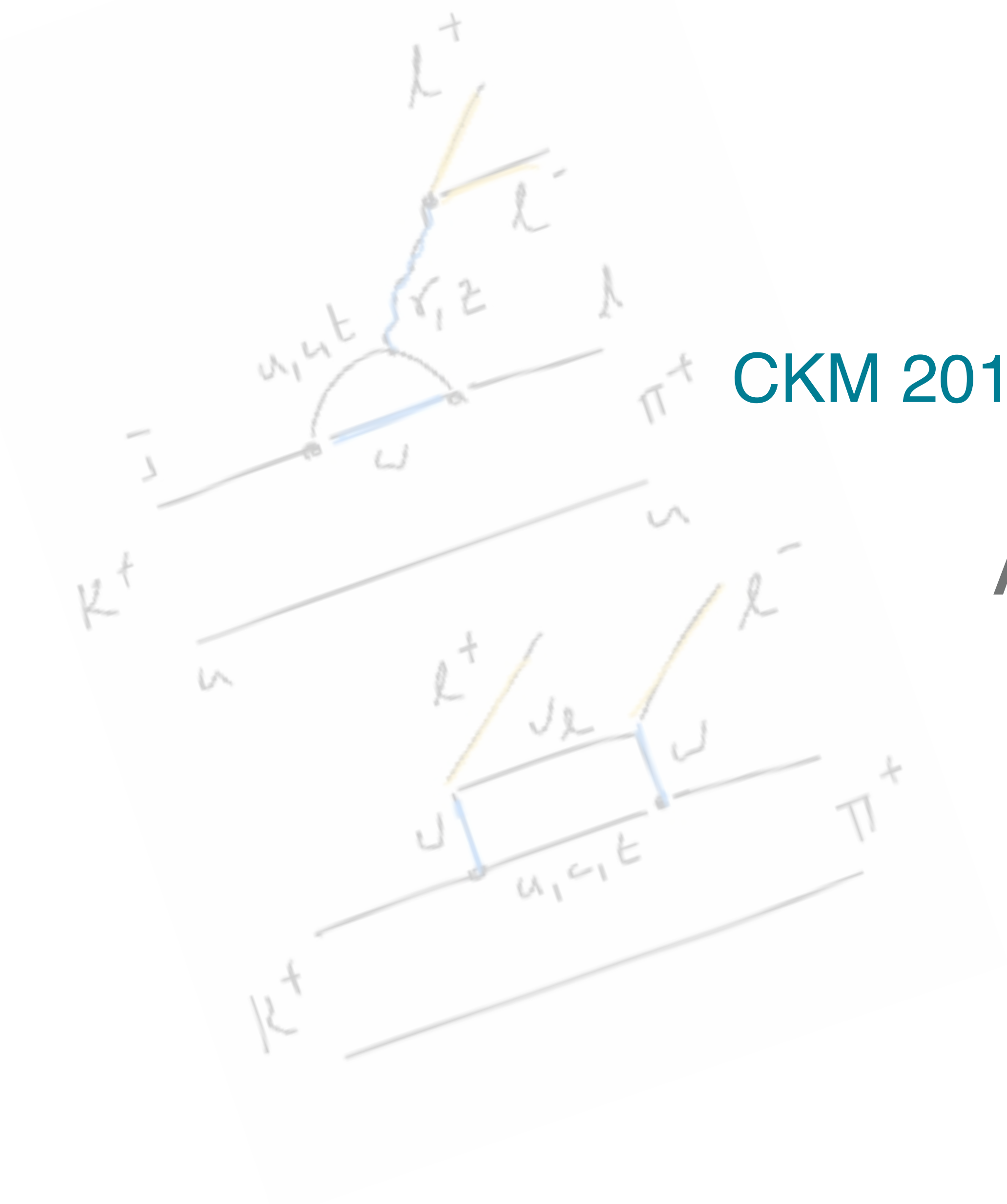
Long distance effects in rare kaon decays



CKM 2018, Heidelberg, September 2018

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Work by the RBC/UKQCD Collaboration

in particular:

Peter Boyle	Edinburgh
Norman Christ	Columbia
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Andreas Jüttner	Southampton
Antonin Portelli	Edinburgh
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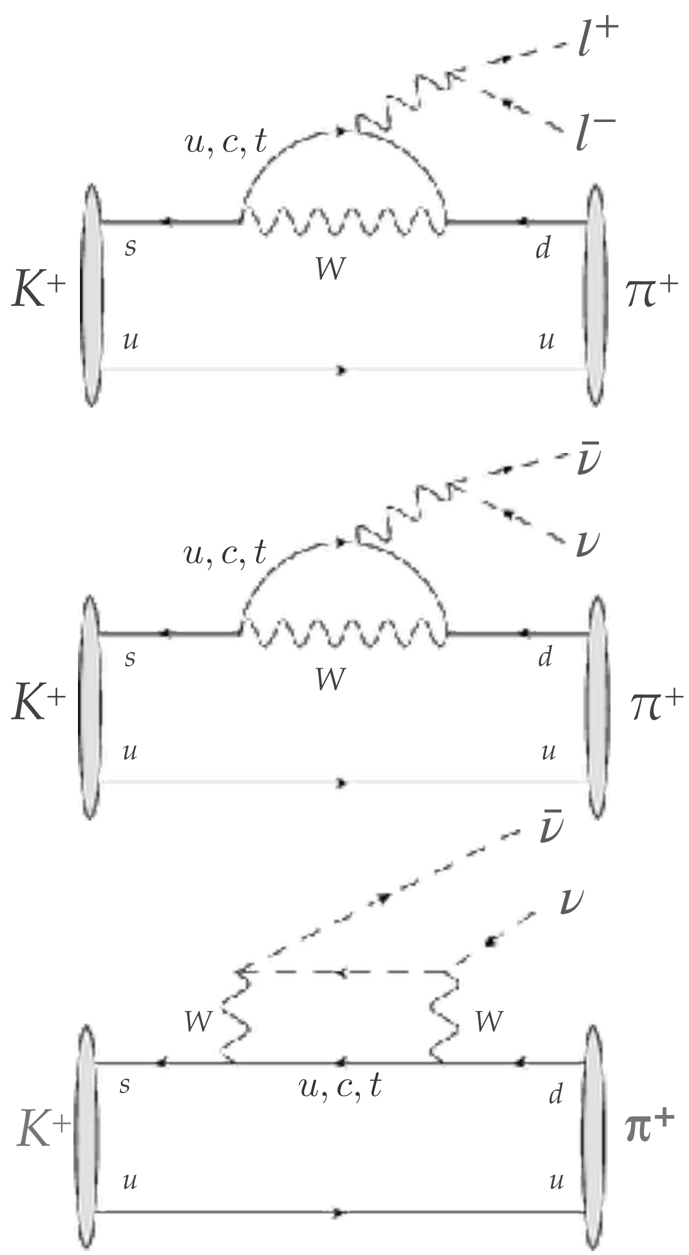
Rare kaon decays



loop suppressed in the SM (FCNC via W-W or γ/Z-exchange diagrams)

hard to observe in nature deep probe into flavour mixing and SM/BSM

J-PARC's KOTO and CERN's NA62 are measuring these decays

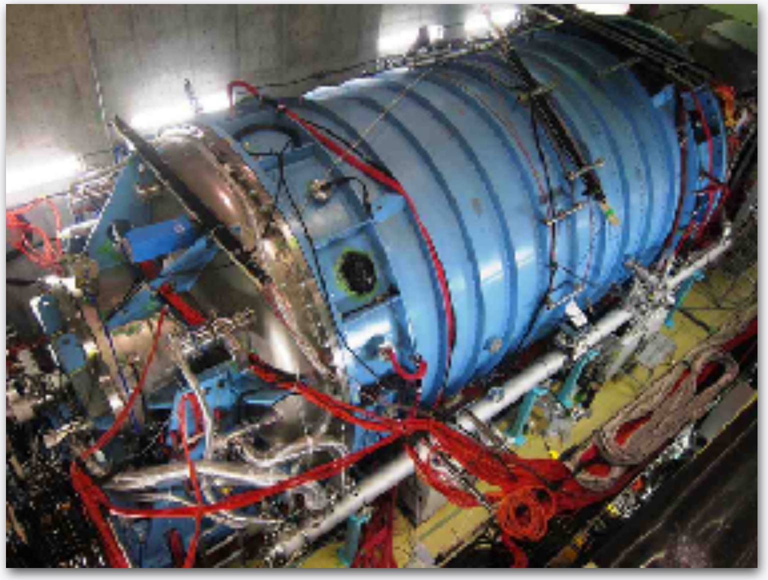
results expected on the time scale of 5 years

see also:
- Kamamoto's talk in plenary session
- Koval's talk

Experiments

$K_L \to \pi^0 \nu \bar{\nu}$

- KOTO (J-PARC)
- direct CP violation
- GIM → top dominated and charm suppressed, pure SD
- phase 2 aims at 10% measurement of BR



$K^+ \to \pi^+ \nu \bar{\nu}$

- NA62 (CERN)
- CP conserving
- small **LD contribution**, candidate for lattice

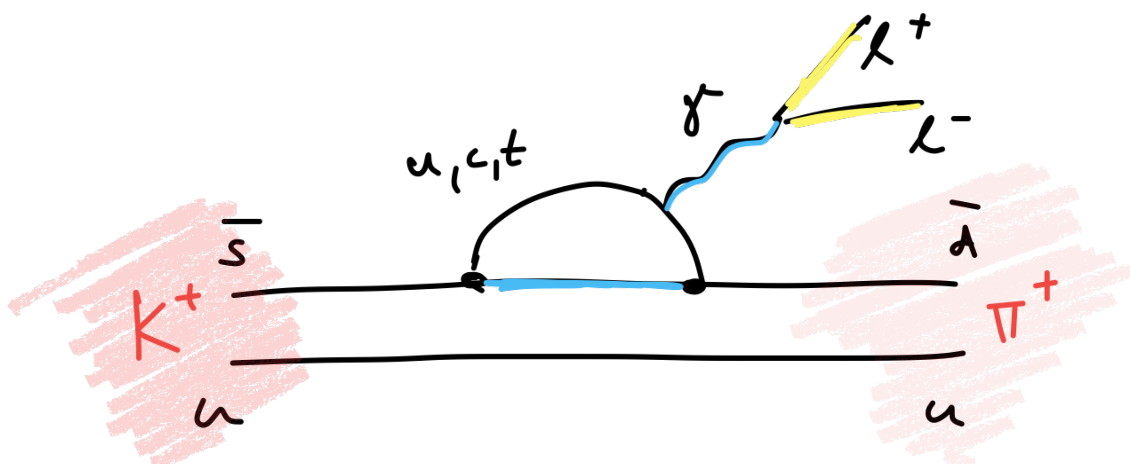
$K^+ \to \pi^+ l^+ l^-$ $K_s \to \pi^0 l^+ l^-$

- 1-photon exchange **LD dom.**
- SM prediction mainly ChPT
- lattice can predict ME and LECs
- well suited for experiment

candidates for lattice computation

2nd order weak processes

consider $K^+ \to \pi^+ l^+ l^-$ with dominant 1-photon contribution:



2nd order weak decay
→ 2 insertions of H_W/J_μ

$$\mathcal{A}_\mu = (q^2)\int d^4x \langle \pi(p)| T\left[J_\mu(0) H_W(x)\right]|K(k)\rangle$$

Aim here: compute non-perturbative physics when *1/x~Λ$_{QCD}$*

Difficulties

1st order weak MEs now bread & butter on the lattice
(see http://flag.unibe.ch)

2nd order weak ME on the lattice new development —
currently we are learning how to do rare kaon decays, ϵ_K, ΔM_K
(similar difficulties also in QCD+QED for decay rate)

Complications

1. **Spectral representation:** Euclidean space intermediate states lead to artefacts that need to be controlled

2. **Renormalisation:** EW operator contact terms lead to UV div.

3. **Finite volume effects:** The finite-volume corrections from intermediate on-shell states can be large

Isidori et al. PLBB 633 (2006) 75-83, **Christ et al.** PRD91 (2015), 114510
RBC/UKQCD PRD92 (2015) 094512, PRD94 (2016) 114516, PRD93 (2016) 114517, PRL118 (2017) 252001, arXiv:1806.11520

Spectral representation - Minkowski

$$\mathcal{A}^c_\mu(q^2) = \int d^4x \langle \pi^c(p)|T\left[J_\mu(0)H_W(x)\right]|K^c(k)\rangle$$

non-strange intermediate states

$$\mathcal{A}^c_\mu(q^2) = i\int_0^\infty dE \frac{\rho(E)}{2E}\frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E + i\epsilon}$$

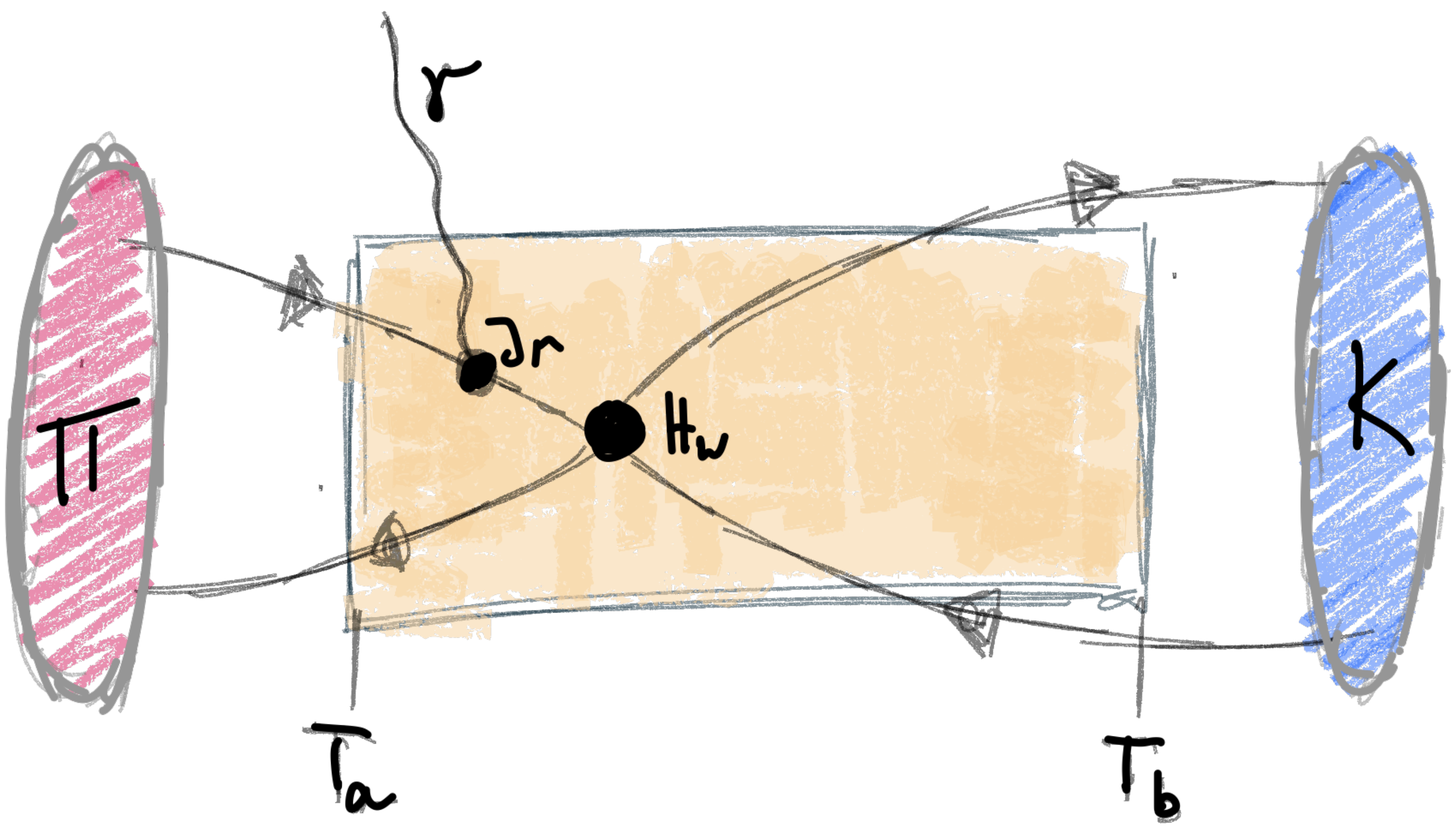
$$-i\int_0^\infty dE \frac{\rho_S(E)}{2E}\frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E - E_\pi(\mathbf{p}) + i\epsilon}$$

strange intermediate states

complications arise when considering the amplitude in **Euclidean space …**

Spectral representation - Euclidean

$$\mathcal{A}_\mu^c(q^2) = \int d^4x \langle \pi^c(p) | T\left[J_\mu(0) H_W(x)\right] | K^c(k) \rangle$$



integrate EW operators over T_a-T_b

see also C. Kelly's talk

Spectral representation - Euclidean

$$A_\mu^c(T_a,T_b,q^2) = \int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+\int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$

exponential in first terms on r.h.s.

- 1st line:
  - $E>E_K$: exponential term vanishes as $T_a \to \infty$
  - $E<E_K$: exponential term grows as $T_a \to \infty$, must be removed (possible intermediate states π, ππ, πππ)
- 2nd line: no problem, all intermediate states E larger E_π

Spectral representation - Euclidean

$$A^c_\mu(T_a, T_b, q^2) = \int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E}\left(1 - e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+ \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E - E_\pi(\mathbf{p})}\left(1 - e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$

subtraction of exponentially increasing states:

- π: either get amplitudes from 2pt and 3pt functions and subtract **or** replace

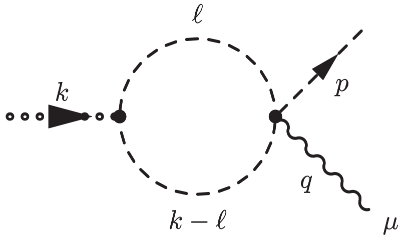
$$H_W(x) \to H'_W(x) = H_W(x) + c_S(\mathbf{k})\bar{s}(x)d(x)$$

where c_S such that $\langle \pi^c(\mathbf{k})|H'_W(0,\mathbf{k})|K^c(\mathbf{k})\rangle = 0$ kills the unwanted divergent contribution and does not contribute to the amplitude itself

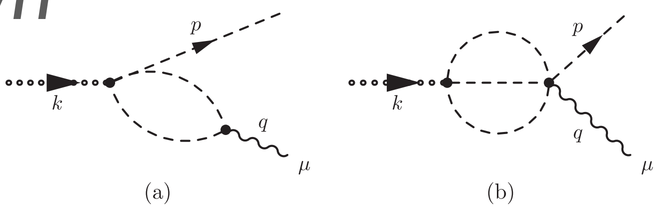
Spectral representation - Euclidean

subtraction of exponentially increasing states:

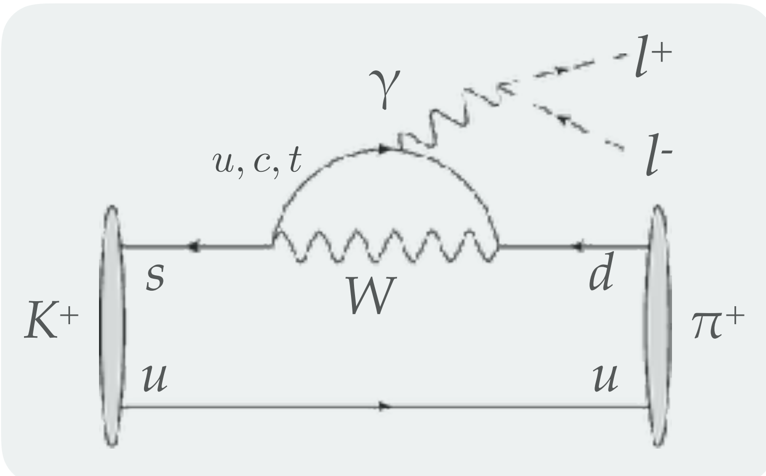
- *ππ: disallowed by O(4) invariance but can be present as discretisation effect — needs to be monitored*



- *πππ: comparison of experimental width (PDG) suggests*
  - *πππ to be highly suppressed wt. respect to ππ*
  - *techniques similar as for ππ possible but it's own research topic (K→πππ)*



Renormalisation

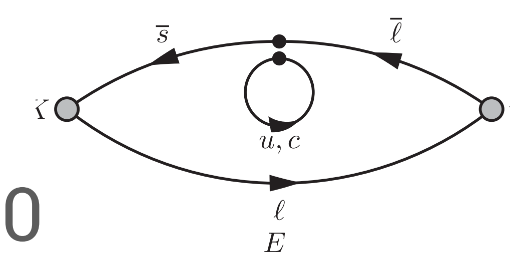


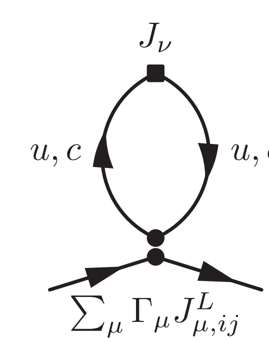
4-flavour

$$\mathcal{A}^c_\mu(q^2) = \int d^4x \langle \pi^c(p) | T\left[J_\mu(0) H_W(x)\right] | K^c(k) \rangle$$

$$H_W(x) = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \left[C_1(Q^u_1 - Q^c_1) + C_2(Q^u_2 - Q^c_2)\right]$$

- Q_1 and Q_2 in H_W renormalise multiplicatively (chiral fermions)
- J_μ conserved
- divergences:
  - quadratic divergence can appear as x→0 but gauge invariance reduces it to a logarithmic one
  - remaining logarithmic divergence cancelled via GIM (→ need charm quark in lattice simulation)

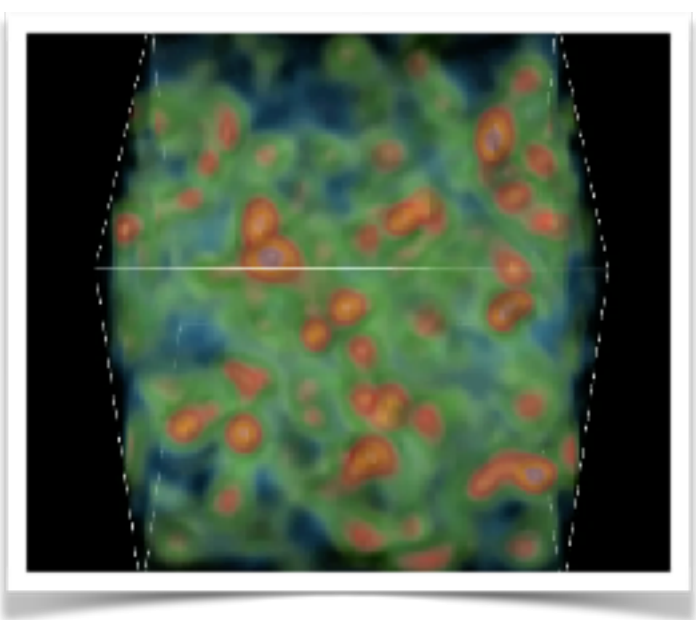




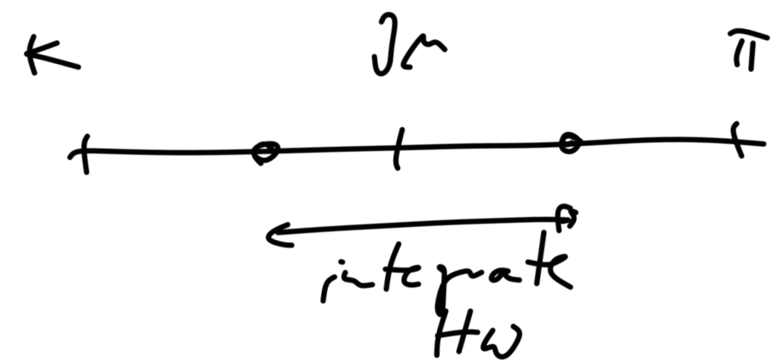
$K^+ \to \pi^+ \nu\bar{\nu}$ more involved due to axial current (also if local vector current)

EXPLORATORY STUDY - Lattice setup

RBC/UKQCD **exploratory** study — unphysical m_π (because it's cheap)



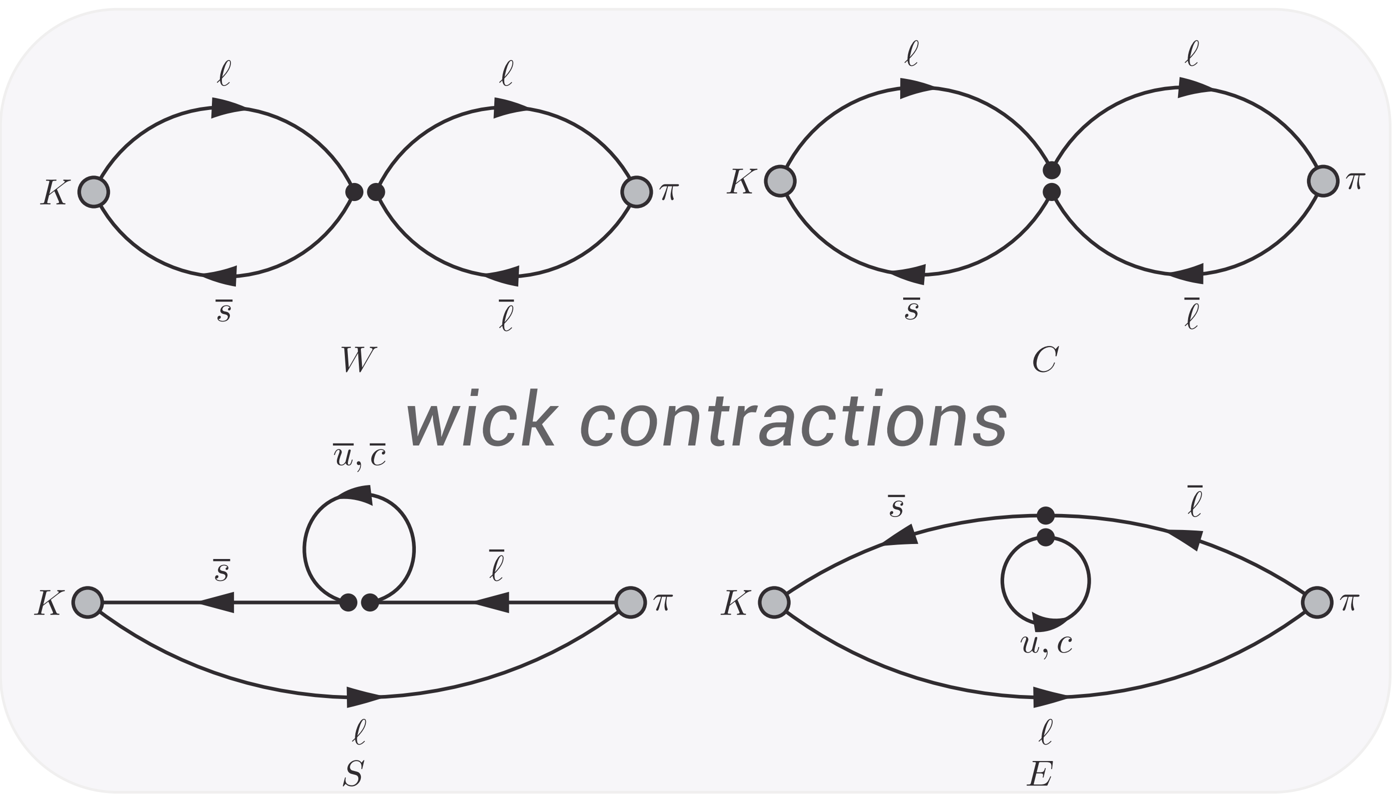
- domain wall fermions (24^3, a~0.12fm)
- m_π~430MeV, m_K~625MeV
  $E_K(\mathbf{k})<2M_\pi \rightarrow$ only one-π intermediate state
- unphysically light charm quark mass
  m_c~533MeV
- no disconnected diagrams
- kaon at rest

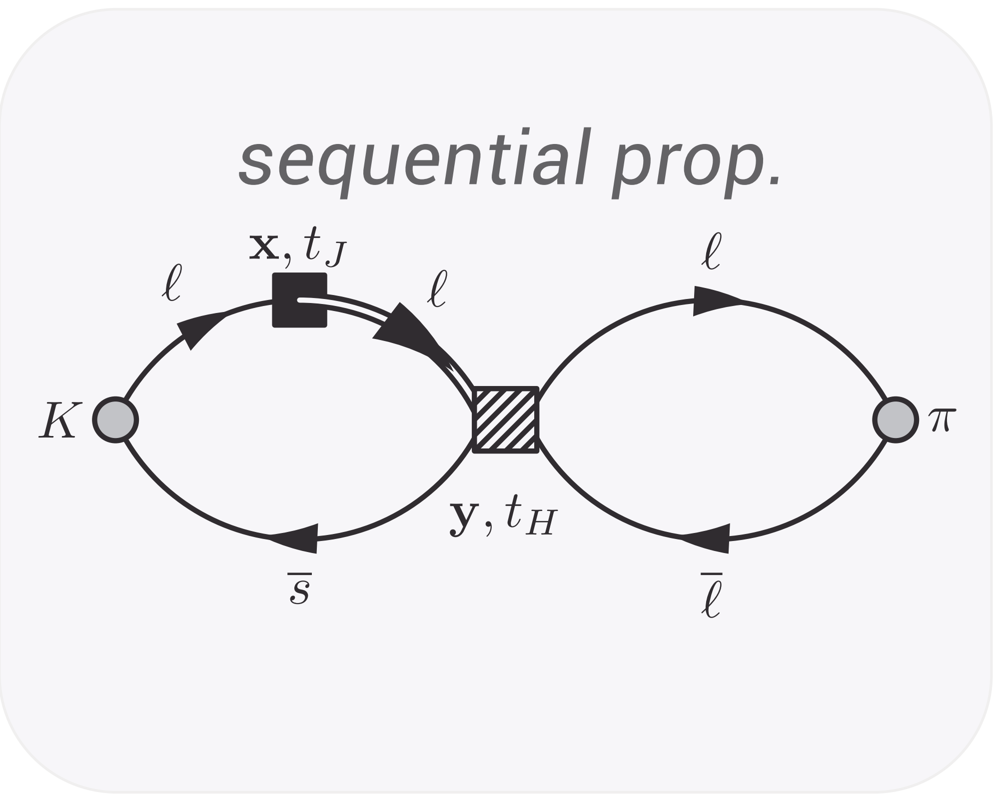


Euclidean correlation functions

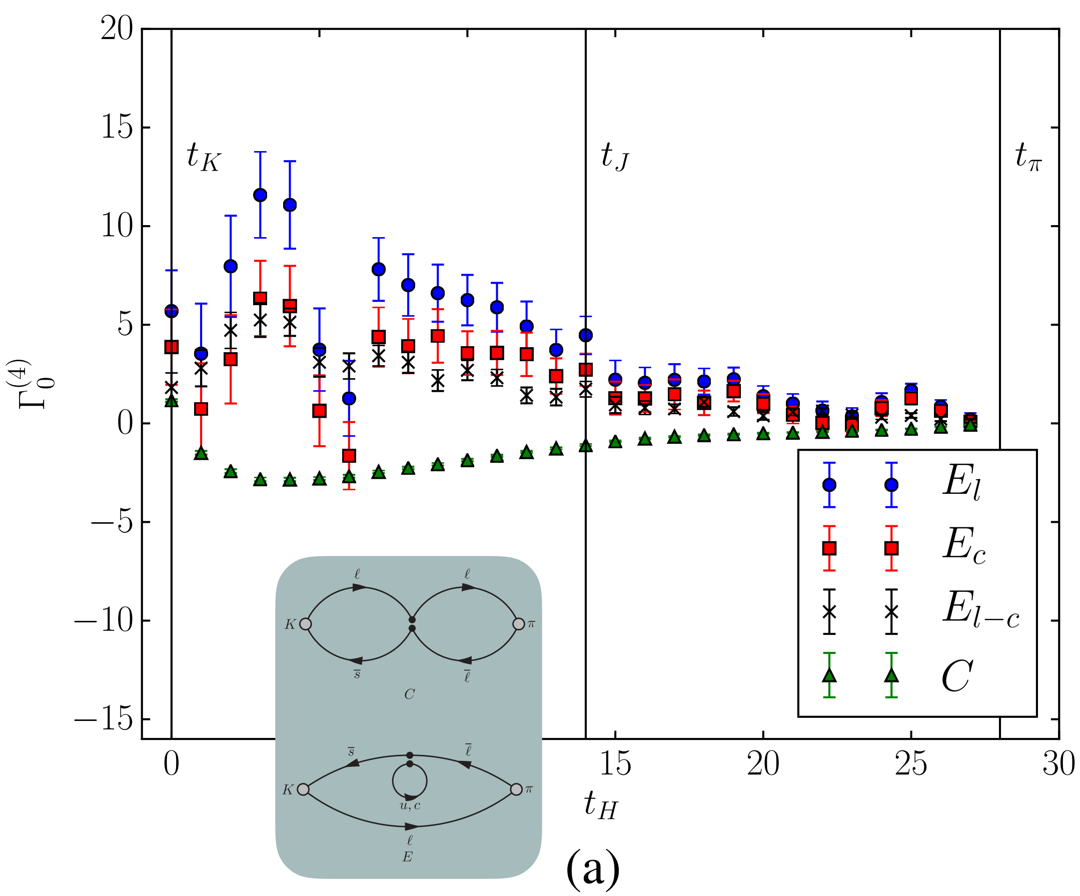
$$\mathcal{A}_\mu^c(q^2) = \int d^4x \langle \pi^c(p) | T\left[J_\mu(0) H_W(x)\right] | K^c(k) \rangle$$

$$\Gamma_\mu^{(4)\,c}(t_H, t_J, \mathbf{k}, \mathbf{p}) = \int d^3\mathbf{x} \int d^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{x}} \langle \phi_{\pi^c}(t_\pi, \mathbf{p}) T\left[J_\mu(t_j, \mathbf{x}) H_W(t_H, \mathbf{y})\right] \phi_{K^c}^\dagger(0, \mathbf{k}) \rangle$$

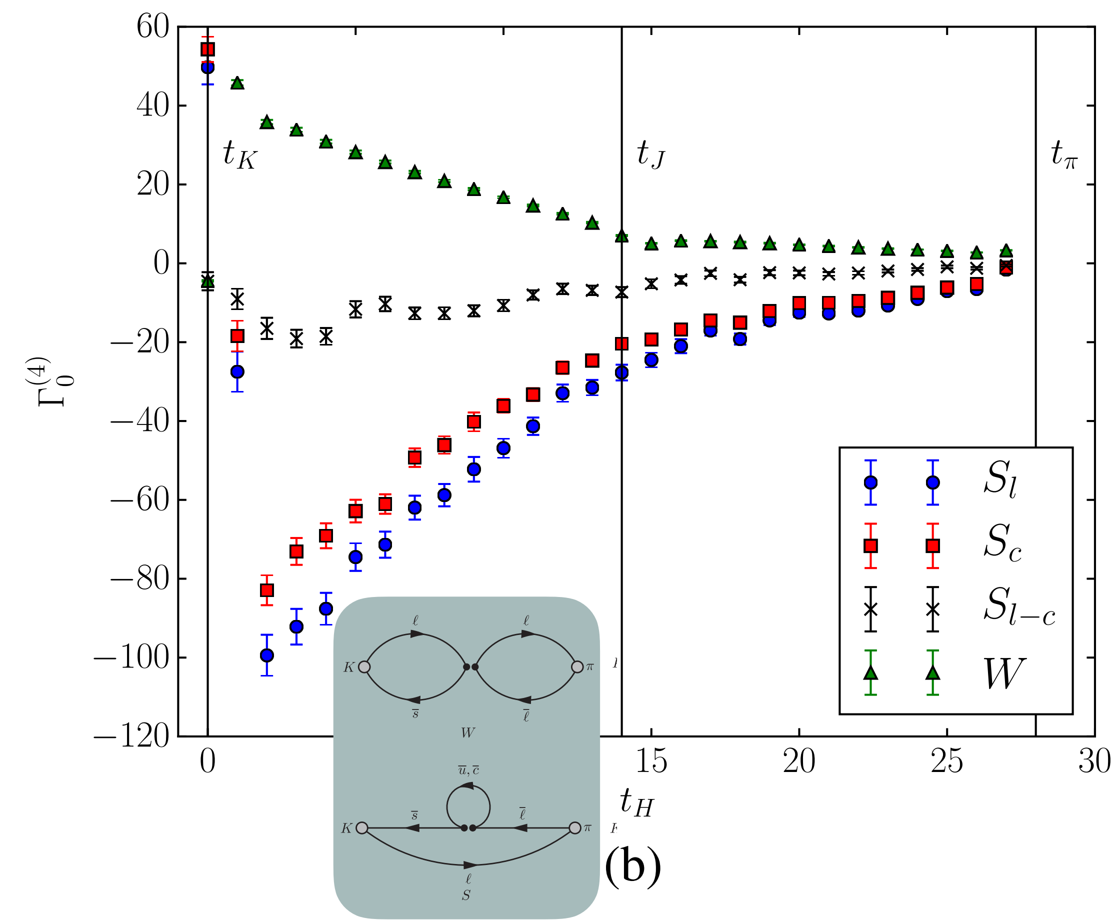




Results - dominant contributions and GIM subtraction



(a)

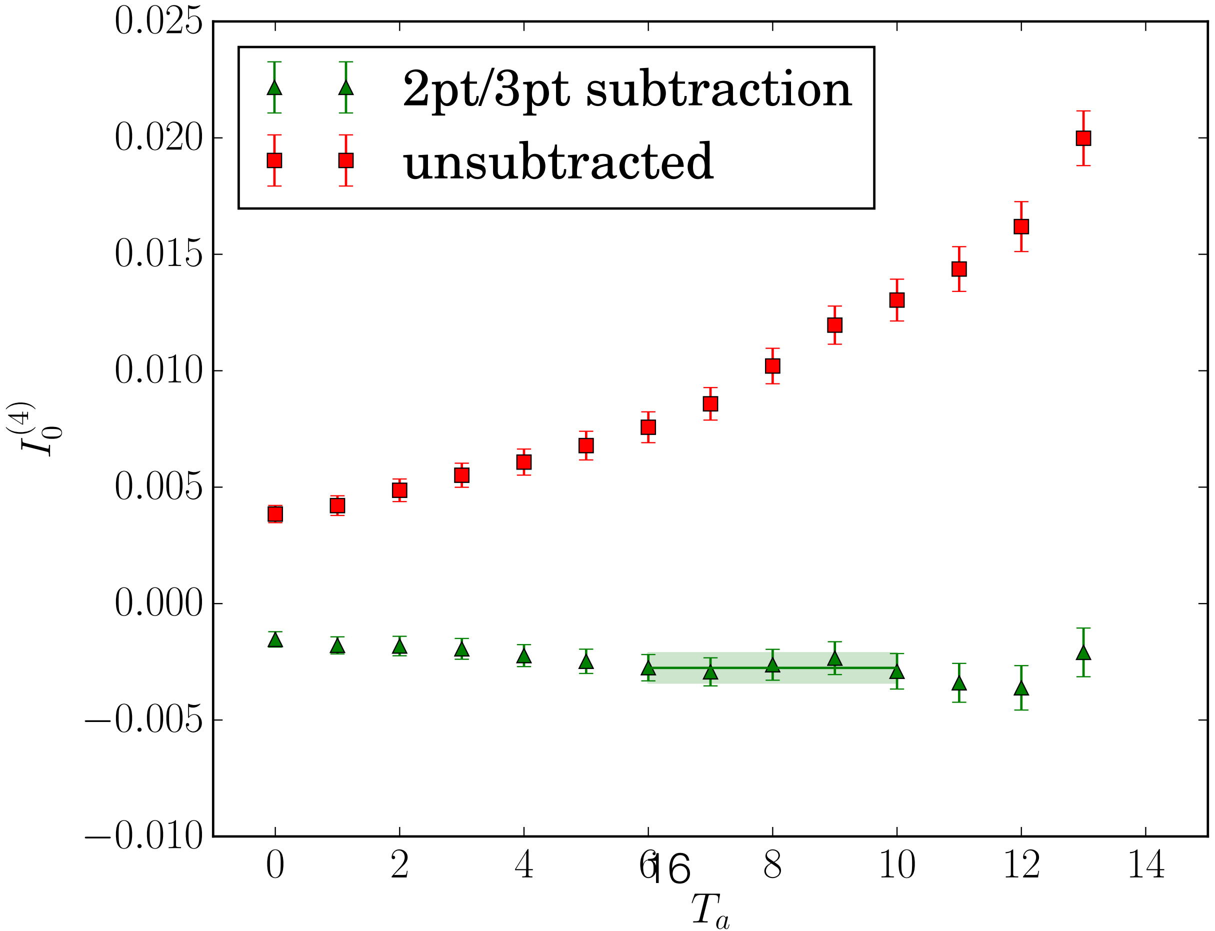


(b)

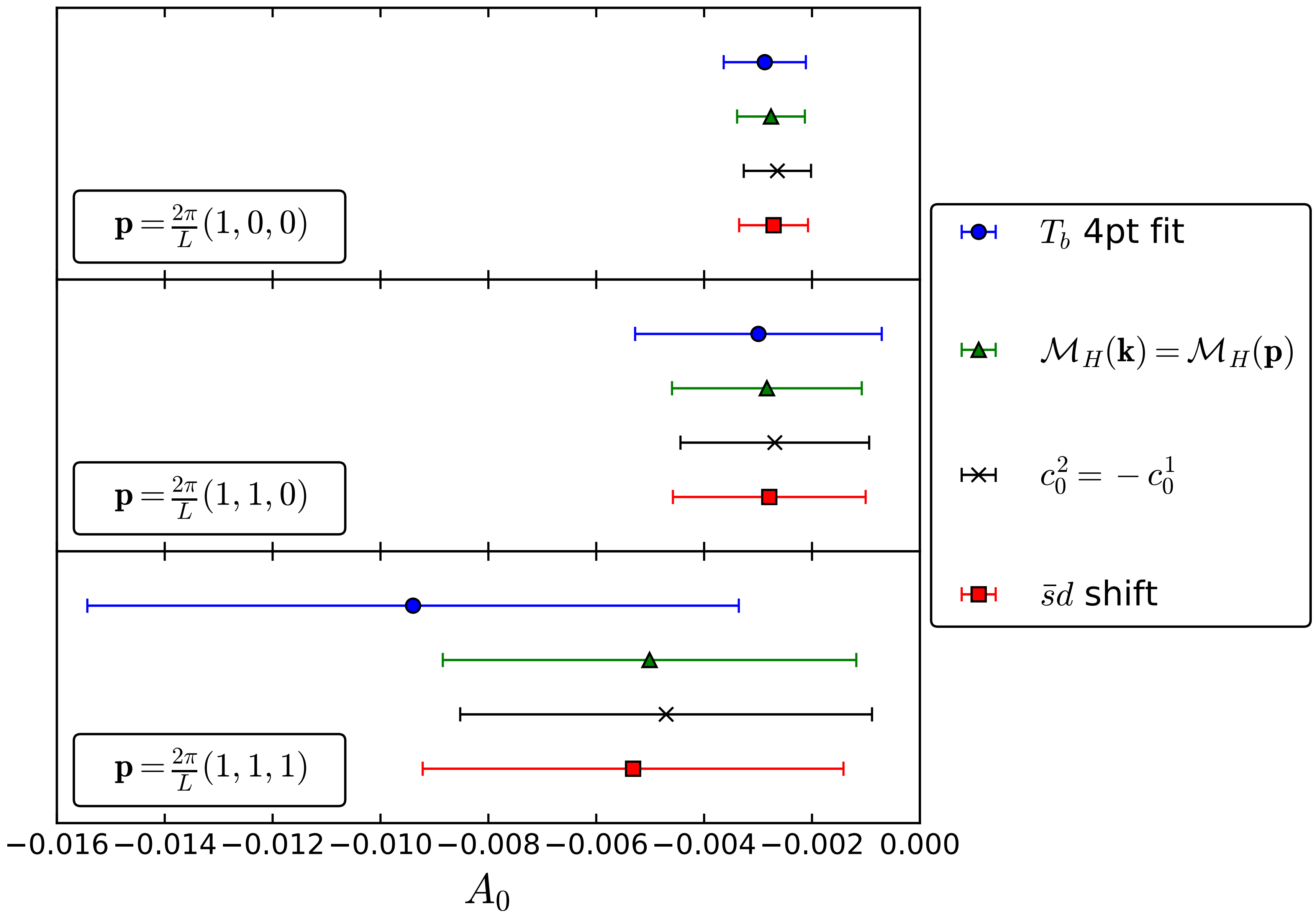
Removing the exponentially rising terms

$$A^c_\mu(T_a,T_b,q^2) = \int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$



Removing the exponentially rising terms - comparison of methods



$K^+ \to \pi^+ l^+ l^-$ form factor

Decay amplitude in terms of elm. transition form factor:

$$\mathcal{A}_\mu^c(q^2) = -i\frac{G_F}{4\pi}^2 \left[q^2(k+p)_\mu - (M_K^2 - M_\pi^2)q_\mu\right] V_c(q^2/M_K^2) \qquad c=+,0$$

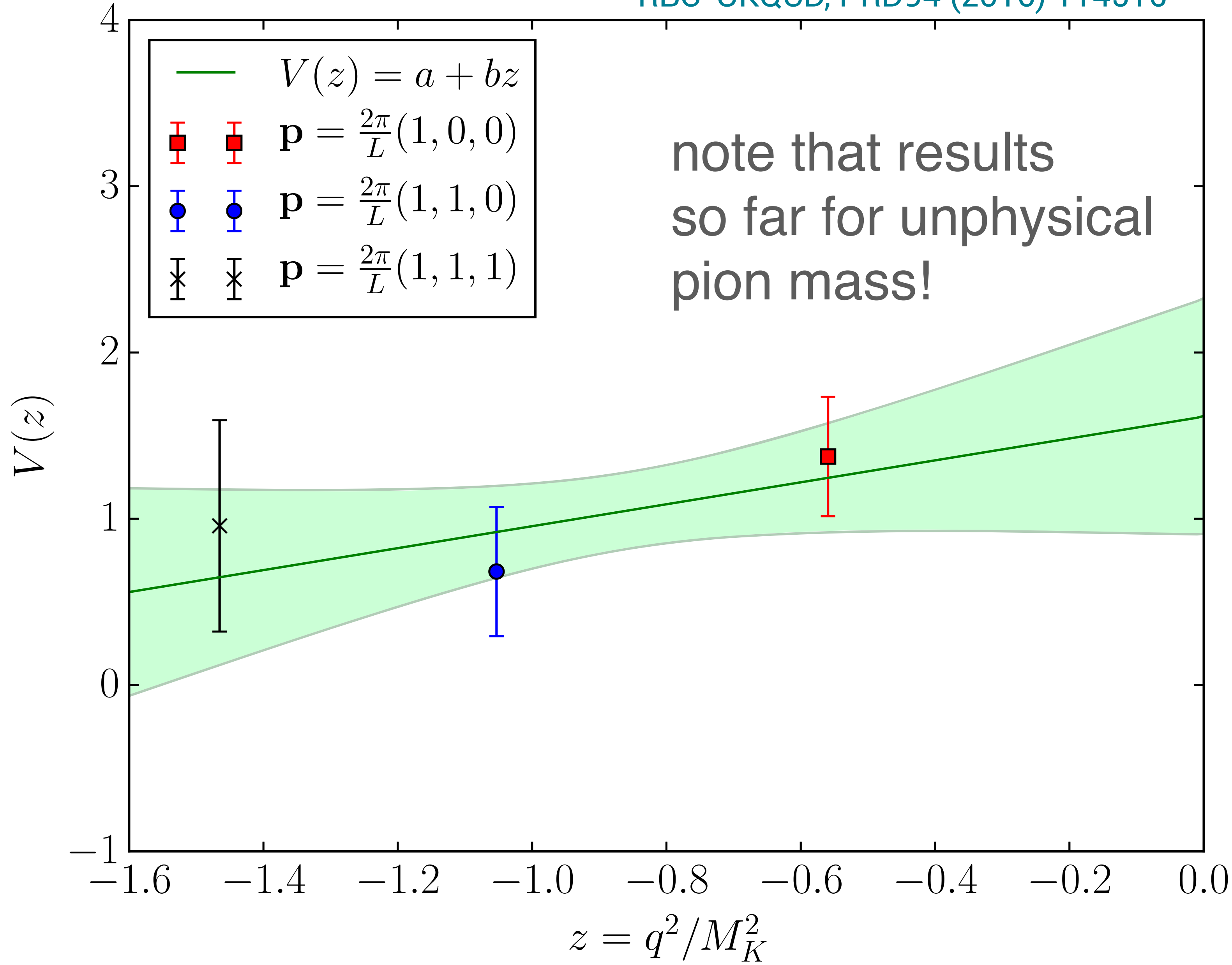
D'Ambrosio et al., JHEP 9808, 004 (1998)

$$V_c(q^2/M_K^2) = a_c + b_c q^2/M_K^2 + V_c^{\pi\pi}(q^2/M_K^2)$$

- the $|a_S|$ and $|a_+|$ can be extracted from branching ratios
- a_S parameterises also the CP-violating contribution to the K_L BR
- sign of a_S unknown - could be predicted by lattice – plays crucial role in BR prediction for $K_L \to \pi^0 e^+e^-/\mu^+\mu^-$

$K^+ \to \pi^+ l^+ l^-$ Results exploratory study

RBC-UKQCD, PRD94 (2016) 114516



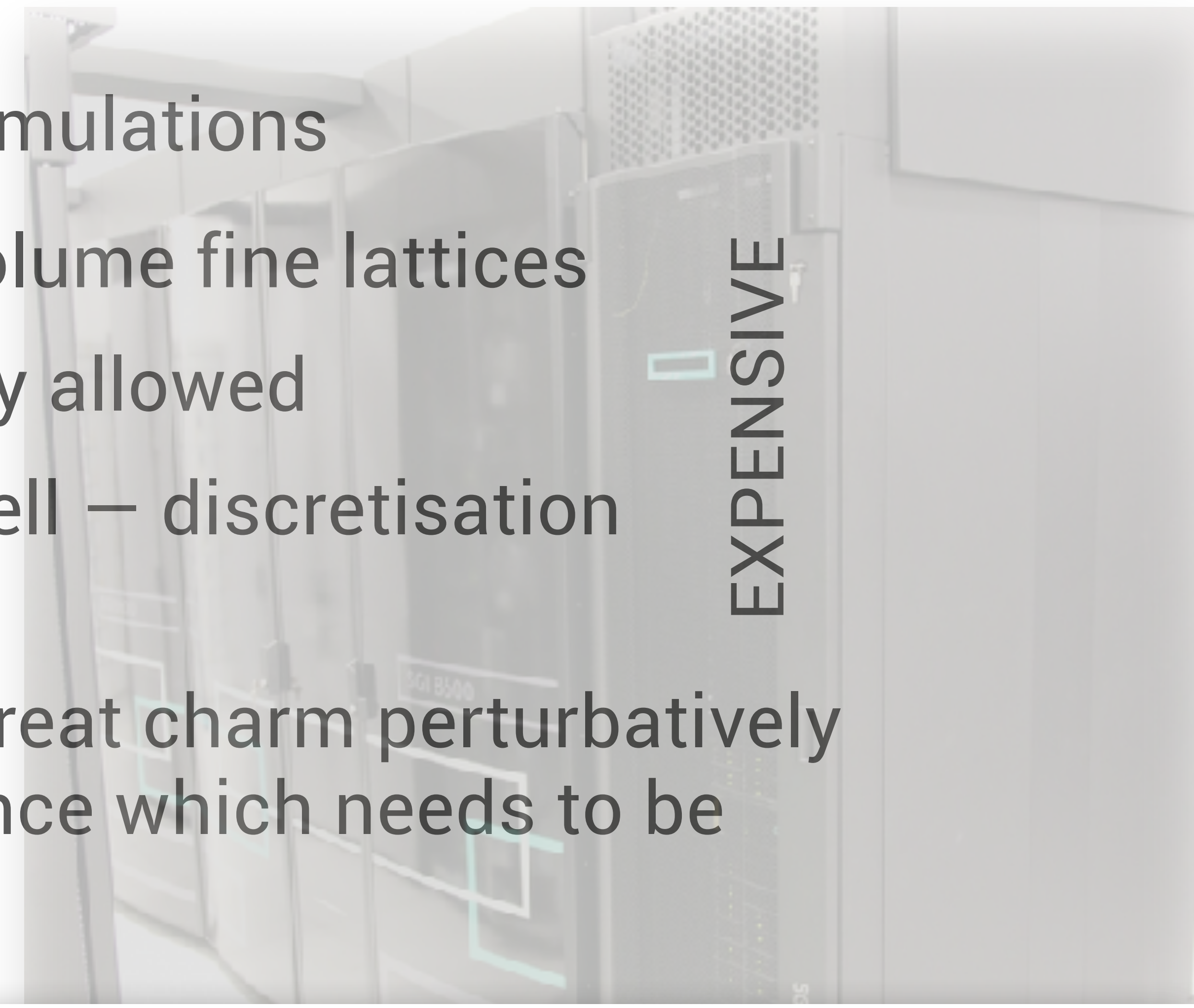
$V_+(z)=a_+ + b_+ \, q^2/m_K^2$ our result: $a_+=1.6(7)$, $b_+=0.7(8)$

pheno fit to exp. data: $a_+=-0.58(2)$, $b_+=0.78(7)$

Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399

$K^+ \to \pi^+ l^+ l^-$ Results exploratory study

- first lattice evaluation of this form factor
- we have shown that it is possible
- we are working on more 'physical' simulations
  - need to reduce m_π on large volume fine lattices
  - $\pi\pi\pi$ state will be kinematically allowed
  - m_c needs to be physical as well — discretisation effects are a concern
- alternatively consider N_f=2+1 H_W — treat charm perturbatively absence of GIM leads to log divergence which needs to be dealt with



very CPU intensive …

DiRAC Distributed Research utilizing Advanced Computing

Science & Technology Facilities Council

$K^+ \to \pi^+\nu\bar{\nu}$ decay

~29% SD
~3% LD

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = \kappa_+(1+\Delta_{\mathrm{EM}}) \cdot \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X_t(x_t)\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}P_c + \frac{\mathrm{Re}\lambda_t}{\lambda^5}X_t(x_t)\right)^2\right]$$

$$= (9.11 \pm 0.72) \times 10^{-11}$$

Buras et al. 2015

~68%

$P_c = P_c^{SD} + \delta P_{c,u}$

Energy $>m_c$

up- and charm-quark loops ~3%?

Buras, Gorbahn, Haisch, Nierste JHEP 0611 (2006) 002

Isidori, Mescia, Smith Nucl.Phys. B718 (2005) 319-338

compute P_c on the lattice in 4-flavour theory thus avoiding PT at around the charm scale

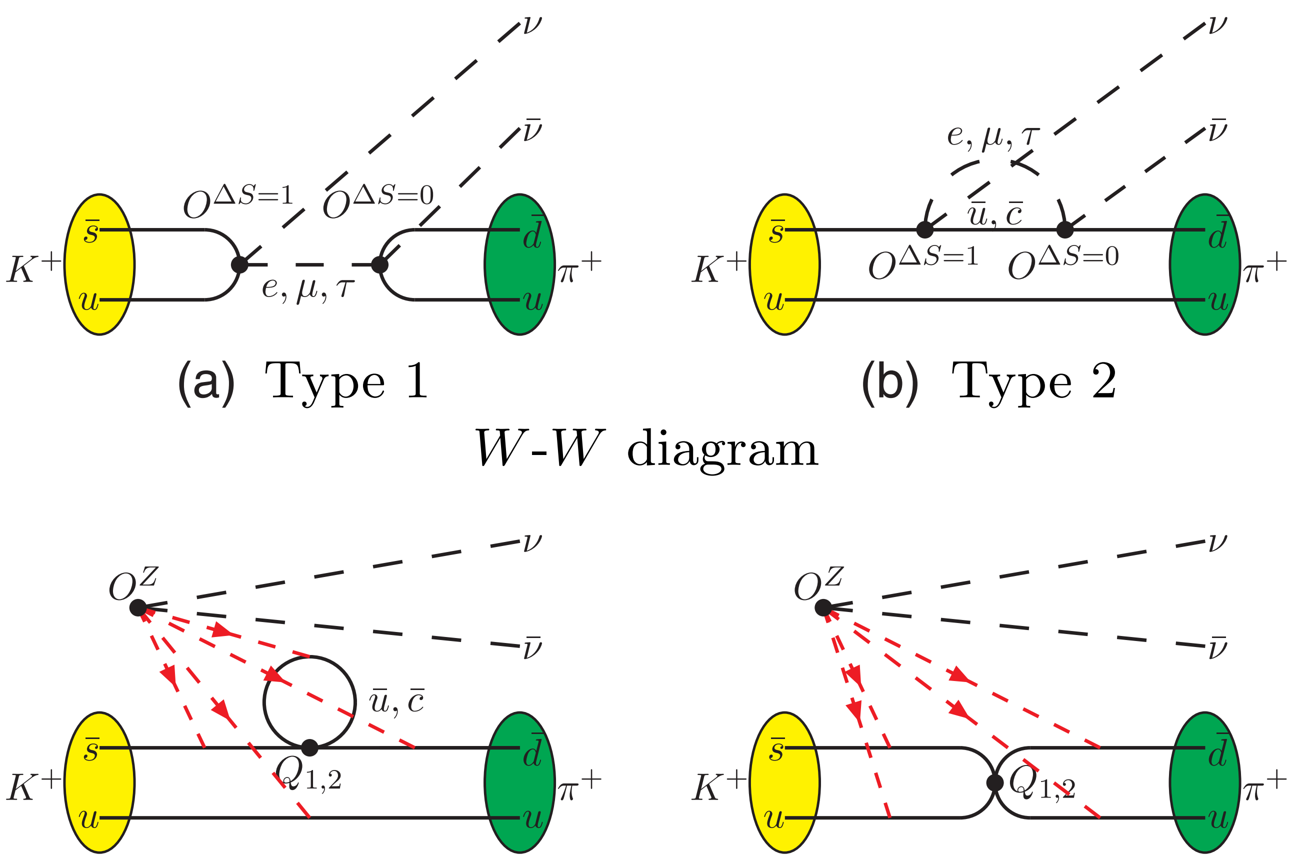
$K^+ \to \pi^+ \nu \bar{\nu}$ decay

involves two genuinely weak operators with V-A structure

V part renormalises similarly to $K^+ \to \pi^+ l^+ l^-$

A-part causes log-div which needs to be subtracted

$$\mathcal{O}(y) = \sum_{A,B} \int d^4x T[C_A Q_A(x) C_B Q_B(y)] + C_0 Q_0(y),$$



(a) Type 1 (b) Type 2

W-W diagram

(c) With closed loop (d) Without closed loop

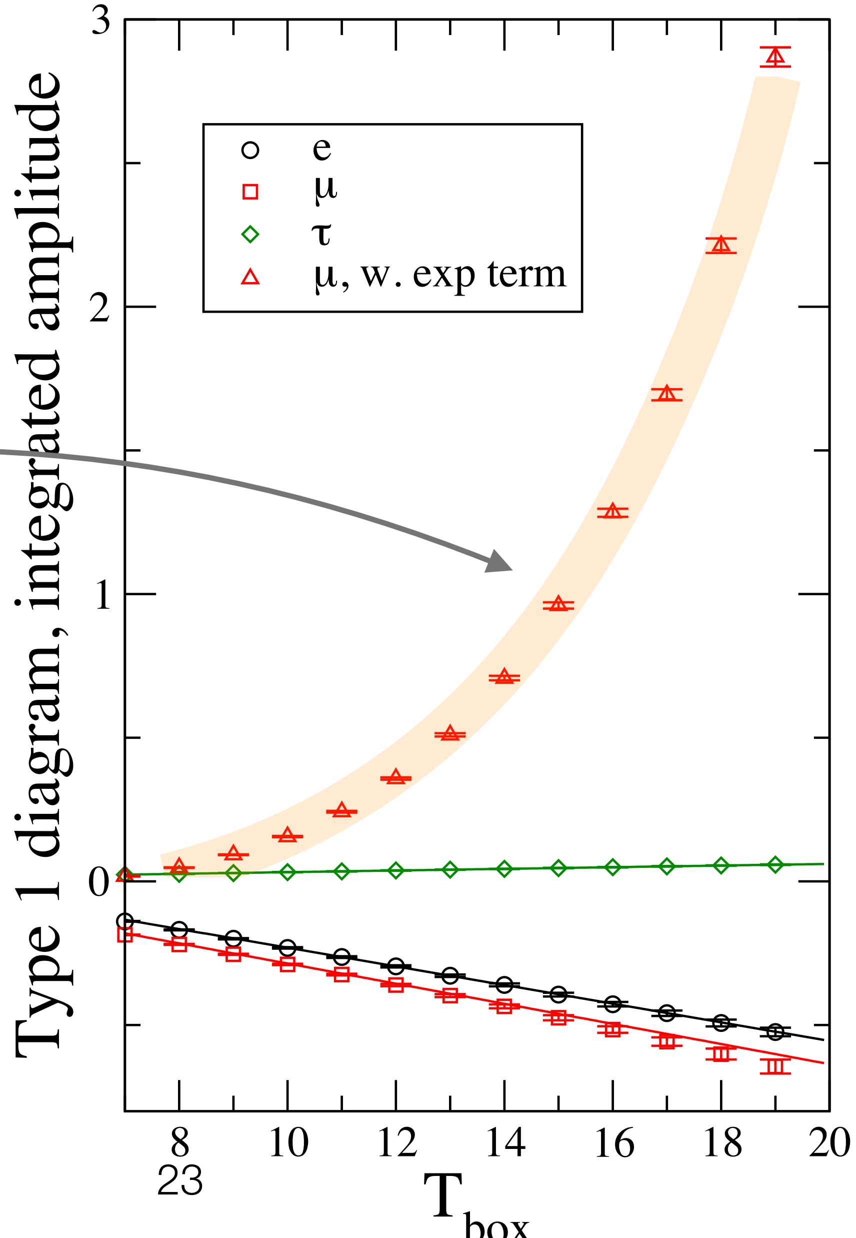
Connected Z-exchange diagram

$K^+ \to \pi^+ \nu\bar{\nu}$ decay

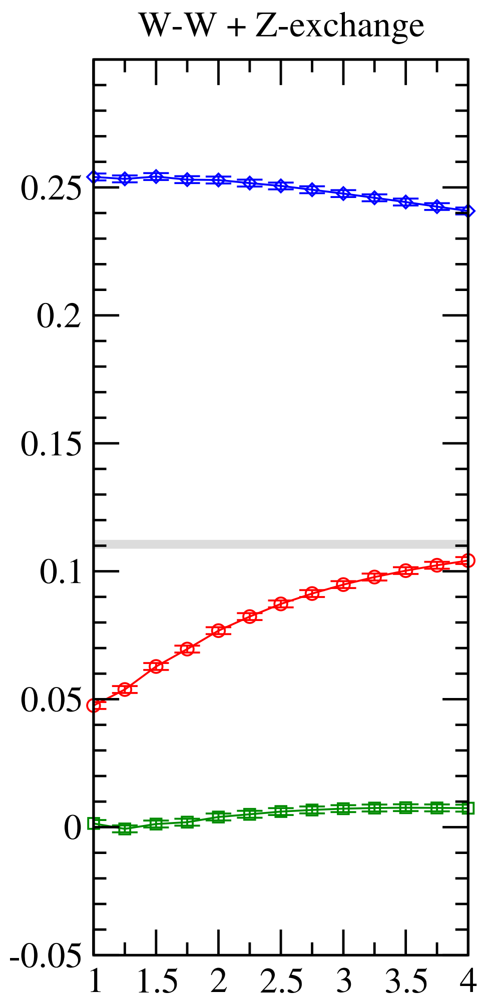
$$\int_{-T_a}^{T_b} dx_0 \langle \pi^+\nu\bar{\nu}|T\{H_A(x_0)H_B(0)\}|K^+\rangle = \sum_n \left\{ \frac{\langle \pi^+\nu\bar{\nu}|H_A|n\rangle\langle n|H_B|K^+\rangle}{E_n - E_K}\left(1 - e^{(E_K - E_n)T_b}\right) + \frac{\langle \pi^+\nu\bar{\nu}|H_B|n\rangle\langle n|H_A|K^+\rangle}{E_n - E_K}\left(1 - e^{(E_K - E_n)T_a}\right) \right\}$$

intermediate states:

$$|n\rangle = |l^+\nu\rangle,\ |\pi^0 l^+\nu\rangle,\ |(\pi^+\pi^0)^{I=2}\rangle$$



$K^+ \to \pi^+ \nu\bar{\nu}$ decay



lattice result for m_π=420MeV, m_c=860MeV

$$P_c = 0.2529(\pm 13)_{stat}(\pm 32)_{scale}(-45)_{FV}$$

$$P_c - P_c^{SD} = 0.0040(\pm 13)_{stat}(\pm 32)_{scale}(-45)_{FV}$$

$$(\mu = 2\text{GeV})$$

- unphysical simulation
- residual scale dependence small
- $P_c - P_c^{SD}$ small due to cancellation between W-W and Z
  will this persist in more physical simulation?

Summary and outlook

- kaon rare decays constitute a new theoretical and technical challenge worthwhile to pursue in view of experimental efforts
- intermediate state subtraction and renormalisation are technical challenges that can be managed
- we are now moving towards *real-world* simulations
- the experiments running, we are looking forward to their results in particular prospect of $K^+ \to \pi^+ l^+ l^-$ @ NA62

- lattice techniques also applicable to other LD effects ΔM_K, ε_K