Review of exclusive semileptonic B meson decays from lattice QCD

Chris Monahan
Institute for Nuclear Theory
University of Washington
Outline

b to q transitions on the lattice
  - systematic uncertainties

Recent results

  - b to u transitions (and $V_{ub}$)
  - b to c transitions and $V_{cb}$ (and $R(D^{(*)}_{s})$)

Summary and outlook
Outline

b to q transitions on the lattice
- systematic uncertainties

Recent results
- b to u transitions (and $V_{ub}$)
- b to c transitions and $V_{cb}$ (and $R(D^{(*)})$)

Summary and outlook

See also:
S. Meinel, Tu. 14:50
J. Komijani, Th. 14:15
A. Kronfel, M. 11:00
A. Vaquero, M. 10:15
O. Witzel, Th. 14:40
Outline

b to q transitions on the lattice
- systematic uncertainties

Recent results
- b to u transitions (and $V_{ub}$)
- b to c transitions and $V_{cb}$ (and $R(D_{s}^{(*)})$)

Summary and outlook

From Jan 2017 to Lattice 2018, excluding FCNC [WG3]
b to q transitions

Paradigm process:

\[
\begin{align*}
\bar{B}^0 & \rightarrow \ell^- \nu \\
& \quad b \rightarrow \ell^- \nu \\
\end{align*}
\]

\[
\frac{d\Gamma(B_s \rightarrow X_s^{(*)})}{dq^2} = \left[ \sum_i C_i(V_{q_b}) \langle X_s^{(*)} | J_i | B_s \rangle \right]^2
\]
b to q transitions

Paradigm process:

\[
\frac{d\Gamma(B_{(s)} \to X_{(s)}^{(*)})}{dq^2} = \left[ \sum_i C_i(V_{qb}) \langle X_{(s)}^{(*)} | J_i | B_{(s)} \rangle \right]^2
\]

\[
X_{(s)}^{(*)} \in \{ \pi, K, D, D_s, D^*, D_s^* \}
\]

C.M. Bouchard, Lattice 2018
Pseudoscalar to pseudoscalar transitions

Differential decay rate

\[ \frac{d\Gamma(B(s) \to X(s))}{dq^2} = \left[ \sum_i C_i(V_{qb}) \langle X(s) | J_i | B(s) \rangle \right]^2 \]

Introduce form factors to parameterise QCD behavior

\[ \langle X(s) (p_{X(s)}) | V^\mu | B(s) (p_{B(s)}) \rangle = f_0(q^2) \frac{M_{B(s)}^2 - M_{X(s)}^2}{q^2} q^\mu \]

\[ + f_+(q^2) \left[ p_{B(s)}^\mu + p_{X(s)}^\mu - \frac{M_{B(s)}^2 - M_{X(s)}^2}{q^2} q^\mu \right] \]

So

\[ \frac{d\Gamma(B(s) \to X(s))}{dq^2} = \frac{G_F^2 |V_{qb}|^2}{24\pi^3 M_{B(s)}^2} \left( 1 - \frac{m_\ell^2}{q^2} \right)^2 |p_{X(s)}| \]

\[ \times \left[ \left( 1 + \frac{m_\ell^2}{2q^2} \right) M_{B(s)}^2 p_{X(s)}^2 |f_+|^2 + \frac{3m_\ell^2}{8q^2} (M_{B(s)}^2 - M_{X(s)}^2) |f_0|^2 \right] \]
Pseudoscalar to light pseudoscalar

For light leptons

$$\frac{d\Gamma(B_{(s)} \rightarrow X_{(s)})}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_{X_{(s)}}|^3 |f_+|^2$$

In principle: extract $V_{qb}$ directly

In practice: experimental and lattice data most precise in different kinematic regions
Pseudoscalar to heavy pseudoscalar/vector

Differential branching fraction typically written

\[
\frac{d\Gamma(B_s \rightarrow D_s)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 |\eta_{EW}|^2}{48\pi^3} M_{D(s)}^3 (M_{B(s)} + M_{D(s)})^2 (\omega - 1)^{3/2} |\mathcal{G}(\omega)|^2
\]

\[
\frac{d\Gamma(B_s \rightarrow D_s)^*}{dq^2} = \frac{G_F^2 |V_{cb}|^2 |\eta_{EW}|^2}{4\pi^3} M_{D(s)^*}^3 (M_{B(s)} - M_{D(s)^*})^2 (\omega - 1)^{1/2} \chi(\omega) |\mathcal{F}(\omega)|^2
\]

Here

\[\omega = v_{B(s)}^{(*)} \cdot v_{D(s)}^{(*)}\]

\[v^\mu = \frac{p^\mu}{M}\]

In general: form factor parameterisation just a choice

Many choices, but only a few independent ones

Physical form factors functions of single kinematic variable

\[\mathcal{F}(1) = h_{A_1}(1)\]
b to q transitions on the lattice

Determine matrix element using lattice QCD

\[ \langle X^{(*)}_{(s)} | J_i | B_{(s)} \rangle \]
Systematic uncertainties on the lattice

In general, systematic uncertainties arise from:

- discretisation effects
- finite volume effects
- unphysical quark masses

But, for semileptonic B meson decays in particular

- heavy quark effects
- renormalisation
- form factor parameterisation

See slides by A. Vaquero, Tu. 10:10
Quarks on the lattice

Some freedom in discretisation of the Dirac Lagrangian

Symanzik improvement reduces discretisation effects

- $a^2$ tadpole-improved staggered (AsqTad)
- highly improved staggered quarks (HISQ)
- twisted mass (TwM)
- domain-wall (DWF)

Cost increases as lattice spacing decreases

Lattice spacing too coarse to resolve heavy quarks

Relativistic quark actions have uncertainties $\sim (am_b)^n$
Heavy quarks on the lattice

Two approaches

1. Effective theories
   - heavy quark effective theory (HQET)
   - nonrelativistic QCD (NRQCD)
   - relativistic heavy quarks (RHQ)

2. Relativistic actions extrapolated to physical b quark mass

See slides by A. Vaquero, Tu. 10:10
(O. Witzel, Th. 14:40)
Heavy-light current renormalisation

EFT current operators require renormalisation

Three approaches

1. Nonperturbative schemes
   ● PCAC relations
   ● RI/(S)MOM-type schemes

2. Lattice perturbation theory

3. “Mostly nonperturbative”

Relativistic formulations, e.g. “heavy HISQ”, avoid this issue
Kinematic extrapolation

Lattice calculations restricted to large momentum transfer

Two approaches

1. Model dependent, e.g.
   - Becirevic-Kaidalov
   - Ball-Zwicky
   - Hill

2. Model independent \( z \)-parameterisation
   - Caprini, Lellouch, and Neubert (CLN)
   - Boyd, Grinstein, and Lebed (BGL)
   - Bourrely, Caprini, and Lellouch (BCL)

\[
f(q^2) = \frac{1}{B(q^2)\phi(q^2)} \sum_{n=0}^{\infty} a_n z(q^2)^n
\]

\[
z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_- - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_- - t_0}}
\]

\[
t_{\pm} = (M_{B(s)}^2 \pm M_{D(s)}^{(*)2})
\]
Kinematic extrapolation

Lattice calculations restricted to large momentum transfer

Two approaches

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\[ f(q^2) = \frac{1}{B(q^2)\phi(q^2)} \sum_{n=0}^{\infty} a_n z(q^2)^n \]

\[ z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_- - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_- - t_0}} \]

\[ t_\pm = (M_{B(s)}^2 \pm M_{D(s)}^2) \]

See slides by K. Lieret, Tu. 09:50
Outline

- b to q transitions on the lattice
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Recent results

- b to u transitions (and $V_{ub}$)
- b to c transitions and $V_{cb}$ (and $R(D^{(*)}_{s})$)

Summary and outlook
### b to u transitions: B to $\pi$ results

#### FLAG 2017

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<td>$2 + 1 + 1$</td>
<td>HPQCD 2015</td>
<td>HISQ</td>
<td>NRQCD-HISQ</td>
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<td>$2 + 1$</td>
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<td>Riggio [ETMC]</td>
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**B to π: 2+1+1 progress**

FNAL/MILC studying $B_{(s)}$ to pseudoscalars on MILC 2+1+1 HISQ ensembles

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*Plot from Z. Gelzer Lattice 2018*
B to π: 2+1+1 progress

FNAL/MILC: first (prelim.) 2+1+1 results away from zero recoil

Plots from Z. Gelzer Lattice 2018 Update of 1710.09442
B to $\pi$: 2+1 updates

HPQCD preliminary results on MILC 2+1 AsqTad ensembles

NRQCD-HISQ currents with perturbative renormalisation.

Includes pion momenta up to (1,1,1), with chiral/continuum extrapolation via hard pion SU(2) ChPT, and kinematic extrapolation via BCL z-expansion.

Study of larger momenta underway
B to $\pi$: 2+1 updates

JLQCD preliminary results on 2+1 DWF ensembles

$\alpha^{-1} = 2.453(4)$ GeV
$\alpha^{-1} = 3.610(9)$ GeV
$\alpha^{-1} = 4.496(9)$ GeV
★ Physical

Plots from B Colquhoun Lattice 2018
B to $\pi$: 2+1 updates

JLQCD Preliminary

Plot from B Colquhoun Lattice 2018
## b to u transitions: $B_s$ to $K$ results

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<td>$f_{0,+}^{\text{blind}}(q^2)$</td>
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**B_s to K: 2+1 updates**

FNAL/MILC prelim. results on MILC 2+1 AsqTad ensembles

RHQ-AsqTad currents with mostly non perturbative renormalisation.

Chiral/continuum extrapolation via NNLO HMrSChPT, and kinematic extrapolation via BCL z-expansion.

Plot from Z Gelzer Lattice 2018
B_\text{s} to K: 2+1 updates

RBC/UKQCD preliminary results on 2+1 DWF ensembles

RHQ-DWF currents with mostly non perturbative renormalisation.

Chiral/continuum extrapolation via SU(2) hard kaon ChPT, and kinematic extrapolation via BCL z-expansion.

Plot from O Witzel Lattice 2018
$B_s$ to $K$: 2+1 updates

Plots from Z Gelzer and O Witzel Lattice 2018
b to u transitions: $V_{ub}$

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**FLAG2017**

**Graph:**
- **Y-axis:** $B(q^2)\phi(q^2)f(q^2)$
- **X-axis:** $z(q^2, t_{opt})$

- **Data Points:**
  - $f_0$ BCL fit
  - $f_+$ BCL fit
  - $f_+$ HPQCD 06
  - $f_+$ FNAL/MILC 15
  - $f_+$ RBC/UKQCD 15
  - $f_0$ FNAL/MILC 15
  - $f_0$ RBC/UKQCD 15
  - BaBar untagged 12bin
  - BaBar untagged 6bin
  - Belle tagged 13bin
  - Belle untagged 13bin
  - Belle tagged 7bin
b to u transitions: $V_{ub}$

Plot from Y. Liu et al., 1711.08085
### b to c transitions: B to D results

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<td>FNAL/MILC 2015C HPQCD 2015</td>
<td>AsqTad</td>
<td>RHQ-AsqTad</td>
<td>$f_{0,+}(q^2)$</td>
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<td>1310.5238</td>
<td>TwM</td>
<td>NRQCD-AsqTad</td>
<td>$f_{0,+}(q^2)$</td>
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<tr>
<td>2</td>
<td>TwM</td>
<td>$G(1)$</td>
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<tr>
<td>1711.03487 [HPQCD] HISQ NRQCD-HISQ</td>
<td>$f_{0,+}^{\text{prelim.}}(q^2)$</td>
<td>$f_{0,+}^{\text{prelim.}}(q^2)$</td>
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B to D: 2+1+1 results

HPQCD: first (prelim.) 2+1+1 results, on MILC HISQ ensembles

NRQCD-HISQ currents with perturbative renormalisation.

Chiral/continuum and kinematic extrapolation via modified z-expansion.

Plot from E McLean 1711.03487
**b to c transitions: B to D* results**

**FLAG 2017**

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Since then:

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<th>HPQCD 18</th>
<th>HISQ</th>
<th>NRQCD-HISQ</th>
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<tr>
<td>1711.01786 [SWME]</td>
<td>HISQ</td>
<td>OK-HISQ</td>
<td>$h_{A_1}^{\text{prelim.}}(1)/\rho_{A_1}$</td>
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**Lattice 2018:**

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<td>2 + 1</td>
<td>Vaquero [FNAL/MILC]</td>
<td>AsqTad</td>
<td>RHQ-AsqTad</td>
<td>$h_{A_1}^{\text{blind}}(\omega)$</td>
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A. Vaquero, Tu. 10:15
B to D*: 2+1+1 results

Experimentally more precise than B to D, but one lattice calc.

HPQCD: first 2+1+1 results, on MILC HISQ ensembles

NRQCD-HISQ currents with perturbative renormalisation

Uncertainties dominated by truncation uncertainty

Results in good agreement with FNAL/MILC 2+1 value

\[ h_{A_1}^{\text{HPQCD}}(1) = 0.895(10)_{\text{stat.}}(24)_{\text{sys.}} \]
\[ h_{A_1}^{\text{FNAL/MILC}}(1) = 0.906(4)_{\text{stat.}}(12)_{\text{sys.}} \]
B to D*: 2+1 results

Experimentally more precise than B to D

FNAL/MILC: first (blind) 2+1 results, on MILC AsqTad ensembles

First result for R(D*) soon...
b to c transitions: $B_s$ to $D_s$ results

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| $2 + 1 + 1$ | 1710.03487 [HPQCD] | HISQ         | NRQCD-HISQ | $f_0^{\text{prelim.}}(q^2)$ |
| $2 + 1$     | HPQCD 2017        | AsqTad       | NRQCD-AsqTad | $f_0^{\text{prelim.}}(q^2)$ |

Lattice 2018:

| $2 + 1 + 1$ | McLean [HPQCD]    | HISQ         | HISQ       | $f_0^{\text{prelim.}}(q^2)$ |
| $2 + 1$     | Witzel [RBC/UKQCD]| DWF          | RHQ-DWF    | $f_0^{\text{prelim.}}(q^2)$ |

O. Witzel, Th. 14:40
**B_s to D_s: 2+1+1 results**

HPQCD: first (prelim.) 2+1+1 results, on MILC HISQ ensembles

NRQCD-HISQ and heavy HISQ preliminary results now available

Heavy HISQ agrees with 2+1 NRQCD at endpoint

\[
f_0^{\text{HISQ}}(0) = 0.650(26) \\
\]

\[
f_0^{\text{NRQCD}}(0) = 0.656(31)
\]
**B_{s} to D_{s}: 2+1 results**

HPQCD: first results away from zero recoil

NRQCD-AsqTad operators with perturbative renormalisation

Chiral/continuum and kinematic extrapolation via modified z-expansion

Fragmentation fraction results agree with FNAL/MILC

\[ \frac{f_{s}}{f_{d}}^{(\text{HPQCD})} = 0.307(16)(21)(23)(44) \quad \frac{f_{s}}{f_{d}}^{(\text{FNAL/MILC})} = 0.286(16)(21)(22)(26) \]
$B_s$ to $D_s$: 2+1 results

RBC/UKQCD preliminary results on 2+1 DWF ensembles

RHQ-DWF currents with mostly non perturbative renormalisation.

Chiral/continuum extrapolation via SU(2) hard kaon ChPT, and kinematic extrapolation via BCL $z$-expansion.

Plot from O Witzel Lattice 2018
$B_s$ to $D_s$: 2+1 results

Plots from C Monahan et al., PRD 95 (2017) 114506 and O Witzel Lattice 2018
$B_s$ to $D_s$: 2+1+1 and 2+1 results

Plots from C Monahan et al., PRD 95 (2017) 114506, E McLean Lattice 2018 and O Witzel Lattice 2018
b to c transitions: $B_s$ to $D_s^*$ results

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**$B_s$ to $D_s^*$: 2+1+1 results**

HPQCD: first results, on MILC 2+1+1 HISQ ensembles

\[ f_0(q^2) / (f_{H_c} \sqrt{M_{H_c}}) \text{ [GeV}^{-3/2}] \]

- $a \approx 0.09$ fm
- $a \approx 0.06$ fm
- $a \approx 0.045$ fm
- NRQCD [1703.09728][1503.05762]

\[ h_{A_1}^{NRQCD}(1) = 0.883(12)_{\text{stat.}}(28)_{\text{sys.}} \]

Harrison et al., PRD 97 (2018) 054502

Plot from E McLean Lattice 2018

Preliminary heavy HISQ in agreement with NRQCD-HISQ
b to c transitions: $V_{cb}$
b to c transitions: $V_{cb}$

\[
|V_{cb}|^{\text{FLAG17,2+1}} = 40.1(1.0) \times 10^{-3}
\]

\[
|V_{cb}|^{\text{FLAG17,2+1}} = 39.27(74) \times 10^{-3}
\]
**b to c transitions: $V_{cb}$**

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<th>$N_r = 2 + 1$</th>
<th>$B \rightarrow D \ell \nu$</th>
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<tr>
<td>$N_r = 2$</td>
<td>$B \rightarrow D^* \ell \nu$</td>
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**HFAG inclusive**

\[
\begin{align*}
|V_{cb}|^{\text{FLAG17,2+1}} & = 40.1(1.0) \times 10^{-3} \\
|V_{cb}|^{\text{FLAG17,2+1}} & = 39.27(74) \times 10^{-3}
\end{align*}
\]

\[
|V_{cb}| = 41.3(2.2) \times 10^{-3}
\]

*Harrison et al., PRD 97 (2018) 054502*
b to c transitions: $R(D_{(s)})$

\[ R(D_{(s)}) = \frac{\mathcal{B}(D_{(s)} \rightarrow \tau \nu)}{\mathcal{B}(D_{(s)} \rightarrow \ell \nu)} \]

\[ R(D) = 0.299(3) \]
Gambino and Bigi., PRD 94 (2016) 094008

\[ R(D_s) = 0.314(6) \]
C Monahan et al., PRD 95 (2017) 114506

- **BaBar had. tag**
  \[ 0.440 \pm 0.058 \pm 0.042 \]

- **Belle had. tag**
  \[ 0.375 \pm 0.064 \pm 0.026 \]

- **Average**
  \[ 0.407 \pm 0.039 \pm 0.024 \]

- **PRD94,094008(2016)**
  \[ 0.299 \pm 0.003 \]

- **FNAL/MILC (2015)**
  \[ 0.299 \pm 0.011 \]

- **HPQCD (2015)**
  \[ 0.300 \pm 0.008 \]
b to u/b to c transitions

HPQCD: first results for ratio of $B_s$ to $K$ and $B_s$ to $D_s$

Anticipating LHCb $B_s$ to $K/B_s$ to $D_s$ results

Lattice community eagerly awaits $B_s$ results!
b to q: tldr

B to $\pi$
- more groups now studying decay (3 -> 5)
- soon: 2+1+1 results at nonzero recoil

$B_s$ to K
- more groups now studying decay (2 -> 3)
- soon: 2+1+1 results

B to D
- now: preliminary 2+1+1 results

B to D*
- more groups now studying decay (1 -> 3)
- now: published 2+1+1 result [HPQCD] and new $V_{cb} = 0.00413(22)$
- soon: 2+1 results at nonzero recoil

$B_s$ to $D_s$
- more groups now studying decay (1 -> 3)
- now: published 2+1 result at nonzero recoil [HPQCD], inc. $R(D_s)$
- soon: 2+1+1 results

$B_s$ to $D_s^*$
- more groups now studying decay (0 -> 2)
- now: published 2+1+1 result [HPQCD]
Outlook

Next few years will see many more lattice results
- expect new Vub from ETMC, FNAL/MILC, HPQCD, JLQCD
- expect new Vcb from FNAL/MILC, HPQCD, SWME

Heavy HISQ (and DWF) results are very promising
- allows entirely nonperturbative current renormalisation
- should facilitate sub-1% precision

Anticipating exp. results, $B_s$ decays a real growth industry
- HPQCD, RBC/UKQCD, SWME
- but further progress really requires experimental data

Moving beyond ~0.5% precision will require
- isospin breaking effects
- QED effects
Thank you

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