Inputs for $\gamma/\phi_3$ from charm decays

Eva Gersabeck

on behalf of the BESIII collaboration

with input from Cleo-c, BELLEII and LHCb

CKM 2018, 17-21 September 2018, Heidelberg
The CKM angle $\gamma$

CPV is an interference effect of two amplitudes: $A_1$ and $A_2$

$$|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\delta_s + \varphi_{CPV})$$

$\delta_s$ strong phase between $A_1$ and $A_2$  \hspace{1cm} $\varphi_{CPV}$ weak CPV phase

SM benchmark: only CKM angle accessible at tree level
(b → u and b → c transitions in $B \rightarrow DK$ decays)

$D^0$ and $\bar{D}^0$ decays to a common final state: interference

weak phase $\gamma = \arg(\frac{-V_{ub}V_{ud}^*}{V_{cd}V_{cd}^*})$
The CKM angle $\gamma$

- **SM benchmark:** sensitive to New Physics effects

- Theory uncertainty on $\gamma$ is very small $\delta \gamma / \gamma \approx O(10^{-7})$

- $\gamma$ can probe for new physics at extremely high-energy scales $\sim O(10^2-10^3)$

- NP can lead to a sizeable 4° effect
  - **Over-constrain the triangle to test for NP**

$$\gamma = (73.5_{-5.0}^{+4.3})^\circ$$

**Direct (WA)**

$$\Delta m_d \text{ and } \Delta m_s$$

**Direct (LHCb)**

$$\gamma = (74.0_{-5.8}^{+5.0})^\circ$$

**Indirect**

$$\gamma = (65.3_{-2.5}^{+1.0})^\circ$$

**Pre-LHCb**

$$\gamma = (73_{-25}^{+22})^\circ$$
Future precision on $\gamma$

Combined sensitivity with 50 ab$^{-1}$: 1.6°

Projected sensitivity for the LHCb $\gamma$ combination

* external input uncertainty of 2°

H. Atmacan - ICHEP18

arXiv:1808.08865v2

2029

Eva Gersabeck, Inputs for $\gamma/\phi_3$ from charm decays
The methods for measuring $\gamma$

- Measure CPV observables in many D modes in $B \to D K$ decays (but not only)

- Intermediate states include $D \to hh'$, $D \to K_{S}hh$, $D \to K3\pi$, etc.

- Different methods depending on the D decay final state used:
  - **GLW**: CP eigenstates e.g. $D \to KK$, $D \to \pi\pi$
  - **ADS**: Cabibbo favoured or doubly suppressed e.g. $D \to K\pi$
  - **GGSZ**: three body final state e.g. $D \to K_{S}hh$

see the talk of Alberto Correa Dos Reis
γ with multibody decays

- D can decay either to two-body or multibody final state
- Multibody decays can be analysed with **model-dependent** and **model-independent** techniques
- **Modelling of the phase space**: irreducible systematics due to the choice of the model
- **Model-independent techniques** require external input for the hadronic parameters of the D decays (e.g. strong phases, coherence factors etc.)
- → Hence the need for quantum-correlated (Q.C.) charm threshold data: CLEO-c (0.8/fb) and BESIII (2.9/fb on tape, more to come)
- Over-constraining of the parameters also possible

Experiments at charm threshold

Stopped taking data in 2008*
*several recent legacy-data publications

Still taking data
A closer look at BESIII

hermetic

barrel \(|\cos \theta| < 0.83\)

endcap \(0.85 < |\cos \theta| < 0.93\)

@ threshold = full kinematic constraint of the decays

Excellent for neutral and invisible particles

Optimised for flavour physics in the tau-charm region

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Physics in the tau-charm region

**BESIII data sets**

- Hadron form factors
- $\Upsilon(2175)$
- $Z_s$ states?
- QCD tests
- Light hadron spectroscopy
- Glueballs, hybrids, exotics
- Rare decays
- $\tau$ physics
- XYZ
- $D$ and $D_s$ physics
- $f_D, f_{D_s}$; mixing, CPV
- Charmed baryons

**XYZ scan**
4190-4280
3.8 fb$^{-1}$

$J/\psi$
$1.3 \times 10^9$

$\psi'$
$0.5 \times 10^9$

$\psi''$
$2.9 \text{ fb}^{-1}$

$\Upsilon(2S)$

+ $\sim$5 billion more in 2018
Charm quantum correlated data at 3.77 GeV

- The production mechanism leads to a coherent state
- \( e^+e^- \rightarrow \Psi(3770) \rightarrow 1/\sqrt{2}(|D_0\rangle|\bar{D}_0\rangle - |\bar{D}_0\rangle|D_0\rangle) \)
- \( e^+e^- \rightarrow \Psi(3770) \rightarrow \bar{D}^0D^0 \)
- \( D_{CP\pm} = [D^0 \pm \bar{D}^0]/\sqrt{2} \)
- no energy for one single additional pion
- Unique access to relative strong phases, CP content
- Use flavour tags: e.g. \( D^0 \rightarrow K^-\pi^+; D^0 \rightarrow K^-\mu^+\nu \)
- Use CP tags: e.g. \( D^0 \rightarrow K^-K^+ \)
- \( \sigma(c\bar{c}) = 3 \text{ nb}, N(c\bar{c}) \sim O(10^7) \)
- Largest data sample of quantum entangled charm particles 2.93 fb\(^{-1}\)
Strong phase in $D^0 \rightarrow K\pi$ decays

- Strong $K\pi$ scattering phase shift, $\delta_{K\pi}$ accessible at BESIII
- Most precise direct measurement


$$\cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01.$$  

- Contributes to extracting the charm parameters $x$ and $y$ from $x'$ and $y'$
- Can contribute to a more precise determination of $\gamma$
• **GLW-like**: quasi-CP eigenstates e.g. $D \rightarrow KK\pi^0$: need CP content $F^+$

$$A_{CP} = \frac{\Gamma(B^- \rightarrow D_{CP}^0 K^-) - \Gamma(B^+ \rightarrow D_{CP}^0 K^+)}{\Gamma(B^- \rightarrow D_{CP}^0 K^-) + \Gamma(B^+ \rightarrow D_{CP}^0 K^+)} = \frac{\pm 2r_B(2F^+ + 1)\sin(\delta_B)\sin(\gamma)}{1 + r_B^2 + 2r_B(2F^+ + 1)\cos(\delta_B)\cos(\gamma)}$$

$$R_{CP} = \frac{\Gamma(B^- \rightarrow D_{CP}^0 K^-) - \Gamma(B^+ \rightarrow D_{CP}^0 K^+)}{\Gamma(B^- \rightarrow D^0 K^-) + \Gamma(B^+ \rightarrow D^0 K^+)} = 1 + r_B^2 + 2r_B(2F^+ + 1)\cos(\delta_B)\cos(\gamma)$$

• **ADS**: involving multibody doubly suppressed e.g. $D \rightarrow K\pi\pi^0$: need dilution from interference $k_D$ (and $r_D, \delta_D$)

$$A_{ADS} = \frac{\Gamma(B^- \rightarrow [K^+\pi^-]D_K^-) - \Gamma(B^+ \rightarrow [K^-\pi^+]D_K^+)}{\Gamma(B^- \rightarrow [K^+\pi^-]D_K^-) + \Gamma(B^+ \rightarrow [K^-\pi^+]D_K^+)} = \frac{2r_Br_Dk_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2r_Br_Dk_D \cos(\delta_B + \delta_D)\cos(\gamma)}$$

$$R_{ADS} = \frac{\Gamma(B^- \rightarrow [K^+\pi^-]D_K^-) - \Gamma(B^+ \rightarrow [K^-\pi^+]D_K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+]D_K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-]D_K^+)} = \frac{r_B^2 + r_D^2 + 2r_Br_Dk_D \cos(\delta_B + \delta_D)\cos(\gamma)}{r_B^2 + r_D^2 + 2r_Br_Dk_D \cos(\delta_B + \delta_D)\cos(\gamma)}$$

• **GGSZ**: three body final state e.g. $D \rightarrow K_S\pi\pi$: look at partial rate as function of the Dalitz position: need strong phase input

$$d\Gamma_{B^\pm}(x) = A_{(\pm,\mp)}^2 + r_B^2A_{(\mp,\pm)}^2 + 2A_{(\pm,\mp)}A_{(\mp,\pm)}[r_B \cos(\delta_B \pm \gamma) \cos(\delta_{D(\pm,\mp)}) + r_B \sin(\delta_B \pm \gamma) \sin(\delta_{D(\pm,\mp)})]$$

where $x^\pm, c(\pm,\mp), y^\pm, s(\pm,\mp)$ are defined.
Measure strong phases

e.g. probe strong-phase distribution of multibody decays...

\[ D^{**} \rightarrow D^0 \pi^+ \text{ or } D^0 \rightarrow K \pi \]

\[ D^0 \rightarrow K_S \pi^+ \pi^- \]

\[ \psi'' \rightarrow D_a D_b \]

\[ D_a \rightarrow K K \quad \text{eg. CP}^+ \]

\[ D_b \rightarrow K_S \pi^+ \pi^- \]

Flavour tagged Distribution \( \sim |D^0|^2 \) or \( |D^0|^2 \)

Unique access to relative strong phases

- Use CP tags: reconstruct one meson as a CP eigenstate
- Project the other meson as a superposition of \( D^0 \) and \( \bar{D}^0 \)

\[ \text{CP-tagged} \sim |D^0|^2 + |D^0|^2 \pm 2 D^0 \bar{D}^0 \cos \delta \]
Definition of $c_i$ and $s_i$

- Phase-space dependent amplitudes for
  - $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays: $A$
  - $\bar{D}^0 \rightarrow K_S^0 \pi^- \pi^+$ decays: $B$

- Fraction of $D^0$ events in bin $i$ $\rightarrow T_i = \int_i |A|^2 dm_+^2 dm_-^2$

- Interference terms between amplitudes $A$ and $B$

$$c_i \equiv \frac{1}{\sqrt{T_i T_i}} \int_i |A^*||B| \cos(\Delta \delta_D) dm_+^2 dm_-^2$$

$$s_i \equiv \frac{1}{\sqrt{T_i T_i}} \int_i |A^*||B| \sin(\Delta \delta_D) dm_+^2 dm_-^2$$

\[\text{strong phase difference}\]
Measure strong phases

e.g. probe strong-phase distribution of multibody decays...

$D^{*+} \rightarrow D^0 \pi^+$ or $D^0 \rightarrow K \pi \nu$

$D^0 \rightarrow K_s \pi^+ \pi^-$

$\psi'' \rightarrow D_a D_b$

$D_a \rightarrow K K$

$D_b \rightarrow K_s \pi^+ \pi^-$

Flavour tagged Distribution $\sim |D^0|^2$ or $|\overline{D^0}|^2$

$D_{CP}$

CP-tagged $\sim |D^0|^2 + |\overline{D^0}|^2 \pm 2 D^0 \overline{D^0}$

$\cos \delta$

2.93 fb$^{-1}$ @ 3.773 GeV

Dan Ambrose, APS 2014

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QC status at BESIII (strong phases, CP content)

- $D^0 \rightarrow K_S \pi \pi^+$ strong phase differences $c_i$ and $s_i$

<table>
<thead>
<tr>
<th>Bins</th>
<th>BES-III $c_i$</th>
<th>CLEO-c $c_i$</th>
<th>BES-III $s_i$</th>
<th>CLEO-c $s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.066 ± 0.066</td>
<td>-0.009 ± 0.088</td>
<td>-0.843 ± 0.119</td>
<td>-0.438 ± 0.184</td>
</tr>
<tr>
<td>2</td>
<td>0.796 ± 0.061</td>
<td>0.900 ± 0.106</td>
<td>-0.357 ± 0.148</td>
<td>-0.490 ± 0.295</td>
</tr>
<tr>
<td>3</td>
<td>0.361 ± 0.125</td>
<td>0.292 ± 0.168</td>
<td>-0.962 ± 0.258</td>
<td>-1.243 ± 0.341</td>
</tr>
<tr>
<td>4</td>
<td>-0.985 ± 0.017</td>
<td>-0.890 ± 0.041</td>
<td>-0.090 ± 0.093</td>
<td>-0.119 ± 0.141</td>
</tr>
<tr>
<td>5</td>
<td>-0.278 ± 0.056</td>
<td>-0.208 ± 0.085</td>
<td>0.778 ± 0.092</td>
<td>0.853 ± 0.123</td>
</tr>
<tr>
<td>6</td>
<td>0.267 ± 0.119</td>
<td>0.258 ± 0.155</td>
<td>0.635 ± 0.293</td>
<td>0.984 ± 0.357</td>
</tr>
<tr>
<td>7</td>
<td>0.902 ± 0.017</td>
<td>0.869 ± 0.034</td>
<td>-0.018 ± 0.103</td>
<td>-0.041 ± 0.132</td>
</tr>
<tr>
<td>8</td>
<td>0.888 ± 0.036</td>
<td>0.798 ± 0.070</td>
<td>-0.301 ± 0.140</td>
<td>-0.107 ± 0.240</td>
</tr>
</tbody>
</table>

CLEO-c results can be found in Phys.Rev. D82 (2010) 112006

- Check the talk of P. Weidenkaff tomorrow as well

- Analyses of $D^0 \rightarrow K_L \pi \pi^+$ and $K_S K^- K^+$ also underway
Binning optimisation

- Optimise the ratio of the statistical sensitivity wrt the unbinned case

\[ Q^2|_{x=y=0} = \frac{\sum (c_i^2 + s_i^2)N_i}{\sum N_i} \]

- **Optimal**: Split the phase space in 16 bins with similar strong phase differences

- Bins symmetric around \( m^2(\pi\pi^+) \) axis

- Binned measurements provided by Cleo-c for various amplitude models

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the choice of the model doesn’t bias the result; it can affect the sensitivity

<table>
<thead>
<tr>
<th>Binning</th>
<th>( Q )</th>
<th>((K_S^0\pi^+\pi^-)^2)-stat. err.</th>
<th>( \sigma_x )</th>
<th>( \sigma_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{N} = 8 ) (uniform)</td>
<td>0.57</td>
<td>0.015</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>( \mathcal{N} = 8 ) (( \Delta\delta_D ))</td>
<td>0.79</td>
<td>0.005</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>( \mathcal{N} = 8 ) (optimal)</td>
<td>0.89</td>
<td>0.008</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>( \mathcal{N} = 19 ) (uniform)</td>
<td>0.69</td>
<td>0.013</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>( \mathcal{N} = 20 ) (( \Delta\delta_D ))</td>
<td>0.82</td>
<td>0.004</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>( \mathcal{N} = 20 ) (optimal)</td>
<td>0.96</td>
<td>0.004</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>Unbinned</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Importance of this measurement

- Most precise determination of $\gamma$ from a single channel from $B \rightarrow DK$ with $D \rightarrow K_{shh}$
  
  $\gamma = (80.0^{+10.0}_{-9.0})^\circ$  \textit{JHEP 08 176}

- Uncertainty due to strong-phase inputs (CLEO-c) $4^\circ >$ uncertainty due to experimental systematic effects $2^\circ$

- Input important for $B \rightarrow DK\pi$ with $D \rightarrow K_{shh}$, precision of $2^\circ$ achievable after the upgrade \textit{Craik et al.,arXiv:1712.0853}

\[ \sigma(\gamma) \]

\[ \text{Integrated Luminosity [fb}^{-1}] \]

$3^\circ$ with 50 ab$^{-1}$ at BELLEII

\textit{P. Krishnan, FPCP2018}
CP content in multibody decays


\[ D^0 \rightarrow \pi\pi\pi^0 \text{ tagged with a CP-even eigenstate} \]

\[ D^0 \rightarrow \pi\pi\pi^0 \text{ tagged with a CP-odd eigenstate} \]

Overwhelmingly CP-even state

\[ F^+ = N^+/N^+ + N^- = 0.973 \pm 0.017 \]

Similarly, for \( D^0 \rightarrow KK\pi^0 \)

\[ F^+ = 0.732 \pm 0.055 \]

CLEO-c data
CP content and $c_i$ and $s_i$ for $D^0 \to \pi^+ \pi^- \pi^+ \pi^-$

5D binning inspired by an amplitude model

\[ F^+ = 0.769 \pm 0.021 \pm 0.010 \]

compatible with results from model

\[ = 0.729 \pm 0.009 \text{ (stat)} \pm 0.015 \text{ (syst)} \pm 0.010 \text{ (model)} \]

d’Argent, EG, et al. JHEP 1705 (2017) 143

- $D^0 \to \pi^+ \pi^- \pi^+ \pi^-$ (5D space):
  - expected Run 2 precision $\sim 10^\circ$;
  - external CLEO-c input uncertainty $\sim 7^\circ$
D⁰→Kₛπ⁺π⁻π⁺π⁰ decays

• D⁰→Kₚπ⁺π⁻π⁰: interesting rich resonant structure
• Some intermediate decays are CF modes (ADS), others CP-eigenstates (GLW-like)
• The phase-space is binned around the resonances

<table>
<thead>
<tr>
<th>Bin</th>
<th>resonance</th>
<th>c_i</th>
<th>s_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ω</td>
<td>-1.11 ± 0.09^{+0.02}_{-0.01}</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>K*⁻ρ⁺</td>
<td>-0.30 ± 0.05 ± 0.01</td>
<td>-0.03 ± 0.09^{+0.01}_{-0.01}</td>
</tr>
<tr>
<td>3</td>
<td>K*⁺ρ⁻</td>
<td>-0.41 ± 0.07^{+0.02}_{-0.01}</td>
<td>0.04 ± 0.12^{+0.01}_{-0.02}</td>
</tr>
<tr>
<td>4</td>
<td>K*⁻</td>
<td>-0.79 ± 0.09 ± 0.05</td>
<td>-0.44 ± 0.18 ± 0.06</td>
</tr>
<tr>
<td>5</td>
<td>K⁺</td>
<td>-0.62 ± 0.12^{+0.03}_{-0.02}</td>
<td>0.42 ± 0.20 ± 0.06</td>
</tr>
<tr>
<td>6</td>
<td>K*⁰</td>
<td>-0.19 ± 0.11 ± 0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>ρ⁺</td>
<td>-0.82 ± 0.11 ± 0.03</td>
<td>-0.11 ± 0.19^{+0.04}_{-0.03}</td>
</tr>
<tr>
<td>8</td>
<td>ρ⁻</td>
<td>-0.63 ± 0.18 ± 0.03</td>
<td>0.23 ± 0.41^{+0.04}_{-0.03}</td>
</tr>
<tr>
<td>9</td>
<td>remainder</td>
<td>-0.69 ± 0.15^{+0.15}_{-0.12}</td>
<td>0.00</td>
</tr>
</tbody>
</table>

• No LHCb analysis (challenging at LHCb because of the soft π⁰ but high priority for BELLEII) but with 60 k (50 ab⁻¹ at BELLEII) ~ 4.4°;
• CLEO-c data contribute uncertainty of 1.5°
Charm input to $\gamma$ from $D^0 \to K^3\pi$

The coherence factor and the average strong phase needed for the determination of $\gamma$

$$R_{K3\pi} e^{-i\delta_D^{K3\pi}} = \int \frac{A_{K+\pi^+\pi^-\pi^-}(x)A_{K+\pi^-\pi^+\pi^-}(x)dx}{A_{K+\pi^+\pi^-\pi^-}A_{K-\pi^+\pi^-\pi^-}}$$

Use interference effects in charm as input to $\gamma$

$$\Gamma(D^0 \to f) \sim \Gamma e^{-\Gamma} \left[ |A_f|^2 + \Gamma t \cdot R^{K3\pi}_{K\pi^+\pi^-\pi^-} \left| \frac{q}{p} \right| |A_f| |A_f| (y \cos(\delta^{K3\pi}_f - \phi) + x \sin(\delta^{K3\pi}_f - \phi)) \right]$$

Scan from $D\bar{D}$ superpositions at CLEO-c

Input from charm mixing (LHCb)

Combination: CLEO-c and mixing.

$$\Delta\chi^2 = 1, 4, 9$$


PRL 116 (2016) no. 24, 241801

study of the time-dependence of the ratio between $D^0 \to K^+\pi^-\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^-\pi^-$ decay rates

+ additional LHCb constraints

Eva Gersabeck, Inputs for $\gamma/\varphi_3$ from charm decays
Future projections for $D^0 \to K3\pi$

### Results of the combined fit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fitted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{K3\pi}$</td>
<td>$0.43^{+0.17}_{-0.13}$</td>
</tr>
<tr>
<td>$\delta^{K3\pi}_D$</td>
<td>$(128^{+28}_{-17})^\circ$</td>
</tr>
<tr>
<td>$r^{K3\pi}_D$</td>
<td>$(5.49 \pm 0.06) \times 10^{-2}$</td>
</tr>
</tbody>
</table>

- Excellent prospects for $D^0 \to K3\pi$ (5D space):
  - expected Run 2 precision $\sim 5.5^\circ$;
  - urgently need BESIII input

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G. Wilkinson BESIII-LHCb joint workshop 2018

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### Shopping list

**Priority ordered for LHCb**

Different priorities and different measurements possible at BELLEII

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Quantity of interest</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D \to K_s^0 \pi^+ \pi^-$</td>
<td>$c_i$ and $s_i$</td>
<td>Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$ data sets, it might be worthwhile to explore alternative binning.</td>
</tr>
<tr>
<td>$D \to K_s^0 K^+ K^-$</td>
<td>$c_i$ and $s_i$</td>
<td>Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$ data sets, it might be worthwhile to explore alternative binning.</td>
</tr>
<tr>
<td>$D \to K^{\pm} \pi^\mp \pi^+ \pi^-$</td>
<td>$R$, $\delta$</td>
<td>In bins guided by amplitude models, currently under development by LHCb.</td>
</tr>
<tr>
<td>$D \to K^+ K^- \pi^+ \pi^-$</td>
<td>$c_i$ and $s_i$</td>
<td>Binning scheme can be guided by the CLEO model [18] or potentially an improved model from LHCb in the future.</td>
</tr>
<tr>
<td>$D \to \pi^+ \pi^- \pi^+ \pi^-$</td>
<td>$F_+$ or $c_i$ and $s_i$</td>
<td>Unbinned measurement of $F_+$. Measurements of $F_+$ in bins or $c_i$ and $s_i$ in bins could be explored.</td>
</tr>
<tr>
<td>$D \to K^{\pm} \pi^\mp \pi^0$</td>
<td>$R$, $\delta$</td>
<td>Simple 2-3 bin scheme could be considered.</td>
</tr>
<tr>
<td>$D \to K_s^0 K^{\pm} \pi^\mp$</td>
<td>$R$, $\delta$</td>
<td>Simple 2 bin scheme where one bin encloses the $K^*$ resonance.</td>
</tr>
<tr>
<td>$D \to \pi^+ \pi^- \pi^0$</td>
<td>$F_+$</td>
<td>No binning required as $F_+ \sim 1$.</td>
</tr>
<tr>
<td>$D \to K_s^0 \pi^+ \pi^- \pi^0$</td>
<td>$F_+$ and $c_i$ and $s_i$</td>
<td>Unbinned measurement of $F_+$ required. Additional measurements of $F_+$ or $c_i$ and $s_i$ in bins could be explored.</td>
</tr>
<tr>
<td>$D \to K^+ K^- \pi^0$</td>
<td>$F_+$</td>
<td>Unbinned measurement required. Extensions to binned measurements of either $F_+$ or $c_i$ and $s_i$ possible.</td>
</tr>
<tr>
<td>$D \to K^{\pm} \pi^\mp$</td>
<td>$\delta$</td>
<td>Of low priority due to good precision available through charm-mixing analyses.</td>
</tr>
</tbody>
</table>
Summary

- The $\gamma$ determination represents a great opportunity for synergy between LHCb/ BELLEII and BESIII

- Measurements of strong phase differences from quantum correlated measurements will play an important role in future CPV measurements

- Sub-degree precision is attainable – but only if LHCb and BESIII work together!

- More $\Psi(3770)$ data are required to exploit fully the very large future samples at LHCb and BELLEII

- Several analyses ongoing at BESIII
BACKUP
### Measurements with CLEO-c data (* indicates legacy-data publication)

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_S\pi\pi$, $K_SKK$ binned $c_i$, $s_i$</td>
<td>PRD 82 (2010) 112006, arXiv:1010.2817</td>
</tr>
<tr>
<td>$\pi\pi\pi\pi⁰$, $KK\pi⁰$ CP-content</td>
<td>PLB 747 (2015) 9, arXiv:1504.05878 *</td>
</tr>
<tr>
<td>4π CP-content &amp; binned $c_i$,$s_i$</td>
<td>JHEP 01 (2018) 144, arXiv:1709.03467 *</td>
</tr>
<tr>
<td>$K_S\pi\pi\pi\pi⁰$ $c_i$, $s_i$</td>
<td>JHEP 01 (2018) 982, arXiv:1710.10086 *</td>
</tr>
</tbody>
</table>

### BESIII measurements

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Reference</th>
</tr>
</thead>
</table>
Dalitz plot

- A decay of a pseudo-scalar (e.g. D) into a final state with three pseudo-scalars (e.g. K, π) can be parametrised as

\[
\Gamma = \frac{1}{(2\pi)^3 32 \sqrt{s^3}} |M|^2 \, dm_{12}^2 \, dm_{23}^2
\]

\( M \) defines dynamic substructure

Kinematic boundaries depending on mother and daughter masses

Additional information about the spin of the resonances
The interference and the phases

GLW

strong phase

weak phase

ADS

measure CP asymmetries and ratios of decay rates

Eva Gersabeck, Inputs for $\gamma/\varphi_3$ from charm decays
**CKM matrix**

- Unitary matrix combining flavour and mass eigenstates

\[
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix} = \begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}
\]

- Unitarity relations lead to triangles in complex plane

\[
\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0
\]

**B_d triangle**

One of the main goals of flavour physics is to determine the angle \(\gamma/\varphi_3\) of the B_d CKM triangle.
$Q$ — a ratio of a statistical sensitivity to that in the unbinned case. Specifically, $Q$ relates the number of standard deviations by which the number of events in bins is changed by varying parameters $x$ and $y$, to the number of standard deviations if the Dalitz plot is divided into infinitely small regions (the unbinned case):

$$Q^2 = \frac{\sum_i \left( \frac{1}{\sqrt{F_i}} \frac{dF_i}{dx} \right)^2 + \left( \frac{1}{\sqrt{F_i}} \frac{dF_i}{dy} \right)^2}{\int_{\mathcal{D}} \left[ \left( \frac{1}{\sqrt{|f_B|^2}} \frac{d|f_B|^2}{dx} \right)^2 + \left( \frac{1}{\sqrt{|f_B|^2}} \frac{d|f_B|^2}{dy} \right)^2 \right] d\mathcal{D}},$$

where $f_B = f_D + (x + iy)\bar{f}_D$, $F_i = \int_{\mathcal{D}_i} |f_B|^2 d\mathcal{D}$. (11)