Effect of QED corrections on $R(D)$

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in collaboration with
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Semileptonic $B$ decay

- **Semileptonic $B$-meson decays induced by $b \to c l \nu$ transitions** play an important role for testing the Standard Model at low energy: $|V_{cb}|$ and **lepton flavor universality**

- **Lepton flavor universality** is violated by only tau lepton mass which leads to smaller phase space and subleading scalar form factors $f_0(q^2)$ and $A_0(q^2)$

- $SU(2)_L$ gauge symmetry

- light lepton universalities in kaon, pion and $\tau$ decays have been checked

$$K^+ \to \pi^0 \ell^+ \nu(\gamma) \quad r_{\mu e}(K^+) = 0.998(9)$$
$$K_L \to \pi^- \ell^+ \nu(\gamma) \quad r_{\mu e}(K_L) = 1.003(5)$$
$$\pi^+ \to \ell^+ \nu(\gamma) \quad r_{\mu e}(\pi^+) = 1.0042(33)$$
$$\tau^+ \to \ell^+ \nu \bar{\nu}(\gamma) \quad r_{\mu e}(\tau^+) = 1.000(4)$$

$$r_{\mu e}^{SM} = \left(\frac{g_{W\mu\bar{\nu}}}{g_{W\ell\bar{\nu}}}ight)^2 = 1$$

[Rainer Wanke, KAON 2007; Cristina Lazzeroni, IoP Nuclear and Particle Divisional Conference, 2011]
Violation of the lepton flavour universality has been announced in $R(D)$ and $R(D^*)$

\[
R(D^{(*)+}) = \frac{\mathcal{B}(\bar{B}^0 \to D^{(*)+} \tau^- \bar{\nu})}{\mathcal{B}(\bar{B}^0 \to D^{(*)+} \ell^- \bar{\nu})}
\]

\[
R(D^{(*)0}) = \frac{\mathcal{B}(B^- \to D^{(*)0} \tau^- \bar{\nu})}{\mathcal{B}(B^- \to D^{(*)0} \ell^- \bar{\nu})}
\]

Theoretically clean: dominant uncertainty from LD QCD in form factors is partially canceled

CKM dependence ($|V_{cb}|$) is canceled

We separately define $R(D^+)$ and $R(D^0)$ to distinguish different QED corrections in neutral and charged $B$ decays
Status of $R(D)$ and $R(D^*)$

LHCb can not measure $D^{*0}$ precisely $[D^{*0} \rightarrow D^0 \gamma, D^0 \pi^0 (\rightarrow \gamma \gamma)]$
Status of $R(D)$ and $R(D^*)$
Effect of QED corrections on $R(D)$ and $R(D^*)$.

No QED corrections

soft-photon corrections are partially subtracted by PHOTOS MC simulation
long-distance QED corrections

Effect of QED corrections on $R(D)$

Soft photons

- Physics in vacuum
  - invisible photon energy
  - sum of invisible emissions and one-loop corrections gives physical quantum correction
    = long-distance QED corrections

Bremsstrahlung

- Physics in detector
  - Emitted photon energy corresponds to dropped lepton energy
  - small bremsstrahlung in $\mu$ and $\tau$
Soft-photon corrections

- At large distance ($\mu \lesssim \Lambda_{\text{QCD}}$), the QED interactions of the charged scalar mesons are well described by the scalar QED.

- We distinguish neutral and charged-$B$ decay

\[
\bar{B}^0 \rightarrow D^+ \ell \bar{\nu} \quad [R(D^+)]
\]

\[
B^- \rightarrow D^0 \ell \bar{\nu} \quad [R(D^0)]
\]
Soft-photon corrections

- $E_{\text{max}}$ is the maximum total energy of undetected soft photons in the rest frame of the $B$-meson: $E_{\text{max}} = 20$–$30$ MeV in current photon detectors
  → NOTE: the explicit photon cut is not set in the semileptonic search but the hard photons do not contribute to the fit of data

- The soft-photon approximation is used for analytic evaluation: we keep $O(\ln E_{\text{max}})$ and $O(E_{\text{max}}^0)$ and drop $O(E_{\text{max}})$, which is valid only $l = \tau$ and $\mu$

- Real soft emissions = $O(\ln E_{\text{max}}) + O(E_{\text{max}}^0)$
  - $O(\ln E_{\text{max}})$ terms are resumed
  - Finite terms [$O(E_{\text{max}}^0)$] are numerically comparable to $O(\ln E_{\text{max}})$

- Vertex corrections = $O(E_{\text{max}}^0)$ and $\mu$-dependent; $\mu \lesssim \Lambda_{\text{QCD}}$ for scalar QED
  - We separate $l_{\mu}=0$ contribution and the other ($l$ is loop momentum)
  - Coulomb pole ($\alpha/v_{\text{rel}}$) exits only in $R(D^+)$ case, and we resumed them

- Both of contributions depend on lepton kinematics → source of LFU violation
Result [de Boer, TK, Nisandzic, PRL120 (2018) no.26, 261804]

\[
\frac{d^2 \Gamma}{d q^2 d s_{D\ell}} = \frac{d^2 \Gamma_0}{d q^2 d s_{D\ell}} \Omega_B^+ \Omega_C \left[1 + \frac{\alpha}{\pi} \left(F_D + F_{\ell} - 2F_{D\ell} - 2H_{D\ell}\right)\right] + \frac{\alpha}{\pi} \frac{d^2 \tilde{\Gamma}^D}{d q^2 d s_{D\ell}},
\]

with

\[
\Omega_B^+ = \left(\frac{2E_{\text{max}}}{\sqrt{m_D m_{\ell}}}\right)^{-\frac{2\alpha}{\pi} (1-2b_{D\ell})}, \quad \Omega_B^0 = \left(\frac{2E_{\text{max}}}{\sqrt{m_B m_{\ell}}}\right)^{-\frac{2\alpha}{\pi} (1-2b_{B\ell})}, \quad \Omega_C = -\frac{2\pi\alpha}{\beta_{D\ell}} \frac{1}{e^{-\frac{2\pi\alpha}{\beta_{D\ell}}} - 1}
\]

\[
F_i = \frac{1}{2\beta_{B_i}} \ln \frac{1 + \beta_{B_i}}{1 - \beta_{B_i}},
\]

\[
F_{D\ell} = \frac{1}{2} \frac{m_D m_{\ell}}{\sqrt{1 - \beta_{D\ell}^2}} \int_0^1 \frac{dz}{P(z)} \frac{E(z)}{[E(z)^2 - P(z)^2]} \ln \frac{E(z) + P(z)}{E(z) - P(z)},
\]

\[
F_{B\ell} = \frac{1}{4\beta_B} \left\{ \text{Li}_2 \left(\frac{1 - \beta_{B\ell}}{2}\right) - \text{Li}_2 \left(\frac{1 + \beta_{B\ell}}{2}\right) + 4\text{Li}_2 (\beta_{B\ell}) - \text{Li}_2 (\beta_{B\ell}^2) + \ln 2 \ln \frac{1 + \beta_{B\ell}}{1 - \beta_{B\ell}} \right. \\
\left. + \frac{1}{2} \ln^2 (1 - \beta_{B\ell}) - \frac{1}{2} \ln^2 (1 + \beta_{B\ell}) \right\},
\]

\[
H_{ij} = -\frac{1}{2\beta_{ij}} \left\{ \frac{1}{2} \ln^2 \frac{m_i}{m_j} \left(1 + \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - \frac{1}{2} \ln^2 \frac{\Delta_{ij} + \Delta_{ij} \beta_{ij}}{\Delta_{ij}^2 + \Delta_{ij} \beta_{ij}} \right) - \text{Li}_2 \left(\frac{2\Delta_{ij} \beta_{ij}}{\Delta_{ij}^2 + \Delta_{ij} \beta_{ij}}\right) \right. \\
\left. + \frac{1}{4} \ln \frac{m_i m_j}{\mu^2} - \frac{1}{2} - \frac{m_i^2 - m_j^2}{4s_{ij}} \ln \frac{m_i}{m_j} - \frac{1}{4} \Delta_{ij} \beta_{ij} \ln \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - \frac{\Delta_{ij}^2}{4\beta_{ij}} \ln \frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right. \\
\left. - \frac{\Delta_{ij}^2}{4\beta_{ij}} \ln \frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right\},
\]

where

\[
b_{D\ell} = \frac{1}{4\beta_{D\ell}} \ln \frac{1 + \beta_{D\ell}}{1 - \beta_{D\ell}}, \quad \beta_{D\ell} = \left[1 - \frac{4m_D^2 m_{D\ell}^2}{(s_{D\ell} - m_D^2 - m_{D\ell}^2)^2}\right]^\frac{1}{2}, \quad \beta_{B\ell} = \left(1 - \frac{m_{D\ell}^2}{E_{\ell}^2}\right)^\frac{1}{2}, \quad E_{\ell} = \frac{s_{D\ell} + q^2 - m_B^2}{2m_B},
\]

\[
\Delta_{ij} = \frac{s_{ij} - m_i^2 - m_j^2}{2s_{ij}}, \quad \Delta_{ij}^i = \frac{s_{ij} + m_i^2 - m_j^2}{2s_{ij}},
\]
what we did

- We obtained analytic full-one loop QED long-distance correction, which is three parameter function; $E_{\text{max}}$ and 2 Dalitz variables $[(p_D + p_e)^2, (p_B - p_D)^2]$

- We resumed two potentially large contributions;
  $(\alpha \ln E_{\text{max}})^n$ from an arbitrary number of real photon emissions
  $(\pi \alpha / \beta_{D\ell})^n$ from ladder of photons

- IR-divergence ($m_{\gamma} \rightarrow 0$) cancels between real emissions$^2$ and loops (vertex and wave-function corrections) : We checked analytically

- numerically crosschecked with LoopTools and Package-X [Hahn, Perez-Victoria '99; Patel '15]

- analytically crosschecked with $P \rightarrow PPP$ QED correction (e.g., $K \rightarrow 3\pi$) when spin dependent terms are dropped [Isidori '08]
Result

We find $E_{\text{max}}$ dependence is suppressed in $\tau$ modes which comes from $\tau$ non-relativistic velocity

[de Boer, TK, Nisandzic, PRL120 (2018) no.26, 261804]
We conclude that the QED corrections to $R(D^+)$ and $R(D^0)$ are different at 1-1.5%.
\[ B^0 \rightarrow D^+ \ell^- \nu \]

Note: naive size of QED corrections \( \sim O(\alpha/\pi) \sim 0.3\% \)

<table>
<thead>
<tr>
<th>( B^0 \rightarrow D^+ \ell^- \nu )</th>
<th>( \Omega_B ) ( \text{log}(E_{\text{max}}) )</th>
<th>( \Omega_C )</th>
<th>( F_D )</th>
<th>( F_I )</th>
<th>( F_{DI} )</th>
<th>( H_{DI} ) ( (l=0) )</th>
<th>( H_{DI} ) ( (l&gt;0) ) ( (\mu=200 \text{ MeV}) )</th>
<th>IB-loop ( (\mu=200 \text{ MeV}) )</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Br(\tau) )</td>
<td>-0.72</td>
<td>3.22</td>
<td>0.26</td>
<td>0.27</td>
<td>-0.65</td>
<td>-0.64</td>
<td>-0.10</td>
<td>-0.10</td>
<td>1.41</td>
</tr>
<tr>
<td>( Br(\mu) )</td>
<td>-3.14</td>
<td>2.31</td>
<td>0.28</td>
<td>0.73</td>
<td>-3.10</td>
<td>-1.40</td>
<td>1.25</td>
<td>0.24</td>
<td>-2.84</td>
</tr>
<tr>
<td>( R(D^+) )</td>
<td>2.50</td>
<td>0.89</td>
<td>-0.02</td>
<td>-0.45</td>
<td>2.52</td>
<td>0.77</td>
<td>-1.33</td>
<td>-0.34</td>
<td>4.38</td>
</tr>
</tbody>
</table>

\( \Omega_B \): log\( (E_{\text{max}}) \) contributions from full real emissions
\( \Omega_C \): Coulomb correction
\( F_D \): finite terms \( \sim O(E_{\text{max}}^0) \) of real emission from \( D^+ \)
\( F_I \): finite terms of real emission from \( \ell \)
\( F_{DI} \): finite terms of interference between real emissions from \( D^+ \) and \( \ell \)
\( H_{DI} \): loop correction between \( D^+ \) and \( \ell \)
IB-loop: loop correction containing Inner-Bremsstrahlung vertex
Note: naive size of QED corrections $\sim O(\alpha/\pi) \sim 0.3\%$

<table>
<thead>
<tr>
<th>$B^{-} \rightarrow D^{0} \tau \bar{\nu}$</th>
<th>$\Omega_{B}$ ($E_{\text{max}} = 20\text{MeV}$)</th>
<th>$\Omega_{C}$</th>
<th>$F_{B}$</th>
<th>$F_{I}$</th>
<th>$F_{\text{BI}}$</th>
<th>$H_{\text{BI}}$ ($l=0$)</th>
<th>$H_{\text{BI}}$ ($l&gt;0$) ($\mu = 200\text{MeV}$)</th>
<th>IB-loop ($\mu = 200\text{MeV}$)</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Br($\tau$)</td>
<td>-0.22</td>
<td>-</td>
<td>0.23</td>
<td>0.27</td>
<td>-0.53</td>
<td>0.03</td>
<td>-0.09</td>
<td>0.37</td>
<td>0.03</td>
</tr>
<tr>
<td>Br($\mu$)</td>
<td>-2.93</td>
<td>-</td>
<td>0.23</td>
<td>0.74</td>
<td>-2.79</td>
<td>0.35</td>
<td>1.11</td>
<td>0.19</td>
<td>-2.98</td>
</tr>
<tr>
<td>R($D^{0}$)</td>
<td>2.79</td>
<td>-</td>
<td>0.00</td>
<td>-0.46</td>
<td>2.32</td>
<td>-0.31</td>
<td>-1.19</td>
<td>0.18</td>
<td>3.11</td>
</tr>
</tbody>
</table>

$\Omega_{B}$: log($E_{\text{max}}$) contributions from full real emissions
$\Omega_{C}$: Coulomb correction
$F_{B}$: finite terms $[=O(E_{\text{max}}^{0})]$ of real emission from $B^{-}$
$F_{I}$: finite terms of real emission from $l$
$F_{\text{BI}}$: finite terms of interference between real emissions from $B^{-}$ and $l$
$H_{\text{BI}}$: finite terms of interference between real emissions from $B^{-}$ and $l$
IB-loop: loop correction containing Inner-Bremsstrahlung vertex
PHOTOS MC simulation

[Barberio, Eijk, Was, ’91; Barberio, Was, ’94; Davidson, Przedzinski, Was ’16]

- PHOTOS Monte-Carlo generator can simulate modifications of the kinematic variables induced by final-state photon radiations
- PHOTOS is utilized in Belle/BaBar/LHCb for $B$ semileptonic decay search
- For general decay processes, PHOTOS can simulate final-state radiation in the leading-logarithmic collinear approximation
  - All virtual corrections including Coulomb pole are not covered in PHOTOS results
  - Quantum interference between two real photon emissions are not covered in PHOTOS (< version 2.13) results
    - LHCb does include the interference between two different real photons from different final charged particle
- Numerical comparison with PHOTOS analysis is required to predict QED corrections to $R(D_{0,+})_{SM}$. Ongoing discussion with a few LHCb colleagues to size up the accuracy of PHOTOS
Missing mass squared analysis

- The experiments (Belle/BaBar/LHCb) have not explicitly utilized the photon cut $E_{\text{max}}$ for event selections for $B$ semileptonic decay.

- The experiments rely on **missing mass squared analysis** for the event selection

\[ M^2_{\text{miss}} \equiv (p_{e^-} + p_{e^+} - p_{B_{\text{tag}}} - p_D - p_{\ell})^2 = p_{\nu}^2 = 0 \quad \text{(ideal)} \]

\[ = \text{Gaussian distribution around 0 by detector resolution} \]

If single undetectable photon exists...

\[ = (p_{\nu} + p_{\gamma})^2 = 2E_{\nu}E_{\gamma}(1 - \cos \theta_{\nu\gamma}) > 0 \rightarrow \text{positive shift of the distribution} \]

\[ E_\gamma \lesssim E_{\text{max}} \approx \frac{m_B}{m_B - s_{D\ell}} \hat{M}^2_{\text{miss,} \gamma} \]

$\hat{M}^2_{\text{miss,} \gamma} = 0.1 \text{ GeV}^2$ and $s_{D\ell} = 10 \text{ GeV} \rightarrow E_{\text{max}} \approx 30 \text{ MeV}$

[de Boer, TK, Nisandzic, PRL]

Data implies slight asymmetry of the distribution

<table>
<thead>
<tr>
<th>QED corr. (%)</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(D^*\mu)$</td>
<td>-2.8</td>
<td>-1.9</td>
<td>-1.0</td>
</tr>
<tr>
<td>$B(D^0\mu)$</td>
<td>-2.9</td>
<td>-2.3</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

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Teppei Kitahara: Karlsruhe Institute of Technology, CKM 2018, September 18, 2018, Heidelberg University
Conclusions

- We analytically evaluated soft-photon corrections to $B \to D\tau\nu$ and $B \to D\mu\nu$ using the soft-photon approximation.

- Soft-photon corrections depend on lepton’s kinematics: mass and velocity and hence can violate lepton flavor universality, which is larger than the QCD uncertainty of form factors.

- Numerical comparison with PHOTOS analysis is required to predict QED corrections to $R(D^{0,+})_{\text{SM}}$.

Outlook

- **Beyond soft-photon approximation** (including electron mode, $E_{\text{max}}>100\text{MeV}$ (structure dependence contributions), missing mass squared analysis in 4-body phase space).

- **soft-photon corrections to** $B \to D^{*}\ell\nu$ [$R(D^*)$]

- **soft-photon corrections to exclusive** $|V_{cbl}|$

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