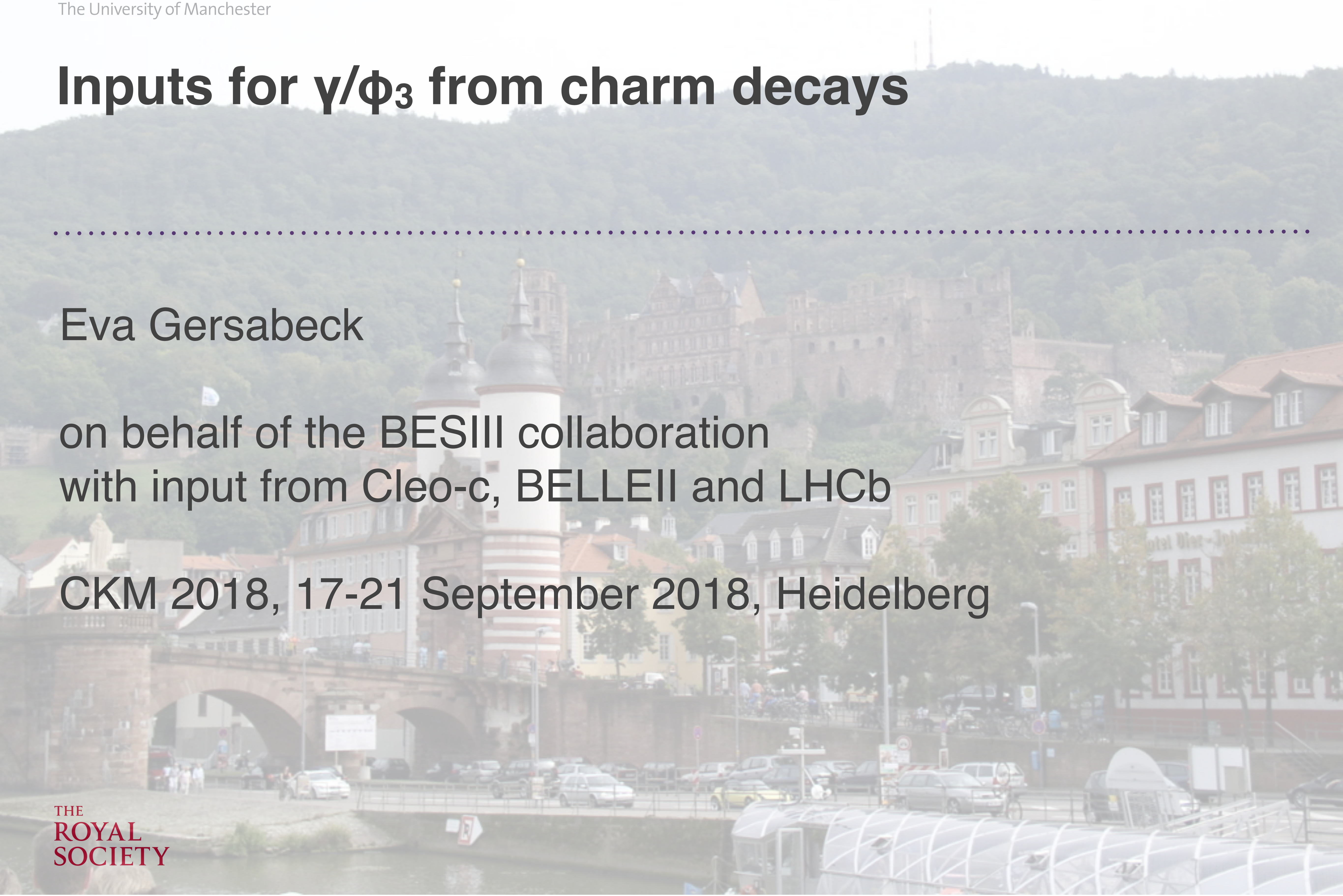
MANCHESTER 1824
The University of Manchester

Inputs for γ/ϕ_3 from charm decays



Eva Gersabeck

on behalf of the BESIII collaboration
with input from Cleo-c, BELLEII and LHCb

CKM 2018, 17-21 September 2018, Heidelberg

THE ROYAL SOCIETY

MANCHESTER 1824
The University of Manchester

The CKM angle γ

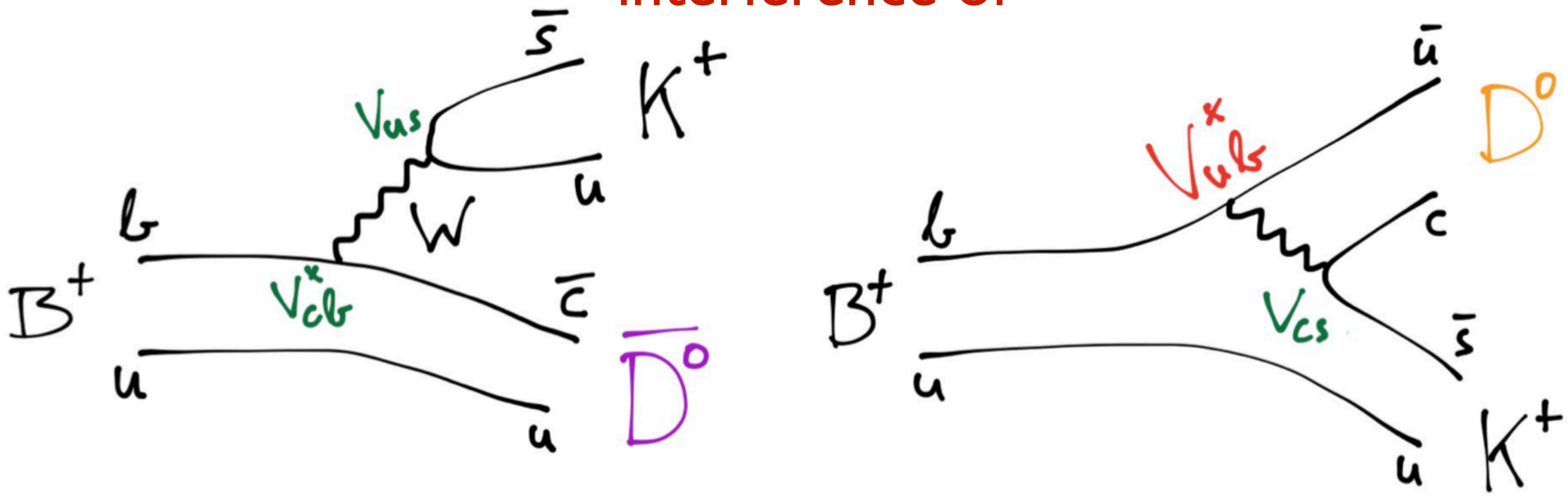
CPV is an interference effect of two amplitudes: A_1 and A_2

$$|A_1|^2+|A_2|^2+2|A_1||A_2|\cos(\delta_s + \varphi_{CPV})$$

δ_s strong phase between A_1 and A_2

φ_{CPV} weak CPV phase

interference of



SM benchmark: only CKM angle accessible at tree level
(b→u and b→c transitions in B→DK decays)
D^0 and $\bar{D}^0$ decays to a common final state: interference

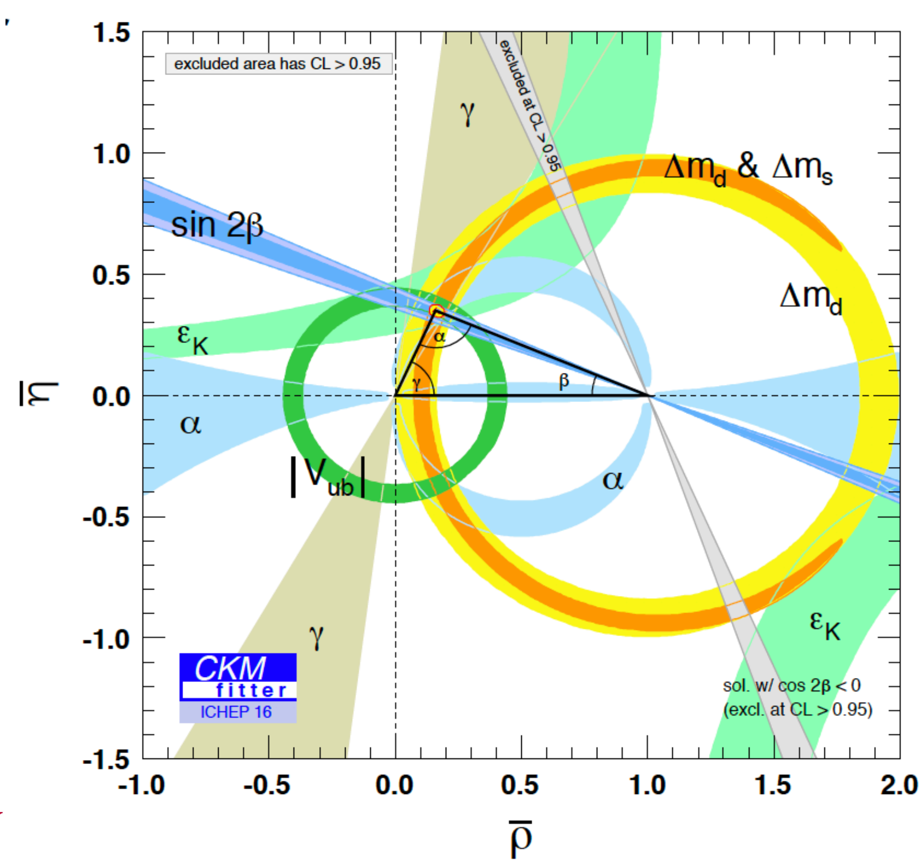
weak phase γ = arg($-V_{ub}V_{ud}^*/V_{cd}V_{cd}^*$)

THE ROYAL SOCIETY

MANCHESTER 1824
The University of Manchester

The CKM angle γ

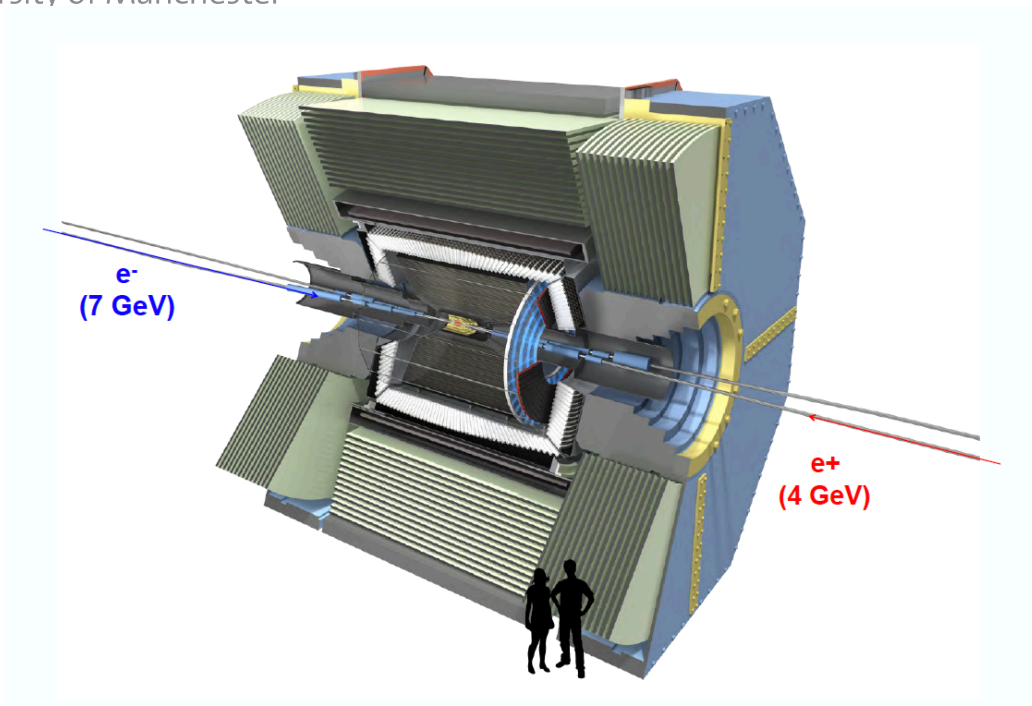
- SM benchmark: sensitive to New Physics effects
- Theory uncertainty on γ is very small $\delta\gamma/\gamma \approx O(10^{-7})$ *Zupan and Brod'13 arXiv:1308.5663*
- γ can probe for new physics at extremely high-energy scales ~$O(10^2\text{-}10^3)$
- NP can lead to a sizeable 4° effect *Brod, Lenz et al'14 arXiv:1412.1446*
  - Over-constrain the triangle to test for NP

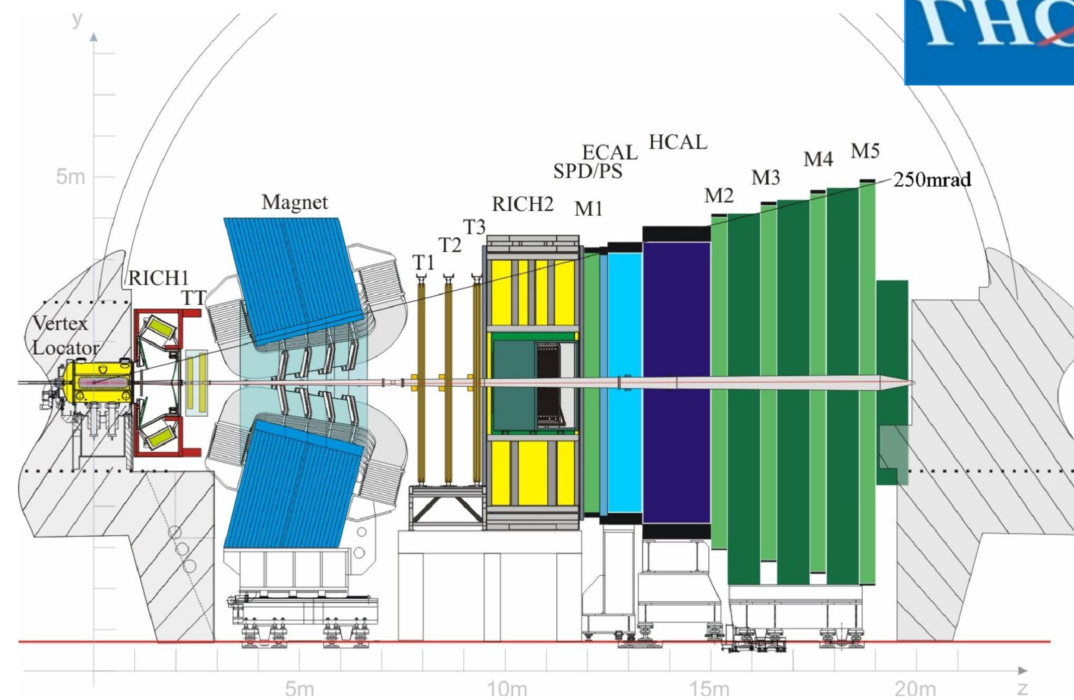


Direct (WA)	$\gamma = \left(73.5^{+4.3}_{-5.0}\right)^\circ$
Direct (LHCb) LHCb-CONF-2018-002	$\gamma = \left(74.0^{+5.0}_{-5.8}\right)^\circ$
Indirect	$\gamma = \left(65.3^{+1.0}_{-2.5}\right)^\circ$
Pre-LHCb	$\gamma = \left(73^{+22}_{-25}\right)^\circ$

THE ROYAL SOCIETY

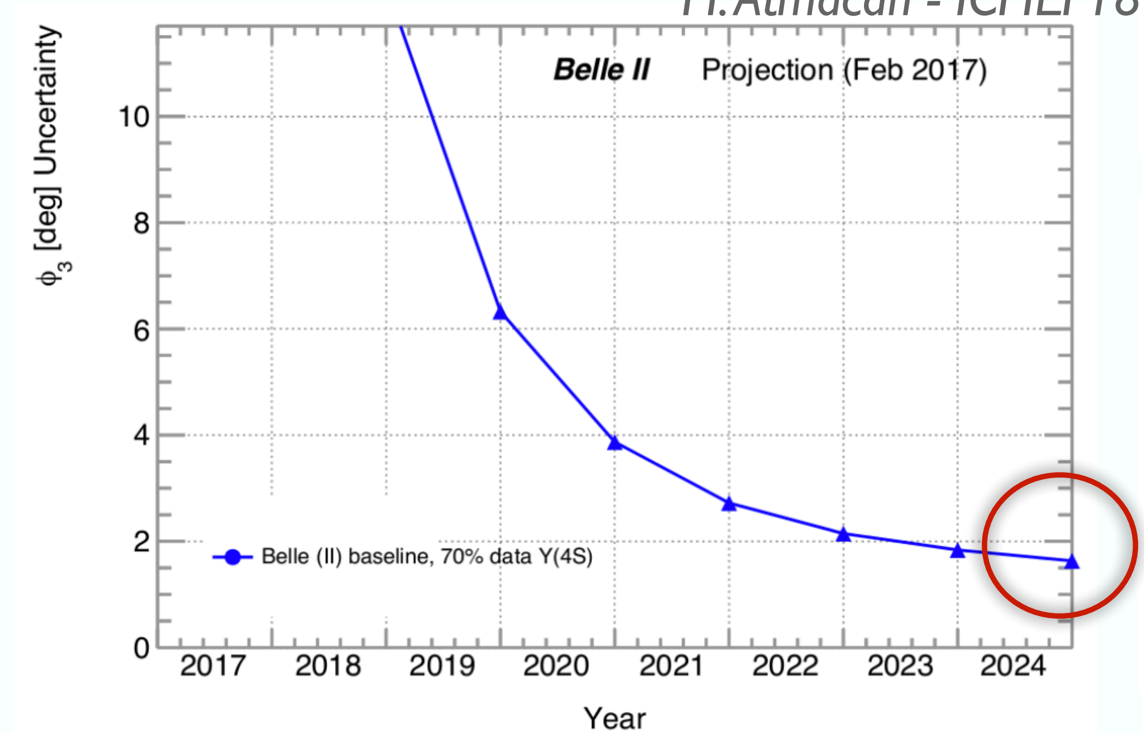
Future precision on γ





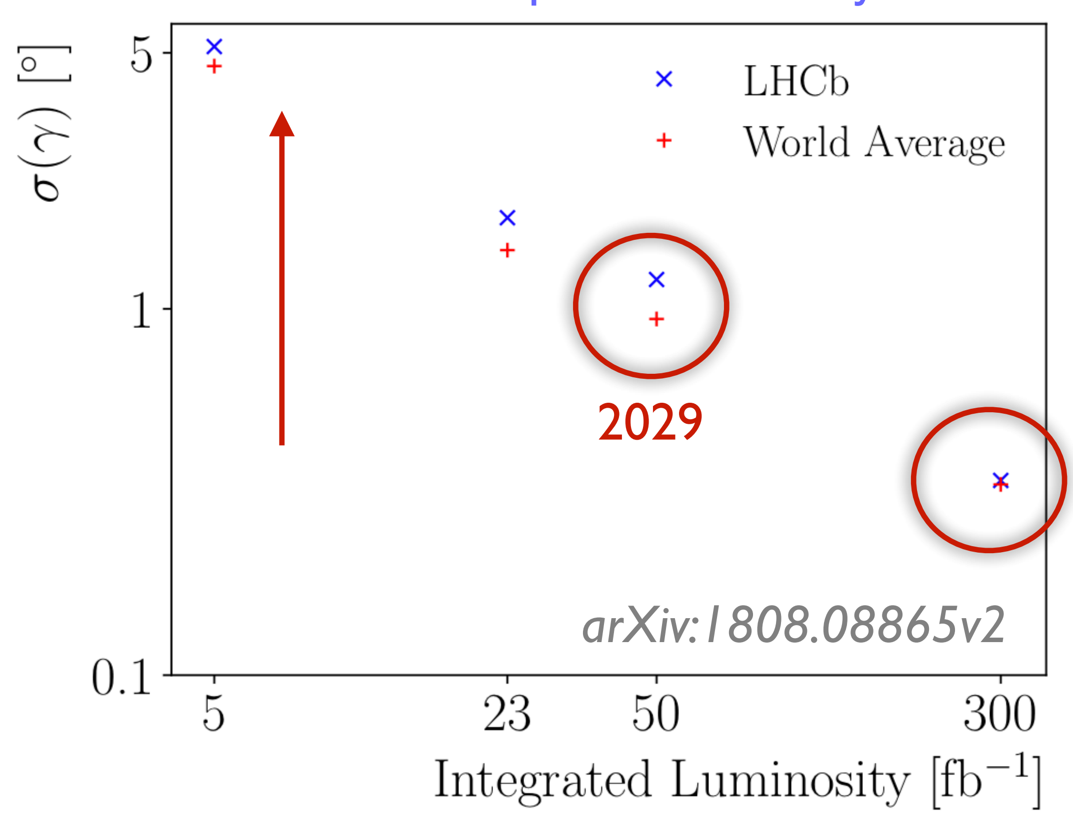
Combined sensitivity with 50 ab^{-1}: 1.6°

H.Atmacan - ICHEP18



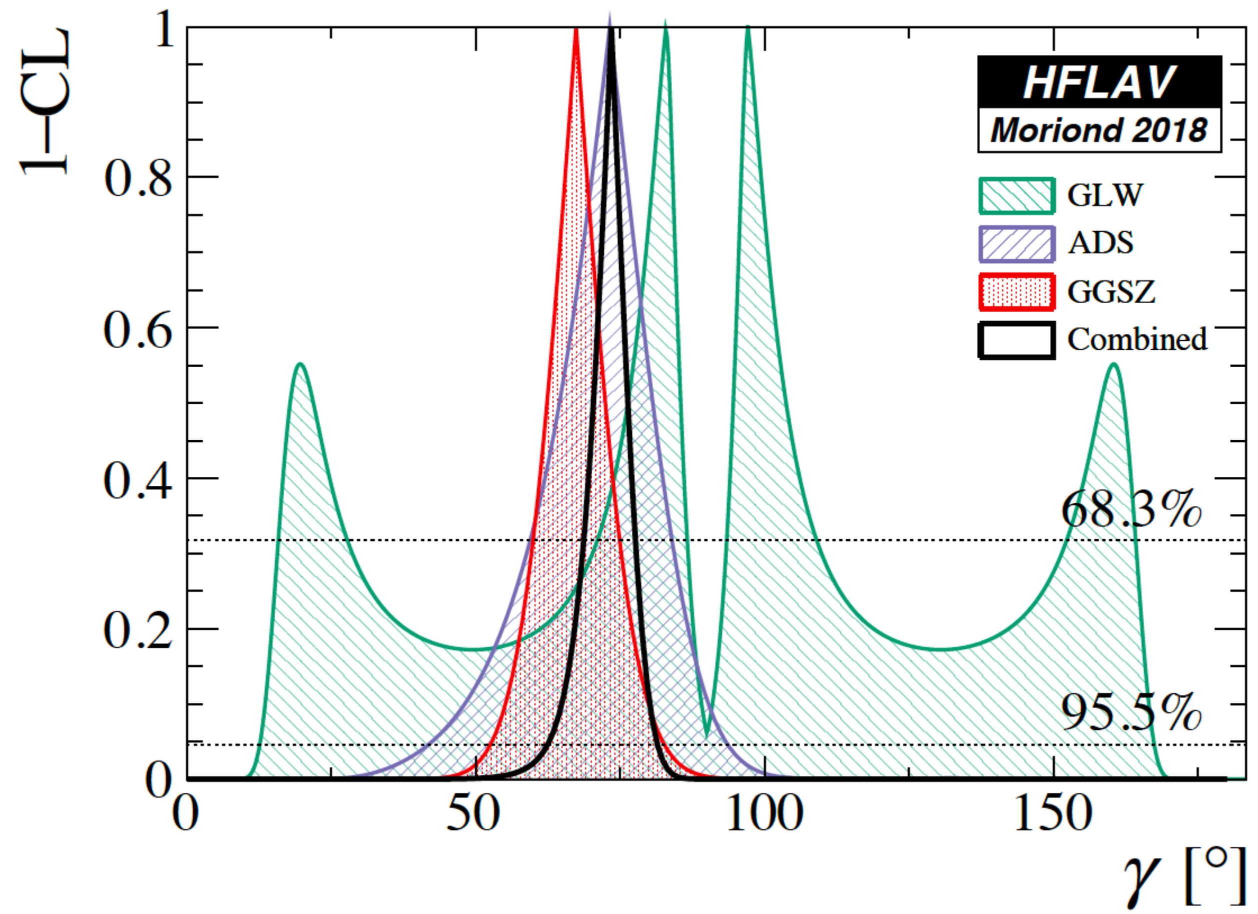
Projected sensitivity for the LHCb γ combination

- external input uncertainty of 2°



The methods for measuring γ

- Measure CPV observables in many D modes in $B \to DK$ decays (but not only)
- Intermediate states include $D \to hh^{(')}$, $D \to K_S hh$, $D \to K3\pi$, etc.
- Different methods depending on the D decay final state used:



- GLW: CP eigenstates e.g. $D \to KK$, $D \to \pi\pi$
- ADS: Cabibbo favoured or doubly suppressed e.g. $D \to K\pi$
- GGSZ: three body final state e.g. $D \to K_S hh$

see the talk of Alberto Correa Dos Reis

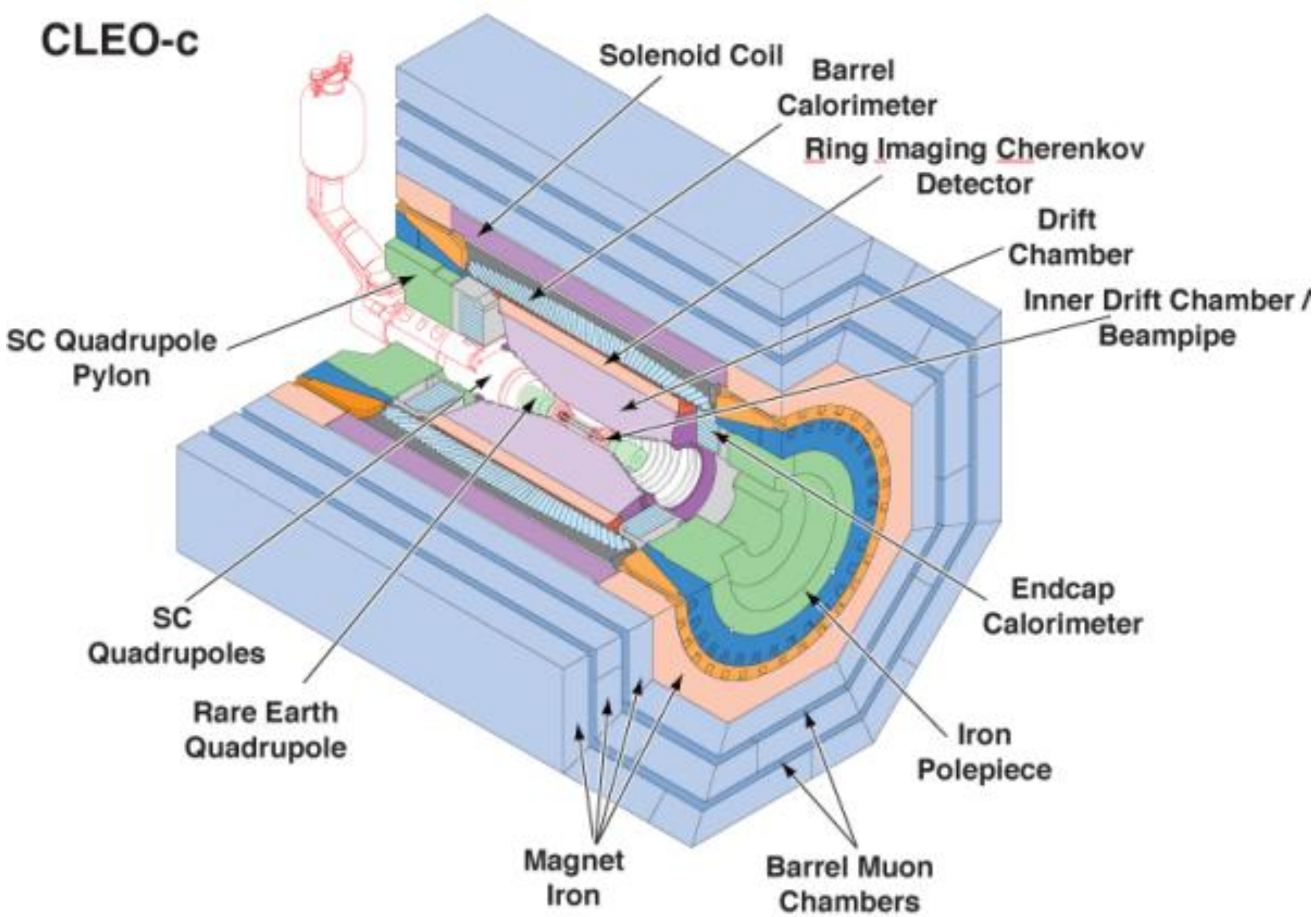
The University of Manchester

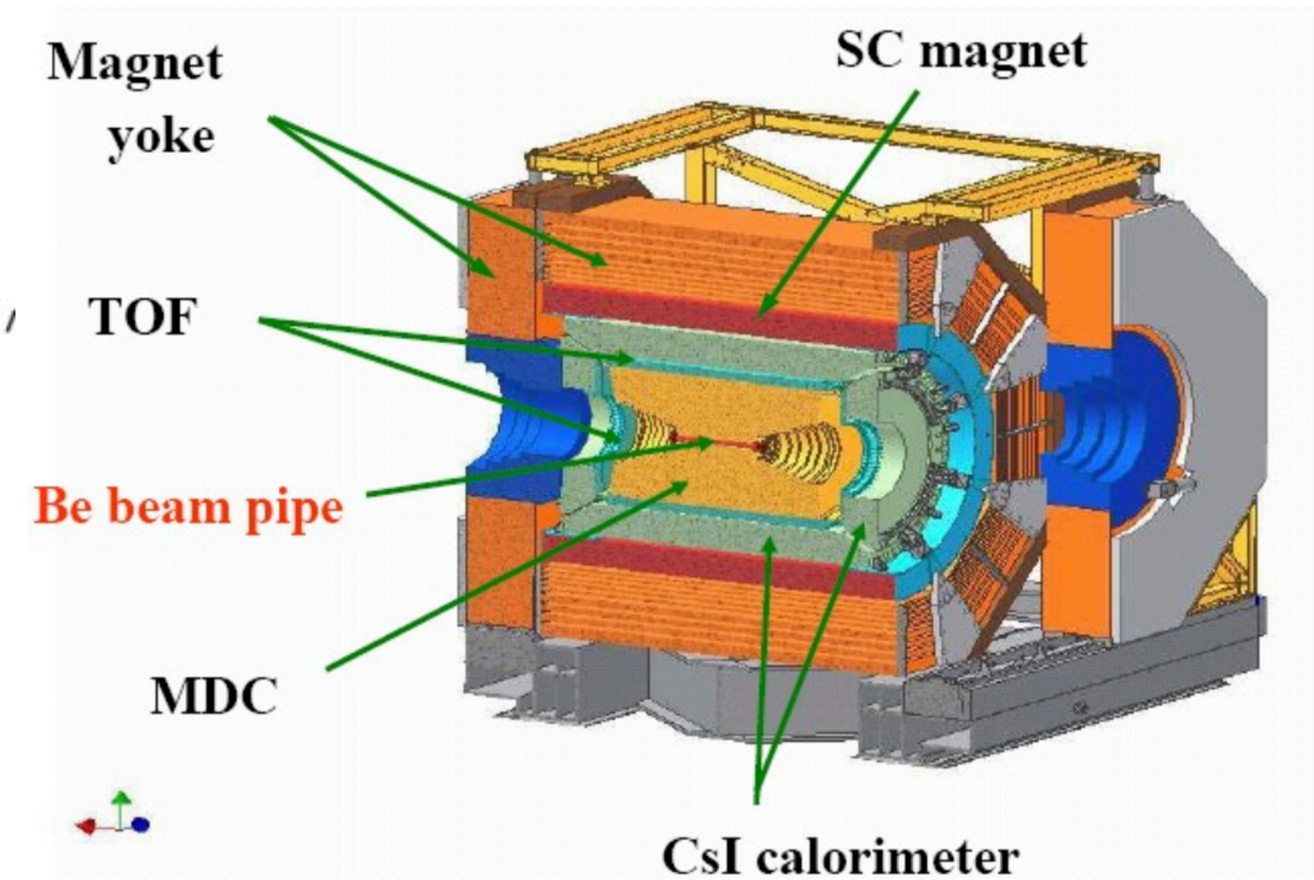
γ with multibody decays

- D can decay either to two-body or multibody final state
- Multibody decays can be analysed with model-dependent and model-independent techniques
- **Modelling of the phase space**: irreducible systematics due to the choice of the model
- ***Model-independent techniques*** require external input for the hadronic parameters of the D decays (*e.g.* strong phases, coherence factors *etc.*)
  - → Hence the need for quantum-correlated (Q.C.) charm threshold data: CLEO-c (0.8/fb) and BESIII (2.9/fb on tape, more to come)
  - Over-constraining of the parameters also possible
    Eur.Phys.J. C78 (2018) no.2, 121

THE ROYAL SOCIETY

Experiments at charm threshold

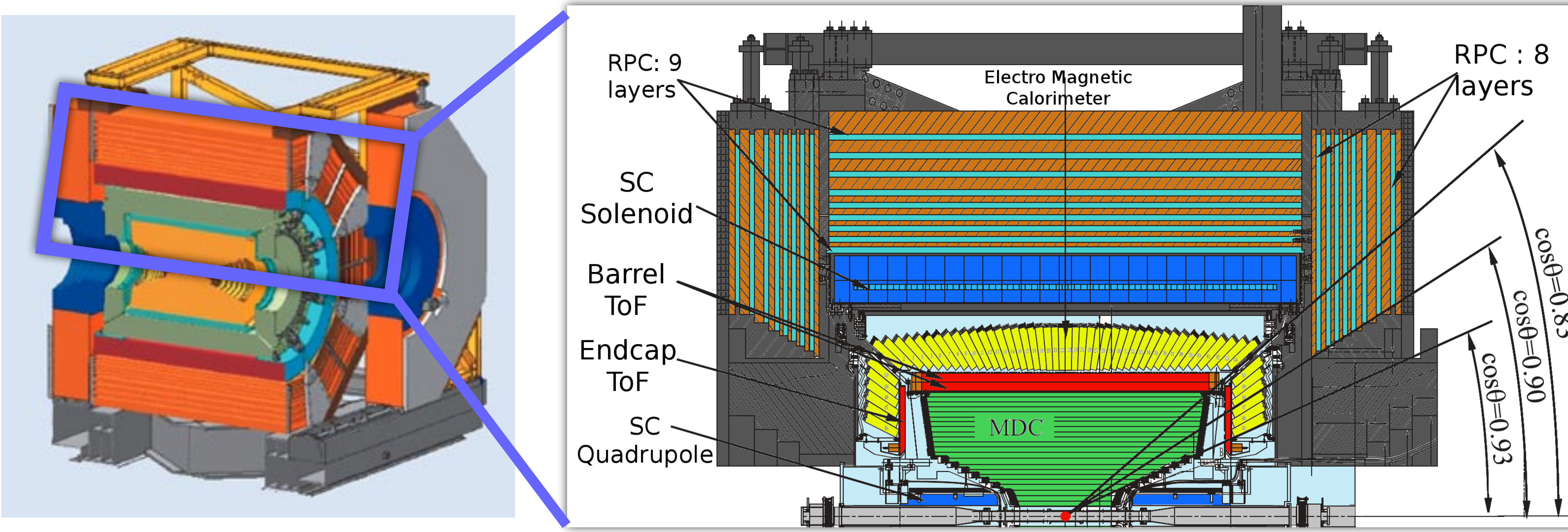




Stopped taking data in 2008*
*several recent legacy-data publications

Still taking data

A closer look at BESIII



hermetic

barrel |cosθ| <0.83

endcap 0.85 < |cosθ| < 0.93

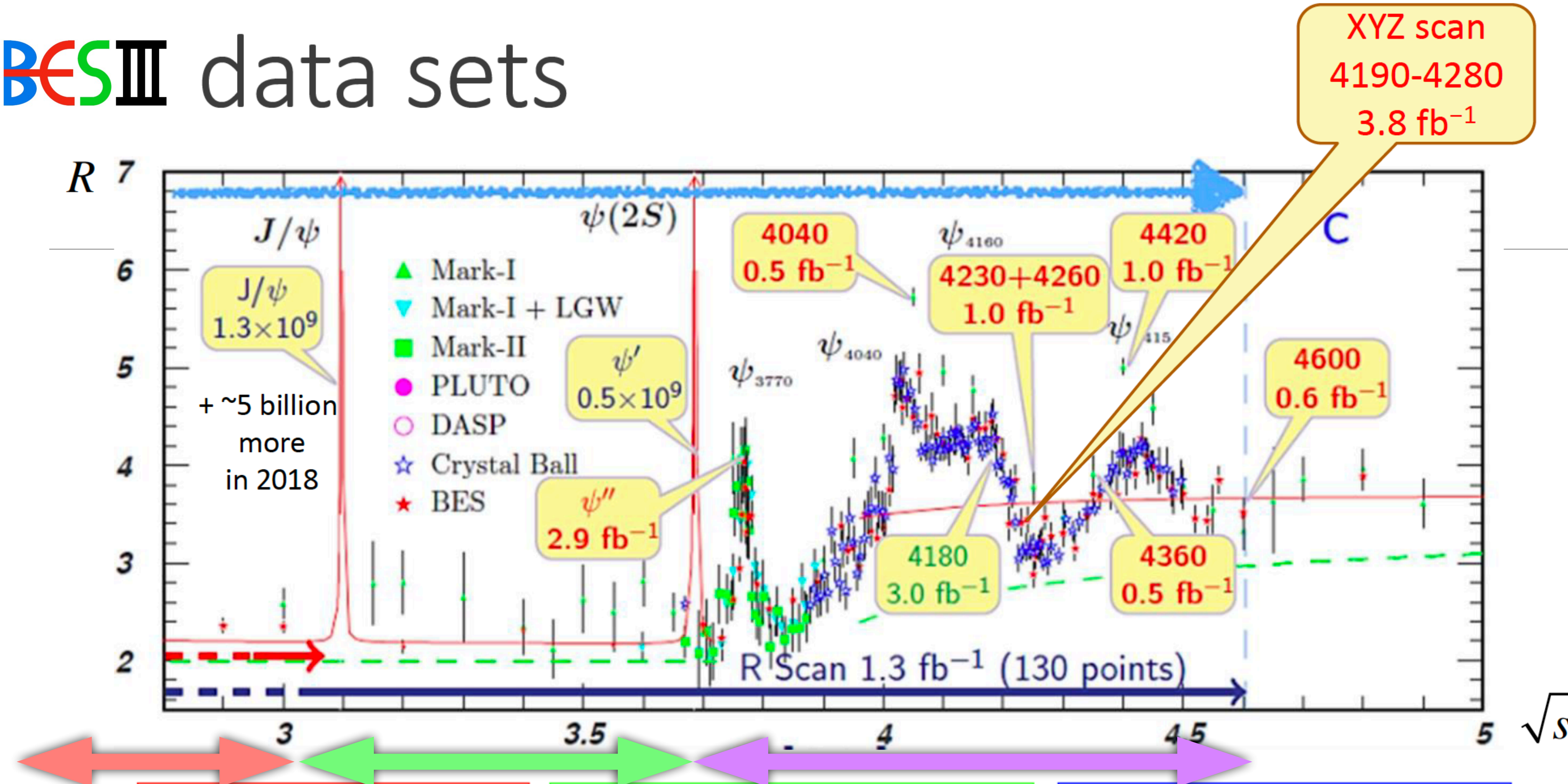
@ threshold = full kinematic constraint of the decays

Excellent for neutral and invisible particles

Optimised for flavour physics in the tau-charm region

Physics in the tau-charm region

BESIII data sets



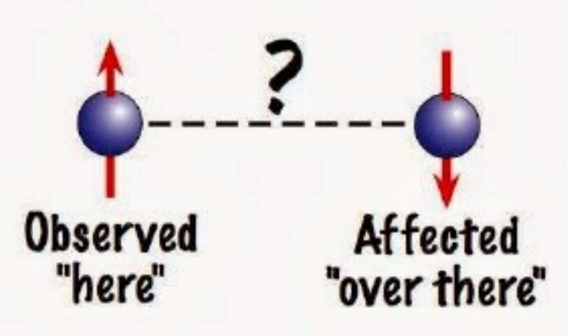
- Hadron form factors
- $Y(2175)$
- Z_s states?
- QCD tests

- Light hadron spectroscopy
- Glueballs, hybrids, exotics
- Rare decays
- τ physics

- XYZ
- D and D_s physics
- f_D, f_{D_s}; mixing, CPV
- Charmed baryons

Charm quantum correlated data at 3.77 GeV

- The production mechanism leads to a coherent state
- $e^+e^- \to \Psi(3770) \to 1/\sqrt{(2)}(|D0\rangle|\bar{D}0\rangle - |\bar{D}0\rangle|D0\rangle)$



- $e^+e^- \to \Psi(3770) \to \bar{D}^0 D^0$
- $D_{CP\pm} = [D^0 \pm \bar{D}^0]/\sqrt{(2)}$

- no energy for one single additional pion
- Unique access to relative strong phases, CP content
- Use flavour tags: e.g $D^0 \to K^-\pi^+$;$D^0 \to K^-\mu^+\nu$
- Use CP tags: e.g. $D^0 \to K^-K^+$

- $\sigma(c\bar{c})$ = 3 nb, N $(c\bar{c}) \sim O(10^7)$
- Largest data sample of quantum entangled charm particles 2.93 fb^{-1}

The University of Manchester

Strong phase in $D^0 \to K\pi$ decays

- Strong $K\pi$ scattering phase shift, $\delta_{K\pi}$ accessible at BESIII
- Most precise direct measurement

Phys. Lett. B 734 (2014) 227

$$\cos\delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01.$$

- Contributes to extracting the charm parameters x and y from x' and y'
- Can contribute to a more precise determination of γ

THE ROYAL SOCIETY

External input

- GLW-like: quasi-CP eigenstates e.g. D→KKπ[0]: need CP content F^+

$$A_{CP} = \frac{\Gamma(B^- \to D^0_{CP}K^-) - \Gamma(B^+ \to D^0_{CP}K^+)}{\Gamma(B^- \to D^0_{CP}K^-) + \Gamma(B^+ \to D^0_{CP}K^+)} = \frac{\pm 2 r_B (2F^+ + 1)\sin(\delta_B)\sin(\gamma)}{1 + r_B^2 + 2 r_B (2F^+ + 1)\cos(\delta_B)\cos(\gamma)}$$

$$R_{CP} = \frac{\Gamma(B^- \to D^0_{CP}K^-) - \Gamma(B^+ \to D^0_{CP}K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to D^0 K^+)} = 1 + r_B^2 + 2 r_B (2F^+ + 1)\cos(\delta_B)\cos(\gamma)$$

- ADS: involving multibody doubly suppressed e.g. D→Kππ[0]: need dilution from interference k_D (and r_D, δ_D)

$$A_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} = \frac{2 r_B r_D k_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2 r_B r_D k_D \cos(\delta_B + \delta_D)\cos(\gamma)}$$

$$R_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)} = r_B^2 + r_D^2 + 2 r_B r_D k_D \cos(\delta_B + \delta_D)\cos(\gamma)$$

- GGSZ: three body final state e.g. D→$K_S$hh: look at partial rate as function of the Dalitz position: need strong phase input

$$d\Gamma_{B^\pm}(x) = A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} + 2A_{(\pm,\mp)}A_{(\mp,\pm)}[\underbrace{r_B\cos(\delta_B \pm \gamma)}_{x^\pm}\underbrace{\cos(\delta_{D(\pm,\mp)})}_{c(\pm,\mp)} + \underbrace{r_B\sin(\delta_B \pm \gamma)}_{y^\pm}\underbrace{\sin(\delta_{D(\pm,\mp)})}_{s(\pm,\mp)}]$$

Measure strong phases

e.g. probe strong-phase distribution of multibody decays…

$D^{*+} \to D^0 \pi^+$ or $D^0 \to Kl\nu$

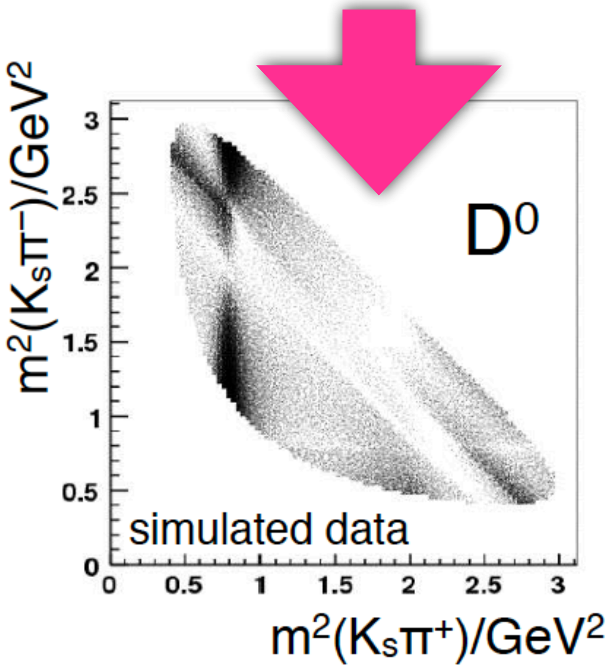
$D^0 \to K_S \pi^+ \pi^-$

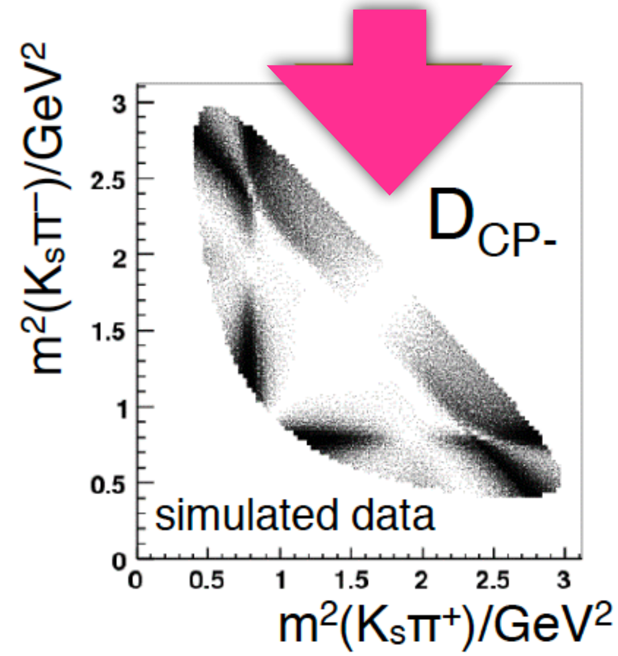
$\psi'' \to D_a D_b$

$D_a \to KK$ eg. CP+

$D_b \to K_S \pi^+ \pi^-$

Flavour tagged Distribution ~ $|D^0|^2$ or $|\bar{D}^0|^2$





CP-tagged ~ $|D^0|^2 + |\bar{D}^0|^2 \pm 2\, D^0 \bar{D}^0 \cos\delta$

- Unique access to relative strong phases
  - Use CP tags: reconstruct one meson as a CP eigenstate
  - Project the other meson as a superposition of D^0 and $\bar{D}^0$

MANCHESTER 1824
The University of Manchester

Definition of c_i and s_i

- Phase-space dependent amplitudes for
  - $D^0 \to K^0_S \pi^+ \pi^-$ decays: $\mathcal{A}$
  - $\overline{D}^0 \to K^0_S \pi^- \pi^+$ decays: $\mathcal{B}$
- Fraction of D^0 events in bin i → $T_i = \int_i |\mathcal{A}|^2 dm^2_+ dm^2_-$
- Interference terms between amplitudes $\mathcal{A}$ and $\mathcal{B}$

$$c_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |\mathcal{A}^*||\mathcal{B}| \cos(\Delta\delta_D) dm^2_+ dm^2_-$$

strong phase difference

$$s_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |\mathcal{A}^*||\mathcal{B}| \sin(\Delta\delta_D) dm^2_+ dm^2_-$$

THE ROYAL SOCIETY

Measure strong phases

MANCHESTER 1824
The University of Manchester

e.g. probe strong-phase distribution of multibody decays…

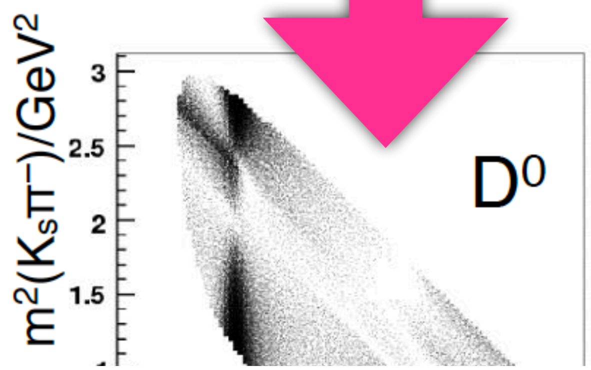
$D^{*+} \rightarrow D^0 \pi^+$ or $D^0 \rightarrow Kl\nu$

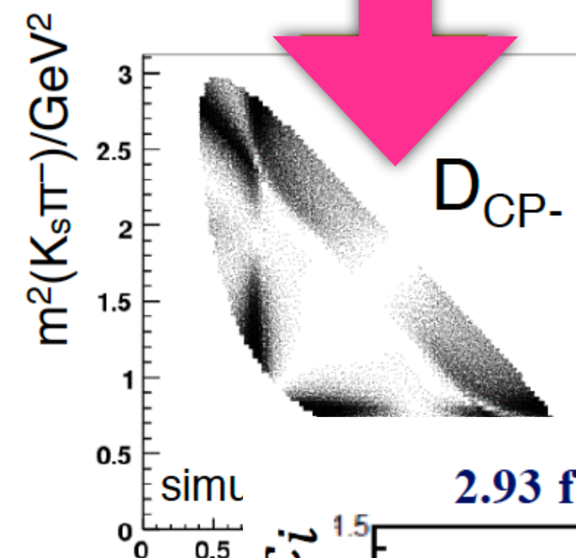
$D^0 \rightarrow K_S\pi^+\pi^-$

$\psi'' \rightarrow D_a D_b$

$D_a \rightarrow KK$ eg. CP+

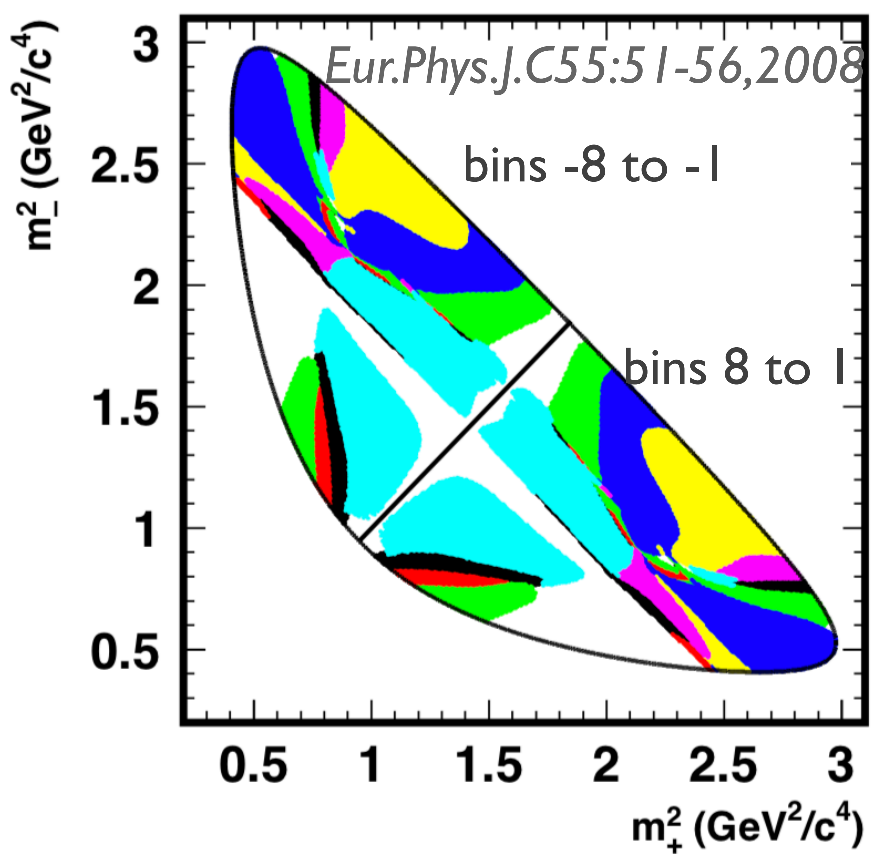
$D_b \rightarrow K_S\pi^+\pi^-$

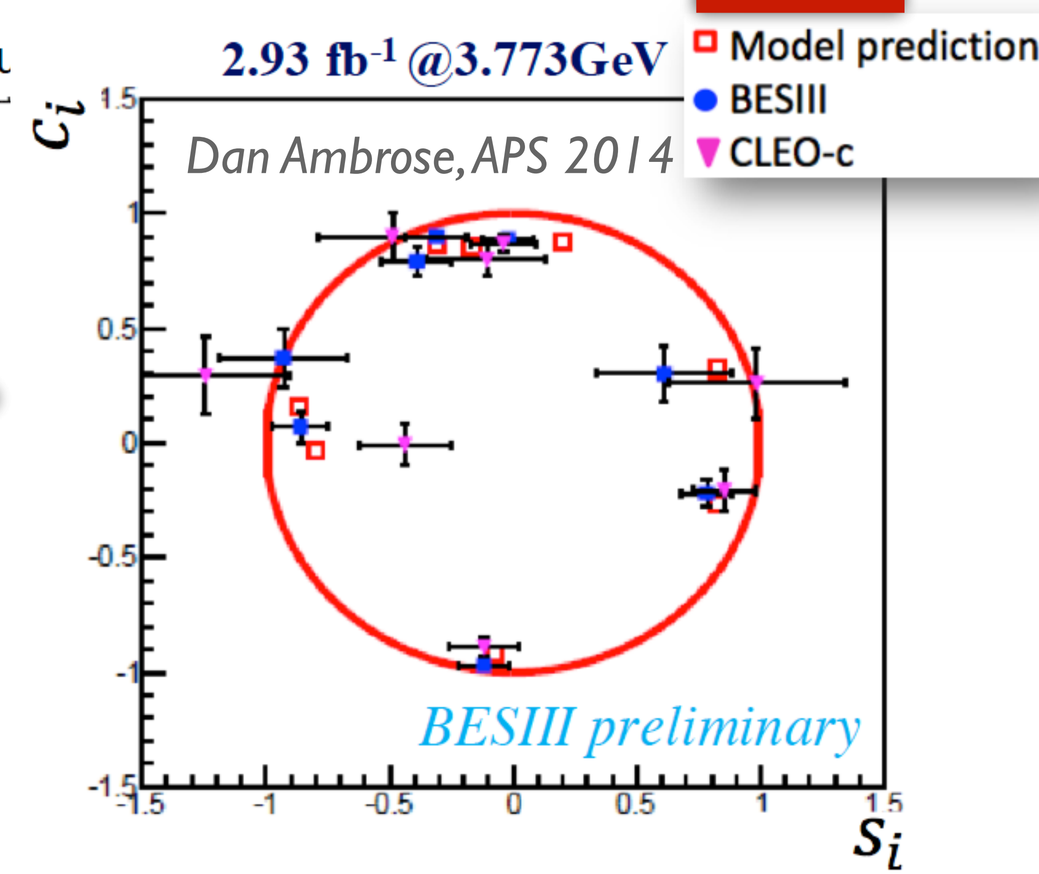




Flavour tagged Distribution ~ $|D^0|^2$ or $|\bar{D}^0|^2$

CP-tagged ~ $|D^0|^2 + |\bar{D}^0|^2 \pm 2\, D^0\bar{D}^0 \cos\delta$





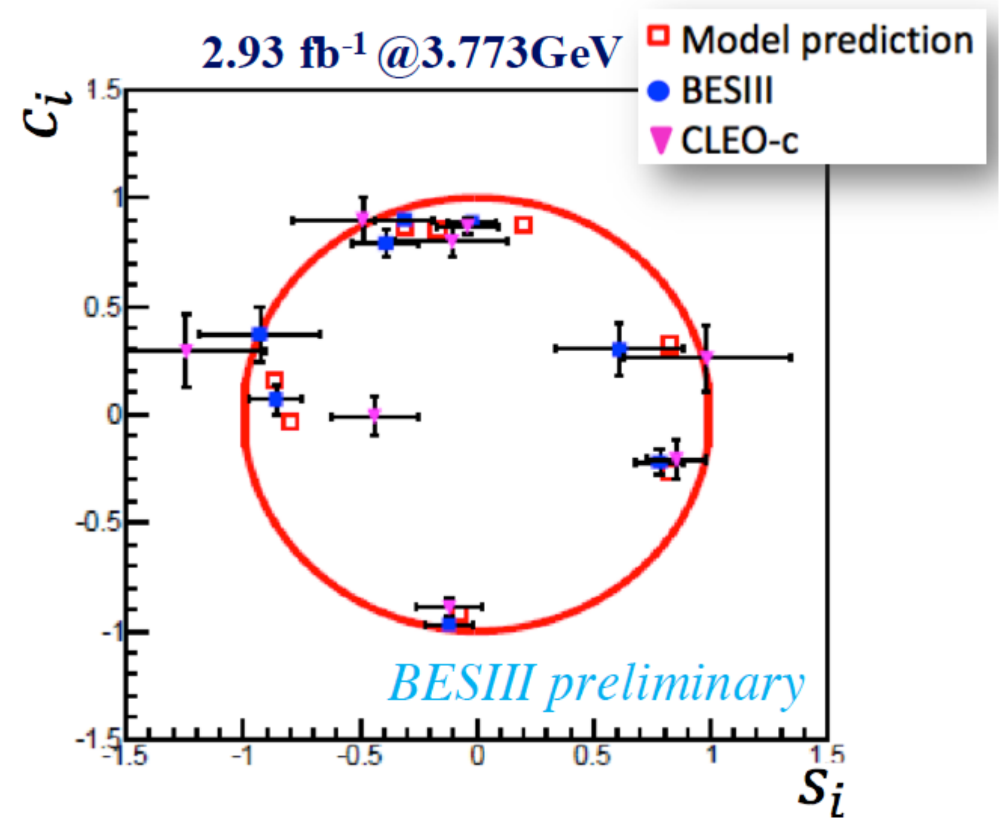
THE ROYAL SOCIETY

QC status at BESIII (strong phases, CP content)

- $D^0 \to K_S \pi^- \pi^+$ strong phase differences c_i and s_i

	c_i		s_i	
Bins	BES-III	CLEO-c	BES-III	CLEO-c
1	0.066 ± 0.066	-0.009 ± 0.088	-0.843 ± 0.119	-0.438 ± 0.184
2	0.796 ± 0.061	0.900 ± 0.106	-0.357 ± 0.148	-0.490 ± 0.295
3	0.361 ± 0.125	0.292 ± 0.168	-0.962 ± 0.258	-1.243 ± 0.341
4	-0.985 ± 0.017	-0.890 ± 0.041	-0.090 ± 0.093	-0.119 ± 0.141
5	-0.278 ± 0.056	-0.208 ± 0.085	0.778 ± 0.092	0.853 ± 0.123
6	0.267 ± 0.119	0.258 ± 0.155	0.635 ± 0.293	0.984 ± 0.357
7	0.902 ± 0.017	0.869 ± 0.034	-0.018 ± 0.103	-0.041 ± 0.132
8	0.888 ± 0.036	0.798 ± 0.070	-0.301 ± 0.140	-0.107 ± 0.240

CLEO-c results can be found in Phys.Rev. D82 (2010) 112006



Dan Ambrose, APS 2014

- Check the talk of P. Weidenkaff tomorrow as well

- Analyses of $D^0 \to K_L \pi^- \pi^+$ and $K_S K^- K^+$ also underway

MANCHESTER 1824
The University of Manchester

Binning optimisation

Eur.Phys.J.C55:51-56,2008

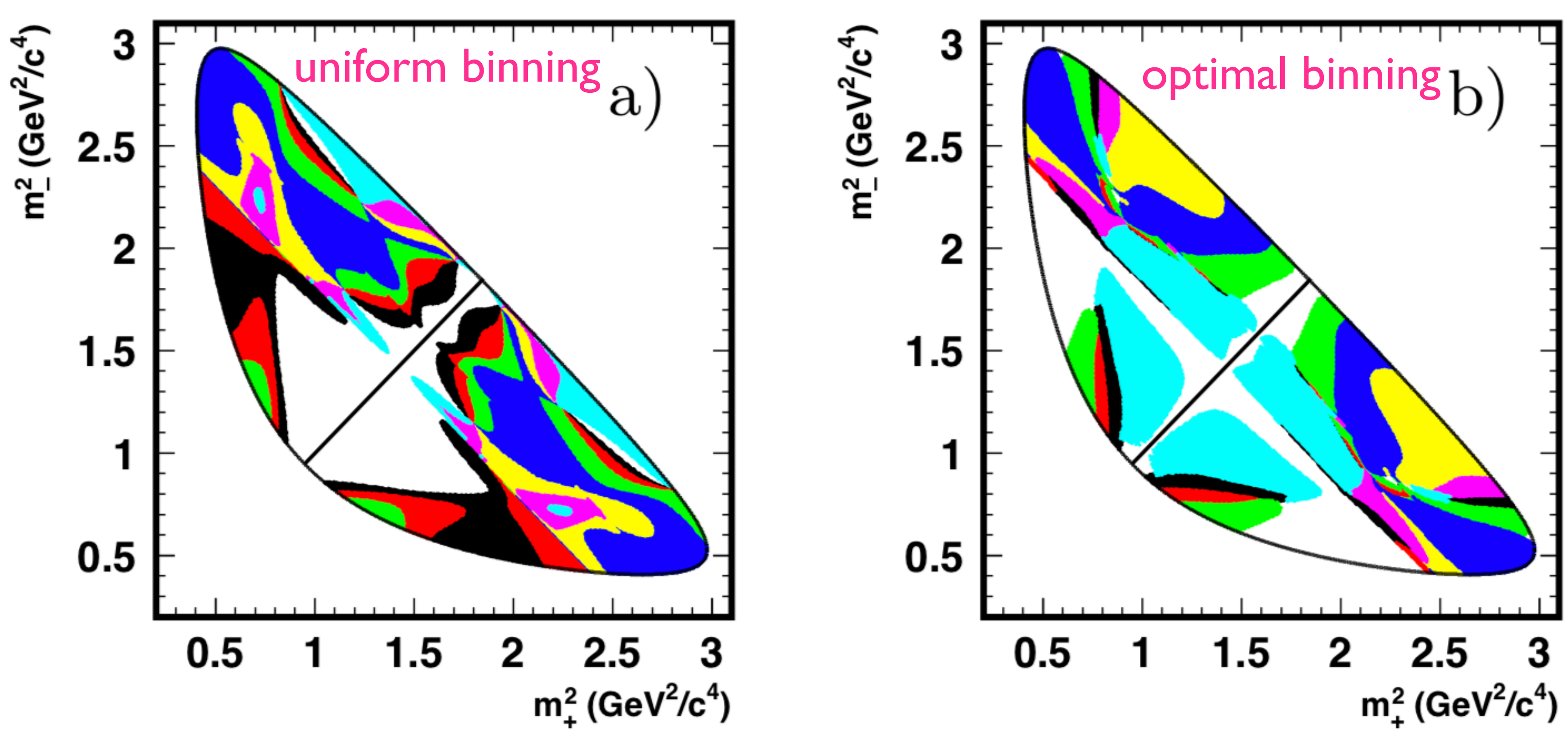
- Optimise the ratio of the statistical sensitivity sensitivity wrt the unbinned case

$$Q^2|_{x=y=0} = \sum_i (c_i^2 + s_i^2) N_i \Big/ \sum_i N_i$$

- Optimal: Split the phase space in 16 bins with similar strong phase differences
- Bins symmetric around $m^2(\pi^-\pi^+)$ axis
- Binned measurements provided by Cleo-c for various amplitude models

the choice of the model doesn't bias the result; it can affect the sensitivity

Binning	Q	$(K_S^0\pi^+\pi^-)^2$-stat. err.	
		σ_x	σ_y
$\mathcal{N} = 8$ (uniform)	0.57	0.015	0.032
$\mathcal{N} = 8$ ($\Delta\delta_D$)	0.79	0.005	0.010
$\mathcal{N} = 8$ (optimal)	0.89	0.008	0.011
$\mathcal{N} = 19$ (uniform)	0.69	0.013	0.019
$\mathcal{N} = 20$ ($\Delta\delta_D$)	0.82	0.004	0.008
$\mathcal{N} = 20$ (optimal)	0.96	0.004	0.010
Unbinned	-	-	-

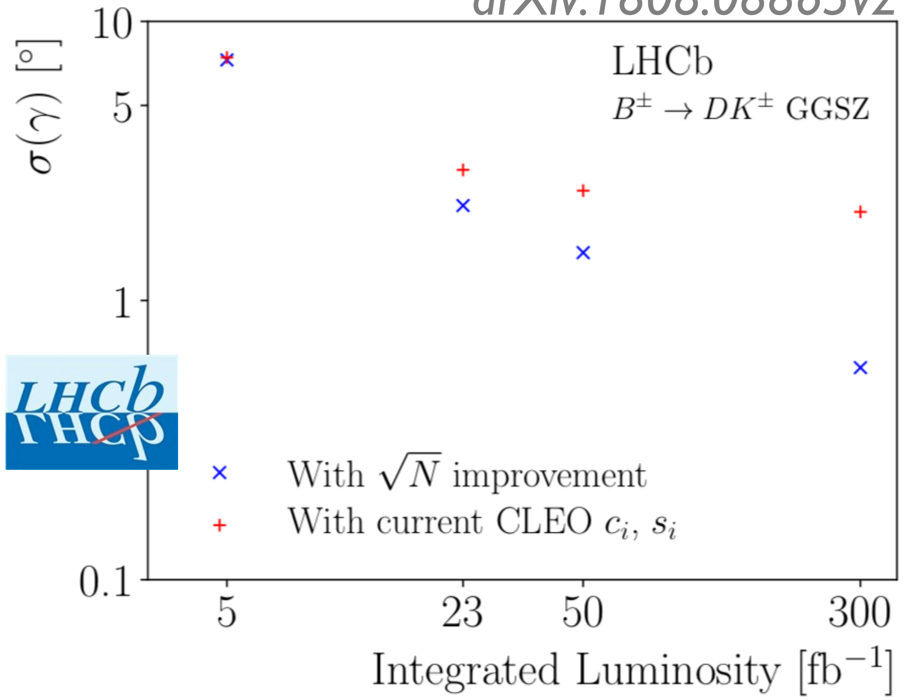


THE ROYAL SOCIETY

Importance of this measurement

- Most precise determination of γ from a single channel from B→DK with D→$K_S$hh $\gamma = \left(80.0^{+10.0}_{-9.0}\right)^{\circ}$ *JHEP 08 176*
- Uncertainty due to strong-phase inputs (CLEO-c) 4° > uncertainty due to experimental systematic effects 2°

arXiv:1808.08865v2



3° with 50 ab^{-1} at BELLEII

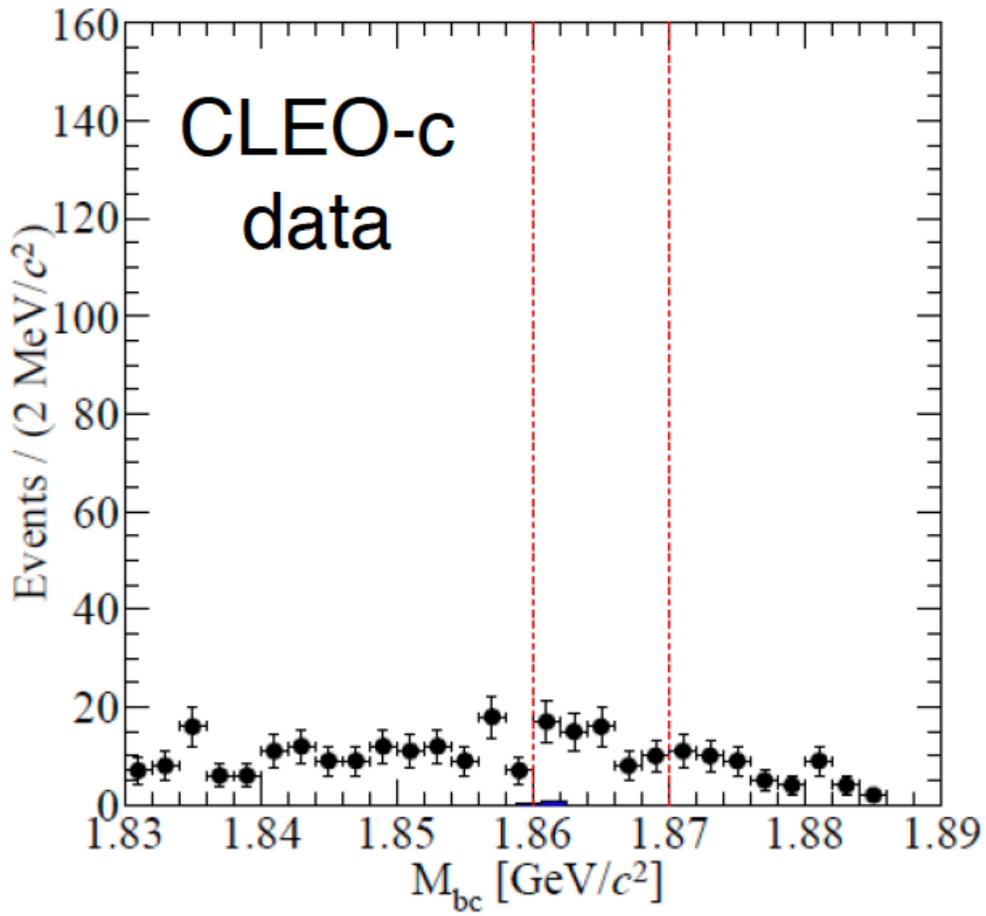
P. Krishnan, FPCP2018

- Input important for B→DKπ with D→$K_S$hh, precision of 2° achievable after the upgrade *Craik et al.,arXiv:1712.0853*

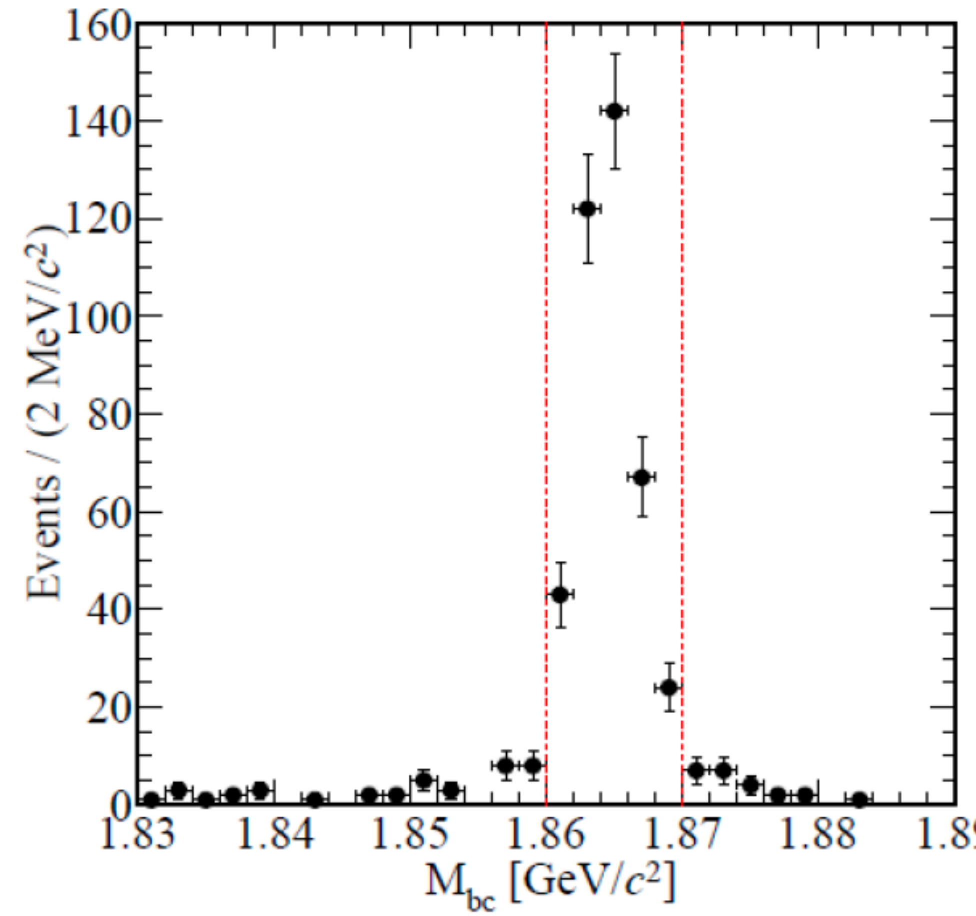
CP content in multibody decays

S. Malde et al. Phys.Lett. B747 (2015) 9-17

$D^0 \to \pi\pi\pi^0$ tagged with a CP-even eigenstate



$D^0 \to \pi\pi\pi^0$ tagged with a CP-odd eigenstate



Overwhelmingly CP-even state

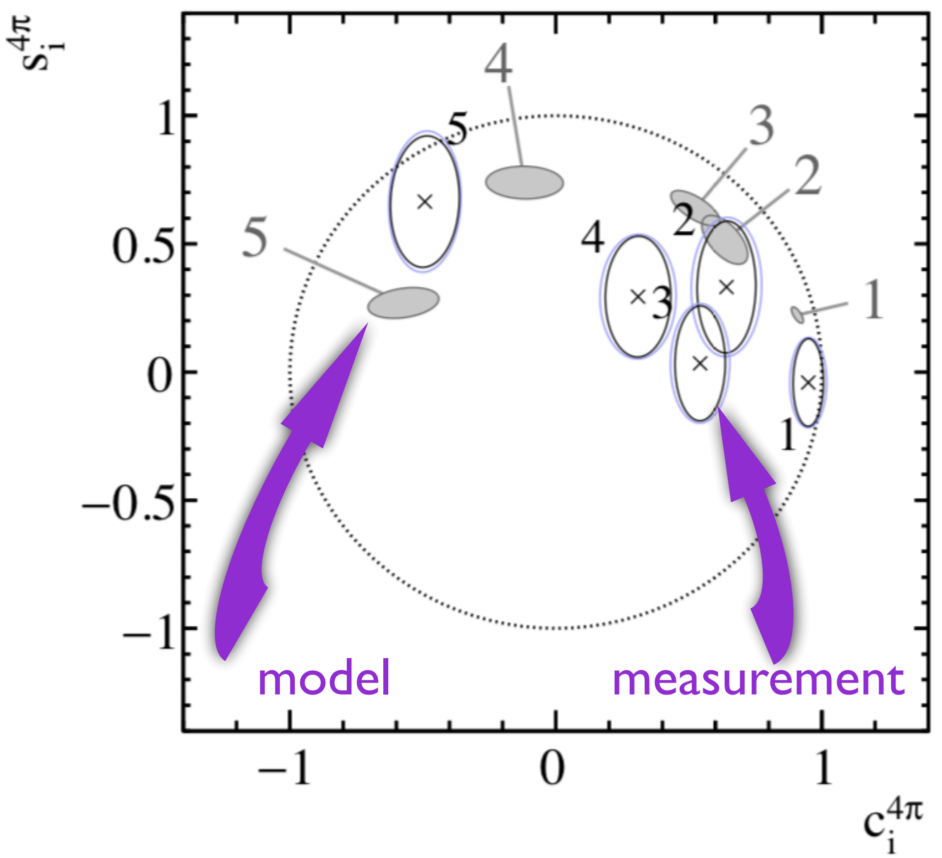
$F^+ = N^+/N^+ + N^- = 0.973 \pm 0.017$

Similarly, for $D^0 \to KK\pi^0$

$F^+ = 0.732 \pm 0.055$

CP content and c_i and s_i for $D^0 \to \pi^-\pi^+\pi^-\pi^+$

S. Harnew et al. JHEP 01, 144 (2018)

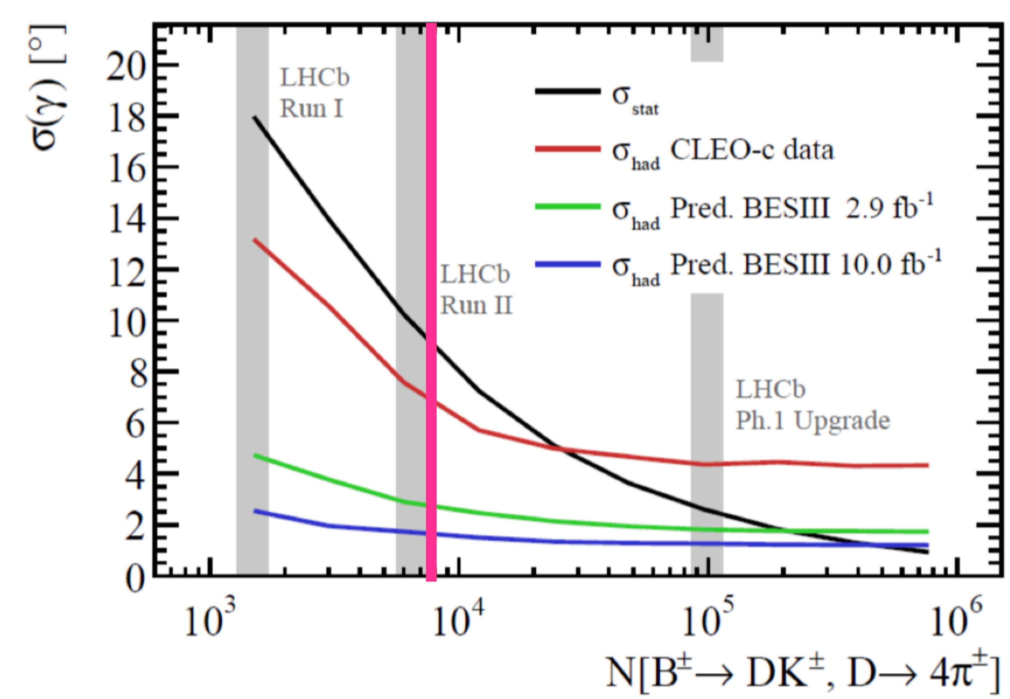


5D binning inspired by an amplitude model
$F^+ = 0.769 \pm 0.021 \pm 0.010$

compatible with results from model
$= 0.729 \pm 0.009\ (\text{stat}) \pm 0.015\ (\text{syst}) \pm 0.010\ (\text{model})$

d'Argent, EG, et al. JHEP 1705 (2017) 143

- $D^0 \to \pi^+\pi^-\pi^+\pi^-$ (5D space):
    - expected Run 2 precision ~ 10°;
    - external CLEO-c input uncertainty ~ 7°

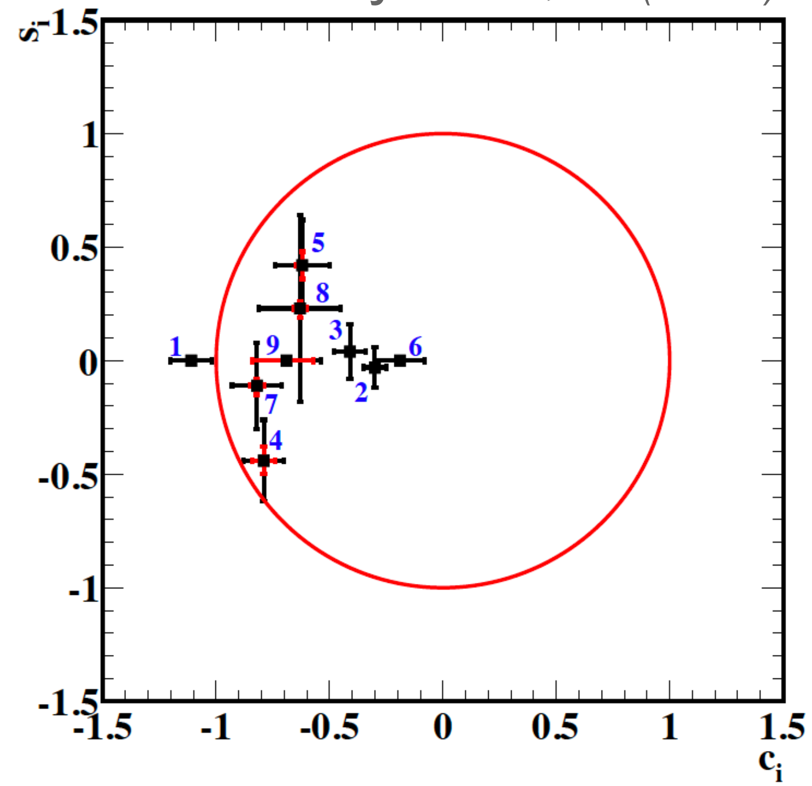


$D^0 \to K_S \pi^+ \pi^- \pi^+ \pi^0$ decays

- $D^0 \to K_S \pi^+ \pi^- \pi^0$: interesting rich resonant structure
- Some intermediate decays are CF modes(ADS), others CP-eigenstates (GLW-like)
- The phase-space is binned around the resonances

Bin	resonance	c_i	s_i
1	ω	$-1.11 \pm 0.09^{+0.02}_{-0.01}$	0.00
2	$K^{*-}\rho^+$	$-0.30 \pm 0.05 \pm 0.01$	$-0.03 \pm 0.09^{+0.01}_{-0.02}$
3	$K^{*+}\rho^-$	$-0.41 \pm 0.07^{+0.02}_{-0.01}$	$0.04 \pm 0.12^{+0.01}_{-0.02}$
4	K^{*-}	$-0.79 \pm 0.09 \pm 0.05$	$-0.44 \pm 0.18 \pm 0.06$
5	K^{*+}	$-0.62 \pm 0.12^{+0.03}_{-0.02}$	$0.42 \pm 0.20 \pm 0.06$
6	K^{*0}	$-0.19 \pm 0.11 \pm 0.02$	0.00
7	ρ^+	$-0.82 \pm 0.11 \pm 0.03$	$-0.11 \pm 0.19^{+0.04}_{-0.03}$
8	ρ^-	$-0.63 \pm 0.18 \pm 0.03$	$0.23 \pm 0.41^{+0.04}_{-0.03}$
9	remainder	$-0.69 \pm 0.15^{+0.15}_{-0.12}$	0.00

P.K.Resmi et al. JHEP 01, 82 (2018)



- No LHCb analysis (challenging at LHCb because of the soft π^0 but high priority for BELLEII) but with 60 k (50 ab^{-1} at BELLEII) ~ 4.4°;
- CLEO-c data contribute uncertainty of 1.5°

Charm input to γ from $D^0 \to K3\pi$

The coherence factor and the average strong phase needed for the determination of γ

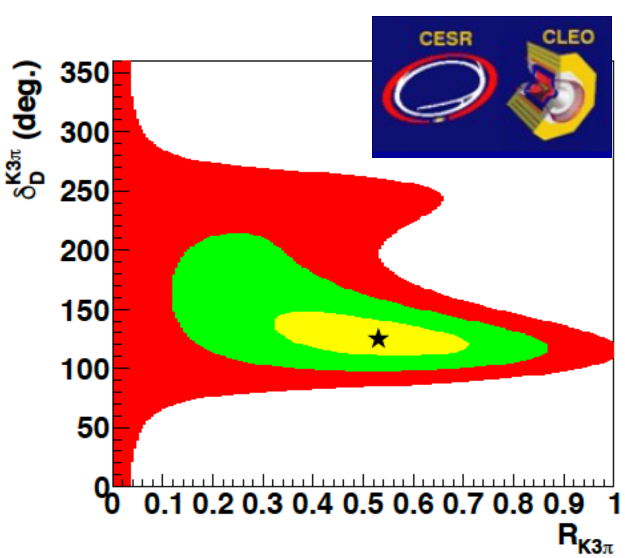
$$R_{K3\pi} e^{-i\delta_D^{K3\pi}} = \frac{\int \mathcal{A}^*_{K^-\pi^+\pi^+\pi^-}(\mathbf{x}) \mathcal{A}_{K^+\pi^-\pi^+\pi^-}(\mathbf{x}) \mathrm{d}\mathbf{x}}{A_{K^-\pi^+\pi^+\pi^-} A_{K^+\pi^-\pi^+\pi^-}}$$

$$Re^{-i\delta_D} = c_i + is_i$$

Use interference effects in charm as input to γ

$$\Gamma(D^0 \to f) \sim \Gamma e^{-\Gamma} \left[|A_f|^2 + \Gamma t \cdot R^{K3\pi} \left|\frac{q}{p}\right| |A_f| \left|\overline{A}_f\right| \left(y\cos(\delta^{K3\pi} - \phi) + x\sin(\delta^{K3\pi} - \phi) \right) \right]$$

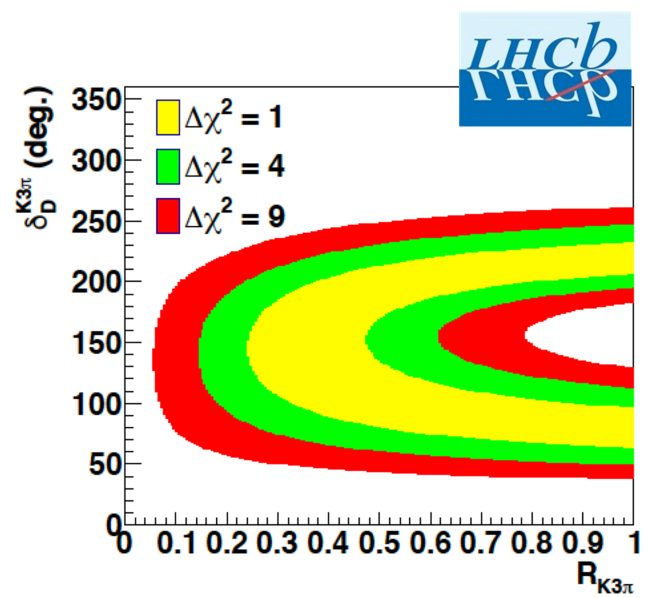
Scan from D-D̄ superpositions at CLEO-c



Phys.Lett. B757 (2016) 520-527

+

Input from charm mixing (LHCb)

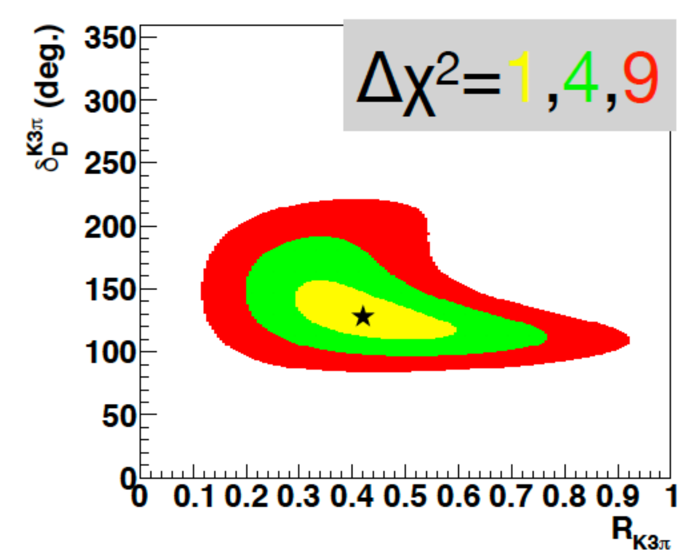


PRL 116 (2016) no.24, 241801

study of the time-dependence of the ratio between $D^0 \to K^+\pi^-\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^+\pi^-$ decay rates

=

Combination:CLEO-c and mixing.



Phys.Lett. B757 (2016) 520-527

+ additional LHCb constraints

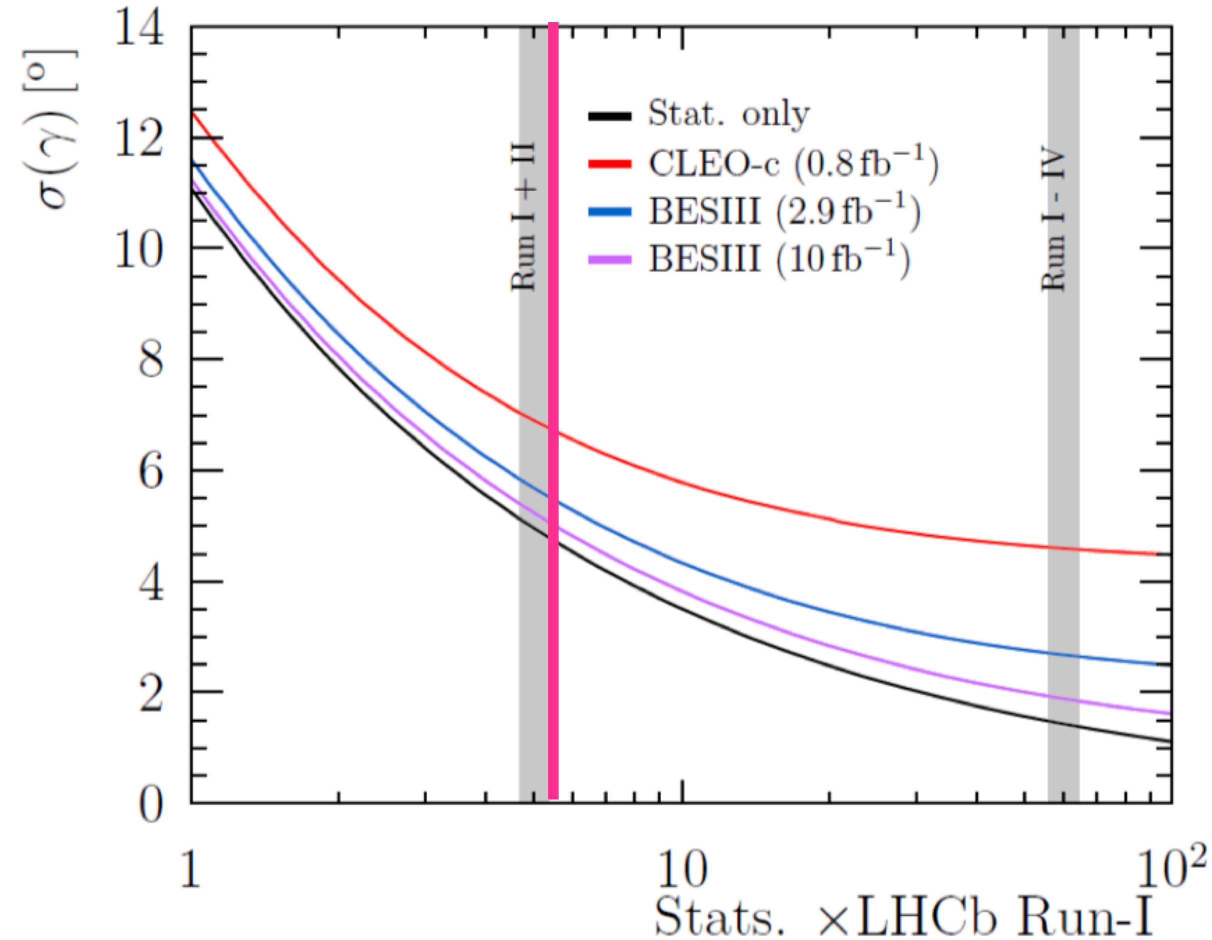
Future projections for $D^0\to K3\pi$

Results of the combined fit

Parameter	Fitted value
$R_{K3\pi}$	$0.43^{+0.17}_{-0.13}$
$\delta_D^{K3\pi}$	$(128^{+28}_{-17})^\circ$
$r_D^{K3\pi}$	$(5.49\pm0.06)\times10^{-2}$

- Excellent prospects for $D^0\to K3\pi$ (5D space):
  - expected Run 2 precision ~ 5.5°; LHCb
  - urgently need BESIII input

$D^0\to K3\pi$



G.Wilkinson BESIII-LHCb joint workshop 2018

Shopping list

Priority ordered for LHCb

Different priorities and different measurements possible at BELLEII

Decay mode	Quantity of interest	Comments
$D \to K_S^0\pi^+\pi^-$	c_i and s_i	Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$data sets, it might be worthwhile to explore alternative binning.
$D \to K_S^0K^+K^-$	c_i and s_i	Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$data sets, it might be worthwhile to explore alternative binning.
$D \to K^\pm\pi^\mp\pi^+\pi^-$	R, δ	In bins guided by amplitude models, currently under development by LHCb.
$D \to K^+K^-\pi^+\pi^-$	c_i and s_i	Binning scheme can be guided by the CLEO model [18] or potentially an improved model from LHCb in the future.
$D \to \pi^+\pi^-\pi^+\pi^-$	F_+ or c_i and s_i	Unbinned measurement of F_+. Measurements of F_+ in bins or c_i and s_i in bins could be explored.
$D \to K^\pm\pi^\mp\pi^0$	R, δ	Simple 2-3 bin scheme could be considered.
$D \to K_S^0K^\pm\pi^\mp$	R, δ	Simple 2 bin scheme where one bin encloses the K^* resonance.
$D \to \pi^+\pi^-\pi^0$	F_+	No binning required as $F_+ \sim 1$.
$D \to K_S^0\pi^+\pi^-\pi^0$	F_+ and c_i and s_i	Unbinned measurement of F_+ required. Additional measurements of F_+ or c_i and s_i in bins could be explored.
$D \to K^+K^-\pi^0$	F_+	Unbinned measurement required. Extensions to binned measurements of either F_+ or c_i and s_i possible.
$D \to K^\pm\pi^\mp$	δ	Of low priority due to good precision available through charm-mixing analyses.

LHCb-PUB-2016-025

Summary

- The γ determination represents a great opportunity for synergy between LHCb/ BELLEII and BESIII
- Measurements of strong phase differences from quantum correlated measurements will play an important role in future CPV measurements
- Sub-degree precision is attainable – but only if LHCb and BESIII work together !
- More Ψ(3770) data are required to exploit fully the very large future samples at LHCb and BELLEII
- Several analyses ongoing at BESIII

MANCHESTER
1824
The University of Manchester

BACKUP

Measurements with CLEO-c data (* indicates legacy-data publication)

$K\pi$ strong phase	PRD 86 (2012) 112001, arXiv:1210.0939
$K_S\pi\pi$, K_SKK binned c_i, s_i	PRD 82 (2010) 112006, arXiv:1010.2817
$K3\pi$ and $K\pi\pi^0$ coherence factor & strong phase	PLB 757 (2016) 520, Corrigendum ibid. 765 (2017) 402, arXiv:1602.07430 *
$K_SK\pi$ coherence factor & strong phase	PRD 85 (2012) 092016, arXiv:1203.3804
$\pi\pi\pi^0$, $KK\pi^0$ CP-content	PLB 747 (2015) 9, arXiv:1504.05878 *
4π CP-content & binned c_i,s_i	JHEP 01 (2018) 144, arXiv:1709.03467 *
$K_S\pi\pi\pi^0$ c_i, s_i	JHEP 01 (2018) 982, arXiv:1710.10086 *

BESIII measurements

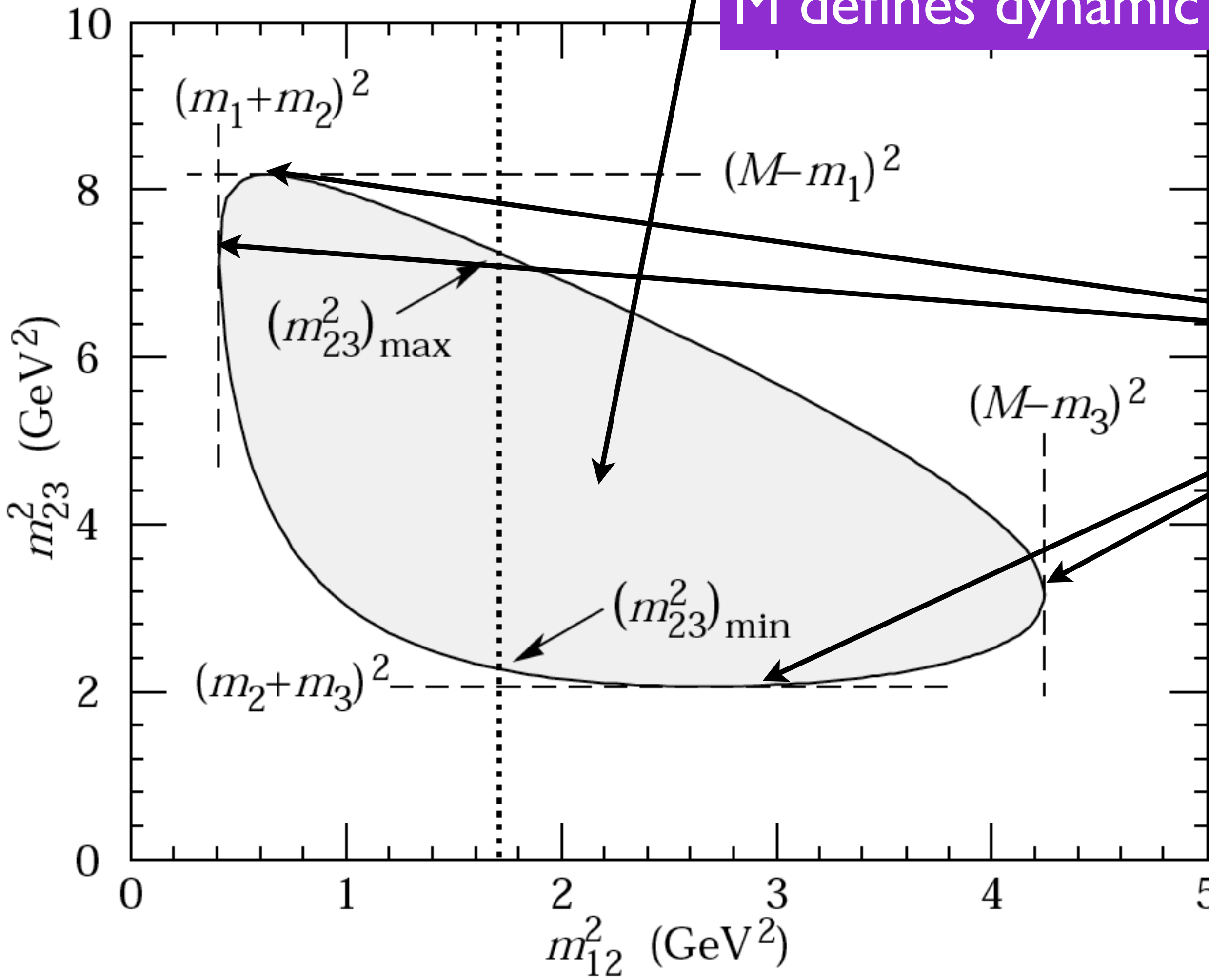
$K\pi$ strong phase	PLB 734 (2014) 227, arXiv:1404.4691
Measurement of y_{CP} with KK	PLB 744 (2015) 339, arXiv:1501.01378

Dalitz plot

- A decay of a pseudo-scalar (e.g. D) into a final state with three pseudo-scalars (e.g. K, π) can be parametrised as

$$\Gamma = \frac{1}{(2\pi)^3 32\sqrt{s^3}} \left|\mathcal{M}\right|^2 dm_{12}^2 \, dm_{23}^2$$

M defines dynamic substructure



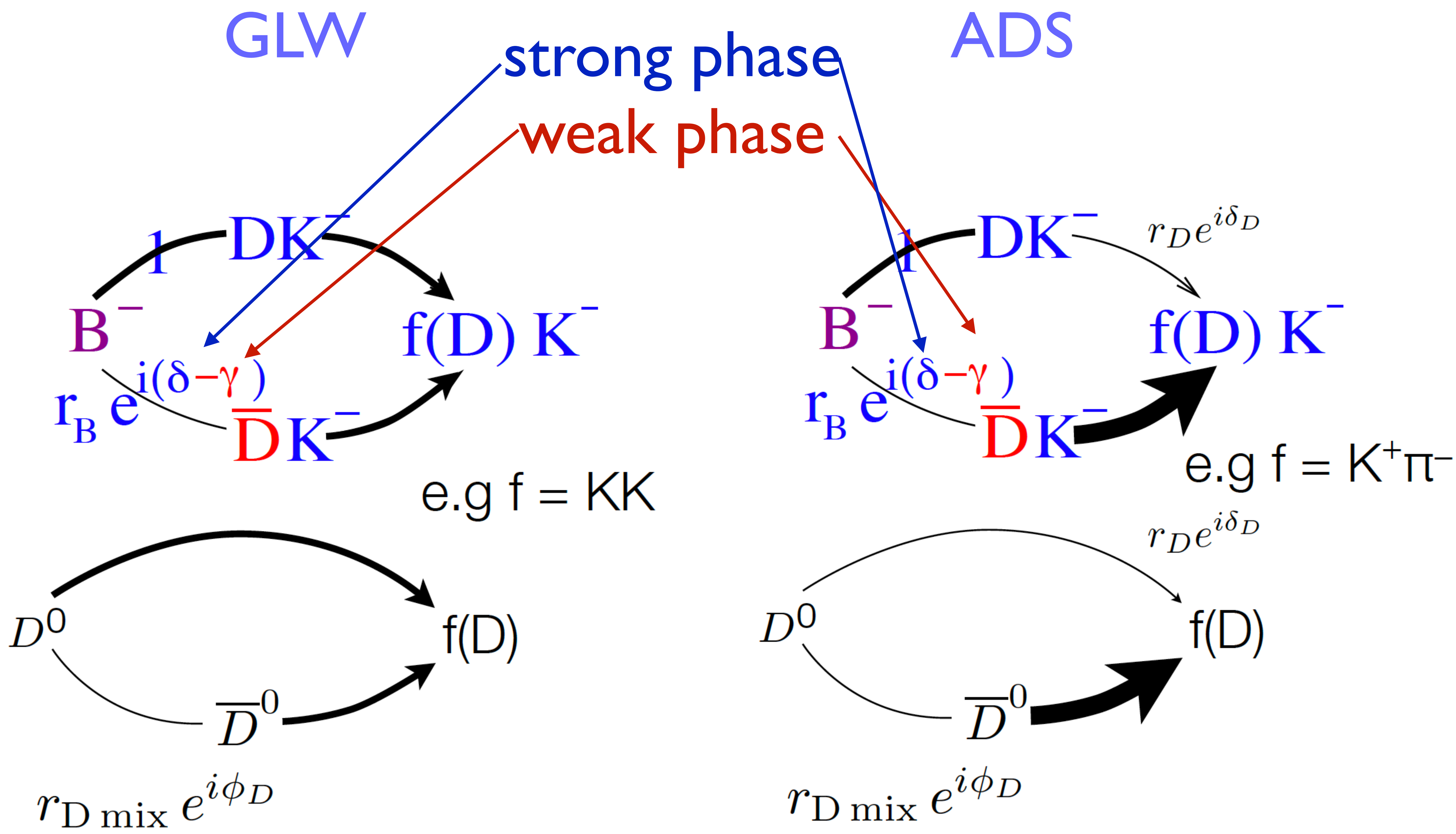
kinematic boundaries depending on mother and daughter masses

Dalitz plot

Additional information about the spin of the resonances

MANCHESTER 1824
The University of Manchester

The interference and the phases



measure CP asymmetries and ratios of decay rates

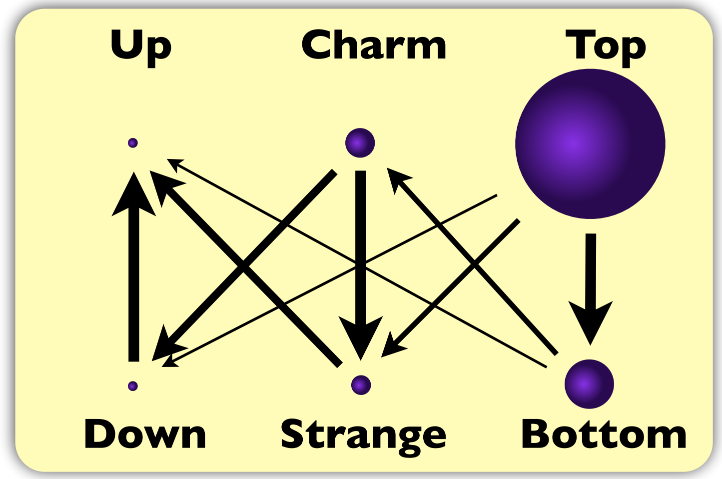
THE ROYAL SOCIETY

MANCHESTER 1824
The University of Manchester

CKM matrix

- Unitary matrix combining flavour and mass eigenstates

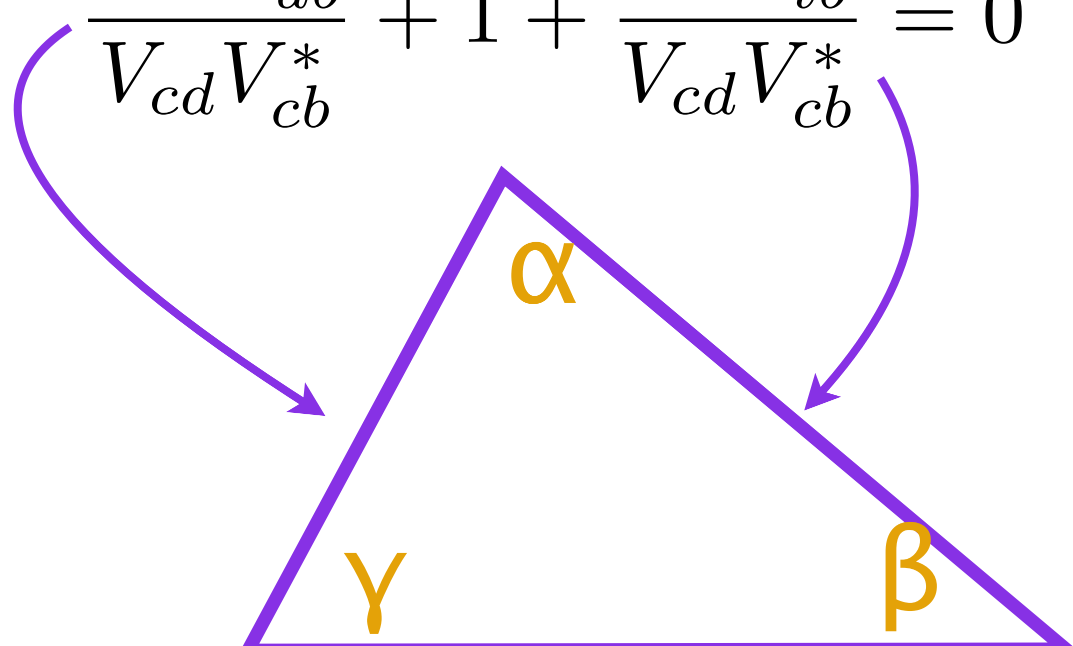
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- Unitarity relations lead to triangles in complex plane

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

B_d triangle



One of the main goals of flavour physics is to determine the angle γ/φ3 of the Bd CKM triangle

THE ROYAL SOCIETY

MANCHESTER 1824
The University of Manchester

Q — a ratio of a statistical sensitivity to that in the unbinned case. Specifically, Q relates the number of standard deviations by which the number of events in bins is changed by varying parameters x and y, to the number of standard deviations if the Dalitz plot is divided into infinitely small regions (the unbinned case):

$$Q^2 = \frac{\sum\limits_i \left(\frac{1}{\sqrt{F_i}}\frac{dF_i}{dx}\right)^2 + \left(\frac{1}{\sqrt{F_i}}\frac{dF_i}{dy}\right)^2}{\int\limits_{\mathcal{D}} \left[\left(\frac{1}{\sqrt{|f_B|^2}}\frac{d|f_B|^2}{dx}\right)^2 + \left(\frac{1}{\sqrt{|f_B|^2}}\frac{d|f_B|^2}{dy}\right)^2\right] d\mathcal{D}}, \tag{11}$$

where $f_B = f_D + (x + iy)\bar{f}_D$, $F_i = \int_{\mathcal{D}_i} |f_B|^2 d\mathcal{D}$.

THE ROYAL SOCIETY