INCLUSIVE SEMILEPTONIC B DECAYS and the DETERMINATION of V_{cb}

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The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

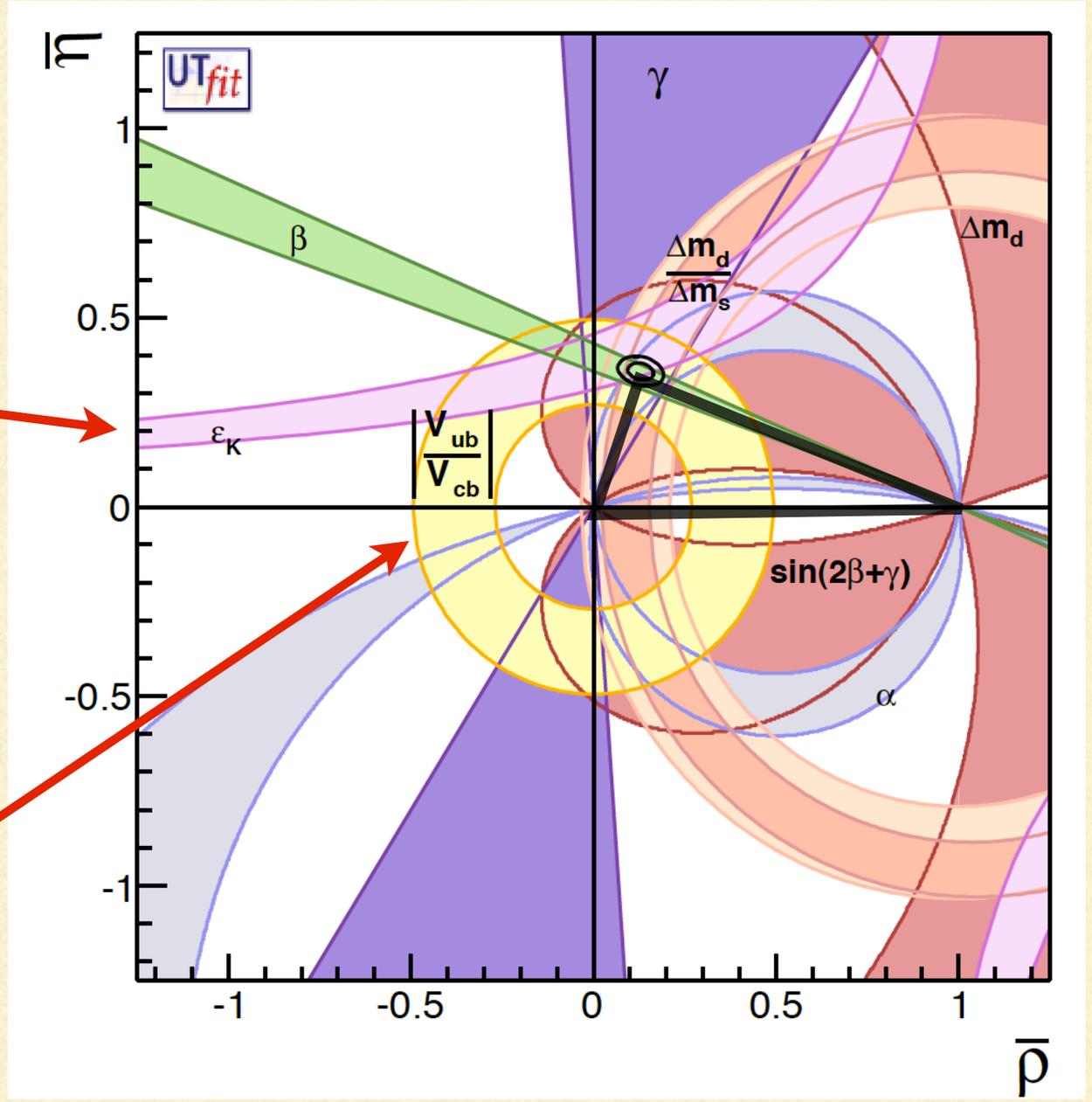
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

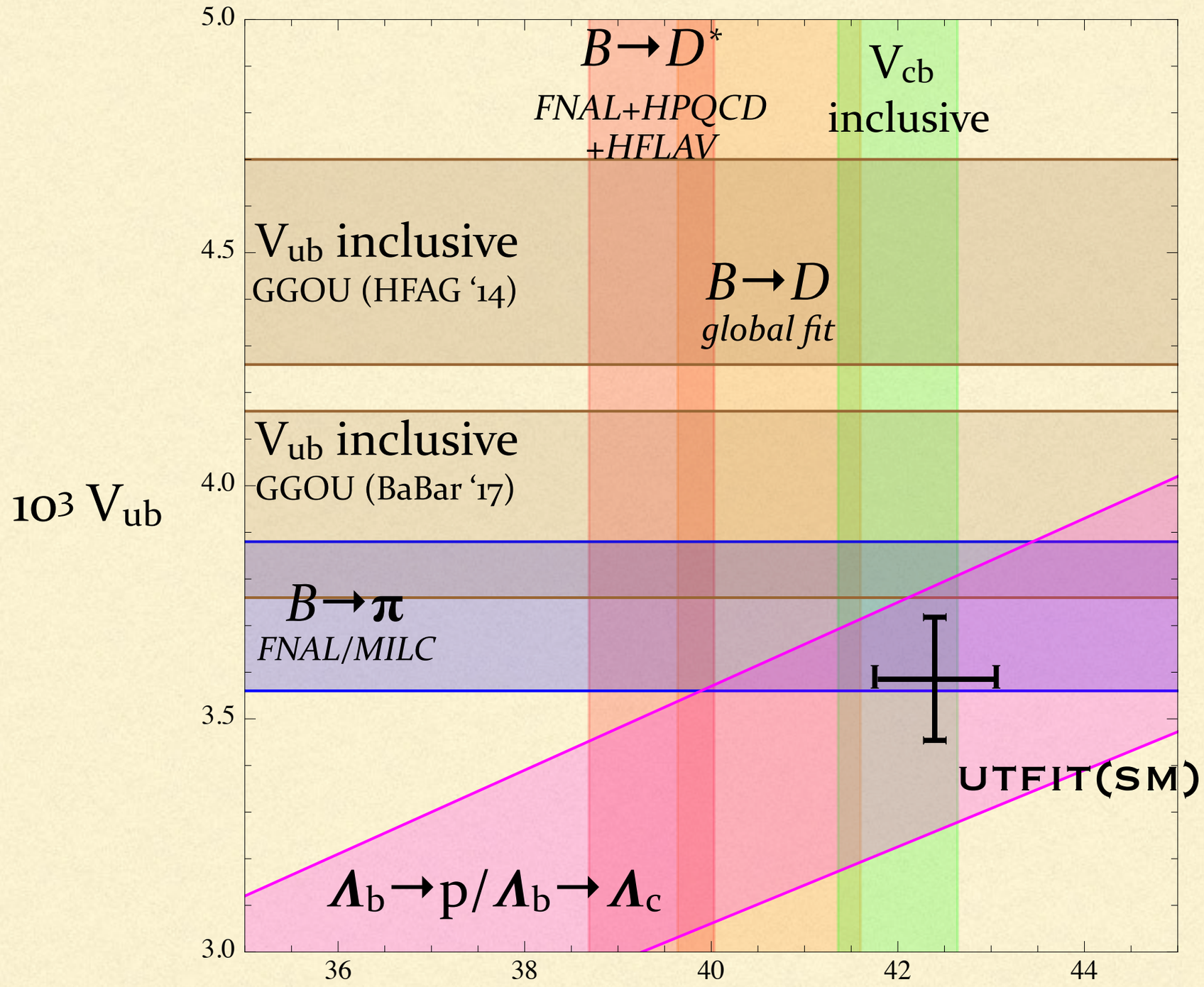
$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty.
V_{ub}/V_{cb} constrains directly the UT

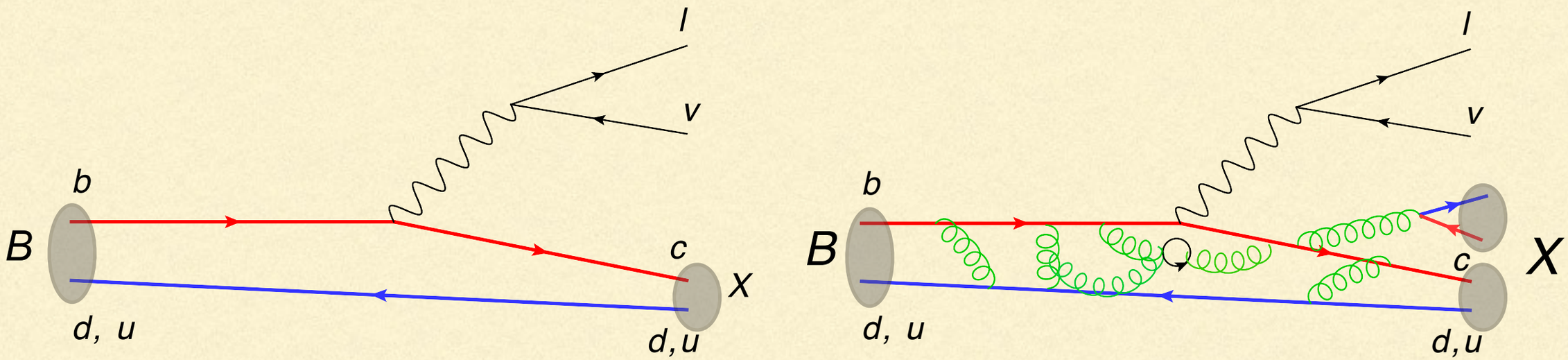


Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

STATUS of V_{cb} and V_{ub}



INCLUSIVE DECAYS: BASICS



- Simple idea: inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators

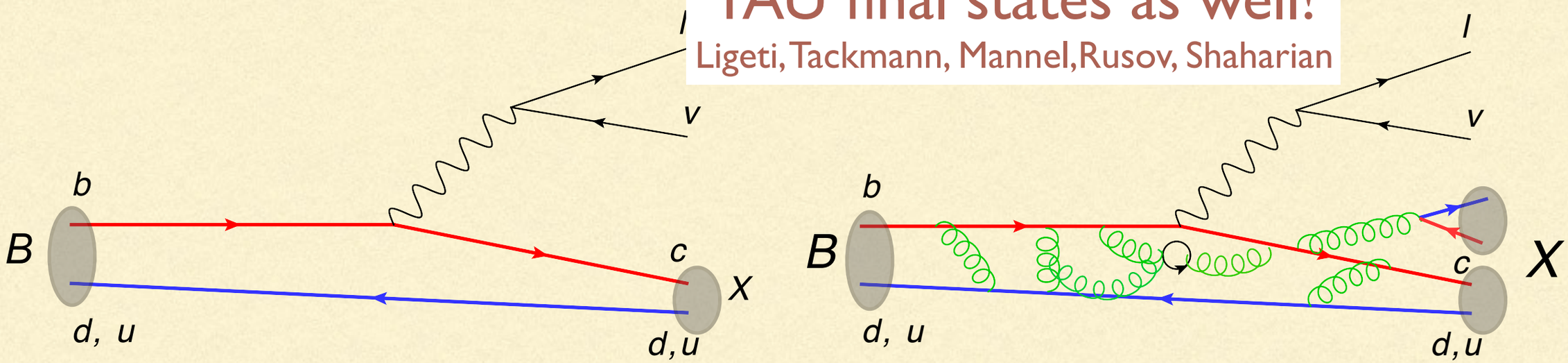
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: double series in α_s, Λ/m_b

- Lowest order: decay of a free b, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

INCLUSIVE DECAYS: BASICS

TAU final states as well!

Ligeti, Tackmann, Mannel, Rusov, Shaharian



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INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b}_v \left(i\vec{D}\right)^2 b_v \right| B \right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v \right| B \right\rangle_\mu$$

OPE valid for inclusive enough measurements, away from perturbative singularities: semileptonic width, moments

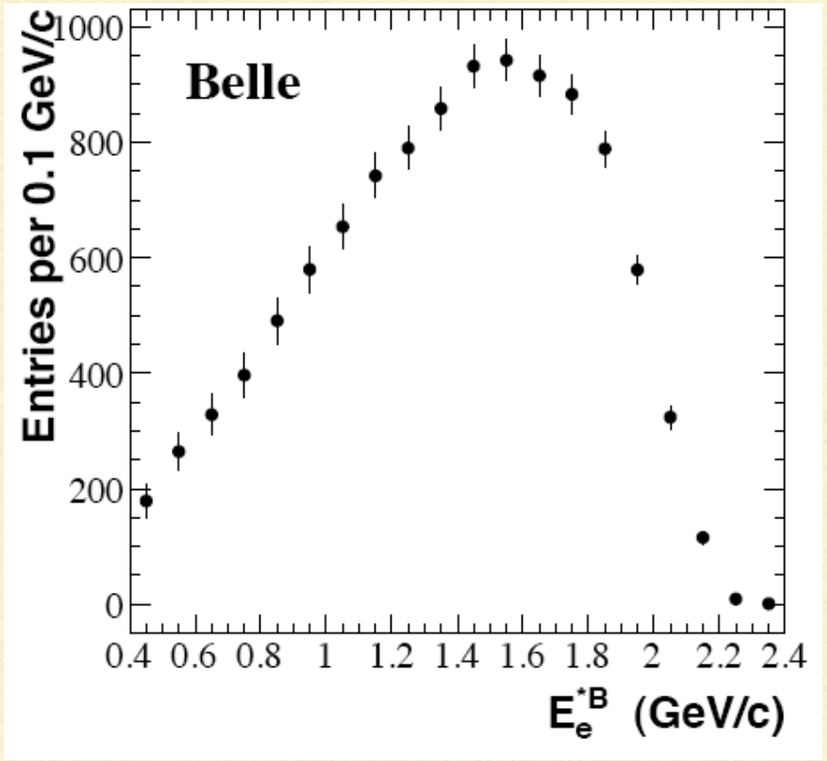
Most fits include 6 non-pert parameters

m_b m_c $\mu_{\pi,G}^2$ $\rho_{D,LS}^3$

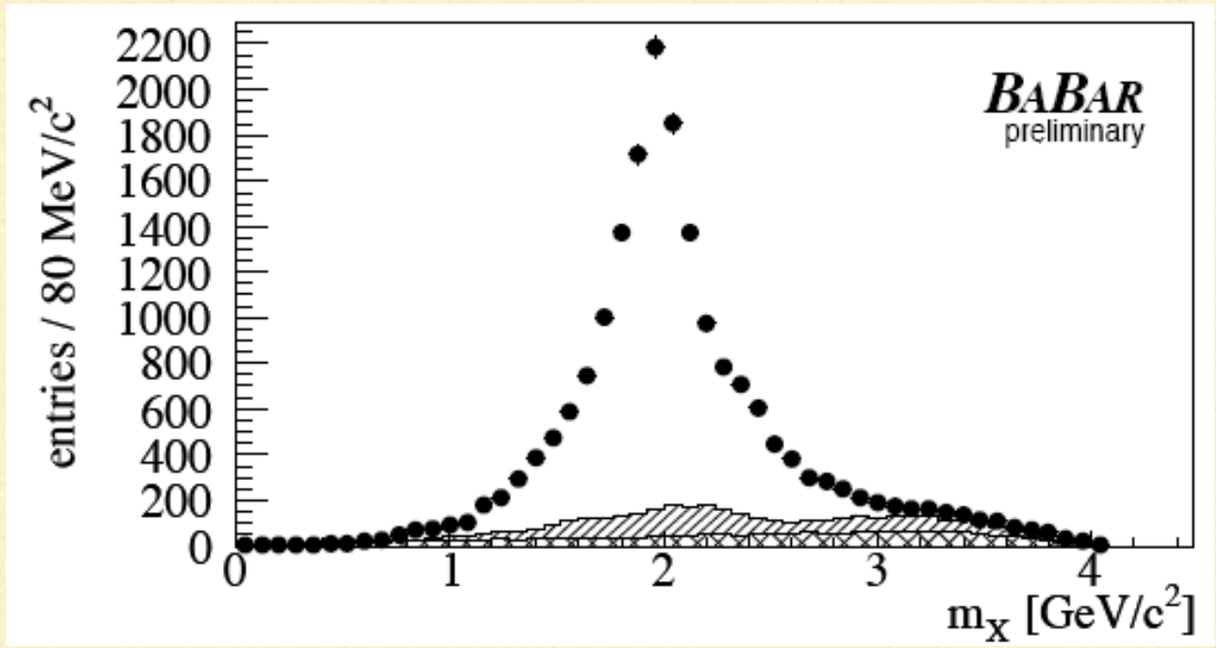
and all known corrections up to $O(\Lambda^3/m_b^3)$

EXTRACTION OF THE OPE PARAMETERS

lepton energy spectrum



hadronic mass spectrum



Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

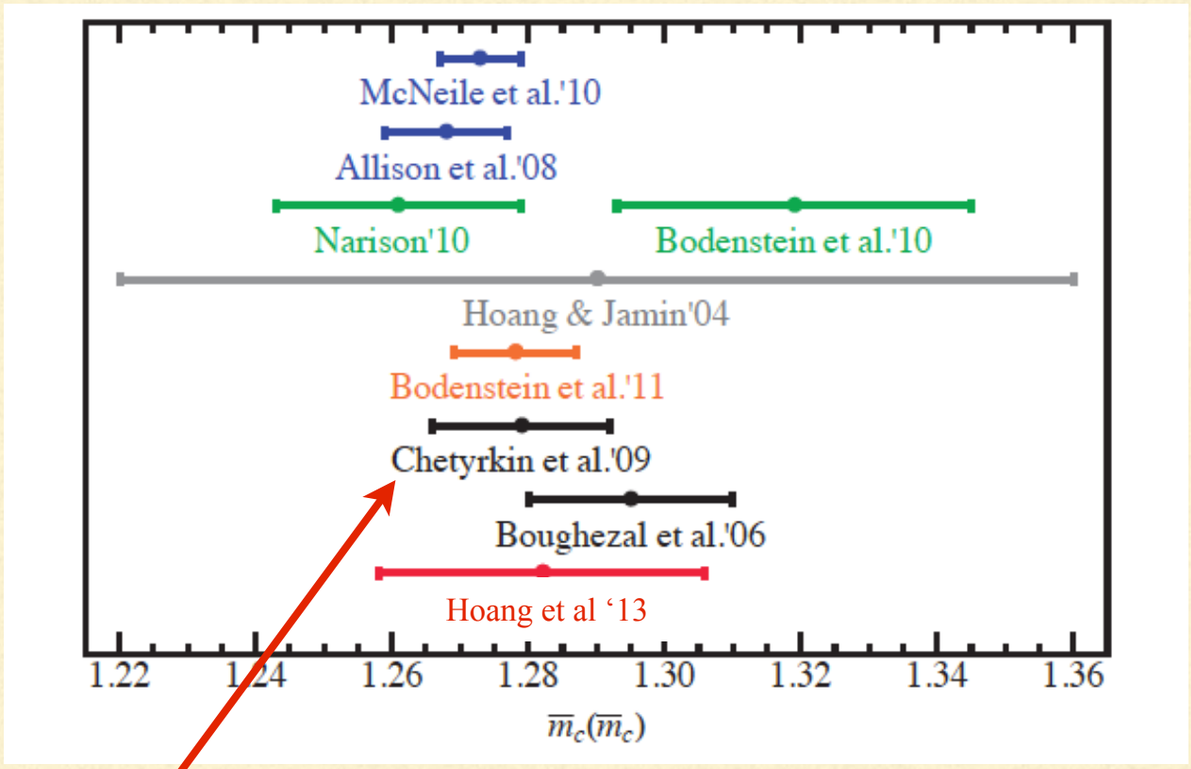
OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

THE CURRENT HFLAV S.L. FIT

- **kinetic scheme** calculation based on 1411.6560,1107.3100; hep-ph/0401063
- includes all $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$ corrections Melnikov, Biswas, Czarnecki, Pak, Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, Mannel, Pivovarov, Rosenthal
- Reliability of the method depends on our control of higher order effects. **Quark-hadron duality violation** would manifest as inconsistency in the fit.
- reassessment of theoretical errors, realistic theoretical correlations following Schwanda, PG, 1307.4551
- external constraints: precise heavy quark mass determinations, mild constraints on μ^2_G from hyperfine splitting and ρ^3_{LS} from sum rules

Older fits, like Buchmuller & Flaecher (2005), Bauer et al (2004) (1S scheme) are not up-to-date

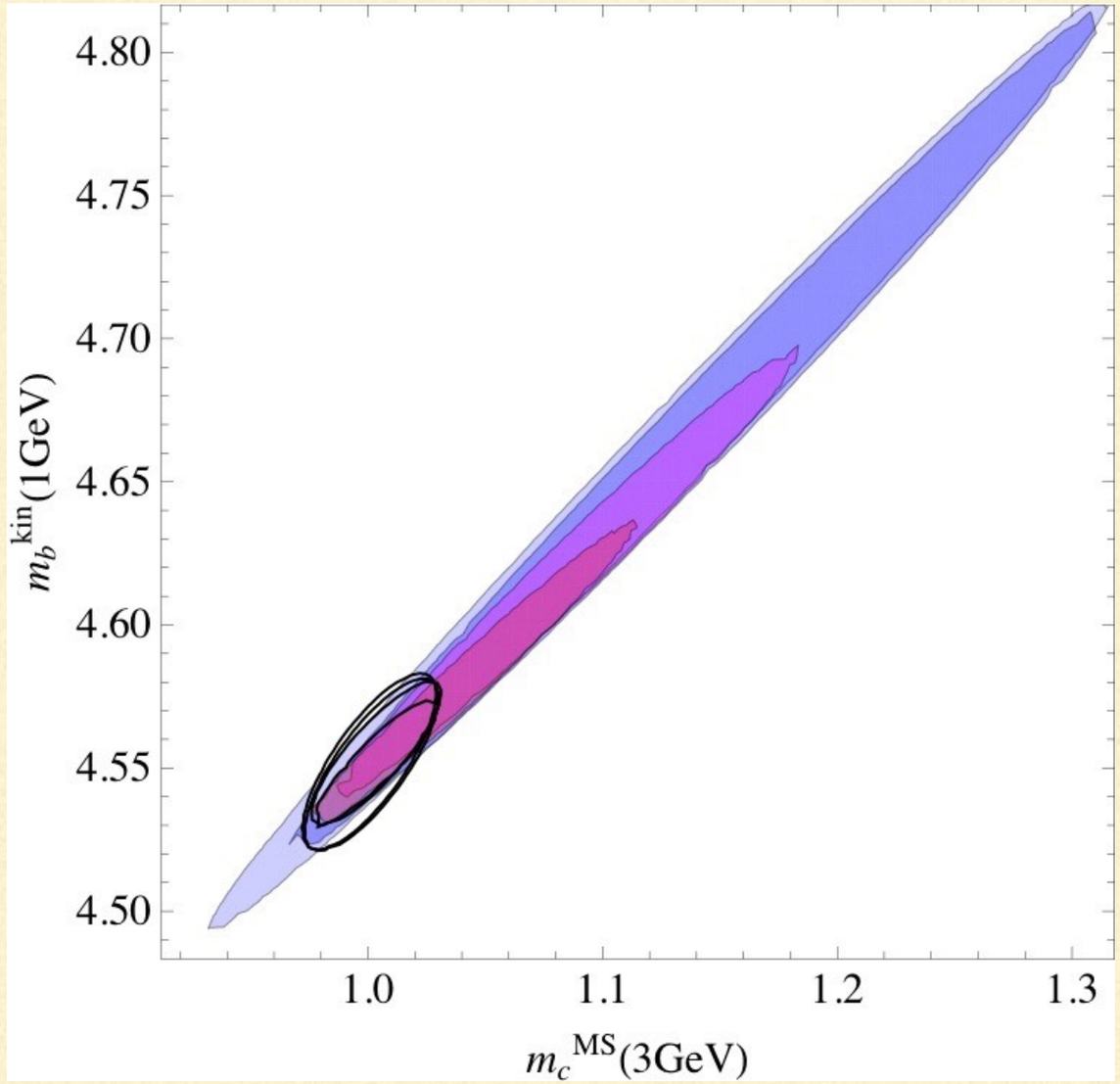
CHARM MASS



our default choice

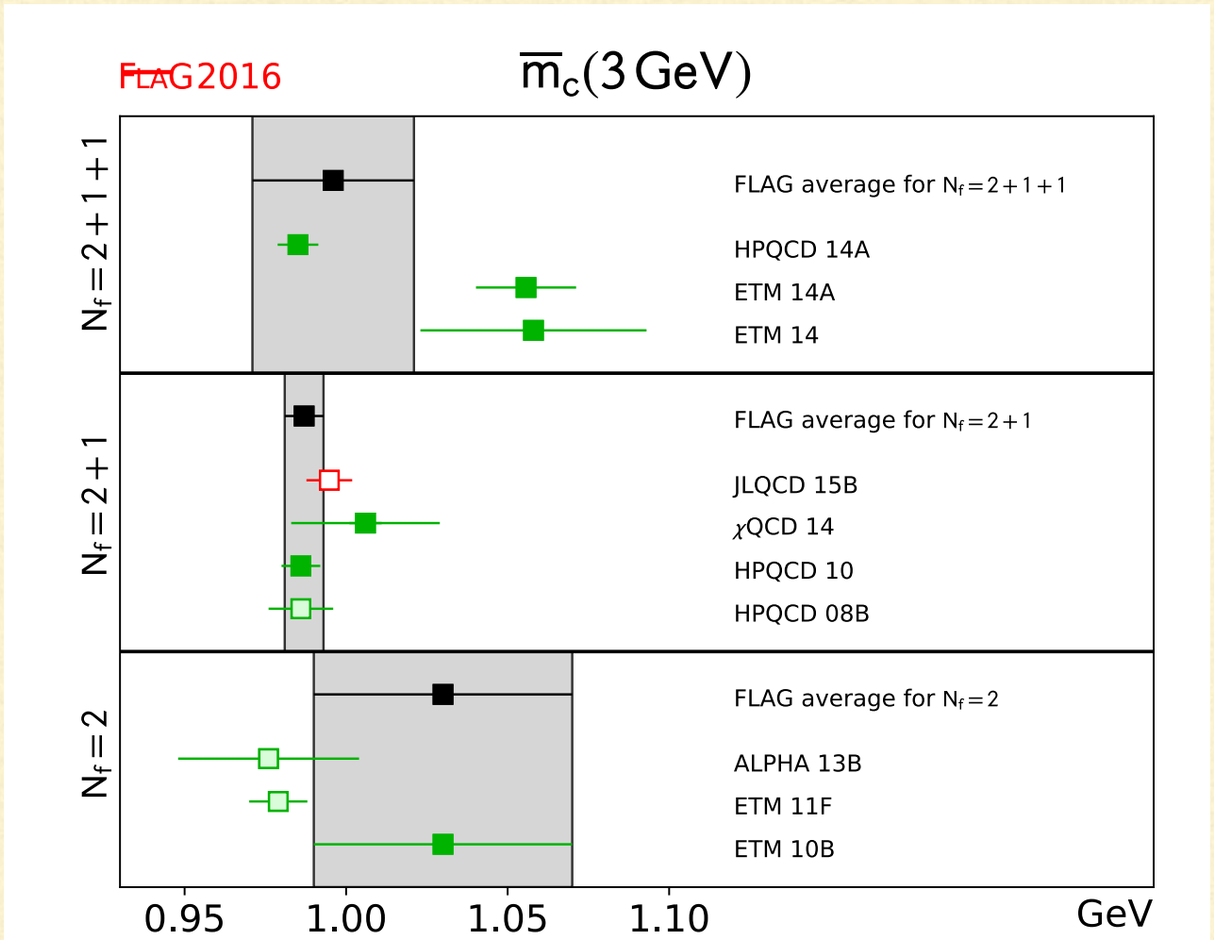
sum rules studies of $\sigma(e^+e^- \to \text{hadrons})$

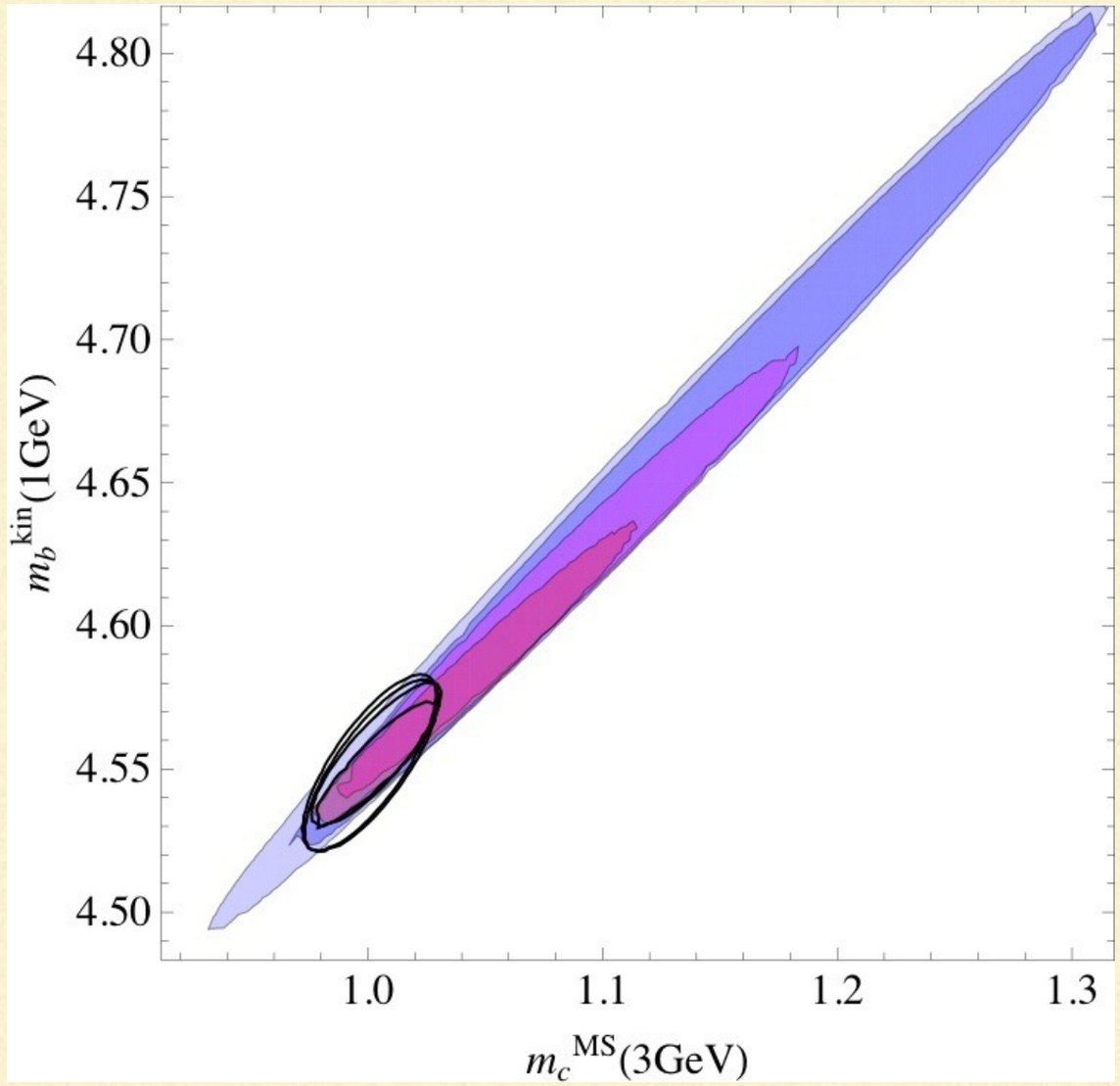
almost all at NNNLO



Remarkable improvement in the last decade. We used m_c as input to fix m_b , but higher precision on m_b should be exploited

CHARM MASS





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FIT RESULTS

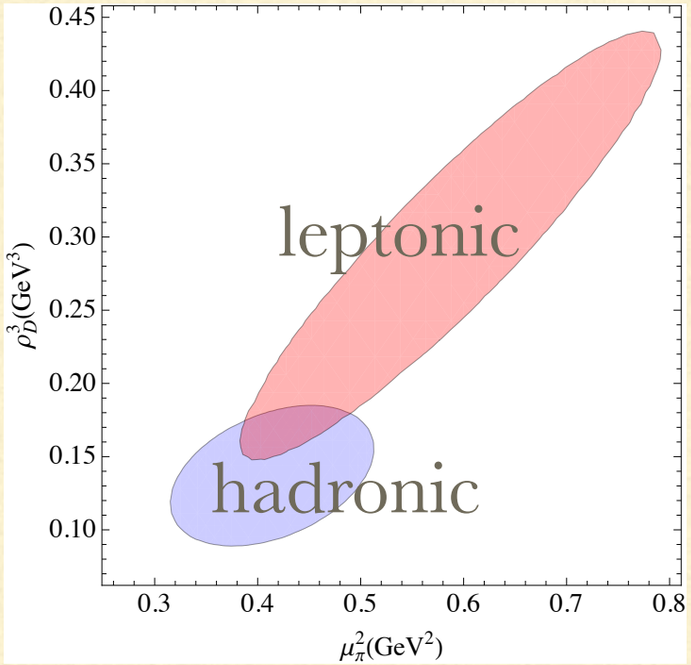
m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

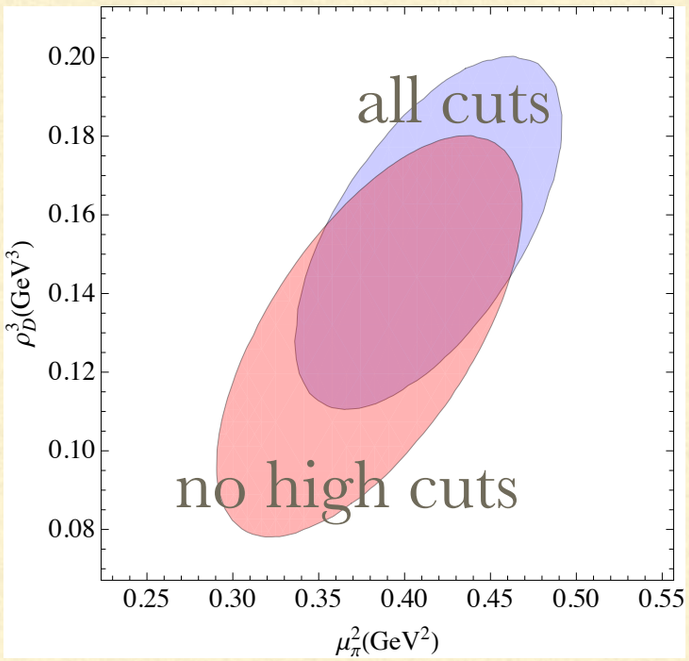
Alberti, Healey, Nandi, PG, 1411.6560

WITHOUT MASS CONSTRAINTS

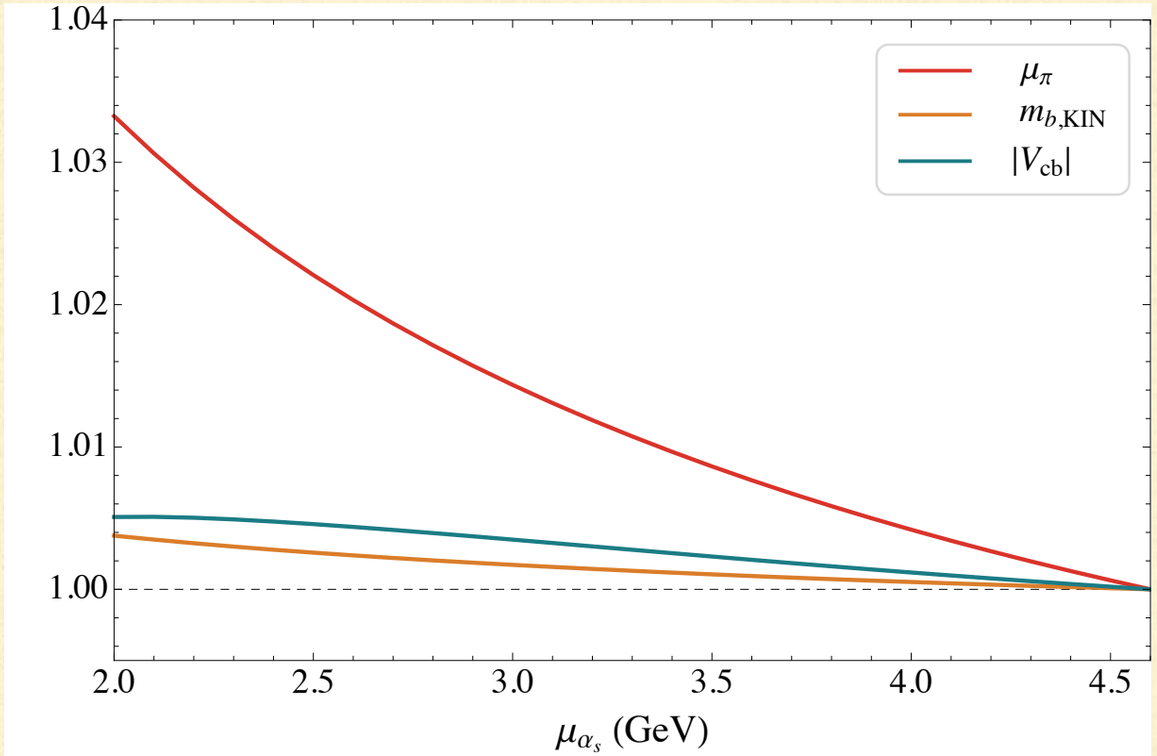
$$m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results

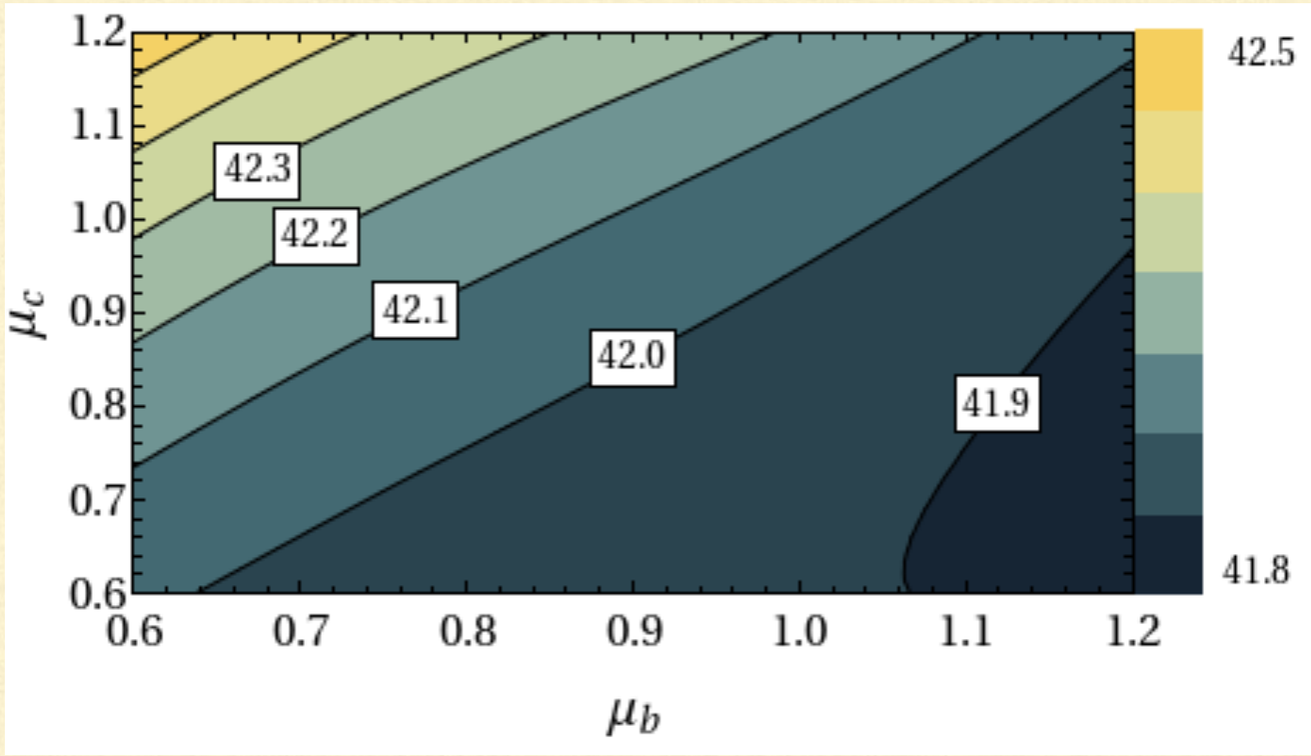




SCALE DEPENDENCE



dependence on the scale of α_s



dependence on the kin scale of m_c and m_b

HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

Mannel, Turczyk, Uraltsev 1009.4622

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen, Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\, \mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\, \mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

INCLUDING HIGHER ORDERS

Healey, Turczyk, PG 1606.06174

- Using LLSA one observes a good convergence of the HQE in width and moments
- We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers
- The rest of the fit is unchanged, with slightly smaller theoretical errors

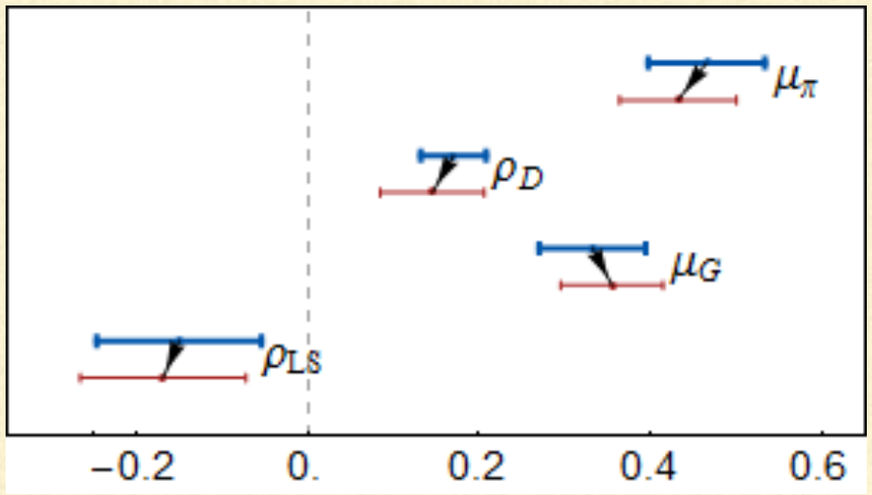
$$|V_{cb}| = 42.11(74) \times 10^{-3} \qquad \text{(0.25\% reduction)}$$

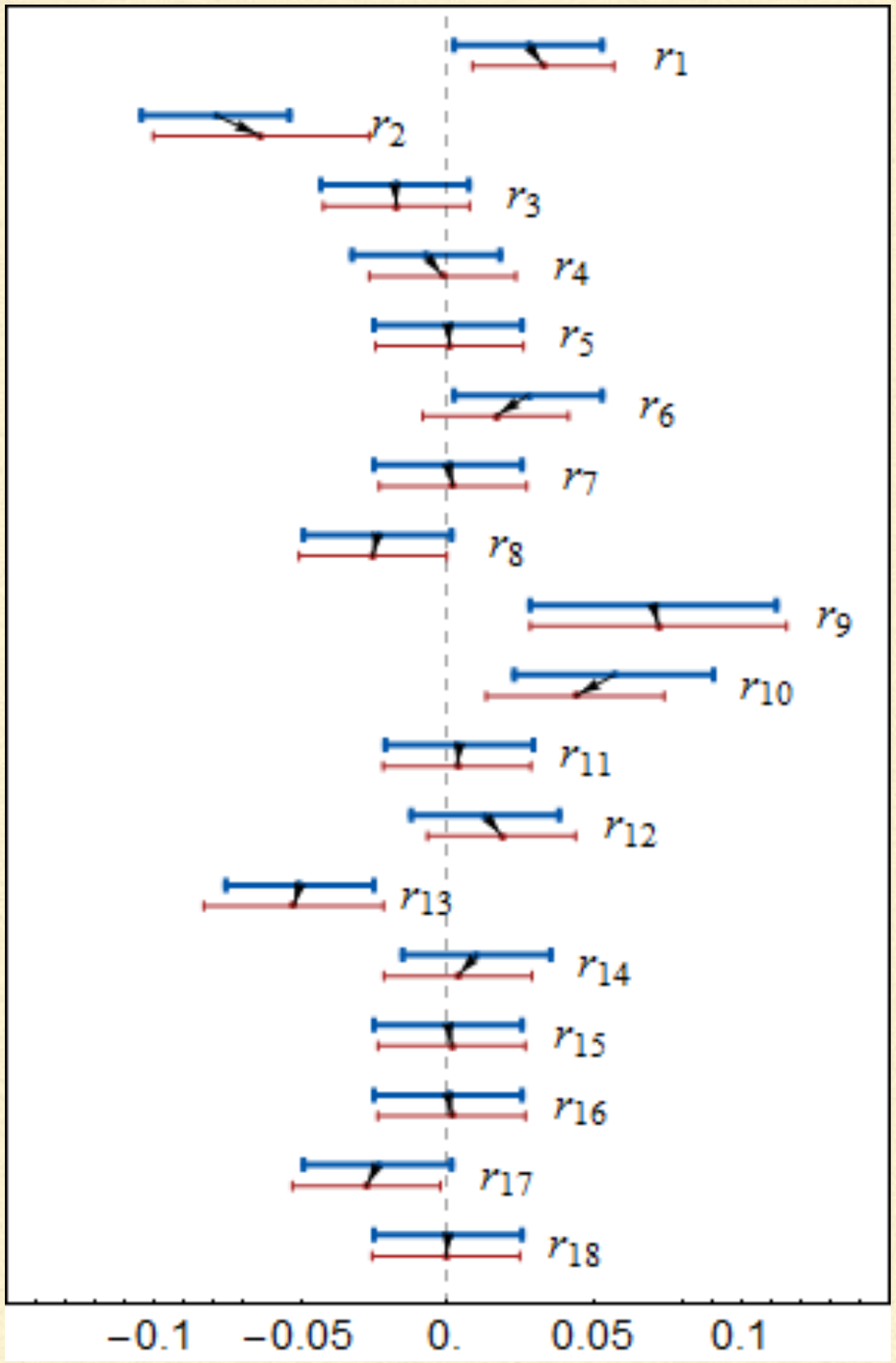
Using m_c(2GeV) and including the PDG average for m_b, 1.5% uncertainty on V_{cb}

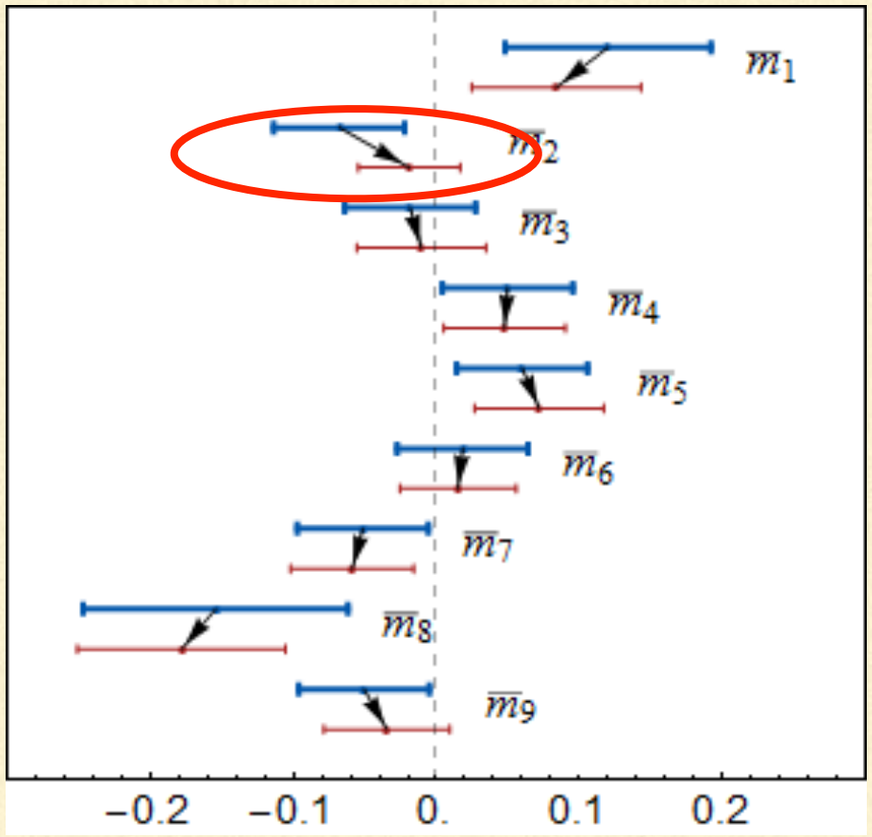
$$\boxed{|V_{cb}| = 42.00(64) \times 10^{-3}}$$

SENSITIVITY TO HIGHER POWER CORRECTIONS

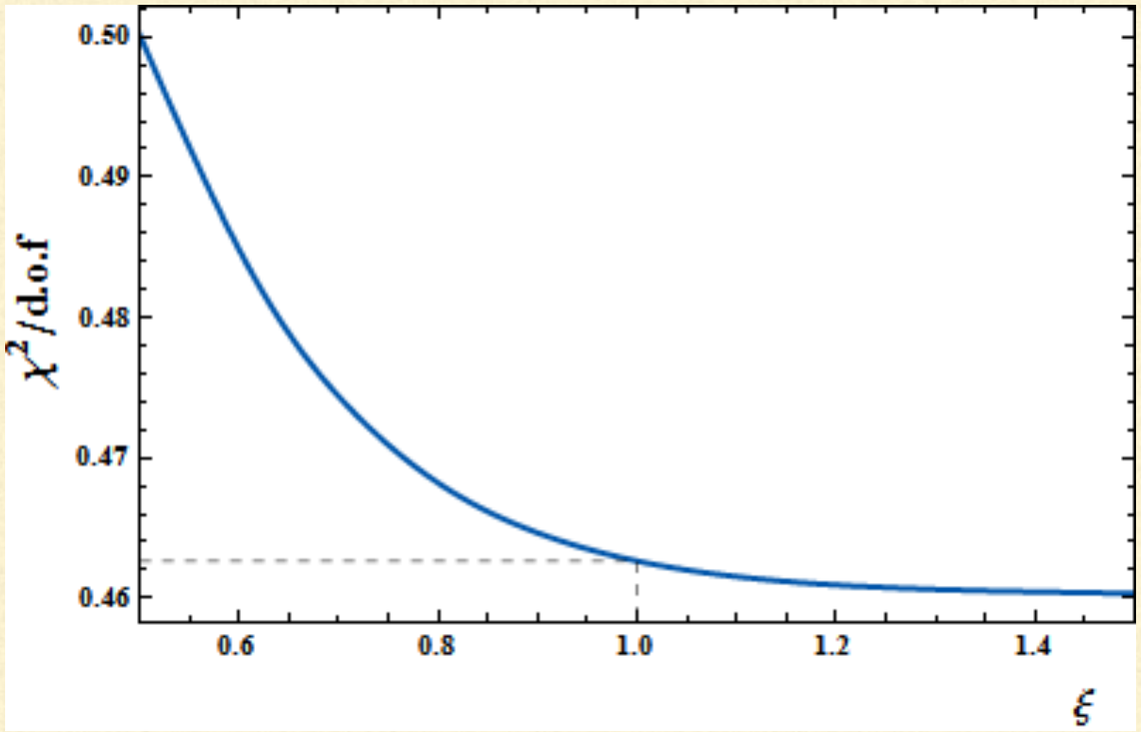
Moderate changes to the leading HQE m.e.

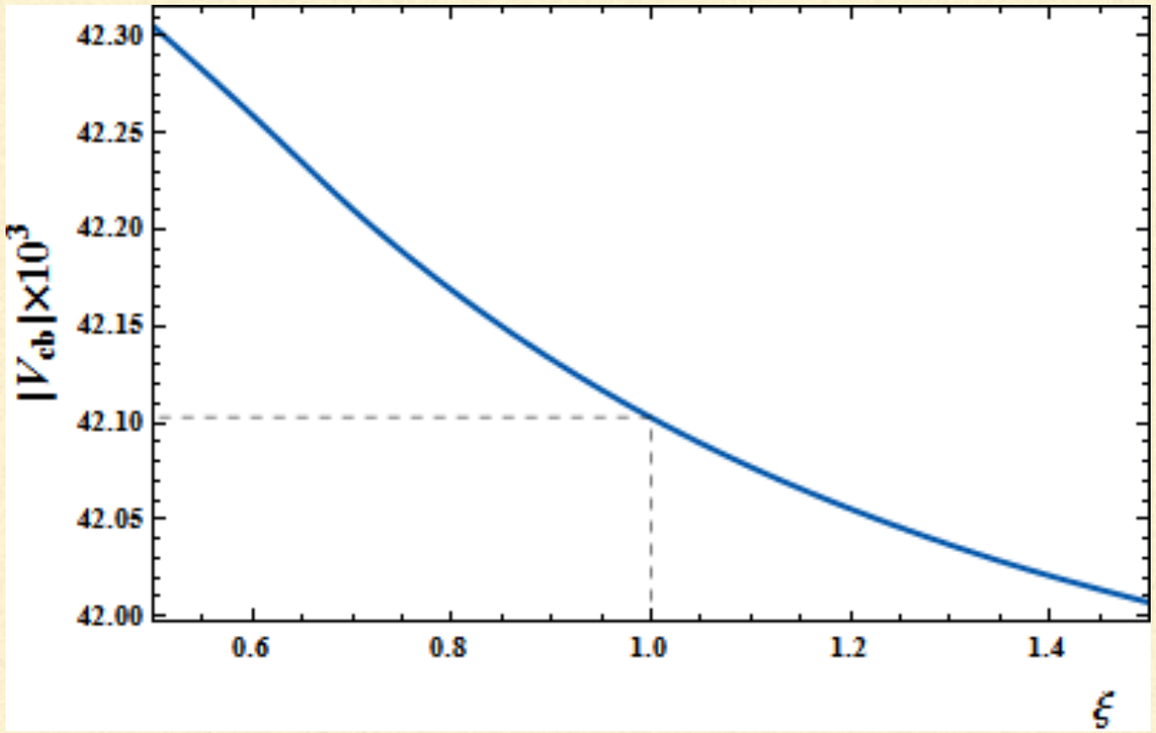






DEPENDENCE ON LLSA UNCERTAINTY



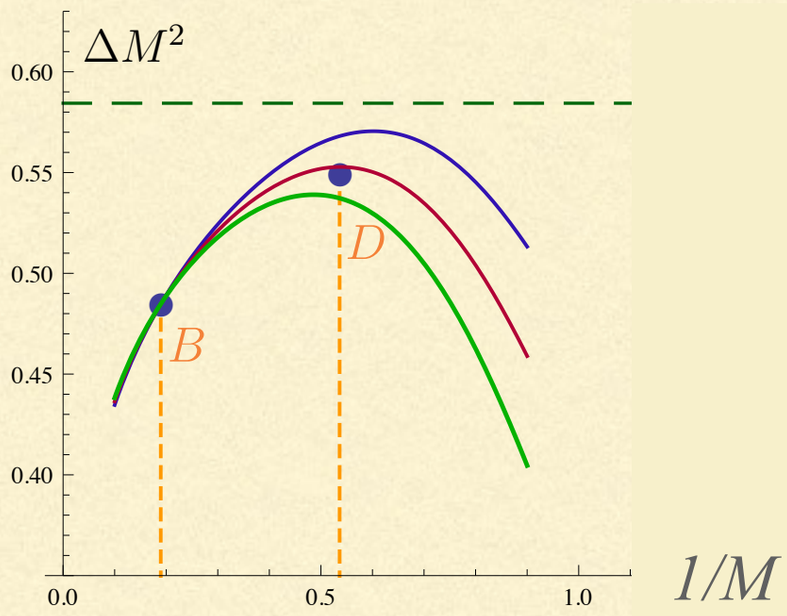


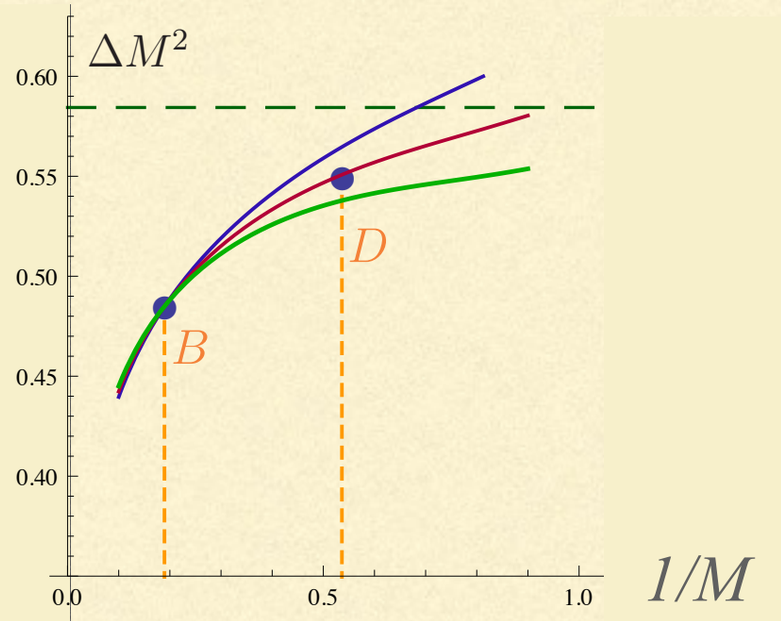
if we rescale all LLSA uncertainties by a factor **ξ** the results change very little.
Similar tiny deviations if we vary **ε** between 0.3 and 0.5 GeV

PROSPECTS for INCLUSIVE V_{cb}

- Theoretical uncertainties already dominant
- theoretical correlations between different moments?
- $O(\alpha_s/m_b^3)$ calculation under way
- $O(1/m_Q^{4,5})$ effects need further investigation but small effect on V_{cb}
- NNNLO corrections to total width feasible, needed for 1% uncertainty?
- Electroweak (QED) corrections require attention
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now
- Lattice QCD information on local matrix elements is the next frontier, e.g.

HQE PARAMETERS FROM MESON MASSES





$$\Delta M_B = M_B^* - M_B = \frac{2}{3}\frac{\mu_G^2}{m_b} + \frac{-\rho_{LS}^3 + \rho_{\pi G}^3 + \rho_A^3}{3\,m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)$$

- heavy mass expansion for hyperfine splitting
- but only 2 heavy quarks in nature: c and b
- on the lattice one can compute mesons for arbitrary quark masses

Kronfeld & Simone hep-ph/0006345, 1802.04248

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

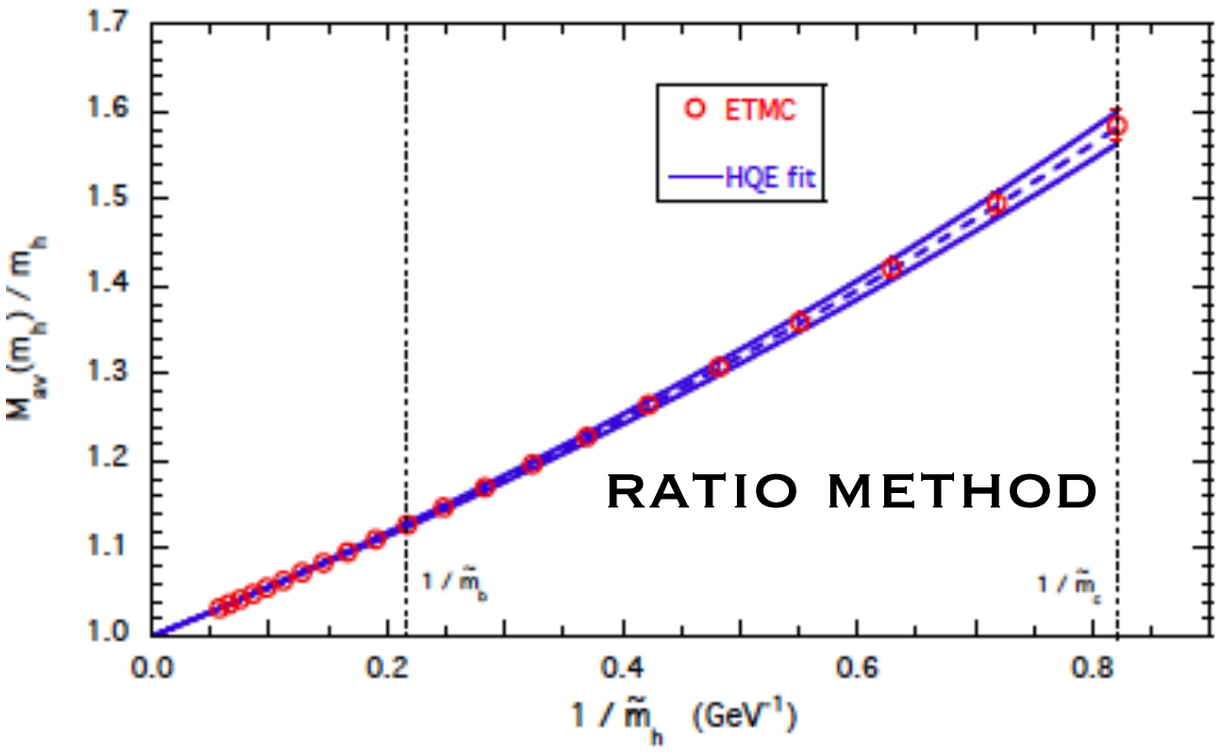
$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \dots$$

- Both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections

HQE expansion parameters from $M_{av}(\tilde{m}_h)=(M_{PS}+3M_V)/4$

We extend the chain equation beyond the b quark point, we choose $n \sim 20$ ($\tilde{m}_h \simeq 4\tilde{m}_b$),

$$\frac{M_{av}(\tilde{m}_h^{(n)})}{\tilde{m}_h^{(n)}} = \frac{M_{av}(\tilde{m}_c)}{\tilde{m}_c} \prod_{i=2}^{n} \overline{y}_M(\tilde{m}_h^{(i)}, \lambda).$$



Correlated fit based on the HQE expansion

$$\frac{M_{av}(\tilde{m}_h)}{\tilde{m}_h} = 1 + \frac{\overline{\Lambda}}{\tilde{m}_h} + \frac{\mu_\pi^2}{2\tilde{m}_h^2} + \frac{\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3}{4\tilde{m}_h^3} + \frac{\sigma^4}{\tilde{m}_h^4}$$

Results of the dim-6 fit $\tilde{m}_h > \tilde{m}_c$

$$\overline{\Lambda} = 0.551\ (13)_{\rm stat}\ (2)_{\rm syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.314\ (14)_{\rm stat}\ (2)_{\rm syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.174\ (12)_{\rm stat}\ (2)_{\rm syst}\ \text{GeV}^3$$

Results of the dim-6 fit $\tilde{m}_h > 2\tilde{m}_c$

$$\overline{\Lambda} = 0.552\ (13)\ \text{GeV}$$
$$\mu_\pi^2 = 0.323\ (16)\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.153\ (24)\ \text{GeV}^3$$

Results of the dim-7 fit

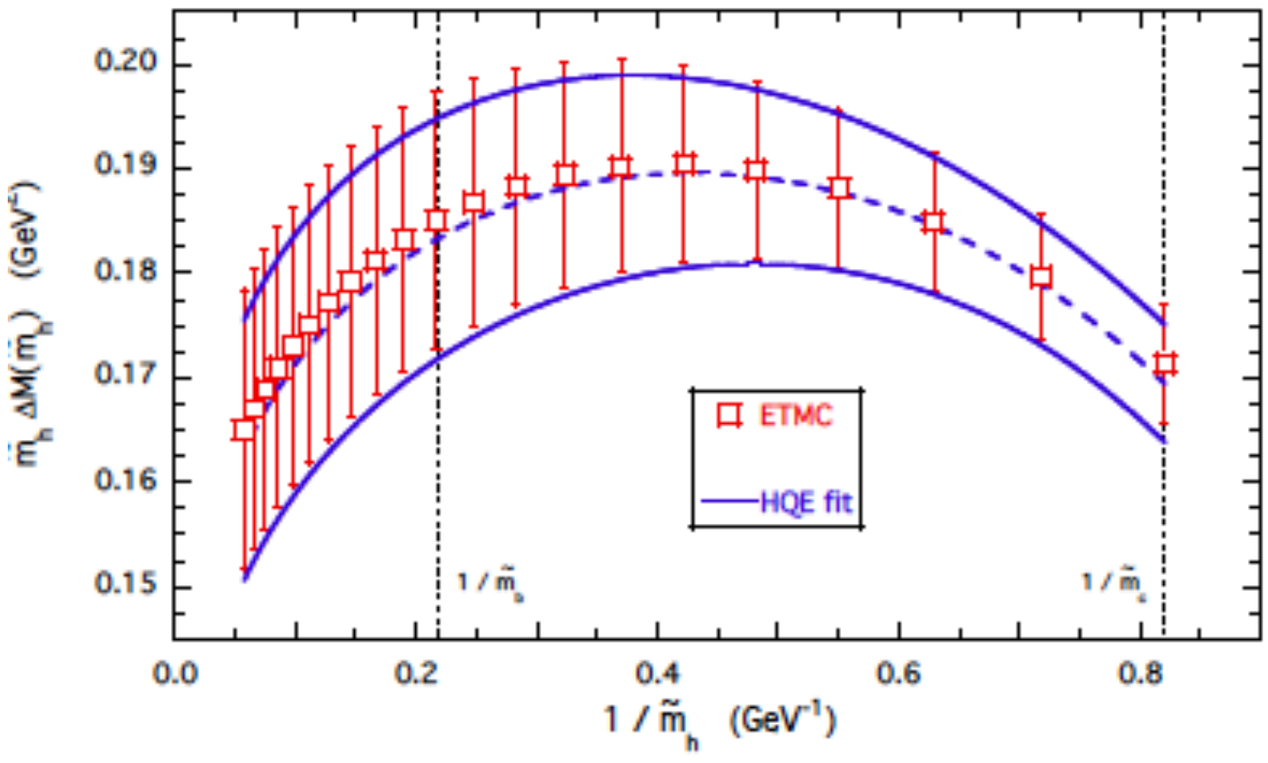
$$\overline{\Lambda} = 0.552\ (13)_{\rm stat}\ (2)_{\rm syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.325\ (17)_{\rm stat}\ (3)_{\rm syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.133\ (34)_{\rm stat}\ (6)_{\rm syst}\ \text{GeV}^3$$
$$\sigma^4 = 0.0071\ (55)_{\rm stat}\ (10)_{\rm syst}\ \text{GeV}^4$$

Courtesy of A. Melis

HQE expansion parameters from $\Delta M(\tilde{m}_h)$

The chain equation can be extended beyond the b quark point, we choose $n \sim 20$ ($\tilde{m}_h \simeq 4\tilde{m}_b$),

$$\tilde{m}_h^{(n)} \frac{\Delta M(\tilde{m}_h^{(n)})}{c_G(\tilde{m}_h^{(n)}, \tilde{m}_b)} = \tilde{m}_c \frac{\Delta M(\tilde{m}_c)}{c_G(\tilde{m}_c, \tilde{m}_b)} \prod_{i=2}^{n} \overline{y}_{\Delta M}(\tilde{m}_h^{(i)}, \lambda) \ .$$



We apply a correlated fit based on the HQE expansion

$$\tilde{m}_h \Delta M(\tilde{m}_h) = \frac{2}{3} c_G(\tilde{m}_h, \tilde{m}_b) \mu_G^2(\tilde{m}_b) + \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3}{3\tilde{m}_h} + \frac{\Delta\sigma^4}{\tilde{m}_h^4} \ .$$

Results of the dim-6 fit $\tilde{m}_h > \tilde{m}_c$

$$\mu_G^2(\tilde{m}_b) = 0.250\ (18)_{\rm stat}\ (8)_{\rm syst}\ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143\ (57)_{\rm stat}\ (21)_{\rm syst}\ \text{GeV}^3$$

Results of the dim-6 fit $\tilde{m}_h > 2\tilde{m}_c$

$$\mu_G^2(\tilde{m}_b) = 0.254\ (22)\ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.158\ (70)\ \text{GeV}^3$$

Results of the dim-7 fit

$$\mu_G^2(\tilde{m}_b) = 0.254\ (20)_{\rm stat}\ (9)_{\rm syst}\ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.173\ (74)_{\rm stat}\ (25)_{\rm syst}\ \text{GeV}^3$$
$$\Delta\sigma^4 = 0.0092\ (58)_{\rm stat}\ (14)_{\rm syst}\ \text{GeV}^4$$

Courtesy of A. Melis

RESULTS AND IMPLICATIONS

$$m_b^{\rm kin}(1\ {\rm GeV}) \;=\; 4.61(20)\ {\rm GeV}$$

$$\overline{\Lambda} \;=\; 0.552\ (26)\ {\rm GeV}\ ,$$

$$\mu_\pi^2 \;=\; 0.321\ (32)\ {\rm GeV}^2\ ,$$

$$\mu_G^2(m_b) \;=\; 0.253\ (25)\ {\rm GeV}^2\ .$$

$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 \;=\; 0.153\ (34)\ {\rm GeV}^3\ ,$$

$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 \;=\; -0.158\ (84)\ {\rm GeV}^3\ .$$

10% (30%) precision on dim 5 and 6 m.e. Competitive with the moments fits?

$$\mu_\pi^2|_B \;=\; \mu_\pi^2|_\infty - \frac{\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2)\ ,$$

$$\mu_G^2(m_b)|_B \;=\; \mu_G^2(m_b)|_\infty + \frac{\rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2)$$

- comparison with fits shows that $\rho_{\pi\pi}^3 + \dfrac{1}{2}\rho_{\pi G}^3 \approx -0.51(35)\,\mathrm{GeV}^3$
- since $\rho_{\pi\pi}^3 + \rho_{\pi G}^3 + \rho_A^3 + \rho_S^3 \geq 0$
- it follows that $\mu_G^2(m_b) \geq 0.36(8)\mathrm{GeV}^2$
- good consistency with fits, but only 4 combinations are determined

CONCLUSIONS

- The efforts to improve the theory of inclusive semileptonic B decays continue. No sign of inconsistency so far, several ways to improve and decrease uncertainty.

- We understand better higher power contributions and have checked their impact on the fit. They seem to have been tamed.

- LQCD determination of HQE parameters from masses is promising: various improvements under way for higher precision, with implications for heavy quark sum rules.

- The V_{cb} puzzle is not yet over. For sure something wasn't OK in the exclusive extraction, but we need more LQCD results to understand if there is really agreement with the inclusive one.

BACKUP

heavy-light (u/d) mesons

dimension-6 fit

$$\bar{\Lambda} = 0.551\left(13\right)_{stat}\left(2\right)_{syst}\ GeV$$
$$\mu_\pi^2 = 0.314\left(14\right)_{stat}\left(2\right)_{syst}\ GeV^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.174\left(12\right)_{stat}\left(2\right)_{syst}\ GeV^3$$
$$\mu_G^2\left(\tilde{m}_b\right) = 0.250\left(18\right)_{stat}\left(8\right)_{syst}\ GeV^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143\left(57\right)_{stat}\left(21\right)_{syst}\ GeV^3$$

dimension-7 fit

$$\bar{\Lambda} = 0.552\left(13\right)_{stat}\left(2\right)_{syst}\ GeV$$
$$\mu_\pi^2 = 0.325\left(17\right)_{stat}\left(2\right)_{syst}\ GeV^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.133\left(34\right)_{stat}\left(6\right)_{syst}\ GeV^3$$
$$\sigma^4 = 0.0072\left(55\right)_{stat}\left(10\right)_{syst}\ GeV^4$$
$$\mu_G^2\left(\tilde{m}_b\right) = 0.254\left(20\right)_{stat}\left(9\right)_{syst}\ GeV^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.173\left(74\right)_{stat}\left(25\right)_{syst}\ GeV^3$$
$$\Delta\sigma^4 = 0.0092\left(58\right)_{stat}\left(14\right)_{syst}\ GeV^4$$

heavy-strange mesons

dimension-6 fit

$$\bar{\Lambda} = 0.637\left(15\right)_{stat}\left(6\right)_{syst}\ GeV$$
$$\mu_\pi^2 = 0.414\left(19\right)_{stat}\left(8\right)_{syst}\ GeV^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.281\left(20\right)_{stat}\left(9\right)_{syst}\ GeV^3$$
$$\mu_G^2\left(\tilde{m}_b\right) = 0.299\left(14\right)_{stat}\left(5\right)_{syst}\ GeV^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.289\left(46\right)_{stat}\left(14\right)_{syst}\ GeV^3$$

dimension-7 fit

$$\bar{\Lambda} = 0.636\left(15\right)_{stat}\left(6\right)_{syst}\ GeV$$
$$\mu_\pi^2 = 0.431\left(21\right)_{stat}\left(10\right)_{syst}\ GeV^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.204\left(20\right)_{stat}\left(18\right)_{syst}\ GeV^3$$
$$\sigma^4 = 0.0128\left(30\right)_{stat}\left(30\right)_{syst}\ GeV^4$$
$$\mu_G^2\left(\tilde{m}_b\right) = 0.303\left(15\right)_{stat}\left(5\right)_{syst}\ GeV^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.359\left(58\right)_{stat}\left(14\right)_{syst}\ GeV^3$$
$$\Delta\sigma^4 = 0.0272\left(55\right)_{stat}\left(11\right)_{syst}\ GeV^4$$

SU(3) breaking

$\sim 15\%$ for $\bar{\Lambda}$

$\sim 25\%$ for μ_π^2

$\sim 40\%$ for $\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3$

$\sim 15\%$ for μ_G^2

$\sim 50\%$ for $\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3$

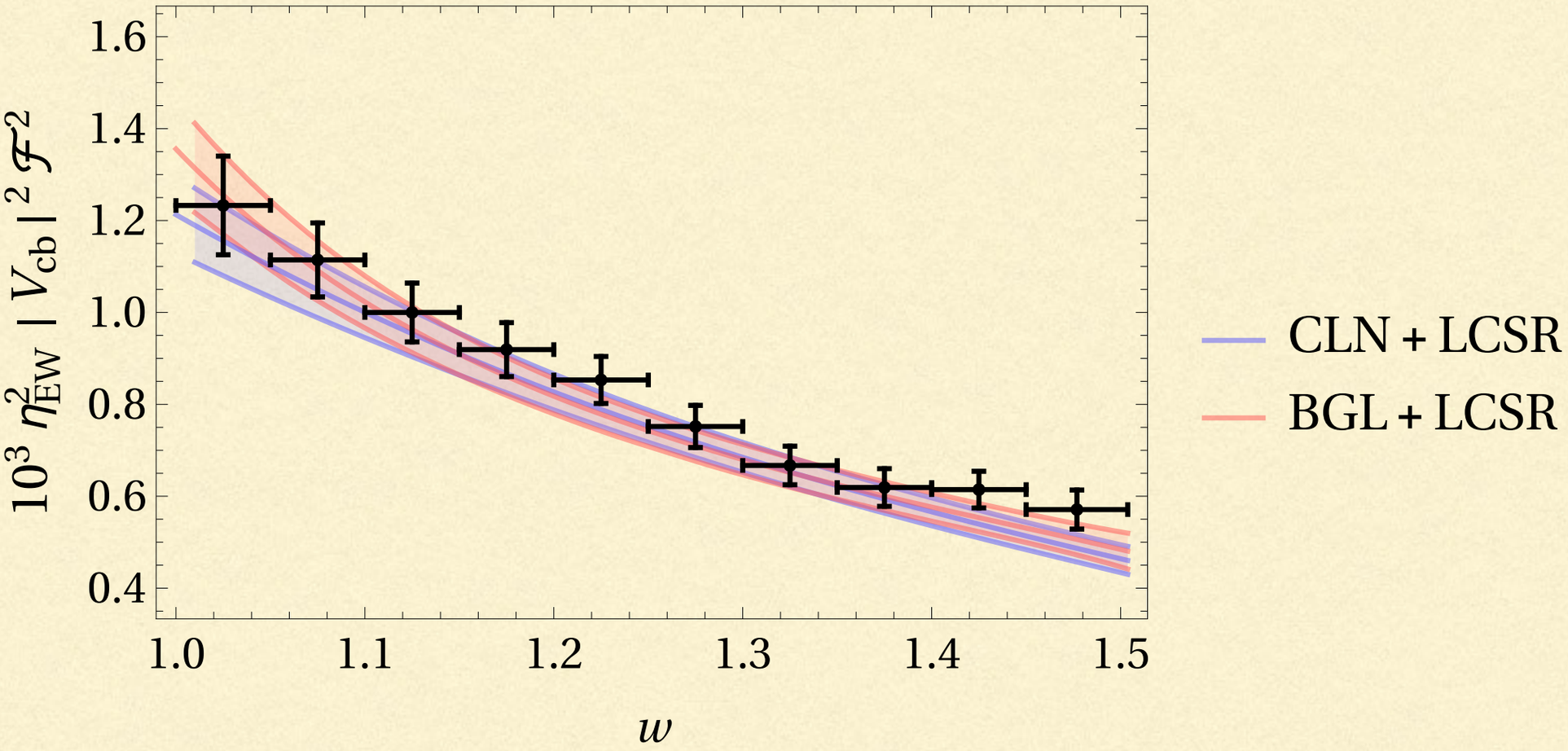
$$M_{av}\left[\tilde{m}_b^{phys}\right] = 5.403\left(12\right)GeV$$
$$\frac{M_{B_s} + 3M_{B_s^*}}{4} = 5.403\left(2\right)GeV \quad \left(\text{PDG}\right)$$

$$\Delta M\left[\tilde{m}_b^{phys}\right] = 45.1\left(1.1\right)MeV$$
$$M_{B_s^*} - M_{B_s} = 46.1\left(1.5\right)MeV \quad \left(\text{PDG}\right)$$

RECENT DEVELOPMENTS

- New Belle $B \rightarrow D^* l\nu$ tagged analysis (deconvoluted spectra)
- Strong dependence of V_{cb} on FF parametrisation (CLN vs BGL)
- BGL alone may be equally misleading: role of HQS constraints
- New Belle $B \rightarrow D^* l\nu$ tagged analysis with both CLN and BGL
- Upcoming FNAL/MILC results on all $B \rightarrow D^* l\nu$ FFs at non-zero recoil

BGL vs CLN


1.6
1.4
1.2
1.0
0.8
0.6
0.4
$10^3\,\eta_{\rm EW}^2\,|\,V_{cb}\,|^2\,\mathcal{F}^2$
1.0
1.1
1.2
1.3
1.4
1.5
w
CLN + LCSR
BGL + LCSR

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \dots\right] \qquad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$c_{b,c}$ can be computed using subleading IW functions from QCD sumrules
Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

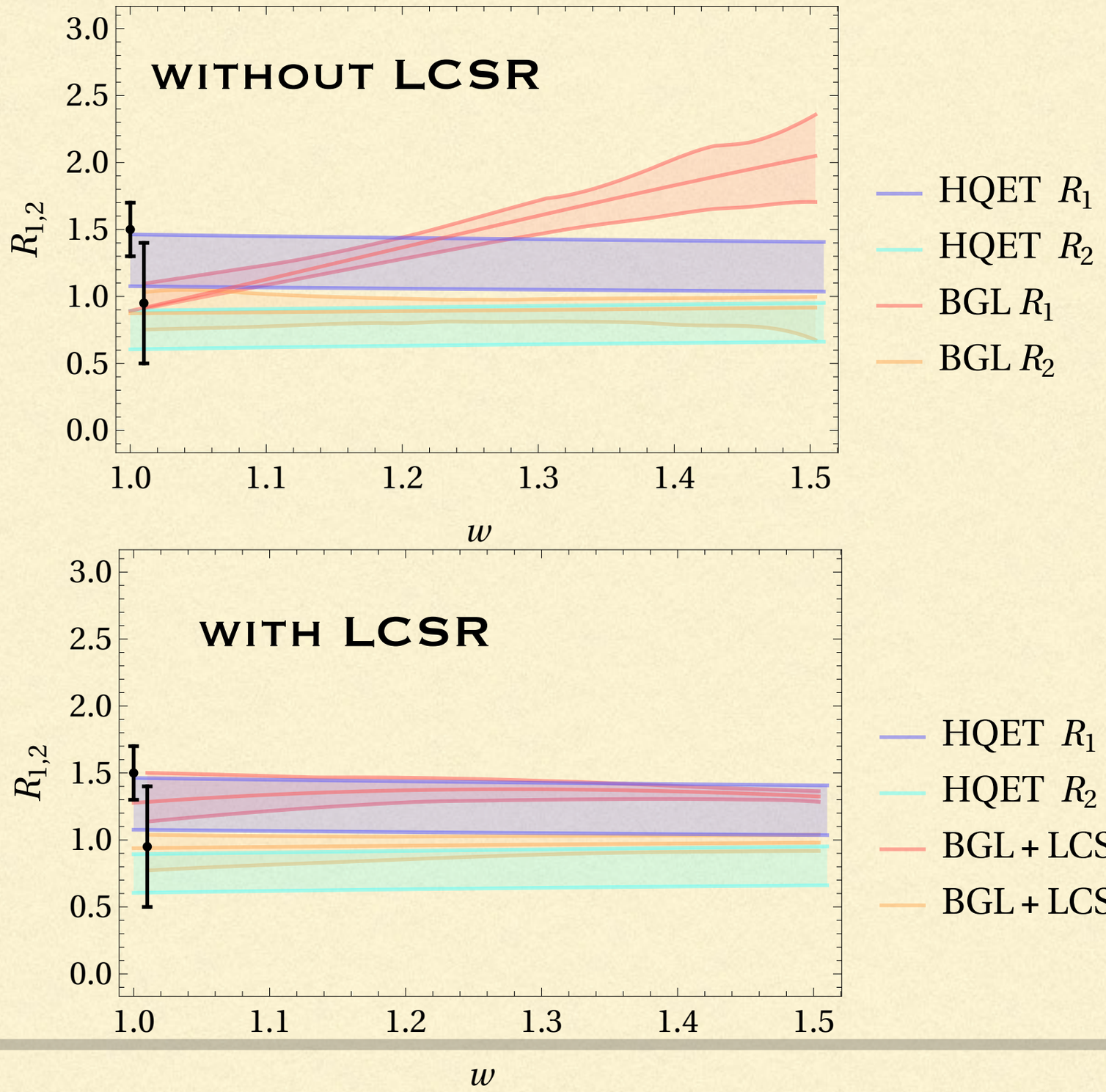
RATIOS $\dfrac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \dots\right] \qquad w_1 = w - 1$

Roughly $\epsilon_c \sim 0.25, \qquad \epsilon_c^2 \sim 0.06$ but coefficients??

In a few cases we can compare these ratios with recent lattice results: there are 5-13% differences, always > NLO correction. For ex.:

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET} = 0.966(28)$$

CONSISTENCY WITH HQET

Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)

black points from preliminary FNAL-MILC calculation according to Bernlochner et al 1708.07134 (before continuum and chiral extrapolations…)