$\bar{B} \longrightarrow D^{(*)}\ell^{-}\bar{\nu}_{\ell}$ at Belle and Belle II

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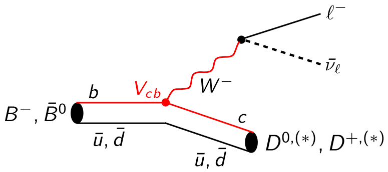
Measuring $|V_{cb}|$

- Pure leptonic: $B_c \longrightarrow \ell\bar{\nu}$. Unavailable @$B$-factories
- Pure hadronic: Theoretical difficulties
- **Semileptonic**:
  Theory: Small EW corrections; QCD uncertainties under control
  Experiment: Only one neutrino missing, good BRs ($\approx 10\%$)
  $\Longrightarrow$ Best opportunity to measure $|V_{cb}|$

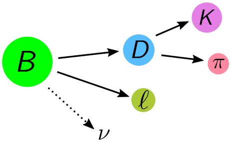
$\bar{B} \longrightarrow D^{(*)}\ell^{-}\bar{\nu}_{\ell}$



Exclusive vs Inclusive Tension

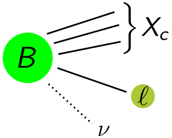
Exclusive Analysis

- Specific decay modes

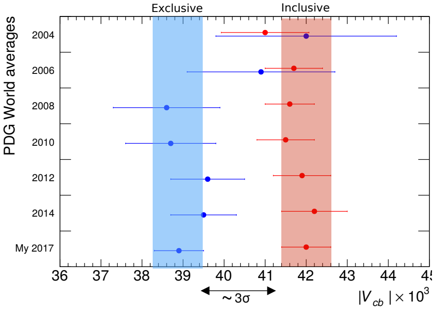


Inclusive Analysis

- Anything $+\ \ell\nu$



[Bernlocher: SM@LHC 2017]



$\longleftarrow$ Long standing tension between exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$!

However:
Current PDG review (Oct. 2017):

$$|V_{cb}| = (41.9 \pm 2.0) \times 10^{-3} \quad \text{(excl.)}$$

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3} \quad \text{(incl.)}$$

$|V_{cb}|$ extraction from $\bar{B} \longrightarrow D^{(*)} \ell^- \bar{\nu}_\ell$

Assuming $m_\ell = 0$

$\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ HFLAV Eur.Phys.J. C77 no.12, 895

$$\frac{d\Gamma(\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell)}{dw} = \frac{G_F^2 m_{D^*}^3}{48\pi^3}(m_B - m_{D^*})^2 \eta_{EW}^2 \chi(w) \mathcal{F}^2(w) |V_{cb}|^2,$$

$$\text{with} \quad \chi(w)\mathcal{F}^2(w) = h_{A_1}^2(w)\sqrt{w^2-1}(w+1)^2 \times$$

$$\times \left(2 \left(\frac{1 - 2wr + r^2}{(1-r)^2} \right) \left(1 + R_1^2(w) \frac{w^2-1}{w+1} \right) + \left(1 + (1 - R_2(w)) \frac{w-1}{1-r} \right)^2 \right)$$

$\bar{B} \longrightarrow D \ell^- \bar{\nu}_\ell$ HFLAV Eur.Phys.J. C77 no.12, 895

$$\frac{d\Gamma(\bar{B} \longrightarrow D \ell^- \bar{\nu}_\ell)}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2 (w^2 - 1)^{3/2} \eta_{EW}^2 \mathcal{G}^2(w) |V_{cb}|^2$$

$$\text{with} \quad \mathcal{G}^2(w) = \frac{4r}{(1+r)^2} f_+^2(w)$$

Where,

- $w = \vec{v}_B \vec{v}_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$
- $r = m_{D^{(*)}} / m_B$
- η_{EW}^2: Small EW correction (+ long distance EM radiation effect = Coulomb correction)

Form factor (FF) parametrizations

Different FF parametrization $\Longrightarrow$ Different $|V_{cb}|$ $\Longrightarrow$ Might solve incl. vs. excl. tension!

CLN (Caprini, Lellouch, Neubert)
Nucl.Phys. B530 153-181

HQET relations + corrections in powers of $\Lambda_{\mathrm{QCD}}/m_b$, α_s (effect of higher order corrections poorly known).

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

For $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$:

$$h_{A_1}(w) = h_{A_1}(1)\Big(-z^3\left(231\rho_{D^*}^2 - 91\right) + z^2\left(53\rho_{D^*}^2 - 15\right) - 8z\rho_{D^*}^2 + 1\Big),$$

$$R_1(w) = R_1(1) + 0.05(w-1)^2 - 0.12(w-1),$$

$$R_2(w) = R_2(1) - 0.06(w-1)^2 + 0.11(w-1)$$

For $\bar{B} \longrightarrow D \ell^- \bar{\nu}_\ell$:

$$\frac{(1+r)^2}{4r} f_+^2(w) = \mathcal{G}(1)(1 - 8\rho_D^2 z + (51\rho_D^2 - 10)z^2 + (252\rho_D^2 - 84)z^3)$$

BGL (Boyd, Grinstein, Lebed)
Nucl.Phys. B461 493-511

No HQET input (some questions remain, see e.g. 1708.07134)

For $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$: Phys.Lett. B769 441-445

$$h_{A_1}(w) = \frac{f(w)}{\sqrt{m_B m_{D^*}}(1+w)}$$

$$R_1(w) = (w+1) m_B m_{D^*} \frac{g(w)}{f(w)}$$

$$R_2(w) = \frac{w-r}{w-1} - \frac{\mathcal{F}_1(w)}{m_B(w-1)f(w)}$$

For $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ and $\bar{B} \longrightarrow D \ell^- \bar{\nu}_\ell$:

f_+ and $f, g, \mathcal{F}_1$ are parametrized as

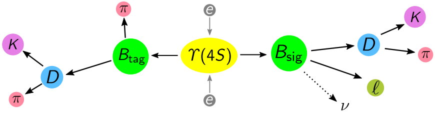
$$\frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_{i,n} z^n$$

Cut off at $N = 2, 3, \ldots$ (when χ^2/ndf is satisfying).

Tagged vs Untagged Analyses

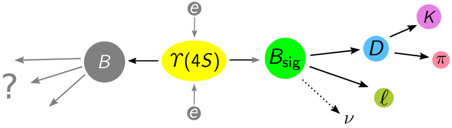
Tagged Analysis

+ High purity
− Low efficiency
(0.3% @Belle → 0.55% @Belle II)



Untagged Analysis

− Low purity
+ High efficiency



Basic analysis steps

1. Reconstruction
2. Projection into bins of kinematic variable
3. Fitting signal yields
4. Compare measured yields to expected yields
   = Fit to determine $|V_{cb}|$ and form factors

Improved hadronic tagging @Belle II

Hadronic tagging @Belle II:
Around 5000 channels!
2.5× efficiency!

Algorithm	Efficiency @0.25 Purity
Belle Cut	0.1%
Belle NB	0.2%
Belle II FEI	0.5%

Calibration for tagged $\bar{B} \longrightarrow D^{(*)}\ell^{-}\bar{\nu}_{\ell}$: $B \longrightarrow X\ell\nu$
(systematics limited)

1808.10567 Red: New reconstruction modes

B^+ modes	B^0 modes
$B^+ \to \bar{D}^0\pi^+$	$B^0 \to D^-\pi^+$
$B^+ \to \bar{D}^0\pi^+\pi^0$	$B^0 \to D^-\pi^+\pi^0$
$B^+ \to \bar{D}^0\pi^+\pi^0\pi^0$	$B^0 \to D^-\pi^+\pi^+\pi^-$
$B^+ \to \bar{D}^0\pi^+\pi^+\pi^-$	$B^0 \to D_s^+D^-$
$B^+ \to D_s^+\bar{D}^0$	$B^0 \to D^{*-}\pi^+$
$B^+ \to \bar{D}^{*0}\pi^+$	$B^0 \to D^{*-}\pi^+\pi^0$
$B^+ \to \bar{D}^{*0}\pi^+\pi^0$	$B^0 \to D^{*-}\pi^+\pi^+\pi^-$
$B^+ \to \bar{D}^{*0}\pi^+\pi^+\pi^-$	$B^0 \to D^{*-}\pi^+\pi^+\pi^-\pi^0$
$B^+ \to \bar{D}^{*0}\pi^+\pi^+\pi^-\pi^0$	$B^0 \to D_s^{*+}D^-$
$B^+ \to D_s^{*+}\bar{D}^0$	$B^0 \to D_s^+D^{*-}$
$B^+ \to D_s^+\bar{D}^{*0}$	$B^0 \to D_s^{*+}D^{*-}$
$B^+ \to \bar{D}^0K^+$	$B^0 \to J/\psi K_S^0$
$B^+ \to D^-\pi^+\pi^+$	$B^0 \to J/\psi K^+\pi^+$
$B^+ \to J/\psi K^+$	$B^0 \to J/\psi K_S^0\pi^+\pi^-$
$B^+ \to J/\psi K^+\pi^+\pi^-$	
$B^+ \to J/\psi K^+\pi^0$	
$B^+ \to D^-\pi^+\pi^+\pi^0$	$B^0 \to D^-\pi^+\pi^0\pi^0$
$B^+ \to \bar{D}^0\pi^+\pi^+\pi^-\pi^0$	$B^0 \to D^-\pi^+\pi^+\pi^-\pi^0$
$B^+ \to \bar{D}^0D^+$	$B^0 \to \bar{D}^0\pi^+\pi^-$
$B^+ \to \bar{D}^0D^+K_S^0$	$B^0 \to D^-D^0K^+$
$B^+ \to \bar{D}^{*0}D^+K_S^0$	$B^0 \to D^-D^{*0}K^+$
$B^+ \to \bar{D}^0D^{*+}K_S^0$	$B^0 \to D^{*-}D^0K^+$
$B^+ \to \bar{D}^{*0}D^{*+}K_S^0$	$B^0 \to D^{*-}D^{*0}K^+$
$B^+ \to \bar{D}^0D^0K^+$	$B^0 \to D^-D^+K_S^0$
$B^+ \to \bar{D}^{*0}D^0K^+$	$B^0 \to D^{*-}D^+K_S^0$
$B^+ \to \bar{D}^0D^{*0}K^+$	$B^0 \to D^-D^{*+}K_S^0$
$B^+ \to \bar{D}^{*0}D^{*0}K^+$	$B^0 \to D^{*-}D^{*+}K_S^0$
$B^+ \to \bar{D}^{*0}\pi^+\pi^0\pi^0$	$B^0 \to D^{*-}\pi^+\pi^0\pi^0$

D^+, D^{*+}, D_s^+ modes	D^0,D^{*0} modes
$D^+ \to K^-\pi^+\pi^+$	$D^0 \to K^-\pi^+$
$D^+ \to K^-\pi^+\pi^+\pi^0$	$D^0 \to K^-\pi^+\pi^0$
$D^+ \to K^-K^+\pi^+$	$D^0 \to K^-\pi^+\pi^+\pi^-$
$D^+ \to K^-K^+\pi^+\pi^0$	$D^0 \to \pi^-\pi^+$
$D^+ \to K_S^0\pi^+$	$D^0 \to \pi^-\pi^+\pi^0$
$D^+ \to K_S^0\pi^+\pi^0$	$D^0 \to K_S^0\pi^0$
$D^+ \to K_S^0\pi^+\pi^+\pi^-$	$D^0 \to K_S^0\pi^+\pi^-$
$D^{*+} \to D^0\pi^+$	$D^0 \to K_S^0\pi^+\pi^-\pi^0$
$D^{*+} \to D^+\pi^0$	$D^0 \to K^-K^+$
$D_s^+ \to K^+K_S^0$	$D^0 \to K^-K^+K_S^0$
$D_s^+ \to K^+\pi^+\pi^-$	$D^{*0} \to D^0\pi^0$
$D_s^+ \to K^+K^-\pi^+$	$D^{*0} \to D^0\gamma$
$D_s^+ \to K^+K^-\pi^+\pi^0$	
$D_s^+ \to K^+K_S^0\pi^+\pi^-$	
$D_s^+ \to K^-K_S^0\pi^+\pi^+$	
$D_s^+ \to K^+K^-\pi^+\pi^+\pi^-$	
$D_s^+ \to \pi^+\pi^+\pi^-$	
$D_s^{*+} \to D_s^+\pi^0$	
$D^+ \to \pi^+\pi^0$	$D^0 \to K^-\pi^+\pi^0\pi^0$
$D^+ \to \pi^+\pi^+\pi^-$	$D^0 \to K^-\pi^+\pi^+\pi^-\pi^0$
$D^+ \to \pi^+\pi^+\pi^-\pi^0$	$D^0 \to \pi^-\pi^+\pi^+\pi^-$
$D^+ \to K^+K_S^0K_S^0$	$D^0 \to \pi^-\pi^+\pi^0\pi^0$
$D^{*+} \to D^+\gamma$	$D^0 \to K^-K^+\pi^0$
$D_s^+ \to K_S^0\pi^+$	
$D_s^+ \to K_S^0\pi^+\pi^0$	
$D_s^{*+} \to D_s^+\pi^0$	

Reconstruction signal side

π, K meson

Identification via PID likelihood ratio, impact parameters.

π^0: From γ candidates (clusters in calorimeter not matched to any track)

D, D^* meson

arXiv	Signal	Tag	D^0 modes	D^+ modes	D^{*-} modes
Phys.Rev. D82 112007 1809.03290	$D^* \ell^- \bar{\nu}_\ell$	No	$K^-\pi^+$		$D^-\pi^-$
1702.01521	$D^* \ell^- \bar{\nu}_\ell$	Had.	$K^-\pi^+(\pi)(\pi)$	$K^-\pi^+\pi^+$	$\bar{D}^0\pi^-$, $D^-\pi^0$
Phys.Rev. D93 no.3, 032006	$D \ell^- \bar{\nu}_\ell$	Had.	$K^-\pi^+(\pi)(\pi)$, $K^0_S\pi^+\pi^-(\pi^0)$, $K^0_S\pi^0$, K^+K^-, $\pi^+\pi^-(\pi^0)$, $K^0_SK^0_S$, $\pi^0\pi^0$, $K^0_S\pi^0\pi^0$, $K^-\pi^+\pi^+\pi^-\pi^0$	$K^-\pi^+\pi^+(\pi^0)$, $K^0_S\pi^+(\pi^0)$, $K^+K^-\pi^+$, $K^0_SK^+$, $K^0_S\pi^+\pi^+\pi^-$, $\pi^+\pi(\pi)$, $K^-\pi^+\pi^+\pi^+\pi^-$	

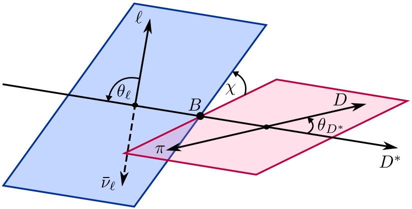
Projection in bins of kinematic variable

$\bar{B} \longrightarrow D \ell^- \bar{\nu}_\ell$

10 equal-size bins in w.
Good resolution (0.005) vs bin width (0.06) $\Longrightarrow$ Bin migration neglected

$\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$

- 10 equal size bins in w, χ, $\cos\theta_\ell$, $\cos\theta_{D^*}$ (Projections)
- Correlation between the 4 distributions ($\longrightarrow$ toy experiments)
- Finite resolution $\Longrightarrow$ Migration!
  $\longrightarrow$ Mig. matrix from truth vs reco MC.
  $\longrightarrow$ Fold theory (easy) or unfold measurement (hard)



Fit variables

Tagged analyses

Fit variable: $m^2_{\text{miss}} := (p_B - p_{D^{(*)}} - p_\ell)^2$ with $p_B = p_{\text{LER}} + p_{\text{HER}} - p_{\text{tag}}$

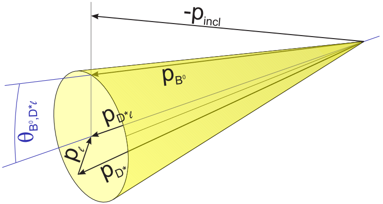
Correct reco $\Longrightarrow$ Peak at 0; Missed particles $\Longrightarrow$ Peak > 0; Particles from tag side $\Longrightarrow$ Peak < 0

Untagged analyses

Fit variables:

- $\cos\theta_{B,D^*\ell} := \dfrac{2E^*_B E^*_{D^*\ell} - m^2_B - m^2_{D^*\ell}}{2|\vec{p}^*_B||\vec{p}^*_{D^*\ell}|}$

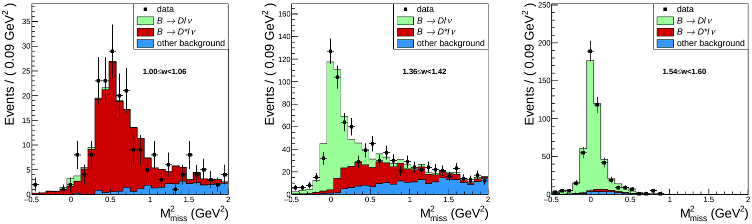
  Correct reco $\Longrightarrow -1 \le \cos\theta_{B,D^*\ell} \le 1$
- $\Delta m = m_{D^*} - m_D$
- p_ℓ



Fit strategies and backgrounds

Belle tagged $\bar{B} \longrightarrow D\ell^- \bar{\nu}_\ell$ Phys.Rev. D93 (2016) no.3, 032006

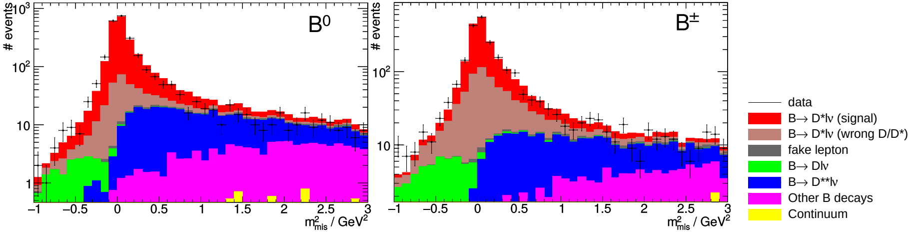
$B = B^0,\ \ell = e$



Fit	Binned extended likelihood fit (Barlow, Beeston, 1993)
Fit variable	$m^2_{\text{miss}} := (p_B - p_D - p_\ell)^2$
Templates	From MC
Fixed Norm.	"other" background from MC
Float. Norm.	**2**: $\bar{B} \longrightarrow D\ell^- \bar{\nu}_\ell$ and $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ normalization

Fit strategies and backgrounds

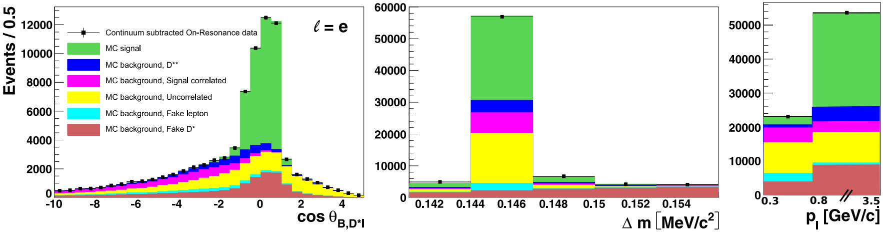
Belle tagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ 1702.01521



Fit	Unbinned likelihood fit
Fit variable	$m^2_{\text{miss}} := (p_B - p_{D^*} - p_\ell)^2$
Templates	From MC
Fixed Norm.	Ratios of background normalizations from MC
Float. Norm.	**2**: Correctly reco. sig. + sig. with $D^{(*)}$ wrongly reco. norm.; total background normalization

Fit strategies and backgrounds

Belle untagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ Phys.Rev. D82 (2010) 112007

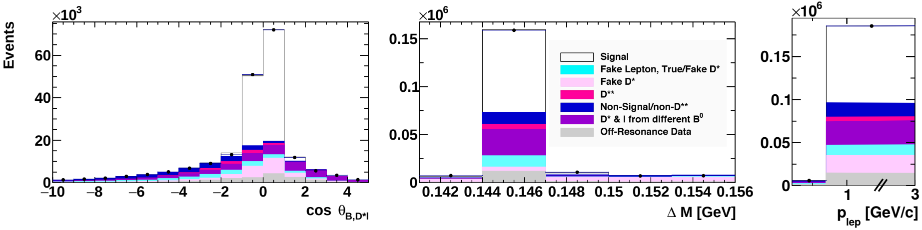


Fit	Binned likelihood fit
Fit variable	$\cos\theta_{B,D^*\ell}$, Δm, p_ℓ
Templates	Continuum from off-resonance, rest from MC[1]
Fixed Norm.	Continuum from off-resonance (corrected for $1/s$ dependency)
Float. Norm.	**6**: Normalizations for signal and backgrounds

[1]For $\ell = \mu$: Shape of fake ℓ corr. with data from $K_s^0 \longrightarrow \pi^+\pi^-$; ℓ PID eff. corr. with data from $2\gamma \longrightarrow e^+e^-/\mu^+\mu^-$

Fit strategies and backgrounds

Belle untagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ (new) 1809.03290



Fit	Binned likelihood fit
Fit variable	$\cos\theta_{B,D^*\ell}$, Δm, p_ℓ
Templates	Continuum from off-resonance, rest from MC[1]
Fixed Norm.	Continuum from off-resonance (corrected for $1/s$ dependency, kinematics)
Float. Norm.	**6**: Normalizations for signal and backgrounds

[1] Shape of fake ℓ corr. with data from $D^* \longrightarrow D^0\pi$, $D^0 \longrightarrow K\pi$; lepton PID eff. corr. with data from $ee \longrightarrow ee\gamma$, $ee \longrightarrow \mu\mu(\gamma)$ and $J/\psi \longrightarrow \ell^+\ell^-$; low momentum track reco. eff. corr. with control sample of $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$

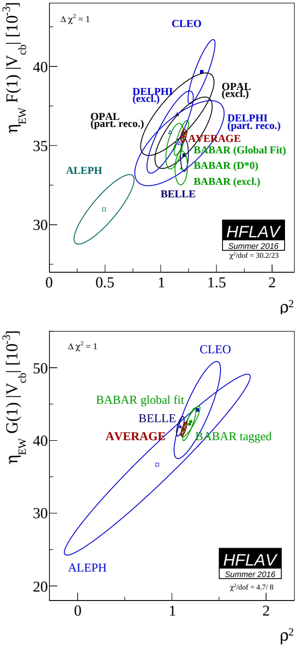
$|V_{cb}|$ and form factors fit

χ^2 fit

$$\chi^2 = \left(\vec{\nu}_{\text{sig}} - \vec{\nu}_{\text{sig}}^{\text{pred}}\right) C^{-1} \left(\vec{\nu}_{\text{sig}} - \vec{\nu}_{\text{sig}}^{\text{pred}}\right) + \chi^2_{\text{nuisance}},$$

where:

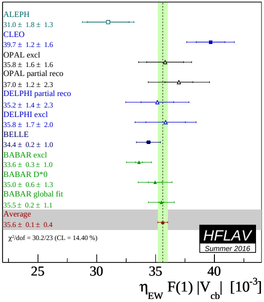
- $\vec{\nu}_{\text{sig}}$ yields in bins of kinematic variables ($D\ell^-\bar{\nu}_\ell$: w, $D^*\ell^-\bar{\nu}_\ell$: w, χ, $\cos\theta_\ell$, $\cos\theta_{D*}$)
- $\vec{\nu}_{\text{sig}}^{\text{pred}} = (\epsilon_{\text{reco}}\epsilon_{\text{tag}})\mathcal{M}_{\text{mig}}\vec{\Delta\Gamma}$
  $\Delta\Gamma_i$: theory expectation diff. CS in bin i (depends on FF param, $|V_{cb}|$),
  $\mathcal{M}_{\text{mig}}$: migration matrix
- C: Covariance matrix
- χ^2_{nuisance}: Account for multiplicative factors degenerate with $|V_{cb}|$

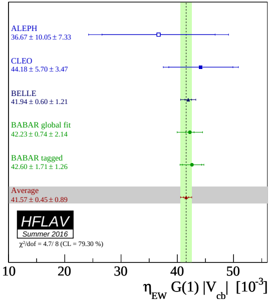


Results and Outlook @Belle

Link	Channel	Tag	$\lvert V_{cb}\rvert \times 10^3$ (CLN)	$\lvert V_{cb}\rvert \times 10^3$ (BGL)	Unfold	Notes
Phys.Rev. D82 112007	$D^*\ell^-\bar{\nu}_\ell$	No	35.5 ± 1.5			
1809.03290	$D^*\ell^-\bar{\nu}_\ell$	No	38.4 ± 0.9	42.5 ± 1.0	Soon	
1702.01521	$D^*\ell^-\bar{\nu}_\ell$	Had.	37.4 ± 1.3		Yes	Soon: Separate results $\ell = e$ and $\ell = \mu$
Phys.Rev. D93 no.3, 032006	$D\ell^-\bar{\nu}_\ell$	Had.	39.9 ± 1.3	40.8 ± 1.1		

cf. current PDG: $V_{cb,\text{incl.}} = (42.2 \pm 0.8) \times 10^{-3}$



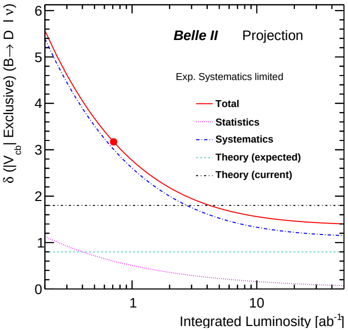


Prospects @Belle II

	ab^{-1}	Stat.	Syst (Red.,Irred.)	$\sum$Exp	Theory	$\sum$
$D^*\ell^-\bar{\nu}_\ell$	0.7	0.6%	(2.8, 1.1)%	3.1%	1.8%	3.6%
	5	0.2%	(1.1, 1.1)%	1.5%	1.0%	1.8%
	50	0.1%	(0.3, 1.1)%	1.2%	0.8%*	1.4%
$D\ell^-\bar{\nu}_\ell$	0.4	4.5%	(3.1, 1.2)%	5.6%	2.2%	6.0%
	5	1.3%	(0.9, 1.2)%	2.0%	1.5%*	2.7%
	50	0.6%	(0.4, 1.2)%	1.4%	1.0%*	1.7%

Table 1: Excpected errors on $|V_{cb}|$

Fig. 1: Excpected errors on $|V_{cb}|$



Light lepton flavor universality tests:

$$\mathcal{R}^{(*)}_{e/\mu} := \frac{\mathrm{BR}(\bar{B} \longrightarrow D^{(*)} e^- \bar{\nu}_e)}{\mathrm{BR}(\bar{B} \longrightarrow D^{(*)} \mu^- \bar{\nu}_\mu)}$$

Observable	Belle	Belle II
$\mathcal{R}^*_{e/\mu}$	5%	1%
$\mathcal{R}_{e/\mu}$	$\approx$ 6%	1%

1607.04918

Backup

Systematic uncertainties: Tagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ 1702.01521

Error Source	$\Delta\mathcal{B}$ [%]
Tagging Calibration	3.6
Tracking Efficiency	1.6
$N_{B\bar{B}}$	1.4
f_{+0}	1.1
PDF shapes	0.9
π^0 Efficiency	0.5
$\mathcal{B}(D \to K\pi(\pi)(\pi))$	0.4
$\mathcal{B}(D^* \to D\,\pi)$	0.2
$\mathcal{B}(\bar{B} \to D^{**}\,\ell\,\bar{\nu}_\ell)$	0.2
e PID	0.2
μ PID	0.1
π_{slow} Eff.	0.1
$\mathcal{B}(\bar{B} \to D\,\ell\,\bar{\nu}_\ell)$	< 0.1
$\bar{B} \to D^{(*,**)}\,\ell\,\bar{\nu}_\ell$ FFs	< 0.1
Lepton Fakerates	< 0.1
K PID	< 0.1
Total Systematic	**4.5**
Statistics	**2.2**

Table 2: Summary of the relative systematic errors ordered by importance in the total branching fraction measurement.

Systematic uncertainties: Untagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ 1809.03290

Systematic Uncertainties	ρ^2	$R_1(1)$	$R_2(1)$	$\mathcal{F}(1)\lvert V_{cb}\rvert$ [%]	B.F. [%]
Slow pion efficiency	0.005	0.002	0.001	0.65	1.29
Lepton ID combined	0.001	0.006	0.004	0.68	1.38
$\mathcal{B}(B \to D^{**}\ell\nu)$	0.002	0.001	0.002	0.26	0.52
$B \to D^{**}\ell\nu$ Form factors	0.003	0.001	0.004	0.10	0.22
f_{+-}/f_{00}	0.001	0.002	0.002	0.52	1.06
Fake e/μ	0.004	0.006	0.001	0.11	0.21
Norm. continuum	0.002	0.002	0.001	0.01	0.06
Fast track efficiency	-	-	-	0.53	1.05
N($\Upsilon(4S)$)	-	-	-	0.68	1.37
B^0 life time	-	-	-	0.13	0.26
K/π ID	-	-	-	0.39	0.77
$\mathcal{B}(D^{*+} \to D^0\pi_s^+)$	-	-	-	0.37	0.74
$\mathcal{B}(D^0 \to K\pi)$	-	-	-	0.51	1.02
Total Systematic	**0.008**	**0.009**	**0.007**	**1.60**	**3.21**

Table 3: Systematic uncertainty breakdown for $\mathcal{F}(1)|V_{cb}|$, branching fraction and form factor parameters in the CLN form factor parameterisation. For $\mathcal{F}(1)|V_{cb}|$ and the branching fraction, the *relative* errors are shown.

Systematic uncertainties: Untagged $\bar{B} \longrightarrow D^* \ell^- \bar{\nu}_\ell$ Phys.Rev. D82 112007

	ρ^2	$R_1(1)$	$R_2(1)$	$\mathcal{F}(1)\|V_{cb}\| \times 10^3$	$\mathcal{B}(B^0 \to D^* \ell \nu)$ [%]
Fast track efficiency				−0.78	−0.206
Slow track efficiency	+0.002	+0.003	−0.004	−0.28	−0.059
ρ_{π_s} stability	+0.001	−0.001	+0.000	−0.03	−0.003
LeptonID	+0.002	+0.006	−0.002	−0.38	−0.100
Norm - D^{**}	+0.001	+0.001	−0.001	−0.03	−0.008
Norm - Signal Corr.	+0.002	−0.003	+0.002	+0.02	+0.006
Norm - Uncorr	+0.002	+0.008	−0.003	−0.02	−0.001
Norm - Fake ℓ	+0.003	−0.003	−0.001	−0.01	−0.003
Norm - Fake D^*	+0.001	−0.001	+0.000	+0.00	+0.003
Norm - Continuum	+0.002	+0.002	−0.001	+0.00	−0.003
D^{**} composition	+0.004	+0.009	−0.003	−0.10	−0.025
D^{**} shape	+0.003	+0.005	−0.002	−0.04	−0.011
$N(\Upsilon(4S))$				−0.24	−0.063
f_{+-}/f_{00}	+0.004	−0.009	+0.003	+0.24	+0.062
B^0 life time				−0.10	−0.027
$\mathcal{B}(D^* \to D^0 \pi_s)$				−0.13	−0.034
$\mathcal{B}(D^0 \to K\pi)$				−0.22	−0.059
Value	**1.214**	**1.401**	**0.864**	**34.6**	**4.58**
Statistical Error	**0.034**	**0.034**	**0.024**	**0.2**	**0.03**
Systematic Error	**0.009**	**0.018**	**0.008**	**1.0**	**0.26**

Table 4: The breakup of the systematic uncertainty in the result of the fit to the full sample. The sign + (−) implies whether the fit result moves to larger (smaller) values, if the value of the corresponding systematic parameter is increased.

Systematic uncertainties: Tagged $\bar{B} \longrightarrow D\ell^-\bar{\nu}_\ell$ Phys.Rev. D93 no.3, 032006

	$\sigma(\Delta\Gamma_i)$ [%]									
	0	1	2	3	4	5	6	7	8	9
Tag correction	3.0	3.2	3.3	3.4	3.4	3.4	3.4	3.3	3.3	3.2
Charged tracks	1.7	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
$\mathcal{B}(D \to \text{hadronic})$	2.0	1.8	1.8	1.8	1.8	1.8	1.8	1.9	1.9	1.9
$\mathcal{B}(B \to D^{*(*)}\ell\nu)$	1.3	0.8	0.8	0.9	0.8	0.7	0.5	0.2	0.2	0.4
$\mathcal{B}(B \to X_u\ell\nu)$	0.4	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
FF$(B \to D^*\ell\nu)$	0.4	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2
FF$(B \to D^{**}\ell\nu)$	2.5	1.2	0.9	0.7	0.5	0.5	0.7	0.5	0.1	0.4
Signal shape	5.0	0.8	0.6	0.5	0.5	0.4	0.3	0.3	0.2	0.1
Lifetimes	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
π^0 efficiency	0.9	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.7
K/π efficiency	1.1	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0
K_S efficiency	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Luminosity	1.4	1.4	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.4
Total	**7.3**	**4.7**	**4.7**	**4.7**	**4.7**	**4.6**	**4.7**	**4.6**	**4.5**	**4.5**

Table 5: Itemization of the systematic uncertainty in $\Delta\Gamma_i$ in each w bin.