Heavy meson mixing and lifetimes from sum rules

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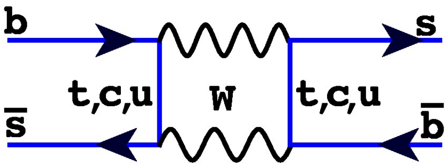
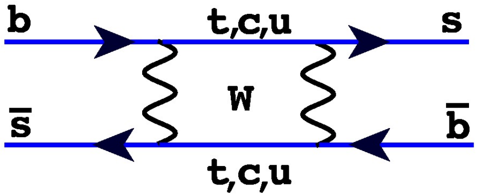
Based on work in collaboration with
D. King, M. Kirk and A. Lenz

Mixing in the SM

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle\end{pmatrix} = \left(\hat{M}^s - \frac{i}{2}\hat{\Gamma}^s\right)\begin{pmatrix}|B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle\end{pmatrix}$$





Factorizes into perturbative Wilson coefficients and hadronic matrix elements:

$$\begin{aligned}
M_{12}^q &= \frac{G_F^2}{16\pi^2}\lambda_t^2 M_W^2 S_0(x_t)\hat{\eta}_B\,\frac{\langle\overline{B}_q|Q_1|B_q\rangle}{2M_{B_q}} \\
\Gamma_{12}^q &= -\frac{G_F^2 m_b^2}{24\pi M_{B_q}}\sum_{x=u,c}\sum_{y=u,c}\left[G_1^{q,xy}\langle\overline{B}_q|Q_1|B_q\rangle - G_2^{q,xy}\langle\overline{B}_q|Q_2|B_q\rangle\right] + \mathcal{O}(1/m_b)
\end{aligned}$$

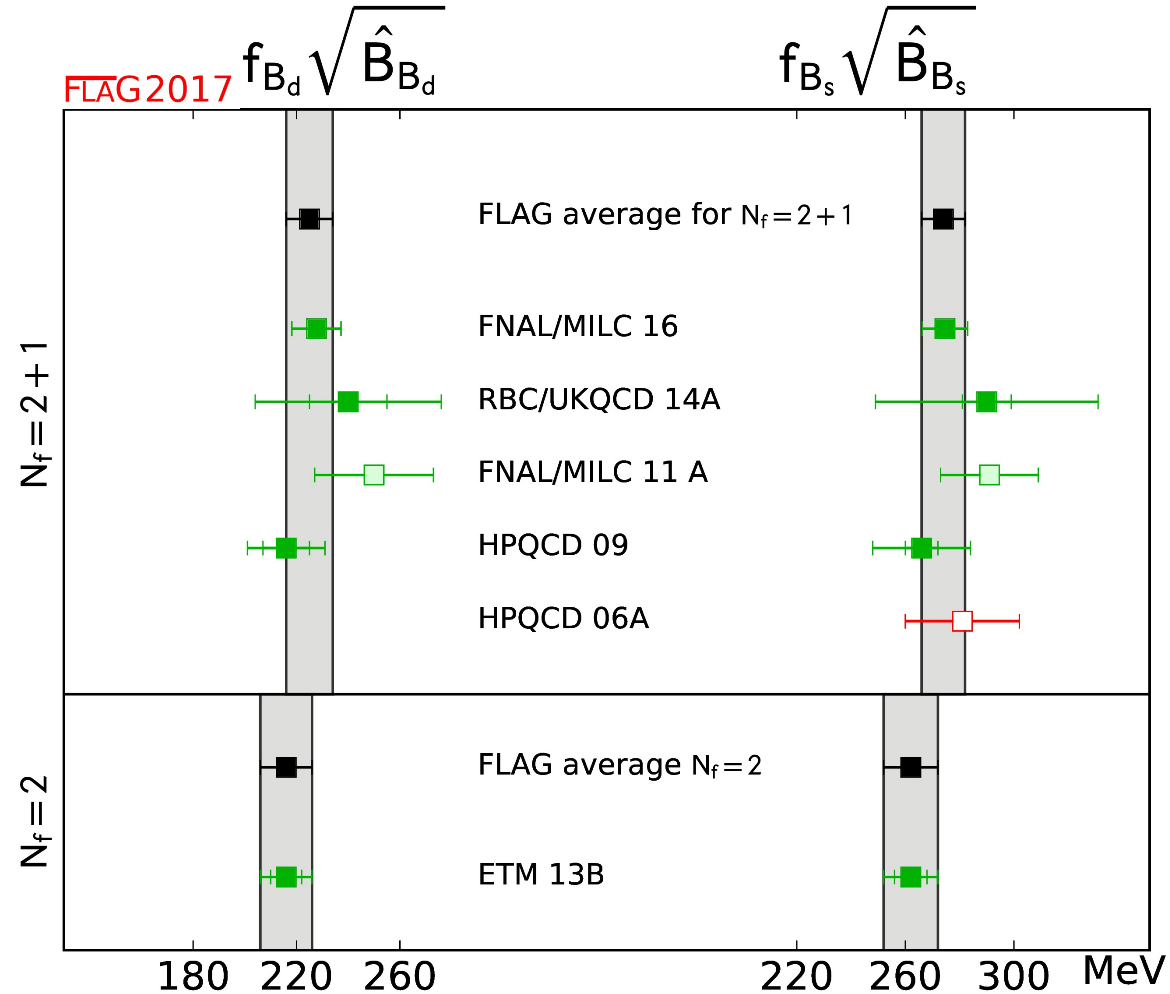
Full basis of dimension-six operators (SM + BSM):

$$\begin{aligned}
Q_1 &= \bar{b}_i\gamma_\mu(1-\gamma^5)q_i\ \bar{b}_j\gamma^\mu(1-\gamma^5)q_j, \\
Q_2 &= \bar{b}_i(1-\gamma^5)q_i\ \bar{b}_j(1-\gamma^5)q_j, \qquad Q_3 = \bar{b}_i(1-\gamma^5)q_j\ \bar{b}_j(1-\gamma^5)q_i, \\
Q_4 &= \bar{b}_i(1-\gamma^5)q_i\ \bar{b}_j(1+\gamma^5)q_j, \qquad Q_5 = \bar{b}_i(1-\gamma^5)q_j\ \bar{b}_j(1+\gamma^5)q_i.
\end{aligned}$$

Lattice results

Matrix elements can be determined on the lattice. Currently dominated by one result FNAL/MILC 16

We want an independent determination!



$$\langle Q(\mu)\rangle = A_Q\, f_B^2 M_B^2\, B_Q(\mu)$$

$$A_{Q_1} = 2 + \frac{2}{N_c},$$

$$A_{Q_2} = \frac{M_B^2}{(m_b+m_q)^2}\left(-2 + \frac{1}{N_c}\right), \qquad A_{Q_3} = \frac{M_B^2}{(m_b+m_q)^2}\left(1 - \frac{2}{N_c}\right),$$

$$A_{Q_4} = \frac{2M_B^2}{(m_b+m_q)^2} + \frac{1}{N_c}, \qquad A_{Q_5} = 1 + \frac{2M_B^2}{N_c(m_b+m_q)^2},$$

HQET sum rules: decay constant

Sum rules give results which are truly independent from the lattice. Based on:

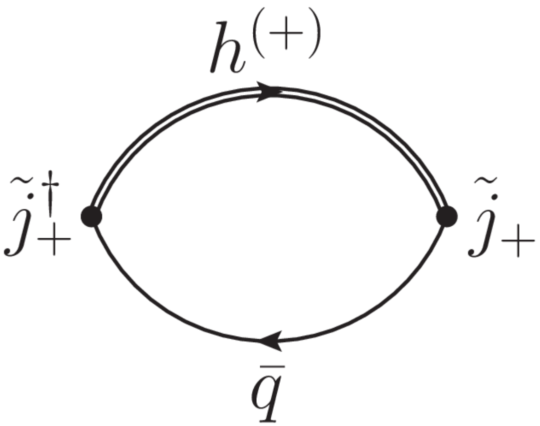
- Analyticity of correlation functions
- Quark-hadron duality

[Shifman, Vainshtein, Zakharov '79]

First consider the sum rule for the decay constant.
Based on the two-point correlator:

$$\Pi(\omega) = i \int d^d x e^{ipx} \left\langle 0 \middle| \mathrm{T} \left[\tilde{j}_+^\dagger(0) \tilde{j}_+(x) \right] \middle| 0 \right\rangle$$

$$\tilde{j}_+ = \bar{q}\gamma^5 h^{(+)} \qquad \omega = p \cdot v$$

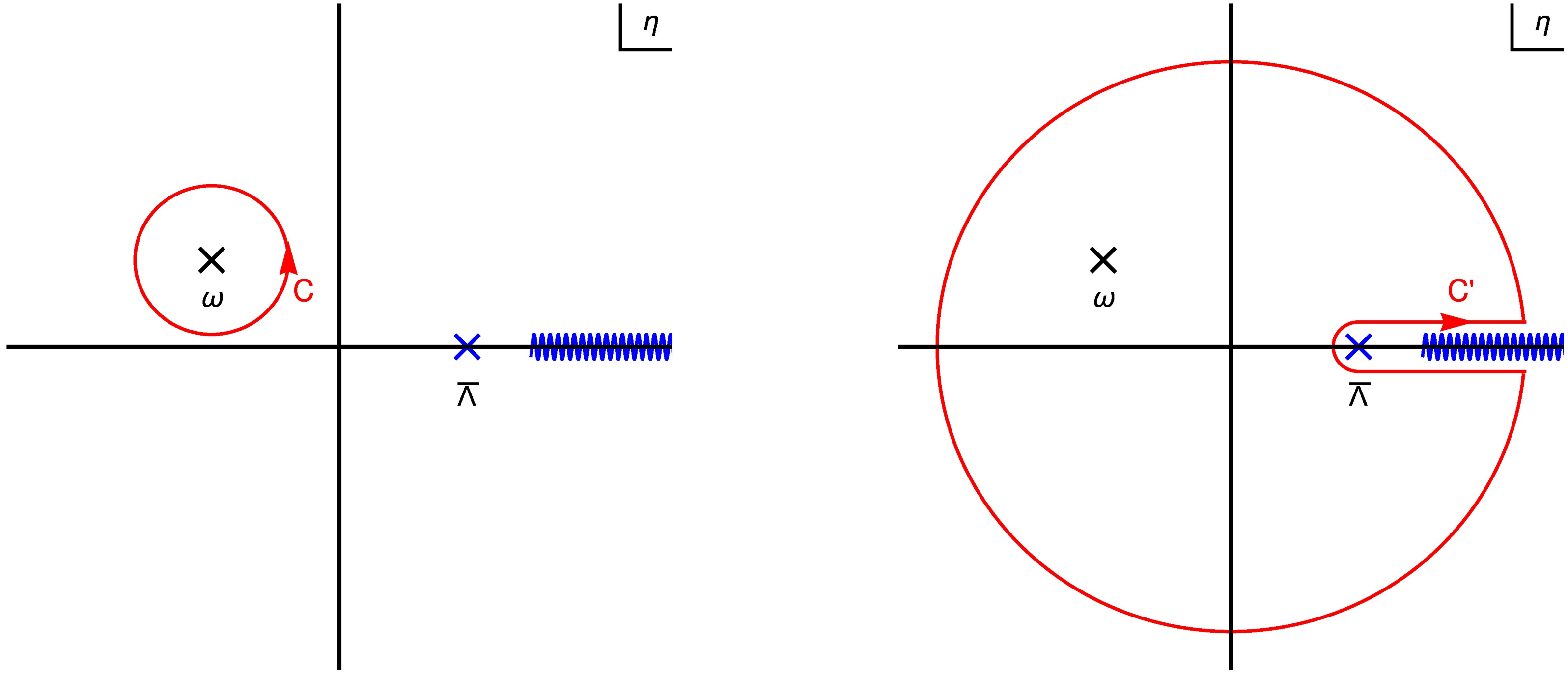


Use Cauchy's residue theorem to derive a dispersion relation:

$$\Pi(\omega) = \frac{1}{2\pi i} \oint_C d\eta \, \frac{\Pi(\eta)}{\eta - \omega}$$

HQET sum rules: decay constant

Deform the contour:



$$\Pi(\omega) = \int_0^\infty d\eta\, \frac{\rho_\Pi(\eta)}{\eta - \omega} + \oint d\eta\, \frac{\Pi(\eta)}{\eta - \omega}$$

HQET sum rules: decay constant

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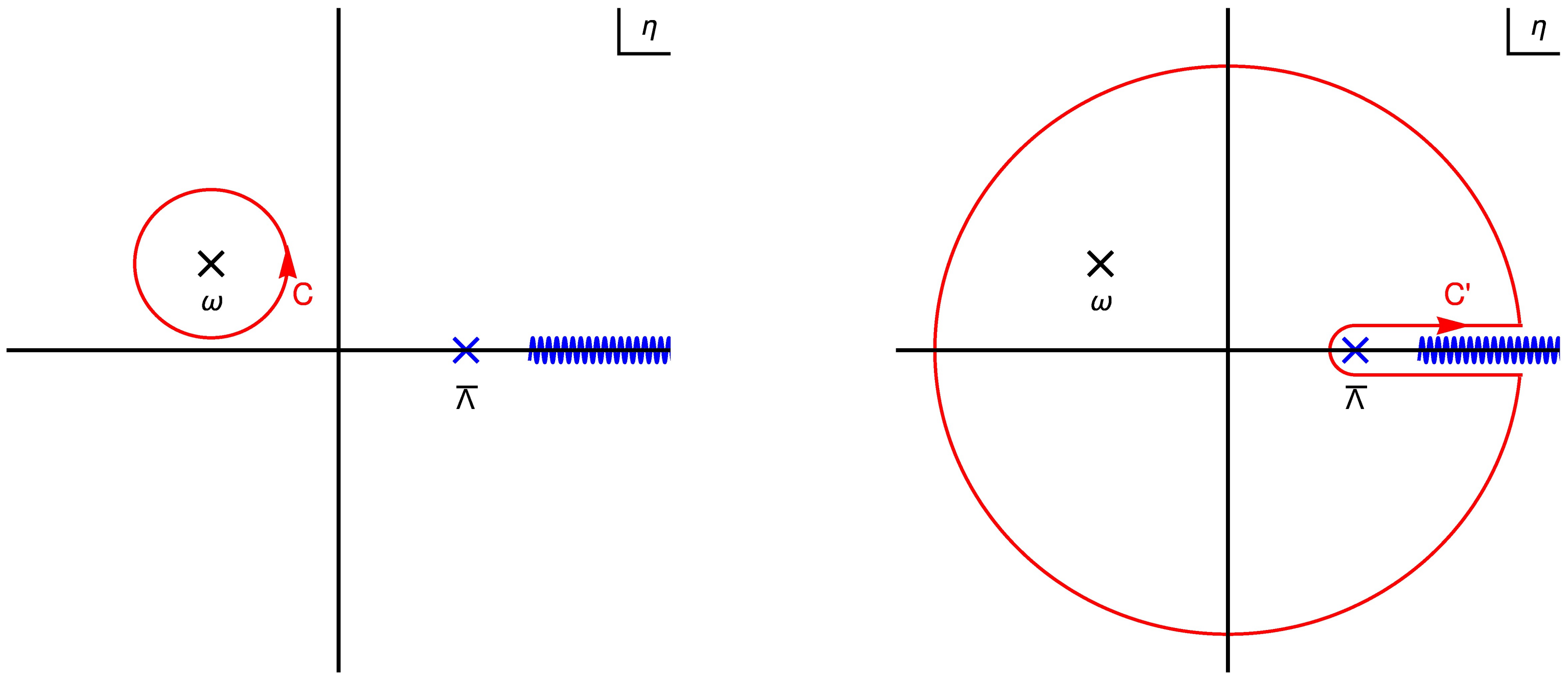

$$\Pi(\omega) = \int_0^\infty d\eta \, \frac{\rho_\Pi(\eta)}{\eta - \omega} + \oint d\eta \, \frac{\Pi(\eta)}{\eta - \omega}$$

Can be computed with an OPE when ω is far away from the physical cut

HQET sum rules: decay constant

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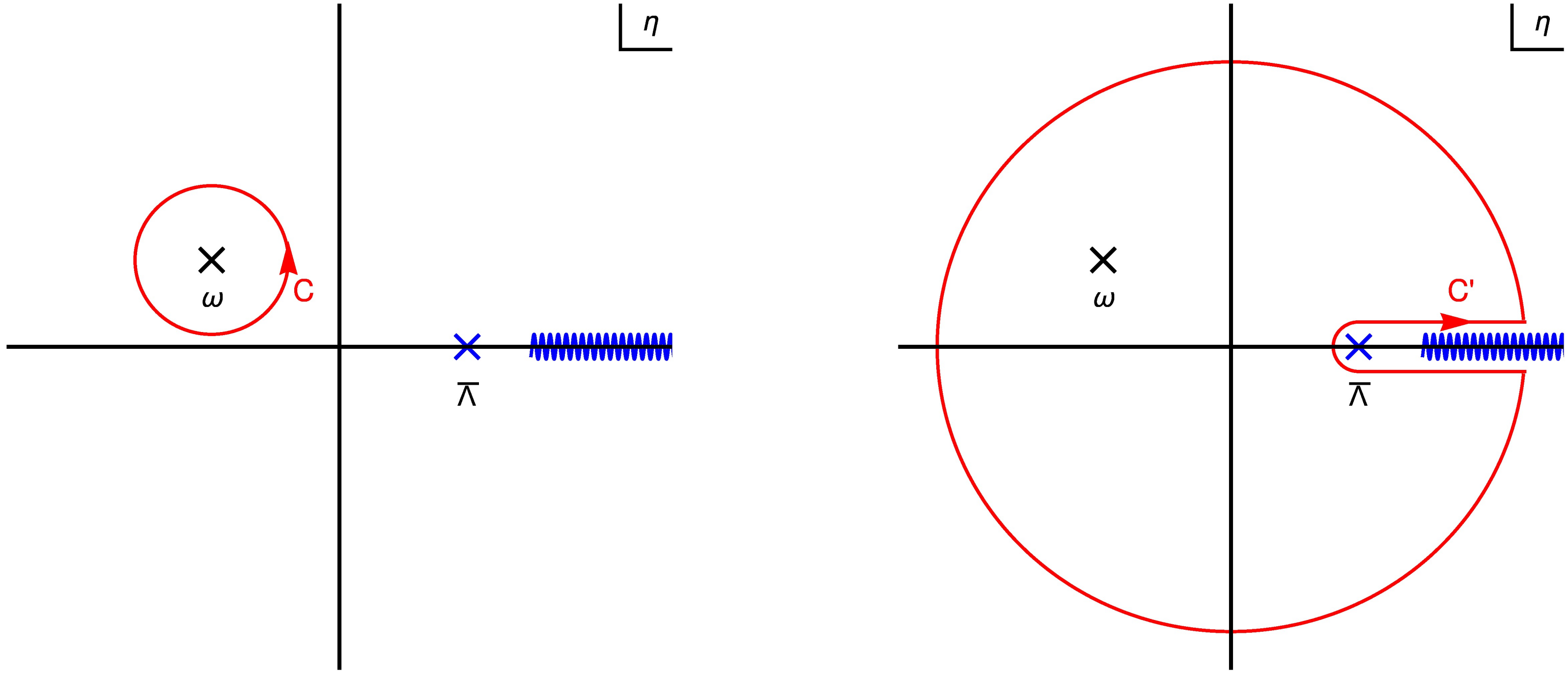

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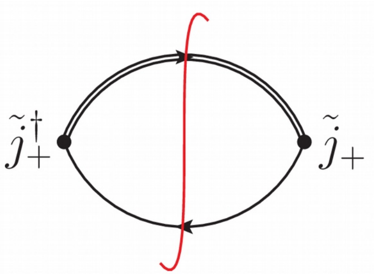
Discontinuity



$$\rho_\Pi^{\text{had}}(\omega) = F^2(\mu)\delta(\omega - \overline{\Lambda}) + \rho_\Pi^{\text{cont}}(\omega)$$

HQET decay constant

HQET sum rules: decay constant

Applying a Borel transform and a cutoff on the continuum part we obtain:

$$F^2(\mu)e^{-\frac{\overline{\Lambda}}{t}} = \int_0^{\omega_c} d\omega e^{-\frac{\omega}{t}} \rho_\Pi^{\mathrm{OPE}}(\omega)$$

[Broadhurst,Grozin '92; Bagan, Ball, Braun,Dosch '92; Neubert '92]

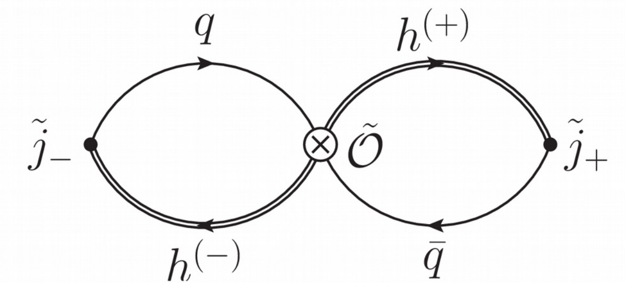
[PDG '16]

Reference	Method	N_f	f_{B^+}(MeV)	f_{B_s}(MeV)	f_{B_s}/f_{B^+}
ETM 13 [85] *,†	LQCD	2+1+1	196(9)	235(9)	1.201(25)
HPQCD 13 [86]	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Average	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Aoki 14 [87] *,‡	LQCD	2+1	218.8(6.5)(30.8)	263.5(4.8)(36.7)	1.193(20)(44)
RBC/UKQCD 14 [88]	LQCD	2+1	195.6(6.4)(13.3)	235.4(5.2)(11.1)	1.223(14)(70)
HPQCD 12 [89] *	LQCD	2+1	191(1)(8)	228(3)(10)	1.188(12)(13)
HPQCD 12 [89] *	LQCD	2+1	189(3)(3)*	–	–
HPQCD 11 [90]	LQCD	2+1	–	225(3)(3)	–
Fermilab/MILC 11 [69]	LQCD	2+1	196.9(5.5)(7.0)	242.0(5.1)(8.0)	1.229(13)(23)
Average	LQCD	2+1	189.9(4.2)	228.6(3.8)	1.210(15)
Our average	LQCD	Both	187.1(4.2)	227.2(3.4)	1.215(7)
Wang 15 [71] §	QCD SR		194(15)	231(16)	1.19(10)
Baker 13 [91]	QCD SR		186(14)	222 (12)	1.19(4)
Lucha 13 [92]	QCD SR		192.0(14.6)	228.0(19.8)	1.184(24)
Gelhausen 13 [72]	QCD SR		$207(^{+17}_{-9})$	$242(^{+17}_{-12})$	$1.17(^{+3}_{-4})$
Narison 12 [73]	QCD SR		206(7)	234(5)	1.14(3)
Hwang 09 [75]	LFQM		–	270.0(42.8)¶	1.32(8)

Sum rules are in good agreement with lattice, but have larger uncertainties

HQET sum rules: Bag parameters

Consider the three-point correlator:



$$K_{\tilde{Q}}(\omega_1,\omega_2) = \int d^dx_1 d^dx_2 e^{ip_1\cdot x_1 - ip_2\cdot x_2} \left\langle 0 \left| \mathrm{T}\left[\tilde{j}_+(x_2)\tilde{Q}(0)\tilde{j}_-(x_1)\right]\right| 0 \right\rangle$$

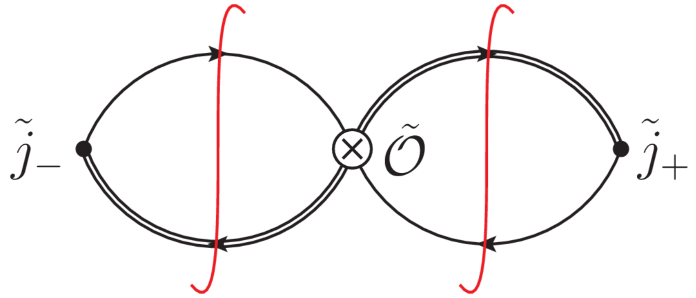
Going through the same steps one obtains the sum rule:

[Chetyrkin, Kataev, Krasulin, Pivovarov '86]

$$F^2(\mu)\langle\tilde{Q}(\mu)\rangle e^{-\frac{\bar{\Lambda}}{t_1}-\frac{\bar{\Lambda}}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} \rho_{\tilde{Q}}^{\mathrm{OPE}}(\omega_1,\omega_2)$$

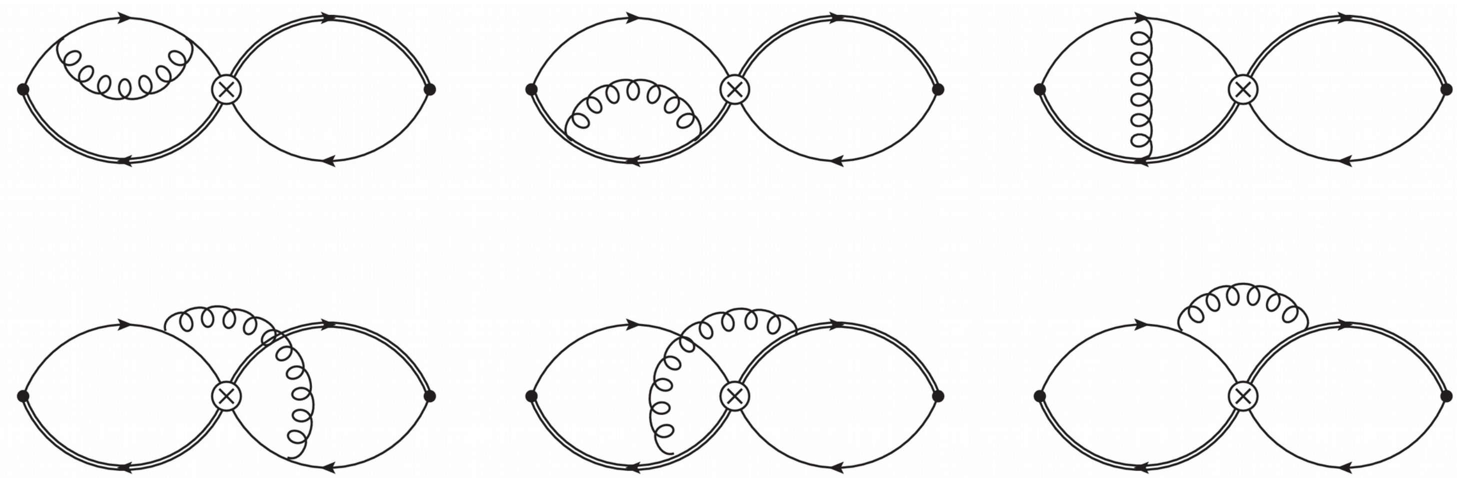
$$\rho_{\tilde{Q}}^{\mathrm{OPE}}(\omega_1,\omega_2) = \rho_{\tilde{Q}}^{\mathrm{pert}}(\omega_1,\omega_2) + \rho_{\tilde{Q}}^{\langle\bar{q}q\rangle}(\omega_1,\omega_2)\langle\bar{q}q\rangle + \rho_{\tilde{Q}}^{\langle\alpha_s G^2\rangle}(\omega_1,\omega_2)\langle\alpha_s G^2\rangle + \ldots$$

In practise we compute the correlator and then take its double discontinuity



Three-point correlator

NLO accuracy in the perturbative part requires a three-loop calculation:



Master integrals:
[Grozin, Lee '08]

Operator Q1:
[Grozin, Mannel, Klein, Pivovarov '16]

All dimension six operators:
[Kirk, Lenz, TR '17]

$$\rho^{\text{pert}}_{\tilde{Q}_i}(\omega_1,\omega_2) = A_{\tilde{Q}_i}\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \frac{\omega_1^2\omega_2^2}{\pi^4}\frac{\alpha_s}{4\pi}\, r_{\tilde{Q}_i}\left(\frac{\omega_2}{\omega_1}, L_\omega\right)$$

Non-factorizable contribution

Factorizable contribution, reproduces the vacuum saturation approximation B=1 (VSA)

$$\begin{aligned}
r_{\tilde{Q}_1}(x,L_\omega) &= 8 - \frac{a_2}{2} - \frac{8\pi^2}{3}, \\
r_{\tilde{Q}_2}(x,L_\omega) &= 25 + \frac{a_1}{2} - \frac{4\pi^2}{3} + 6L_\omega + \phi(x), \\
r_{\tilde{Q}_4}(x,L_\omega) &= 16 - \frac{a_3}{4} - \frac{4\pi^2}{3} + 3L_\omega + \frac{\phi(x)}{2}, \\
r_{\tilde{Q}_5}(x,L_\omega) &= 29 - \frac{a_3}{2} - \frac{8\pi^2}{3} + 6L_\omega + \phi(x).
\end{aligned}$$

Sum rule for Bag parameters

Formulate sum rule for deviation $\Delta B_{\tilde{Q}}(\mu) = B_{\tilde{Q}}(\mu) - 1$ from the HQET Bag parameters $\langle \tilde{Q}(\mu) \rangle = A_{\tilde{Q}}\, F^2(\mu)\, B_{\tilde{Q}}(\mu)$.

$$
\begin{aligned}
\Delta B_{\tilde{Q}_i} &= \frac{1}{A_{\tilde{Q}_i} F(\mu)^4} \int_0^{\omega_c} d\omega_1 d\omega_2 e^{\frac{\overline{\Lambda}-\omega_1}{t_1} + \frac{\overline{\Lambda}-\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1, \omega_2) \\
&= \frac{1}{A_{\tilde{Q}_i}} \frac{\int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1, \omega_2)}{\left(\int_0^{\omega_c} d\omega_1 e^{-\frac{\omega_1}{t_1}} \rho_\Pi(\omega_1)\right) \left(\int_0^{\omega_c} d\omega_2 e^{-\frac{\omega_2}{t_2}} \rho_\Pi(\omega_2)\right)}.
\end{aligned}
$$

Dispersion relation is not violated by arbitrary analytical weight function (Note of caution: Duality breaks down for pathological choices)

$$
F^4(\mu) e^{-\frac{\overline{\Lambda}}{t_1} - \frac{\overline{\Lambda}}{t_2}} w(\overline{\Lambda}, \overline{\Lambda}) = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} w(\omega_1, \omega_2) \rho_\Pi(\omega_1) \rho_\Pi(\omega_2) + \ldots .
$$

With an appropriate choice we obtain an analytic result for the pert contribution:

$$
\Delta B^{\rm pert}_{\tilde{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} r_{\tilde{Q}_i} \left(1, \log \frac{\mu_\rho^2}{4\overline{\Lambda}^2}\right).
$$

SU(3) breaking effects

The exact calculation with non-zero strange-quark mass is very challenging. We need to resort to an expansion in m_s. This yields

$$\Delta B^{\mathrm{pert}}_{\tilde{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} \Bigg\{ r^{(0)}_{\tilde{Q}_i}(1) + \frac{2m_s}{\overline{\Lambda}+m_s}\left[r^{(1)}_{\tilde{Q}_i}(1) - r^{(0)}_{\tilde{Q}_i}(1)\right] + \frac{2m_s^2}{(\overline{\Lambda}+m_s)^2}\left[r^{(2)}_{\tilde{Q}_i}(1) - 2r^{(1)}_{\tilde{Q}_i}(1) + 2r^{(0)}_{\tilde{Q}_i}(1)\right] + \dots \Bigg\},$$

where

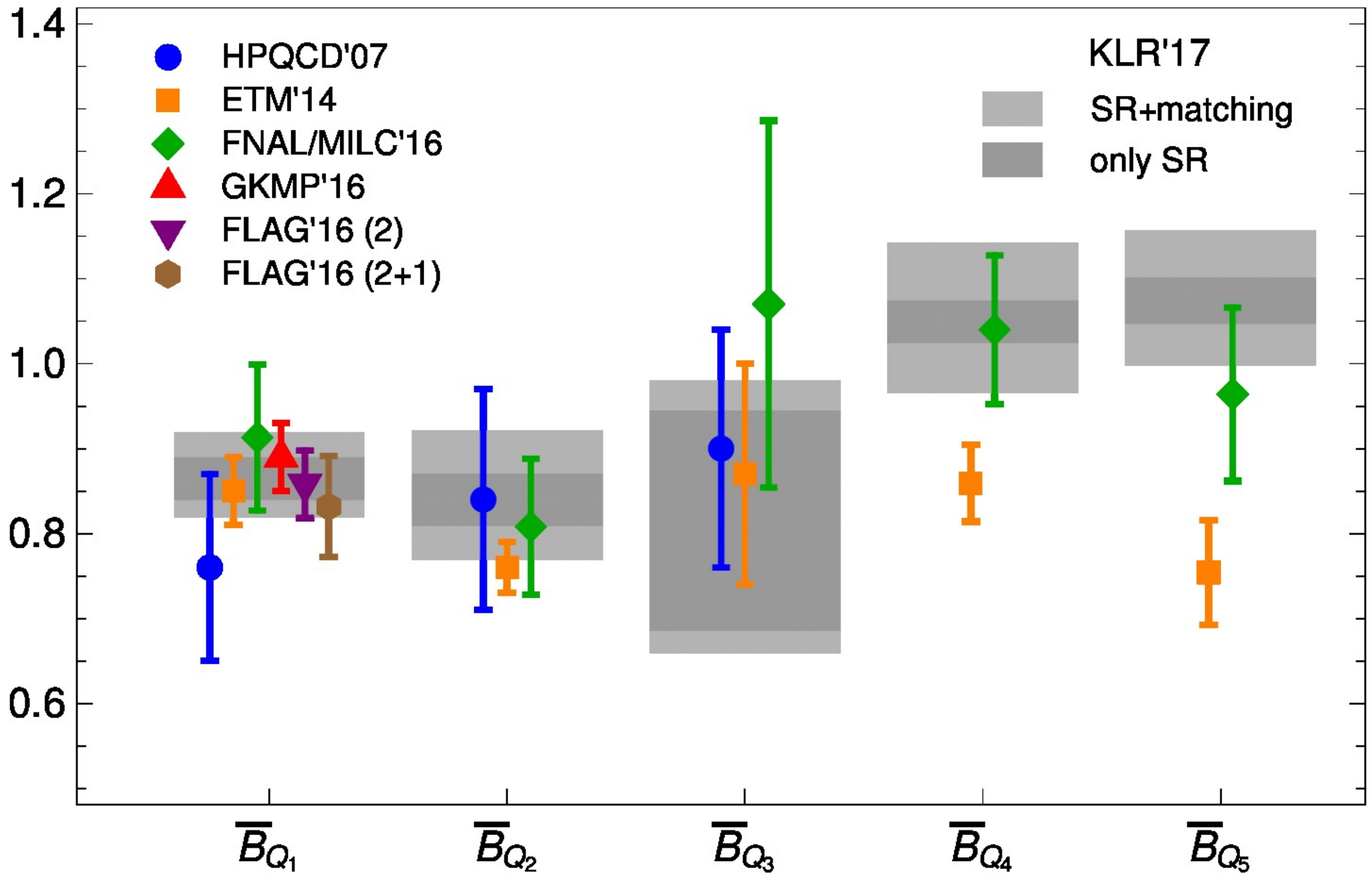
$$\Delta\rho_{\tilde{Q}_i} \equiv \frac{N_c C_F}{4}\frac{\omega_1^2\omega_2^2}{\pi^4}\frac{\alpha_s}{4\pi}\Bigg[r^{(0)}_{\tilde{Q}_i}(x, L_\omega) + \left(\frac{m_s}{\omega_1}+\frac{m_s}{\omega_2}\right) r^{(1)}_{\tilde{Q}_i}(x, L_\omega) + \left(\frac{m_s^2}{\omega_1^2}+\frac{m_s^2}{\omega_2^2}\right) r^{(2)}_{\tilde{Q}_i}(x, L_\omega) + \dots \Bigg].$$

Expanded correlator can be computed by the method of regions. Only the ‘hard’ region contributes up to quadratic order.

[King, Lenz, TR: WIP]

Matrix elements for Bd mixing

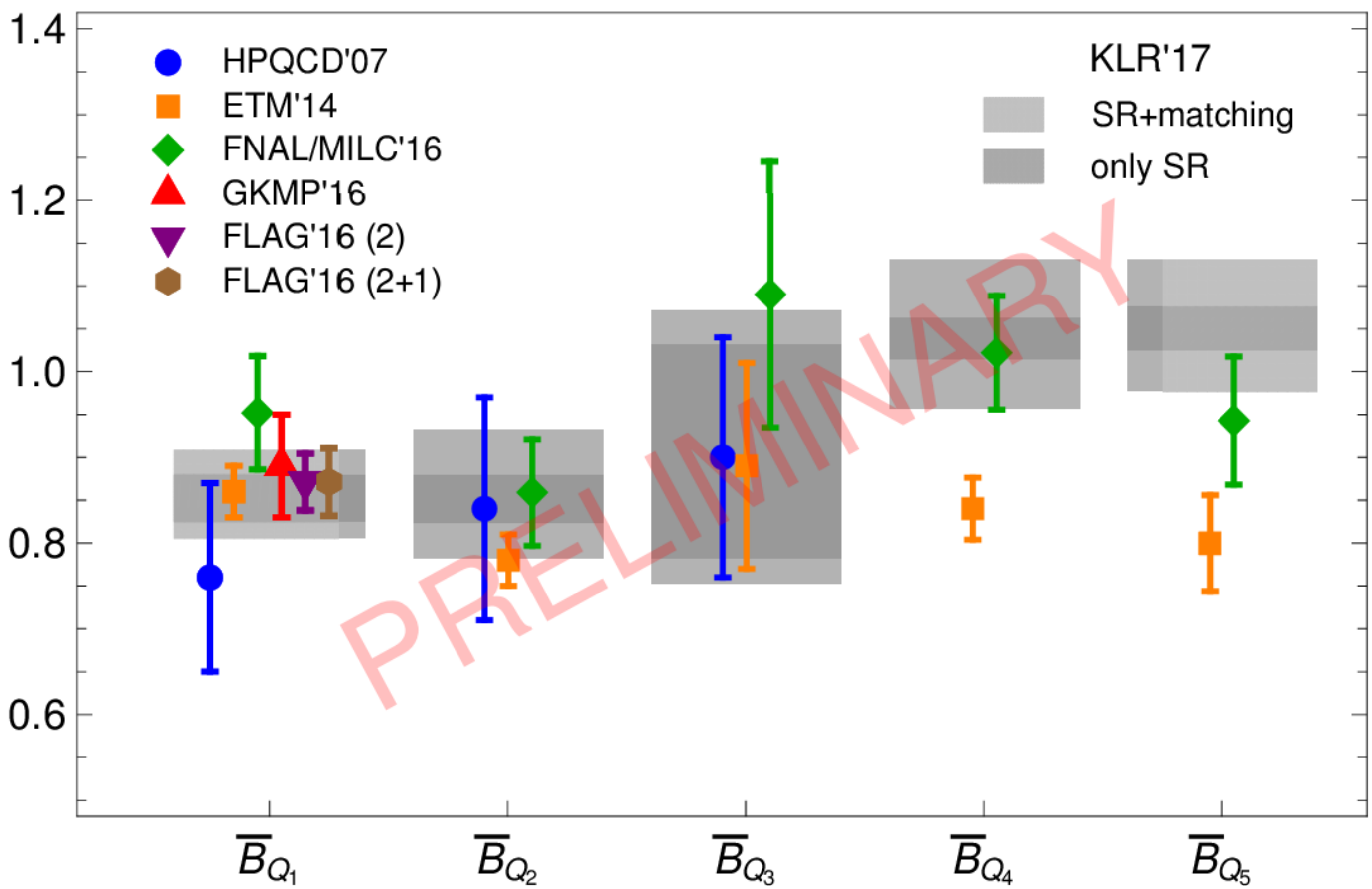
- Determine HQET Bag parameters at low scale $\mu_\rho \sim 1.5~\mathrm{GeV}$ from sum rules
- Run up to $\mu_m \sim m_b$ and match to QCD Bag parameters at NLO
- Detailed analysis performed in 1711.02100



[Kirk, Lenz, TR '17]

Matrix elements for Bs mixing

- Determine HQET Bag parameters at low scale $\mu_\rho \sim 1.5\ \mathrm{GeV}$ from sum rules
- Run up to $\mu_m \sim m_b$ and match to QCD Bag parameters at NLO
- Includes SU(3) breaking effects up to m_s^2



[King, Lenz, TR: WIP]

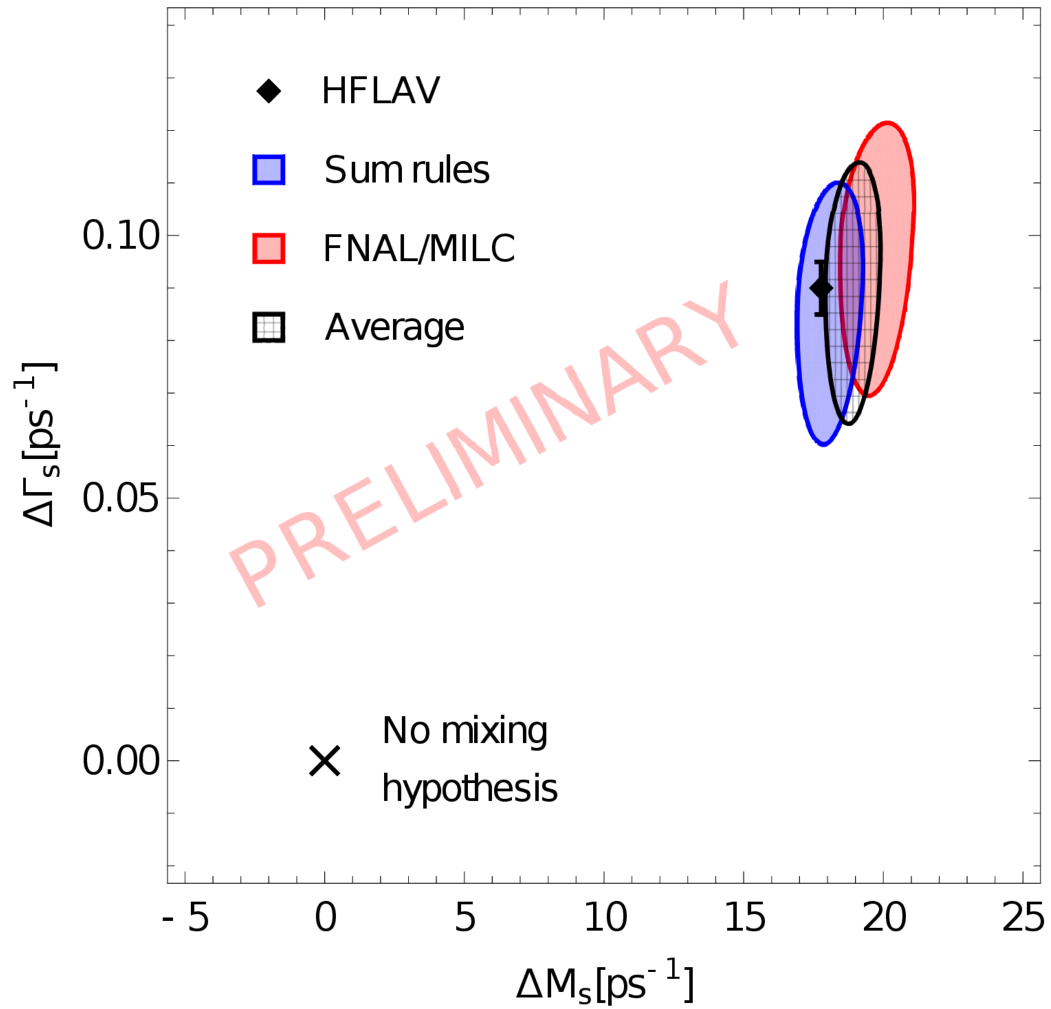
Bs-mixing observables

Update of 1711.02100 with CKM elements from CKMFitter, new decay constants from [FNAL/MILC '17] and including SU(3) breaking effects:

$$\Delta M_s^{\rm exp} = (17.757 \pm 0.021)\,{\rm ps}^{-1},$$
$$\Delta M_s^{\rm SM} = (18.1 \pm 1.1\,({\rm had.}) \pm 0.1\,({\rm scale}) {}^{+0.2}_{-0.5}\,({\rm param.}))\,{\rm ps}^{-1},$$

$$\Delta\Gamma_s^{\rm exp} = (0.090 \pm 0.005)\,{\rm ps}^{-1},$$
$$\Delta\Gamma_s^{\rm PS} = (0.089 \pm 0.020\,({\rm had.}) {}^{+0.008}_{-0.021}\,({\rm scale}) {}^{+0.001}_{-0.003}\,({\rm param.}))\,{\rm ps}^{-1},$$

$$a_{\rm sl}^{s,\,\rm exp} = (-60 \pm 280) \cdot 10^{-5},$$
$$a_{\rm sl}^{s,\,\rm PS} = (1.8 \pm 0.0\,({\rm had.}) {}^{+0.0}_{-0.1}\,({\rm scale}) \pm 0.1\,({\rm param.})) \cdot 10^{-5},$$



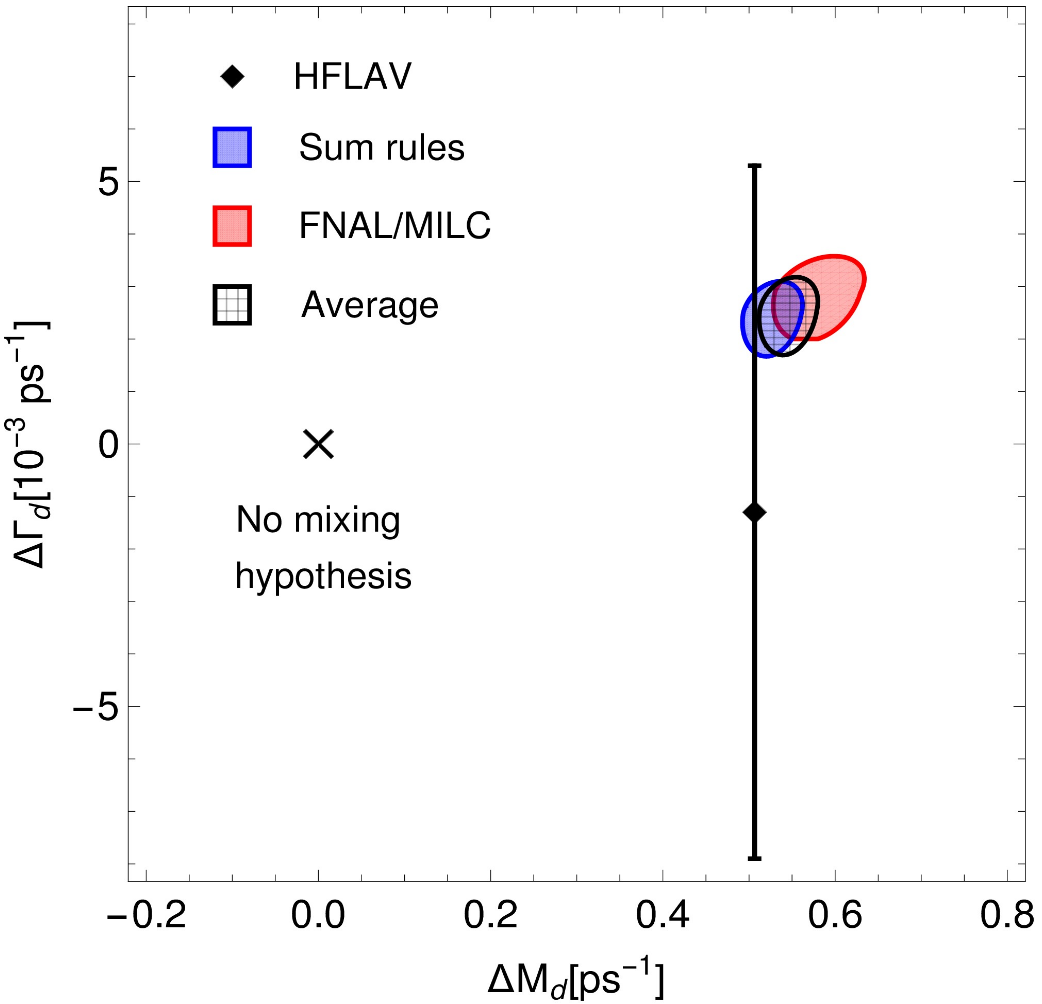
Bd-mixing observables

Update of 1711.02100 with CKM elements from CKMFitter and new decay constants from [FNAL/MILC '17]:

$$
\begin{aligned}
\Delta M_d^{\rm exp} &= (0.5065 \pm 0.0019)\,{\rm ps}^{-1},\\
\Delta M_d^{\rm SM} &= (0.53 \pm 0.03\,({\rm had.})\\
&\quad \pm 0.00\,({\rm scale})\\
&\quad {}^{+0.01}_{-0.02}\,({\rm param.}))\,{\rm ps}^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\Delta\Gamma_d^{\rm exp} &= (-1.3 \pm 6.6)\cdot 10^{-3}\,{\rm ps}^{-1},\\
\Delta\Gamma_d^{\rm PS} &= (2.5 \pm 0.6\,({\rm had.})\\
&\quad {}^{+0.2}_{-0.6}\,({\rm scale})\\
&\quad \pm 0.1\,({\rm param.}))\cdot 10^{-3}\,{\rm ps}^{-1},
\end{aligned}
$$

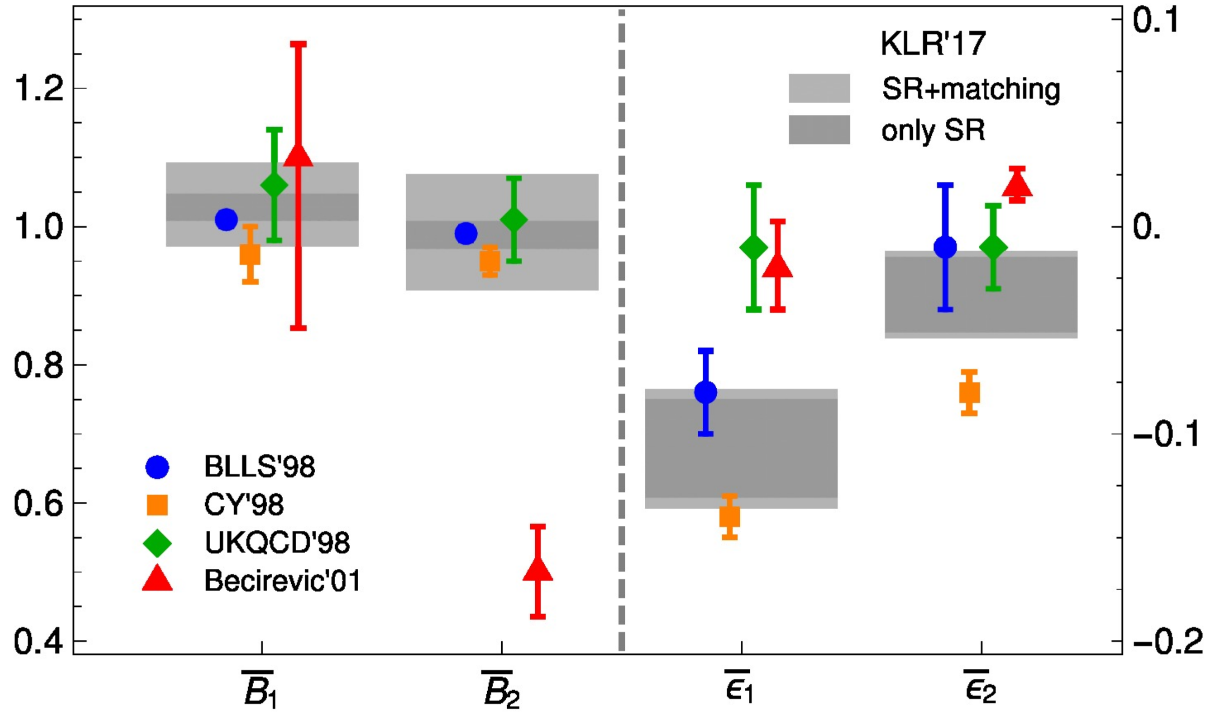
$$
\begin{aligned}
a_{\rm sl}^{d,\,\rm exp} &= (-21 \pm 17)\cdot 10^{-4},\\
a_{\rm sl}^{d,\,\rm PS} &= (-4.2 \pm 0.1\,({\rm had.})\\
&\quad {}^{+0.2}_{-0.1}\,({\rm scale})\\
&\quad \pm 0.2\,({\rm param.}))\cdot 10^{-4},
\end{aligned}
$$



B meson lifetimes

$\Delta B = 0$ Bag parameters

[Kirk, Lenz, TR '17]



$$\left.\frac{\tau(B^+)}{\tau(B^0)}\right|_{\text{exp}} = 1.076 \pm 0.004,$$
$$\left.\frac{\tau(B^+)}{\tau(B^0)}\right|_{\text{PS}} = 1.082 \pm 0.021\,(\text{had.})\,{}^{+0.000}_{-0.015}\,(\text{scale}) \pm 0.006\,(\text{param.}),$$

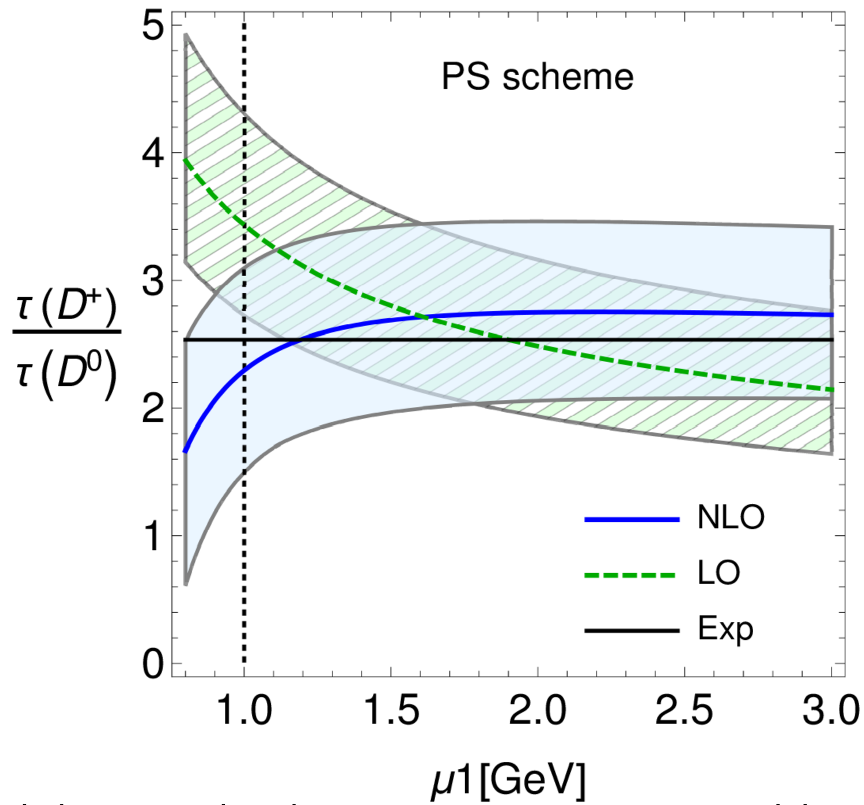
$$\left.\frac{\tau(B_s^0)}{\tau(B^0)}\right|_{\text{exp}} = 0.994 \pm 0.004,$$
$$\left.\frac{\tau(B_s^0)}{\tau(B^0)}\right|_{\overline{\text{MS}}} = 0.9994 \pm 0.0014\,(\text{had.}) \pm 0.0006\,(\text{scale}) \pm 0.0020\,(1/m_b^4),$$

D lifetimes as test of HQE

HQE even provides good description of lifetimes in charm sector:

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\mathrm{exp}} = 2.536 \pm 0.019\,, \qquad \left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\mathrm{HQE}} = 2.7^{+0.7}_{-0.8}\,,$$ [Kirk, Lenz, TR '17]

$$\left.\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right|_{\mathrm{exp}} = 1.292 \pm 0.019\,, \qquad \left.\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right|_{\mathrm{HQE}} = 1.19 \pm 0.13\,.$$ [Lenz, TR '13]

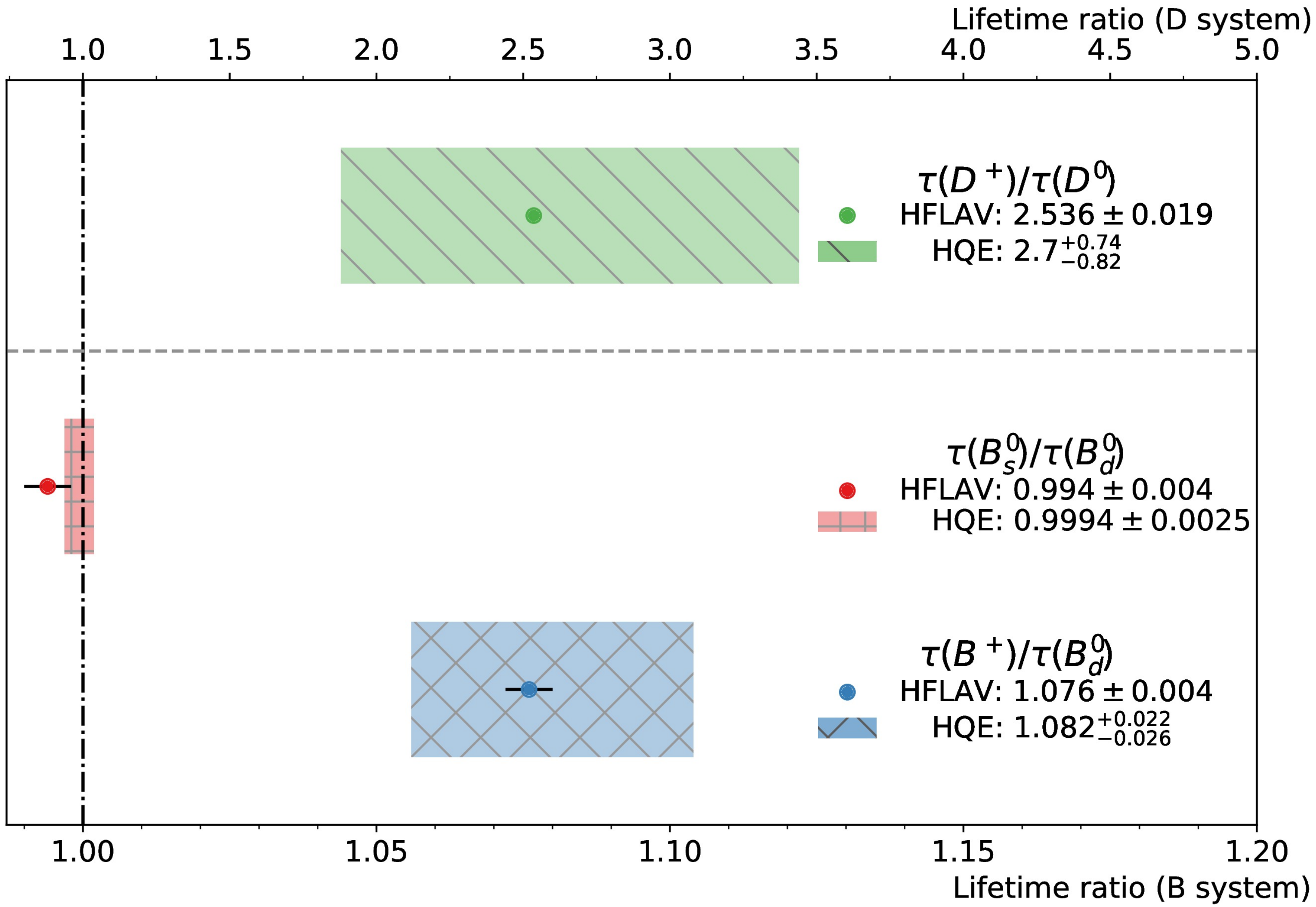


Good convergence:

NLO QCD +28%, 1/mc -34%.

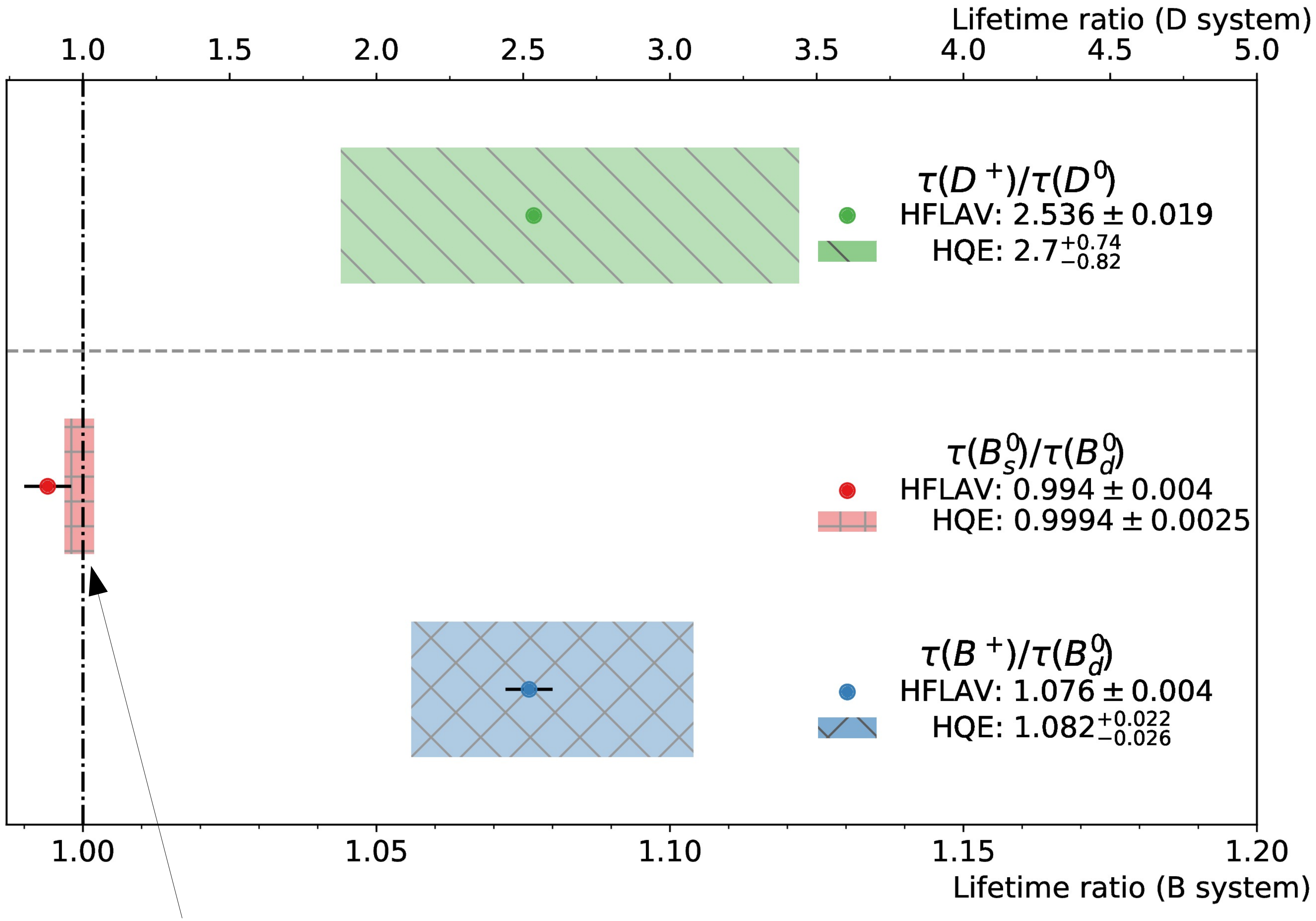
Good behaviour under scale variation above about 1 GeV.

Overview of lifetime ratios from sum rules



[Kirk, Lenz, TR '17]

Overview of lifetime ratios from sum rules



SU(3) breaking effects in matrix elements can also be determined with the strategy of ms-expansion

[Kirk, Lenz, TR '17]

Conclusions & outlook

- Sum rules provide highly competitive alternative to lattice simulations for the matrix elements of 4-quark operators and truly independent comparisons.
- The HQE is in terrific shape. Lifetimes even look promising in the charm sector.
- Mixing gives strong constraints on models that are frequently invoked to explain the current 'anomalies'. [cf. talk by L. Di Luzio in WG4 Tue 10:20]
- First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice would be interesting.

SINCE YEARS OF BEGGING DID NOT HELP - IT'S TIME TO PROVOKE

Lifetimes are too heavy for lattice physicists!

The strongest lattice researcher alive

Arbitrary sum rule researcher

Matrix elements for lifetimes of HEAVY mesons

[Lenz Implications '17]

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- Mixing gives strong constraints on models that are frequently invoked to explain the current 'anomalies'. [cf. talk by L. Di Luzio in WG4 Tue 10:20]
- First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice would be interesting.
- NNLO QCD-HQET matching calculations can significantly decrease uncertainties for dimension-six operators. Q1: [Grozin, Mannel, Pivovarev '17,18, cf talk by T. Mannel in WG4 Tue 9:55]
- Uncertainties in decay rate difference and lifetimes can be reduced considerably by a sum rule determination of the dimension seven matrix elements.

Heavy quark expansion in charm?

B-physics: HQE is well established approach, $\Lambda/m_b \sim 0.2 \ll 1$

D-physics: HQE commonly dismissed, $\Lambda/m_c \sim 0.2\, m_b/m_c \sim 0.7 \approx 1$

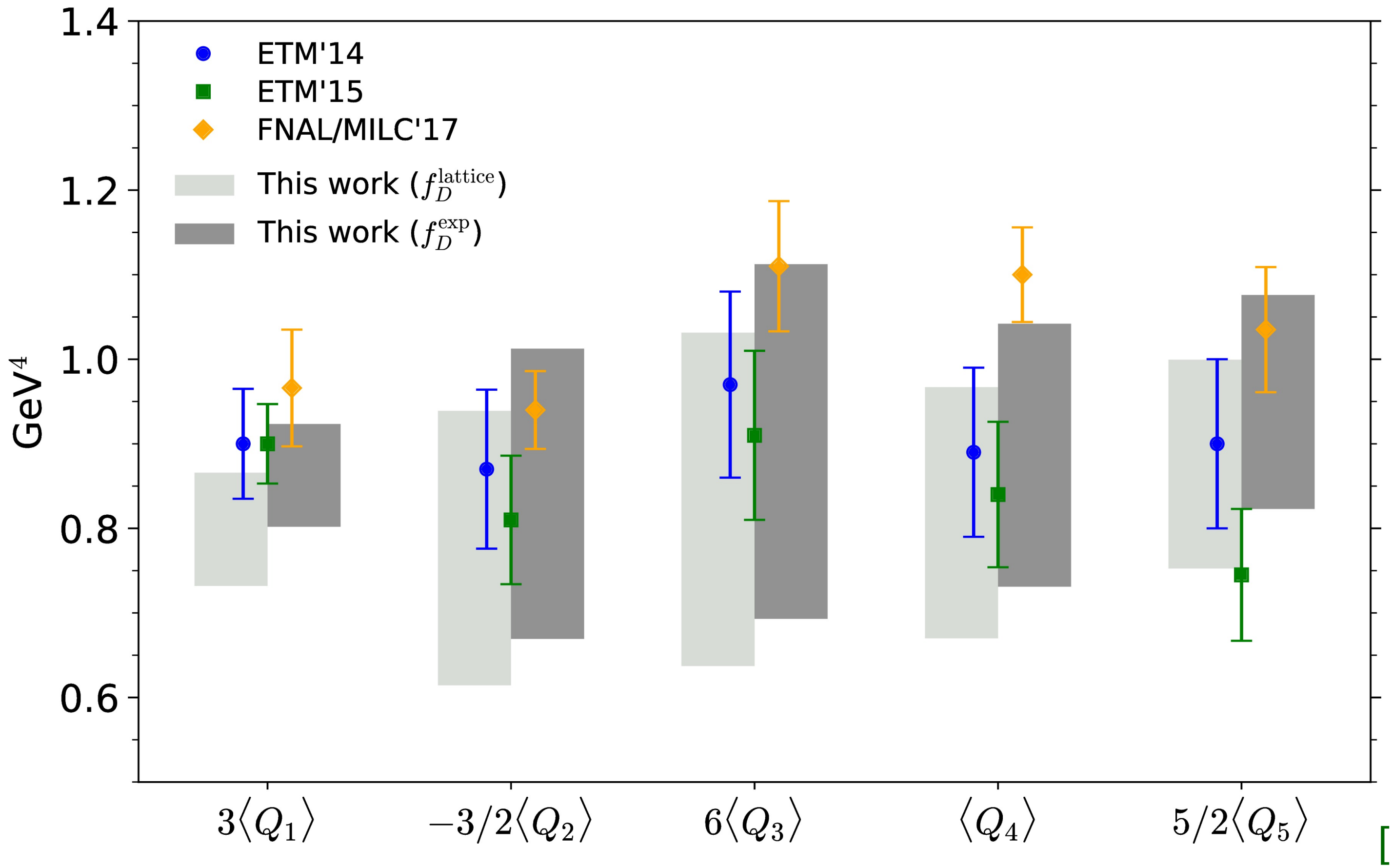
BUT: HQE is really an expansion in $\Lambda/\mathrm{momentum\ release}$

- $\Delta\Gamma_s$ dominated by $\mathrm{D}_s^{(*)+} D_s^{(*)-}$ final state, momentum release $\sim 3.5\ \mathrm{GeV}$
- D decays dominated by $\mathrm{K}\pi^{(1-3)}$ final state, momentum release $\sim 1.7\ \mathrm{GeV}$
- expected expansion parameter is of the order 0.4

Small enough for convergence?

Shut up and calculate!

Matrix elements



[Kirk, Lenz, TR '17]

- Good agreement with lattice (using lattice results for the decay constant)
- Larger uncertainties due to lower matching scale
- Also: first determination of $\Delta C = 0$ matrix elements in 1711.02100

Uncertainties

$\Delta B = 2$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_b$	μ_m	a_i
$\overline{B}_{Q_1}$	${}^{+0.001}_{-0.002}$	± 0.018	± 0.004	${}^{+0.011}_{-0.022}$	± 0.010	${}^{+0.045}_{-0.039}$	${}^{+0.007}_{-0.007}$
$\overline{B}_{Q_2}$	${}^{+0.014}_{-0.017}$	∓ 0.020	± 0.004	${}^{+0.012}_{-0.019}$	± 0.010	${}^{+0.071}_{-0.062}$	${}^{+0.015}_{-0.015}$
$\overline{B}_{Q_3}$	${}^{+0.060}_{-0.074}$	± 0.107	± 0.023	${}^{+0.016}_{-0.008}$	± 0.010	${}^{+0.086}_{-0.069}$	${}^{+0.053}_{-0.052}$
$\overline{B}_{Q_4}$	${}^{+0.007}_{-0.006}$	± 0.021	± 0.011	${}^{+0.003}_{-0.003}$	± 0.010	${}^{+0.088}_{-0.079}$	${}^{+0.005}_{-0.006}$
$\overline{B}_{Q_5}$	${}^{+0.019}_{-0.015}$	± 0.018	± 0.009	${}^{+0.004}_{-0.006}$	± 0.010	${}^{+0.077}_{-0.068}$	${}^{+0.012}_{-0.012}$

$\Delta B = 0$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_b$	μ_m	a_i
$\overline{B}_1$	${}^{+0.003}_{-0.002}$	± 0.019	± 0.002	${}^{+0.002}_{-0.002}$	± 0.010	${}^{+0.060}_{-0.052}$	${}^{+0.002}_{-0.003}$
$\overline{B}_2$	${}^{+0.001}_{-0.001}$	∓ 0.020	± 0.002	${}^{+0.000}_{-0.001}$	± 0.010	${}^{+0.084}_{-0.076}$	${}^{+0.001}_{-0.002}$
$\overline{\epsilon}_1$	${}^{+0.006}_{-0.007}$	± 0.022	± 0.003	${}^{+0.003}_{-0.003}$	± 0.010	${}^{+0.010}_{-0.012}$	${}^{+0.006}_{-0.007}$
$\overline{\epsilon}_2$	${}^{+0.005}_{-0.006}$	± 0.017	± 0.003	${}^{+0.002}_{-0.001}$	± 0.010	${}^{+0.001}_{-0.002}$	${}^{+0.003}_{-0.004}$

Uncertainties

$\Delta C = 2$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_c$	μ_m	a_i
$\overline{B}_{Q_1}$	${}^{+0.001}_{-0.002}$	± 0.013	± 0.003	${}^{+0.009}_{-0.021}$	± 0.030	${}^{+0.039}_{-0.021}$	± 0.003
$\overline{B}_{Q_2}$	${}^{+0.011}_{-0.014}$	∓ 0.015	± 0.003	${}^{+0.010}_{-0.016}$	± 0.030	${}^{+0.092}_{-0.050}$	± 0.012
$\overline{B}_{Q_3}$	${}^{+0.037}_{-0.045}$	± 0.059	± 0.013	${}^{+0.016}_{-0.016}$	± 0.030	${}^{+0.116}_{-0.059}$	± 0.016
$\overline{B}_{Q_4}$	${}^{+0.006}_{-0.005}$	± 0.017	± 0.009	${}^{+0.003}_{-0.003}$	± 0.030	${}^{+0.131}_{-0.071}$	± 0.004
$\overline{B}_{Q_5}$	${}^{+0.014}_{-0.012}$	± 0.014	± 0.007	${}^{+0.004}_{-0.005}$	± 0.030	${}^{+0.127}_{-0.069}$	± 0.004

$\Delta C = 0$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_c$	μ_m	a_i
$\overline{B}_1$	${}^{+0.004}_{-0.003}$	± 0.017	± 0.002	${}^{+0.002}_{-0.002}$	± 0.030	${}^{+0.068}_{-0.037}$	${}^{+0.003}_{-0.005}$
$\overline{B}_2$	${}^{+0.001}_{-0.000}$	∓ 0.015	± 0.001	${}^{+0.000}_{-0.000}$	± 0.030	${}^{+0.120}_{-0.065}$	${}^{+0.000}_{-0.001}$
$\overline{\epsilon}_1$	${}^{+0.007}_{-0.008}$	± 0.024	± 0.004	${}^{+0.003}_{-0.004}$	± 0.030	${}^{+0.012}_{-0.022}$	${}^{+0.006}_{-0.008}$
$\overline{\epsilon}_2$	${}^{+0.003}_{-0.004}$	± 0.011	± 0.002	${}^{+0.001}_{-0.001}$	± 0.030	${}^{+0.000}_{-0.000}$	${}^{+0.001}_{-0.002}$

Uncertainties

	ΔM_s^{SM} [ps^{-1}]	$\Delta\Gamma_s^{\mathrm{PS}}$ [ps^{-1}]	$a_{\mathrm{sl}}^{s,\,\mathrm{PS}}$ [10^{-5}]
$\overline{B}_{Q_1}$	± 1.1	± 0.005	± 0.01
$\overline{B}_{Q_3}$	± 0.0	± 0.005	± 0.01
$\overline{B}_{R_0}$	± 0.0	± 0.003	± 0.00
$\overline{B}_{R_1}$	± 0.0	± 0.000	± 0.00
$\overline{B}_{R_1'}$	± 0.0	± 0.000	± 0.00
$\overline{B}_{R_2}$	± 0.0	± 0.016	± 0.00
$\overline{B}_{R_3}$	± 0.0	± 0.001	± 0.02
$\overline{B}_{R_3'}$	± 0.0	± 0.000	± 0.05
f_{B_s}	± 0.5	± 0.002	± 0.00
μ_1	± 0.0	${}^{+0.007}_{-0.018}$	${}^{+0.04}_{-0.08}$
μ_2	± 0.1	${}^{+0.000}_{-0.002}$	± 0.01
m_b	± 0.0	${}^{+0.000}_{-0.001}$	± 0.01
m_c	± 0.0	${}^{+0.000}_{-0.001}$	± 0.06
α_s	± 0.0	± 0.000	± 0.04
CKM	${}^{+1.4}_{-1.3}$	± 0.006	${}^{+0.21}_{-0.22}$

	ΔM_d^{SM} [ps^{-1}]	$\Delta\Gamma_d^{\mathrm{PS}}$ [10^{-3} ps^{-1}]	$a_{\mathrm{sl}}^{d,\,\mathrm{PS}}$ [10^{-4}]
$\overline{B}_{Q_1}$	${}^{+0.04}_{-0.03}$	± 0.16	± 0.02
$\overline{B}_{Q_3}$	± 0.00	${}^{+0.17}_{-0.16}$	± 0.03
$\overline{B}_{R_0}$	± 0.00	± 0.11	± 0.01
$\overline{B}_{R_1}$	± 0.00	± 0.01	± 0.00
$\overline{B}_{R_1'}$	± 0.00	± 0.01	± 0.00
$\overline{B}_{R_2}$	± 0.00	± 0.54	± 0.00
$\overline{B}_{R_3}$	± 0.00	± 0.00	± 0.04
$\overline{B}_{R_3'}$	± 0.00	± 0.01	± 0.09
f_B	± 0.03	± 0.11	± 0.00
μ_1	± 0.00	${}^{+0.24}_{-0.62}$	${}^{+0.17}_{-0.07}$
μ_2	± 0.00	${}^{+0.00}_{-0.08}$	${}^{+0.01}_{-0.03}$
m_b	± 0.00	${}^{+0.01}_{-0.03}$	${}^{+0.01}_{-0.03}$
m_c	± 0.00	${}^{+0.01}_{-0.02}$	± 0.13
α_s	± 0.00	± 0.01	± 0.08
CKM	± 0.08	${}^{+0.38}_{-0.37}$	${}^{+0.47}_{-0.44}$

Uncertainties

$\overline{B}_1$	$\overline{B}_2$	$\bar{\epsilon}_1$	$\bar{\epsilon}_2$	ρ_3	ρ_4	σ_3	σ_4
± 0.002	± 0.000	${}^{+0.016}_{-0.015}$	± 0.004	± 0.001	± 0.000	± 0.013	± 0.000
f_B	μ_1	μ_0	m_b	m_c	α_s	CKM	
${}^{+0.004}_{-0.003}$	${}^{+0.000}_{-0.013}$	${}^{+0.000}_{-0.006}$	${}^{+0.000}_{-0.001}$	± 0.000	± 0.002	± 0.006	

Table 8: Individual errors for the ratio $\tau(B^+)/\tau(B^0)$ in the PS mass scheme.

$\overline{B}_1$	$\overline{B}_2$	$\bar{\epsilon}_1$	$\bar{\epsilon}_2$	ρ_3	ρ_4	σ_3	σ_4
${}^{+0.07}_{-0.05}$	± 0.00	${}^{+0.52}_{-0.47}$	± 0.017	± 0.05	± 0.00	± 0.46	± 0.00
f_B	μ_1	μ_0	m_c	m_s	α_s	CKM	
± 0.08	${}^{+0.07}_{-0.40}$	${}^{+0.08}_{-0.21}$	± 0.08	± 0.00	${}^{+0.07}_{0.06}$	± 0.00	

Table 9: Individual errors for the ratio $\tau(D^+)/\tau(D^0)$ in the PS mass scheme.