pi^2}{3}, \\
r_{\tilde{Q}_2}(x, L_\omega) &= 25 + \frac{a_1}{2} - \frac{4\pi^2}{3} + 6L_\omega + \phi(x), \\
r_{\tilde{Q}_4}(x, L_\omega) &= 16 - \frac{a_3}{4} - \frac{4\pi^2}{3} + 3L_\omega + \frac{\phi(x)}{2}, \\
r_{\tilde{Q}_5}(x, L_\omega) &= 29 - \frac{a_3}{2} - \frac{8\pi^2}{3} + 6L_\omega + \phi(x).
\end{align*}
\]
Sum rule for Bag parameters

Formulate sum rule for deviation $\Delta B_{\bar{Q}_i}(\mu) = B_{\bar{Q}_i}(\mu) - 1$ from the HQET Bag parameters $\langle \bar{Q}(\mu) \rangle = A_{\bar{Q}_i} F^2(\mu) B_{\bar{Q}_i}(\mu)$.

$$\Delta B_{\bar{Q}_i} = \frac{1}{A_{\bar{Q}_i} F(\mu)^4 \int_d \omega_1 d\omega_2 e^{\frac{\Lambda_1 - \omega_1}{t_1} + \frac{\Lambda_2 - \omega_2}{t_2}} \Delta \rho_{\bar{Q}_i}(\omega_1, \omega_2)}$$

$$= \frac{1}{A_{\bar{Q}_i}} \left( \int^\omega_1 d\omega_2 e^{\frac{\omega_1}{t_1}} - \rho(\omega_1) \rho(\omega_2) \right) \left( \int^\omega_2 d\omega_1 e^{\frac{\omega_2}{t_2}} \rho(\omega_1) \rho(\omega_2) \right).$$

Dispersion relation is not violated by arbitrary analytical weight function (Note of caution: Duality breaks down for pathological choices)

$$F^4(\mu) e^{-\frac{\Lambda_1}{t_1} - \frac{\Lambda_2}{t_2}} w(\Lambda, \bar{\Lambda}) = \int d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} w(\omega_1, \omega_2) \rho(\omega_1) \rho(\omega_2) + \ldots.$$ 

With an appropriate choice we obtain an analytic result for the pert contribution:

$$\Delta B_{\bar{Q}_i}^{\text{pert}}(\mu, \rho) = \frac{C_F}{N_c A_{\bar{Q}_i}} \frac{\alpha_s(\mu, \rho)}{4\pi} r_{\bar{Q}_i} \left( 1 + \log \frac{\mu^2}{4\Lambda^2} \right).$$
SU(3) breaking effects

The exact calculation with non-zero strange-quark mass is very challenging. We need to resort to an expansion in $m_s$. This yields

$$\Delta B^{\text{pert}}_{\bar{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\bar{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} \left\{ r^{(0)}_{\bar{Q}_i}(1) + \frac{2m_s}{\Lambda + m_s} \left[ r^{(1)}_{\bar{Q}_i}(1) - r^{(0)}_{\bar{Q}_i}(1) \right] ight\} + \frac{2m^2_s}{(\Lambda + m_s)^2} \left[ r^{(2)}_{\bar{Q}_i}(1) - 2r^{(1)}_{\bar{Q}_i}(1) + 2r^{(0)}_{\bar{Q}_i}(1) \right] + \ldots$$

where

$$\Delta \rho_{\bar{Q}_i} \equiv \frac{N_c C_F \omega_1^2 \omega_2^2}{4} \frac{\alpha_s}{4\pi} \left[ r^{(0)}_{\bar{Q}_i}(x, L_\omega) + \left( \frac{m_s}{\omega_1} + \frac{m_s}{\omega_2} \right) r^{(1)}_{\bar{Q}_i}(x, L_\omega) \right. \left. + \left( \frac{m^2_s}{\omega_1^2} + \frac{m^2_s}{\omega_2^2} \right) r^{(2)}_{\bar{Q}_i}(x, L_\omega) + \ldots \right] .$$

Expanded correlator can be computed by the method of regions. Only the ‘hard’ region contributes up to quadratic order.

[King, Lenz, TR: WIP]
Matrix elements for Bd mixing

- Determine HQET Bag parameters at low scale $\mu_\rho \sim 1.5$ GeV from sum rules
- Run up to $\mu_m \sim m_b$ and match to QCD Bag parameters at NLO
- Detailed analysis performed in 1711.02100

[Kirk, Lenz, TR '17]
Matrix elements for Bs mixing

- Determine HQET Bag parameters at low scale $\mu_\rho \sim 1.5$ GeV from sum rules.
- Run up to $\mu_m \sim m_b$ and match to QCD Bag parameters at NLO.
- Includes SU(3) breaking effects up to $m_s^2$.

[King, Lenz, TR: WIP]
Bs-mixing observables

Update of 1711.02100 with CKM elements from CKMFitter, new decay constants from [FNAL/MILC '17] and including SU(3) breaking effects:

\[
\Delta M_s^{\exp} = (17.757 \pm 0.021) \, \text{ps}^{-1}, \\
\Delta M_s^{\text{SM}} = (18.1 \pm 1.1 \, \text{(had.)}) \\
\quad \pm 0.1 \, \text{(scale)} \\
\quad ^{+0.2}_{-0.5} \, \text{(param.}) \, \text{ps}^{-1},
\]

\[
\Delta \Gamma_s^{\exp} = (0.090 \pm 0.005) \, \text{ps}^{-1}, \\
\Delta \Gamma_s^{\text{PS}} = (0.089 \pm 0.020 \, \text{(had.)}) \\
\quad ^{+0.008}_{-0.021} \, \text{(scale)} \\
\quad ^{+0.001}_{-0.003} \, \text{(param.}) \, \text{ps}^{-1},
\]

\[
a_{s,\text{exp}}^{s,\text{PS}} = (-60 \pm 280) \cdot 10^{-5}, \\
a_{s,\text{PS}}^{s,\text{PS}} = (1.8 \pm 0.0 \, \text{(had.)}) \\
\quad ^{+0.0}_{-0.1} \, \text{(scale)} \\
\quad \pm 0.1 \, \text{(param.}) \cdot 10^{-5},
\]
Bd-mixing observables

Update of 1711.02100 with CKM elements from CKMFitter and new decay constants from [FNAL/MILC '17]:

\[
\Delta M_{d}^{\exp} = (0.5065 \pm 0.0019) \text{ ps}^{-1}, \\
\Delta M_{d}^{\text{SM}} = (0.53 \pm 0.03 \text{ (had.)} \\
\pm 0.00 \text{ (scale)} \\
\pm 0.02 \text{ (param.)}) \text{ ps}^{-1},
\]

\[
\Delta \Gamma_{d}^{\exp} = (-1.3 \pm 6.6) \cdot 10^{-3} \text{ ps}^{-1}, \\
\Delta \Gamma_{d}^{\text{PS}} = (2.5 \pm 0.6 \text{ (had.)} \\
\pm 0.2 \text{ (scale)} \\
\pm 0.1 \text{ (param.)}) \cdot 10^{-3} \text{ ps}^{-1},
\]

\[
a_{s_{l}}^{d, \exp} = (-21 \pm 17) \cdot 10^{-4}, \\
a_{s_{l}}^{d, \text{PS}} = (-4.2 \pm 0.1 \text{ (had.)} \\
\pm 0.2 \text{ (scale)} \\
\pm 0.2 \text{ (param.)}) \cdot 10^{-4},
\]
\[ \Delta B = 0 \text{ Bag parameters} \]

[Kirk, Lenz, TR '17]

\[
\begin{align*}
\frac{\tau(B^+)}{\tau(B^0)} \bigg|_{\text{exp}} & = 1.076 \pm 0.004, \\
\frac{\tau(B^+)}{\tau(B^0)} \bigg|_{\text{PS}} & = 1.082 \pm 0.021 \text{ (had.)} +0.000 \text{ (scale)} \pm 0.006 \text{ (param.)}, \\
\frac{\tau(B_s^0)}{\tau(B_s^0)} \bigg|_{\text{exp}} & = 0.994 \pm 0.004, \\
\frac{\tau(B_s^0)}{\tau(B_s^0)} \bigg|_{\text{MS}} & = 0.9994 \pm 0.0014 \text{ (had.)} \pm 0.0006 \text{ (scale)} \pm 0.0020 \left(\frac{1}{m_b^4}\right),
\end{align*}
\]
D lifetimes as test of HQE

HQE even provides good description of lifetimes in charm sector:

\[
\frac{\tau(D^+)}{\tau(D^0)} \bigg|_{\text{exp}} = 2.536 \pm 0.019, \quad \frac{\tau(D^+)}{\tau(D^0)} \bigg|_{\text{HQE}} = 2.7^{+0.7}_{-0.8}, \quad [\text{Kirk, Lenz, TR '17}]
\]

\[
\frac{\tau(D^+)}{\tau(D^0)} \bigg|_{\text{exp}} = 1.292 \pm 0.019, \quad \frac{\tau(D^+)}{\tau(D^0)} \bigg|_{\text{HQE}} = 1.19 \pm 0.13. \quad [\text{Lenz, TR '13}]
\]

Good convergence:
NLO QCD $+28\%$, $1/mc -34\%$.
Good behaviour under scale variation above about 1 GeV.
Overview of lifetime ratios from sum rules

\[ \tau(D^+)/\tau(D^0) \]
- HFLAV: 2.536 ± 0.019
- HQE: 2.7^{+0.74}_{-0.82}

\[ \tau(B_s^0)/\tau(B_d^0) \]
- HFLAV: 0.994 ± 0.004
- HQE: 0.9994 ± 0.0025

\[ \tau(B^+)/\tau(B_d^0) \]
- HFLAV: 1.076 ± 0.004
- HQE: 1.082^{+0.022}_{-0.026}

[Kirk, Lenz, TR '17]
Overview of lifetime ratios from sum rules

SU(3) breaking effects in matrix elements can also be determined with the strategy of ms-expansion

[Kirk, Lenz, TR '17]
Conclusions & outlook

• Sum rules provide highly competitive alternative to lattice simulations for the matrix elements of 4-quark operators and truly independent comparisons.

• The HQE is in terrific shape. Lifetimes even look promising in the charm sector.

• Mixing gives strong constraints on models that are frequently invoked to explain the current 'anomalies'. [cf. talk by L. Di Luzio in WG4 Tue 10:20]

• First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice would be interesting.
SINCE YEARS OF BEGGING DID NOT HELP – IT’S TIME TO PROVOKE

Lifetimes are too heavy for lattice physicists!

The strongest lattice researcher alive

Arbitrary sum rule researcher

Matrix elements for lifetimes of HEAVY mesons

[Lenz Implications '17]
Conclusions & outlook

- Sum rules provide highly competitive alternative to lattice simulations for the matrix elements of 4-quark operators and truly independent comparisons.
- The HQE is in terrific shape. Lifetimes even look promising in the charm sector.
- Mixing gives strong constraints on models that are frequently invoked to explain the current 'anomalies'. [cf. talk by L. Di Luzio in WG4 Tue 10:20]
- First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice would be interesting.

- NNLO QCD-HQET matching calculations can significantly decrease uncertainties for dimension-six operators. Q1: [Grozin, Mannel, Pivovarev '17,18, cf talk by T. Mannel in WG4 Tue 9:55]

- Uncertainties in decay rate difference and lifetimes can be reduced considerably by a sum rule determination of the dimension seven matrix elements.
Heavy quark expansion in charm?

B-physics: HQE is well established approach, $\Lambda/m_b \sim 0.2 \ll 1$

D-physics: HQE commonly dismissed, $\Lambda/m_c \sim 0.2 m_b/m_c \sim 0.7 \approx 1$

**BUT:** HQE is really an expansion in $\Lambda$/momentum release
- $\Delta \Gamma_s$ dominated by $D_s^{(*)+} D_s^{(*)-}$ final state, momentum release $\sim 3.5$ GeV
- D decays dominated by $K\pi^{(1-3)}$ final state, momentum release $\sim 1.7$ GeV
- expected expansion parameter is of the order 0.4

Small enough for convergence?

Shut up and calculate!
Matrix elements

- Good agreement with lattice (using lattice results for the decay constant)
- Larger uncertainties due to lower matching scale
- Also: first determination of $\Delta C = 0$ matrix elements in 1711.02100
## Uncertainties

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<th>$\Delta B = 2$</th>
<th>$\overline{\Lambda}$</th>
<th>intrinsic SR</th>
<th>condensates</th>
<th>$\mu_p$</th>
<th>$1/m_b$</th>
<th>$\mu_m$</th>
<th>$a_i$</th>
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<td>$\overline{B}_{Q_1}$</td>
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<td>$\pm 0.018$</td>
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## Uncertainties

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Uncertainties

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Table 8: Individual errors for the ratio $\tau(B^+)/\tau(B^0)$ in the PS mass scheme.

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Table 9: Individual errors for the ratio $\tau(D^+)/\tau(D^0)$ in the PS mass scheme.