# Studies on $D^0 \to P_1^+ P_2^- \ell^+ \ell^-$: SM prediction and windows on NP

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# *Motivation*

- FCNC processes suppressed in the SM: window to new physics. $D$ physics has a very efficient GIM suppression but long-distance dominated.

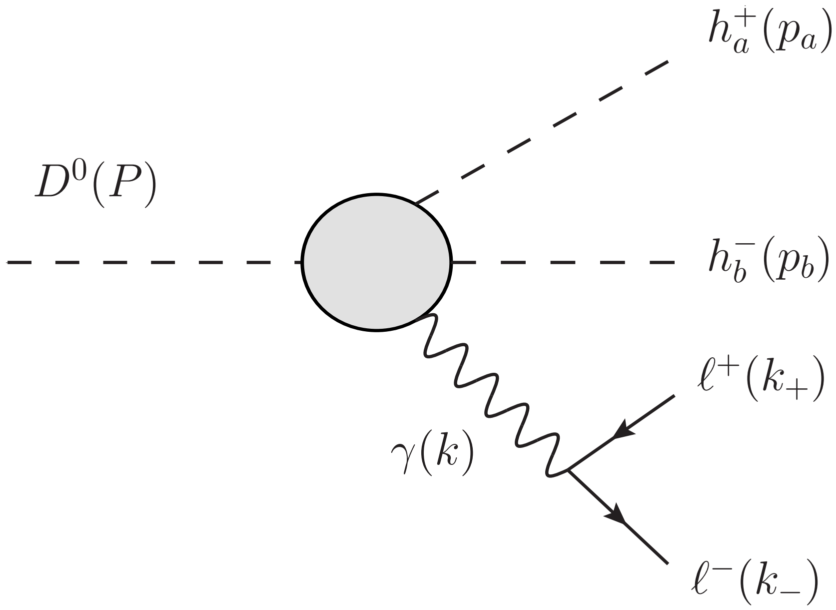
- As opposed to $B$ and $K$ physics, no meaningful expansion works. EFT language not very useful: estimation of hadronic form factors through lattice or sum rules.

- Many intermediate states allowed by phase space. Resonant contributions dominate.

- Rich kinematics.

- Some of the channels have already been measured. [arXiv:1510.08367; 1707.08377; 1808.09680]

# *Long distances*



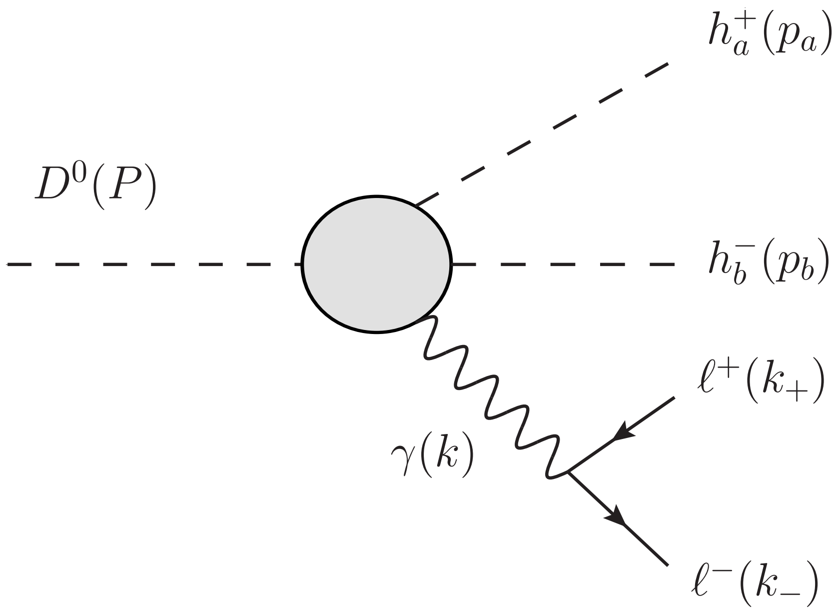

- Bremsstrahlung: pure QED effects, calculable with Low's theorem:

$$\mathcal{M}_b(D^0 \to h_1^+ h_2^- \gamma) = 2e \left[ \frac{p_1 \cdot \epsilon}{2p_1 \cdot q + q^2} - \frac{p_2 \cdot \epsilon}{2p_2 \cdot q + q^2} \right] \mathcal{M}(D^0 \to h_1^+ h_2^-)$$

- Resonant contributions (dominant effects, estimated at $\sim 10^{-6}$). [Burdman et al'02].
  For comparison, short distance SM contribution estimated at $\sim 10^{-9}$. [Bigi et al'11].

- Charge radius (suppressed, not discussed in this talk).

# *Long distances*




Generic kinematical parametrization:

$$\mathcal{M}_{ab} = \frac{e}{k^2}\left[\bar{u}(k_-)\gamma^\mu v(k_+)\right] H^\mu_{ab}(p_a, p_b; k)$$

with

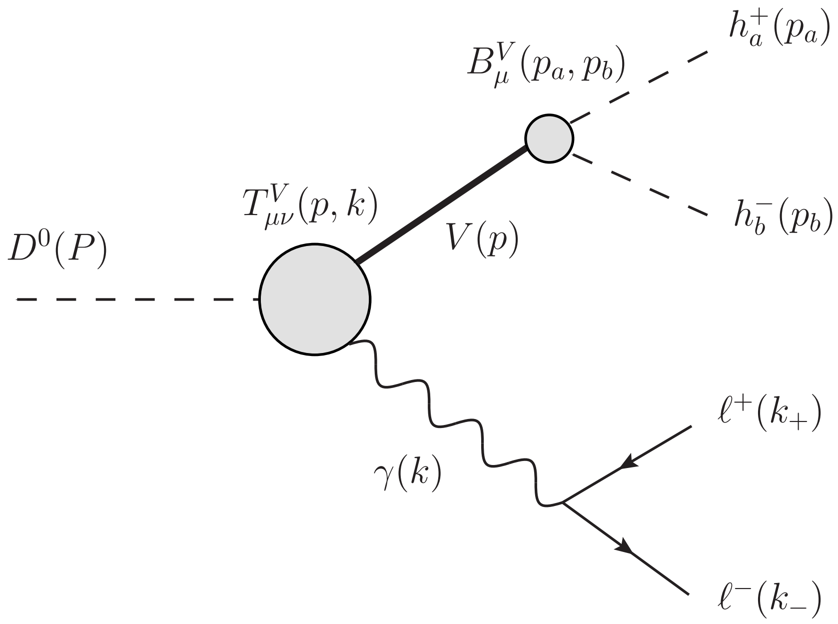
$$H^\mu_{ab}(p_a, p_b; k) = F_1^{(ab)} p_a^\mu + F_2^{(ab)} p_b^\mu + F_3^{(ab)} \epsilon^{\mu\nu\lambda\rho} p_{a\nu} p_{b\lambda} k_\rho$$

Matrix elements:

$$\sum_{\text{spins}} |\mathcal{M}_{ab}|^2 = \frac{2e^2}{q^4}\left[\sum_i^3 |F_i^{(ab)}|^2 T_{ii} + 2\text{Re}\sum_{i<j}^3 (F_i^{(ab)})^* F_j^{(ab)} T_{ij}\right]$$

# *Resonant contribution*

[Cappiello, OC, D'Ambrosio'12]




$$H_{ab}^\mu(p_a,p_b;k) = \langle h_a^+ h_b^- \gamma^* | \mathcal{H} | D^0 \rangle \varepsilon_\gamma^\mu = \sum_k \langle h_a^+ h_b^- | \mathcal{H} | V_k \rangle \frac{\varepsilon_\gamma^\mu}{P_k(p^2)} \langle V_k \gamma^* | \mathcal{H} | D^0 \rangle$$

where

$$\langle h_a^+ h_b^- | \mathcal{H} | V \rangle \equiv B_V^\mu(p_a,p_b) \varepsilon_\mu^V(p); \qquad \langle V\gamma^* | \mathcal{H} | D^0 \rangle \equiv T_V^{\mu\nu}(p,k) \varepsilon_\mu^{V*}(p) \varepsilon_\nu^{\gamma*}(k)$$

Relevant kinematic invariants:

$$B_V^\mu(p_a,p_b) = b^V(p^2)(p_a-p_b)^\mu; \qquad T_{\mu\nu}^V(p,k) = t_1^V g_{\mu\nu} + t_2^V k_\mu p_\nu + t_3^V \epsilon_{\mu\nu\lambda\rho} p_\lambda k_\rho$$

# *Weak form factors*

At hadronic scales,

$$\mathcal{H}_{\Delta c=1} = \sum_{j=1,2} \left[\mathcal{H}_j^{CF} + \mathcal{H}_j^{SCS} + \mathcal{H}_j^{DCS}\right]$$

where

$$\mathcal{H}_j^{CF} = \frac{G_F}{\sqrt{2}} \left[\lambda_{sd} C_j^{(sd)} Q_{sd}^{(j)}\right] \qquad (K^-\pi^+)$$

$$\mathcal{H}_j^{SCS} = \frac{G_F}{\sqrt{2}} \left[\lambda_d C_j^{(d)} Q_d^{(j)} + \lambda_s C_j^{(s)} Q_s^{(j)}\right] \qquad (\pi^+\pi^-; K^+K^-)$$

$$\mathcal{H}_j^{DCS} = \frac{G_F}{\sqrt{2}} \left[\lambda_{ds} C_j^{(ds)} Q_{ds}^{(j)}\right] \qquad (K^+\pi^-)$$

and

$$Q_{sd}^{(1)} = (\bar{s}\gamma^\mu c)_L (\bar{u}\gamma_\mu d)_L; \qquad Q_{sd}^{(2)} = (\bar{u}\gamma_\mu c)_L (\bar{s}\gamma^\mu d)_L$$

$$Q_d^{(1)} = (\bar{d}\gamma^\mu c)_L (\bar{u}\gamma_\mu d)_L; \qquad Q_d^{(2)} = (\bar{u}\gamma_\mu c)_L (\bar{d}\gamma^\mu d)_L$$

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Penguin and semileptonic operator contributions not considered (expected to be subdominant).

# Weak form factors

MAIN ASSUMPTIONS:

(i) Photon leg dominated by (lowest-lying) vector exchange:

$$\langle R_i \gamma^*(k)|\mathcal{H}|R_j\rangle = \sum_{V=\rho,\omega,\phi} \frac{\langle \gamma^*|\mathcal{H}_{\rm EM}|V\rangle}{P_V(k^2)} \langle R_i V|\mathcal{H}|R_j\rangle$$

(ii) Factorization of weak matrix elements (current-current operators)

$$\langle R^+ R^-|J^\mu_{ik} J^{jc}_\mu|D^0\rangle = \langle R^+|J^\mu_{ik}|0\rangle\langle R^-|J^{jc}_\mu|D^0\rangle + \langle R^-|J^\mu_{ik}|0\rangle\langle R^+|J^{jc}_\mu|D^0\rangle$$

The weak tensor $T_{\mu\nu}$ can then be related to the $D^0 \to V$ transitions:

$$\langle V(p)|J^{\bar{u}c}_\mu|D^0(P)\rangle = P_0^V(k^2) k\cdot\varepsilon^* \frac{(m_D^2 - m_V^2)}{k^2} k^\mu + A_1^V(k^2)(m_D^2 - m_V^2)\left[\varepsilon^{*\mu} - \frac{k\cdot\varepsilon^*}{k^2} k^\mu\right]$$
$$+ A_2^V(k^2) k\cdot\varepsilon^* \left[p_+^\mu - \frac{(m_D^2 - m_V^2)}{k^2} k^\mu\right] + iV^V(k^2)\epsilon^{\mu\nu\lambda\rho} p_{+\nu} k_\lambda \varepsilon^*_\rho$$

The form factors are determined using single pole exchange, e.g. [Wirbel et al'85]

$$V(k^2) \sim \frac{h_{V1} m_{V1}^2}{m_{V1}^2 - k^2}$$

with $m_{V1} = 2110$ MeV and residues measured from $D^0 \to K^*$.

# Form factors

One eventually finds

$$t_1^V(p^2,k^2) = -ie\xi_2^V(m_D+m_\rho)\left[J^V(k^2)\hat{A}_1(p^2) + \delta_{V,\rho}W(k^2)\hat{A}_1(k^2)\right]$$

$$t_2^V(p^2,k^2) = \frac{2ie\xi_2^V}{m_D+m_\rho}\left[J^V(k^2)\hat{A}_2(p^2) + \delta_{V,\rho}W(k^2)\hat{A}_2(k^2)\right]$$

$$t_3^V(p^2,k^2) = -\frac{2e\xi_2^V}{m_D+m_\rho}\left[J^V(k^2)\hat{V}(p^2) + \delta_{V,\rho}W(k^2)\hat{V}(k^2)\right]$$

with

$$J^V(k^2) = k^2\left(\frac{f_\rho}{m_\rho P_\rho(k^2)} + \frac{f_\omega}{3m_\omega P_\omega(k^2)}\right) f_V m_V$$

$$W(k^2) = k^2\left(\frac{f_\rho^2}{P_\rho(k^2)} + \frac{f_\omega^2}{3P_\omega(k^2)} - \frac{\xi_2^s}{\xi_2^d}\frac{\sqrt{2}f_\phi^2}{3P_\phi(k^2)}\right)$$

The strong parameters $b_V$ can be determined from $V \to e^+e^-$ decay (experimental input).

$$b_\rho \sim 6.0; \qquad b_{K^*} \sim 5.4; \qquad b_\phi \sim 4.5$$

# *Form factors*

In terms of the hadronic form factors:

$$F_{1V}^{(ab)}(t,b) = \left[ (t_2^V [k \cdot (p_b - p_a)] - t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2} (t_2^V p \cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{2V}^{(ab)}(t,b) = \left[ (t_2^V [k \cdot (p_b - p_a)] + t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2} (t_2^V p \cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{3V}^{(ab)}(t,b) = 2t_3^V \frac{b_V}{P_V(p^2)}$$

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- $t_i$ from $D^0 \to V$ transitions.

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$$F_{3V}^{(ab)}(t,b) = 2t_3^V \frac{b_V}{P_V(p^2)}$$

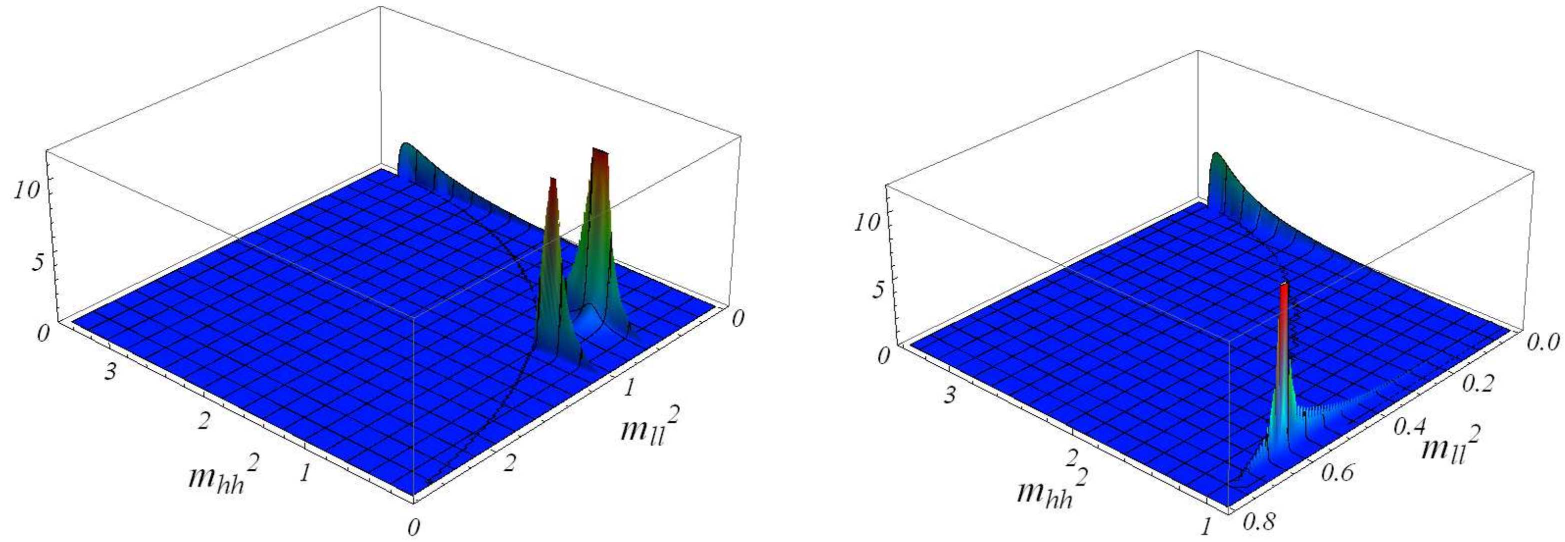
- $t_i$ from $D^0 \to V$ transitions.
- $b_V$ from $V \to e^+e^-$ decays.

# Dalitz plots

$$F_1^{(\pi\pi)} = \frac{2ie}{2q\cdot p_1 + q^2}\mathcal{M}_{(D\to\pi\pi)} + b_\rho \frac{t_2^\rho[q\cdot(p_1-p_2)] + t_1^\rho}{P_\rho(p^2)}$$

$$F_2^{(\pi\pi)} = -\frac{2ie}{2q\cdot p_2 + q^2}\mathcal{M}_{(D\to\pi\pi)} + b_\rho \frac{t_2^\rho[q\cdot(p_1-p_2)] - t_1^\rho}{P_\rho(p^2)}$$

$$F_3^{(\pi\pi)} = -2b_\rho \frac{t_3^\rho}{P_\rho(p^2)}$$




Bremsstrahlung and resonant regions far apart: interference negligible.

# *Branching ratios*

| Decay mode | Bremsstrahlung | Direct emission (E) | Direct emission (M) |
|---|---|---|---|
| $D^0 \to K^-\pi^+e^+e^-$ | $9.9 \cdot 10^{-6}$ | $6.2 \cdot 10^{-6}$ | $4.8 \cdot 10^{-7}$ |
| $D^0 \to \pi^+\pi^-e^+e^-$ | $5.3 \cdot 10^{-7}$ | $1.3 \cdot 10^{-6}$ | $1.3 \cdot 10^{-7}$ |
| $D^0 \to K^+K^-e^+e^-$ | $5.4 \cdot 10^{-7}$ | $1.1 \cdot 10^{-7}$ | $5.0 \cdot 10^{-9}$ |
| $D^0 \to K^+\pi^-e^+e^-$ | $3.7 \cdot 10^{-8}$ | $1.7 \cdot 10^{-8}$ | $1.3 \cdot 10^{-9}$ |
| $D^0 \to K^-\pi^+\mu^+\mu^-$ | $8.6 \cdot 10^{-8}$ | $6.2 \cdot 10^{-6}$ | $4.8 \cdot 10^{-7}$ |
| $D^0 \to \pi^+\pi^-\mu^+\mu^-$ | $5.6 \cdot 10^{-9}$ | $1.3 \cdot 10^{-6}$ | $1.3 \cdot 10^{-7}$ |
| $D^0 \to K^+K^-\mu^+\mu^-$ | $3.3 \cdot 10^{-9}$ | $1.1 \cdot 10^{-7}$ | $5.0 \cdot 10^{-9}$ |
| $D^0 \to K^+\pi^-\mu^+\mu^-$ | $3.3 \cdot 10^{-10}$ | $1.7 \cdot 10^{-8}$ | $1.3 \cdot 10^{-9}$ |

- Experimental results (LHCb and BaBar): [arXiv:1510.08367; arXiv:1707.08377; arXiv:1808.09680]

$$\mathcal{B}(D^0 \to K^-\pi^+\mu^+\mu^-) = (4.17 \pm 0.42) \cdot 10^{-6}$$

$$\mathcal{B}(D^0 \to \pi^+\pi^-\mu^+\mu^-) = (9.64 \pm 0.48 \pm 0.51 \pm 0.97) \cdot 10^{-7}$$

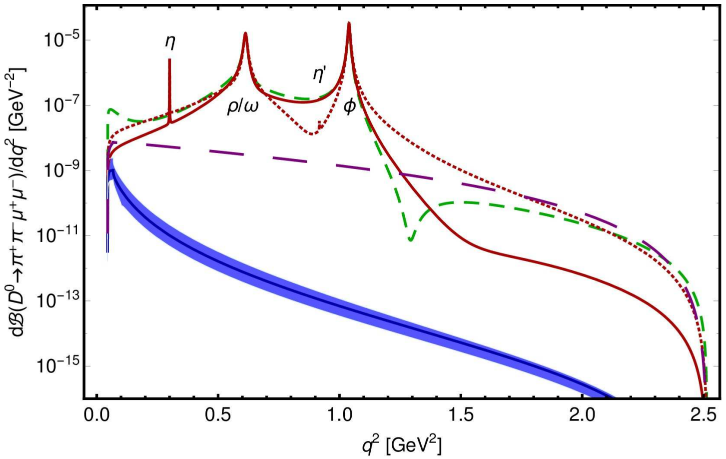
$$\mathcal{B}(D^0 \to K^+K^-\mu^+\mu^-) = (1.54 \pm 0.27 \pm 0.09 \pm 0.16) \cdot 10^{-7}$$

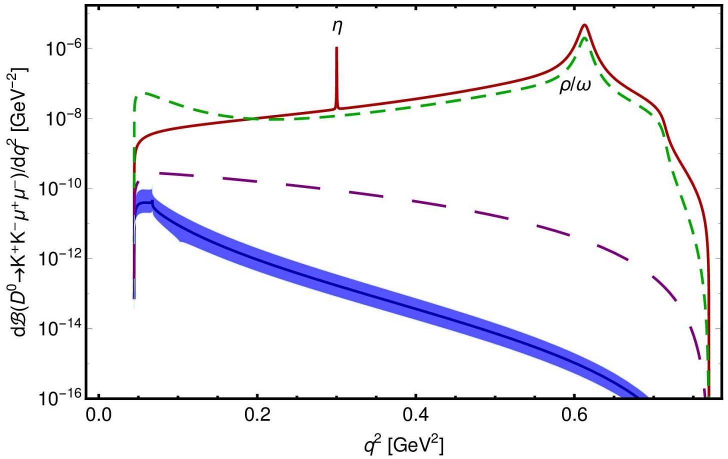
$$\mathcal{B}(D^0 \to K^-\pi^+e^+e^-) = (4.0 \pm 0.5 \pm 0.2 \pm 0.1) \cdot 10^{-6}$$

- Good agreement overall. Approximations capture the bulk of the effect.

# *Branching ratios*

The numbers for the SCS modes have been recently confirmed as well as the dilepton invariant mass distribution: [de Boer, Hiller'18]

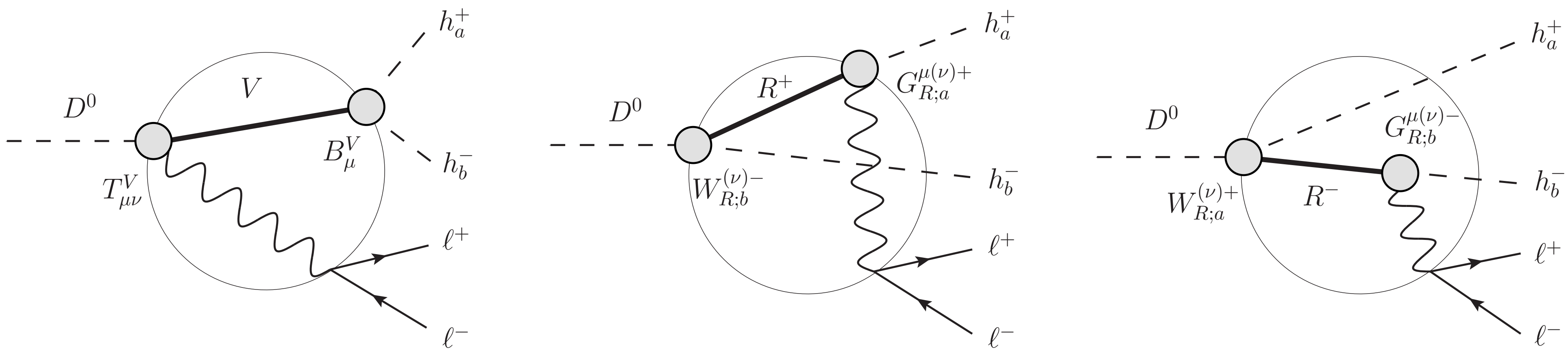



Similar techniques are employed but different numerical input. Nonresonant effects also estimated (negligible).

# *Additional resonant contributions*

[OC, D'Ambrosio'18]

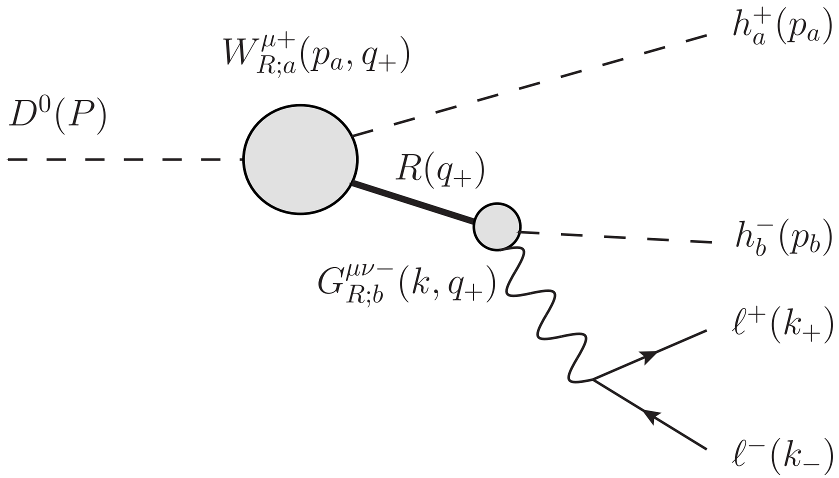


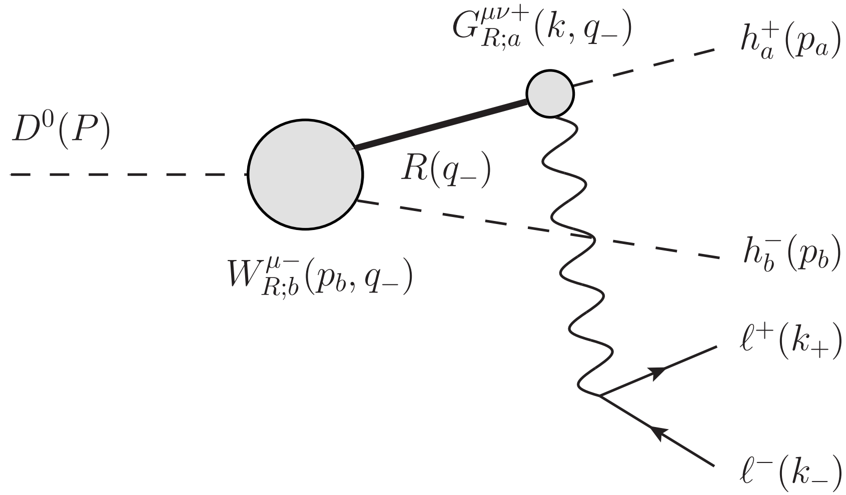

- Extra topologies have intermediate axials ($a_1(1230)$ and $K_1(1272)$). A priori sizeable.

| Decay mode | Bremsstrahlung | Resonant V. (E) | Resonant V. (M) | Resonant A. (E) |
|---|---|---|---|---|
| $D^0 \to K^+K^-\mu^+\mu^-$ | $3.3 \cdot 10^{-9}$ | $1.1 \cdot 10^{-7}$ | $5.0 \cdot 10^{-9}$ | $3.3 \cdot 10^{-8}$ |

Preliminary estimate ($\sim 30\%$ of the vector contribution).

# *Extraction of form factors*




$$F^{(ab)}_{1A} = \frac{w_1^{A;a}}{P_A(q_+^2)} g^-_{1A;b} + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[ g^+_{2A;a} \left( p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g^+_{1A;a} \frac{p_b \cdot q_-}{m_A^2} \right]$$

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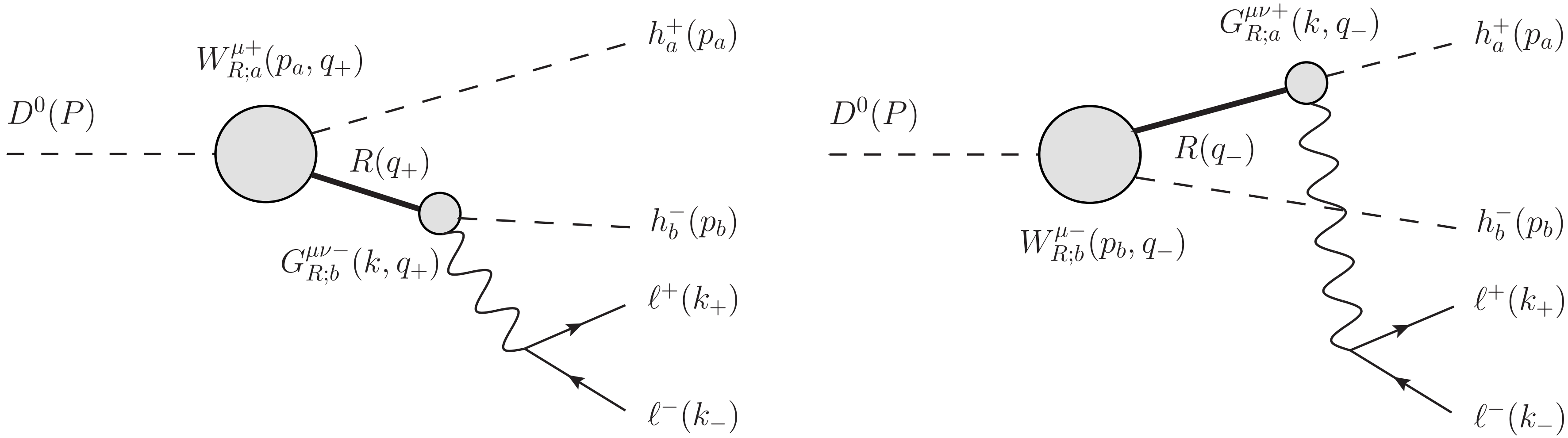
$$F^{(ab)}_{3V} = \frac{w_1^{V;a}}{P_V(q_+^2)} g^-_{3V:b} - \frac{w_1^{V;b}}{P_V(q_-^2)} g^+_{3V:a}$$

# Extraction of form factors




$$F^{(ab)}_{1A} = \frac{w_1^{A;a}}{P_A(q_+^2)} g^-_{1A;b} + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[ g^+_{2A;a} \left( p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g^+_{1A;a} \frac{p_b \cdot q_-}{m_A^2} \right]$$

$$F^{(ab)}_{2A} = \frac{w_1^{A;b}}{P_A(q_-^2)} g^+_{1A;a} + \frac{w_1^{A;a}}{P_A(q_+^2)} \left[ g^-_{2A;b} \left( p_a \cdot k - \frac{(k \cdot q_+)(p_a \cdot q_+)}{m_A^2} \right) - g^-_{1A;b} \frac{p_a \cdot q_+}{m_A^2} \right]$$

$$F^{(ab)}_{3V} = \frac{w_1^{V;a}}{P_V(q_+^2)} g^-_{3V:b} - \frac{w_1^{V;b}}{P_V(q_-^2)} g^+_{3V:a}$$

- $w_1$ from $D^0 \to P, A$ transitions (applying factorization). $D^0 \to P$ well-known, $D^0 \to A$ not measured, so only an educated estimate is possible.

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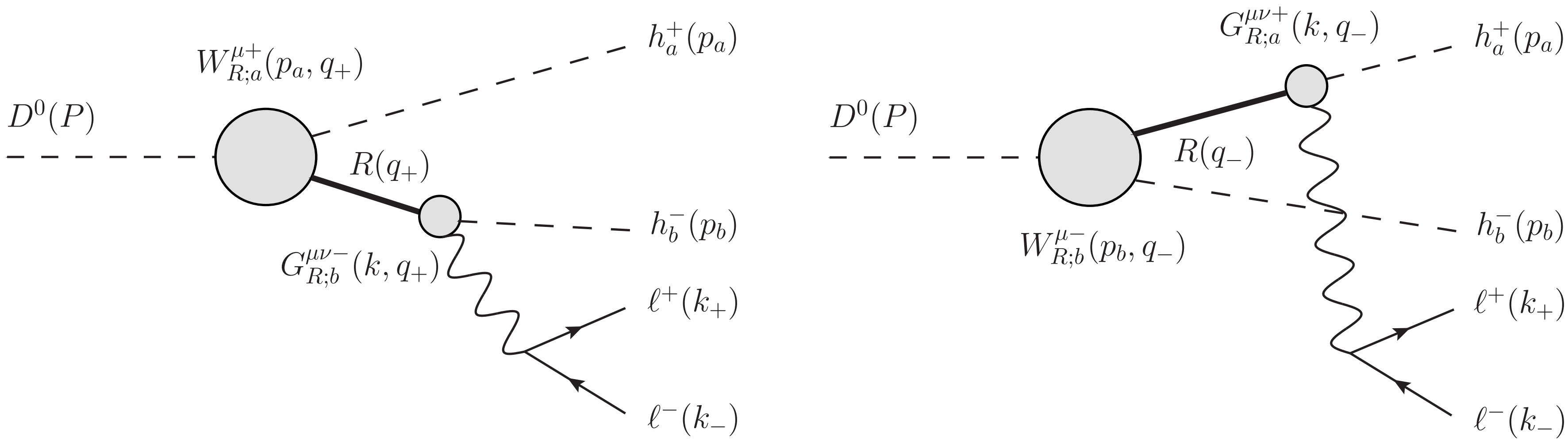


$$F^{(ab)}_{1A} = \frac{w_1^{A;a}}{P_A(q_+^2)} g^-_{1A;b} + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[ g^+_{2A;a} \left( p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g^+_{1A;a} \frac{p_b \cdot q_-}{m_A^2} \right]$$

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$$F^{(ab)}_{3V} = \frac{w_1^{V;a}}{P_V(q_+^2)} g^-_{3V:b} - \frac{w_1^{V;b}}{P_V(q_-^2)} g^+_{3V:a}$$

- $w_1$ from $D^0 \to P, A$ transitions (applying factorization). $D^0 \to P$ well-known, $D^0 \to A$ not measured, so only an educated estimate is possible.
- $g_i$ from $VAP$ correlator [Moussallam'98, Knecht et al'01], once properly LSZ-reduced.

# *Short distances*

- The most general angular distribution in terms of 9 structures:

$$\frac{d^5\Gamma}{dxdy} = \mathcal{A}_1(x) + \mathcal{A}_2(x)s_\ell^2 + \mathcal{A}_3(x)s_\ell^2 c_\phi^2 + \mathcal{A}_4(x)s_{2\ell}c_\phi$$
$$+ \mathcal{A}_5(x)s_\ell c_\phi + \mathcal{A}_6(x)c_\ell + \mathcal{A}_7(x)s_\ell s_\phi$$
$$+ \mathcal{A}_8(x)s_{2\ell}s_\phi + \mathcal{A}_9(x)s_\ell^2 s_{2\phi}$$

  Only the first line contributes to the decay width. The remaining structures through angular asymmetries.

- Null tests especially interesting. Examples:

$$A_\phi = \langle \mathrm{sgn}(s_\phi c_\phi)\rangle = \frac{1}{\Gamma}\int_0^{2\pi}\frac{d\Gamma}{d\phi}d\phi^*, \qquad \int_0^{2\pi} d\phi^* \equiv \left[\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right] d\phi$$

$$A_{FB} = \langle \mathrm{sgn}(c_\ell)\rangle = \frac{1}{\Gamma}\left[\int_0^1 dy\frac{d\Gamma}{dy} - \int_{-1}^0 dy\frac{d\Gamma}{dy}\right]$$

  Recently measured: [LHCb, arXiv:1806.10793]

$$A_{CP}(\pi\pi) = (4.9 \pm 3.8 \pm 0.7)\%, \qquad A_{CP}(KK) = (0 \pm 11 \pm 2)\%$$
$$A_\phi(\pi\pi) = (-0.6 \pm 3.7 \pm 0.6)\%, \qquad A_\phi(KK) = (9 \pm 11 \pm 1)\%$$
$$A_{FB}(\pi\pi) = (3.3 \pm 3.7 \pm 0.6)\%, \qquad A_{FB}(KK) = (0 \pm 11 \pm 2)\%$$

- Exhaustive analysis of null tests recently performed. [de Boer, Hiller'18]

# Conclusions

- Long distance estimate of $D^0 \to V(h^+h^-)\ell^+\ell^-$ in good agreement with experiment. Slight tension in $D^0 \to K^-\pi^+\mu^+\mu^-$ should be resolved.

- Determination of the DCS mode $D^0 \to K^+\pi^-\mu^+\mu^-$ and the modes decaying into electrons would be interesting.

- Other contributions (charged axials) to long distances are present, a priori as important as neutral vectors.

- Short distances: probes of new physics not restricted to charge asymmetry. Clean angular asymmetries exist. Distributions are peaked at the resonant poles. Reanalysis of short distances with axials in mind important.

- The role of resonant axial states could be also interesting in $B \to P_1 P_2 \ell^+\ell^-$ decays, where there is better theoretical control.