WG2: Vub, Vcb and semileptonic/leptonic b decays including tau

Effect of QED corrections on $R(D)$

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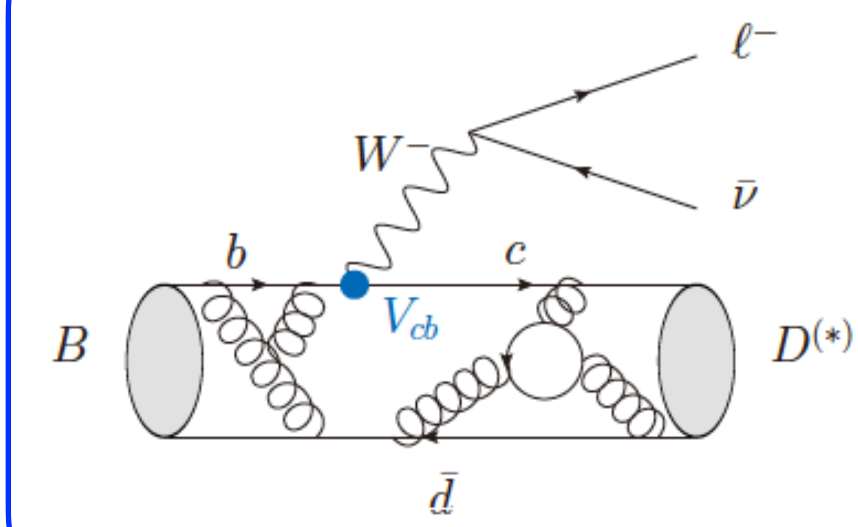
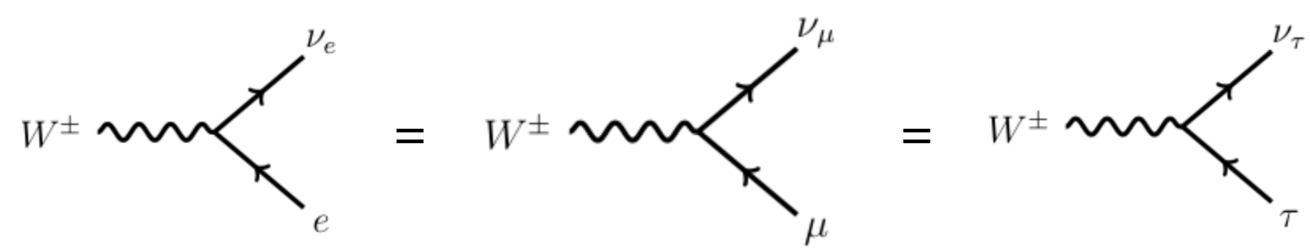
Semileptonic B decay



- **Semileptonic B-meson decays induced by $b \to cl\nu$ transitions** play an important role for testing the Standard Model at low energy: **$|V_{cb}|$** and **lepton flavor universality**

- **Lepton flavor universality** is violated by only tau lepton mass which leads to smaller phase space and subleading scalar form factors $f_0(q^2)$ and $A_0(q^2)$

SU(2)$_L$ gauge symmetry



- light lepton universalities in kaon, pion and τ decays have been checked

$K^+ \to \pi^0 \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(K^+) = 0.998(9)$

$K_L \to \pi^- \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(K_L) = 1.003(5)$

$\pi^+ \to \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(\pi^+) = 1.0042(33)$

$\tau^+ \to \ell^+ \nu \bar{\nu}(\gamma)$ $\quad r_{\mu e}(\tau^+) = 1.000(4)$

$$r_{\mu e}^{\mathrm{SM}} = (g_{W\mu\bar{\nu}}/g_{We\bar{\nu}})^2 = 1$$

[Rainer Wanke, KAON 2007; Cristina Lazzeroni, IoP Nuclear and Particle Divisional Conference, 2011]

$R(D)$ and $R(D^*)$

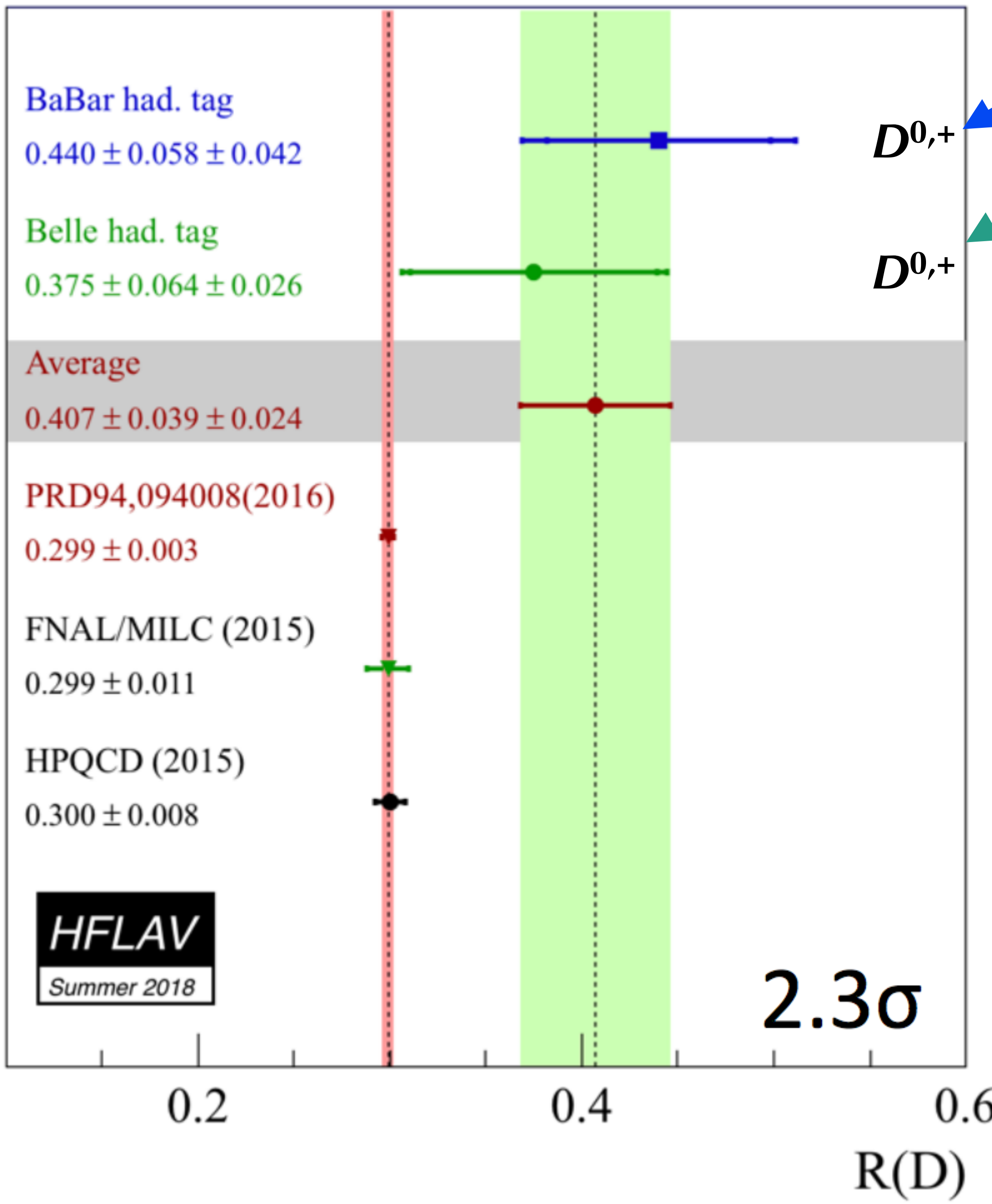
- **Violation of the lepton flavour universality** has been announced in $R(D)$ and $R(D^*)$

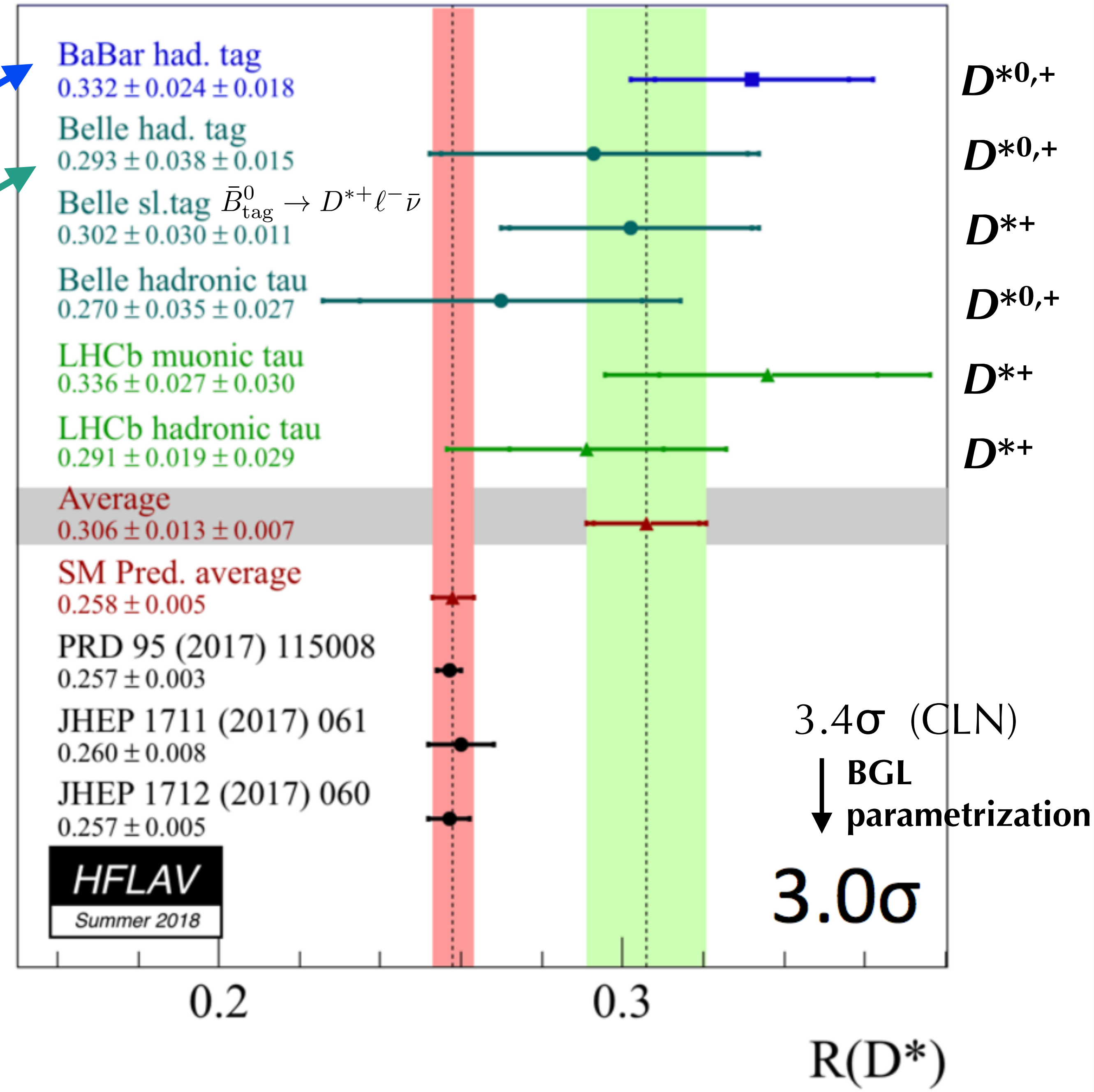
$$R(D^{(*)+}) = \frac{\mathcal{B}(\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu})}{\mathcal{B}(\bar{B}^0 \to D^{(*)+}\ell^-\bar{\nu})}$$

$$R(D^{(*)0}) = \frac{\mathcal{B}(B^- \to D^{(*)0}\tau^-\bar{\nu})}{\mathcal{B}(B^- \to D^{(*)0}\ell^-\bar{\nu})}$$

- **Theoretically clean:** dominant uncertainty from LD QCD in form factors is partially canceled
- CKM dependence ($|V_{cb}|$) is canceled
- We separately define $R(D^+)$ and $R(D^0)$ to distinguish different QED corrections in neutral and charged B decays

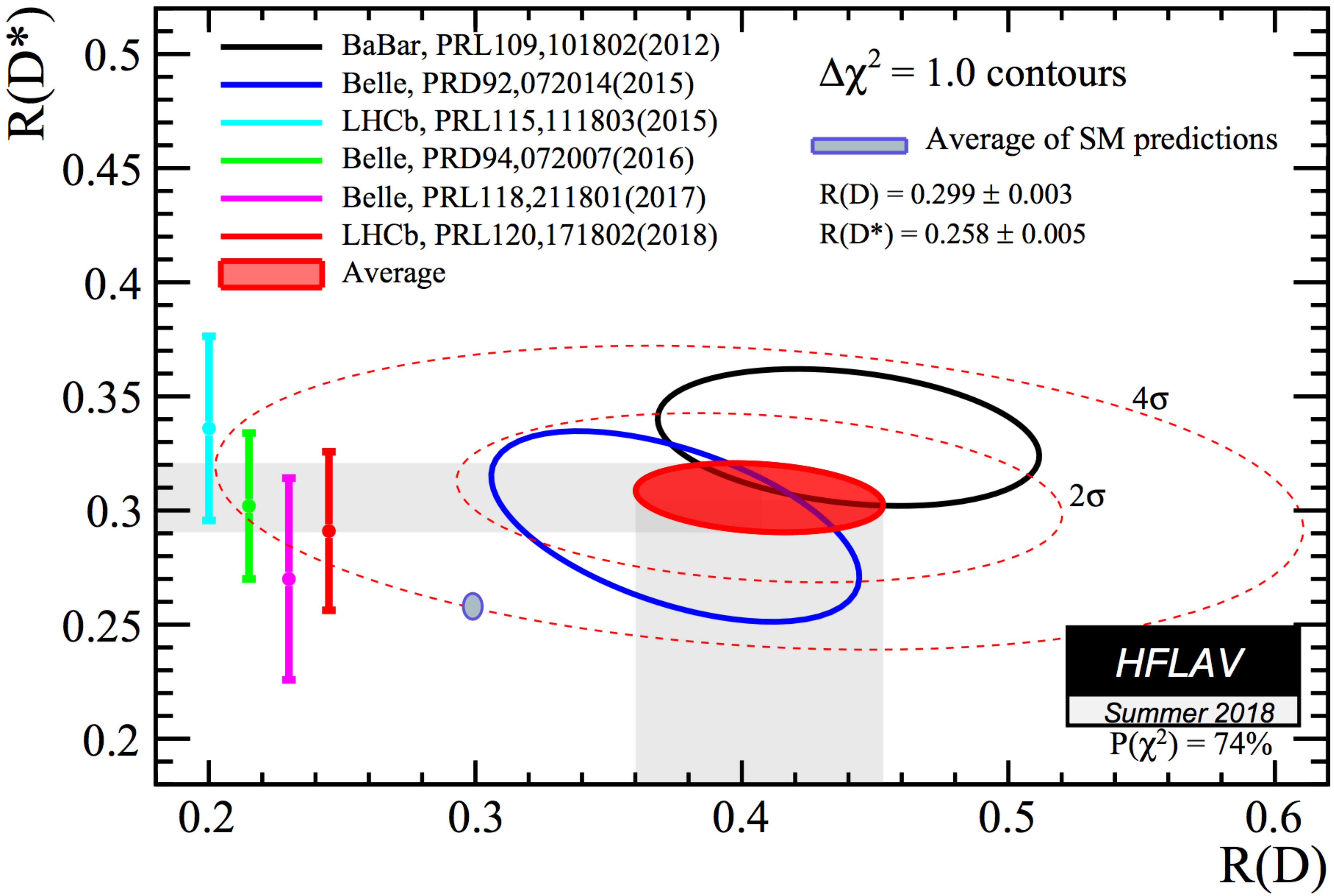
Status of $R(D)$ and $R(D^*)$





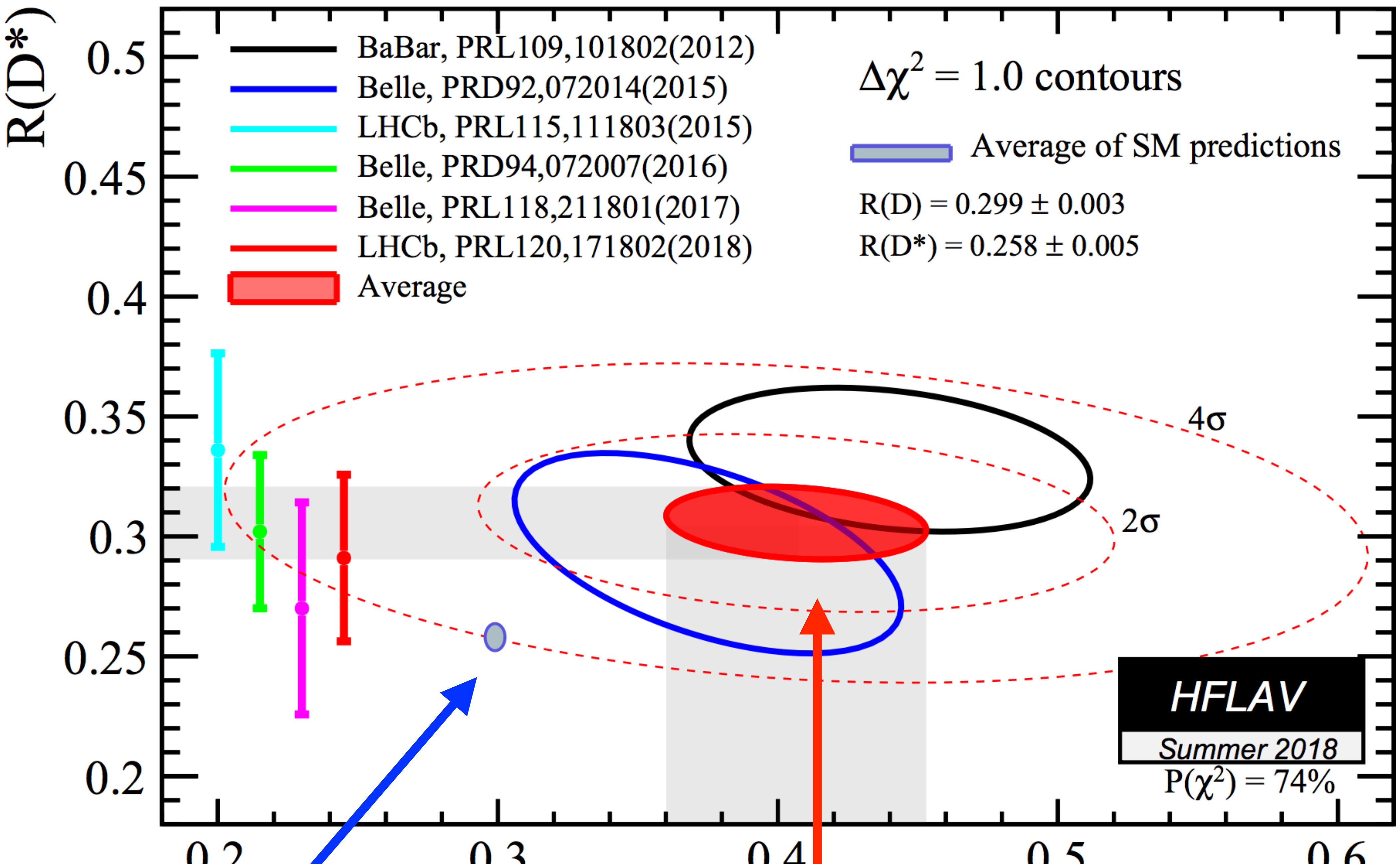
LHCb can not measure D^{*0} precisely [$D^{*0} \to D^0\gamma$, $D^0\pi^0\,(\to\gamma\gamma)$]

Status of $R(D)$ and $R(D^*)$



—> 3.9σ deviation including all correlations <—

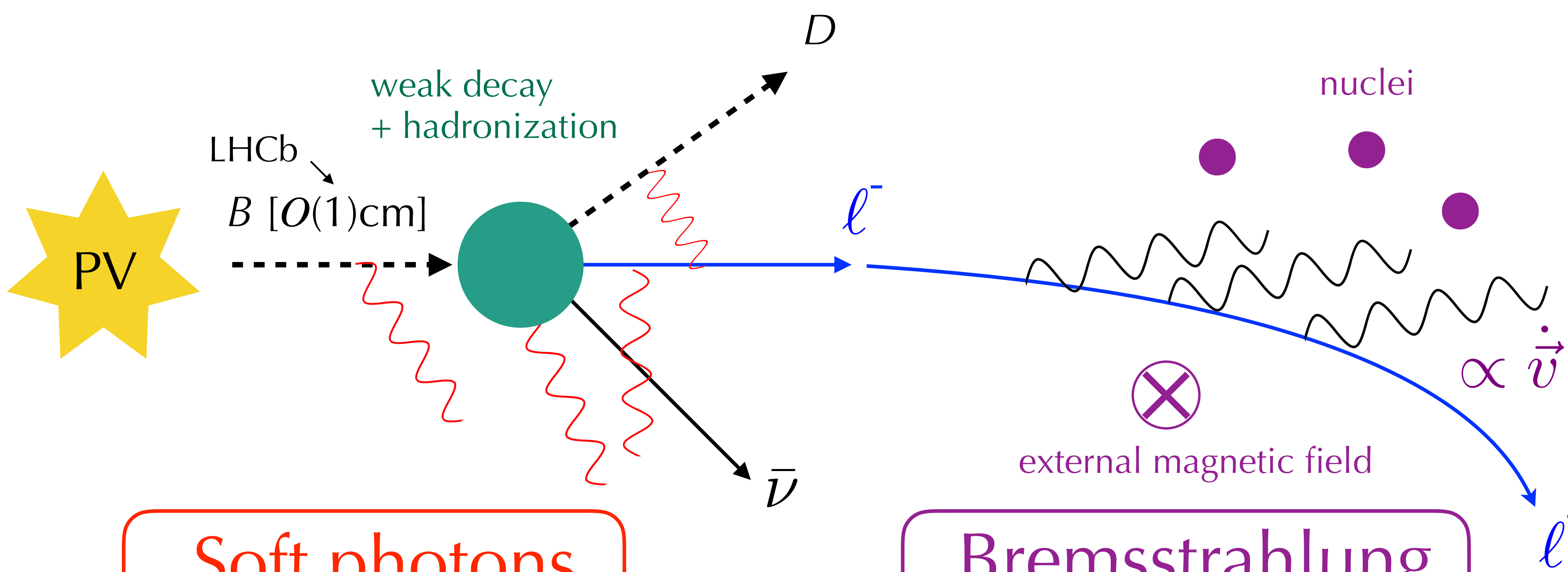
Status of $R(D)$ and $R(D^*)$



No QED corrections

soft-photon corrections are partially subtracted by PHOTOS MC simulation

long-distance QED corrections



Soft photons

Physics **in vacuum**

invisible photon energy

our work → sum of invisible emissions and one-loop corrections gives physical quantum correction = **long-distance QED corrections**

Bremsstrahlung

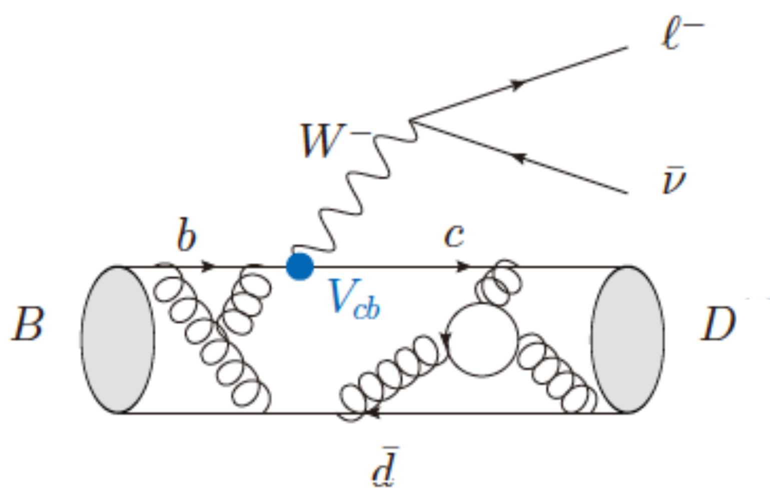
Physics **in detector**

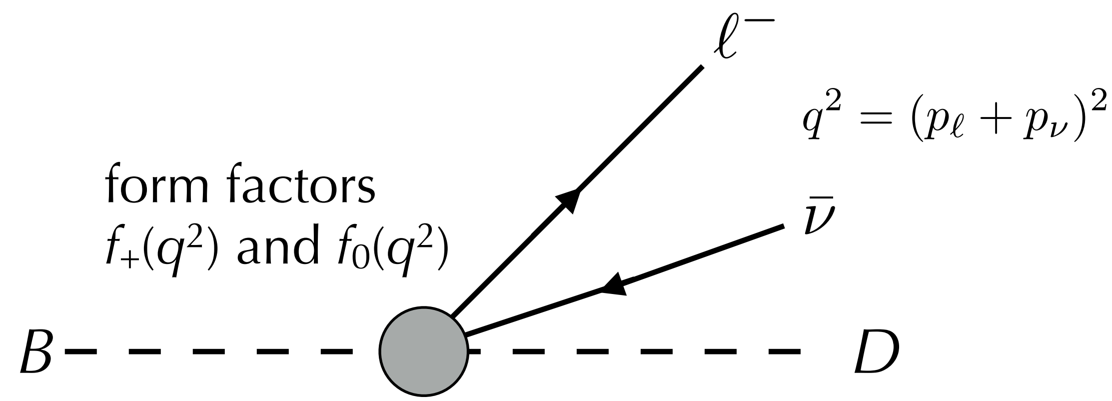
Emitted photon energy corresponds to dropped lepton energy

small bremsstrahlung in μ^- and τ^-

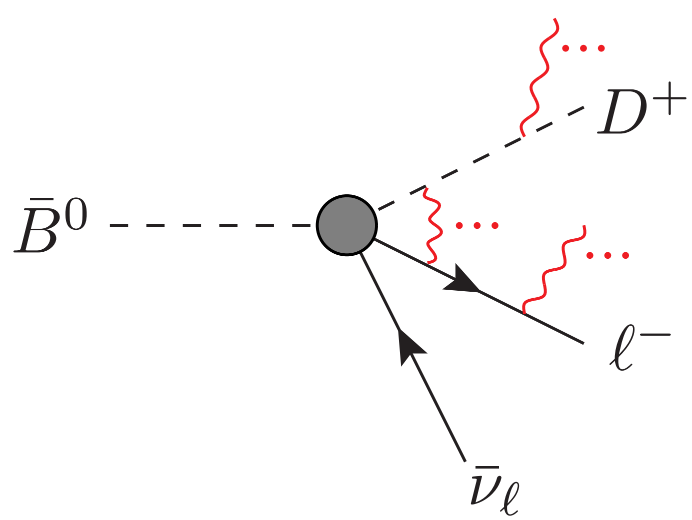
Soft-photon corrections

- At large distance ($\mu \lesssim \Lambda_{\rm QCD}$), the QED interactions of the charged scalar mesons are well described by the **scalar QED**.

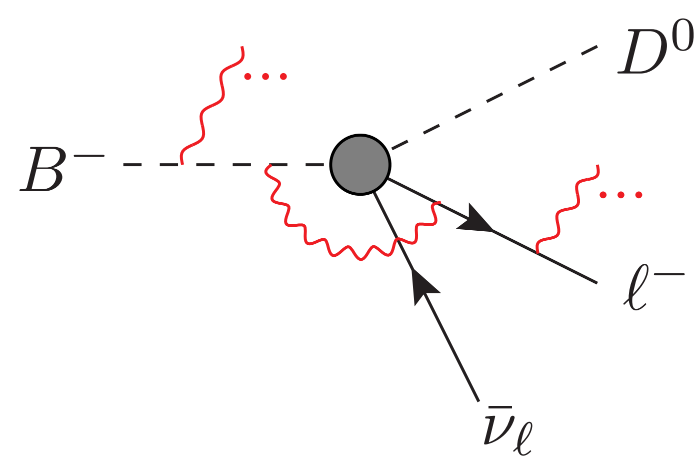




- We distinguish neutral and charged-B decay



$\bar{B}^0 \to D^+ \ell \bar{\nu}$ $[R(D^+)]$



$B^- \to D^0 \ell \bar{\nu}$ $[R(D^0)]$

Soft-photon corrections

- $E_{\max}$ **is the maximum total energy of undetected soft photons in the rest frame of the B-meson**: $E_{\max} = 20\sim30$ MeV in current photon detectors

  →NOTE: the explicit photon cut is not set in the semileptonic search but the hard photons do not contribute to the fit of data

- **The soft-photon approximation** is used for analytic evaluation: we keep $O(\ln E_{\max})$ and $O(E^0_{\max})$ and drop $O(E_{\max})$, which is **valid only $l = \tau$ and μ**

- Real soft emissions = $O(\ln E_{\max}) + O(E^0_{\max})$

  - $O(\ln E_{\max})$ terms are **resumed**

  - Finite terms $[O(E^0_{\max})]$ are numerically comparable to $O(\ln E_{\max})$

- Vertex corrections = $O(E^0_{\max})$ and **μ-dependent** ; $\mu \lesssim \Lambda_{\rm QCD}$ for scalar QED

  - We separate l_μ=0 contribution and the other (l is loop momentum)

  - Coulomb pole $(\alpha/v_{\rm rel})$ exits only in $R(D^+)$ case, and we **resumed** them

- Both of contributions depend on lepton kinematics → **source of LFU violation**

Result [de Boer, TK, Nisandzic, PRL120 (2018) no.26, 261804]

$$\frac{d^2\Gamma}{dq^2 ds_{D\ell}} = \frac{d^2\Gamma_0}{dq^2 ds_{D\ell}} \Omega_B^{D^+} \Omega_C \left[1 + \frac{\alpha}{\pi}\left(F_D + F_\ell - 2F_{D\ell} - 2H_{D\ell}\right)\right] + \frac{\alpha}{\pi} \frac{d^2\tilde{\Gamma}^{D^+}}{dq^2 ds_{D\ell}},$$

with
$$\Omega_B^{D^+} = \left(\frac{2E_{\max}}{\sqrt{m_D m_\ell}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{D\ell})}, \quad \Omega_B^{D^0} = \left(\frac{2E_{\max}}{\sqrt{m_B m_\ell}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{B\ell})}, \quad \Omega_C = -\frac{2\pi\alpha}{\beta_{D\ell}} \frac{1}{e^{-\frac{2\pi\alpha}{\beta_{D\ell}}} - 1}$$

$$F_i = \frac{1}{2\beta_{Bi}} \ln \frac{1+\beta_{Bi}}{1-\beta_{Bi}},$$

$$F_{D\ell} = \frac{1}{2} \frac{m_D m_\ell}{\sqrt{1-\beta_{D\ell}^2}} \int_0^1 dz \frac{E(z)}{P(z)\left[E(z)^2 - P(z)^2\right]} \ln \frac{E(z)+P(z)}{E(z)-P(z)},$$

$$F_{B\ell} = \frac{1}{4\beta_{B\ell}} \left\{ \mathrm{Li}_2\left(\frac{1-\beta_{B\ell}}{2}\right) - \mathrm{Li}_2\left(\frac{1+\beta_{B\ell}}{2}\right) + 4\mathrm{Li}_2\left(\beta_{B\ell}\right) - \mathrm{Li}_2\left(\beta_{B\ell}^2\right) + \ln 2 \ln \frac{1+\beta_{B\ell}}{1-\beta_{B\ell}} + \frac{1}{2}\ln^2\left(1-\beta_{B\ell}\right) - \frac{1}{2}\ln^2\left(1+\beta_{B\ell}\right) \right\},$$

$$H_{ij} = -\frac{1}{2\beta_{ij}} \left\{ \frac{1}{2}\ln^2 \frac{m_i}{m_j} - \frac{1}{8}\ln^2 \frac{1+\beta_{ij}}{1-\beta_{ij}} - \frac{1}{2}\ln^2 \left|\frac{\Delta_{ij}^i + \Delta_{ij}\beta_{ij}}{\Delta_{ij}^j + \Delta_{ij}\beta_{ij}}\right| - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_{ij}^i + \Delta_{ij}\beta_{ij}}\right) - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_{ij}^j + \Delta_{ij}\beta_{ij}}\right) \right\}$$
$$+ \frac{1}{4}\ln \frac{m_i m_j}{\mu^2} - \frac{1}{2} - \frac{m_i^2 - m_j^2}{4s_{ij}} \ln \frac{m_i}{m_j} - \frac{1}{4}\Delta_{ij}\beta_{ij} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}} - \frac{\Delta_{ij}}{2} \ln \frac{m_i}{m_j} - \frac{\Delta_{ij}^i}{4\beta_{ij}} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}},$$

where
$$b_{i\ell} = \frac{1}{4\beta_{i\ell}} \ln \frac{1+\beta_{i\ell}}{1-\beta_{i\ell}}, \quad \beta_{D\ell} = \left[1 - \frac{4m_D^2 m_\ell^2}{\left(s_{D\ell} - m_D^2 - m_\ell^2\right)^2}\right]^{\frac{1}{2}}, \quad \beta_{B\ell} = \left(1 - \frac{m_\ell^2}{E_\ell^2}\right)^{\frac{1}{2}}, \quad E_\ell = \frac{s_{D\ell} + q^2 - m_D^2}{2m_B},$$

$$\Delta_{ij} = \frac{s_{ij} - m_i^2 - m_j^2}{2s_{ij}}, \quad \Delta_{ij}^{i,j} = \frac{s_{ij} + m_{i,j}^2 - m_{j,i}^2}{2s_{ij}},$$

what we did

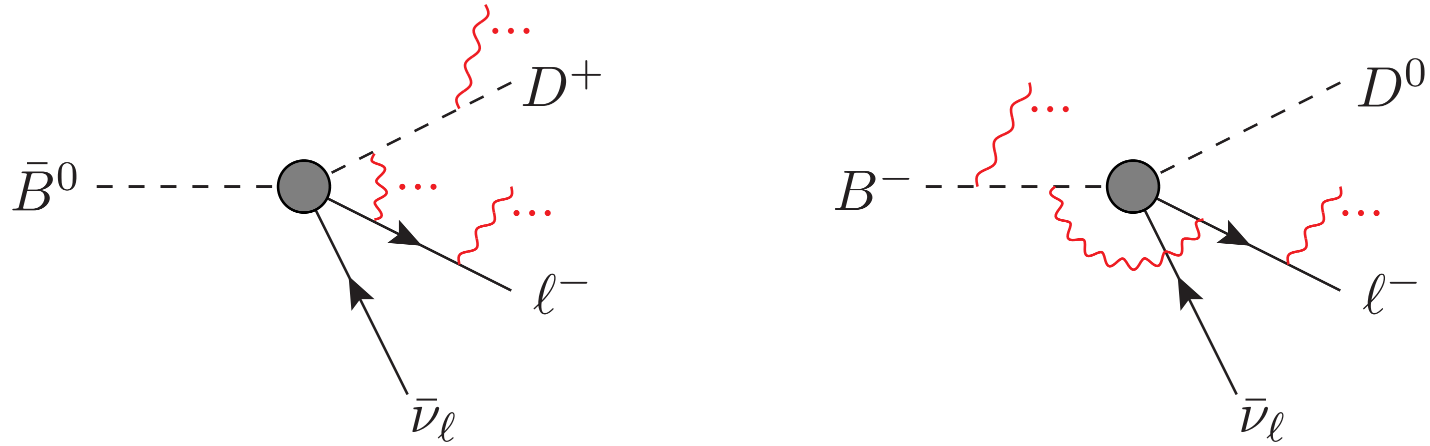
- We obtained analytic full-one loop QED long-distance correction, which is three parameter function; $\boldsymbol{E_{\mathrm{max}}}$ and **2 Dalitz variables** [$(p_D+p_\ell)^2$, $(p_B-p_D)^2$]
- We resumed two potentially large contributions;
  $(\alpha \ln E_{\mathrm{max}})^n$ from an arbitrary number of real photon emissions
  $(\pi\alpha/\beta_{D\ell})^n$ from ladder of photons

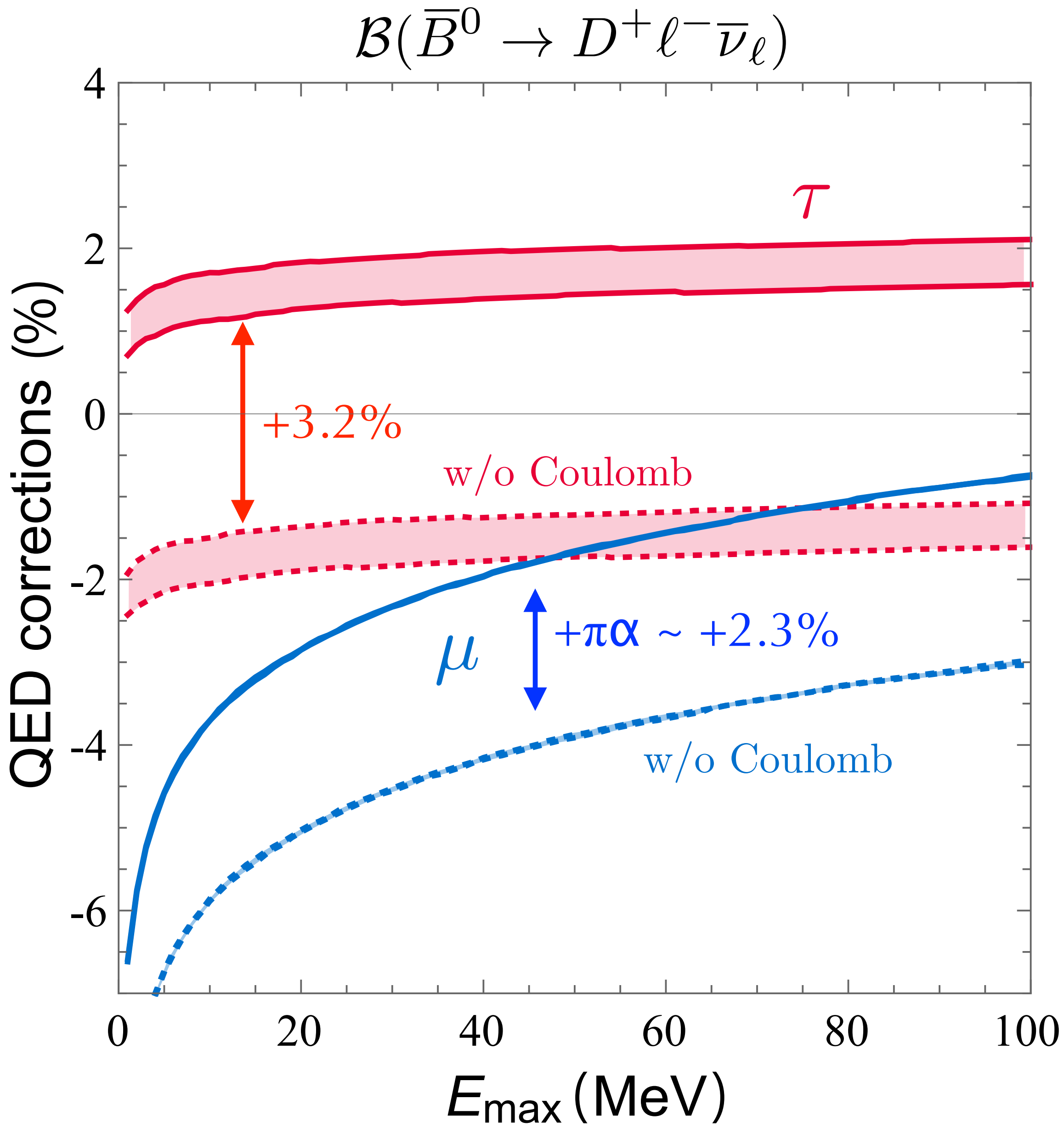
☑ IR-divergence ($m_\gamma \to 0$) cancels between |real emissions|^2 and loops (vertex and wave-function corrections) : **We checked analytically**

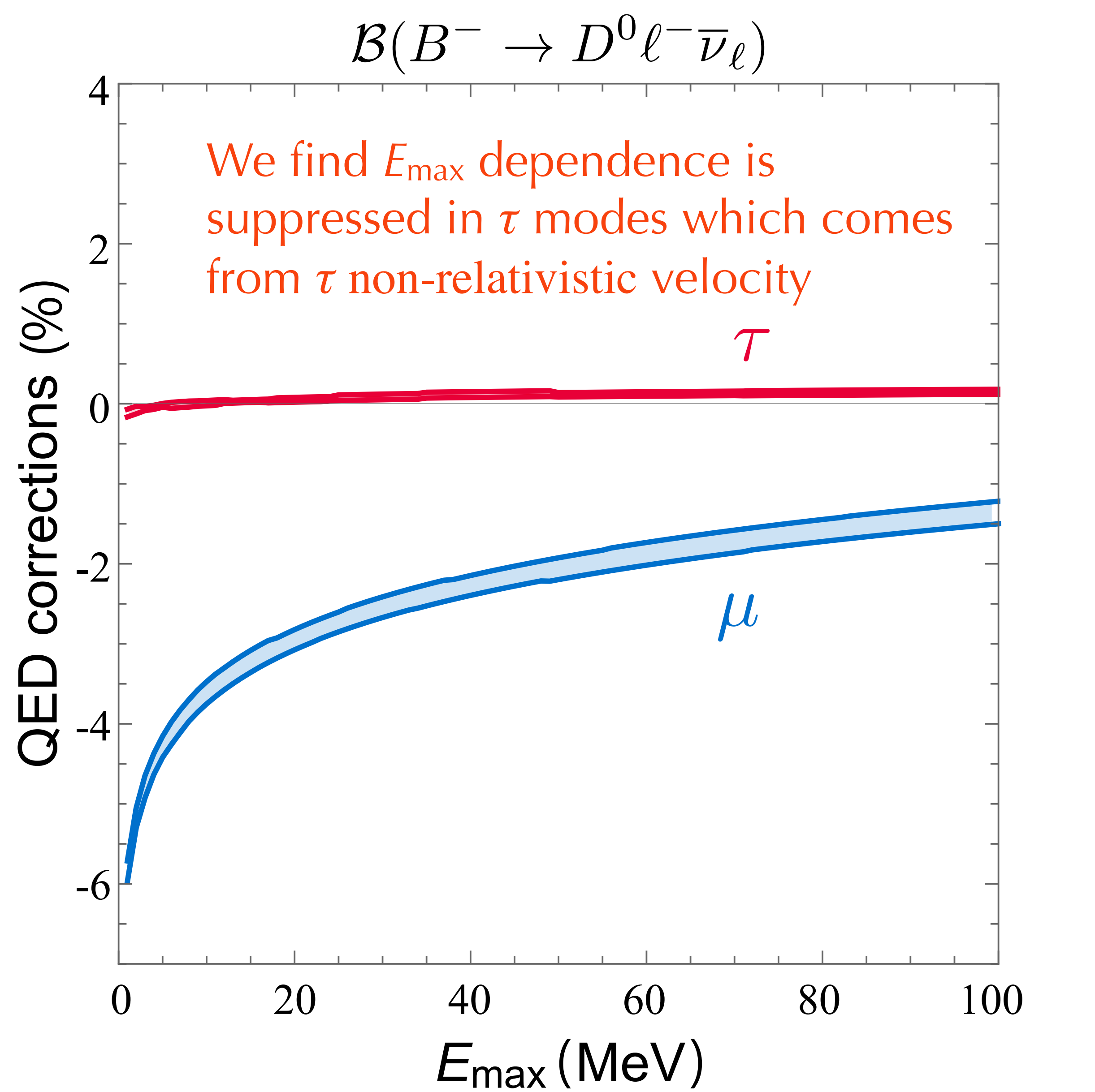
☑ numerically crosschecked with LoopTools and Package-X [Hahn, Perez-Victoria '99; Patel '15]

☑ analytically crosschecked with $P \to PPP$ QED correction (e.g., $K \to 3\pi$) when spin dependent terms are dropped [Isidori '08]

Result



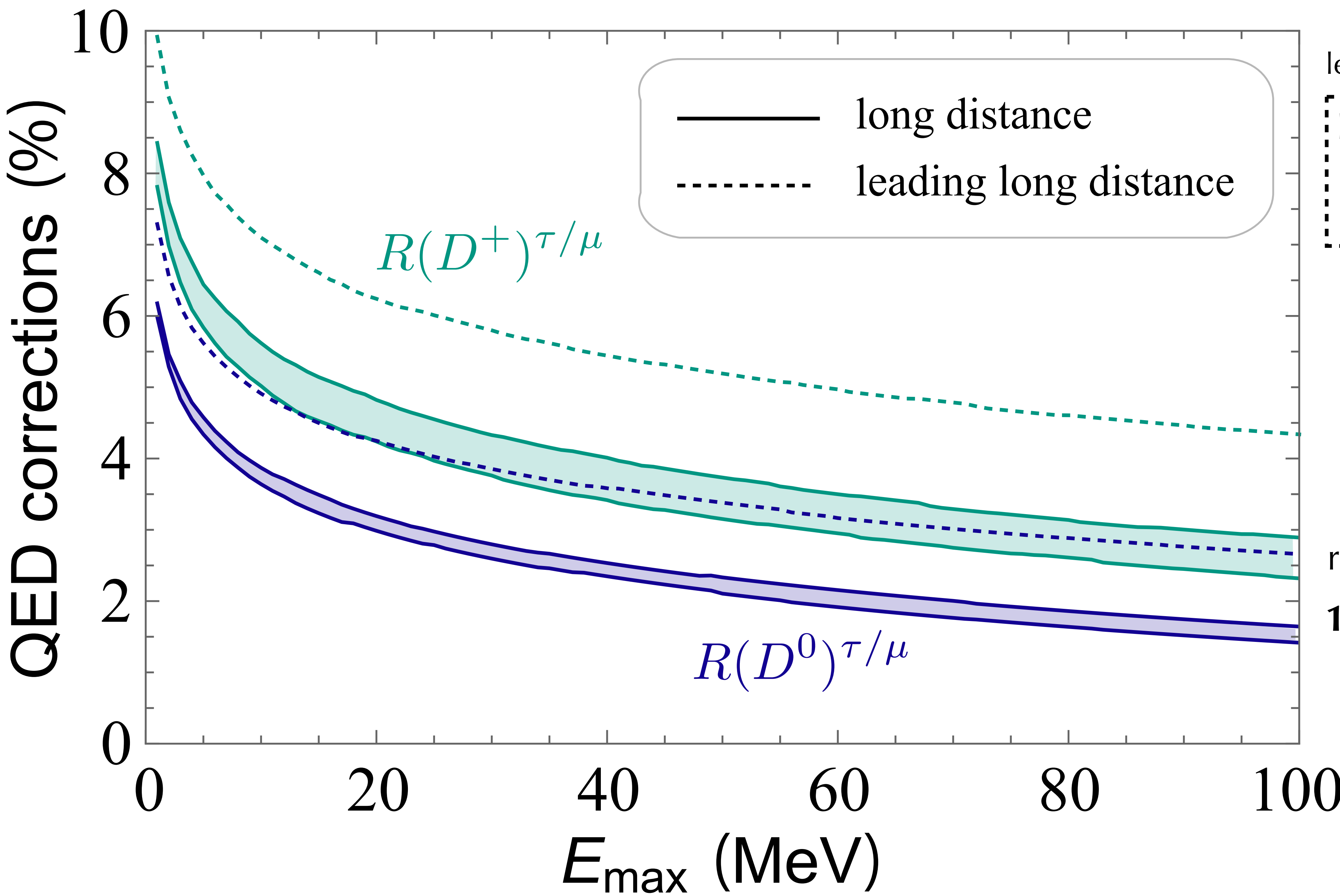




[de Boer, TK, Nisandzic, PRL120 (2018) no.26, 261804]

Result

[de Boer, TK, Nisandzic, PRL120 (2018) no.26, 261804]



leading long distance

Real emissions
+ l_μ=0 contribution in the loop

= μ-independent

renormalization scale

100MeV < μ < 1GeV

We conclude that the QED corrections to $R(D^+)$ and $R(D^0)$ are different at 1-1.5%

$B^0 \to D^+ l^- \nu$

Note: naive size of QED corrections ~ O(α/π) ~ 0.3%

QED corrections (%)

$B^0 \to D^+ l^- \nu$	Ω_B (E_{max}=20MeV)	Ω_C	F_D	F_l	F_{Dl}	H_{Dl} (l=0)	H_{Dl} (l>0) (μ=200 MeV)	IB-loop (μ=200 MeV)	TOTAL
Br(τ)	-0.72	3.22	0.26	0.27	-0.65	-0.64	-0.10	-0.10	1.41
Br(μ)	-3.14	2.31	0.28	0.73	-3.10	-1.40	1.25	0.24	-2.84
R(D^+)	**2.50**	**0.89**	-0.02	-0.45	**2.52**	**0.77**	**-1.33**	-0.34	**4.38**

ΩB: log(E_{max}) contributions from full real emissions
ΩC: Coulomb correction
FD: finite terms [=$O(E^0_{max})$] of real emission from D^+
FI: finite terms of real emission from l
FDI: finite terms of interference between real emissions from D^+ and l
HDI: loop correction between D^+ and l
IB-loop: loop correction containing Inner-Bremsstrahlung vertex

$B^- \to D^0 \ell^- \nu$

Note: naive size of QED corrections ~ $O(\alpha/\pi)$ ~ 0.3%

QED corrections (%)

$B^- \to D^0 \ell^- \nu$	Ω_B (E_{max}=20MeV)	Ω_C	F_B	F_l	F_{Bl}	H_{Bl} (l=0)	H_{Bl} (l>0) (μ=200 MeV)	IB-loop (μ=200 MeV)	TOTAL
Br(τ)	-0.22	-	0.23	0.27	-0.53	0.03	-0.09	0.37	0.03
Br(μ)	-2.93	-	0.23	0.74	-2.79	0.35	1.11	0.19	-2.98
R(D^0)	**2.79**	-	0.00	-0.46	**2.32**	-0.31	**-1.19**	0.18	**3.11**

ΩB: log(E_{max}) contributions from full real emissions

ΩC: Coulomb correction

FB: finite terms [$=O(E^0_{max})$] of real emission from B^-

Fl: finite terms of real emission from l

FBl: finite terms of interference between real emissions from B^- and l

HBl: loop correction between B^- and l

IB-loop: loop correction containing Inner-Bremsstrahlung vertex

PHOTOS MC simulation

[Barberio, Eijk, Was, '91; Barberio, Was, '94; Davidson, Przedzinski, Was '16]

- PHOTOS Monte-Carlo generator can simulate modifications of the kinematic variables induced by **final-state** photon radiations
- PHOTOS is utilized in Belle/BaBar/LHCb for B semileptonic decay search
- For general decay processes, **PHOTOS can simulate final-state radiation in the leading-logarithmic collinear approximation**
  - **All** virtual corrections **including Coulomb pole** are not covered in PHOTOS results
  - Quantum interference between two real photon emissions are not covered in PHOTOS (< version 2.13) results

    LHCb *does* **include** the **interference** between two different real photons from different **final charged particle**
- Numerical comparison with PHOTOS analysis is required to predict QED corrections to $R(D^{0,+})_{\rm SM}$. Ongoing discussion with a few LHCb colleagues to size up the accuracy of PHOTOS

Missing mass squared analysis

- The experiments (Belle/BaBar/LHCb) have not explicitly utilized the photon cut $\boldsymbol{E_{max}}$ for event selections for B semileptonic decay

- The experiments rely on **missing mass squared analysis** for the event selection

$$M^2_{\rm miss} \equiv \left(p_{e^+e^-} - p_{B_{\rm tag}} - p_D - p_\ell\right)^2 \quad = p_{\bar\nu}^2 = 0 \quad \text{(ideal)}$$

= Gaussian distribution around 0 by detector resolution

If single undetectable photon exists…

$$= (p_\nu + p_\gamma)^2 = 2E_\nu E_\gamma \left(1 - \cos\theta_{\nu\gamma}\right) > 0$$

→ positive shift of the distribution

$\langle \cos\theta_{\nu\gamma}\rangle \sim 0$

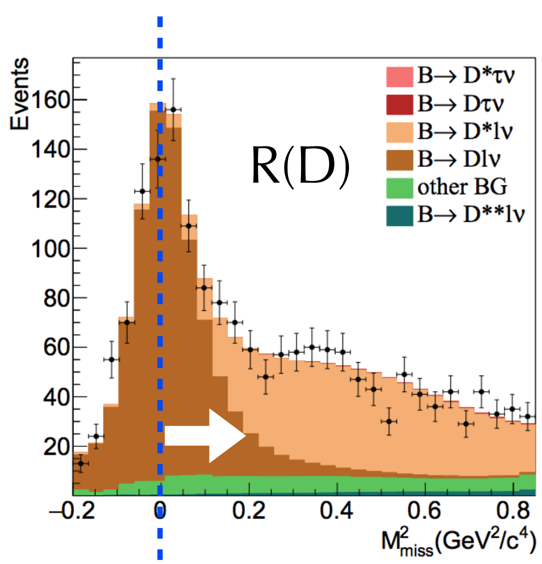
$$E_\gamma \lesssim E_{\rm max} \approx \frac{m_B}{m_B^2 - s_{D\ell}} \hat{M}^2_{{\rm miss},\gamma}$$

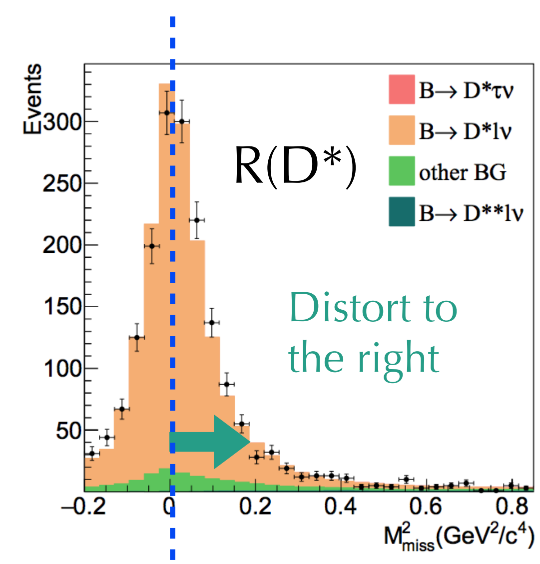
$\hat{M}^2_{{\rm miss},\gamma} = 0.1\,{\rm GeV}^2$ and $s_{D\ell} = 10\,{\rm GeV}$ → $E_{\rm max} \approx 30\,{\rm MeV}$

[de Boer, TK, Nisandzic, PRL]

Miss^2 distribution of selected events

[Belle, PRD92 (2015) no.7, 072014]





data implies slight asymmetry of the distribution

QED corr. (%)

$\hat{M}^2_{{\rm miss},\gamma}$	0.05	0.1	0.2
***B*(D⁺μ)**	-2.8	-1.9	-1.0
***B*(D⁰μ)**	-2.9	-2.3	-1.6

Effect of QED corrections on $R(D)$

Conclusions

- We analytically evaluated soft-photon corrections to $B \to D\tau\nu$ and $B \to D\mu\nu$ using the soft-photon approximation
- Soft-photon corrections depend on lepton's kinematics: **mass and velocity** and hence **can violate lepton flavor universality**, which is **larger than the QCD uncertainty** of form factors
- Numerical comparison with PHOTOS analysis is required to predict QED corrections to $R(D^{0,+})_{SM}$

Outlook

- **Beyond soft-photon approximation** (including electron mode, E_{max}>100MeV (structure dependence contributions), missing mass squared analysis in 4-body phase space)
- **soft-photon corrections to $B \to D^* l\nu$ [$R(D^*)$]**
- **soft-photon corrections to exclusive $|V_{cb}|$**